Triangle Analysis in Square $ABCD$

Given a square $ABCD$ with points E and F being the midpoints of sides AD and CD respectively, and point G being the intersection of segments CE and BF, we will analyze the properties of triangles CDG, ADG, ABG, and BCG.

Step-by-Step Analysis

1. Coordinates Assignment

Let the vertices of square $ABCD$ be:

$$A(0,1), \quad B(1,1), \quad C(1,0), \quad D(0,0)$$

The midpoints E and F are calculated as follows:

$$E\,(\text{midpoint of } AD): \quad E\left(\frac{0+0}{2}, \frac{1+0}{2}\right) = E(0,0.5)$$

$$F\,(\text{midpoint of } CD): \quad F\left(\frac{1+0}{2}, \frac{0+0}{2}\right) = F(0.5,0)$$

2. Finding the Intersection Point G

The line segment CE can be determined using the coordinates of C and E:

$$\text{Slope from } C(1,0) \text{ to } E(0,0.5): \quad \text{slope} = \frac{0.5-0}{0-1} = -0.5$$

Using point-slope form, the equation of line CE is:

$$y - 0 = -0.5(x-1) \implies y = -0.5x + 0.5$$

For segment BF from $B(1,1)$ to $F(0.5,0)$:

$$\text{Slope}: \quad \frac{0-1}{0.5-1} = 2$$

Its equation is:

$$y - 1 = 2(x-1) \implies y = 2x - 1$$

To find G, solve the equations:

$$-0.5x + 0.5 = 2x - 1$$

$$2.5x = 1.5 \implies x = 0.6$$

Substituting x back to find y:

$$y = 2(0.6) - 1 = 0.2$$

Thus, $G(0.6, 0.2)$.

Analyzing the Triangles

a) Triangle CDG is Isosceles The lengths CD, CG, and DG need to be compared.

$$CD = 1, \quad CG = \sqrt{(1-0.6)^2+(0-0.2)^2} = \sqrt{0.16+0.04} = \sqrt{0.2}, \quad DG = \sqrt{(0-0.6)^2+(0-0.2)^2} = \sqrt{0.36+0.04} =$$

Since CG and DG are not equal, triangle CDG is not isosceles. **Answer: No.**

b) Triangle ADG is Right Check if the slopes of AD and AG are negative reciprocals.

$$\text{Slope of } AD = 0 \quad (\text{horizontal line}), \quad \text{Slope of } AG = \frac{0.2-1}{0.6-0} = \frac{-0.8}{0.6} = -\frac{4}{3}$$

Since they are not negative reciprocals, ADG is not a right triangle. **Answer: No.**

c) Triangle ABG is Isosceles Compare lengths AB, AG, and BG:

$$AB = 1, \quad AG = \sqrt{(0.6-0)^2 + (0.2-1)^2} = \sqrt{0.36+0.64} = \sqrt{1} = 1, \quad BG = \sqrt{(0.6-1)^2 + (0.2-1)^2} = \sqrt{0.16+0.64} =$$

Since $AB = AG$, triangle ABG is isosceles. **Answer: Yes.**

d) Triangle BCG is Right Check slopes of BC and BG:

$$\text{Slope of } BC = \frac{0-1}{1-1} \quad \text{(undefined, vertical line)}$$

$$\text{Slope of } BG = \frac{0.2-1}{0.6-1} = \frac{-0.8}{-0.4} = 2$$

Since one slope is undefined (vertical line) and the other is defined, triangle BCG is a right triangle. **Answer: Yes.**

Summary of Results

- Triangle CDG is isosceles: **No.**
- Triangle ADG is right: **No.**
- Triangle ABG is isosceles: **Yes.**
- Triangle BCG is right: **Yes.**