# plotting (pnas)

#### April 1, 2021

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[82]: %pylab inline

Populating the interactive namespace from numpy and matplotlib

/home/eddie/anaconda3/envs/forests/lib/python3.8/sitepackages/IPython/core/magics/pylab.py:159: UserWarning: pylab import has clobbered these variables: ['imread'] `%matplotlib` prevents importing \* from pylab and numpy warn("pylab import has clobbered these variables: %s" % clobbered +

[83]: plt.rc('font', size=20)

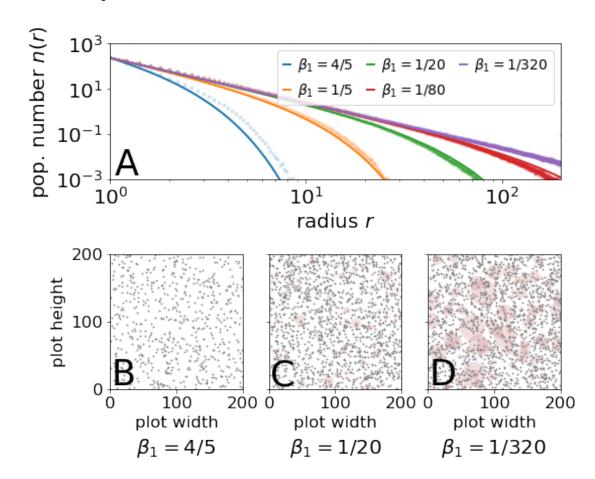
[84]: # import necessary standard Python modules
from matplotlib.patches import Circle
from matplotlib.collections import PatchCollection

```
[85]: load_pickle('cache/biomass_scaling_w_compet_nu=2.5.p.bak')
      for k in nk.keys():
          nk[k] = np.concatenate([i[-1000:,:,None] for i in nk[k]], axis=2).mean(2)
      kappa = (1.8 - 4/3) * (nu-1)
      f = forest.coeffs['sharing fraction']
      rhobar = forest.coeffs['resource efficiency']
      cr = forest.coeffs['root']**2 * np.pi
      lastn = 1000
      gs = GridSpec(2, 3, hspace=.55)
      fig = plt.figure(figsize=(8,6.3))
      ax = fig.add_subplot(gs[0,:])
      h = \prod
      for i, (rk_, nk_) in enumerate(zip(rk.values(), nk.values())):
          h.append(ax.plot(rk_[0], nk_[-lastn:].mean(0), '.', zorder=0, alpha=.14))
          beta1 = basalRange[i]
          vareps = forest.coeffs['resource efficiency']
          assert nu>2
          xi0 = (nu-2)**(1/(2-nu))
          B = (beta1 * xi0 / vareps / rhobar / cr / (1-f))**(nu-1) / (nu-1)
          s = forest.coeffs['area competition']
          dx = rk_[0][1] - rk_[0][0]
          n0 = g0 / (Abar + cg/dx + dt * r0**kappa)
```

```
y = (n0 * (rk_[0]/r0)**(-1/3 - Abar/cg) *
         np.exp(-B * s / cg / (kappa+2/3) * (rk_[0]**(kappa+2/3) - r0**(kappa+2/3))
 →3))))
    h.append(ax.loglog(rk_[0], y, '-', lw=2, c=f'C{i}', zorder=1)[0])
ax.text(1.05, 1.5e-3, 'A', fontsize=35)
ax.set(xlabel=r'radius $r$',
       ylabel=r'pop. number $n(r)$',
       ylim=(1e-3, 1e3),
       xlim=(1,199)
ax.legend(h[1::2],
          [r'$\beta_1=4/5$', r'$\beta_1=1/5$', r'$\beta_1=1/20$', r'$\beta_1=1/
\rightarrow 80\$', r'\$\beta_1=1/320\$'
          fontsize='x-small',
          handlelength=.5, ncol=3, loc=1, columnspacing=1, handletextpad=.5)
load_pickle('plotting/biomass_scaling_w_compet_nu=2.5.p')
ax = [fig.add_subplot(gs[1,i]) for i in range(3)]
forest[basalRange[0]].plot(ax=ax[0],
                           show_canopy=False,
                           show_center=True,
                           center_kw={'c':'gray','ms':1.5})
ax[0].text(2, 6, 'B', fontsize=35)
[el.set fontsize('small') for el in ax[0].yaxis.get ticklabels()]
[el.set fontsize('small') for el in ax[0].xaxis.get ticklabels()]
ax[0].set_title(r'\$\beta_1=4/5\$', y=-.52, fontsize=20)
ax[0].set_xlabel('plot width', fontsize='small')
ax[0].set_ylabel('plot height', fontsize='small')
forest[basalRange[2]].plot(ax=ax[1],
                           show canopy=False,
                           show_center=True,
                           center_kw={'c':'gray','ms':1.5},
                           plot_kw={'yticklabels':[]})
ax[1].text(2, 6, 'C', fontsize=35)
[el.set_fontsize('small') for el in ax[1].xaxis.get_ticklabels()]
ax[1].set_title(r'$\beta_1=1/20$', y=-.52, fontsize=20)
ax[1].set_xlabel('plot width', fontsize='small')
forest[basalRange[4]].plot(ax=ax[2],
                           show_canopy=False,
                           show center=True,
                           center_kw={'c':'gray','ms':1.5},
                           plot_kw={'yticklabels':[]})
```

