1) Use these returns to estimate the vector of mean returns and the covariance matrix of returns for the ten industry portfolios.

Vector of mean returns:

```
vector_mean:
[[0.90283333]
 [0.73333333]
 [1.01283333]
 [1.23116667]
 [0.76625   ]
 [0.88141667]
 [0.91633333]
 [0.78383333]
 [0.90716667]
 [0.48908333]]
```

Covariance matrix of returns:

	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
NoDur	11.193422	18.449666	14.104907	10.531341	12.922949	11.968078	10.170832	9.953112	7.866653	14.438409
Durbl	18.449666	69.920577	39.178097	27.019794	35.466652	27.490543	27.444731	16.824003	12.746136	39.361987
Manuf	14.104907	39.178097	28.198970	23.145380	24.618739	19.550150	17.622867	13.596447	11.440612	26.313423
Enrgy	10.531341	27.019794	23.145380	36.984933	19.267276	15.366817	11.297800	9.630327	14.027168	18.320469
HiTec	12.922949	35.466652	24.618739	19.267276	28.957220	18.708273	17.837115	13.254064	10.304187	23.855470
Telcm	11.968078	27.490543	19.550150	15.366817	18.708273	19.787227	14.169356	11.506599	10.991596	19.610836
Shops	10.170832	27.444731	17.622867	11.297800	17.837115	14.169356	16.759084	10.178849	6.694350	19.226524
Hlth	9.953112	16.824003	13.596447	9.630327	13.254064	11.506599	10.178849	14.342669	7.475036	14.864553
Utils	7.866653	12.746136	11.440612	14.027168	10.304187	10.991596	6.694350	7.475036	13.703052	9.992960
Other	14.438409	39.361987	26.313423	18.320469	23.855470	19.610836	19.226524	14.864553	9.992960	31.163771

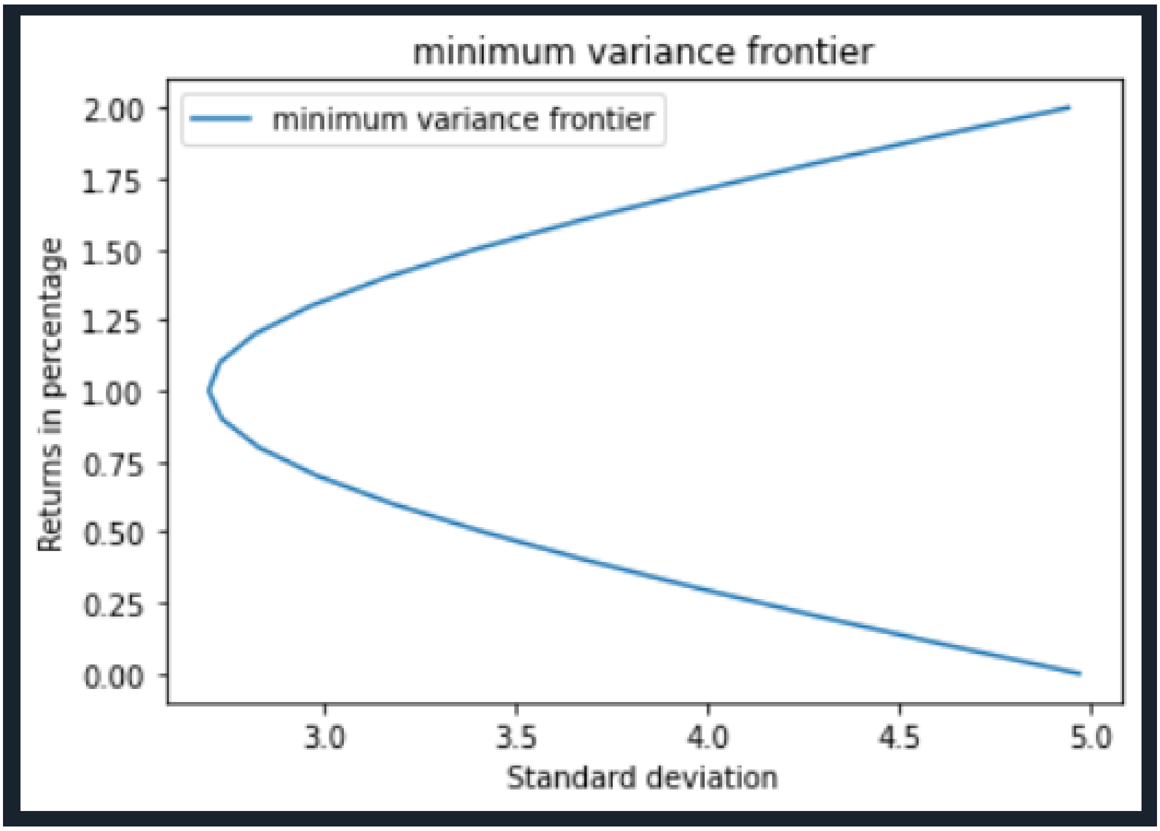
2) Create a table showing the mean return and standard deviation of return for the ten industry portfolios.

