CSE 373 Hw 1

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February 14, 2016

1). Big-O Notation Prove each of the following using the definition of big-O notation (find constants c and n_o such that $f(n) \leq c * g(n)$ for $n > n_0$

- $3n^3 + 9n^2 + n + 1 = O(n^3)$

$$\begin{gathered} \text{make n = 1,000 and c = 4} \\ 3(1000)^3 + 9(1000)^2 + (1000) + 1 = 4 * O((1000)^3) \\ 3000000000 + 9000000 + 1000 + 1 \leq 4 * 1000000000 \\ 3009001001 \leq 4000000000 \end{gathered}$$

- $5nlog_2n + 8n - 200 = O(nlog_2n)$

$$\begin{gathered} \text{make c = 10 and n = 1024} \\ 5 * 1024 * log_2(1024) + 8(1024) - 200 = 10(1024(log_2(1024))) \\ 51200 + 8192 - 200 \leq 102400 \\ 59192 \leq 102400 \end{gathered}$$

2). More Big-O Order the following by their growth rates from smallest to largest.

1. $O(n^{1.9})$
2. $O(n^3)$
3. $O(logn)$
4. $O(n)$
5. $O(n^n)$
6. $O(\sqrt{n})$
7. $O(2^n)$
8. $O(n!)$
9. $O(nlogn)$

Solution:

$$O(logn),\ O(\sqrt{n}),\ O(n)$$
$$O(nlogn),\ O(n^{1.9}),\ O(n^3)$$
$$O(2^n),\ O(n!),\ O(n^n)$$

5). Recurrences Solve the following recurrences. Assume $T(n) \leq c$ for some constant c and for all $n \leq 10$.

- $2T(\frac{n}{4}) + n^{0.3}$

Let t = 0.3

$T(n) = 2T(\frac{n}{4}) + n^t$

$T(n/4) = 2T(\frac{n}{4^2}) + (\frac{n}{4})^t$

$T(n/4^2) = 2T(\frac{n}{4^3}) + (\frac{n}{4^2})^t$

$T(n) = 2T(\frac{n}{4}) + n^t$
for i = 1

$T(n) = 2[2T(\frac{n}{4^2}) + (\frac{n}{4})^t] + n^t$

$T(n) = 2^2T(\frac{n}{4^2}) + \frac{2}{4^t}n^t$ for i = 2

$T(n) = 2^2[2T(\frac{n}{4^3}) + (\frac{n}{4^2})^t] + (\frac{2}{4^t})n^t + n^t$

$T(n) = 2^3T(\frac{n}{4^3}) + (\frac{2}{4^t})^2n^t + (\frac{2}{4^t})n^t + n^t$ for i = 3

Using the geometric series*

$T(n) = 2^iT(\frac{n}{4^i}) + (\frac{1-(\frac{2}{4^t})^i}{1-(\frac{2}{4^t})})n^t$

Since "t" is a constant $c1 = \frac{1}{1-(\frac{2}{4^t})}$

$T(n) \leq 2^iT(\frac{n}{4^i}) + c1(1 - (\frac{2}{4^t})^i)n^t$

$T(n) = 2^iT(\frac{n}{4^i}) + c1n^t - c1(\frac{2}{4^t})^in^t$

$= \frac{n}{4^i} = 1; n = 4^i; i = log_4n$

$= 2^iT(\frac{n}{4^i}) + c1n^t - c1(\frac{2}{4^t})^in^t$

$= 2^{log_4n}T(\frac{4^i}{4^i}) + c1n^t - c1(\frac{2}{4^t})^{log_4n}n^t$

$= c * 2^{log_4n} + c1n^t - c1(\frac{2}{4^t})^{log_4n}n^t$

$\leq c * 2^{log_4 n} + c1n^t - c1(\frac{2^{log_4 n}}{4^{tlog_4 n}})n^t$

$\leq c * 2^{log_4 n} + c1n^t - c1(\frac{2^{log_4 n}}{n^t})n^t$

$\leq c * 2^{log_4 n} + c1n^t - c1(2^{log_4 n})$

$\leq (c - c1)2^{log_4 n} + c1n^t$

$Since : 2^{log_4 n} = (n^{log_n 2})^{log_4 n} = n^{log_n 2 * log_4 n} = n^{log_4 2}$

$T(n) \leq (c - c1)n^{log_4 2} + c1n^t$

$log_4 2 \geq t$

$thus : T(n) = O(n^{log_4 2})$

7). Partition The following array has been partitioned. Which elements could have been the pivot value?

[16, 25, 8, 40, 32, 42, 55, 67, 59, 73]

Solution: In a partition, all values less than the pivot are located on one side of the pivot value (in any order) and all the values greater than the pivot are on the opposite side (to the right). With this constraint the only possible values that could have been pivot values are 42, 55 and 73