Kinematic Analysis of a Designed Gough-Stewart Platform using CAD and Mathematics Software

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Abstract

This project assesses the strengths and weaknesses of a gough stewart platform and discusses is application for both commercial and industrial uses. It goes over in detail a potential use for the platform if it were to be used for directing jet engine thrust by being implemented into a possible vectoring nozzle assembly. There are currently various versions of vectoring nozzles attached to current engines and aircrafts, such as the American fighter jet F-22 raptor with a variable rectangular vectoring nozzle profile or the European typhoon fighter jet, which uses twin engines with a vectoring nozzle being controlled by four pneumatic actuators. Details of mathematical calculations of inverse and forward kinematics of a gough stewart platform, and simulations of various configurations are within the report, illustrating how the platform could be used to direct thrust. Certain characteristics of potential limitations, size, weight, materials and other qualities of such project are discussed to demonstrate the viability and application of the proposed idea. An example gough stewart platform with selected dimensions and positions are used to demonstrate the orientation a vectoring nozzle could be facing.

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1 Introduction

Throughout the 20th century, the advancement of the robotics industry has enabled it to spread throughout the world, where its influence has practically touched every single industry. Applications such as manufacturing, research, testing and recreational purposes have allowed the robotics industry to flourish. Industrial robotics have allowed manufacturers to create new products with such speed and efficiency far exceeding any physical manpower is capable of, and provides flexibility to the assembly line, where it can be reprogrammed to do different tasks. Robotics have enabled researchers to create large leaps in science. It enables researchers to carefully control tools, machines and environments to conduct experiments with high amounts of control, accuracy and precision. The robotics industry is slowly growing into recreational purposes such as video games and at home machinery such as 3d printing. The industrial robotics have become so widespread that in 2015, the industrial robotics industry alone spent \$51 billion in 2015, with projected estimates of \$135 billion in 2019.

An industrial robot is automated, programmable and capable of movement on two or more axes. These qualities allow movement where precision and strength far exceed anything a human could control or operate. They can be used to move in an infinite number of configurations within its feasible work space, unlike most automated systems, where they can only be used for specific tasks. These qualities can be altered and tailored for any given scenario, and are primarily dependant on their architectures, materials and actuators. Most robotic manipulators operate in within 3 to 6 degrees of freedom (DoF), where each DoF indicates the number of actuated joints. A 6 DoF provides the manipulator with the maximum amount of movement and can orientate itself in almost any position while limiting the number actuators required.

The most common robotic manipulators are in an open loop system. Open looped systems can provide a large range of movement in all directions and orientation, and generally have a larger workspace when minimizing its own dimensions. In comparison, closed looped systems have a much more limited range and use more actuators. Some widely used models of closed looped systems are delta manipulators and gough stewart platforms.

1.1 Gough Stewart Platform

A gough stewart platform is a type of planar parallel platforms. It differentiates itself from other planar platforms by using two platforms, six chains composed of a universal joint, actuated prismatic joints, and spherical joints. The prismatic joints have independent motion from each other, yielding a total of 6-DoF of motion (3 translations – 3 orientations). The strength of a robotic manipulator is referring to its ability to lift heavy loads. The stability of a robotic manipulator refers to its ability to resist displacement and vibrations due to outside forces and impacts. Both strength and sturdiness of any robotic manipulator can be determined by its material selection, structure of the manipulator, actuator torque and actuator power.

A certain advantage a gough-stewart platform has compared to other manipulators is its structure. It has a sturdy frame structure due to its use of three 4 bar link mechanisms to support the center of the moving platform. When compared to any other manipulator made with the same materials and actuators, a gough stewart platforms structure provides the system with more strength and stability.

The structure of a gough stewart platform acts similarly to that of a universal revolute joint, however it has a small limited range of linear motion. It allows for the center of the mobile platform to orientate itself in a numerous amount of directions from a numerous amount of points. However, its structure is also what limits the manipulators range of motion. It cannot orientate itself any further than 180 degrees without any of the prismatic actuators crossing through one another.

Current materials and prismatic actuators limit the size of gough stewart platforms, making it unsuitable for small scale applications, although not impossible. Since the architecture of the gough stewart platform moves a platform and is generally used for moving large and heavy objects.

