Surgical Iatromathic Model for Procedures (SIM-P)

Master Formalism, Normative Definitions, and Operational Guidelines

SIM Project Core

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Version	Date	Changelog / Notes
1.0.0	2023-01-10	Initial Release (Frozen Normative Definition).
1.1.0	2023-06-15	Integration of Layer E (PCP Bridge).
1.2.0	**January 23, 2026**	**Refinement of SemVer auditing and visual consistency.**

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1 Introduction

Surgery can be understood, in its most general form, as a **highly structured technical process**, composed of a finite sequence of elementary actions executed upon a complex, variable, and not completely controllable biological system.

Unlike other technical processes, the surgical process unfolds in an environment where **uncertainty, irreversibility, and the coupling between local decisions and global consequences** are inherent. A minimal action executed poorly can compromise an entire step, and a deficient step can alter the course of the entire intervention, potentially manifesting effects in later phases or during the postoperative period.

From an operational standpoint, a surgical procedure is organized into **steps** and **sub-actions** (*surgits*), some strictly sequential and others potentially interchangeable. Additionally, there exist actions specifically designed to **mitigate risks** (security surgits or steps), external pauses that interrupt flow without modifying intrinsic risk, and structural modifications mandated by conditions external to the surgeon.

The process is further affected by uncontrollable factors, such as patient characteristics or events external to the procedure, which amplify the probability of deviation even when the technique is correct. These properties render the surgical process irreducible to a sum of independent events or a simple linear sequence of tasks.

Translating these characteristics into mathematical language requires recognizing that the surgical process possesses **non-compensable**, **non-extensive**, and **history-dependent** properties. Elementary actions behave as ideal binary events (performed or not performed) but carry an inherent probability of deviation. The correct execution of a step depends on the joint fulfillment of all its constituent actions, while deviation can accumulate and propagate non-locally, affecting non-consecutive steps.

Furthermore, the normative structure of the procedure defines which actions are possible, in what order, and under what security conditions, while the clinical state of the system evolves dynamically as the intervention progresses, retaining a memory of prior events.

In this work, we propose the **Surgical Iatromathic Model (SIM)**, a formal framework that models the surgical process as a normative, dynamic, and non-extensive system. The SIM integrates a logical structure defining permitted actions (Petri Nets), an expanded state dynamics capturing clinical evolution and causal memory (Markovian State), and a global non-extensive metric (Tsallis Entropy) quantifying the fragility and risk of the procedure.

Glossary of Symbols

General Indices and Sets

t	Index of current surgical step ($t \in \{1, \dots, T\}$).
s	Index of surgit (elementary surgical action).
k	Index of a **past** surgical step (used in provenance vectors).
$\mathcal{S}_t$	Set of surgits composing step t.
T	Total number of surgical steps in the procedure.

Surgit-Level Probabilities (Layer A)

$\pi_{t,s}$	Planned (ideal) probability of correct execution of surgit s in step t.
$\delta_{t,s}$	Probability of deviation of surgit s in step t, with $\pi_{t,s} + \delta_{t,s} = 1$.
$\delta_{t,s}^{(\mathrm{intr})}$	Intrinsic deviation probability (procedure-related).
$\delta_{t,s}^{(\mathrm{pat})}$	Deviation amplification due to patient-related noise.
$\delta_{t,s}^{(\mathrm{ext})}$	Deviation amplification due to external noise.
$\delta_{t,s}^{(\mathrm{tot})}$	Total deviation probability before security mitigation.
$\delta_{t,s}^{(\mathrm{final})}$	Final deviation probability after security mitigation.

Noise and Context Modifiers

n_t	Patient-related noise factor affecting step t ($n_t \geq 1$).
e_t	External noise factor affecting step t ($e_t \geq 1$).
ϕ_t	External pause indicator (neutral event).

Security and Mitigation Operators (Layer A')

$\sigma_{t,s}$	Security factor (mitigation strength) applied to surgit s, where $0 < \sigma \leq 1$.
$\mathbb{S}_{t,s}$	The security operator acting on the deviation.
$\mathrm{Scope}(\mathbb{S})$	Defines if effect is immediate (imm), residual (res), or postoperative (pcp).

