

Please submit your solutions (both tex and PDF files) to WyoCourses; and hand in a printed version in class. If you are a graduate student, please also finish Problem 6 and 7.

The following problems are mainly selected from Ross's book.

1. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?

Solution: Let C be the event of a C section and S be the event of survival. From the information given, we know $P(S) = .98$, $P(C) = .15$, $P(S|C) = .96$, $P(C^c) = 1 - P(C) = 1 - .15 = .85$ and we are trying to find $P(S|C^c)$.

$$\begin{aligned}
 P(S|C^c) &= \frac{P(SC^c)}{P(C^c)} \\
 &= \frac{P(S) - P(SC)}{P(C^c)} \\
 &= \frac{P(S) - P(S|C)P(C)}{P(C^c)} \\
 &= \frac{0.98 - (0.96)(0.15)}{.85} \\
 &= \frac{.836}{.85} \\
 &\approx .9835
 \end{aligned}$$

2. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random.

Given that this person voted in the local election, what is the probability that he or she is

- (a) an Independent?
- (b) a Liberal?
- (c) a Conservative?
- (d) What fraction of voters participated in the local election?

Solution: Let V be the event that a voter voted in the local election, I be the event that a voter is Independent, L be the event that a voter is Liberal, and C be the event that a voter is Conservative. From the information given, we know $P(I) = .46$, $P(L) = .3$, $P(C) = .24$, and $P(V|I) = .35$, $P(V|L) = .62$, $P(V|C) = .58$. The problem is asking (a) $P(I|V) = ?$, (b) $P(L|V) = ?$, (c) $P(C|V) = ?$, and (d) $P(V) = ?$. Because this information will be needed to solve the other parts, I will start by solving (d).

- (d)

$$\begin{aligned}
 P(V) &= P(V|I)P(I) + P(V|L)P(L) + P(V|C)P(C) \\
 &= (.35)(.46) + (.62)(.3) + (.58)(.24) \\
 &= .161 + .186 + .1392 \\
 &= .4862
 \end{aligned}$$

(a)

$$\begin{aligned}
 P(I|V) &= \frac{P(IV)}{P(V)} \\
 &= \frac{P(V|I)P(I)}{P(V)} \\
 &= \frac{(.35)(.46)}{.4862} \\
 &= \frac{.161}{.4862} \\
 &\approx .3311
 \end{aligned}$$

(b)

$$\begin{aligned}
 P(L|V) &= \frac{P(LV)}{P(V)} \\
 &= \frac{P(V|L)P(L)}{P(V)} \\
 &= \frac{(.62)(.3)}{.4862} \\
 &= \frac{.186}{.4862} \\
 &\approx .3826
 \end{aligned}$$

(c)

$$\begin{aligned}
 P(C|V) &= \frac{P(CV)}{P(V)} \\
 &= \frac{P(V|C)P(C)}{P(V)} \\
 &= \frac{(.58)(.24)}{.4862} \\
 &= \frac{.1392}{.4862} \\
 &\approx .2863
 \end{aligned}$$

3. A total of 48 percent of the women and 37 percent of the men that took a certain “quit smoking” class remained nonsmokers for at least one year after completing the class. These people then attended a success party at the end of a year. If 62 percent of the original class was male, what percentage of the original class attended the party?

Solution: Let S be the event that a person in the class attends the success party, M be the event that a person in the class is male, and F be the event that a person in the class is female. From the information given, we know $P(S|F) = .48$, $P(S|M) = .37$, $P(M) = .62$ and we are trying to find $P(S) = ?$. Because M and F are mutually exclusive, we also know $P(F) = 1 - P(M) = 1 - .62 = .38$.

$$\begin{aligned}
 P(S) &= P(S|M)P(M) + P(S|F)P(F) \\
 &= (.37)(.62) + (.48)(.38) \\
 &= .2294 + .1824 \\
 &= .4118
 \end{aligned}$$

4. With probability .6, the present was hidden by mom; with probability .4, it was hidden by dad. When mom hides the present, she hides it upstairs 70 percent of the time and downstairs 30 percent of the time. Dad is equally likely to hide it upstairs or downstairs.

