AMPLITUDE MODULATION
DOUBLE SIDEBAND FULL CARRIER
SIGNAL GENERATION

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Contents

AM DSB-FC

We have message signal:

$$V_m(t) = V_m \cos(2\pi f_m t) \tag{1}$$

Which V_m is amplitude of message signal and f_m is frequency of message signal. And then, we have carrier signal:

$$V_c(t) = V_c \cos(2\pi f_c t) \tag{2}$$

Which V_c is amplitude of carrier signal and f_c is frequency of carrier signal. Modulation index of AM is:

$$M_{AM} = \frac{V_m}{V_c} \tag{3}$$

Then, the AM modulated signal can be generated by:

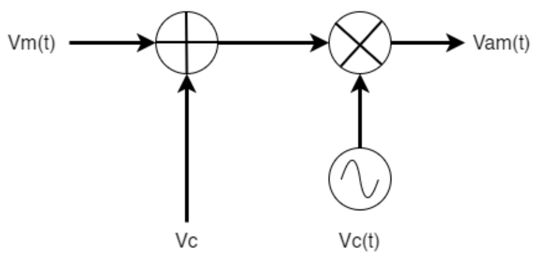

Figure 1. AM DSB-FC block diagram

Then, the AM modulated signal is:

$$V_{AM}(t) = (V_c + V_m(t)) \cos(2\pi f_c t) \tag{4}$$

Substituting (1) to (4), we got:

$$V_{AM}(t) = (V_c + V_m \cos(2\pi f_m t)\ ) \cos(2\pi f_c t) \tag{5}$$

Taking out V_c from parentheses, we got:

$$V_{AM}(t) = V_c\ (1 + \frac{V_m}{V_c} \cos(2\pi f_m t)\ ) \cos(2\pi f_c t) \tag{6}$$

Substituting (3) to (6), we got AM DSB-FC signal equation:

$$V_{AM}(t) = V_c\ (1 + M_{AM} \cos(2\pi f_m t)\ ) \cos(2\pi f_c t) \tag{7}$$

Example: $M_{AM} < 1$

For example purpose, let's assume $V_m = 1\ volt$, $f_m = 2\ Hz$, $V_c = 2\ volt$, and $f_c = 10\ Hz$. We got message signal:

$$V_m(t) = \cos(2\pi\ 2t)$$

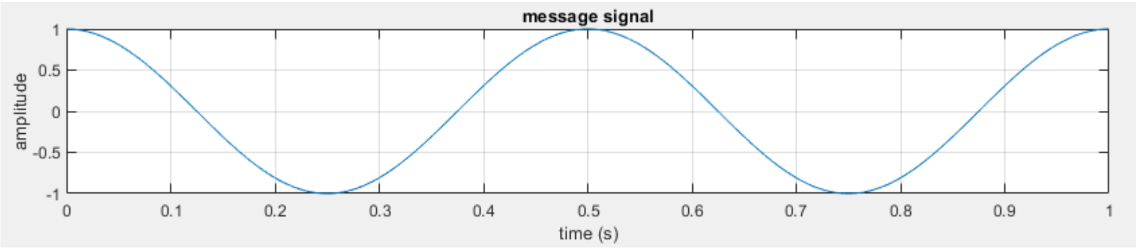

Figure 2. Message signal

Carrier signal:

$$V_c(t) = 2 \cos(2\pi\ 10t)$$

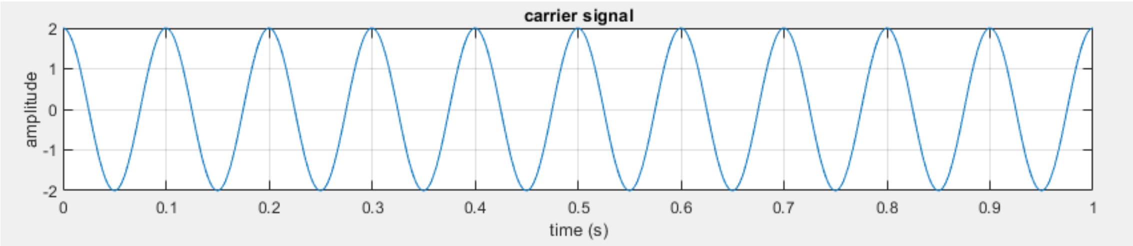

Figure 3. Carrier signal

Modulation index:

