Session 14

Introduction to Mathematics

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Task 2

How to solve nonlinear matrices in Algebra

A matrix is a rectangular array of numbers which are called entries

$$A = a_{ij} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \end{array}$$

System of Linear Equation

$$2.0x + 4.0y + 6.0z = 18$$

$$4.0x + 5.0y + 6.0z = 24$$

$$3.0x + 1y - 2.0z = 4$$

Matrix representation

$$A = \begin{bmatrix} 2.0 & 4.0 & 6.0 \\ 4.0 & 5.0 & 6.0 \\ 3.0 & 1.0 & -2.0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 18.0 \\ 24.0 \\ 4.0 \end{bmatrix}$$

[i]NSTANT

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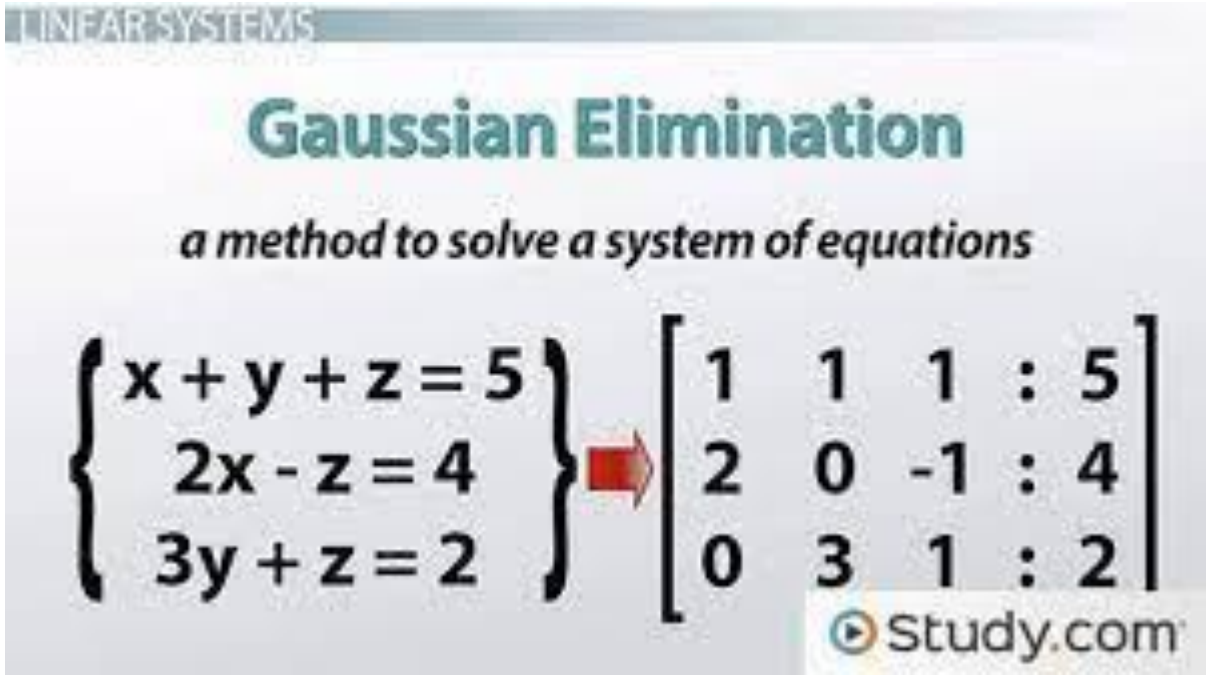


Gauss-Jordan elimination

- Reduction to REF = "Gaussian Elimination"
- Reduction to RREF = "Gauss-Jordan Elimination"

GENERAL IDEA : Start from bottom pivot then work up

back to example :

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -3 & 3 \end{array}\right]$$

pivot $\neq 1$, so scalar multiply

$R_3 \to -\frac{1}{3} R_3$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 2 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] R_3$$

$R_1 \to R_1 + \boxed{-2} R_3$

$R_2 \to R_2 + \boxed{1} R_3$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

$R_1 \to R_1 + \boxed{1} R_2$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

