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The Quicksand of Problem Four

Background

Bill Baker is a TA assigned to handle discussion sections for Calculus I. The main lectures are given by a professor in a large classroom on Mondays, Wednesdays, and Fridays. Bill's discussion sections review, in smaller groups of 15 or 20, the material covered in lectures. Bill is supposed to make sure the students understand the homework problems, and he has some grading responsibilities as well.

Narrative

Bill briefly goes over in his mind his plans for today's discussion section. This is the fourth discussion section of the semester, and it seems to Bill that things are going reasonably well. He has been trying to conduct the section with plenty of interaction, and in general feels that the students are speaking up and participating in discussions. He knows from the syllabus that the professor talked about instantaneous and average velocity in the lecture the day before, and his plan is to talk about the relationship between these two topics with the students. He especially wants to talk about the nature of the limiting process which relates the two ideas. He figures he'll spend a few minutes answering homework questions, after which he can present some examples he's worked out showing the significance of instantaneous velocity in physical problems.

Entering the room, he begins as planned.

BILL: Yesterday in class the lecturer went over the idea of instantaneous velocity and talked about limits. I figured we could spend today in discussion section looking at some of the problems from this section and reviewing some of these ideas. Before I begin, does anyone have any questions from the lecture?

Bill pauses and looks around the room. No one raises a hand, so he continues speaking.

BILL: OK, so let's start with one of the homework problems. Does anyone have a problem they'd like to look at?

Again Bill looks around the room. He notices some of the students shifting in their seats, but no one says anything or raises their hands.

BILL: Really? Are you sure there are no questions?

At this point, one of the students opens his book and speaks up.

JIM: Yeah, can you do problem 3 on page 85?

Bill looks at his book and sees that problem 3 on page 85 is the first assigned problem from the section. The problem is

> *3. Alice travels from Saint Louis to Chicago, a distance of 300 miles. It takes her 5 hours and 45 minutes to cover this distance. What is her average velocity over this time period?*

Bill thinks this seems pretty straightforward. He turns to the class.

BILL: OK, how do you do this problem? Anyone have any suggestions?

He looks at the class and waits what seems to him to be an eternity for someone to volunteer something, but no one speaks.

BILL: Come on, guys, someone here must have looked at this problem...you do know you're supposed to try the homework before coming to discussion section if you want to get anything out of this. Someone here must have some suggestion.

Kathy raises her hand and Bill feels some of the pressure come off.

BILL: Thanks, Kathy, what do you have in mind?

KATHY: Well, you should use the formula for average velocity.

BILL: OK, and what's that?

KATHY (reading from her notebook): It's $(s_1 - s_2)/(t_1 - t_2)$.

BILL: Right! So in this case, we have $s_1 = 0$, that's St. Louis, and $s_2 = 300$, that's Chicago; (he writes on the board: $s_1 = 0$ and $s_2 = 300$); and we have $t_1 = 0$ at St. Louis and $t_2 = 5.75$ at Chicago, (he writes on the board $t_1 = 0$ and $t_2 = 5.75$) and if we use Kathy's formula we get $v_{\text{avg}} = (300)/5.75 =$ 52.17 miles per hour (he writes $v_{\text{avg}} = 300/5.72 = 52.17$ on the board). Does that make sense to everyone?

He looks at the class, making eye contact with Kathy; she nods; Jim, who asked the original question, nods too and starts to write in his notebook. He catches a few other looks from others in the class and decides that he's gotten through that problem without trouble. Now for the next...

BILL: OK, that wasn't too bad. Any more homework problems you'd like to look at?

Jim raises his hand right away.

JIM: How about the next one? Problem 4 on the same page?

Bill figures Jim just wants him to do the homework assignment for him. He remembers his TA training course, where they made a big deal about NOT just doing the problems for the students.

BILL: Well, Jim, you got your question answered last time; how about giving someone else a turn.

Almost immediately a couple of hands go up. Fred speaks out loud:

FRED: I had trouble with Problem 4 too—could you please go over it?

Some of the other students, who have their hands up, make encouraging noises as well. Bill figures he'd better go over the problem—after all, the other thing he remembers from his TA training course is that he should try to respond to the students' interests and difficulties. He decides, though, that he won't just give them the answers; they'll work the problem together.

