Trigonometry in Arithmetic Sequences

Determine whether the following three numbers form an arithmetic sequence in the given order:

1. $\cos^2 34°, \cos^2 45°, \cos^2 56°$;
2. $\sin 10°, \sin 30°, \sin 50°$;
3. $\tan 150°, \tan 30°, \tan 60°$;
4. $\cos 0°, \cos 60°, \cos 90°$.

Solution

An arithmetic sequence is defined by the condition that the difference between consecutive terms remains constant. That is, for three numbers a, b, c, they form an arithmetic sequence if and only if:

$$2b = a + c.$$

We check this condition for each case.

(a) Checking $\cos^2 34°, \cos^2 45°, \cos^2 56°$

We verify whether:

$$2\cos^2 45° = \cos^2 34° + \cos^2 56°.$$

Since:

$$\cos 45° = \frac{\sqrt{2}}{2}, \quad \cos^2 45° = \frac{1}{2},$$

and using trigonometric identities,

$$\cos 34° = \sin 56°, \quad \cos 56° = \sin 34°,$$

we find that the equation holds. Thus, these numbers **form** an arithmetic sequence.

(b) Checking $\sin 10°, \sin 30°, \sin 50°$

We verify whether:

$$2\sin 30° = \sin 10° + \sin 50°.$$

Since:

$$\sin 30° = \frac{1}{2},$$

and using the sum-to-product identity:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

for $A = 50^\circ$ and $B = 10^\circ$,

$$\sin 50^\circ + \sin 10^\circ = 2 \sin 30^\circ \cos 20^\circ = 2 \times \frac{1}{2} \times \cos 20^\circ = \cos 20^\circ.$$

Since $\cos 20^\circ \neq 1$, the equality does not hold, meaning these numbers **do not form** an arithmetic sequence.

(c) Checking $\tan 150^\circ, \tan 30^\circ, \tan 60^\circ$

We verify whether:

$$2 \tan 30^\circ = \tan 150^\circ + \tan 60^\circ.$$

Since:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \tan 60^\circ = \sqrt{3}, \quad \tan 150^\circ = -\frac{1}{\sqrt{3}},$$

we check:

$$2 \times \frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} + \sqrt{3}.$$

This simplifies to:

$$\frac{2}{\sqrt{3}} = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}},$$

which is true. Thus, these numbers **form** an arithmetic sequence.

(d) Checking $\cos 0^\circ, \cos 60^\circ, \cos 90^\circ$

We verify whether:

$$2 \cos 60^\circ = \cos 0^\circ + \cos 90^\circ.$$

Since:

$$\cos 0^\circ = 1, \quad \cos 60^\circ = \frac{1}{2}, \quad \cos 90^\circ = 0,$$

we check:

$$2 \times \frac{1}{2} = 1 + 0.$$

This simplifies to:

$$1 = 1,$$

which is true. Thus, these numbers **form** an arithmetic sequence.

Final Answer

1. Yes
2. No
3. Yes
4. Yes