Table of mean return and standard deviations:

	industry	mean_return	standard_deviation
0	NoDur	0.902833	3.34566
1	Durbl	0.733333	8.36185
2	Manuf	1.01283	5.31027
3	Enrgy	1.23117	6.08152
4	HiTec	0.76625	5.38119
5	Telcm	0.881417	4.44828
6	Shops	0.916333	4.09379
7	Hlth	0.783833	3.78717
8	Utils	0.907167	3.70176
9	Other	0.489083	5.58245

3) Plot the minimum-variance frontier (without the riskless asset) generated by the ten industry portfolios, with expected (monthly) return on the vertical axis and standard deviation of (monthly) return on the horizontal axis. This plot should cover the range from 0% to 2% on the vertical axis, in increments of 0.1% (or less).

Minimum Variance Frontier:

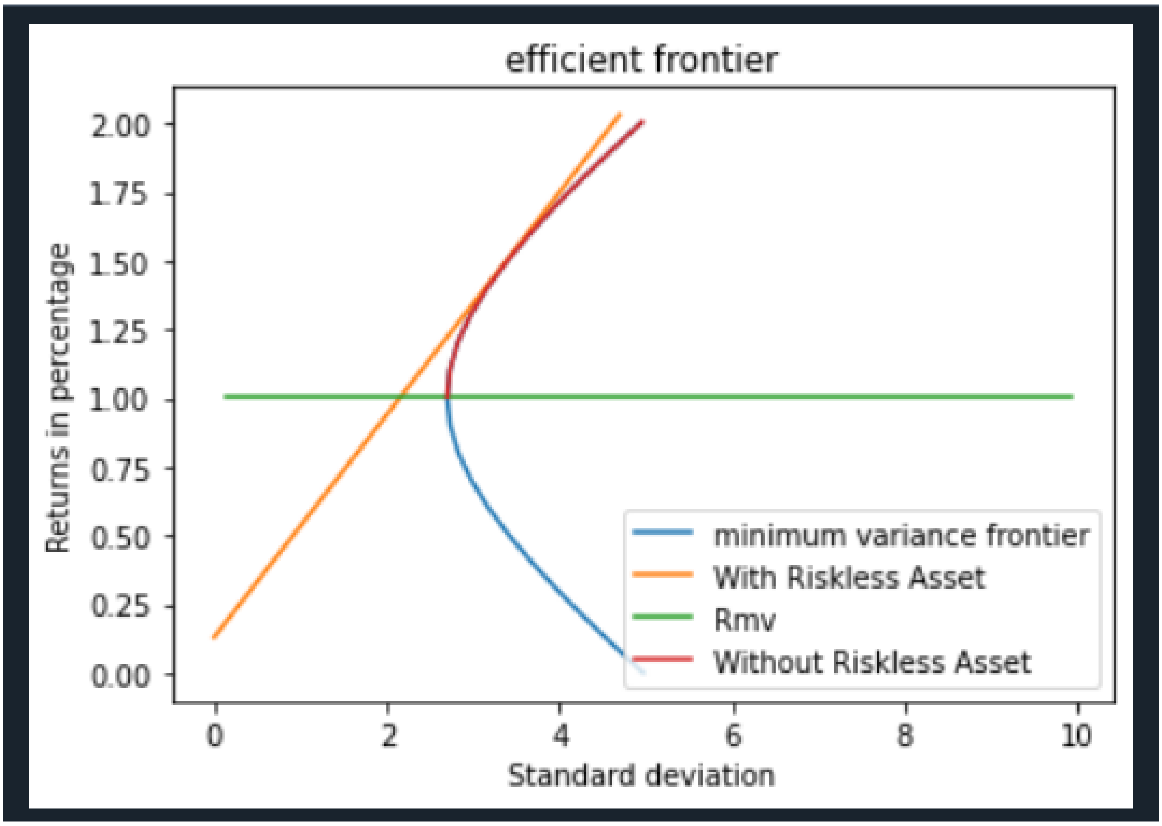


4) Briefly explain the economic significance and relevance of the minimum-variance frontier to an investor.

Different investors have different risk appetite or known as different utility. Given a pool of stocks to choose from. Minimum variance frontier shows the minimum variance that can be achieved with a given level of expected return. With minimum variance frontier, investor is able to choose the optimal portfolio from the list of stocks. The portfolio which has the least risk is the global minimum variance portfolio. Investor can maximize expected return or minimize risk. The outcome of combination of weights of different stocks gives you the minimum variance frontier. It involves diversifying your holdings to reduce volatility. The assets which are itself risky turns out to balance each other when they are put together in the same portfolio. Returns are plotted on the y-axis and standard deviations are plotted on the x-axis. Historical data are used to estimate the covariance, standard deviation and mean for formation of portfolio. Investor should not have a portfolio that fall outside of minimum variance point. The higher returns are along the positively sloped part of the minimum-variance frontier whereas the lower risk is at the slope to the left side of the minimum variance frontier. Risk-averse investor is to pick a portfolio within the minimum variance frontier with the lowest risk given a desired return.

5) Plot the efficient frontier (with the riskless asset) on the same plot as the minimum-variance frontier generated by the ten industry portfolios.

Efficient Frontier:



6) Briefly explain the economic significance and relevance of the efficient frontier to an investor.