> *4. Suppose Alice drives the 300 miles from Chicago to St. Louis in 5 hours total driving time. For the first three hours, Alice travels 55 miles per hour. Then she decides she is taking too long to cover the distance. She speeds up to 80 miles per hour for the next hour, when she sees a state patrolman by the side of the road; this makes her slow down and finish the drive at a constant speed.*
>
> *a. Draw a graph showing the distance Alice has traveled as a function of time.*
>
> *b. What is Alice's average velocity for the entire trip?*
>
> *c. Draw a line whose slope shows Alice's average velocity over the entire time.*
>
> *d. Draw another graph showing Alice's instantaneous velocity as a function of time.*

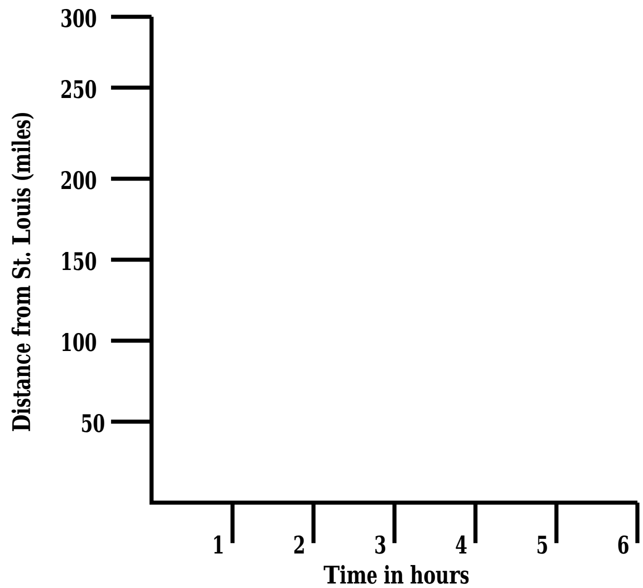
BILL: Let's start out with part (a), where we are supposed to draw a graph of Alice's position as a function of time. Let's start out by looking at the first three hour period. The problem says that Alice travels at 55 miles an hour for these three hours. So what does the graph of position look like for those three hours?

Bill looks around the class. Sarah, who Bill remembers from the past few sections has had reasonable things to say, raises her hand. Before Bill can call on her, though, John interrupts.

JOHN: It's a line.

BILL: Right, it's a line.

Bill turns to the blackboard and puts up a set of axes, labeling the x-axis time (in hours) and the y-axis distance from St. Louis (in miles). Then he turns back to John.



BILL: What line do you have in mind?

JOHN: It's a straight line at height 55.

Bill is surprised at this answer. He is sure the lecturer has reminded the class the day before that motion at a constant speed produces a line whose slope is given by the speed. And anyway, this is pretty basic stuff. He wants to get through this so he can talk about the interesting things. He decides, however, to try to keep working interactively.

BILL: What do other people think. Is that correct?

Sarah raises her hand again, and this time Bill calls on her.

SARAH: It's a line with slope 55 miles per hour.

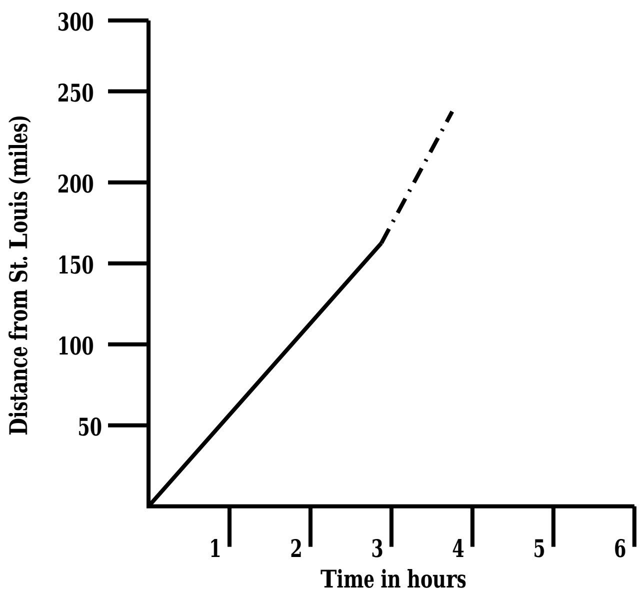
Bill is relieved. This is more like it.

BILL: Right, it's a line with slope 55. Remember that when travelling at a constant rate, distance is rate times time; and in that situation the graph of distance is a line where the slope gives the rate.

