II.4. Prove by contradiction that for any integer n if n^2 is odd then n is odd.

Let the statement be $\boldsymbol{P}$

<u>Proof by Contradiction:</u>

Assume $\neg P$ is true; this is:

n^2 is odd and $\boldsymbol{n}$ **is even**.

Since n is even, by definition of even numbers we have:

$$\exists k : n = 2k \quad , \quad k \in \mathbb{Z}$$

Then,

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$$

Let $x = 2k^2$

Since $k \in \mathbb{Z}$, it follows that $x \in \mathbb{Z}$

Hence,

$n^2 = 2x$, which shows that n^2 is even.

This contradicts our assumption $\neg P \rightarrow True$ (n^2 is odd)

Therefore, our assumption $(\neg P \rightarrow T)$ must be false, and the statement P must be true.

Thus, **if $\boldsymbol{n^2}$ is odd, then $\boldsymbol{n}$ is odd.** ■