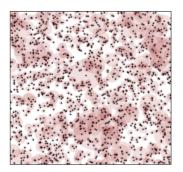
```
ax[2].text(2, 6, 'D', fontsize=35)
[el.set_fontsize('small') for el in ax[2].xaxis.get_ticklabels()]
ax[2].set_title(r'$\beta_1=1/320$', y=-.52, fontsize=20)
ax[2].set_xlabel('plot width', fontsize='small')
fig.savefig(f'{imgdr}/competition_example.pdf', bbox_inches='tight')
```

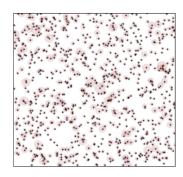
[85]: Text(0.5, 0, 'plot width')

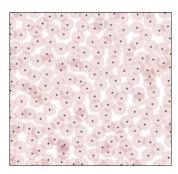


```
[27]: fig, ax = plt.subplots(figsize=(14,4), ncols=3, sharex=True, sharey=True)
    ax = ax[::-1]

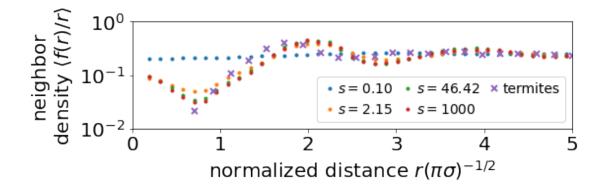
load_pickle('cache/spacing_with_Abar_no_growth.p')
    allxy = xy
    xy = vstack(allxy[AbarRange[-1]][areaDeathRateRange[-1]][0])
    data.plot(xy, [8.5]*len(xy), 200, ax=ax[0], show_center=True,
```