Step-Level and Global Metrics

π_t	Probability of perfect linear execution of step t (all surgits correct).
δ_t	Total deviation probability of step t.
$S_q(t)$	Tsallis entropy of step t (structural fragility).
$\mathrm{Score}_{\mathrm{SIM}}$	Global Surgical Iatromathic Score.

Structure and Dynamics (Layers B & C)

$\mathcal{N}$	Petri Net representing the normative structure.
$\mathbf{Z}_t$	Expanded Global State vector at step t.
$\mathbf{H}_t$	Provenance vector (causal memory of past deviations).
$\mathbf{X}_t$	Clinical degradation state vector.

SIM–PCP Bridge (Layer E)

$\mathcal{K}$	Set of postoperative complication types (PCP types), indexed by k.
$P_k(\mathrm{PCP} \mid \mathrm{SIM})$	Predicted probability of postoperative complication type k given SIM metrics.
$\mathrm{logit}(p)$	Log-odds transform, $\mathrm{logit}(p) = \ln\left(\frac{p}{1-p}\right)$.
$\sigma(\eta)$	Logistic inverse link, $\sigma(\eta) = \frac{1}{1+e^{-\eta}}$.
η_k	Linear predictor (log-odds) for complication type k.
α_k	Intercept (baseline log-odds) for complication type k.
$\beta_k^{(\Delta)}$	Coefficient linking global deviation metric Δ_{SIM} to PCP log-odds (type k).
$\beta_k^{(S)}$	Coefficient linking global entropy $S_q(\mathrm{SIM})$ to PCP log-odds (type k).
$\gamma_{k,t}$	Step-level coefficient linking δ_t to PCP log-odds for type k.
$\xi_{k,t}$	Step-level coefficient linking ρ_t to PCP log-odds for type k.

Δ_{SIM}	Global deviation aggregate used in the bridge (defined by the procedure template).
$v_k(\mathcal{U})$	Value function for Shapley attribution of PCP k using step subset $\mathcal{U} \subseteq \{1, \dots, T\}$.
$\phi_{k,t}$	Shapley contribution of step t to complication type k (PCP attribution).
$\Delta\eta_k$	Difference in log-odds between two techniques for complication k.

2 Layer A: Primitive (Axiomatic) Definitions

A1. Surgit as an Ideal Binary Action

A surgit s belonging to surgical step t is defined as an ideal elementary surgical action required in the normative model.

$$\pi_{t,s} + \delta_{t,s} = 1 \tag{1}$$

A2. Intrinsic Deviation

Each surgit possesses an intrinsic deviation probability associated with its technical nature:

$$\delta_{t,s}^{(\mathrm{intr})} \in [0, 1] \tag{2}$$

A3. Patient-Related Noise

Patient-related noise is a multiplicative amplification factor $n_t \geq 1$. The amplified deviation is defined by the saturation model:

$$\delta_{t,s}^{(\mathrm{pat})} = 1 - \left(1 - \delta_{t,s}^{(\mathrm{intr})}\right)^{n_t} \tag{3}$$

A4. External Noise

External noise $e_t \geq 1$ acts on the patient-amplified deviation:

$$\delta_{t,s}^{(\mathrm{ext})} = 1 - \left(1 - \delta_{t,s}^{(\mathrm{pat})}\right)^{e_t} \tag{4}$$

A5. Total Surgit Deviation (Pre-Mitigation)

The total deviation probability aggregating all noise sources is:

$$\delta_{t,s}^{(\mathrm{tot})} = 1 - \left(1 - \delta_{t,s}^{(\mathrm{intr})}\right)^{n_t\, e_t} \tag{5}$$

A6. Security Surgit (Mitigation)