$$M_{AM} = \frac{1\ volt}{2\ volt} = 0.5$$

AM modulated signal:

$$V_{AM}(t) = 2\ (1 + 0.5\cos(2\pi\ 2t)\ )\cos(2\pi\ 10t)$$
$$V_{AM}(t) = 2\cos(2\pi\ 10t) + 2(0.5)\cos(2\pi\ 2t)\cos(2\pi\ 10t)$$

With trigonometry identity we know that:

$$\cos(x)\ \cos(y) = 0.5\ [\ \cos(x - y) + \cos(x + y)\ ]$$

And then, the AM modulated signal:

$$V_{AM}(t) = 2\cos(2\pi\ 10t) + \cos(2\pi\ 2t)\cos(2\pi\ 10t)$$
$$V_{AM}(t) = 2\cos(2\pi\ 10t) + 0.5\ [\ \cos(2\pi(10 - 2)t) + \cos(2\pi(10 + 2)t)\ ]$$
$$V_{AM}(t) = 2\cos(2\pi\ 10t) + 0.5\ \cos(2\pi\ 8t) + 0.5\cos(2\pi\ 12t)$$

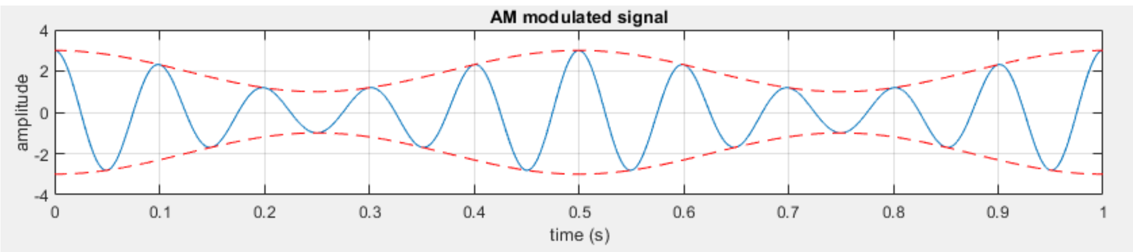

Figure 4. AM modulated signal with $M_{AM} < 1$

Bandwidth:

$$B = 2 \,.\, f_m \tag{8}$$

$$B = 2 \,.\, 2 = 4 \; Hz$$

With Fourier transformation:

$$F\{\, \cos(2\pi \, At) \,\} = 0.5 \, [\, \delta(f - A) + \delta(f + A) \,] \tag{9}$$

We got representation of AM modulated signal in frequency domain:

$$V_{AM}(t) = 2\cos(2\pi\, 10t) + 0.5\, \cos(2\pi\, 8t) + 0.5\cos(2\pi\, 12t)$$

$$V_{AM}(f) = 2(0.5)\, [\, \delta(f - 10) + \delta(f + 10) \,] + 0.5(0.5)\, [\, \delta(f - 8) + \delta(f + 8) \,]$$
$$+\, 0.5(0.5)\, [\, \delta(f - 12) + \delta(f + 12) \,]$$

$$V_{AM}(f) = [\, \delta(f - 10) + \delta(f + 10) \,] + 0.25\, [\, \delta(f - 8) + \delta(f + 8) \,]$$
$$+0.25\, [\, \delta(f - 12) + \delta(f + 12) \,]$$