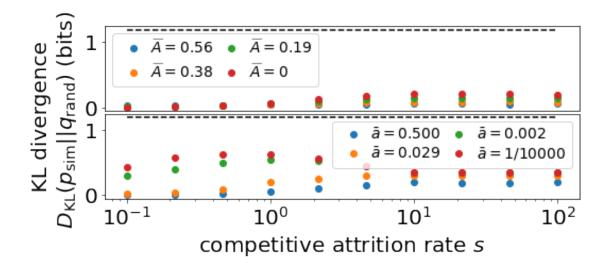


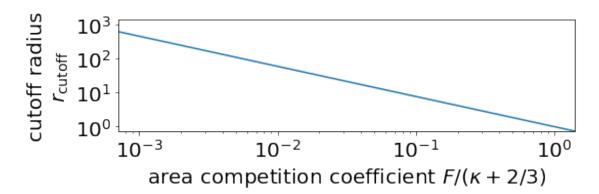
[86]: <matplotlib.legend.Legend at 0x7f489651e280>



```
[87]: fig, ax = plt.subplots(figsize=(8,3), nrows=2, sharey=True,
                             gridspec_kw={'hspace':0.03})
      load_pickle('plotting/dkl_Abar_range.p')
      for kl_ in kl[1:]:
          ax[0].semilogx(areaDeathRateRange, kl , 'o')
      load_pickle('plotting/dkl_cg_range.p')
      for kl_ in kl:
          ax[1].semilogx(areaDeathRateRange, kl_, 'o')
      ax[0].hlines(1.19, areaDeathRateRange.min(), areaDeathRateRange.max(),
                   linestyles='--')
      ax[1].hlines(1.19, areaDeathRateRange.min(), areaDeathRateRange.max(),
                   linestyles='--')
      ax[1].set(xlabel=r'competitive attrition rate $s$',
                ylabel='
                                     KL divergence\n'r'
                                                                   $D_{\rm KL}(p_{\rm_
      \rightarrowsim\}||q_{\rm m} rand\})$ (bits)')
      # ax[0].xaxis.set_tick_params(length=0)
      ax[0].set(xticks=[])
      ax[0].xaxis.set_ticks_position('none')
      ax[1].set(ylim=ax[0].get_ylim())
```

[87]: <matplotlib.legend.Legend at 0x7f48966252b0>





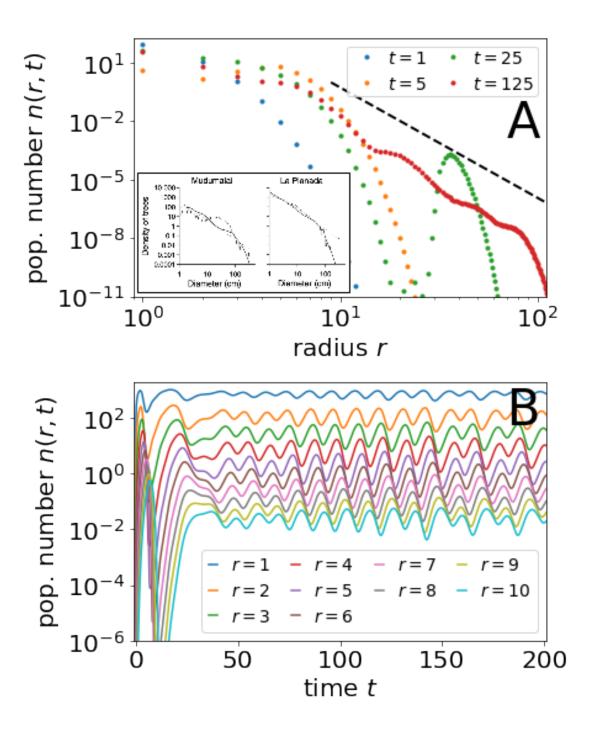
```
[88]: fig, ax = plt.subplots(figsize=(6,9), nrows=2,
                             gridspec_kw={'hspace':.33})
      tplot = [1, 5, 25, 125]
      y = pd.read_csv('../mathematica/nk_oscillations.csv')
      for i, t in enumerate(tplot):
          ax[0].plot(y.columns, y.iloc[i], '.')
      x = array([9, 200])
      y = x**(-6+1/3)
      y *= 1 / y[0]
      ax[0].loglog(x, y, 'k--', lw=2)
      ax[0].set(xscale='log', yscale='log',
             xlabel=r'radius $r$', ylabel=r'pop. number $n(r,t)$')
      ax[0].set(xlim=(.9, 100/.9), ylim=(1e-11, 2e2))
      ax[0].legend([r'$t=%d$'%i for i in tplot],
                   fontsize='x-small',
                   ncol=2,
                   handletextpad=.2,
                   borderpad=.2,
                   columnspacing=.5,
                   loc=1)
      forestex = imread('.../writing/20200723_dynamics_draft/wave_example.png')
      imagebox = OffsetImage(forestex, zoom=.11)
```

```
ab = AnnotationBbox(imagebox, (3.25, 2e-8), pad=.1)
ax[0].add_artist(ab)

y = pd.read_csv('../mathematica/curves.csv')
x = array(y.columns.values).astype(float)
for i in range(10):
    ax[1].semilogy(x, y.iloc[i], '-')
ax[1].set(ylim=(1e-6, 2e3), xlim=(-1,201),
        xlabel=r'time $t$', ylabel=r'pop. number $n(r,t)$')
ax[1].legend([r'$r=%d$'%r for r in range(1,11)],
        fontsize='x-small', handlelength=.5, ncol=4, columnspacing=1.2)

fig.text(.825, .755, 'A', fontsize='40')
fig.text(.825, .395, 'B', fontsize='40')
fig.savefig(f'{imgdr}/pop_waves.pdf', bbox_inches='tight', dpi=200)
```