- (a) What is the probability that the present is upstairs?
 (b) Given that it is downstairs, what is the probability it was hidden by dad?

Solution: Let M be the event that mom hid the present, D be the event that dad hid the present, U be the event that the present is hidden upstairs, and N be the event that present is hidden downstairs. From the information given, we know $P(M) = .6$, $P(D) = .4$, $P(U|M) = .7$, $P(N|M) = .3$, $P(U|D) = .5$, and $P(N|D) = .5$. The problem is asking to find (a) $P(U) = ?$ and (b) $P(D|N) = ?$.

(a)

$$\begin{aligned} P(U) &= P(U|M)P(M) + P(U|D)P(D) \\ &= (.7)(.6) + (.5)(.4) \\ &= .42 + .2 \\ &= .62 \end{aligned}$$

- (b) Using the value we found for $P(U)$ in (a) and knowing that U and N are mutually exclusive, we know $P(N) = 1 - P(U) = 1 - .62 = .38$

$$\begin{aligned} P(D|N) &= \frac{P(DN)}{P(N)} \\ &= \frac{P(N|D)P(D)}{P(N)} \\ &= \frac{(.5)(.4)}{.38} \\ &= \frac{.2}{.38} \\ &\approx .5263 \end{aligned}$$

5. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.

- (a) What is the probability that after 2 days the stock will be at its original price?
 (b) What is the probability that after 3 days the stock's price will have increased by 1 unit?
 (c) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?

Solution: Let E_1 be the event that the price increases by 1 unit on day 1, E_2 be the event that the price increases by 1 unit on day 2 and E_3 be the event that the price increases by 1 unit on day 3. Therefore, E_1^c , E_2^c and E_3^c are the events that the price decreases by 1 unit on their respective day. From the information given in the problem, we know $P(E_1)$, $P(E_2)$, $P(E_3)$ are equal to p , and $P(E_1^c)$, $P(E_2^c)$, and $P(E_3^c)$ are equal to $(p - 1)$.

- (a) For the stock to be at its original price after two days, the price has to increase on day 1 then decrease on day 2 ($E_1E_2^c$), OR decrease on day 1 and increase on day 2 ($E_1^cE_2$). Therefore, the probability is given as

$$\begin{aligned} P(E_1E_2^c \cup E_1^cE_2) &= P(E_1E_2^c) + P(E_1^cE_2) \\ &= P(E_1)P(E_2^c) + P(E_1^c)P(E_2) \\ &= p(1 - p) + p(1 - p) \\ &= 2p(1 - p) \end{aligned}$$

- (b) For the price to increase by 1 unit after 3 days, the price has to increase on both day 1 and 2 then decrease on day 3 ($E_1 E_2 E_3^c$), OR increase on day 1 then decrease on day 2 then increase on day 3 ($E_1 E_2^c E_3$), OR decrease on day 1 then increase on both day 2 and 3 ($E_1^c E_2 E_3$). Therefore, the probability is given as

$$\begin{aligned}
 P(E_1 E_2 E_3^c \cup E_1 E_2^c E_3 \cup E_1^c E_2 E_3) &= P(E_1 E_2 E_3^c) + P(E_1 E_2^c E_3) + P(E_1^c E_2 E_3) \\
 &= P(E_1)P(E_2)P(E_3^c) + P(E_1)P(E_2^c)P(E_3) + P(E_1^c)P(E_2)P(E_3) \\
 &= p \cdot p(p-1) + p \cdot p(p-1) + p \cdot p(p-1) \\
 &= 3p^2(p-1)
 \end{aligned}$$

- (c) Let U be the event that the stock's price has increased by 1 unit after 3 days. From part (b), we know $P(U) = 3p^2(1-p)$.

$$\begin{aligned}
 P(E_1|U) &= \frac{P(E_1 U)}{P(U)} \\
 &= \frac{P(E_1 E_2^c E_3) + P(E_1 E_2 E_3^c)}{P(U)} \\
 &= \frac{p \cdot p(1-p) + p \cdot p(1-p)}{3p^2(1-p)} \\
 &= \frac{2p^2(1-p)}{3p^2(1-p)} \\
 &= \frac{2}{3}
 \end{aligned}$$