## COQ878 – Química Quântica Computacional 2025/3

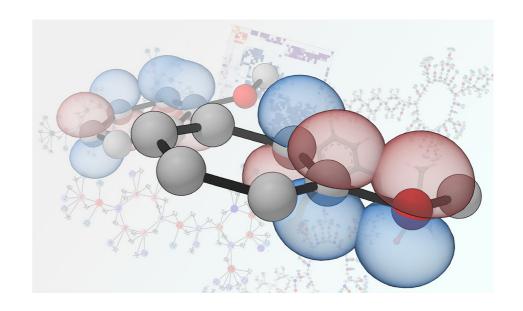




### Aula 02 – Espectro Molecular

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$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle$$
 
$$|\Psi\rangle = |\psi\rangle e^{-iEt/\hbar}$$
 
$$\hat{H} |\psi\rangle = E |\psi\rangle$$

# Aproximação de Born-Oppenheimer

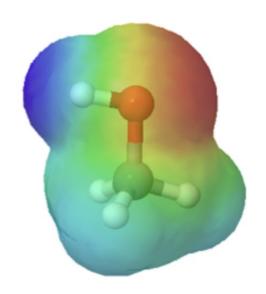
$$\Psi(\{\boldsymbol{r}_i\}, \{\boldsymbol{R}_A\}) = \sum_e \Psi_e(\{\boldsymbol{r}_i\}) \Phi_e(\{\boldsymbol{R}_A\})$$

- Dinâmica dos Elétrons

$$\hat{H}_e |\Psi_e\rangle = E_e |\Psi_e\rangle$$

- Dinâmica dos Núcleos Atômicos

$$\left[ -\frac{1}{2} \sum_{A=1}^{M} \frac{\nabla^2}{M_A} + E_e \right] |\Phi\rangle = E_T |\Phi\rangle$$

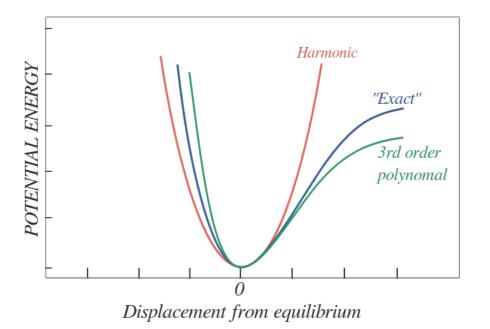


# Aproximação Harmônica

$$E_{e}[\{\mathbf{R}_{A}\}] = E_{e}[\{\mathbf{R}_{A}^{(0)}\}] + \sum_{A=1}^{M} \frac{\partial E_{e}}{\partial \mathbf{R}_{A}} \Big|_{\mathbf{R}_{A} = \mathbf{R}_{A}^{(0)}} (\mathbf{R}_{A} - \mathbf{R}_{A}^{(0)})$$

$$+ \frac{1}{2|} \sum_{A=1}^{M} \sum_{B=1}^{M} (\mathbf{R}_{A} - \mathbf{R}_{A}^{(0)}) \frac{\partial^{2} E_{e}}{\partial \mathbf{R}_{A} \partial \mathbf{R}_{B}} \Big|_{\mathbf{R}_{A} = \mathbf{R}_{A}^{(0)}, \mathbf{R}_{B} = \mathbf{R}_{B}^{(0)}} (\mathbf{R}_{B} - \mathbf{R}_{B}^{(0)})$$





### Hessiana

$$\frac{\partial^2 E_e}{\partial \boldsymbol{R}_A \partial \boldsymbol{R}_B} = \begin{pmatrix} \frac{\partial^2 E_e}{\partial \boldsymbol{R}_1 \partial \boldsymbol{R}_1} & \cdots & \frac{\partial^2 E_e}{\partial \boldsymbol{R}_1 \partial \boldsymbol{R}_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 E_e}{\partial \boldsymbol{R}_M \partial \boldsymbol{R}_1} & \cdots & \frac{\partial^2 E_e}{\partial \boldsymbol{R}_M \partial \boldsymbol{R}_M} \end{pmatrix}$$
3M

3M

H<sub>1</sub> C<sub>2</sub> C<sub>3</sub> H<sub>4</sub> C<sub>5</sub> H<sub>6</sub> C<sub>7</sub> H<sub>8</sub> C<sub>9</sub> H<sub>10</sub> C<sub>11</sub> H<sub>12</sub>