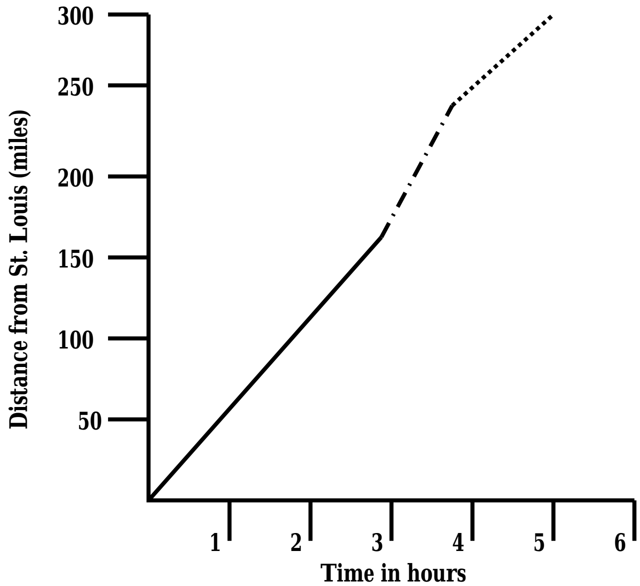
Bill turns to the blackboard and draws in a line with the correct slope. This problem is taking a bit longer than he'd planned—he hadn't really wanted to work through it anyway.

BILL: For the same reason, over the next hour, Alice covers 80 more miles and the graph of her position over the next interval is a line with slope 80.

Bill fills in the next interval. His graph at this point looks like this:



BILL: Now all we need to do is figure out what we need to do to finish the graph and show Alice's arrival in St. Louis. So you see, after 4 hours she's traveled 245 miles; she has to travel 300 miles total; so she has to go another 55 miles. Altogether the trip takes 5 hours. So we can fill in the graph like this:



BILL: Finally, you see, we can get her speed over the last interval by looking at the slope of this line—the change in x being 1 hour (he writes $\Delta x = 1$ on the board) and the change in y being 55 miles (he writes $\Delta y = 55$) for a speed of 55 miles per hour. That covers part a, I think.

Bill turns from the board and looks around the class. Several of the students are writing in their notebooks from his description on the board. Most of the others are sitting passively—Bill figures they know this stuff already and are as bored as he feels. He is also starting to feel a little time pressure—all in all, this was taking a lot longer than he thought it would.

BILL: Now part b tells us that we need to determine Alice's average velocity for this entire trip. We've already done one problem like this. So what is her average velocity?

Again there is quiet in the room.

BILL: Remember problem 3 everyone? We already did one computation of average velocity.

After a long pause, Bill sees Jim do something on his calculator, after which he speaks up.

JIM: Her average velocity is 67 miles per hour.

This answer takes Bill completely by surprise. He has been under the impression that the principles behind average velocity are clear to everyone—it is completely straightforward, plus he has already explained it in Problem 3. Trying to keep his voice from sounding testy, he continues.

BILL: Hmmm... Jim, how did you get 67 miles per hour?

JIM: Well, I averaged 55 and 80 miles per hour. They average out to 67 miles per hour.

This surprises Bill even more. It makes so little sense that he is kind of shocked that anyone would think like this—especially since they have heard a lecture the day before in which average velocity is discussed in detail. Before he can get too worked up, though, he decides that he shouldn't panic just because one student made an inane comment. He figures Jim is just not on top of things, and he shouldn't let him undermine Bill's confidence in the other students.

BILL: I think everyone should realize that the use of the term "average" in the phrase "average velocity" doesn't refer to averaging in the usual sense. Kathy, I think it was you who told us how to figure out average velocity in Problem 3. Can you help us out again with this problem?

Bill looks at Kathy, who has been writing steadily in her notebook. She looks up, startled.

KATHY: Well, uh, I'm not sure.

BILL: As I recall, you gave us the formula $v_{\text{avg}} = (s_1 - s_2)/(t_1 - t_2)$. (He writes this formula on the board.) How do we apply this formula in this context?

Kathy shrugs and looks uncomfortable. Bill scans the room seeing a lot of faces carefully schooled to show no interest. Jim avoids his look. His gaze crosses Sarah's and she raises her eyebrows at him—he realizes she understands this completely, but her face offers no sympathy either for him or her fellow students. She keeps quiet.

Bill starts to suspect that most of the students in the class have no idea what is meant by average velocity. He wonders if more than a very few of them have looked at the homework problems at all. His assumption at the beginning of the class that he could quickly review a few homework problems and then move on to a discussion of limits seems hopelessly naive. In addition, he's gotten himself into the middle of this homework problem, and used up 25 minutes of his 50 minutes, and it looks like it's all been wasted. He starts to feel very depressed, and with a sinking feeling in his stomach, he tries to decide what to do....