

$$a = 8$$

$$A = 8G = (5, 65)$$

$$M = (40, 36)$$

$$b = \frac{g}{B = gg} = \underline{\underline{40}}$$

$$G = (57, 24)$$

$$8(gG)$$

$$AB = 8 \times g = 72$$

$$AB = 72G = (10, 11)$$

$$C = M + AB$$

$$C = (40, 36) + (10, 11)$$

$$x_1 = 40, x_2 = 10, y_1 = 36, y_2 = 11$$

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 36}{10 - 40} = \frac{-25}{-30} = \frac{25}{30} = \frac{5}{6} = 5 \times 6^{-1}$$

$$\lambda = 5 \times 15 = 75 \quad \underline{\underline{\lambda = 75}}$$

$$x_3 = (\lambda)^2 - x_1 - x_2 = (75)^2 - 40 - 10$$

$$= 5625 - 50 = 5575 \bmod 89$$

$$(x_3 = 57)$$

$$V = y_1 + \lambda x_1 = 36 + 75(40) = 3036 \bmod 89$$

$$(V = 74)$$

$$V = y_2 - \lambda x_1 = 36 - 75(40) = 36 - 3000$$

$$= -2964 \bmod 89$$

$$(V = 69)$$

$$y_3 = \lambda x_3 + V = 75(57) + 69$$

$$= 4337 \bmod 89$$

$$= 65.$$

$$(= (57, -65))$$

$$\Rightarrow$$

$$C = (57, 24)$$

Decrypt (57, 24)

$$D = C - bA$$

$$D = (57, 24) - (10, 11)$$

$$D = (57, 24) + (10, -11)$$

$$D \quad x_1 = 57, x_2 = 10, y_1 = 24, y_2 = -11$$

$$\gamma = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - 24}{10 - 57} = \frac{-35}{-47} = \frac{35}{47}$$

$$\gamma = 35 \times 47^{-1} = 35 \times 36 = 1260 \bmod 89$$

$$x_3 = (\gamma)^2 - x_1 - x_2 = (14)^2 - 57 - 10 = 14$$
$$= 196 - 67 = 129 \bmod 89$$
$$\boxed{x_3 = 40}$$

Decrypt 80 (80, 8)

$$D = C - bA$$

$$D = (80, 8) - (10, 11)$$

$$D = (80, 8) + (10, -11)$$

$$\gamma = D = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - 8}{10 - 80} = \frac{-19}{-70} = \frac{19}{70}$$

$$\gamma = 19 \times 70^{-1} = 19 \times 14 = 266 \bmod 89$$

$$\boxed{\gamma = 88}$$

$$x_3 = (\gamma)^2 - x_1 - x_2 = (88)^2 - 80 - 10 = 7744 - 90$$

$$x_3 = 7654 \bmod 89$$

$$x_3 > 0$$

$$V = y_1 - \gamma x_1 = 8 - 88(80) = 8 - 7040$$

$$= -7032 \bmod 89$$

$$\boxed{V = 88}$$

$$AB = BA$$

$$BA = 72G = (70, 7)$$

$$P_1 - P_2 = P_1 + (-P_2)$$
$$= (x_1, y_1) + (x_2, -y_2)$$

$$B \quad x_1 = 57, x_2 = 10, y_1 = 24, y_2 = -11$$

$$\gamma = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - 24}{10 - 57} = \frac{-35}{-47} = \frac{35}{47}$$

$$\gamma = 35 \times 47^{-1} = 35 \times 36 = 1260 \bmod 89$$

$$x_3 = (\gamma)^2 - x_1 - x_2 = (14)^2 - 57 - 10 = 14$$
$$= 196 - 67 = 129 \bmod 89$$
$$\boxed{x_3 = 40}$$

$$y_3 = \gamma x_3 + v = 88(0) + 88 = 88$$
$$\overline{AB} = (0, -88) \Rightarrow B = (0, 1)$$