$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & 10 \\ 1 & -13 & -14 & 13 \end{bmatrix}$$

Det $(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 & 2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix}$$

Det $(A - \lambda I) = 0$

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$$

By Newton Raphson method

$$\lambda_{n+1} = \lambda_n - \frac{f(\lambda_n)}{f'(\lambda_n)}$$

$$f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$$

$$f'(\lambda) = 4\lambda^3 + 39\lambda^3 - 438\lambda - 835$$

$$\Rightarrow \lambda - \frac{(4\lambda^3 + 39\lambda^3 - 438\lambda - 835) - (\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500)}{4\lambda^3 + 39\lambda^3 - 438\lambda - 835}$$

$$\Rightarrow \frac{4\lambda^4 + 39\lambda^3 - 438\lambda^2 - 835\lambda - \lambda^4 - 13\lambda^3 + 219\lambda^2 - 835\lambda + 3500}{4\lambda^3 + 39\lambda^3 - 438\lambda - 835}$$

$$\Rightarrow \frac{3\lambda^4 + 26\lambda^3 - 219\lambda^2 - 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

Impose $\lambda_0 = 2$ (Divisible by 3500)

$$\Rightarrow \frac{3(2)^4 + 26(2)^3 - 219(2)^2 - 3500}{4(2)^3 + 39(2)^2 - 438(2) - 835}$$

$$\Rightarrow 2.675 \Rightarrow \text{A.1.}$$

For $\lambda = 2.675$, $(A - \lambda I)v = 0$

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & 14 & -13 \end{bmatrix} - 2.675 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

By Gaussian Elimination,

$$\left(\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right) \times (0.754)$$

$R_1 / (1.325) \to R_1$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 1 & 13 & -14 & -15.675 & 0 \end{array}\right) \times (1)$$

$R_4 - (-1) \cdot R_1 \to R_4$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -3.509 & -7.018 & 0 \\ 0 & 0 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right) \times (2.518)$$

$R_2 - (-2) \cdot R_1 \to R_2$

$R_2 / (0.397) \to R_2$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right) \times -10$$

$R_3 - 10 R_2 \to R_3$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & 166.683 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right) \times (6.964)$$

$R_4 - 6.94.82 \to R_4$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 90.669 & -10 & 0 \\ 0 & 0 & -76.278 & -17.184 & 0 \end{array}\right) \times (0.084)$$

$R_3 / (90.669) \to R_3$

$$\left(\begin{array}{cccc|c} 1 & 6.036 & -0.754 & & 0 \\ 0 & 1 & -8.834 & & 0 \\ 0 & 0 & 1 & & 0 \\ 0 & 0 & -76.278 & & 0 \end{array}\right) \times (76.278)$$

$R_4 - 76.27 \cdot R_3 \to R_4$

$$\begin{pmatrix} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \times(8.834) \qquad R_2 \to -8.834 R_3 + R_2$$

$$\begin{pmatrix} 1 & 6.036 & 0 & -0.754 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \times(-6.036) \qquad R_1 \to -6.036 R_2 + R_1$$

$$\begin{pmatrix} 1 & 0 & 0 & 8.494 & 0 \\ 0 & 1 & 0 & -1.428 & 0 \\ 0 & 0 & 1 & 1.838 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} t_1 + 8.494 t_4 = 0 \\ t_2 - 1.428 t_4 = 0 \\ t_3 + 1.838 t_4 = 0 \end{cases}$$

$$\Rightarrow t_3 = -1.838 t_4$$
$$t_2 = 1.428 t_4$$
$$t_1 = -8.494 t_4.$$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix} \Rightarrow \begin{pmatrix} -8.494 t_4 \\ 1.428 t_4 \\ 1.838 t_4 \\ t_4 \end{pmatrix} \Rightarrow \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{pmatrix} \Rightarrow t_4 \begin{pmatrix} -8.494 \\ 1.428 \\ 1.838 \\ 1 \end{pmatrix}$$

