ICME Intro to Stats Summer Workshop

Section 3 Solutions

2023-07-24

Section 3

1.

The Answer here is C. Here is an explanation of why the other answers are wrong. Typically, we say that when n $\geq$ 30 we have a large enough sample to assume a normal distribution:

- A. By the Central Limit Theorem, the sum of a large number of observations of a random variable follows a normal distribution, not a uniform distribution, even if the observations themselves follow a uniform distribution.

- B. By the Central Limit Theorem, the sum of a large number of observations of a random variable follows a normal distribution, not an exponential distribution.

- C. By the Central Limit Theorem, the sum of a large number of observations of a random variable follows a normal distribution in which the mean is the sample size times the mean of the original distribution (in this case, $40(1.5) = 60$), and the standard deviation is the square root of the sample size times the original standard deviation (in this case, $\sqrt{40}\sqrt{\frac{9}{12}} \approx 5.477$). We get the standard deviation by following the formula for the standard deviation of a uniform random variable.

$$Var(X) \sim \frac{(b-a)^2}{12} = \frac{(7-4)^2}{12} = \frac{9}{12}$$

- D. By the Central Limit Theorem, the sum of a large number of observations of a random variable follows a normal distribution in which the mean is the sample size times the mean of the original distribution; this answer does not account for the sample size.

2.

$$X \sim N(2, \frac{0.25}{\sqrt{16}})$$

Let X be the random variable for the average carrot length in a single bag. The sampling distribution of the mean Y follows a normal distribution in which the mean of Y is the same as that of X (in this case, 2), and the standard deviation is that of Y divided by the square root of the sample size (in this case, $\frac{0.25}{\sqrt{16}}$).