PIVOTS

RREF

Solv Lin Eqn Inverse Matrix Method:

$$x - 3y = -1$$
$$4x + 3y = 11$$

$$X = A^{-1}B$$

$$A = \begin{bmatrix} 1 & -3 \\ 4 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{D}\begin{bmatrix} 3 & 3 \\ -4 & 1 \end{bmatrix}$$

$$D = \begin{vmatrix} 1 & -3 \\ 4 & 3 \end{vmatrix} = 15$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 3 & 3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 11 \end{bmatrix}$$

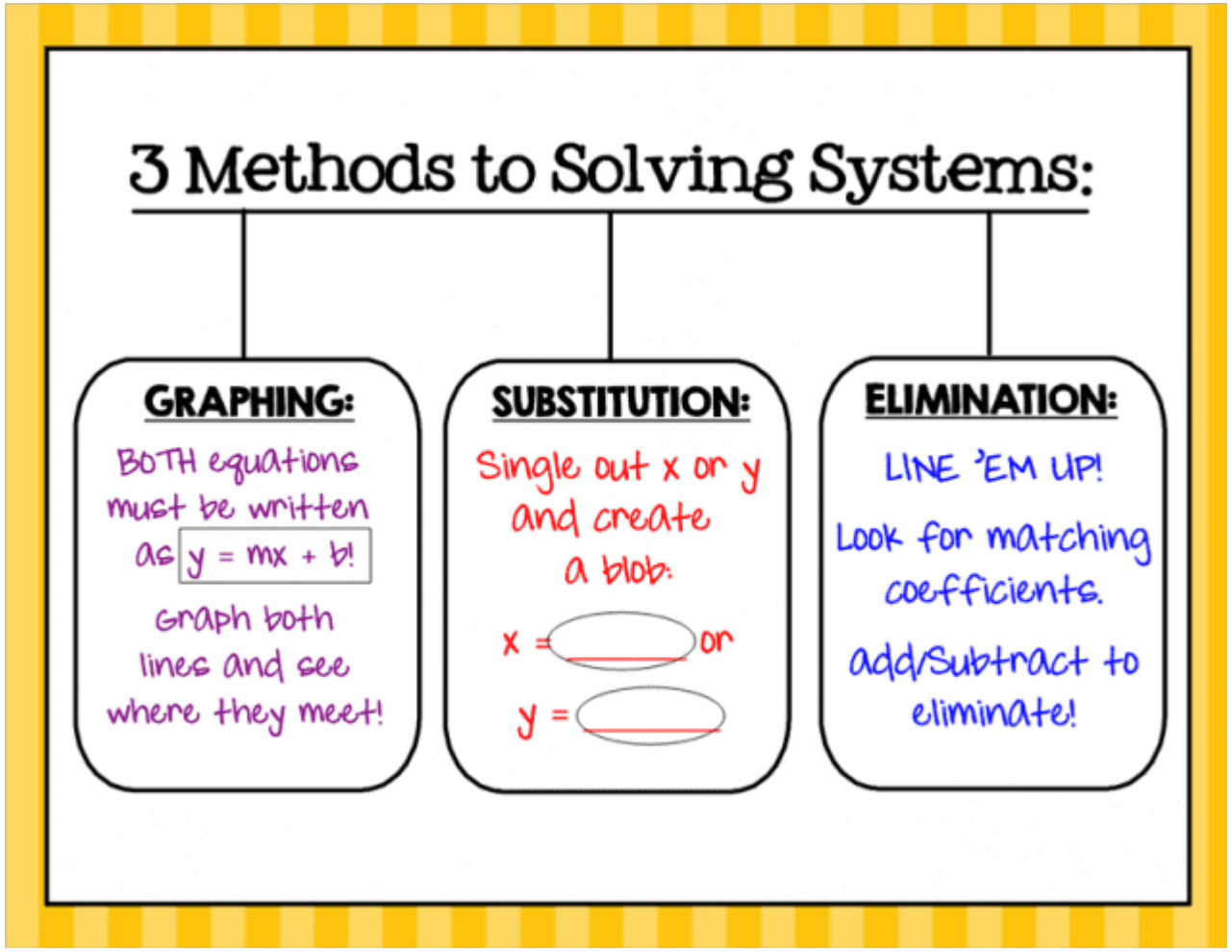
Practice Using Cramer's Rule

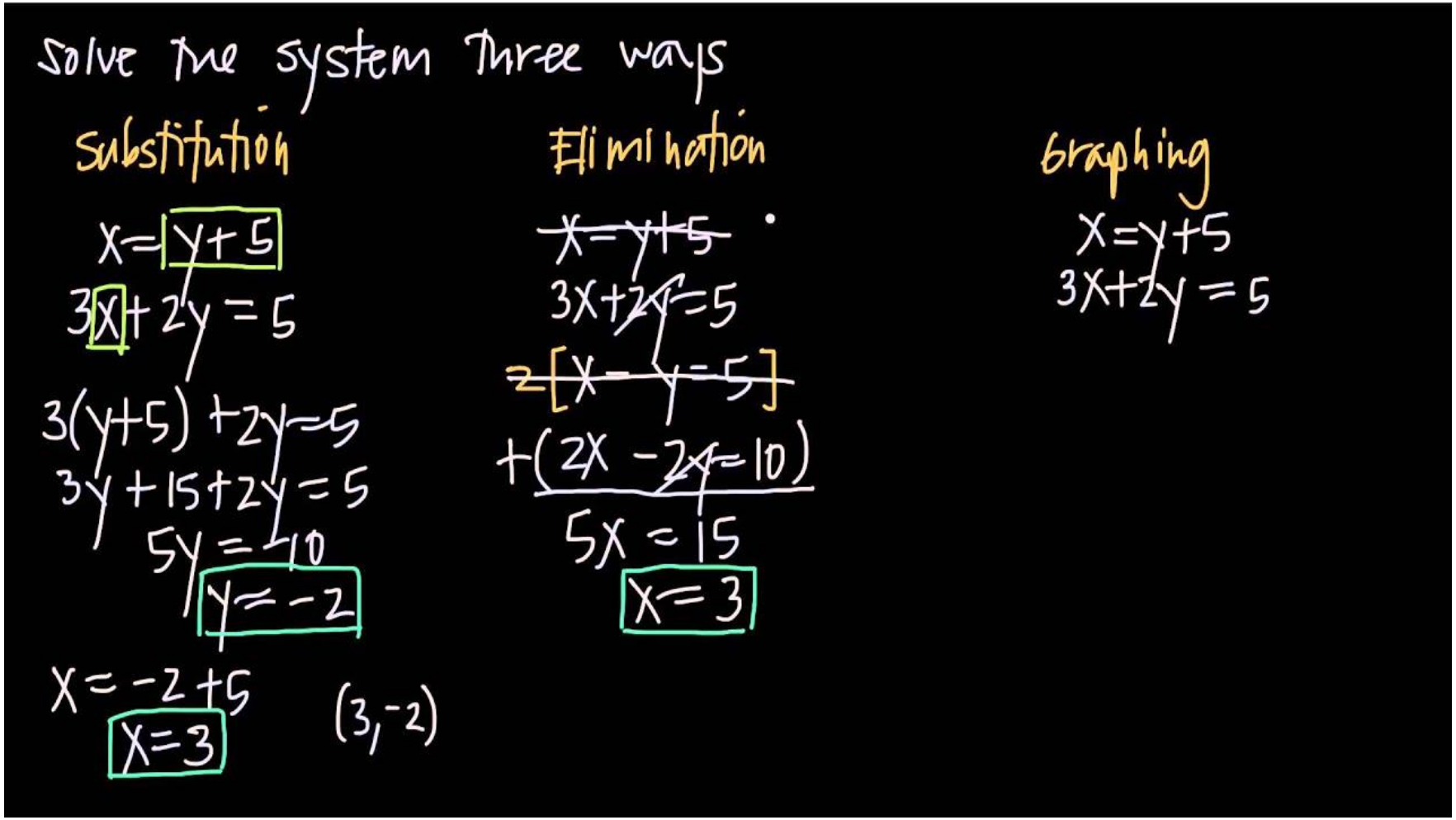
$$x_1 + 3x_2 = 5$$
$$2x_1 + 2x_2 = 6$$

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \quad A_1 = \begin{bmatrix} 5 & 3 \\ 6 & 2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}$$

$$|A| = -4 \qquad |A_1| = -8$$

$$x_1 = 2 \qquad x_2 = \frac{|A_2|}{|A|}$$


3 Methods to Solving Systems:
GRAPHING:
BOTH equations must be written as y = mx + b!
Graph both lines and see where they meet!
SUBSTITUTION:
Single out x or y and create a blob:
x =
or
y =
ELIMINATION:
LINE 'EM UP!
Look for matching coefficients.
add/Subtract to eliminate!


