Please submit your solutions (both tex and PDF files) to WyoCourses; and hand in a printed version in class. If you are a graduate student, please also finish Problem 5 and 6.

The following problems are mainly selected from Ross's book.

1. Let E and F be two events. Prove that

$$P(E \cap F^c) = P(E) - P(E \cap F).$$

Solution:
$E \cap F$ and $E \cap F^c$ are mutually exclusive, therefore

$$P((E \cap F) \cup (E \cap F^c)) = P(E \cap F) + P(E \cap F^c)$$

We know

$$\begin{aligned}(E \cap F) \cup (E \cap F^c) &= E(F \cup F^c) \quad \text{By the distributive laws}\\ &= E(1)\\ &= E\end{aligned}$$

Therefore,

$$\begin{aligned}P(E) &= P((E \cap F) \cup (E \cap F^c))\\ &= P(E \cap F) + P(E \cap F^c)\end{aligned}$$

By rearranging, we get

$$P(E \cap F^c) = P(E) - P(E \cap F)$$

2. Suppose that A and B are mutually exclusive events for with $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that

(a) either A or B occurs?
(b) A occurs but not B?
(c) both A and B occur.

Solution:

(a) $(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8$
(b) $P(A) = 0.3$, because A and B are mutually exclusive, the occurrence of A means that there is no occurrence of B.
(c) $P(A \cap B) = P(\emptyset) = 0$, because A and B are mutually exclusive, A and B cannot occur at the same time.

3. A pair of fair dice are rolled.

(a) Describe the outcome of the experiment and the sample space S by using vectors (in this case, a vector has two coordinates).
(b) Compute the number of elements of the sample space S.
(c) What is the probability to get a double 6?
(d) What is the probability that the second die lands on a higher value than does the first?

Solution:

(a) $S = \{(i, j) : i, j = 1, 2, 3, 4, 5, 6\}$, where the outcome (i, j) is said to occur if i appears on the leftmost die and j on the other die.
(b) $6^2 = 36$
(c) $\frac{1}{6^2} = \frac{1}{36}$

(d) $\frac{\binom{6}{2}}{6^2} = \frac{15}{36} = \frac{5}{12}$

4. Prove *Bonferroni's inequality*, namely,

$$P(EF) \geq P(E) + P(F) - 1.$$

Solution:
From Axiom 1 of probability, we know

$$\begin{aligned} &P(E \cup F) \leq 1 \\ \implies &P(E) + P(F) - P(E \cap F) \leq 1 \\ \implies &P(E) + P(F) - 1 \leq P(E \cap F) \end{aligned}$$

By rearranging, we get

$$P(E \cap F) \geq P(E) + P(F) - 1$$