model_answers

July 27, 2020

1 A mathematical model of a zombie apocalypse

A warm-up activity for UWE Maths and Stats students

1.1 Problem description

Let's model a zombie apocalypse.
n = the number of days since the start of the invasion
S_n = the number of susceptible humans after n days
Z_n = the number of zombies after n days
R_n = the number of zombies killed by humans after n days
a = the proportion of humans that a single zombie will bite each day
b = the proportion of zombie that a human will kill each day

$$S_{n+1} = S_n - aS_nZ_n$$

$$Z_{n+1} = Z_n + (a-b)S_nZ_n$$

$$R_{n+1} = R_n + bS_nZ_n$$

Who will survive?

1.2 Creating a simulation step by step

Let's create our variables, first the parameters of prograpagation of teh zombie epidemy a and b

[1]:
```
a=0.0095
```

[2]:
```
b=0.005
```

Then the lengths of time for the model to run N (in days)

[3]:
```
N=100
```

Now let's create a variable time T, counting the number of days since the begining of the simulation

[4]:
```
T=range(N)
```

S, N and R are vectors of the number of human, zombie and dead at each day. We will store them in table of size N, with $S[i]$ ($N[i]$, $R[i]$) the number of humans (zombie, dead) at time i

[5]:
```
S=[0]*100
```

[6]:
```
Z=[0]*100
```

[7]:
```
R=[0]*100
```

Finally let's set our initial populations sizes, 199 humans and 1 zombie

[8]:
```
S[0]=199
```

[9]:
```
Z[0]=1
```

[10]:
```
R[0]=0
```

The first day:

[11]:
```
S[1]=S[0]-a*S[0]*Z[0]
Z[1]=Z[0]+(a-b)*S[0]*Z[0]
R[1]=R[0]+b*S[0]*Z[0]
```

[12]:
```
(S[1],Z[1],R[1])
```

[12]:
```
(197.1095, 1.8955, 0.995)
```

Let's write this for all time i until N

[13]:
```
for i in range(1,N):
    S[i]=S[i-1]-(a*S[i-1]*Z[i-1])
    Z[i]=Z[i-1]+((a-b)*S[i-1]*Z[i-1])
    R[i]=R[i-1]+(b*S[i-1]*Z[i-1])
```

[14]:
```
S[:10]
```

[14]:
```
[199,
 197.1095,
 193.560099956125,
 186.98301482344482,
 175.0953016543179,
 154.5966946335348,
 122.23730695245068,
 78.85136060992724,
 35.46979607310864,
 9.03110957755094]
```

[15]:
```
Z[:10]
```

[15]:
```
[1,
 1.8955,
 3.5767947576249997,
 6.692256136262974,
 12.323278163744146,
 22.03314464727297,
 37.36127565410229,
 57.9125133952976,
 78.46167554431693,
 90.98526388431794]
```

[16]:
```
R[:10]
```

[16]:
```
[0,
 0.995,
 2.86310528625,
 6.324729040292194,
 12.58142018193794,
 23.370160719192192,
 40.401417393447,
 63.23612599477512,
 86.06852838257439,
 99.98362653813106]
```

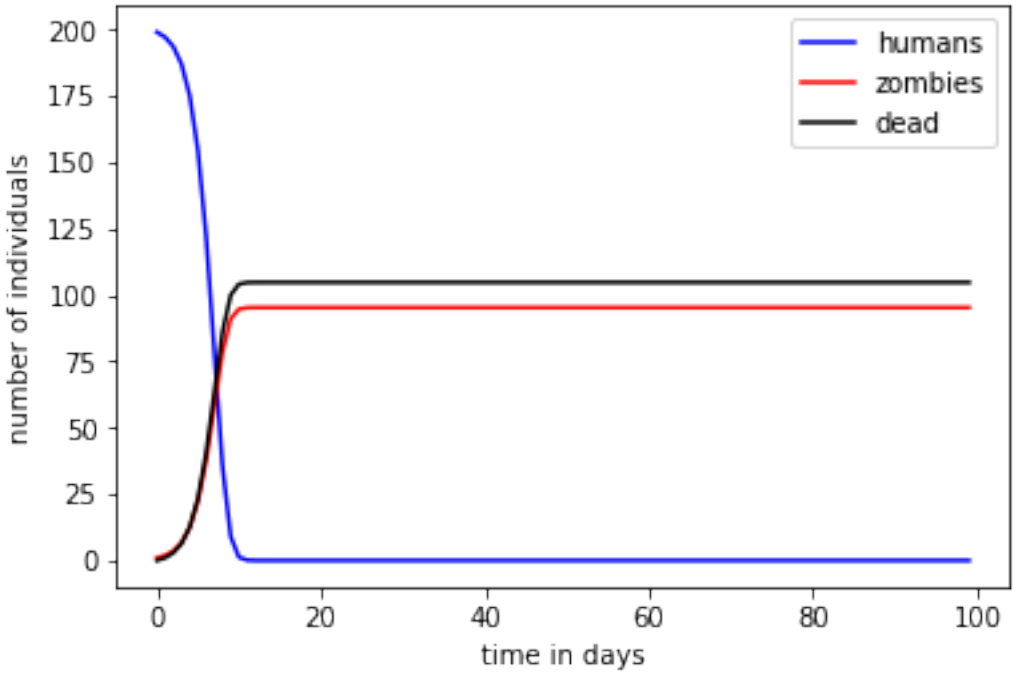
Now let's plot this:

