System of equations
For any real numbers x, y, z satisfying the system of equations:

1. $x + y + z = 3$,
2. $2x + 3y + 4z = 9$,
3. $3x + y - z = 3$,

does the following hold true?

- a) $x = z$ (Yes/No)
- b) $y = 1$ (Yes/No)
- c) $x = 1$ (Yes/No)
- d) $y = z$ (Yes/No)

Step 1: Solve the system of equations
First, we express z in terms of x and y using the first equation:

$$z = 3 - x - y$$

Step 2: Substitute z into the other equations
Substituting z into the second equation:

$$2x + 3y + 4(3 - x - y) = 9$$

Expanding:

$$2x + 3y + 12 - 4x - 4y = 9$$

Combining like terms:

$$-2x - y + 12 = 9$$

Rearranging:

$$-2x - y = -3 \quad \Rightarrow \quad 2x + y = 3 \quad \text{(Equation 4)}$$

Substituting z into the third equation:

$$3x + y - (3 - x - y) = 3$$

Expanding:

$$3x + y - 3 + x + y = 3$$

Combining like terms:

$$4x + 2y - 3 = 3$$

Rearranging:

$$4x + 2y = 6 \quad \Rightarrow \quad 2x + y = 3 \quad \text{(Equation 5)}$$

Step 3: Analyze the relationships

Both Equation 4 and Equation 5 are identical:

$$2x + y = 3$$

Thus, we can express y in terms of x:

$$y = 3 - 2x$$

Step 4: Substitute y back into the equation for z
Substituting y back into the expression for z:

$$z = 3 - x - (3 - 2x) = 3 - x - 3 + 2x = x$$

Step 5: Summary of relationships
From the derived equations, we have:

$$y = 3 - 2x \quad \text{and} \quad z = x$$

Step 6: Evaluate the given options

- a) $x = z$: **Yes** (since $z = x$).
- b) $y = 1$: To check:

  $3-2x = 1 \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1 \quad \Rightarrow \quad y = 1$ (This is true when $x = 1$)

  However, y is not necessarily always equal to 1 for all values of x. Thus, the statement is **No**.
- c) $x = 1$: This is not always true; x can take various values. Thus, the statement is **No**.
- d) $y = z$: To check:

  $3-2x = x \quad \Rightarrow \quad 3 = 3x \quad \Rightarrow \quad x = 1 \quad \Rightarrow \quad y = 1$ and $z = 1$ (This is true when $x = 1$)

  However, this is not true for all x. Thus, the statement is **No**.