1.2 Applications of Gough-Stewart Platforms

Gough-Stewart platforms have been used for multiple applications. The first ever patent was issued to Willard L.V. pollard in 1942 for designing an automatic spray painting. In 1954, Dr. Eric Gough made the first octahedral hexapod for tyre testing and after 11 years, Stewart published a paper to use a six degree of freedom platform as a flight simulator. Klaus Cappel also developed the octahedral hexapod manipulator for the purpose of motion simulator.

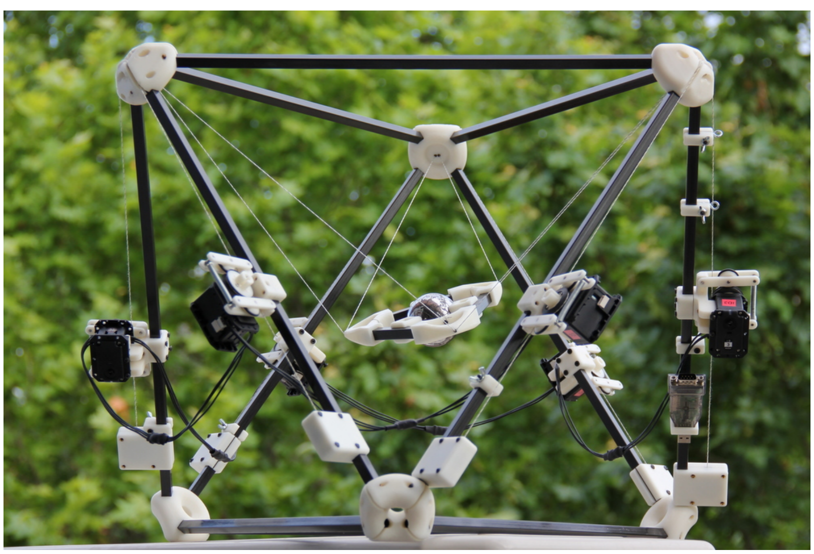

Figure 1: Cable Driven Platform

1. Cabe-Driven Platforms
   Cable-driven robots use cables in place of prismatic joints which helps to increase the workspace area. The cables are supported with motors to change the length and move the end-effector. These cable suspended manipulators can be used for cutting, shaping and finishing, Lifting and positioning. One of the examples is cranes that use it to move heavy load around the work area. Another example could be of cargo handling which is one the biggest area for the use of this system.

2. Space application
   The platform has been used in space to balance the joining area between two components to make them parallel to each other. This is one of the important applications which has made working in space easier where we don't have gravity and objects move with relatively less force. Another use is to move the satellite tracker. The tracker is a large dish which needs to move/rotate to track the satellite position.

3. Medical applications
   There have been many ideas for using the manipulators to work in areas requiring precision and accuracy. One of the areas is surgery as it is hard to control the movement of hands for longer hours which is where robots can be very useful. Another application includes prosthesis to restore functioning in patients with transhumeral amputation.

4. Other applications
   The use of this parallel manipulator has been increasing in different areas. These include window cleaning, surveillance, ship inspection, Oil tank inspection, bridge inspection etc. Welding robot has been used for processing different material in manufacturing. 3D printer, Claw machine, in drones for the movement of the motor in different directions are some of the other ways the platform is being used.

2 Task Description

A jet engine combines fuel and air to generate large amounts of thrust. Since the development of jet engines in the 1950s, most commercial and military airplanes use jet engines, as they can generate a greater amount of thrust and fuel efficient compared to propeller driven aircraft. Any modern aircraft built for speed will use either a jet engine or a rocket to generate its thrust. Outside of commercial aircrafts, jet engines are largely used for military purposes, particularly for fighter aircraft. The purpose of fighter aircraft is to combat enemy aircraft and defend its airspace, and are typically much faster, maneuverable and smaller than other types of aircraft. One method fighter aircrafts use to increase their maneuverability is by using directional thrust generated by the jet engine. This change in thrust direction is done by altering the exhaust path coming out of the end of the jet engine, which is done by using a mechanical system attached to the exhaust port called the thrust vectoring nozzle.