Solve the system three ways
Substitution
x = y + 5
3x + 2y = 5
3(y+5) + 2y = 5
3y + 15 + 2y = 5
5y = -10
y = -2
x = -2 + 5
x = 3
(3, -2)
Elimination
x = y + 5
3x + 2y = 5
-2[x - y = 5]
+(2x - 2y = 10)
5x = 15
x = 3
Graphing
x = y + 5
3x + 2y = 5

Matrix rules

scalar multiplication

$$n\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} na & nb & nc \\ nd & ne & nf \end{bmatrix}$$

matrix addition

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} + \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} a+g & b+h \\ c+i & d+j \\ e+k & f+l \end{bmatrix}$$

matrix multiplication

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{bmatrix}$$

System of Equations in an Augmented Matrix

$$2x + 5y = 3$$
$$-x + 2y = 4$$

$$\longrightarrow \left[\begin{array}{cc|c} 2 & 5 & 3 \\ -1 & 2 & 4 \end{array}\right]$$

Determining Number of Solutions

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THE MATH EXPERT

The homogenous system of linear equations is :

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = 0$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n = 0$$

👉 If det(A) ≠ 0 ,it has only trivial solution(unique)

👉 If det(A) = 0 , it has both trivial and non-trivial solutions(infinite)

$$\text{where det(A)} = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}$$

Example 1: Solve the nonlinear system of equations.

$$\begin{cases} 2 - y = 3x \\ x^2 + y^2 = 16 \end{cases}$$

$2 - y = 3\left(\frac{3-\sqrt{39}}{5}\right)$

$-2 \quad -2$

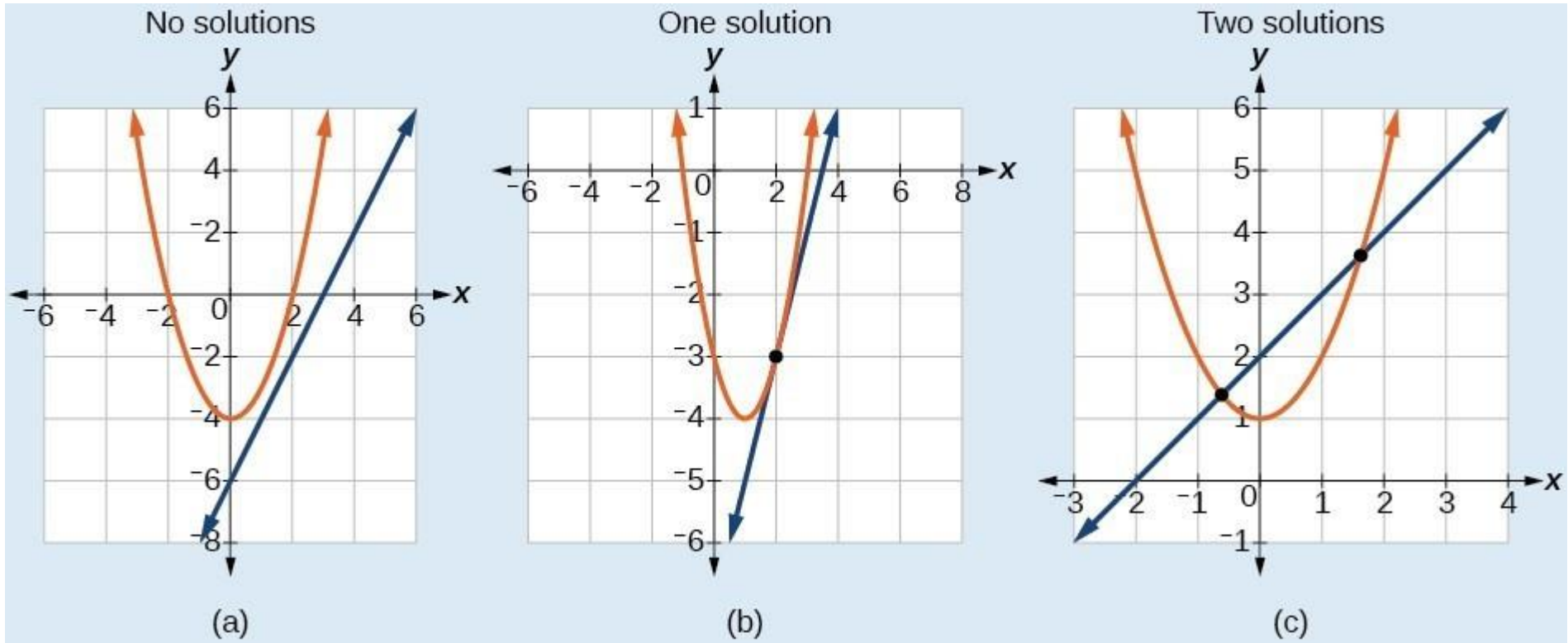
$-y = 3\left(\frac{3-\sqrt{39}}{5}\right) - 2$

