

Pedagogy + A Lil' More Grading

Eric Lybrand, Jeff Rabin, Jacqueline Warren



UC San Diego

Announcements

- ▶ **Homework:** Read article on website. Write 1 or 2 paragraphs reflecting on the piece. Did you find it useful? How can you incorporate what you learned into your teaching? Did you dislike the piece?

Comments on Observations

- ▶ Reflect on problems before, during, and after. Lead students through what you observe. They need to emulate this analysis and build intuition.
- ▶ **Eric's Opinion** It's the TA's job to break problems into manageable pieces and have students address these pieces.

Overview

Example Problems

Example Problem

(6 pts) Suppose A, B are square matrices and $AB = BA$. Let (λ, v) be an eigenpair of A .

1. Show that (λ, Bv) is also an eigenpair for A .
 - ▶ $ABv = BA v = B(\lambda v) = \lambda Bv$.
2. Now suppose that the algebraic multiplicity of λ is equal to one. Use (a) to show that v is also an eigenvector of B .
 - ▶ Algebraic multiplicity \geq geometric multiplicity ≥ 1 .
 - ▶ \implies geometric multiplicity $= 1$.
 - ▶ v, Bv in eigenspace corresponding to λ .
 - ▶ Conclude v and Bv are parallel.

Student Response

Suppose A, B are square matrices and $AB = BA$. Let (λ, v) be an eigenpair of A . Show that (λ, Bv) is also an eigenpair for A .

$$\begin{aligned}
 A \cdot v &= \lambda \cdot v, \quad AB = BA, \quad u = Bv \\
 A \cdot Bv &= \lambda \cdot Bv \\
 A &= \lambda, \quad AB = BA
 \end{aligned}$$

Since $AB = BA$, this means they are invertible, $u = Bv$ is a valid eigenvector. B can be an eigenvector of A because they are inverses.

Student Response

Suppose A, B are square matrices and $AB = BA$. Let (λ, v) be an eigenpair of A . Suppose that the algebraic multiplicity of λ is equal to one. Show that v is also an eigenvector of B .

$$A \cdot Bv = \lambda \cdot Bv$$

$$\lambda = 1$$

$$A \cdot Bv = B \cdot v$$

v is an eigenvector of B because the multiplicity is 1, which simplifies to $A \cdot Bv = Bv$. Since A and B have an inverse, v is a valid eigenvector for B .

Regrade!

Furthermore, in problem number 7, I was given 1 point, but for correct ideas towards proof, but I believe this should be given 2.5 points for mostly correct proof with some minor errors in precision or terminology. I believe that my proof was concrete because I used the equation for eigenvalues and manipulated it by substituting $u = Bv$, and then further elaborating on it and using the inverse definition as well.

Furthermore, I would also like to add that I was extremely close to a passing grade of a 25/50, but I was just short by 3.5 points. Had my final been a passing grade, I would have received a C in the class instead of an F. This is very unfortunate for me because I planned on taking Math 184A during the summer. I had done extremely well on the 1st mid term, scoring a 29/30, and did average on the 2nd mid term, but I fell behind a lot after that because of family and financial issues that affected my ability to study, since my parents went unemployed at that time so I had to help them find a new job, since their English is not very good. Also, CSE 30 was a lot harder than I expected so that took up more than 20 hours a week. I believe that I understood the material now that I look back on it, but sadly it is too late. I did not expect the final to be that difficult and it the Saturday time did not help either unfortunately! Can you please look at my Math 18 final and see if there were any grading mistakes?

Thank you!

Regrade!

- ▶ Does the student bring your attention to anything you missed while grading?
- ▶ Beware the pity parties. Final course grade adjustments should be directed to the professor.

Furthermore, in problem number 7, I was given 1 point, but for correct ideas towards proof, but I believe this should be given 2.5 points for mostly correct proof with some minor errors in precision or terminology. I believe that my proof was concrete because I used the equation for eigenvalues and manipulated it by substituting $u = Bv$, and then further elaborating on it and using the inverse definition as well.

Furthermore, I would also like to add that I was extremely close to a passing grade of a 25/50, but I was just short by 3.5 points. Had my final been a passing grade, I would have received a C in the class instead of an F. This is very unfortunate for me because I planned on taking Math 184A during the summer. I had done extremely well on the 1st mid term, scoring a 29/30, and did average on the 2nd mid term, but I fell behind a lot after that because of family and financial issues that affected my ability to study, since my parents went unemployed at that time so I had to help them find a new job, since their English is not very good. Also, CSE 30 was a lot harder than I expected so that took up more than 20 hours a week. I believe that I understood the material now that I look back on it, but sadly it is too late. I did not expect the final to be that difficult and it the Saturday time did not help either unfortunately! Can you please look at my Math 18 final and see if there were any grading mistakes?

Thank you!

Student Response

Suppose A, B are square matrices and $AB = BA$. Let (λ, v) be an eigenpair of A . Suppose that the algebraic multiplicity of λ is equal to one. Show that v is also an eigenvector of B .

$$\begin{array}{l}
 AB = BA \\
 \vdots \\
 B = B \\
 \vdots \\
 Av = \lambda v \\
 BAv = \lambda Bv \\
 ABv = \lambda Bv \\
 Au = \lambda u
 \end{array}
 \quad
 \begin{array}{l}
 Av = \lambda v \\
 u = Bv \\
 \vdots
 \end{array}
 \quad
 \begin{array}{l}
 \text{prove } Au = \lambda u \\
 ABv = \lambda Bv \\
 \rightarrow BAv = \lambda Bv \\
 B^{-1}BAv = \lambda B^{-1}Bv \\
 Av = \lambda v \\
 + \text{true} \\
 \therefore Au = \lambda u
 \end{array}$$

Student Response

Suppose A, B are square matrices and $AB = BA$. Let (λ, v) be an eigenpair of A . Show that (λ, Bv) is also an eigenpair for A .

Left Side (Correct Approach):

$$Av = \lambda v$$

$$AB = BA$$

$\rightarrow v$ is only eigenvector of λ for A .

$$Av = \lambda v$$

$$BAv = \lambda Bv$$

$$ABv = \lambda Bv$$

Bv is eigenvector of A w/ eigenvalue λ

Since λ has algebraic multiplicity 1, Bv is only eigenvector of λ

$$\therefore Bv = v$$

$$\therefore B = I$$

Right Side (Incorrect Approach):

Show $Bv = \lambda v$

$$Bv = v$$

$V = v$ true

$$Bv = \lambda v$$

only $u = \lambda v$ eigen vectors of I are λv