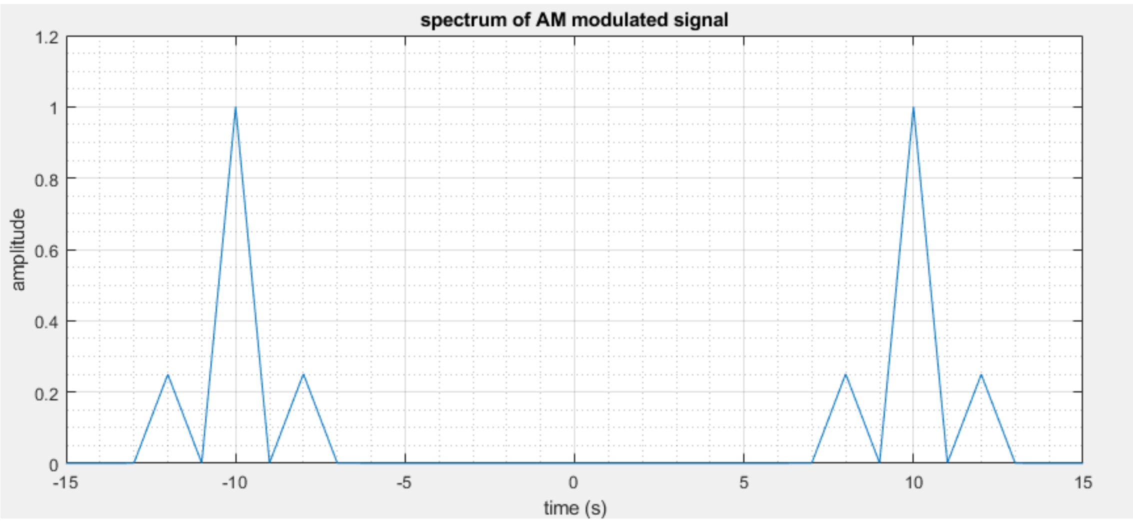

Figure 5. Spectrum of AM modulated signal

Example: $M_{AM} = 1$

For example purpose, let's assume $V_m = 2\; volt$, $f_m = 2\; Hz$, $V_c = 2\; volt$, and $f_c = 10\; Hz$. We got message signal:

$$V_m(t) = 2\; \cos(2\pi\, 2t)$$

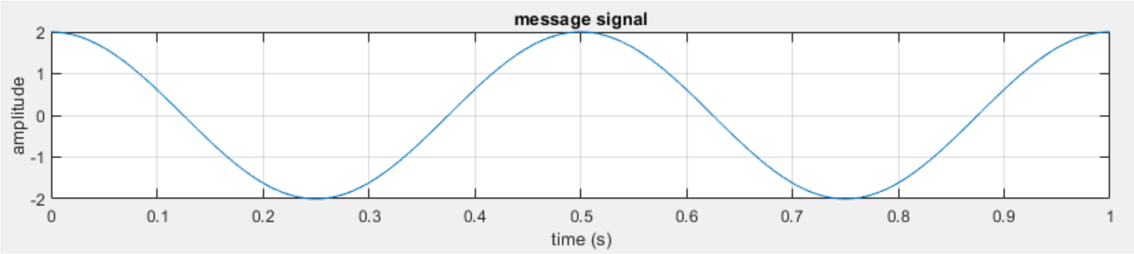

Figure 6. Message signal

For carrier signal is the same with Figure 3. Modulation index:

$$M_{AM} = \frac{2\ volt}{2\ volt} = 1$$

AM modulated signal:

$$V_{AM}(t) = 2\ (1 + \cos(2\pi\ 2t)\ )\ \cos(2\pi\ 10t)$$
$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 2\cos(2\pi\ 2t)\cos(2\pi\ 10t)$$
$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 2(0.5)\ [\cos(2\pi\ (10-2)\ t) + \cos(2\pi\ (10+2)t)\ ]$$
$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + \cos(2\pi\ 8t) + \cos(2\pi\ 12t)$$