With the specified standard deviation, efficient frontier provides the set of portfolios that provides the highest expected return. Efficient frontier is a curved line as when the risk goes higher, the return provided will be relatively lower amount. The curvature signifies diminishing marginal return of risk. Diversification results in lower risk and higher returns which in turns creates the optimal portfolio on the curved line. Portfolios that are below the efficient frontier are not optimal as they do not give the adequate amount of return for the level of risk. Portfolios that are towards the right of the efficient frontier are not good as they have higher risk for the given returns. Standard deviation refers to level of risk and consistency of returns. Returns are plotted on the y-axis whereas standard deviation are plotted on the x-axis. Higher covariance between portfolio provides higher standard deviations. The optimal portfolio is the portfolio whereby the CAL is tangent with the efficient line. The optimal portfolio usually has a high degree of diversification. There is no single efficient frontier for everyone has everyone has different risk adverseness thus everyone's indifference curve is not the same.

7) Calculate the Sharpe ratio for the tangency portfolio, and also the tangency portfolio weights for the ten industry portfolios.

Sharpe ratio:

```
sharpe_ratio: [[0.4035656]]
```

The tangency portfolio weights are:

NoDur	0.567972
Durbl	-0.214073
Manuf	0.714105
Enrgy	0.104087
HiTec	-0.363438
Telcm	-0.095463
Shops	0.991647
Hlth	0.075570
Utils	0.132643
other	-0.913051

8) Briefly explain the economic significance and relevance of the tangency portfolio to an investor.

Tangency portfolio also named as the optimal portfolio. The tangency portfolio includes both risk-free asset and risky asset. The tangency portfolio is the portfolio where the CAL line is tangent to the efficient frontier. Investors includes both risk-free and riskless assets to reduce risks thus creating the Capital Allocation line (CAL). The Sharpe ratio refers to the slope of the line. The highest Sharpe ratio is the Sharpe ratio of the optimal portfolio. Which means the tangency portfolio is the portfolio that generates the highest return which every unit of risk taken. MPT assumes that investor can borrow money at the risk-free rate but in fact in real life, investors cannot. Thus, with higher borrowing rate. As portfolio consist of borrowing portfolio. The margin of return per standard deviation decreases.