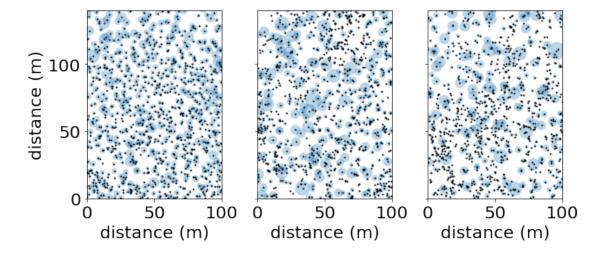
[88]: Text(0.825, 0.395, 'B')



# 5 Supplementary figures

```
[56]: def forest_plot(plotNo, ax):
    df = pd.read_csv('../data/uas/southeast_alaska/Data/Data.csv')
```

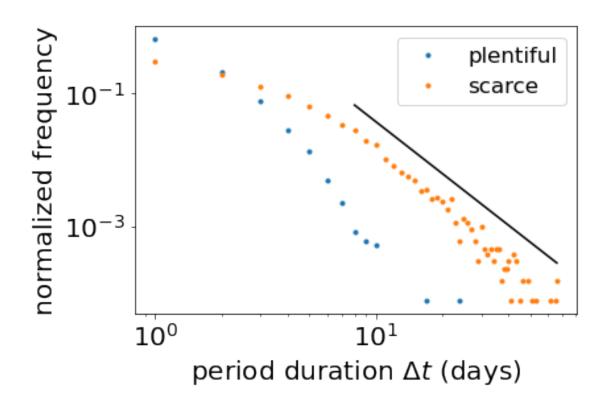
```
ix = df['Plot '] == plotNo # select a patch
    x, y = df['Xcoord.m'].loc[ix], df['Ycoord.m'].loc[ix]
    r = df['DBH.cm'].loc[ix] / 100 * 5
    # there are some weird outliers that must something wrong
    badix = (x<0) | (x>100)
    x = x[~badix]
   y = y[~badix]
    r = r[~badix]
    # create patches for plotting
    p = []
    for x_{,} y_{,} r_{in} zip(x, y, r):
        p.append(Circle((x_,y_), r_))
    patches = PatchCollection(p, alpha=.3)
    ax.plot(x, y, 'k.', ms=2)
    ax.add_collection(patches)
fig, ax = plt.subplots(figsize=(10,4), ncols=3, sharex=True, sharey=True)
ax = ax.ravel()
forest_plot(32, ax[0])
forest_plot(76, ax[1])
forest_plot(130, ax[2])
ax[-1].set(xlim=(0,100), ylim=(0,140))
ax[0].set(ylabel='distance (m)')
for a in ax:
    a.set_aspect('equal')
    a.set(xlabel='distance (m)')
fig.savefig(f'{imgdr}/alaskan_forest.pdf', bbox_inches='tight')
```