First Vector $= \begin{pmatrix} -8.494 \\ 1.428 \\ 1.838 \\ 1 \end{pmatrix}$

$$\frac{3\lambda^4 + 26\lambda^3 - 219\lambda^2 - \cdots}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

$$\lambda = 2.6746$$

$$\Rightarrow \frac{3(2.6746)^4 + 26(2.6746)^3 - 219(2.6746)^2 - 3500}{4(2.6746)^3 + 39(2.6746)^2 - 438(2.6746) - 835}$$

$$\Rightarrow 11.0540 \Rightarrow \lambda_2$$

$\lambda_2 \approx 11.054$

$$A - \lambda_4 I \approx \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix}$$

$Av = \lambda v$

$(A - \lambda I) \cdot v = 0$

Gaussian Elimination:

$$\left(\begin{array}{cccc|c} -7.054 & 8 & -1 & -2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(-0.142)$$

$R_1 / (-7.054) \to R_1$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ -2 & -20.54 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(2)$$

$R_2 (-2(-2) \cdot R_1 - R_2$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.392 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times(1)$$

$R_4 - (-1) \cdot R_1 \to R_4$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.392 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times(-0.045)$$

$R_2 / (-22.392) \to R_2$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times(-10)$$

$R_3 - 10 \cdot R_2 \to R_3$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times(12.771)$$

$R_4 - 14.134 \cdot R_2 \to R_4$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 1 & 0 & -12.771 & -21.597 & 0 \end{array}\right) \times(-0.147)$$

$R_3 / (-6.823) \to R_3$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right) \times(12.771)$$

$R_4 - 12.771 \cdot R_3 \to R_4$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-0.077)$$

$R_2 - 0.077 \cdot R_3 \to R_2$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times(-0.142)$$

$R_1 - 0.142 \cdot R_3 \to R_1$

$$\left(\begin{array}{cccc|c} 1 & -1.134 & 0 & 0.044 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \times (1.134)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0.071 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$f_1 - 1.134 \cdot f_2 \to f_1$

$$\begin{cases} t_1 + 0.071 \cdot v_4 = 0 \\ t_2 + 0.024 \cdot v_4 = 0 \quad (1) \\ t_3 + 1.691 \cdot v_4 = 0 \end{cases}$$

Find the variable v_3 from the equation 3 of the system

$$t_3 = -1.691\, v_4$$

Find the variable v_2 from the equation 2 of the system (1)

$$t_2 = -0.024\, v_4$$

Find the variable v_1 from the equation 1 of the system

$$t_1 = -0.071\, v_4$$

Answer:

$$t_1 = -0.071\, v_4$$
$$t_2 = -0.024\, v_4$$
$$t_3 = -1.691\, v_4$$
$$v_4 = v_4$$

$$t = \begin{pmatrix} -0.071\, v_4 \\ -0.024\, v_4 \\ -1.691\, v_4 \\ v_4 \end{pmatrix}$$

$3\lambda^4 + 26\lambda^3 - 219\lambda^2 - 3500$
$4\lambda^3 + 39\lambda^2 - 438\lambda - 835$

$\lambda = 11.0540$

$\frac{3(11.0540)^4 + 26(11.0540)^3 - 219(11.0540)^2 - 3500}{4(11.0540)^3 + 39(11.0540)^2 - 438(11.0540) - 835}$

-5.6040

$\lambda_3 = -5.6040$

$\lambda_3 = -5.604$

$A - \lambda_3 I = \begin{pmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{pmatrix}$

$AV = \lambda V$

$(A - \lambda I) \cdot V = 0$

$\begin{pmatrix} 9.604 & 8 & -1 & -2 & : 0 \\ -2 & -3.396 & -2 & -4 & : 0 \\ 0 & 10 & 10.604 & -10 & : 0 \\ -1 & -13 & -14 & -7 & : 0 \end{pmatrix} \times (0.104)$

$R_1 / (9.604) \to R_1 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ -2 & -3.396 & -2 & -4 & : 0 \\ 0 & 10 & 10.604 & -10 & : 0 \\ -1 & -13 & -14 & -7.396 & : 0 \end{pmatrix} \times (2)$