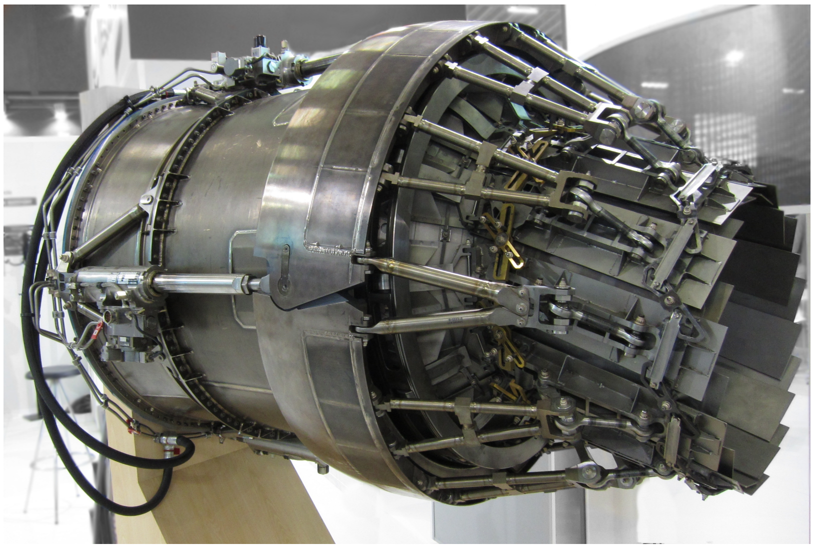

Figure 2: EJ200 Afterburner Vectoring Nozzle

There are many variations of a vectoring nozzle, however most are controlled using a set of pneumatic actuators that alter the direction and orientation of fins surrounding the exit port of the jet engine. For example, a Eurofighter typhoon fighter jet uses an EJ200 jet engine, which can alter the thrust direction by using four pneumatic actuators on a complex chain of panels surrounding the propelling nozzle. These panels can increase and decrease the outlet size and direct the thrust in a small cone shaped area. This orientation of the vectoring nozzle is similar to the reachable workspace of a gough stewart platform. For this project, a gough stewart platform will be used to design a new type of vectoring nozzle.

Figure 3: Fighter Jet Maneuvering

For the project simulation, the vectoring nozzle will simulate what would happen when the pilot decides to make a sharp 90 degree turn upwards, or in other words, bank straight up. The nozzle would change the direction by making the jet engine thrust point upwards, offsetting the balance of the jetfighter and rotating the fighter about its center of mass, which is presumably in front of the propelling nozzle. Once the plane is flying straight up into the sky, the nozzle would orientate itself back to its original position. Assuming the orientation of the moving platform is in the same direction and orientation as the vectoring nozzle, the moving platform will be oriented in the exact same manner as the desired directional thrust sequence.

3 CAD Design

The prismatic link lengths for the lower and upper joint is 1 meter each. Therefore, the prismatic joint length is from 1.05 to 2 meters. The base has a length of 1.5 metre from the centre to the vertex whereas the platform has a length of 1 metre. So the maximum height for the platform is 1.93 m high.

Joint	**Min**	**Max**
Base Joint θ_{1i}	-45 deg	45 deg
Base Joint θ_{2i}	45 deg	135 deg
Prismatic Joint d_i	1000 mm	2000 mm

