Analytical Examination of Given Functions and Graphs

1 Problem Statement

We analyze whether any of the graphs in Figures 1–3 correspond to the given functions:

$$\begin{aligned}
&(a) \quad f(x) = x^2 - |x^2 - 1|, \\
&(b) \quad f(x) = |x + 1| - 1, \\
&(c) \quad f(x) = x^2 + |x^2 - 1|, \\
&(d) \quad f(x) = |x - 1| + 1.
\end{aligned}$$

We will derive each function's expression in piecewise form, analyze their key properties, and compare them with the given graphs.

2 Detailed Function Analysis

2.1 Function (a): $f(x) = x^2 - |x^2 - 1|$

The absolute value expression introduces two cases:

- If $x^2 \geq 1$, then $|x^2 - 1| = x^2 - 1$, so:

$$f(x) = x^2 - (x^2 - 1) = 1.$$

- If $x^2 < 1$, then $|x^2 - 1| = 1 - x^2$, so:

$$f(x) = x^2 - (1 - x^2) = 2x^2 - 1.$$

Thus, the function is given by:

$$f(x) = \begin{cases} 1, & |x| \geq 1, \\ 2x^2 - 1, & |x| < 1. \end{cases}$$

This corresponds to the graph in Figure 2, confirming that function (a) is present among the graphs.

2.2 Function (b): $f(x) = |x + 1| - 1$

The absolute value function gives:

- If $x + 1 \geq 0$ (i.e., $x \geq -1$), then $|x + 1| = x + 1$, so:

$$f(x) = (x + 1) - 1 = x.$$

- If $x + 1 < 0$ (i.e., $x < -1$), then $|x + 1| = -(x + 1)$, so:

$$f(x) = -(x + 1) - 1 = -x - 2.$$

Thus, the function is given by:

$$f(x) = \begin{cases} x, & x \geq -1, \\ -x - 2, & x < -1. \end{cases}$$

This corresponds to the V-shaped graph in Figure 1, confirming that function (b) is present.

2.3 Function (c): $f(x) = x^2 + |x^2 - 1|$

Again, we consider the absolute value cases:

- If $x^2 \geq 1$, then $|x^2 - 1| = x^2 - 1$, so:

$$f(x) = x^2 + (x^2 - 1) = 2x^2 - 1.$$

- If $x^2 < 1$, then $|x^2 - 1| = 1 - x^2$, so:

$$f(x) = x^2 + (1 - x^2) = 1.$$

Thus, the function is given by:

$$f(x) = \begin{cases} 2x^2 - 1, & |x| \geq 1, \\ 1, & |x| < 1. \end{cases}$$

This function does not match any of the given graphs. Thus, function (c) is not present.

2.4 Function (d): $f(x) = |x - 1| + 1$

The absolute value function gives:

- If $x - 1 \geq 0$ (i.e., $x \geq 1$), then $|x - 1| = x - 1$, so:

$$f(x) = (x - 1) + 1 = x.$$

- If $x - 1 < 0$ (i.e., $x < 1$), then $|x - 1| = -(x - 1)$, so:

$$f(x) = -(x - 1) + 1 = -x + 2.$$

Thus, the function is given by:

$$f(x) = \begin{cases} x, & x \geq 1, \\ -x + 2, & x < 1. \end{cases}$$

This function also does not match any of the given graphs, confirming that function (d) is not present.