[17]:
```
%matplotlib inline
import matplotlib.pyplot as plt
```

```
/home/nbuser/anaconda3_501/lib/python3.6/site-
packages/matplotlib/font_manager.py:229: UserWarning: Matplotlib is building the
font cache using fc-list. This may take a moment.
  'Matplotlib is building the font cache using fc-list. '
```

[18]:
```
plt.plot(T,S,color="blue",label="humans")
plt.plot(T,Z,color="red",label="zombies")
plt.plot(T,R,color="black", label="dead")
plt.legend()
plt.xlabel("time in days")
plt.ylabel("number of individuals")
```

[18]:
```
Text(0, 0.5, 'number of individuals')
```



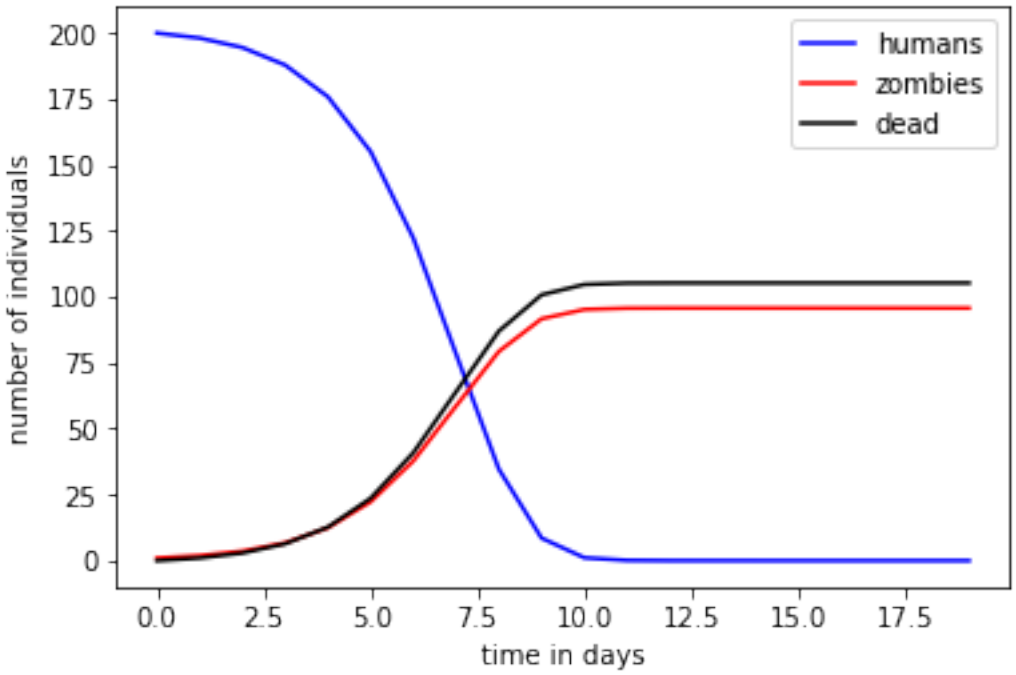
1.3 Generic simulation

Now let's put it all in one place to do different simulations

[19]:
```
a=0.0095
b=0.005
N=20 # here we put 20 since all humans were already dead at 20
T=range(N)
S=[0]*N
Z=[0]*N
R=[0]*N
S[0]=200
Z[0]=1
R[0]=0
```

[20]:
```
for i in range(1,N):
    S[i]=S[i-1]-(a*S[i-1]*Z[i-1])
    Z[i]=Z[i-1]+((a-b)*S[i-1]*Z[i-1])
    R[i]=R[i-1]+(b*S[i-1]*Z[i-1])
plt.plot(T,S,color="blue",label="humans")
plt.plot(T,Z,color="red",label="zombies")
plt.plot(T,R,color="black", label="dead")
plt.legend()
plt.xlabel("time in days")
plt.ylabel("number of individuals")
```

[20]: `Text(0, 0.5, 'number of individuals')`



1.4 Task 2

with $S_0 = 150$, $Z_0 = 50$, $R_0 = 0$, $a = 0.005$ and $b = 0.0095$

[21]:
```
a=0.005
b=0.0095
N=100 # adjust according to simulation
T=range(N)
S=[0]*N
Z=[0]*N
R=[0]*N
S[0]=150
Z[0]=50
R[0]=0
```

[22]:
```
for i in range(1,N):
    S[i]=S[i-1]-(a*S[i-1]*Z[i-1])
    Z[i]=Z[i-1]+((a-b)*S[i-1]*Z[i-1])
    R[i]=R[i-1]+(b*S[i-1]*Z[i-1])
plt.plot(T,S,color="blue",label="humans")
plt.plot(T,Z,color="red",label="zombies")
plt.plot(T,R,color="black", label="dead")
plt.legend()
```