A security surgit is characterized by a mitigation factor $\sigma_{t,s} \in (0,1]$. The **final deviation** is:

$$\delta_{t,s}^{(\text{final})} = \sigma_{t,s} \cdot \delta_{t,s}^{(\text{tot})} \tag{6}$$

A7. Surgit by External Cause

A surgit executed due to an external cause is denoted $s^{(\text{CE})}$. It obeys the same probabilistic structure as any other surgit but carries a categorical label for auditing:

$$\pi_{t,s^{(\text{CE})}} + \delta_{t,s^{(\text{CE})}} = 1 \tag{7}$$

A8. External Pause

An external pause ϕ_t is a neutral categorical event occurring during step t:

$$\phi_t : \quad \Delta t > 0, \quad \Delta\delta = 0 \tag{8}$$

A9. Non-Compensability

Within a step, surgits are non-compensable. The step is linear (ideal) only if all required surgits are ideal:

$$\text{Step } t \text{ is linear} \iff \forall s \in \mathcal{S}_t^{\text{req}},\ \delta_{t,s}^{(\text{final})} = 0 \tag{9}$$

A10. Normative Ideal Constraint

In the ideal surgical model, all surgits and steps are assumed to be executed. Deviation represents departure from ideal execution, not omission.

3 Layer A': Axiomatic Security Operators

A'1. The Security Operator

A security operator $\mathbb{S}$ acts multiplicatively on deviation probabilities:

$$\mathbb{S}_{t,s} : \delta \mapsto \sigma_{t,s}\,\delta, \quad \sigma_{t,s} \in (0,1] \tag{10}$$

A'2. Immediate Security Effect

An immediate security effect mitigates the deviation probability of the current surgit:

$$\delta_{t,s}^{(\text{imm})} = \sigma_{t,s}^{(\text{imm})}\,\delta_{t,s}^{(\text{tot})} \tag{11}$$

A'3. Residual Security Effect

A residual security effect mitigates deviation of subsequent surgits (forward scope):

$$\delta_{t',s'} \;\mapsto\; \sigma^{(\mathrm{res})}_{t,s\to t',s'}\,\delta_{t',s'}, \quad t' \geq t \tag{12}$$

A'4. Security Scope

Each security operator is characterized by its scope:

$$\mathrm{Scope}(\mathbb{S}_{t,s}) \in \{\mathrm{imm},\ \mathrm{res},\ \mathrm{pcp}\} \tag{13}$$

A'5. PCP Operator

A Postoperative Complication Prevention (PCP) operator acts exclusively on postoperative risk variables:

$$\rho \mapsto \sigma^{(\mathrm{pcp})}_{t,s}\,\rho \tag{14}$$

A'6. Independence

Security operators are independent of noise sources: $\partial\sigma/\partial n = 0$.

A'7. Non-Compensatory Nature

Security reduces probabilistic risk but does not compensate for structural violations:

$$\exists \mathbb{S}_{t,s} \;\not\Rightarrow\; \text{Structural Correctness} \tag{15}$$

A'8. Composition

Multiple security operators applied to the same target compose multiplicatively, ensuring monotonic risk reduction.

A'9. Separation of Variables

Intraoperative deviation and postoperative risk are distinct variables:

$$\delta \;\perp\!\!\!\perp\; \rho \tag{16}$$

A'10. Normative Interpretation

Security operators represent intentional design features of the ideal procedure. Their omission increases effective deviation.

4 Layer B: Normative Structural Layer

B1. Petri Net Definition

The surgical process is modeled as a Petri net $\mathcal{N} = (\mathcal{P}, \mathcal{T}, \mathcal{F})$.

B2. Mapping

$$\begin{gathered} \text{surgit } s \longleftrightarrow \text{transition } tr_s \in \mathcal{T} \\ \text{step } t \longleftrightarrow \text{subnet } \mathcal{N}_t \subset \mathcal{N} \end{gathered} \tag{17}$$

B3. Token Semantics

Tokens traversing the net represent the patient: Token = (ID, context).

B4. Sequentiality and Concurrency

Sequentiality is encoded by flow ($s_i \prec s_j$). Non-sequential surgits are represented by concurrent branches.