Appendix:

```
# -*- coding: utf-8 -*-
"""
Created on Thu Sep  1 14:51:26 2022

@author: XuebinLi
"""



import glob
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.dates import DateFormatter, MinuteLocator
import datetime
from datetime import date
from datetime import timedelta
import pandas as pd
from tabulate import tabulate
from numpy.linalg import inv
import math
pd.set_option('display.max_rows', 500)
pd.set_option('display.max_columns', 500)
pd.set_option('display.width', 1000)



df = pd.DataFrame()
```

```
df = pd.read_excel('C:\\Users\\lixue\\OneDrive\\Desktop\\smu\\MQF\\Asset Pricing\\lesson2\\Industry_Portfolios.xlsx')

#df = pd.read_excel('C:\\Users\\XuebinLi\\OneDrive - Linden Shore LLC\\Desktop\\python\\asset_pricing_project\\Industry_Portfolios.xlsx')


NoDur_mean = df['NoDur'].mean()

Durbl_mean = df['Durbl'].mean()

Manuf_mean = df['Manuf'].mean()

Enrgy_mean = df['Enrgy'].mean()

HiTec_mean = df['HiTec'].mean()

Telcm_mean = df['Telcm'].mean()

Shops_mean = df['Shops'].mean()

Hlth_mean = df['Hlth'].mean()

Utils_mean = df['Utils'].mean()

Other_mean = df['Other'].mean()


NoDur_std = df['NoDur'].std()

Durbl_std = df['Durbl'].std()

Manuf_std = df['Manuf'].std()

Enrgy_std = df['Enrgy'].std()

HiTec_std = df['HiTec'].std()

Telcm_std = df['Telcm'].std()

Shops_std = df['Shops'].std()

Hlth_std = df['Hlth'].std()

Utils_std = df['Utils'].std()

Other_std = df['Other'].std()


data = {'industry': ['NoDur', 'Durbl', 'Manuf', 'Enrgy', 'HiTec','Telcm','Shops','Hlth','Utils','Other'],

        'mean_return': [NoDur_mean, Durbl_mean, Manuf_mean, Enrgy_mean, HiTec_mean,Telcm_mean,Shops_mean,Hlth_mean,Utils_mean,Other_mean],
```

```
    'standard_deviation': [NoDur_std, Durbl_std, Manuf_std, Enrgy_std,
HiTec_std,Telcm_std,Shops_std,Hlth_std,Utils_std,Other_std],

    }


df_table_mean_std = pd.DataFrame(data)




#covariance

df_cov = df.iloc[: , 1:]

df_cov = df_cov.cov()

# print(df_cov)

vector_mean = df_table_mean_std[["mean_return"]].to_numpy()

# transpose of returns

vector_mean_transpose = np.transpose(vector_mean)

#inverse covariance

df_cov_inverse = inv(df_cov)

weight = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

weight_transpose = np.transpose(weight)




alpha = np.matmul(vector_mean_transpose, df_cov_inverse)

alpha = np.matmul(alpha, weight)


zelta = np.matmul(vector_mean_transpose, df_cov_inverse)

zelta = np.matmul(zelta, vector_mean)


delta = np.matmul(weight_transpose, df_cov_inverse)
```

```
delta = np.matmul(delta, weight)

rmv = alpha/delta
rf_rate = 0.13


def my_range(start, end, step):
    while start <= end:
        yield start
        start += step


#minimum variance frontier(page 11 and 13 lecture)
yaxis = []
xaxis = []
for x in my_range(0, 2.1, 0.1):
    stdplot = (1/delta) + (delta/(zelta*delta-(alpha*alpha))) * (x - (alpha/delta))**2
    stdplot = math.sqrt(stdplot)
