

**MASTER 203 - M2 - Fixed Income II**  
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We will use the following notations:

- $F(t, T_s, T_e) :=$  Forward fixed at time  $t$  for the period  $[T_s, T_e]$
- $S(t, T_s, T_e) :=$  Swap rate at time  $t$  for a swap spanning over the period  $[T_s, T_e]$
- $r_t :=$  Short rate at time  $t$ ;  $\beta_t := \exp(\int_0^t r_s ds)$  the bank account numeraire and  $\mathbb{Q}^\beta$  the probability measure associated to it.
- $B(t, T) :=$  Price at  $t$  of a Zero-Coupon bond with maturity  $T$  and  $\mathbb{Q}^T$  the probability measure associated to it.
- $FFX(t, T) :=$  The forward FX observed at time  $t$  for an exchange at maturity  $T$ .

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**Assignment Instructions**

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- The input market data required for the coding will be provided in a separate Excel file but your Python code doesn't have to read from the Excel. You can put this data in a csv file or directly in the code.
- It is expected that you submit 3 files:
  - (i) one for the theoretical part
  - (ii) one file for the Python code and
  - (iii) one Excel file with the results
- The files should be named *GroupX.ext* (e.g. the code file will be named *GroupX.ipynb*).
- The submitted code should include clear comments on what you are trying to achieve.
- The submitted code should be done under Jupyter Notebook (or if it's not available to any member of the group, the ".py" can be submitted).
- Please note this is an assignment per group. If 2 groups submit similar pieces of code or answers, it will be penalising for both groups.

**Good luck and stay safe!**

## Topic : CMS Call Price & SABR

The objective of this project is to replicate the CMS Call price using (graphical) discrete basket of cash settled swaptions and study the impact of SABR parameters on such price. We will focus on a call with 5Y maturity on CMS2Y.

The attached Excel file provides the USD market conventions, the bootstrapped curve to read the 3M forwards, the bootstrapped OIS curve for discounting and the SABR parameters for the 5Y2Y swaption.

We assume that rates are always positive.

- (A) Please write a piece of code that allows to do the following (separate functions). Please document the methodology, formulae and assumptions that you are using in the "theoretical" document.
- (A-1) Calculate the Libor forwards. The code should be able to handle the market conventions as specified under "Conventions" tab (holidays' calendar, day count basis,...). One idea is to create an object that represents the USD market conventions call this one in the function. Inputs to this function should be start date of the LIBOR, end date of the LIBOR, the market object and the curve.
- (A-2) Calculate the par swap rate. Inputs to this function should be start date of the swap, end date of the swap, the market object, the LIBOR curve and the OIS curve.
- (A-3) Calculate the price of a Physical swaption. Inputs to this function should be expiry date of the option, start date of the swap, end date of the swap, the market object, the LIBOR curve, the OIS curve, the strike and the volatility parameter (not the one provided in the file but any volatility).
- (A-4) Calculate the price of a Cash-Settled swaption. Inputs to this function should be expiry date of the option, start date of the swap, end date of the swap, the market object, the LIBOR curve, the OIS curve, the strike and the volatility parameter (not the one provided in the file but any volatility).
- (B) We would like now to replicate a call on CMS2Y with maturity 5Y using implied volatilities generated by SABR parameters.
- (B-1) Derive the generic formula for the replication of a CMS Call using the discrete basket of cash-settled swaptions.  
Please write a piece of code that allows to do the following (separate functions):
- (B-2) Calculate the implied volatility and the price of cash-settled swaption for any strike using the provided SABR parameters.
- (B-3) Calculate the discrete replication basket for the CMS2Y call expiry in 5Y using cash-settled swaptions priced using SABR parameters.
- (B-4) Shift each SABR parameter by 1% and recalculate the price of the CMS call. Compare to the original price and comment on the impact of each SABR parameter.