Math, Problem Set #4, Optimization Introduction

Elysa Strunin

July 14, 2017

6.1

Find $\mathbf{w} \in R^n$ in order to
minimize $-e^{-\mathbf{w}^T\mathbf{x}}$,
subject to:
$\mathbf{w}^T(-A\mathbf{w} + A\mathbf{y} + \mathbf{x}) \geq a$,
$\mathbf{w}^T(\mathbf{y} - \mathbf{x}^T) = b$

6.5

Find b, k in order to
minimize $-0.07b - 0.05k$,
subject to:
$2b + k \leq 6,000$,
$4b + 3k \leq 240,000$,
$-b \leq 0$,
$-k \leq 0$

6.6

All critical points of f:
$Df(x,y) = [\frac{df}{dx}, \frac{df}{dy}] = [6xy + 4y^2 + y, 3x^2 + 8xy + x] = [0,0]$
At $(0,0), (-\frac{1}{3}, 0), (0, -\frac{1}{4}), (-\frac{1}{9}, -\frac{1}{12})$
$D^2f(x,y) =$

$$\begin{pmatrix} 6y & 6x + 8y + 1 \\ 6x + 8y + 1 & 8x \end{pmatrix}$$

$(0,0)$:

$$D^2f(0,0) =$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

determinant $= -1 \Rightarrow$ mixed-sign $\lambda \Rightarrow$ saddle point
$(-\frac{1}{3}, 0)$:

$$D^2f(-1/3, 0) =$$

$$\begin{pmatrix} 0 & -1 \\ -1 & -8/3 \end{pmatrix}$$

determinant $= -1 \Rightarrow$ mixed-sign $\lambda \Rightarrow$ saddle point

$(0, -\frac{1}{4})$:

$$D^2f(0, -1/4) =$$

$$\begin{pmatrix} -3/2 & -1 \\ -1 & 0 \end{pmatrix}$$

determinant $= -1 \Rightarrow$ mixed-sign $\lambda \Rightarrow$ saddle point

$(-\frac{1}{9}, -\frac{1}{12})$:

$$D^2 f(-1/9, -1/12) =$$
$$\begin{pmatrix} -1/2 & -1/3 \\ -1/3 & -8/9 \end{pmatrix}$$

determinant $= 1/3 \Rightarrow$ same-sign λ
trace $= 25/18$; both $\lambda < 0 \Rightarrow$ location of local maximum

6.11

$f'(x_0) = 2ax_0 + b, f''(x_0) = 2a$
Via Newton's method: $x_1 = x_0 - f'(x_0)/f''(x_0) = -b/2a$
Via setting derivative to zero and solving: $f'(x_{min}) = 0 = 2ax_{min} + b$
$\Rightarrow x_{min} = -b/2a$, which aligns with Newton's method here

6.14

See Jupyter notebook.