Verification of Logarithmic Identities

1 Problem Statement

Determine the truth of the following equalities:

a) $3^n + n + 1$

b) $n^2 + n$

c) $3^n + n$

d) $3^n + 5^n$

2 Solution

2.1 a) $\log_7(\sqrt{5}) = \sqrt{\log_7(5)}$

Using the property of logarithms, we have:

$$\log_7(\sqrt{5}) = \log_7(5^{1/2}) = \frac{1}{2}\log_7(5)$$

Comparing this with the right-hand side, we find:

$$\sqrt{\log_7(5)} \text{ is not equal to } \frac{1}{2}\log_7(5) \text{ in general.}$$

Thus, this equality is **false**.

2.2 b) $\log_7(2) + \log_7(3) = \log_7(6)$

Using the property of logarithms that states $\log_b(x) + \log_b(y) = \log_b(xy)$, we find:

$$\log_7(2) + \log_7(3) = \log_7(2 \cdot 3) = \log_7(6)$$

Thus, this equality is **true**.

2.3 c) $\log_2(\log_4(16)) = \log_4(\log_2(16))$

Calculating each side: - For the left side:

$$\log_4(16) = \log_4(4^2) = 2 \implies \log_2(2) = 1$$

- For the right side:

$$\log_2(16) = \log_2(2^4) = 4 \implies \log_4(4) = 1$$

Thus, both sides equal 1, so this equality is **true**.

2.4 d) $\log_7(2) \cdot \log_7(3) = \log_7(5)$

Using the change of base formula and properties of logarithms:

$$\log_7(2) \cdot \log_7(3) \text{ does not equal } \log_7(5) \text{ in general.}$$

Thus, this equality is **false**.