

# Notes on 1D plot covariances

March 16, 2023

The number of objects in pixel  $i$  is  $N_i$ .

In the absense of clustering,  $N_i$  is just a poisson sampling so the the covariance matrix is

$$\text{Cov}_{\mathbf{w}=\mathbf{0}}(N_i, N_j) = \delta_{ij} \bar{N} \quad (1)$$

where  $\delta_{ij}$  is the Kronecker delta function, and  $\bar{N}$  is the average number count per pixel.

Since  $w_{\text{true}}(\theta)$  is just the covariance of the overdensity field. When we add clusteirng, the covariance of  $N_i$  looks something like this

$$\text{Cov}(N_i, N_j) = \delta_{ij} \bar{N} + \bar{N} w_{\text{true}}(\theta_{ij}) \quad (2)$$

where  $\theta_{ij}$  is the separation between pixel  $i$  and  $j$ . I think we are assuming the noise on  $\bar{N}$  itself is small in the second term for now. It is a little unclear to me right now what to do when  $i = j$  as there will be some beyond poission noise coming from the sample variance of the clustering. Maybe this depends on the size of the pixel? I will come back to this later.....

For the 1d plots we are summing many pixels in an SP bin  $k$ . We will call this sum  $N^{(SP)}$  and we will call the number of pixels in SP bin  $k$ ,  $N_k^{\text{pix}}$ .

$$N_k^{(SP)} = \sum_{i \text{ in } k} N_i \quad (3)$$

In the absence of clustering, the covariance of this would be

$$\text{Cov}_{\mathbf{w}=\mathbf{0}}(N_k^{SP}, N_l^{SP}) = \delta_{kl} \bar{N} N_k^{\text{pix}} \quad (4)$$

with clustering it is

$$\text{Cov}(N_k^{SP}, N_l^{SP}) = \text{Cov}\left(\sum_{i \text{ in } k} N_i, \sum_{j \text{ in } l} N_j\right) \quad (5)$$

$$\text{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} \text{Cov}(N_i, N_j) \quad (6)$$

$$\text{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} [\delta_{ij} \bar{N} + \bar{N} w_{\text{true}}(\theta_{ij})] \quad (7)$$

$$\text{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl} \bar{N} N_k^{\text{pix}} + \sum_{\theta} N_{kl}^{(\text{pix pairs})}(\theta) \bar{N} w_{\text{true}}(\theta) + (i = j \text{ term?}) \quad (8)$$

where  $N_{\text{pix pairs}}(\theta)$  is the number of pairs of pixels separated by  $\theta$  which can be obtained from treecorr. Assuming your  $w(\theta)$  and pair counts are in discrete bins.

If there is an actual systematic signal, I think we could just use the real number counts as the poisson term

$$\text{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl} N_k^{SP} + \sum_{\theta} N_{kl}^{(\text{pix pairs})}(\theta) \bar{N} w_{\text{true}}(\theta) + (i = j \text{ term?}) \quad (9)$$