Table 1: Joint Limits

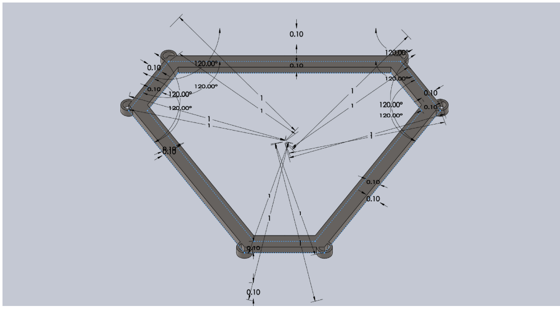

Figure 4: System Platform

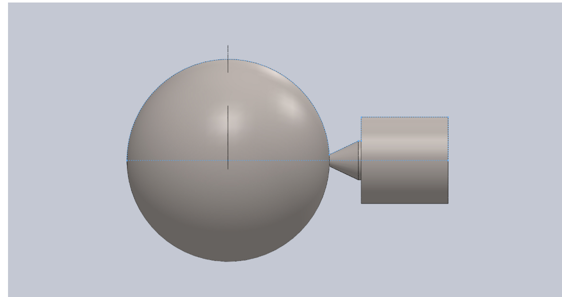

Figure 5: A platform joint

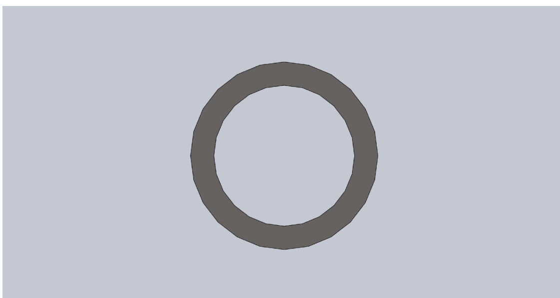

Figure 6: Top part of prismatic joint

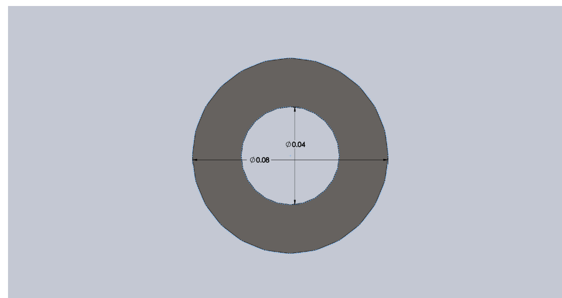

Figure 7: Bottom part of prismatic joint

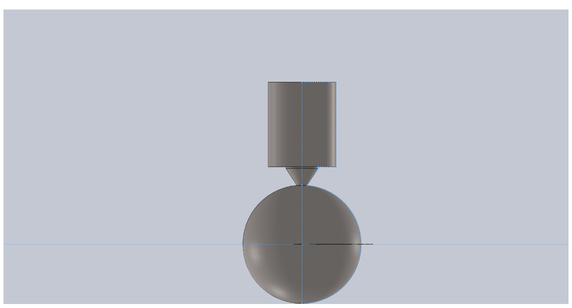

Figure 8: A base joint

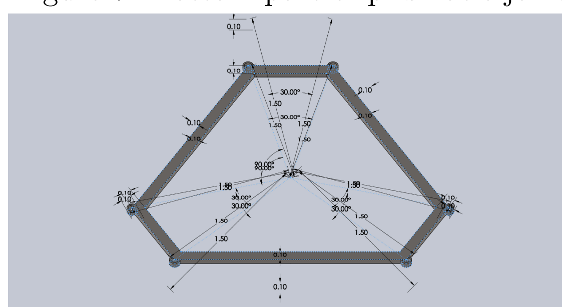

Figure 9: System Base

The Hall Bridge Mars actuator provided by Texas Hydraulic can be used to control the prismatic joint to control the platform.

Below specifies the weights of the parts in the platform.

Part	**Mass (kg)**
Base	190.56
Platform	125.23
Prismatic Joint	19.35
Base Universal Joint	1.09
Platform Spherical Joint	0.9
Total	**443.86**

Table 2: Mechanism part wieghts

The material that will be used is a titanium alloy because it needs to be strong and also need to prevent the heat from the jet engine. For approximation, the metal that is used is *Ti-5Al-2.5Sn Annealed (SS)'* which gives the mass of the structure to be around 443.86 Kg

4 Kinematics

4.1 Inverse Kinematics

For the inverse kinemeatics problem, loop representations was used to first find the link lengths for a given posture and then spherical coorindaeted for the angular displacement of the links [1].

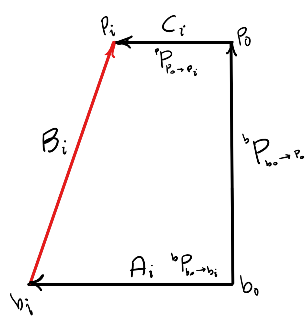

Figure 10: Inverse Kinematics Calculation Reference

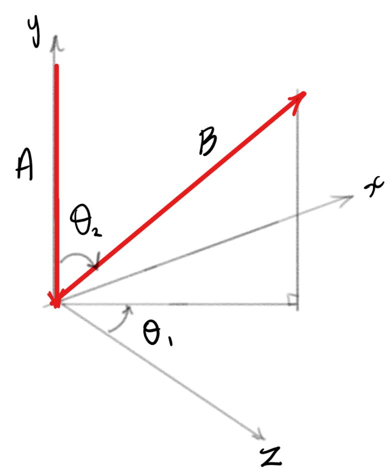

Figure 11: Forward Kinematics Calculation Reference

Loop equations

$$p_i = {}^bP_{b_0 \to p_0} + {}^bR_{p_0} \times {}^pP_{p_0 \to p_i}$$
$$p_i = {}^bP_{b_0 \to b_i} + B_i$$

Hence,

$$B_i = {}^bP_{b_0 \to p_0} + {}^bR_{p_0} \times {}^pP_{p_0 \to p_i} - {}^bP_{b_0 \to b_i}$$

Link length

$$d_i = |B_i|$$

For angular displacement, the spherical coordinate convention is used,

$$y = |B_i| cos(\theta_2)$$
$$z = |B_i| sin(\theta_2) cos(\theta_1)$$
$$x = |B_i| sin(\theta_2) sin(\theta_1)$$

$$c_2 = \frac{y}{|B_i|}$$
$$s_2 = \sqrt{1 - c_2^2}$$
$$c_1 = \frac{z}{|B_i| s_2}$$
$$s_1 = \frac{x}{|B_i| s_2}$$

Finding the angles;

$$\theta_1 = atan2(s_1, c_1)$$
$$\theta_2 = atan2(s_2, c_2)$$

4.2 Forward Kinematics

Forward Kinematics for a single branch

$$
\begin{aligned}
&\text{Given } \alpha,\, d_i,\, \theta_{1i},\, \theta_{2i}\\
&A_i = {}^{b}P_{b_0 \to b_i}\\
\\
&w = \alpha + 90\\
&B_i = \begin{bmatrix} d_i sin(\theta_{2i}) sin(\theta_{1i}) \\ d_i cos(\theta_{2i}) \\ d_i sin(\theta_{2i}) cos(\theta_{1i} \end{bmatrix}\\
&R_z = \begin{bmatrix} cos(w) & -sin(w) & 0 \\ sin(w) & cos(w) & 0 \\ 0 & 0 & 1 \end{bmatrix}\\
&B_0 = R_z B_i\\
\\
&x = |A_i| cos(\alpha) + B_{0x}\\
&y = |A_i| sin(\alpha) + B_{0y}\\
&z = B_{0z}
\end{aligned}
$$

Using the results from above the following platform can be assembled using MATLAB [2].

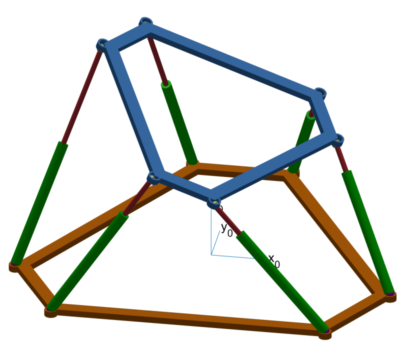

Figure 12: Matlab Assembled Platform

4.3 Reachable Workspace

Based on the joint limits specified in Section 3, and the Forward kinematics result for a single chain obtained in Section 4.2, the reachable workspace can be plotted as below:

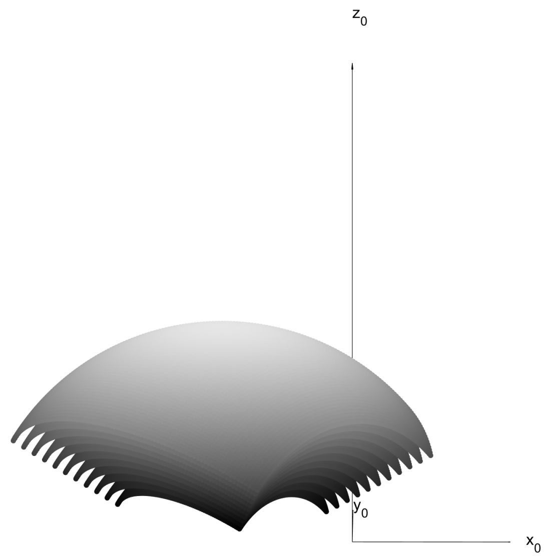

Figure 13: Reachable Workspace (Single Branch)

5 Trajectory Generation

The trajectory of the plaform is defined by moving the thrust direction vector 20-40 degrees towards the north. which is deifined to be the Y-Axis from the reference frame of the platform base.

