Length of the Lateral Edge of a Regular Pyramid with an n-sided Base

Let $k(n)$ be the length of the lateral edge of a regular pyramid with an n-sided base, in which both the height and the length of the base edge are equal to 1. We aim to determine the values of $k(n)$ for $n = 3, 4, 5, 6$.

Step 1: Pyramid Structure and Geometry

Characteristics of the Regular Pyramid

- The base is a regular n-gon.
- The height h of the pyramid is equal to 1.
- The length of each edge of the base polygon is also equal to 1.

Lateral Edge

The lateral edge connects the apex O of the pyramid to a vertex A of the base.

Step 2: Identifying Key Elements

- Let O be the apex of the pyramid and G be the center of the base polygon.
- The length of the lateral edge $k(n)$ can be calculated using the Pythagorean theorem in triangle OGA.

Step 3: Calculating $k(n)$

The length of the lateral edge $k(n)$ can be expressed as:

$$k(n) = \sqrt{h^2 + GA^2}$$

where:

- $h = 1$ (the height of the pyramid).
- GA is the distance from the center G of the base to a vertex A.

Step 3.1: Finding GA

To find GA, we consider the geometry of the regular n-gon inscribed in a circle of radius R:

- The radius R of the circumcircle of the regular n-gon can be found using the formula:
$$R = \frac{a}{2\sin(\pi/n)}$$
where a is the length of a side of the polygon. Since $a = 1$, we have:
$$R = \frac{1}{2\sin(\pi/n)}$$

- This distance GA is equal to R.

Step 4: Evaluating $k(n)$ for $n = 3, 4, 5, 6$

For $n = 3$ (Triangle)

The circumradius R is:

$$R = \frac{1}{2\sin(\pi/3)} = \frac{1}{2 \cdot \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Thus, $GA = R = \frac{1}{\sqrt{3}}$. Therefore, we compute $k(3)$:

$$k(3) = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}.$$

For $n = 4$ (Square)

The circumradius R is:

$$R = \frac{1}{2\sin(\pi/4)} = \frac{1}{2 \cdot \frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}}.$$

Thus, $GA = R = \frac{1}{\sqrt{2}}$. Therefore, we compute $k(4)$:

$$k(4) = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}}.$$

For $n = 5$ (Pentagon)

The circumradius R is:

$$R = \frac{1}{2\sin(\pi/5)}.$$

Since $\sin(\pi/5) \approx 0.5878$, we compute:

$$R \approx \frac{1}{2 \cdot 0.5878} \approx \frac{1}{1.1756} \approx 0.849.$$

Thus, $GA \approx 0.849$. Therefore, we compute $k(5)$:

$$k(5) = \sqrt{1 + (0.849)^2} \approx \sqrt{1 + 0.720} \approx \sqrt{1.720} \approx \sqrt{2}.$$

For $n = 6$ (Hexagon)

The circumradius R is:

$$R = \frac{1}{2\sin(\pi/6)} = \frac{1}{2 \cdot \frac{1}{2}} = 1.$$

Thus, $GA = 1$. Therefore, we compute $k(6)$:

$$k(6) = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}.$$

Summary of Results

The correct values for $k(n)$ are as follows:

- $k(3) = \frac{2}{\sqrt{3}}$ (Correct)
- $k(4) = \frac{\sqrt{3}}{\sqrt{2}}$ (Correct)
- $k(5) = \sqrt{2}$ (Incorrect as noted)
- $k(6) = 1$ (Incorrect, should be $\sqrt{2}$)

Thus, the correct statements from the original problem are:

- a) No
- b) No
- c) Yes
- d) Yes