#### Puerto Rico rainfall

```
[57]: load_pickle('plotting/pr_rainfall.p')
      abovedt = np.concatenate(list(abovedt.values()))
      belowdt = np.concatenate(list(belowdt.values()))
      fig, ax = plt.subplots()
      na = bincount(abovedt)[1:]
      xa = arange(1, na.size+1)
      nb = bincount(belowdt)[1:]
      xb = arange(1, nb.size+1)
      na = na / na.sum()
      nb = nb / nb.sum()
      # data
      ax.loglog(xa, na, '.', c='CO')
      ax.loglog(xb, nb, '.', c='C1')
      # power law comparison
      x = array([8, xb[-1]])
      ax.loglog(x, 15 * x**-2.6, 'k-')
      ax.set(xlabel=r'period duration $\Delta t$ (days)', ylabel='normalized_

¬frequency')
      ax.legend(('plentiful', 'scarce'), fontsize='small')
      fig.savefig(f'{imgdr}/pr_rainfall.pdf', bbox_inches='tight')
```



#### Automaton solution to WEB model

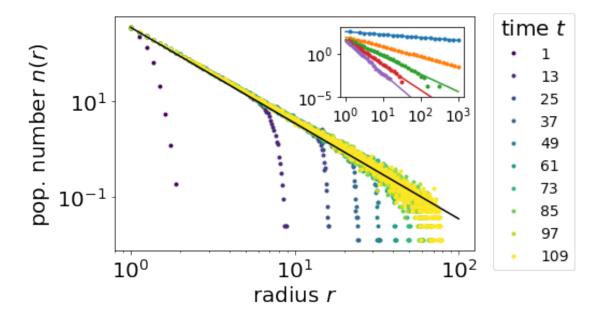
```
[37]: load_pickle('cache/linear_model_exponent_transience.p')
      rk = rk[0]
      nk = np.concatenate([i[:,:,None] for i in nk], axis=2)
      nk = nk.mean(2)
      tRange = range(1, 112, 12)
      fig, ax = plt.subplots()
      h = []
      cmap = colorcycle(10)
      for i in tRange:
          h.append(ax.loglog(rk, nk[i], '.', c=next(cmap))[0])
      dx = (rk[1]-rk[0])
      y = rk**(-1/3 - cm/cg)
      y = g0 / (cg / dx * rk[0]**(2/3) + cm * rk[0]**(-2/3)) / y[0]
      ix = rk<100
      h.append(ax.loglog(rk[ix], y[ix], 'k-')[0])
      ax.set(xlabel=r'radius $r$', ylabel=r'pop. number $n(r)$',
```

```
xlim=(.8, 100/.8))
leg = ax.legend(h, tRange, title=r'time $t$',
                bbox_to_anchor=(1.3,1.05),
                loc=1,
                fontsize='x-small')
load_pickle('cache/linear_model_exponent.p')
# create log bins
nbins = arange(5) * 6 + 25
lognk, logrk, dx = {}, {}, {}
counter = 0
for (k, nk_), rk_ in zip(nk.items(), rk.values()):
   nk_{-} = nk_{-}[-200:].mean(0)
    ix = np.digitize(rk_, np.logspace(0, np.log10(rk_[-1]), nbins[counter]))
    logrk[k] = np.logspace(0, np.log10(rk_[-1]), nbins[counter])
    lognk[k] = np.array([nk_[ix==i].sum() / (ix==i).sum() for i in range(ix.
\rightarrowmax()+1)])
    dx[k] = rk_{1} - rk_{0}
    counter += 1
axins = inset_axes(ax, width=1.6, height=.9)
h = []
for i, (cm, rk_, nk_, dx_) in enumerate(zip(cmRange, logrk.values(), lognk.
→values(), dx.values())):
    h.append(axins.loglog(rk_, nk_[:-1], '.')[0])
    # match up slope prediction to last point
    exponent = 1/3 + cm / cg
    y = rk_**(-exponent)
    y *= g0 / (cg / dx_ + cm) / y[0]
    h.append(axins.loglog(rk_, y, '-', c=f'C{i}')[0])
axins.set(ylim=(1e-5, 1e3), xticks=(1,10,100,1e3))
[el.set_fontsize('x-small') for el in axins.xaxis.get_ticklabels()]
[el.set_fontsize('x-small') for el in axins.yaxis.get_ticklabels()]
fig.savefig(f'{imgdr}/indpt_model.pdf', bbox_inches='tight')
```