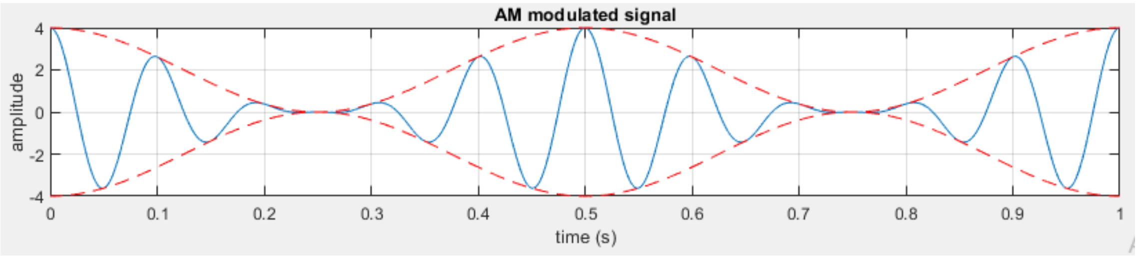

Figure 7. AM modulated signal with $M_{AM} = 1$

Bandwidth is same, 4 Hz.

Representation of AM modulated signal in frequency domain:

$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + \cos(2\pi\ 8t) + \cos(2\pi\ 12t)$$
$$V_{AM}(f) = [\ \delta(f-10) + \delta(f+10)\ ] + 0.5\ [\ \delta(f-8) + \delta(f+8)\ ]$$
$$+0.5\ [\ \delta(f-12) + \delta(f+12)\ ]$$

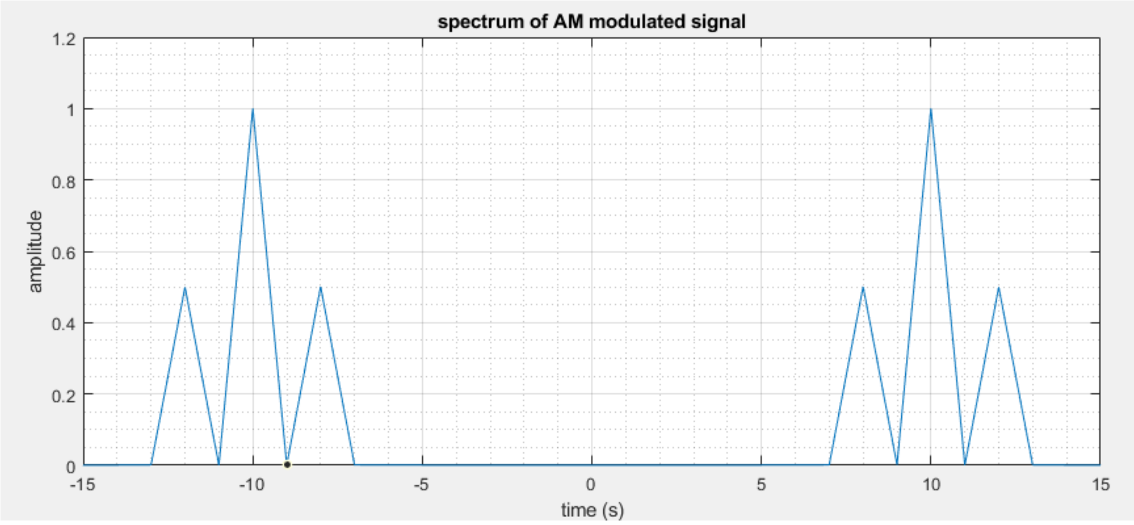

Figure 8. Spectrum of AM modulated signal

Example: $M_{AM} > 1$

For example purpose, let's assume $V_m = 3\ volt$, $f_m = 2\ Hz$, $V_c = 2\ volt$, and $f_c = 10\ Hz$. We got message signal:

