Notes on 1D plot covariances

April 25, 2023

The number of objects in pixel i is N_i.

In the absence of clustering, N_i is just a poisson sampling so the the covariance matrix is

$$\mathrm{Cov}_{\mathbf{w}=\mathbf{0}}(N_i, N_j) = \delta_{ij}\bar{N} \tag{1}$$

where δ_{ij} is the Kronecker delta function, and $\bar{N}$ is the average number count per pixel.

Since $w_{\mathrm{true}}(\theta)$ is just the covariance of the overdensity field. When we add clusteirng, the covariance of N_i looks something like this

$$\mathrm{Cov}(N_i, N_j) = \delta_{ij}\bar{N} + \bar{N}^2 w_{\mathrm{true}}(\theta_{ij}) \tag{2}$$

where θ_{ij} is the separation between pixel i and j. I think we are assuming the noise on $\bar{N}$ itself is small in the second term for now. It is a little unclear to me right now what to do when $i = j$ as there will be some beyond poission noise coming from the sample variance of the clustering. Maybe this depends on the size of the pixel? I will come back to this later.....

For the 1d plots we are summing many pixels in an SP bin k. We will call this sum $N^{(SP)}$ and we will call the number of pixels in SP bin k, N_k^{pix}.

$$N_k^{(SP)} = \sum_{i \text{ in } k} N_i \tag{3}$$

In the absence of clustering, the covariance of this would be

$$\mathrm{Cov}_{\mathbf{w}=\mathbf{0}}(N_k^{SP}, N_l^{SP}) = \delta_{kl}\bar{N}N_k^{\mathrm{pix}} \tag{4}$$

with clustering it is

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \mathrm{Cov}(\sum_{i \text{ in } k} N_i, \sum_{j \text{ in } l} N_j) \tag{5}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} \mathrm{Cov}(N_i, N_j) \tag{6}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \sum_{i \text{ in } k} \sum_{j \text{ in } l} \left[\delta_{ij}\bar{N} + \bar{N}^2 w_{\mathrm{true}}(\theta_{ij})\right] \tag{7}$$

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl}\bar{N}N_k^{\mathrm{pix}} + \sum_{\theta} N_{kl}^{(\mathrm{pix\ pairs})}(\theta)\bar{N}^2 w_{\mathrm{true}}(\theta) + (i = j \text{ term?}) \tag{8}$$

where $N_{\mathrm{pix\ pairs}}(\theta)$ is the number of pairs of pixels separated by θ which can be obtained from treecorr or other pair counting codes. Assuming your $w(\theta)$ and pair counts are in discrete bins.

If there is an actual systematic signal, I think we could just use the real number counts as the poisson term

$$\mathrm{Cov}(N_k^{SP}, N_l^{SP}) = \delta_{kl}N_k^{SP} + \sum_{\theta} N_{kl}^{(\mathrm{pix\ pairs})}(\theta)\bar{N}^2 w_{\mathrm{true}}(\theta) + (i = j \text{ term?}) \tag{9}$$

1 Example on DES Y6 log-normal mocks

I have taken 100 Lognormal mocks designed to match the Y6 maglim sample and computed the covariance of the 1d airmass correlation from the mocks. I then compare this to the above calculation using the same $w(\theta)$ used to generate the mocks.

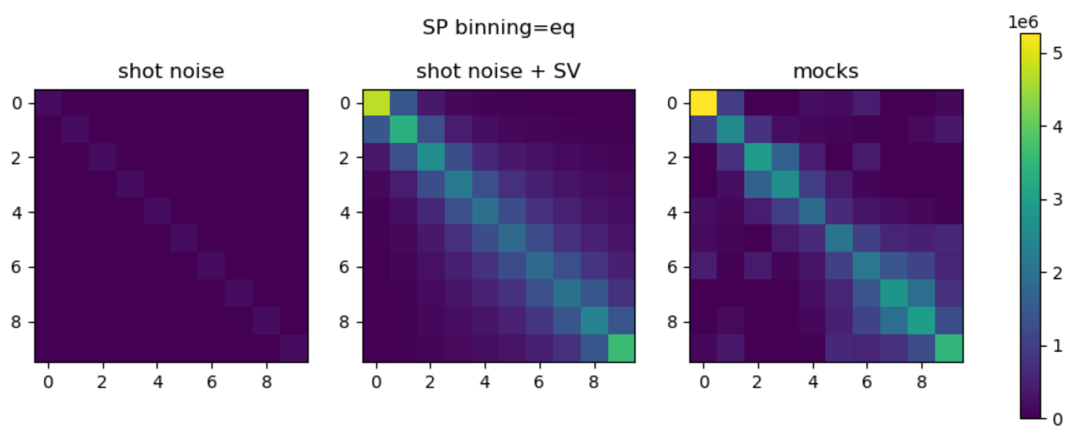

Figure 1:

