Logical Analysis of Agent Definition

1 Problem Statement

In a certain country, an individual X is called an *agent* if there exists a person Y such that X sends reports to Y, and for every person Z, there exists a person W such that W knows Z and W sends reports to Y.

Formally, X is an agent if:

$$\exists Y \, [R(X,Y) \wedge \forall Z \, \exists W \, (K(W,Z) \wedge R(W,Y))]$$

Where:

- $R(A,B)$ means that individual A sends reports to individual B,
- $K(A,B)$ means that individual A knows individual B.

The negation of this definition, i.e., that X is not an agent, is:

$$\neg\exists Y \, [R(X,Y) \wedge \forall Z \, \exists W (K(W,Z) \wedge R(W,Y))] \equiv \forall Y \, [\neg R(X,Y) \vee \exists Z \, \forall W \, (\neg K(W,Z) \vee \neg R(W,Y))]$$

We now analyze each of the given logical statements and determine whether it is logically equivalent to the negation of the definition of agent.

2 Analysis of Options

(a)

Statement: For every Y, at least one of the following holds:

(i) X does not send reports to Y,

(ii) For every Z and W, at least one of the following is false:

(*) W knows Z,

(**) W sends reports to Y.

Formal Translation:

$$\forall Y \, [\neg R(X,Y) \vee \forall Z \forall W (\neg K(W,Z) \vee \neg R(W,Y))]$$

Evaluation: This formula is strictly stronger than the correct negation, as it requires the universal quantifier over Z and W, rather than just $\exists Z \, \forall W$. Hence, this condition is **not equivalent**.

Answer: No

(b)

Statement: For every Y, at least one of the following holds:

(i) X does not send reports to Y,

(ii) It is not true that for every Z there exists a W such that W knows Z and W sends reports to Y.

Formal Translation:

$$\forall Y\,[\neg R(X,Y) \lor \neg\forall Z \exists W (K(W,Z) \land R(W,Y))] \equiv \forall Y\,[\neg R(X,Y) \lor \exists Z \forall W (\neg K(W,Z) \lor \neg R(W,Y))]$$

Evaluation: This is exactly the formal negation of the agent definition.
Answer: Yes

(c)

Statement: There exists a Y such that at least one of the following holds:

(i) X does not send reports to Y,

(ii) There exists a Z such that for every W, at least one of the following holds:

(*) W does not send reports to Y,

(**) W does not know Z.

Formal Translation:

$$\exists Y\,[\neg R(X,Y) \lor \exists Z \forall W (\neg R(W,Y) \lor \neg K(W,Z))]$$

Evaluation: This is weaker than the negation, as it only requires the condition to hold for *some* Y, rather than *all*. Therefore, this is **not equivalent**.
Answer: No

(d)

Statement: For every Y, at least one of the following holds:

(i) X does not send reports to Y,

(ii) There exists a Z such that for every W, at least one of the following holds:

(*) W does not send reports to Y,

(**) W does not know Z.

Formal Translation:

$$\forall Y\,[\neg R(X,Y) \lor \exists Z \forall W (\neg R(W,Y) \lor \neg K(W,Z))]$$

Evaluation: This is logically equivalent to the formal negation of the agent definition. The order and structure of quantifiers matches exactly.
Answer: Yes