$$V_m(t) = 3\ \cos(2\pi\ 2t)$$

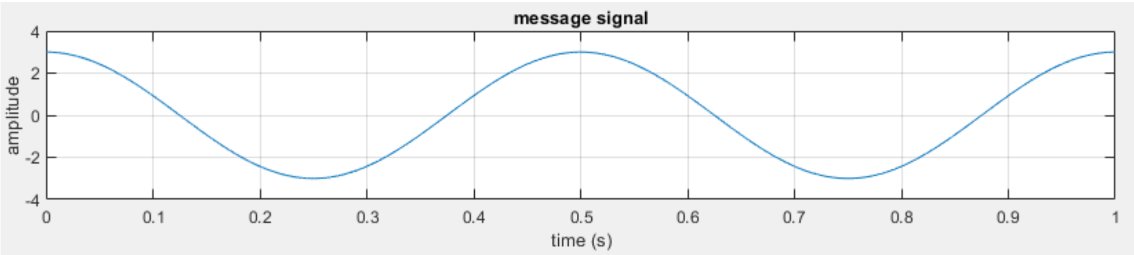

Figure 9. Message signal

For carrier signal is the same with Figure 3. Modulation index:

$$M_{AM} = \frac{3\ volt}{2\ volt} = 1.5$$

AM modulated signal:

$$V_{AM}(t) = 2\ (1 + 1.5\cos(2\pi\ 2t)\ )\cos(2\pi\ 10t)$$

$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 3\cos(2\pi\ 2t)\cos(2\pi\ 10t)$$

$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 3(0.5)\ [\cos(2\pi\ (10-2)\ t) + \cos(2\pi\ (10+2)t)\ ]$$

$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 1.5\cos(2\pi\ 8t) + 1.5\cos(2\pi\ 12t)$$

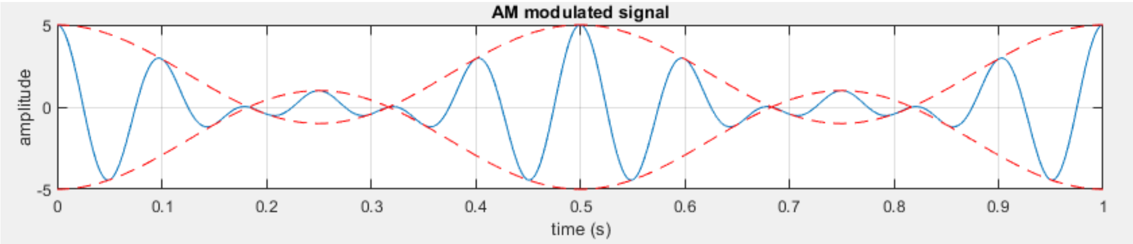

Figure 10. AM modulated signal with $M_{AM} > 1$

Bandwidth is same, 4 Hz.

Representation of AM modulated signal in frequency domain:

$$V_{AM}(t) = 2\ \cos(2\pi\ 10t) + 1.5\ \cos(2\pi\ 8t) + 1.5\cos(2\pi\ 12t)$$

$$V_{AM}(f) = [\ \delta(f-10) + \delta(f+10)\ ] + 0.75\ [\ \delta(f-8) + \delta(f+8)\ ]$$
$$+ 0.75\ [\ \delta(f-12) + \delta(f+12)\ ]$$