B5. Step Boundaries

Each step t is a subnet $\mathcal{N}_t$ with well-defined entry and exit places.

B6. Structural Non-Compensability

All required transitions in a subnet must fire for the step to be valid. There are no bypass paths for mandatory surgits:

$$\forall s \in \mathcal{S}_t^{\text{req}}, \quad tr_s \text{ is structurally mandatory} \tag{18}$$

B7. Security Transitions

Security surgits are explicit transitions $\mathcal{T}^{(\text{sec})} \subset \mathcal{T}$ that apply the factor σ.

B8. Residual Scope

Residual scope is encoded by placing security transitions upstream of multiple downstream targets.

B9. External Cause Transitions

Transitions due to external causes ($tr^{(\mathrm{CE})}$) branch from the ideal path but must converge to preserve validity.

B10. External Pause Places

Pauses are time-annotated places $p^{(\phi)}$ that introduce delay but do not affect reachability.

B11. Structural Validity

A procedure is structurally valid if and only if there exists a valid firing sequence w from start to finish:

$$\exists\, w \text{ such that } M_0 \xrightarrow{w} M_f \tag{19}$$

B12. Forbidden States

Markings reachable only by skipping required transitions are defined as $\mathcal{M}_{\text{forbidden}}$.

B13. Interpretation

Layer B defines the skeleton of admissible executions.

5 Layer C: Expanded Global State Dynamics

C1. Expanded Global State

The state at step t is a composite vector $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \big(\mathbf{X}_t,\ \mathbf{H}_t\big) \tag{20}$$

where $\mathbf{X}_t$ is the clinical degradation vector and $\mathbf{H}_t$ is the provenance vector.

C2. Clinical Degradation Component

$\mathbf{X}_t \in \mathcal{X}$ encodes the accumulated intraoperative burden (e.g., edema, fragility).

C3. Provenance Vector (Causal Memory)

$\mathbf{H}_t$ stores the residual influence of past steps $k \leq t$:

$$\mathbf{H}_t = (h_{1,t}, h_{2,t}, \ldots, h_{t,t}) \tag{21}$$

C4. Initialization

$\mathbf{Z}_0$ is initialized as $\mathbf{X}_0 = \mathbf{0}$ and $\mathbf{H}_0 = \emptyset$.

C5. Transition Kernel

The system evolves according to a Markovian kernel in the expanded space:

$$P(\mathbf{Z}_{t+1} \mid \mathbf{Z}_t) \tag{22}$$

C6. Clinical State Update

$\mathbf{X}_{t+1} = f(\mathbf{X}_t, \delta_t^{(\text{final})}, n_t, e_t)$.

C7. Provenance Update Rule

The provenance vector updates recursively:

$$h_{k,t+1} = \begin{cases} g(h_{k,t}) & k \leq t \quad \text{(decay of past events)} \\ \delta_{t+1}^{(\text{final})} & k = t+1 \quad \text{(new event)} \end{cases} \tag{23}$$

C8. Delayed Causality

Non-zero terms in $\mathbf{H}_t$ for $k < t$ explain delayed complications.

C9. Semi-Markov Extension

The kernel may include dwell times: $P(\mathbf{Z}_{t+1}, \tau_t \mid \mathbf{Z}_t)$.

C10. Independence

Dynamics do not alter structural admissibility (Layer B).

C11. Interpretation

Layer C provides the mathematical basis for causal traceability.

6 Layer D: Global Metric and SIM Score

D1. Step Linearity

The probability that step t is executed linearly (ideal) uses the final mitigated probabilities:

$$\pi_t = \prod_{s \in \mathcal{S}_t} \pi_{t,s}^{(\text{final})} \tag{24}$$

D2. Step Deviation

$\delta_t = 1 - \pi_t$.

D3. Tsallis Entropy (Fragility)

$$S_q(t) = \frac{1}{q-1} \left[1 - (\pi_t^q + \delta_t^q)\right] \tag{25}$$

D4. Maximum Entropy

Occurs at $\pi_t = \delta_t = 1/2$.