<ipython-input-37-4431144a3497>:43: RuntimeWarning: invalid value encountered in

```
double_scalars
  lognk[k] = np.array([nk_[ix==i].sum() / (ix==i).sum() for i in
range(ix.max()+1)])
```

[37]: [None, None, None, None, None, None]



#### KL divergence for solid phase

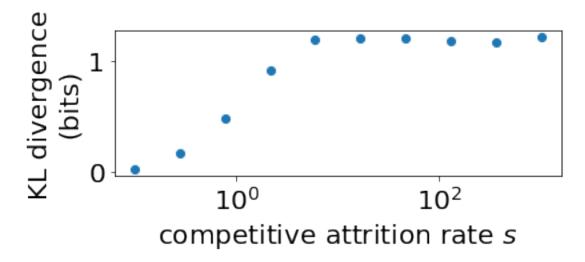
```
[58]: load_pickle('cache/packing_example.p')

kl = np.zeros_like(areaDeathRateRange)
  interpkl = zeros_like(areaDeathRateRange)
  optbinWidth = zeros_like(areaDeathRateRange)

for j, adr in list(enumerate(areaDeathRateRange)):
    dr = concatenate([nn.dist(xy_) for xy_ in xy[adr]])
    N = mean([len(xy_) for xy_ in xy[adr]])

# interpolate DKL curve to extrapolate value
    bins = logspace(log10(dr.max()/dr.size)+2, log10(dr.max()/dr.size)+5, 30)
    d = zeros_like(bins)
    for k in range(d.size):
        d[k] = nn.kl(dr, N, L, bins[k])
    optbinWidth[j] = bins[argmin(d)]

kl[j] = nn.kl(dr, N, L, optbinWidth[j])
```

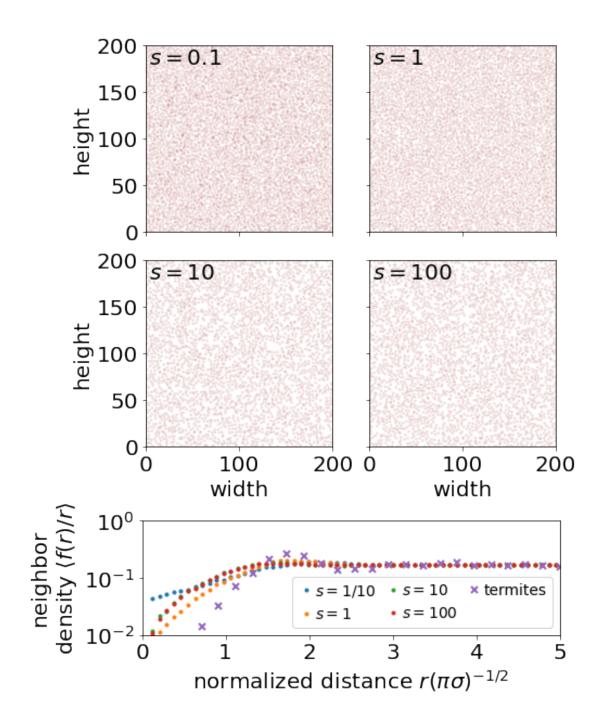


#### Examples of phase regions

```
[48]: load_pickle('cache/spacing_with_cg.p')
      allxy = xy
      fig = plt.figure(figsize=(6.9,10))
      gs = GridSpec(3, 2, wspace=.1, hspace=.5, height_ratios=(1,1,.7))
      gsSub = GridSpecFromSubplotSpec(2,2, gs[:2,:], wspace=.15, hspace=.15)
      ax = (fig.add_subplot(gsSub[0,0], aspect='equal'),
            fig.add_subplot(gsSub[0,1], aspect='equal'),
            fig.add_subplot(gsSub[1,0], aspect='equal'),
            fig.add_subplot(gsSub[1,1], aspect='equal'))
      xy = vstack(allxy[cgRange[-1]][areaDeathRateRange[0]][0])
      data.plot(xy, [1.5]*len(xy), 200, ax=ax[0])
      ax[0].text(3, 180, r'$s=0.1$')
      xy = vstack(allxy[cgRange[-1]][areaDeathRateRange[3]][0])
      data.plot(xy, [1.5]*len(xy), 200, ax=ax[1])
      ax[1].text(3, 180, r'$s=1$')
      xy = vstack(allxy[cgRange[-1]][areaDeathRateRange[6]][0])
```

```
data.plot(xy, [1.5]*len(xy), 200, ax=ax[2])
ax[2].text(3, 180, r'$s=10$')
xy = vstack(allxy[cgRange[-1]][areaDeathRateRange[9]][0])
data.plot(xy, [1.5]*len(xy), 200, ax=ax[3])
ax[3].text(3, 180, r'$s=100$')
ax[0].set(xticklabels=[], ylabel='height')
ax[1].set(xticklabels=[], yticklabels=[])
ax[2].set(xlabel='width', ylabel='height')
ax[3].set(yticklabels=[], xlabel='width')
load_pickle('plotting/spatial_corr_liquid.p')
ax = fig.add_subplot(gs[2,:])
for p_ in list(p.values())[::3]:
    # spatial autocorrelation function
   ax.semilogy(r, p_/r, '.')
namp, namr = data.namibia_corr_fcn()
drRatio = (namr[1]-namr[0]) / (r[1]-r[0])
ax.plot(namr, namp/namr / drRatio, 'x', mew=2)
ax.set(xlim=(0,5), yscale='log',
       xlabel=r"normalized distance $r(\pi\sigma)^{-1/2}$",
\negylabel='neighbor\n'+r"density $\langle f(r)/r\rangle$")
ax.set(ylim=(1e-2,1), yticks=[1e-2,.1,1])
ax.legend([r'$s=1/10$',r'$s=1$',r'$s=10$',r'$s=100$']+['termites'],
          ncol=3, fontsize='x-small', columnspacing=.6, handletextpad=.09,
→handlelength=1, loc=4)
fig.savefig(f'{imgdr}/liquid_phase.pdf', bbox_inches='tight')
```

[48]: <matplotlib.legend.Legend at 0x7f3181c331c0>



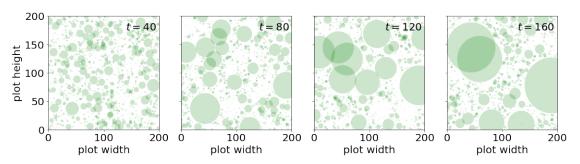
### Canopy simulation example

```
[396]: load_pickle('plotting/oscillation_plot_examples.p')

fig, ax = plt.subplots(figsize=(17.5,4), ncols=4)
for i, (k, t) in enumerate(trees.items()):
    forest.plot(all_trees=t, ax=ax[i])
```

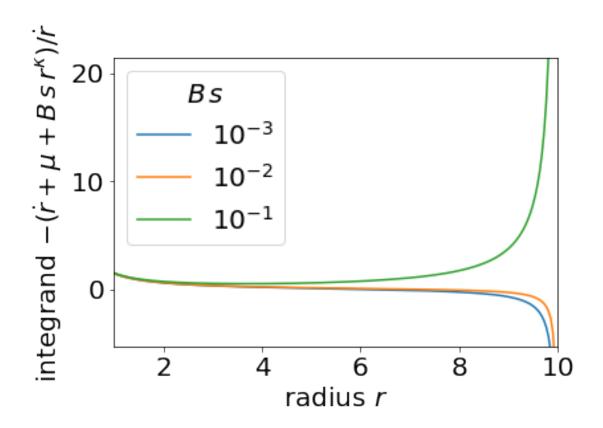
```
ax[i].text(195, 175, r'$t=%d$'%k, ha='right')
if i>0: ax[i].set(yticklabels=[])
else: ax[i].set(ylabel='plot height')
ax[i].set(xlabel='plot width')

fig.savefig(f'{imgdr}/oscillation_plots.pdf', bbox_inches='tight')
```



