

VI.6. How many times must two dice be rolled to guarantee that the same total score is obtained at least three times?

### Solutions:

For Two dice: each die has 1–6  $\rightarrow$  total ranges from 2 to 12

List of possible totals:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

That's 11 possible sums in total.

We want some total to appear at **least 3 times**

Let  $n = \text{number of rolls}$ .

If we distribute  $n$  rolls among 11 totals, by the pigeonhole principle:

The maximum number of rolls without any total appearing 3 times is when each total appears at most 2 times.

That's:

$$11 \times 2 = 22 \text{ rolls.}$$

Thus, to guarantee at least one total appears 3 times, we need:

$$n = 22 + 1 = \mathbf{23 \text{ rolls.}}$$