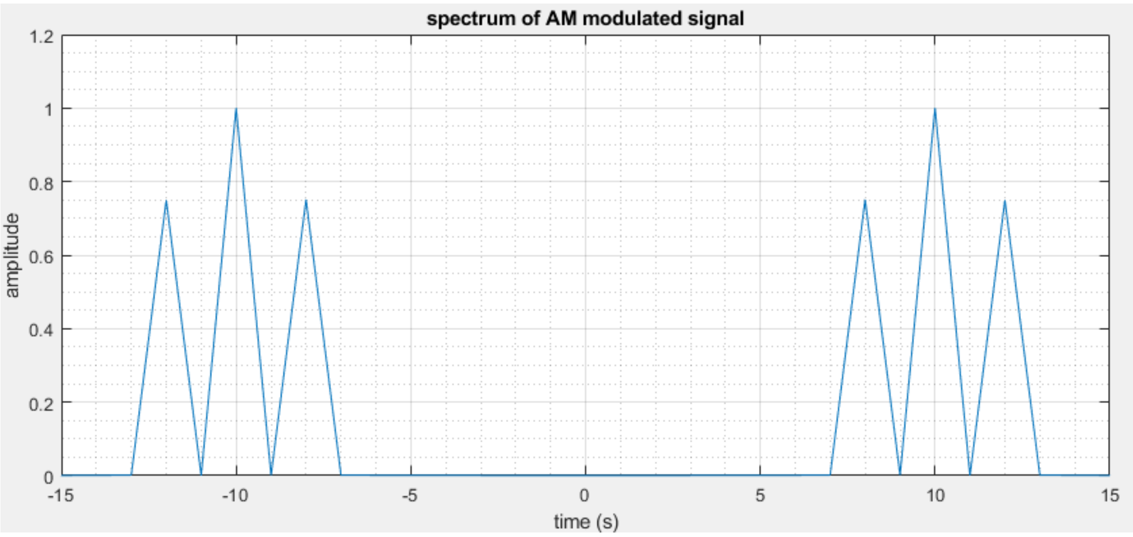

Figure 11. Spectrum of AM modulated signal

Appendix

```
% INTRODUCTION INTO AMPLITUDE MODULATION DSB-FC
clear; clc; close all;

%% I. Variable
fs = 8000;
t = 0:1/fs:1;

% configurable parameters
vm = 1;          % message signal amplitude
vc = 2;          % carrier signal amplitude
fm = 2;          % message signal frequency
fc = 10;         % carrier signal frequency

% visualization settings
msg_mod = 1;     % set to 0: just plot the modulated signal
                 % set to 1: plot modulated & message together
(overlap)

%% II. Message signal
% message signal
vmt = vm*cos(2*pi*fm*t);

% plotting signal
figure(1);
subplot(3,1,1);
plot(t,vmt);
x0=10; y0=10; width=1000; height=600;
set(gcf,'position',[x0,y0,width,height]);
title('message signal');
xlabel('time (s)');
ylabel('amplitude');
grid on;

%% III. Carrier signal
% carrier signal
vct = vc*cos(2*pi*fc*t);

% plotting signal
subplot(3,1,2);
plot(t,vct);
title('carrier signal');
xlabel('time (s)');
ylabel('amplitude');
grid on;

%% IV. Modulation index and bandwidth
Mam = vm/vc;     % modulation index
B = 2*fm;        % bandwidth

% printing resulting parameters
fprintf('modulation index : %.2f\n', Mam);
fprintf('bandwidth        : %d\n\n', B);

%% V. AM modulated signal
% AM modulated signal
```

```
vst = vc.*(1+Mam.*cos(2*pi*fm*t)).*cos(2*pi*fc*t);

% plotting signal
subplot(3,1,3);
plot(t,vst);
if msg_mod == 1
    hold on;
    plot(t,vmt+vc,'--r');
    plot(t,-vmt-vc,'--r');
    hold off;
end
grid on;
title('AM modulated signal');
xlabel('time (s)');
ylabel('amplitude');

%% VI. AM signal frequency spectrum
N = length(vst);
f = fs*[-N/2+0.5:N/2-0.5]/N;
X = fftshift(fft(vst,N));

% plotting spectrum
figure(2);
plot(f,abs(X)/fs);
x0=10; y0=10; width=1000; height=400;
set(gcf,'position',[x0,y0,width,height]);
title('spectrum of AM modulated signal');
xlabel('time (s)');
ylabel('amplitude');
xlim([-1.5*fc 1.5*fc]);
grid on;
grid minor;
```