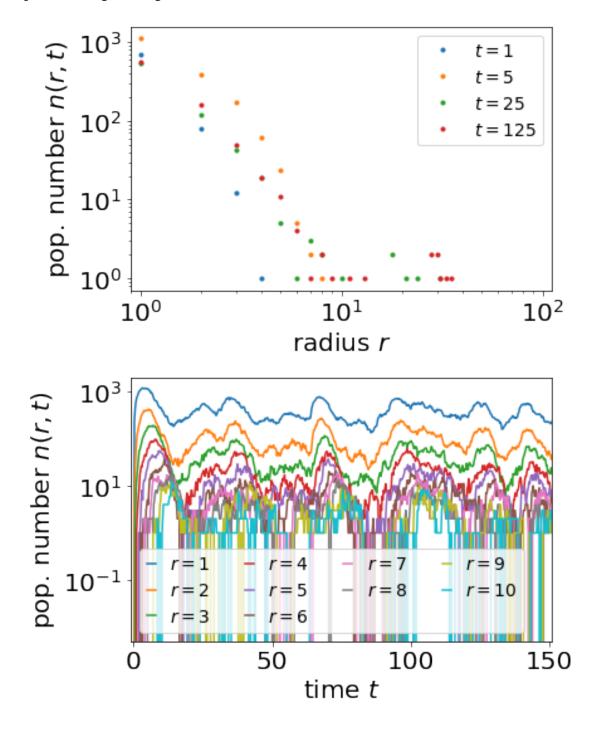
### Non-power-law growth (logistic) example

```
[44]: r0 = 1
      rmax = 10
      a = 1
      thalf = 5
      lam = 1
      mu0 = 1
      kappa = 2
      rdot = lambda r: rmax * a / 2 * cosh(arctanh(2 * r / rmax - 1))**-2
      drdot = lambda r: -2 * a * (2*r / rmax - 1)
      mu = lambda r: mu0 * r**(-2/3)
      bsrange = np.logspace(-3, -1, 3)
      rRange = np.linspace(r0, rmax-.05, 200)
      fig, ax = plt.subplots()
      for bs in bsrange:
          ax.plot(rRange, (drdot(rRange) + mu(rRange) + bs * rRange**kappa)/
       →rdot(rRange))
      ax.set(xlabel=r'radius rs', ylabel='integrand -(\dot{r}+\mu+ B\,s\,r^\kappa)/
       \hookrightarrow \det\{r\}$',
             xlim=(r0, rmax), ylim=(-5.308656142979472, 21.37184385702066))
      ax.legend((r'$10^{-3}$',r'$10^{-2}$',r'$10^{-1}$'), title=r'$B\,s$')
      fig.savefig(f'{imgdr}/pop_tail.pdf', bbox_inches='tight')
```



### Automaton simulation of asymmetric competition

[53]: <matplotlib.legend.Legend at 0x7f3182db28b0>



#### Spatial distribution comparison with termites

```
[295]: df = pd.read_csv('../data/Tarnita/termite_mound_location_field_data/Namib_G1.
        ⇔txt',
                        sep='\t',
                        header=None)
       termitexy = df.values
       load_pickle('cache/packing_example.p')
       allxy = xy
       xy = vstack(allxy[areaDeathRateRange[-1]][0])
       fig = plt.figure(figsize=(10,4))
       ax = fig.add_subplot(121, aspect='equal'), fig.add_subplot(122, aspect='equal')
       data.plot(xy, [4.2]*len(xy), 200, ax=ax[0])
       data.plot(termitexy, [10]*len(termitexy), 520, ax=ax[1])
       ax[0].set(title='simulation', xlabel='width', ylabel='height')
       ax[1].set(title='termite mound data', xlabel='width (m)', ylabel='height (m)')
       fig.subplots_adjust(wspace=.3)
       fig.savefig(f'{imgdr}/termite_mounds.pdf', bbox_inches='tight')
```