$R_2 = (-2) \cdot R_1 - R_2 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & -1.730 & -2.208 & -4.416 & : 0 \\ 0 & 10 & 10.604 & -10 & : 0 \\ -1 & -13 & -14 & -7.396 & : 0 \end{pmatrix} \times (1)$

$R_4 - (-1) \cdot R_1 \to R_4$

$\begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & -1.730 & -2.208 & -4.416 & : 0 \\ 0 & 10 & 10.604 & -10 & : 0 \\ 0 & -12.167 & -14.104 & -7.604 & : 0 \end{pmatrix} \times (-0.578)$

$R_2 / (-1.730) \to R_2 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 1.276 & 2.553 & : 0 \\ 0 & 10 & 10.604 & -10 & : 0 \\ 0 & -12.167 & -14.104 & -7.604 & : 0 \end{pmatrix} \times (10)$

$R_3 - 10 \cdot R_2 \to R_3 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 1.276 & 2.553 & : 0 \\ 0 & 0 & -2.160 & -35.524 & : 0 \\ 0 & -12.167 & -14.104 & -7.604 & : 0 \end{pmatrix} \times (12.167)$

$R_4 - 12.167 \cdot R_2 \to R_4$

$\begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 1.276 & 2.553 & : 0 \\ 0 & 0 & -2.160 & -35.524 & : 0 \\ 0 & 0 & 1.426 & 23.467 & : 0 \end{pmatrix} \times (0.463)$

$R_3 / (-2.160) \to R_3 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 1.276 & 2.553 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 1.426 & 23.467 & : 0 \end{pmatrix} \times (-1.426)$

$R_4 - 1.426 \cdot R_3 \to R_4 \begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 1.276 & 2.553 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 0 & 0 & : 0 \end{pmatrix} \times (-1.276)$

$R_2 - 1.276 \cdot R_3 \to R_2$

$\begin{pmatrix} 1 & 0.833 & -0.104 & -0.208 & : 0 \\ 0 & 1 & 0 & -18.439 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 0 & 0 & : 0 \end{pmatrix} \times (0.104)$

$R_1 - 0.104 \cdot R_3 \to R_1$

$R_1 - 0.833 \cdot R_2 \to R_1 \begin{pmatrix} 1 & 0 & 0 & 16.464 & : 0 \\ 0 & 1 & 0 & -18.439 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 0 & 0 & : 0 \end{pmatrix} \times 0.104$

$R_1 - 0.104 \cdot R_3 \to R_1$

$\begin{pmatrix} 1 & 0.833 & 0 & 1.504 & : 0 \\ 0 & 1 & 0 & -18.4 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 0 & 0 & : 0 \end{pmatrix} \times (-0.833)$

$R_1 - 0.833 \cdot R_2 \to R_1 \begin{pmatrix} 1 & 0 & 0 & 16.364 & : 0 \\ 0 & 1 & 0 & -18.439 & : 0 \\ 0 & 0 & 1 & 16.446 & : 0 \\ 0 & 0 & 0 & 0 & : 0 \end{pmatrix}$

$\begin{cases} t_1 + 16.364 \cdot t_4 = 0 \\ t_2 - 18.439 \cdot t_4 = 0 \quad (1) \\ t_3 + 16.446 \cdot t_4 = 0 \end{cases}$

$t_3 = 16.446\, t_4$

$t_2 = 18.439\, t_4$

$t_1 = -16.864\, t_4$

$t_1 = -16.864\, t_4$

$t_2 = 18.439 t_4$

$t_3 = -16.446\, t_4$

$t_4 = t_4$

$\begin{pmatrix} -16.864\, t_4 \\ 18.439\, t_4 \\ -16.446\, t_4 \end{pmatrix} \Rightarrow \left\{ t_4 \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \end{pmatrix} \right\}$

Let $t_4 = 1$, $V_2 = \begin{pmatrix} -16.864 \\ 18.439 \\ -16.446 \end{pmatrix}$