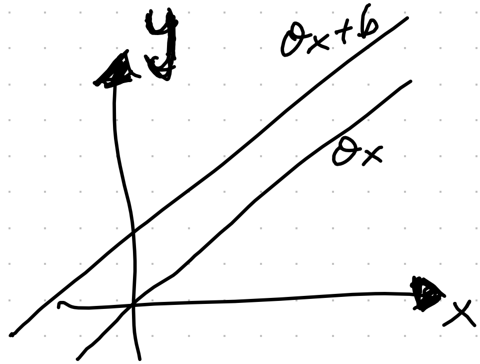
Data $(y_i)_{i=1}^{n}$ at $(x_i)_{i=1}^{n}$

$y = f(x)$

$x \in \mathbb{R}, \quad y \in \mathbb{R}$

$\theta \in \mathbb{R}$

$y = \theta x + b$




$$L_2\left(f; (x_i)_{i=1}^{n}, (y_i)_{i=1}^{n}\right) :=$$

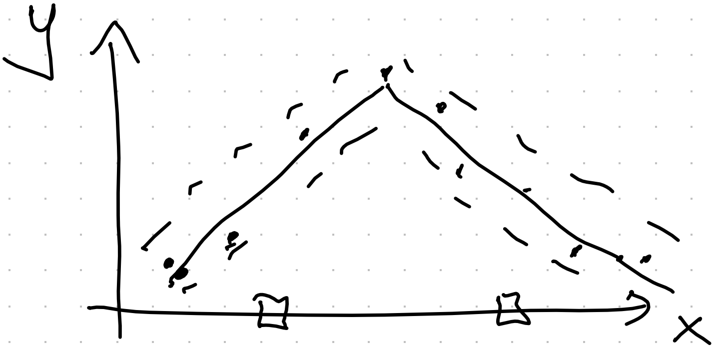
$$:= \left(\sum_{i=1}^{n} (y_i - f(x_i))^2\right)^{\frac{1}{2}} = \begin{pmatrix}\text{linear} \\ \text{model}\end{pmatrix}$$

$$= \left(\sum_{i=1}^{n} (y_i - \theta x_i)^2\right)^{\frac{1}{2}}$$

$$L_1(f; \ldots) := \sum_{i=1}^{n} \left| y_i - f(x_i) \right|$$

$$L_2 + \underbrace{|\theta|}_{\text{regulariser}}$$

$$y = f(x) \sim \mathcal{N}(\mu(x), \sigma(x)^2)$$

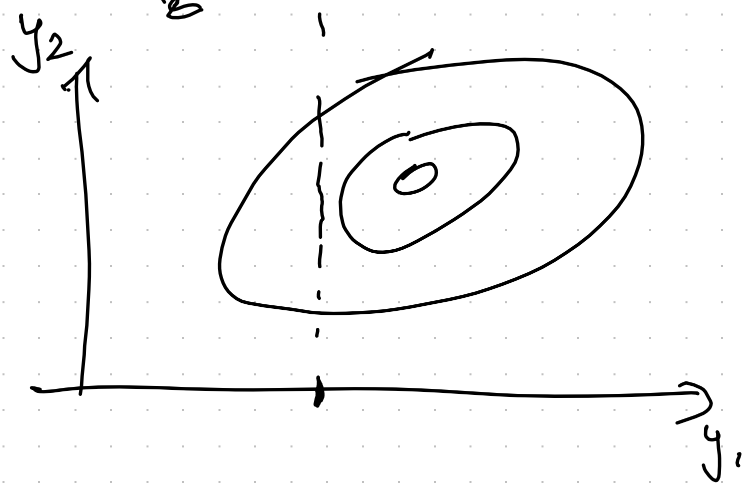



$$y = f(x) \sim GP(\underbrace{\mu(x)}_{\text{mean}}, \underbrace{k(x, x)}_{\text{cov}})$$

$$\begin{pmatrix} f(x) \\ f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix} \sim MVN\left( \begin{pmatrix} \mu(x) \\ \mu(x_1) \\ \vdots \\ \mu(x_n) \end{pmatrix}, \underbrace{\begin{pmatrix} k(x, x) & k(x, x_1) & \cdots & \\ k(x_1, x) & k(x_1, x_1) & k(x_1, x_2) & \\ & \vdots & k(x_2, x_2) & \\ & k(x_n, x_1) & \cdots & \end{pmatrix}}_{K(X, X)} \right)$$

$$f(x) \mid f(x_1) = y_1, \ldots, f(x_n) = y_n$$

$Y = \begin{matrix} Y_1 \\ Y_2 \end{matrix}$




$f(x) \mid f(x_1) = y_1 \; \dots \; f(x_n) = y_n \sim$

$\sim MVN(\bar{\mu}, \bar{\Sigma})$

$$\bar{\mu} := \mu(x) - (k(x, x_1) \dots k(x, x_n)) \cdot \begin{pmatrix} k(x_1, x_1) & k(x_1, x_2) & \dots & k(x_1, x_n) \\ & \ddots & & \\ & & & k(x_n, x_n) \end{pmatrix}^{-1} \cdot \begin{pmatrix} y_1 - \mu(x_1) \\ y_n - \mu(x_n) \end{pmatrix}$$

$K(X, x)$

$$\Sigma = k(x, x) - \overbrace{\left( k(x, x_1) \;\ldots\; k(x, x_n) \right)}^{K_*} \cdot K(X, X)^{-1} K_*^T$$

$$k(x_1, x_2) := \underline{\sigma} \cdot \exp\left( \frac{-|x_1 - x_2|^2}{\underline{\ell}^2} \right)$$