The trajectory is made up of 10 points of 6 parametes which constrain the 6 degrees-of-freedom of the platform. The matrix can be written as:

$$P_{ee} = \begin{bmatrix} x_{ee} \\ y_{ee} \\ z_{ee} \\ \alpha_{ee} \\ \beta_{ee} \\ \gamma_{ee} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 300 & 650 & 900 & 1100 & 1100 & 900 & 650 & 300 & 0 \\ 1500 & 1500 & 1500 & 1500 & 1500 & 1500 & 1500 & 1500 & 1500 & 1500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & -10 & -15 & -20 & -20 & -15 & -10 & -5 & 0 \end{bmatrix}$$

Below are the plots of displacements, velocity and acceleration for the end effector.

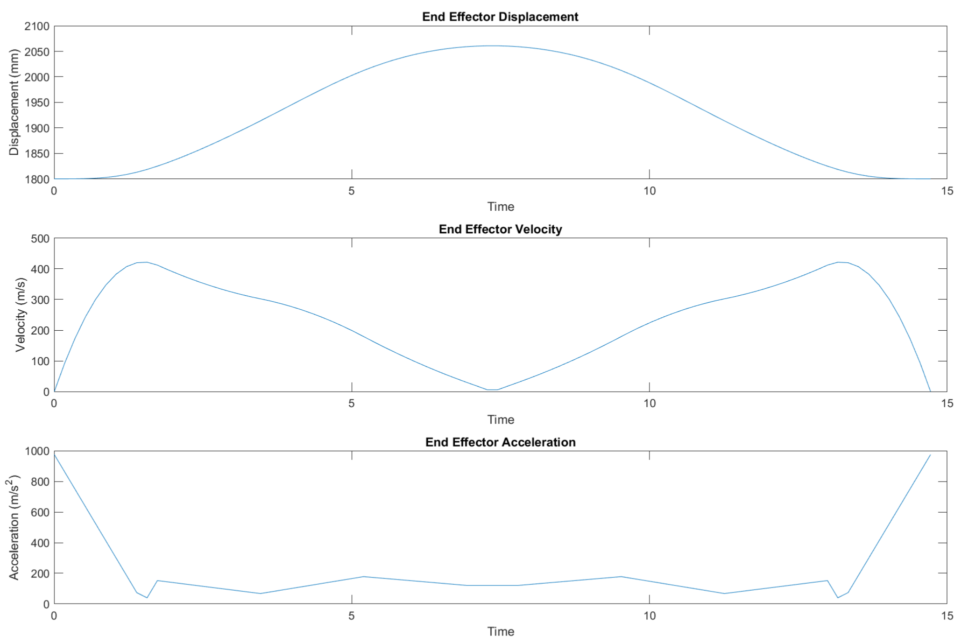

Figure 14: End Effector Plots

Joint displacements of the multiple links of the platform.

