Notes on 1D plot covariances

March 16, 2023

The number of objects in pixel i is N_i.

In the absense of clustering, N_i is just a poisson sampling so the the covariance matrix is

$$\mathrm{Cov}_{\mathbf{w}=\mathbf{0}}(N_i, N_j) = \delta_{ij}\bar{N} \tag{1}$$

where δ_{ij} is the Kronecker delta function, and $\bar{N}$ is the average number count per pixel.

Since $w_{\mathrm{true}}(\theta)$ is just the covariance of the overdensity field. When we add clusteirng, the covariance of N_i looks something like this

$$\mathrm{Cov}(N_i, N_j) = \delta_{ij}\bar{N} + \bar{N}^2 w_{\mathrm{true}}(\theta_{ij}) \tag{2}$$

where θ_{ij} is the separation between pixel i and j. I think we are assuming the noise on $\bar{N}$ itself is small in the second term for now. It is a little unclear to me right now what to do when $i = j$ as there will be some beyond poission noise coming from the sample variance of the clustering. Maybe this depends on the size of the pixel? I will come back to this later.....

For the 1d plots we are summing many pixels in an SP bin k. We will call this sum $N^{(SP)}$ and we will call the number of pixels in SP bin k, N_k^{pix}.

$$N_k^{(SP)} = \sum_{i \text{ in } k} N_i \tag{3}$$

In the absence of clustering, the covariance of this would be

$$\mathrm{Cov}_{\mathbf{w}=\mathbf{0}}(N_k^{SP}, N_l^{SP}) = \delta_{kl}\bar{N}N_k^{\mathrm{pix}} \tag{4}$$

with clustering it is

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \mathrm{Cov}(\sum_{i \text{ in } k} N_i, \sum_{j \text{ in } l} N_j) \tag{5}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} \mathrm{Cov}(N_i, N_j) \tag{6}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} \left[\delta_{ij}\bar{N} + \bar{N}^2 w_{\text{true}}(\theta_{ij})\right] \tag{7}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl}\bar{N}N_k^{\text{pix}} + \sum_{\theta} N_{kl}^{(\text{pix pairs})}(\theta)\bar{N}^2 w_{\text{true}}(\theta) + (i = j \text{ term?}) \tag{8}$$

where $N_{\text{pix pairs}}(\theta)$ is the number of pairs of pixels separated by θ which can be obtained from treecorr. Assuming your $w(\theta)$ and pair counts are in discrete bins.

If there is an actual systematic signal, I think we could just use the real number counts as the poisson term

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl}N_k^{SP} + \sum_{\theta} N_{kl}^{(\text{pix pairs})}(\theta)\bar{N}^2 w_{\text{true}}(\theta) + (i = j \text{ term?}) \tag{9}$$

1 Example on DES Y6 log-normal mocks

I have taken 100 Lognormal mocks designed to match the Y6 maglim sample and computed the covariance of the 1d airmass correlation from the mocks. I then compare this to the above calculation using the same $w(\theta)$ used to generate the mocks.

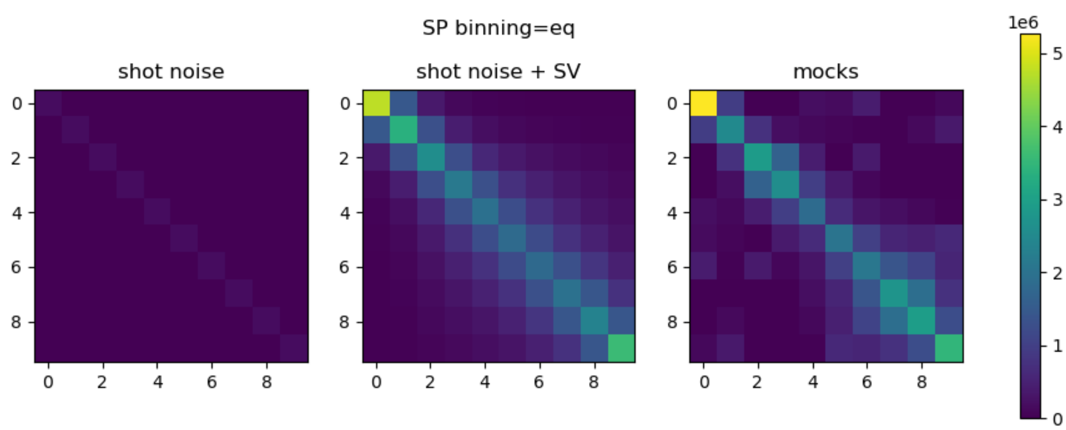

Figure 1:

