Polynomial Divisibility Check
Is the polynomial $x^{100} - 3x^{50} + 2$ divisible by the polynomial

a) $x + 2$;

b) $x + 1$;

c) $x^2 - 3x + 2$;

d) $x - 1$?

To determine whether the polynomial $f(x) = x^{100} - 3x^{50} + 2$ is divisible by the given polynomials, we will use the Remainder Theorem. According to this theorem, a polynomial $f(x)$ is divisible by $x - r$ if and only if $f(r) = 0$.

a) $x + 2$

To check if $f(x)$ is divisible by $x + 2$, we will evaluate $f(-2)$:

$$f(-2) = (-2)^{100} - 3(-2)^{50} + 2$$

Calculating each term:

- $(-2)^{100} = 2^{100}$
- $(-2)^{50} = 2^{50}$

Thus,

$$f(-2) = 2^{100} - 3 \cdot 2^{50} + 2$$

Since $f(-2)$ is not equal to zero, $f(x)$ is not divisible by $x + 2$.

b) $x + 1$

To check if $f(x)$ is divisible by $x + 1$, we evaluate $f(-1)$:

$$f(-1) = (-1)^{100} - 3(-1)^{50} + 2$$

Calculating each term:

- $(-1)^{100} = 1$
- $(-1)^{50} = 1$

Thus,

$$f(-1) = 1 - 3 \cdot 1 + 2 = 1 - 3 + 2 = 0$$

Since $f(-1) = 0$, $f(x)$ is divisible by $x + 1$.

c) $x^2 - 3x + 2$

The polynomial $x^2 - 3x + 2$ can be factored as follows:

$$x^2 - 3x + 2 = (x - 1)(x - 2)$$

To determine if $f(x) = x^{100} - 3x^{50} + 2$ is divisible by $x^2 - 3x + 2$, we need to check if $f(x)$ is equal to zero at the roots of $x^2 - 3x + 2$, which are $x = 1$ and $x = 2$.

Step 1: Evaluate at $x = 1$

We already calculated $f(1)$:

$$f(1) = 1^{100} - 3 \cdot 1^{50} + 2 = 1 - 3 + 2 = 0$$

Since $f(1) = 0$, $f(x)$ is divisible by $x - 1$.

Step 2: Evaluate at $x = 2$

Next, we will evaluate $f(2)$:

$$f(2) = 2^{100} - 3 \cdot 2^{50} + 2$$

Calculating each term:
- 2^{100} is a very large number. - 2^{50} is also a large number, but significantly smaller than 2^{100}.
Now, let's break it down:
1. Calculate 2^{100}: - This is 1267650600228229401496703205376.
2. Calculate $3 \cdot 2^{50}$: - $2^{50} = 1125899906842624$. - Therefore, $3 \cdot 2^{50} = 3 \cdot 1125899906842624 = 3377699720527872$.
Now we can substitute these values into $f(2)$:

$$f(2) = 1267650600228229401496703205376 - 3377699720527872 + 2$$

This simplifies to:

$$f(2) = 1267650600224852901578117607498 \quad \text{(which is not 0)}$$

Conclusion for Part c:

Since $f(2) \neq 0$, we conclude that $f(x)$ is **not divisible** by $x^2 - 3x + 2$.

d) $x - 1$

As calculated above, since $f(1) = 0$, $f(x)$ is divisible by $x - 1$.

Summary of Results:

- a) $x + 2$: Not divisible
- b) $x + 1$: Divisible
- c) $x^2 - 3x + 2$: Not divisible
- d) $x - 1$: Divisible