Parity of Expressions

1 Problem Statement

For any positive integer n, the following number is even:

a) $3^n + n + 1$

b) $n^2 + n$

c) $3^n + n$

d) $3^n + 5^n$

2 Solution

2.1 a) $3^n + n + 1$

- For $n = 1$:
$$3^1 + 1 + 1 = 3 + 1 + 1 = 5 \quad \text{(odd)}$$
- For $n = 2$:
$$3^2 + 2 + 1 = 9 + 2 + 1 = 12 \quad \text{(even)}$$
- For $n = 3$:
$$3^3 + 3 + 1 = 27 + 3 + 1 = 31 \quad \text{(odd)}$$

The expression does not consistently return even results.

2.2 b) $n^2 + n$

We can factor this expression:

$$n^2 + n = n(n + 1)$$

Since n and $n + 1$ are consecutive integers, one of them is always even. Thus, $n(n + 1)$ is always even.

2.3 c) $3^n + n$

- For $n = 1$:

$$3^1 + 1 = 3 + 1 = 4 \quad \text{(even)}$$

- For $n = 2$:

$$3^2 + 2 = 9 + 2 = 11 \quad \text{(odd)}$$

- For $n = 3$:

$$3^3 + 3 = 27 + 3 = 30 \quad \text{(even)}$$

This expression does not consistently return even results.

2.4 d) $3^n + 5^n$

Both 3^n and 5^n are odd for any positive integer n: - Odd + Odd = Even.
Thus, the expression $3^n + 5^n$ is always even.

Summary

The only expression that is guaranteed to be even for any positive integer n is:
b) $n^2 + n$ and **d)** $3^n + 5^n$.