D5. Global Entropy (q-sum)

$$S_q(\text{SIM}) = S_q(1) \oplus_q S_q(2) \oplus_q \cdots \oplus_q S_q(T) \tag{26}$$

where $x \oplus_q y = x + y + (1-q)xy$.

D6. Risk Aggregation

$\rho_{\text{SIM}} = \sum w_t \rho_t$.

D7. Global SIM Score

$$\text{Score}_{\text{SIM}} = \alpha\, \rho_{\text{SIM}} + \beta\, S_q(\text{SIM}) \tag{27}$$

D8. Interpretation

Lower scores indicate higher robustness.

D9. Decomposition Function

$F(S) = \mathrm{Score}_{\mathrm{SIM}}(S)$ for a subset of deviation causes S.

D10. Shapley Value Decomposition

The contribution of deviation cause $i \in \mathcal{C}$ is:

$$\Phi_i = \sum_{S \subseteq \mathcal{C} \setminus \{i\}} \frac{|S|!\,(|\mathcal{C}| - |S| - 1)!}{|\mathcal{C}|!} \left[F(S \cup \{i\}) - F(S)\right] \tag{28}$$

D11. Guarantee

$\sum \Phi_i = \Delta\mathrm{Score}$.

D12. Normative Interpretation

Layer D provides an auditable, non-extensive quantitative evaluation.

7 Layer E: SIM–PCP Bridge (Logit)

E1. Logit Operator (D-Bridge 1)

$$\mathrm{logit}(p) \;:=\; \ln\!\left(\frac{p}{1-p}\right), \qquad p \in (0,1). \tag{29}$$

E2. Logistic Inverse Link (D-Bridge 2)

$$\sigma(\eta) \;:=\; \frac{1}{1+e^{-\eta}}. \tag{30}$$

E3. PCP Probability Model (D-Bridge 3)

Let $k \in \mathcal{K}$ denote a postoperative complication type (PCP type). We define the SIM-to-PCP mapping by:

$$P_k(\mathrm{PCP} \mid \mathrm{SIM}) = \sigma(\eta_k). \tag{31}$$

E4. Linear Predictor (D-Bridge 4)

The log-odds predictor η_k is defined as:

$$\eta_k = \alpha_k + \beta_k^{(\Delta)} \, \Delta_{\mathrm{SIM}} + \beta_k^{(S)} \, S_q(\mathrm{SIM}) + \sum_{t=1}^{T} \gamma_{k,t} \, \delta_t + \sum_{t=1}^{T} \xi_{k,t} \, \rho_t. \tag{32}$$

E5. Two-Technique Log-Odds Difference (D-Bridge 5)

For two techniques (A and B) with estimated or observed PCP rates $P_{k,A}$ and $P_{k,B}$:

$$\Delta\eta_k = \operatorname{logit}(P_{k,B}) - \operatorname{logit}(P_{k,A}), \tag{33}$$

and, under the bridge model,

$$\begin{aligned} \Delta\eta_k \approx \beta_k^{(\Delta)} \big(\Delta_{\mathrm{SIM},B} - \Delta_{\mathrm{SIM},A}\big) + \beta_k^{(S)} \big(S_q(\mathrm{SIM})_B - S_q(\mathrm{SIM})_A\big) \\ + \sum_{t=1}^{T} \gamma_{k,t} \big(\delta_{t,B} - \delta_{t,A}\big) + \sum_{t=1}^{T} \xi_{k,t} \big(\rho_{t,B} - \rho_{t,A}\big). \end{aligned} \tag{34}$$

E6. Step-Level PCP Attribution (Shapley) (D-Bridge 6)

Define a value function $v_k(\mathcal{U})$ for subsets of steps $\mathcal{U} \subseteq \{1, \ldots, T\}$:

$$v_k(\mathcal{U}) := P_k(\mathrm{PCP} \mid \mathrm{SIM\ using\ only\ step\ variables\ from\ } \mathcal{U}). \tag{35}$$

Then the Shapley contribution of step t to PCP type k is:

$$\phi_{k,t} = \sum_{\mathcal{U} \subseteq \{1,\ldots,T\} \setminus \{t\}} \frac{|\mathcal{U}|! \, (T - |\mathcal{U}| - 1)!}{T!} \Big(v_k(\mathcal{U} \cup \{t\}) - v_k(\mathcal{U}) \Big). \tag{36}$$

E7. Interpretation

Layer E is a probabilistic bridge: it does not replace Layers A–D, but translates non-extensive structural and risk metrics into interpretable PCP probabilities and step-level attribution.

Annex I: SIM Implementation Guide (Operational)

A.I.1 Normative Template Initialization (Procedure-Type Level)

For each surgical procedure type, a single normative SIM template must be defined *a priori*. The template shall include:

- The complete set of steps and surgits, including mandatory and safety surgits (Layers A, A').
- The normative structural model as a (Coloured) Petri net (Layer B), including forbidden states and mandatory transitions.
- The expanded-state dynamics specification (Layer C): state components and update/decay functions for provenance.
- The global scoring specification (Layer D): parameter q, weights w_t, and score weights (e.g., α, β), plus the Shapley characteristic-function convention $F(S)$.
- The SIM–PCP bridge specification (Layer E): complication set $\mathcal{K}$, calibration coefficients $(\alpha_k, \beta_k^{(\Delta)}, \beta_k^{(S)}, \gamma_{k,t}, \xi_{k,t})$, and the PCP Shapley convention $v_k(\mathcal{U})$.

A.I.2 Model Freezing Rule (Anti Overfitting / Audit Integrity)

Once published for a given procedure type, the SIM template is **frozen**. It may not be modified to fit a specific audited case or outcome. Updates are permitted only as **versioned releases** at the procedure-type level, with explicit change logs (e.g., SIM–Fundoplication v1.1).

A.I.3 Allowed Case-Specific Instantiation (What May Vary per Surgery)

For each audited real case, the following may vary *without modifying the normative template*:

- Patient noise factors n_t (per step or per phase, as defined by the procedure-type template).
- External noise factors e_t (per step).
- Occurrence and duration of external pauses, recorded by start/end markers.
- Activation of external-cause substitutions or additions, labeled as CE (see A.II.4).
- Observed deviations at the surgit level and their cause attribution (see Annex II).

A.I.4 Structural Validity Check (Before Scoring)

Before any scoring, the audited trace must pass structural validation:

- Petri-net reachability from M_0 to M_f,
- No forbidden markings,
- No bypass of mandatory transitions (normative surgits).

If structural validity fails, the case is flagged as **normatively invalid** and reported separately.

Annex II: Data Acquisition Template for Real Surgeries

A.II.1 Minimum Required Record (Event Log)

Each audited case must provide a timestamped event log with the following minimum fields:

1. Procedure type identifier and SIM version (frozen template reference).
2. Step ID t and surgit ID s (e.g., P7S2), aligned to the normative template.
3. Event timestamp (start and end) for each surgit execution.
4. Indicator whether the surgit is: normal, safety, or CE-labeled.
5. Deviation occurrence indicator at surgit level (binary: deviated / not deviated).
6. Postoperative-risk tags if applicable (SCP/PCP relevance in the template).

A.II.1b Postoperative Outcomes (Required for Calibration and Validation)

For each audited case (when available), record:

- Occurrence (yes/no) of each complication type $k \in \mathcal{K}$.
- Time window for outcome ascertainment (e.g., intraoperative, 30-day, 90-day).
- Severity grade if applicable (e.g., Clavien–Dindo), allowing stratified calibration.

A.II.2 External Pauses (Time-Neutral for Risk)

External pauses must be recorded as paired markers:

$$\phi_{t,k}^{(\text{start})}, \quad \phi_{t,k}^{(\text{end})} \tag{37}$$

with measured duration $\tau(\phi_{t,k})$. Pauses are time-only and must not alter deviation, robustness, or postoperative risk.

A.II.3 Noise Capture (Patient and External)

Noise values must be recorded (at least) at step resolution:

- Patient noise n_t (e.g., fibrosis, adhesions burden, friability proxies).
- External noise e_t (e.g., device malfunction, anesthesia instability, missing supplies).

If noise is absent for a step, record $n_t = 1$ and/or $e_t = 1$ explicitly.

A.II.4 External Cause (CE) Labeling: Substitution vs Addition

External cause (CE) is a **categorical label** indicating that a surgit/step was substituted or added due to constraints external to the surgeon (e.g., missing device, forced alternative technique). CE labeling has two forms:

- **CE-Substitution:** a normative surgit is replaced by an alternative surgit labeled CE, without implying surgeon decision deviation.
- **CE-Addition:** additional surgits/steps are introduced and labeled CE (e.g., suturing instead of stapling), increasing complexity but not automatically implying higher postoperative risk.

All CE surgits still have their own planned probability and deviation probability under the SIM formalism; CE is used for audit interpretability.

A.II.5 Deviation Cause Attribution (For Explainability and Shapley)

For each observed deviation, exactly one cause must be assigned from:

- Intrinsic procedural (intr),
- Patient-related noise (pat),
- External noise (ext),
- Surgeon decision / unjustified (dec).

Important: CE is not a deviation cause. CE is a structural label for substitutions/additions.

A.II.6 Mapping to the Normative Model

The real event log is mapped to the normative SIM template by:

- Aligning executed surgits to normative transitions,
- Labeling substituted/added transitions as CE when applicable,
- Recording pauses as separate events that do not alter risk variables,
- Preserving step boundaries for comparable reporting.

Annex III: SIM Final Report (Standard Output)

A.III.1 Mandatory Output Tables

Each audited case produces:

- Step table: m_t, π_t, δ_t, $S_q(t)$, ρ_t, and step criticality weight w_t.
- Noise table: n_t, e_t per step, plus pause durations $\tau(\phi_{t,k})$.
- CE table: list of CE substitutions/additions by step with counts and timestamps.
- PCP table (Layer E): $P_k(\text{PCP} \mid \text{SIM})$ for each $k \in \mathcal{K}$, plus η_k (log-odds).
- PCP attribution (Layer E): Shapley contributions $\phi_{k,t}$ per complication type k (top-ranked steps reported).

A.III.2 Expanded-State Traceability Summary (Causal Provenance)

The report must include a traceability section showing:

- the expanded-state trajectory summary (Layer C),
- the provenance vector summary (which early steps contributed to later effects),
- at least one explicit example of delayed attribution (e.g., Step 1 contributing to Step 5).

A.III.3 Global Metrics

The report must include:

- Global Tsallis entropy $S_q(\text{SIM})$,
- Aggregated postoperative risk ρ_{SIM},
- Global score $\text{Score}_{\text{SIM}}$.

A.III.4 Shapley Decomposition (Explainable Attribution)

The final report includes a Shapley-based decomposition by deviation cause:

$$\text{Score}_{\text{SIM}} = \text{Score}_{\text{ideal}} + \sum_{i \in \mathcal{C}} \Phi_i \tag{38}$$

where $\mathcal{C} = \{\text{intr}, \text{pat}, \text{ext}, \text{dec}\}$. The report must present both absolute and percentage contributions.

A.III.5 Interpretive Audit Conclusions

The report must explicitly distinguish:

- Deviations explained by patient noise (n_t),
- Deviations explained by external noise (e_t),
- Structural modifications by CE (substitution/addition),
- Residual deviations lacking justification (candidate surgeon decision component),
- The effect of safety surgits/steps on intraoperative deviation and postoperative risk (by scope).

A.III.6 Normative Disclaimer

SIM does not replace clinical judgment. It provides a formal, explainable, and reproducible mathematical framework to support surgical quality analysis, auditing, and procedure design under a frozen normative reference.