Determining Rationality of Given Numbers

Problem

Is the number rational for each of the following cases?

- a) $\sqrt{(\sqrt{3}-2)^2 - \sqrt{3}}$
- b) $\sqrt{(1-\sqrt{2})^2} + \sqrt{2}$
- c) $\sqrt{(2-\sqrt{3})^2} + \sqrt{3}$
- d) $\sqrt{(\sqrt{2}-1)^2} - \sqrt{2}$

Solution

To determine whether the given numbers are rational, we need to check if they can be simplified to a form that is a rational number (i.e., a number that can be expressed as a ratio of two integers).

a) $\sqrt{(\sqrt{3}-2)^2 - \sqrt{3}}$

1. First, calculate $(\sqrt{3}-2)^2$:

$$(\sqrt{3}-2)^2 = 3 - 4\sqrt{3} + 4 = 7 - 4\sqrt{3}$$

2. Now subtract $\sqrt{3}$:

$$7 - 4\sqrt{3} - \sqrt{3} = 7 - 5\sqrt{3}$$

3. So we have:

$$\sqrt{7 - 5\sqrt{3}}$$

This number is irrational because $7 - 5\sqrt{3}$ is not a perfect square of a rational number.

b) $\sqrt{(1-\sqrt{2})^2}+\sqrt{2}$

1. Calculate $(1-\sqrt{2})^2$:

$$(1-\sqrt{2})^2 = 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$$

2. Note that $\sqrt{(1-\sqrt{2})^2} = |1-\sqrt{2}|$. Since $\sqrt{2} > 1$, we have:

$$|1-\sqrt{2}| = \sqrt{2} - 1$$

3. Add $\sqrt{2}$:

$$(\sqrt{2}-1) + \sqrt{2} = 2\sqrt{2} - 1$$

This number is irrational because $\sqrt{2}$ is irrational.

c) $\sqrt{(2-\sqrt{3})^2}+\sqrt{3}$

1. Calculate $(2-\sqrt{3})^2$:

$$(2-\sqrt{3})^2 = 4 - 4\sqrt{3} + 3 = 7 - 4\sqrt{3}$$

2. We have:

$$\sqrt{(2-\sqrt{3})^2} = |2-\sqrt{3}|$$

Since $2 > \sqrt{3}$, we have:

$$|2-\sqrt{3}| = 2 - \sqrt{3}$$

3. Add $\sqrt{3}$:

$$(2-\sqrt{3}) + \sqrt{3} = 2$$

Since 2 is an integer, it is also a rational number.

d) $\sqrt{(\sqrt{2}-1)^2}-\sqrt{2}$

1. Calculate $(\sqrt{2}-1)^2$:

$$(\sqrt{2}-1)^2 = 2 - 2\sqrt{2} + 1 = 3 - 2\sqrt{2}$$

2. We have:

$$\sqrt{(\sqrt{2}-1)^2} = |\sqrt{2}-1|$$

Since $\sqrt{2} > 1$, we have:

$$|\sqrt{2}-1| = \sqrt{2} - 1$$

3. Subtract $\sqrt{2}$:

$$(\sqrt{2}-1) - \sqrt{2} = -1$$

Since -1 is an integer, it is also a rational number.