Analysis of Function Transformations

1 Introduction

Given the function $f(x)$ depicted in Figure 4, we analyze and derive the transformations leading to the graphs shown in Figures 5 through 8. Each transformation is considered in detail with its analytical properties and effects on the function.

2 Absolute Value Transformation: $f_1(x) = |f(x)|$

The function $f_1(x) = |f(x)|$ modifies the original function such that all negative values of $f(x)$ are reflected above the x-axis. Mathematically, we define:

$$f_1(x) = \begin{cases} f(x), & \text{if } f(x) \geq 0, \\ -f(x), & \text{if } f(x) < 0. \end{cases}$$

This operation does not alter the domain of the function but ensures that all function values remain non-negative. The points where $f(x) = 0$ remain unchanged.
Correct Answer: Yes

3 Reflection Across the x-axis: $f_2(x) = -f(x)$

The transformation $f_2(x) = -f(x)$ reflects the graph of $f(x)$ across the x-axis. This operation affects all function values by negating them:

$$f_2(x) = -f(x) \quad \forall x.$$

Zeros of the function remain unchanged, while positive values become negative and vice versa.
Correct Answer: Yes

4 Even Function Extension: $f_3(x) = f(|x|)$

Applying $f_3(x) = f(|x|)$ modifies the function by making it symmetric with respect to the y-axis. Mathematically,

$$f_3(x) = \begin{cases} f(x), & \text{if } x \geq 0, \\ f(-x), & \text{if } x < 0. \end{cases}$$

This operation preserves function values for $x \geq 0$ but mirrors them into the negative x region. However, based on the given figures, this transformation does not match the expected graph.
Correct Answer: No

5 Reflection Across the y-axis: $f_4(x) = f(-x)$

The transformation $f_4(x) = f(-x)$ reflects the graph across the y-axis, which means:

$$f_4(x) = f(-x) \quad \forall x.$$

This preserves function values symmetrically but swaps left and right sides.
Correct Answer: Yes

6 Conclusion

Each of these transformations provides a fundamental way to manipulate functions, preserving key properties such as symmetry, zeros, and behavior under reflection. Understanding these modifications is essential in various applications of mathematical analysis and function modeling.