Septembroon polymers

H<sub>1</sub> C<sub>2</sub> C<sub>3</sub> H<sub>4</sub> C<sub>5</sub> H<sub>6</sub> C<sub>7</sub> H<sub>8</sub> C<sub>9</sub> H<sub>10</sub> C<sub>11</sub> H<sub>12</sub>

H<sub>1</sub> C<sub>2</sub> C<sub>3</sub> H<sub>4</sub> C<sub>5</sub> H<sub>6</sub> C<sub>7</sub> H<sub>8</sub> C<sub>9</sub> H<sub>10</sub> C<sub>11</sub> H<sub>12</sub>

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Flattened coordinates

## **Separando Modos**

#### Função de Onda Nuclear

$$|\Phi\rangle = |\Phi_t\rangle \otimes |\Phi_r\rangle \otimes |\Phi_v\rangle$$

#### Translação

$$\hat{\boldsymbol{P}}_{cm} \ket{\Phi_t} = \boldsymbol{P}_{cm} \ket{\Phi_t} \longrightarrow$$

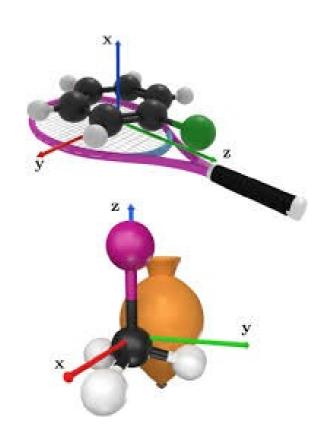
$$\hat{m{P}}_{cm}\ket{\Phi_t}=m{P}_{cm}\ket{\Phi_t} ~ \longrightarrow ~ egin{array}{c} \hat{m{P}}_{cm}^2 \ 2M_T\ket{\Phi_t}=E_t\ket{\Phi_t} \end{array}$$

#### Rotação

$$\hat{J}_z \left| \Phi_r \right\rangle = m_j \left| \Phi_r \right\rangle \qquad \longrightarrow \qquad$$

$$\hat{J}^2 |\Phi_r\rangle = j(j+1) |\Phi_r\rangle - - -$$

$$\frac{\hat{J}^2}{2I} |\Phi_r\rangle = E_r |\Phi_r\rangle$$



### **Modos Vibracionais**

3 modos de translação

3 modos de rotação (2 modos para moléculas lineares e 1 modo para molécula pontual)

- Coordenadas de vibração

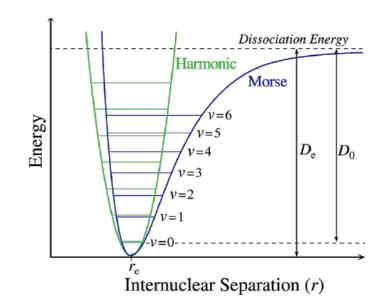
$$oldsymbol{Q}_lpha = \sum_{A=1}^M \lambda_{lpha A} (oldsymbol{R}_A - oldsymbol{R}_A^{(0)})$$

- Hamiltoniano de vibração

$$\left[-\frac{1}{2}\sum_{\alpha}^{3M-6}\frac{\nabla_{\alpha}^{2}}{\nabla_{\alpha}^{2}} + \frac{1}{2}k_{\alpha}Q_{\alpha}^{2}\right]|\Phi_{v}\rangle =$$

Restam 3M-6 modos vibracionais na Hessiana

(3M-5 para moléculas lineares e 3M-4 para molécula pontual)



$$\omega_{\alpha} = (k_{\alpha}/M_{\alpha})^{1/2}$$

$$\left[ -\frac{1}{2} \sum_{\alpha=1}^{3M-6} \frac{\nabla_{\alpha}^{2}}{M_{\alpha}} + \frac{1}{2} k_{\alpha} Q_{\alpha}^{2} \right] |\Phi_{v}\rangle = E_{v} |\Phi_{v}\rangle \longrightarrow E_{v} = \sum_{\alpha=1}^{3M-6} \hbar \omega_{\alpha} \left( n_{\alpha} + \frac{1}{2} \right)$$

### **Modos Vibracionais**

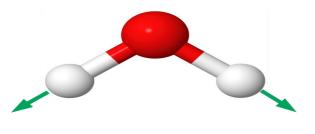
Symmetric stretch 1388 cm<sup>-1</sup> Antisymmetric stretch 2349 cm<sup>-1</sup>







## **Modos Vibracionais**



symmetric stretching



asymmetric stretching



bending

$$3652 \text{ cm}^{-1}$$

$$3756 \text{ cm}^{-1}$$

$$1595 \text{ cm}^{-1}$$