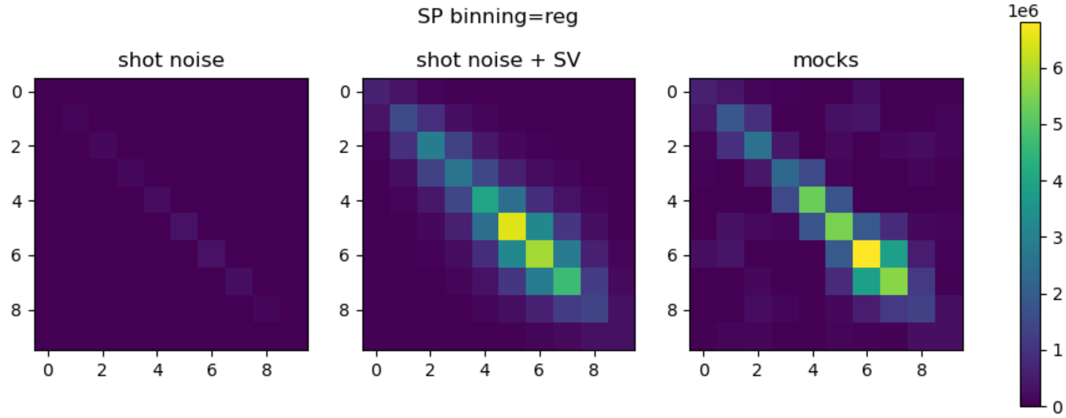

Figure 2:

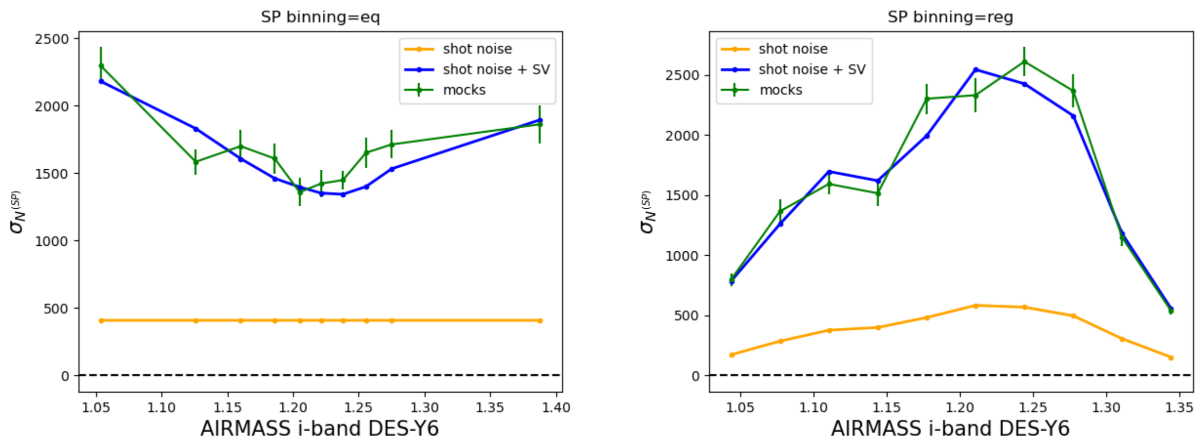

Figure 3:

2 Things to check

Does the $i = j$ term matter? I'm guessing it will matter more for large pixels

What are the requirements wtheta binning, and max/min separation

How important is the $k = l$ term (additional to the diagonal) vs the off diagonal terms, in terms of impact on the χ^2.

Can this be extended to multiple maps simultaniously? e.g could we make a data vector that contains the correlations with many SP maps, fit them simultaniously and use this method to compute the large covariance matrix (e.g. 10 SP maps, 10 bins per map, we make a 100x100 covariance)

3 Multiple SP maps

What if we have two SP maps and we want to know the covariance between the number counts in a given SP bin for map α vs the counts in another bin in map β. i.e. $\mathrm{Cov}(N_k^{SP\alpha}, N_l^{SP\beta})$.

To derive this the first three steps are the same as above. I think the only thing that changes is the shot noise term.

$$\mathrm{Cov}(N_k^{SP\alpha}, N_l^{SP\beta}) = N_{kl}^{SP} + \sum_{\theta} N_{kl}^{(\text{pix pairs})}(\theta)\bar{N}^2 w_{\text{true}}(\theta) + (i = j \text{ term?}) \tag{10}$$

where N_{kl}^{SP} is the number of objects in both patch k and l