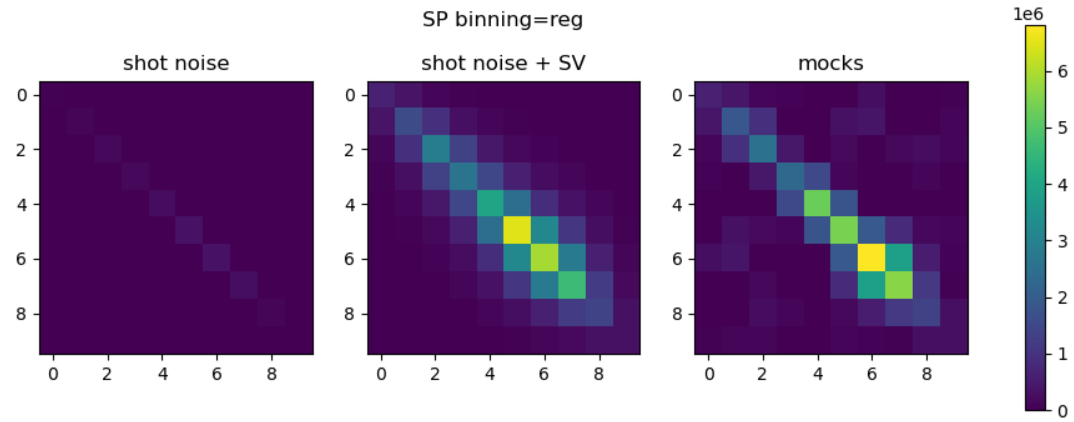

Figure 2:

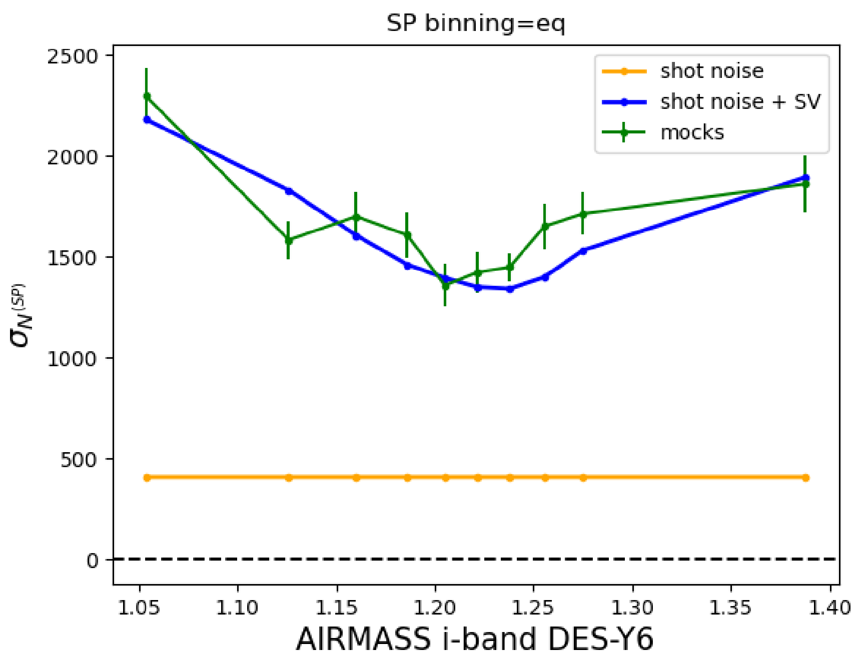

Figure 3:

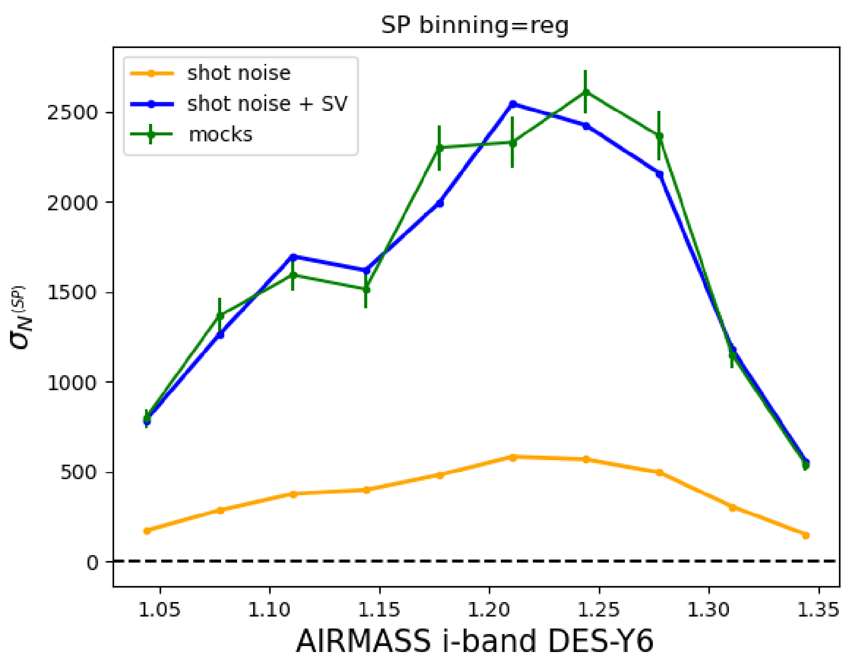
SP binning=reg
shot noise
shot noise + SV
mocks
$\sigma_{N^{(SP)}}$
2500
2000
1500
1000
500
0
1.05
1.10
1.15
1.20
1.25
1.30
1.35
AIRMASS i-band DES-Y6

Figure 4: