Elyas Aga

1.) Solve the following recurrence relation using repeated substitution $T(n) = 3T(n/4) + 4n$

Replace T with recurrence relation

$3T(n/16) + 4(n/4)$

$T(n) = 3[3T(n/16) + 4(n/4)] + 4n$

$T(n) = 3^2T(n/16) + 3 \cdot 4(n/4) + 4n$

$3^2T(n/16) + 3n + 4n$

$3^2T(n/16) + 7n$

$9T(n/16) + 7n$

$T(n/16) =$

$3T(\frac{n}{4^3}) + 4(\frac{n}{16})$

$3^3T(n/64) +$

$3^2n + 7n$

$3^3T(n/64) + 16n$

pattern k substitution

$T(n) = 3^kT(n/4^k) + 4n \cdot \sum_{i=0}^{k-1} (3/4)^i$

recursive term $\sum_{i=0}^{k-1} (3/4)^i$

$n = 4^k$ so $k = \log_4 n$

$\frac{3^k - 1}{3-1} = \frac{3^k - 1}{2}$

$T(n) = O(n^{\log_4 3})$

Master theorem

$T(n) = aT(n/b) + f(n)$

$a = 3$ $b = 4$ $f(n) = 4n$

$\log_4 3$ n $\log 3/4 = \frac{0.477}{0.602} = 0.793$

$n^{\log_4 3}$

$n^{0.793}$

$f(n) = O(n)$ f(n) grows faster than

$n^{\log_4 3}$ Case 3: $f(n) = O(f(n)) = O(n)$

Substitution

non-recursive, Master theorem

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Algorithms

2.) Master theorem   a = recursive   f(n) = outside work

$T(n) = aT(n/b) + f(n)$ b = decreasing

a.) $T(n) = 3T(n/5) + n^2$

$a = 3$ $b = 5$, $f(n) = n^2$

$\log_5 3 = 0.6826$

$O(n^2) > O(n^{0.6826})$

Case #3 $T(n) = O(n^2)$

b.) $T(n) = 4T(n/3) + 7n$

$a = 4$ $b = 3$ $f(n) = 7n$

$\log_3 4 = 1.2618$

$O(n) < O(n^{1.2618})$

Case #1 $T(n) = O(n^{1.2618})$

c.) $T(n) = 5T(n/4) + 10$

$a = 5$ $b = 4$ $f(n) = O(1)$

$\log_4 5 = 1.1609$

$f(n) = O(1) < O(n^{1.1609})$

Case #1 $T(n) = O(n^{1.1609})$

d.) $T(n) = 9T(n/3) + n^4$

$a = 9$ $b = 3$ $f(n) = n^4$

$\log_3 9 = 2$

$O(n^4) > O(n^2)$

Case #3 $T(n) = O(n^4)$

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e.) $T(n) = 6T(n/8) + n^3$

$a = 6 \quad b = 8 \quad f(n) = n^3$

$\log_8 6 = 0.903$

$f(n) = O(n^3) > O(n^{0.903})$

Case # 3 $= O(n^3)$

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Algoritm

3.) Cap, COL, USD, SUN, JPY, VEE, ROW, JOB, COX, LOL, RAT, WOW, DOD, CAR, FIG, PIG, VIS, LOW, LOX, VEA, CAD, DOG, TSL

- Sort by last letter
- Sort by second letter
- Sort by first letter.

3 passes because 3 letter string.

* RightMost

A = VEA
B = JOB
C
D = USD, DOD, CAD
E = VEE
F = FIG, PIG, DOG
G
H
I
J
K
L = COL, LOL, TSL
M
N = SUN
O
P = CAP
Q
R = CAR
S = VIS
T = RAT
U
V
W = ROW, WOW, LOW
X = COX, LOX
Y = JPY
Z

VEA, JOB, USD, DOD, CAD, VEE, fig, Pig, Dog, Col, LOL, TSL, Sun, CAP, CAR, VIS, RAT, Row, WOW, Low, COX, LOX, JPY.

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-second pass go through second letter.

A: CAP, RAT, CAB, CAD,
E= VEE, VEA
I= FIG, PIG, VIS
O= COL, ROW, JOB, COX, LOL, WOW, DOD, LOW, LOX,
POG
P= JPY
S= USD, TSL
U= SUN

CAP, RAT, CAP, CAB, VEE, VEA, FIG, PIG, VIS, COL, ROW,
JOB, COX, LOL, WOW, DOD, LOW, LOX, POG, JPY, USD,
TSL, SUN

-Third sort first letter

*CAD, RAT, CAP, CAB, COL, COX, DOD, FIG, Job, Jpy, LOL,
LOW, LOX, PIG, POG, ROW, RAT, Sun, TSL, USD, VEE,
VEA, WOW

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Algorithm

4.) [25, 14, 9, 7, 5, 3, 0, 21, 6, 33, 25, 42, 24, 107]

```
int h1 (int key)
  int x = (key+19) * (key+11);
  x = x / 15;
  x = x + key;
  x = x % m;
  return x;
```

* $h1(25) = ((25+19) \times (25+11)) / 15 + 25$ mod $13 = 0$ $25 \longleftrightarrow 52$

* $h(14) = (14+19) \times (14+11) / 15 + 14$ Mod $13 = 4$ $14 \longleftrightarrow 41$

* $h(9) = (9+19) \cdot (9+11) / 15 + 9$ mod $13 = 7$

* $h1(7) = (7+19) \cdot (7+11) / 15 + 7$ mod $13 = 12$ $7 \longleftrightarrow 7$

* $h1(5) = (5+19) \cdot (5+11) / 15 + 5$ mod $13 = 4$ $5 \longleftrightarrow 5$

* $h1(3) = (3+19) \cdot (3+11) / 15 + 3$ mod $13 = 10$ $3 \longleftrightarrow 3$

* $h1(0) = (0+19) \cdot (0+11) / 15 + 0$ mod $13 = 6$ $0 \longleftrightarrow 0$

* $h1(21) = (21+19) \cdot (21+11) / 15 + 21$ mod $13 = 2$ $21 \longleftrightarrow 12$

* $h1(6) = (6+19) \cdot (6+11) / 15 + 6$ Mod $13 = 8$ $6 \longleftrightarrow 6$

* $h1(33) = (33+19) \cdot (33+11) / 15 + 33$ mod $13 = 3$ $33 \longleftrightarrow 33$

Elyas
Algorithms

#4 Continued

* $h_1(25) = (25+19) \cdot (25+11) / 15 + 25$
  $\bmod 13 = 0$ $\quad 25 \longleftrightarrow 52$
* $h_1(42) = (42+19) \cdot (42+11) / 15 + 42$
  $\bmod 13 = 10$ $\quad 42 \longleftrightarrow 24$
* $h_1(24) = (24+19) \cdot (24+11) / 15 + 24$
  $\bmod 13 = 7$ $\quad 24 \longleftrightarrow 42$
* $h_1(107) = (107+19) \cdot (107+11) / 15 + 107$
  $\bmod 13 = 6$ $\quad 107 \longleftrightarrow 701$