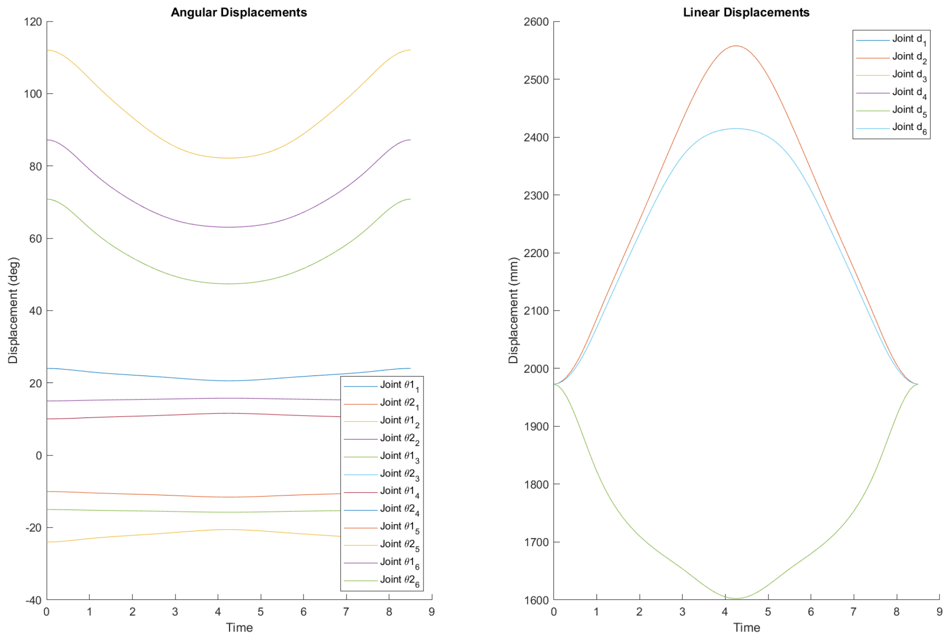

Figure 15: Joint Displacements

3D Plot of the end-effector's motion using the **comet3** [2] command.

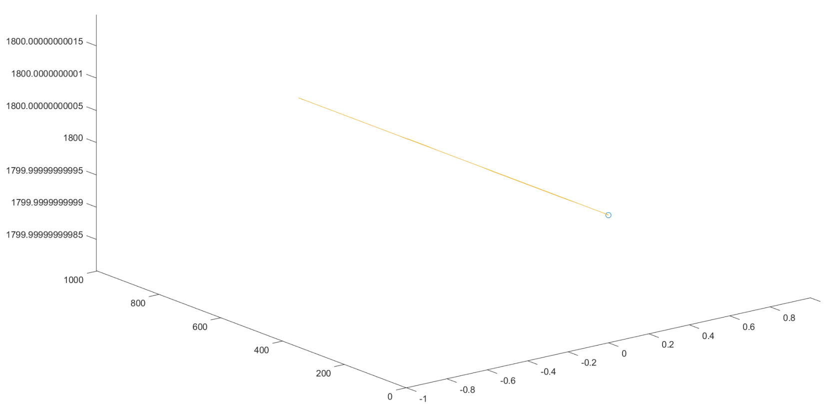

Figure 16: End Effector Motion

The joint velocities and the torques for each arm of the gough stewart platform is displayed below.

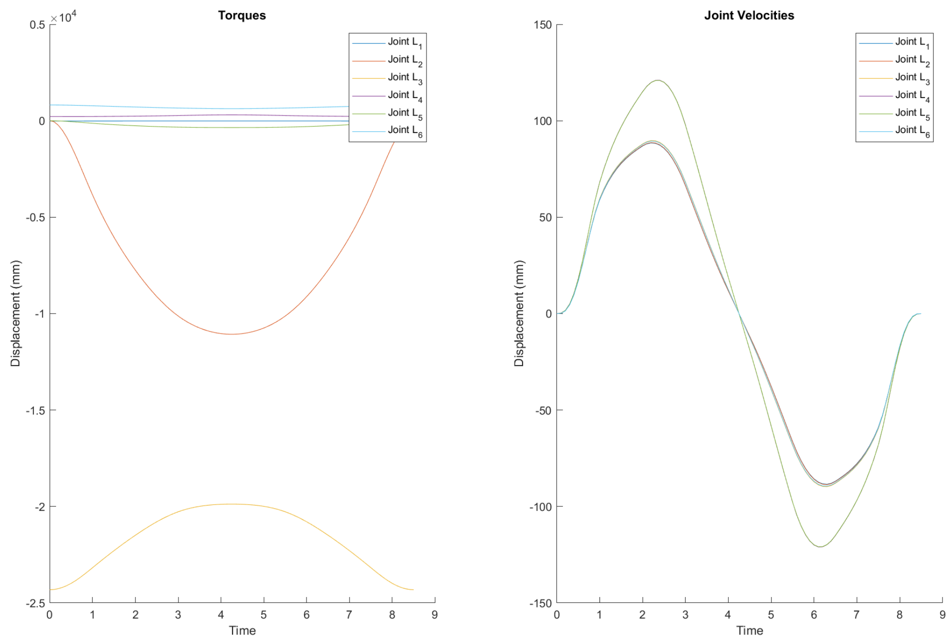

Figure 17: Torque and Joint Velocity plots

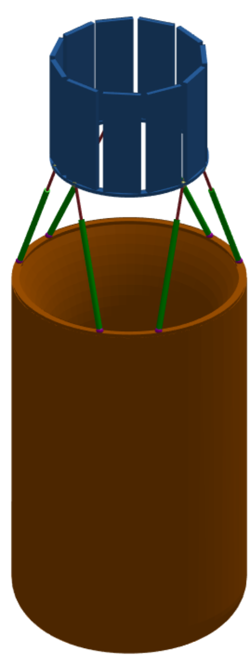

Figure 18: Designed Platform

6 Jacobians

A Jacobian is a matrix that gives us a relation between joint variables and position and the orientation of the moving platform. The jacobians of the manipulator are calculated by using a set of equations that illustrate a closed loop for each prismatic joint. The closed loop is taken from the center of the base and the platform to each prismatic joint. This is represented by the equation given below.