    xaxis += [stdplot]
    yaxis += [x]
plt.plot(xaxis,yaxis,label="minimum variance frontier")
plt.xlabel("Standard deviation")
plt.ylabel("Returns in percentage")
plt.title("minimum variance frontier")
plt.legend()


#risk-free line(PAGE 25 lecture)
yaxis2 = []
xaxis2 = []
for x2 in my_range(rf_rate, 2.1, 0.1):
```

```
    stdplot2 = ((x2 - rf_rate)**2)/(zelta - 2*alpha*rf_rate+zelta*rf_rate*rf_rate)
    stdplot2 = math.sqrt(stdplot2)
    xaxis2 += [stdplot2]
    yaxis2 += [x2]
plt.plot(xaxis2,yaxis2,label="With Riskless Asset")
plt.xlabel("Standard deviation")
plt.ylabel("Returns in percentage")
plt.legend()

#mid line
yaxis3 = []
xaxis3 = []
for x3 in my_range(rf_rate, 10, 0.1):
    stdplot3 = alpha/delta
    xaxis3 += [stdplot3]
    yaxis3 += [x3]
plt.plot(yaxis3,xaxis3,label='Rmv')
plt.xlabel("Standard deviation")
plt.ylabel("Returns in percentage")
plt.legend()



df.to_excel (r'C:\\Users\\lixue\\OneDrive\\Desktop\\smu\\MQF\\Asset Pricing\\lesson2\\std_mean.xlsx')
#df.to_excel (r'C:\\Users\\XuebinLi\\OneDrive - Linden Shore LLC\\Desktop\\python\\asset_pricing_project\\std_mean.xlsx')



#efficient frontier
yaxis4 = []
xaxis4 = []
```

```
for x4 in my_range(1.004, 2.1, 0.1):
    stdplot4 = (1/delta) + (delta/(zelta*delta-(alpha*alpha))) * (x4 - (alpha/delta))**2
    stdplot4 = math.sqrt(stdplot4)
    xaxis4 += [stdplot4]
    yaxis4 += [x4]
plt.plot(xaxis4,yaxis4 ,label="Without Riskless Asset")
plt.xlabel("Standard deviation")
plt.ylabel("Returns in percentage")
plt.title("efficient frontier")
plt.legend()


#tangency portfolio
Rtg = (alpha * rf_rate - zelta)/(delta*rf_rate - alpha)
Rtg = Rtg[0][0]
print(Rtg)

Stg = -((zelta-2*alpha*rf_rate+delta*rf_rate*rf_rate)**0.5)/(delta*(rf_rate-rmv))

# Stg = Stg[0][0]
print("STG:",Stg)
# Rtg = Rtg[0][0]
print("Rtg",Rtg)


#sharpe ratio
sharpe_ratio = (Rtg - rf_rate)/Stg
```

```
print("sharpe_ratio:", sharpe_ratio )



#weight of optimal portfolio
lmda = (Rtg - rf_rate)/(zelta - 2*alpha*rf_rate+delta*rf_rate*rf_rate)
lmda = lmda[0][0]
lmda = [lmda,lmda,lmda,lmda,lmda,lmda,lmda,lmda,lmda,lmda]
weight_Rtg1 = np.dot(lmda,df_cov_inverse)
weight_Rtg1 = lmda * df_cov_inverse
weight_Rtg2 = np.dot(rf_rate,weight)
weight_Rtg2 = np.reshape(weight_Rtg2, (10, 1))
weight_Rtg3 = np.subtract(vector_mean,weight_Rtg2)
weight_final = np.dot(weight_Rtg1,weight_Rtg3)
print("weight of optimal portfolio")
print(weight_final)



header = ['NoDur', 'Durbl', 'Manuf', 'Enrgy', 'HiTec', 'Telcm','Shops','Hlth','Utils','other']
frame = pd.DataFrame(weight_final, index=header)
print(frame)

#question1: vector of mean and covariance
print("vector_mean:")
print(vector_mean)
print("covariance:")
print(df_cov)
#question2: table with mean and std
print(tabulate(df_table_mean_std, headers = 'keys', tablefmt = 'psql'))
#df_table_mean_std.plot(x='standard_deviation', y='mean_return', kind='scatter');
```