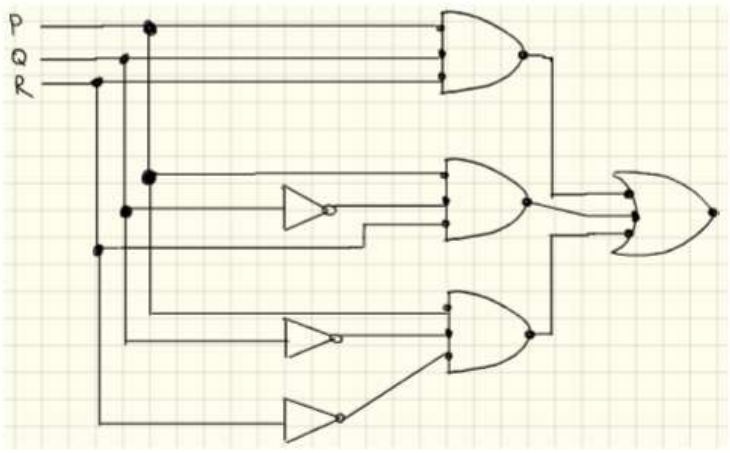
IX.2. For the circuit given below



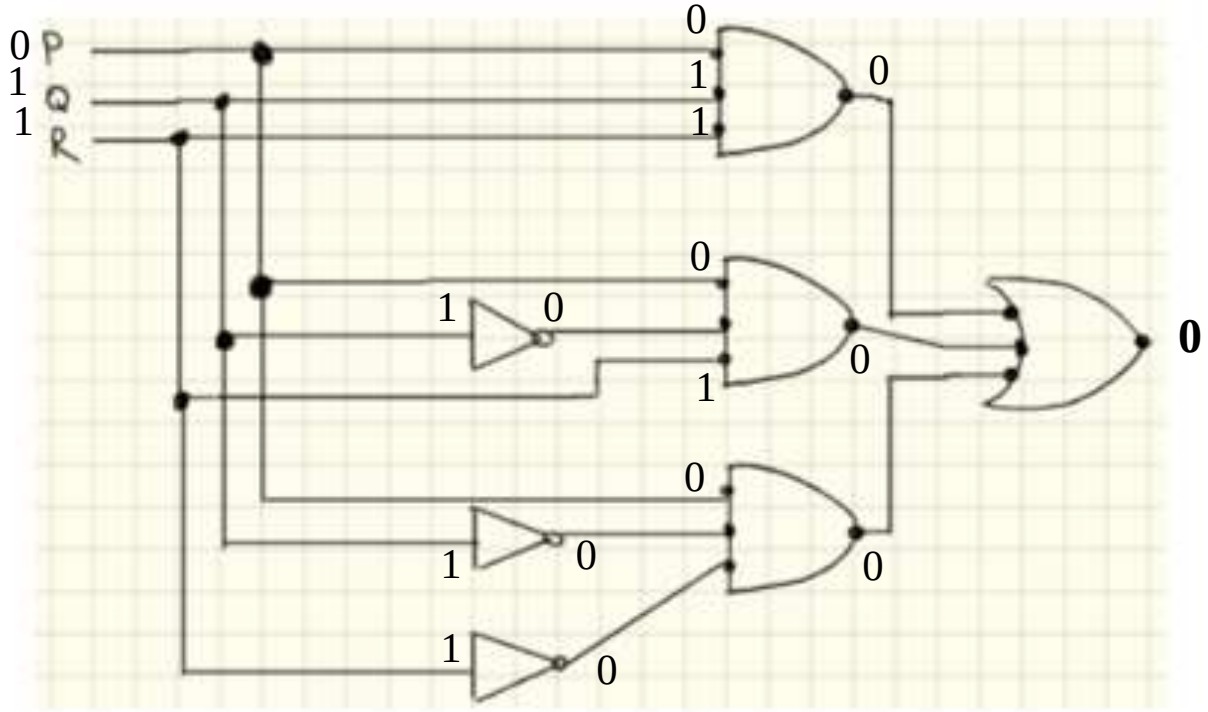
(a) What is the output signal for the input signals $P = 0, Q = 1, R = 1$?

(b) What logical formula in Disjunctive Normal Form (DNF) describes the behaviour of the circuit?

(c) Use equivalence laws to simplify the formula from item (b).

Solution:

(a):



(b):

$$(\boldsymbol{P} \wedge \boldsymbol{Q} \wedge \boldsymbol{R}) \vee (\boldsymbol{P} \wedge \neg \boldsymbol{Q} \wedge \boldsymbol{R}) \vee (\boldsymbol{P} \wedge \neg \boldsymbol{Q} \wedge \neg \boldsymbol{R})$$

(c):

$$(P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R)$$

$$= P \wedge \big((Q \wedge R) \vee (\neg Q \wedge R) \vee (\neg Q \wedge \neg R)\big)$$

$$= P \wedge \Big((Q \wedge R) \vee \big(\neg Q \wedge \underbrace{(R \vee \neg R)}_{True}\big)\Big)$$

$$= P \wedge \big((Q \wedge R) \vee \underbrace{(\neg Q \wedge T)}_{\neg Q}\big)$$

$$= P \wedge \big((Q \wedge R) \vee \neg Q\big)$$

$$= P \wedge \big(\underbrace{(\neg Q \vee Q)}_{True} \wedge (\neg Q \vee R)\big)$$

$$= P \wedge \big(T \wedge (\neg Q \vee R)\big)$$

$$= \boldsymbol{P} \wedge (\neg \boldsymbol{Q} \vee \boldsymbol{R})$$