$$L_i = P_{b_o \to p_o} + P_{p_o \to p_i} - P_{b_o \to b_i}$$

To calculate the jacobian, we calculate J_x and J_q. They are calculated by taKiing a derivative of the equation of the square of the prismatic length which is derived by the equation above. J_x is calculated by taKiing the derivative of the equations by the changing variables i.e. the position and the orientation variables. J_q is calculated by taKiing the derivative w.r.t. The prismatic joint length.

After calculation J_q and J_x, we can finally calculate the Jacobian which is given by the equation below.

$$J = J_q^{-1} J_x$$

This Jacobian is very useful for the purpose of understanding the system. One of the main use is to calculate the singularity. A singularity is basically the positions which are not included in the workspace area. Following shows on how can we calculate the singularities with thee help of the Jacobian. All singularities occur when all prismatic joints lie on the same plane.

1. **Singularity type I**
   It occurs when J_q is singular, therefore that occurs when the prismatic joint lengths are all equal to zero. Another way to prove it is when $det(J_q) = 0$. Since, J_q is a diagonal matrix, the system does not have a Singularity type I. For the design illustrated in this report, this singularity will never occur, as the minimum lengths of the prismatic joints must be greater than 1.5m.

2. **Singularity type II**
   This occurs when J_x is singular. The only method to determine is when $det(J_x) = 0$.

7 Conclusion

This project report depicts the current and potential uses of a gough stewart platform and has proposed another use for the platform. The application of a gough stewart platform for a vectoring nozzle would be a possibility due to the inherent advantages and range its configuration offers and can perfectly mimic the desired thrust direction. It illustrates the mathematics involved, such as the forward and inverse kinematics of the platform, to position the proper configuration and how it can be used to model the actual vectoring nozzle. Other estimates such as possible actuators and material composition of the platform has also been proposed, given its certain situation for operating around a jet engine.

References

[1] J. W. Eaton, D. Bateman, S. Hauberg, and R. Wehbring, *GNU Octave version 4.2.1 manual: a high-level interactive language for numerical computations*, 2017. [Online]. Available: https://www.gnu.org/software/octave/doc/v4.2.1/

[2] MATLAB, *version 7.10.0 (R2017a)*. Natick, Massachusetts: The MathWorks Inc., 2017.

[3] F. Firmani, *MATLAB Starter Code Project Manual: MSE 429 Project Fall 2018 Starter code*, 2018. [Online]. Available: https://www.sfu.ca/mechatronics/people/faculty/

[4] J. van den Berg (https://math.stackexchange.com/users/91768/jur-van-den berg), “Calculate rotation matrix to align vector a to vector b in 3d?” Mathematics Stack Exchange, uRL:https://math.stackexchange.com/q/476311 (version: 2016-09-01). [Online]. Available: https://math.stackexchange.com/q/476311

Appendices

A MATLAB Codes

1. *main_project1.m*
   Main project file which collates all the solid part MAT files and creates a configuration of the platform [3].

2. *findrot.m*
   Finds a rotation matrix which aligns one vector to another [4].

3. *rotmat.m*
   This function creates a rotation matrix, based on the fixed angle representation. It takes in three inputs the α: rotation about z axis, β: rotation about the y-axis and γ: rotation about the z-axis.

4. *workspace_single_branch.m*
   Plotting the reachable workspace of a single branch of the platform using the limits specified in Section 3.

B Converted STL Stereolithography Files

1. *Platform* - The mobile platform.

2. *JointTop* - The joint connected to the platfrom.

3. *TubeTop* - The top part of the prismatic joint.

4. *TubeBottom* - The bottom part of the prismatic joint.

5. *JointBottom* - The joints connected to the base.

6. *Base* - The base of the system and fixed part.