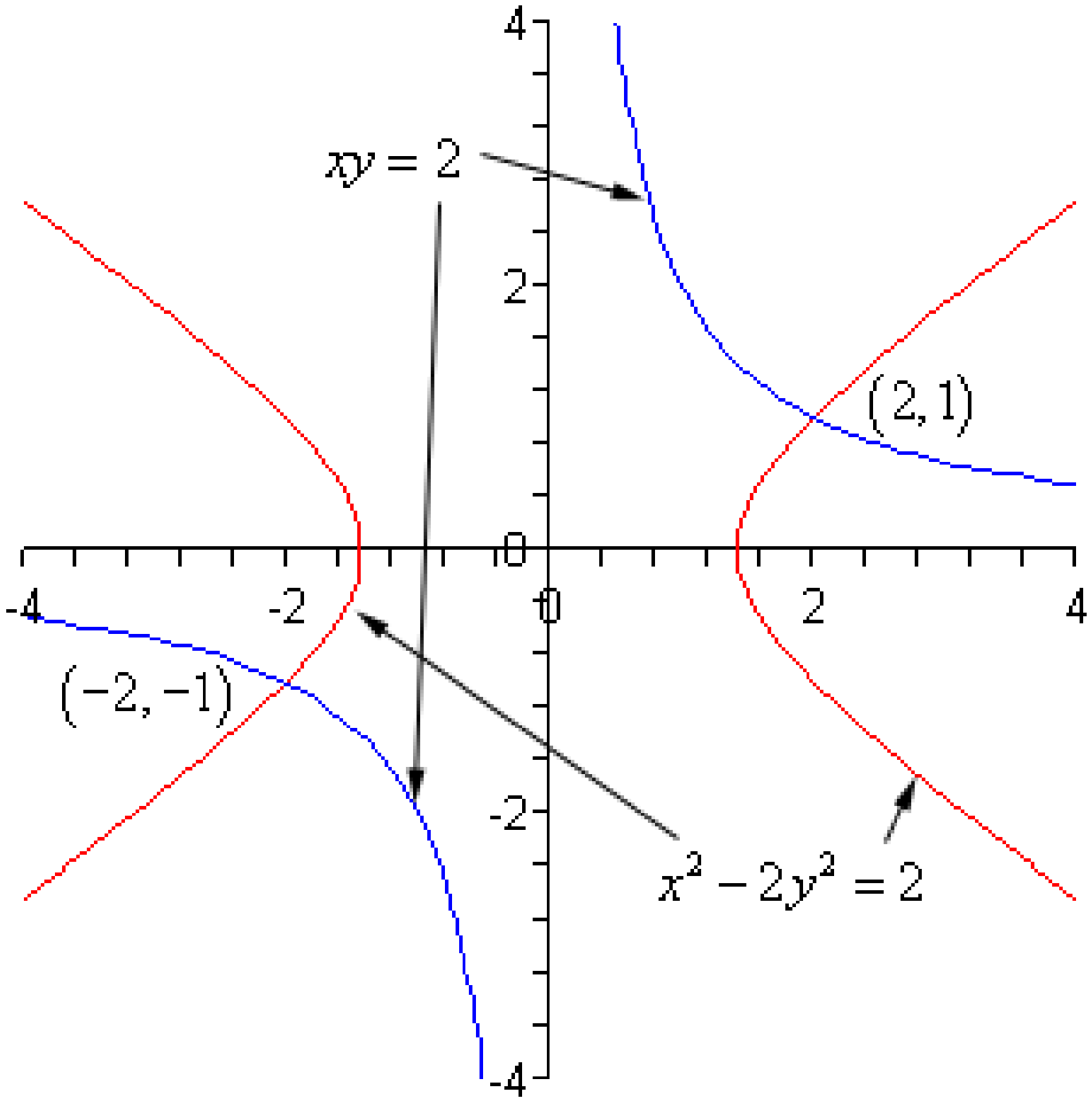
$-1 \quad -1 \quad -1$

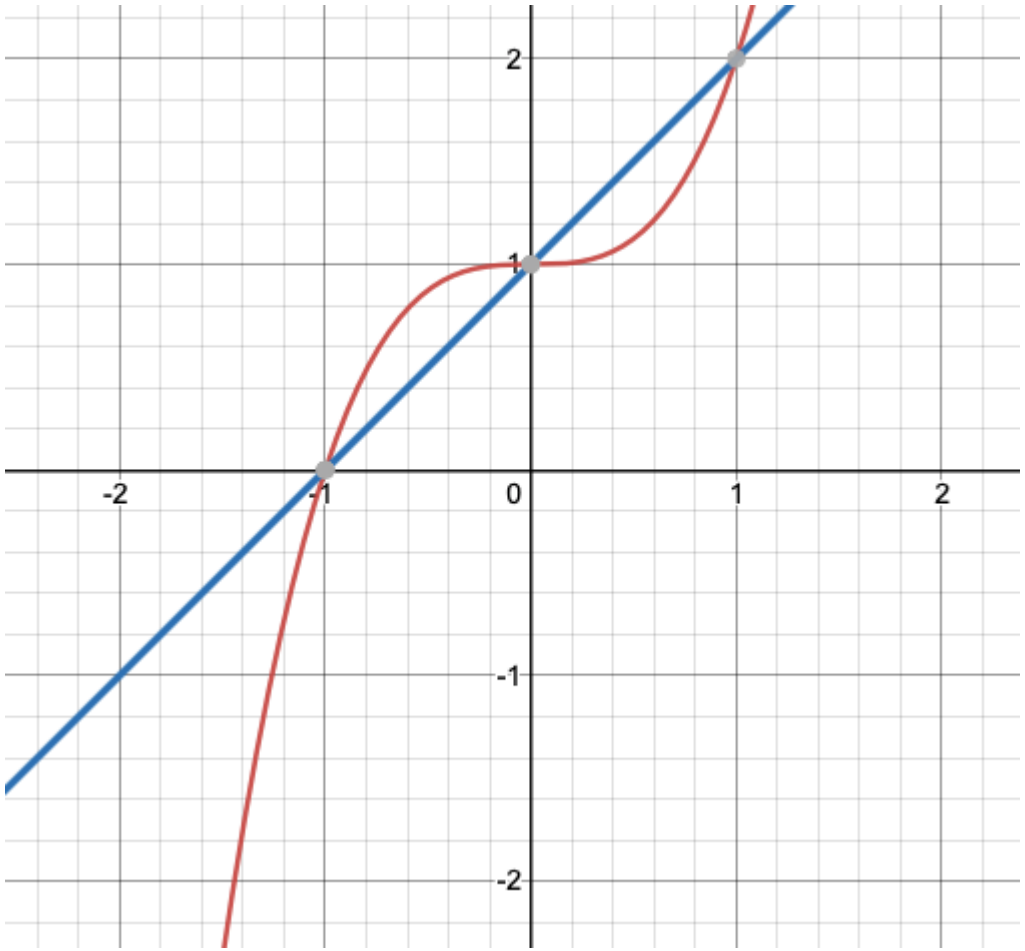
$y = -3\left(\frac{3-\sqrt{39}}{5}\right) + 2 = \frac{-9+3\sqrt{39}}{5} + \frac{2(5)}{5} = \frac{1+3\sqrt{39}}{5}$

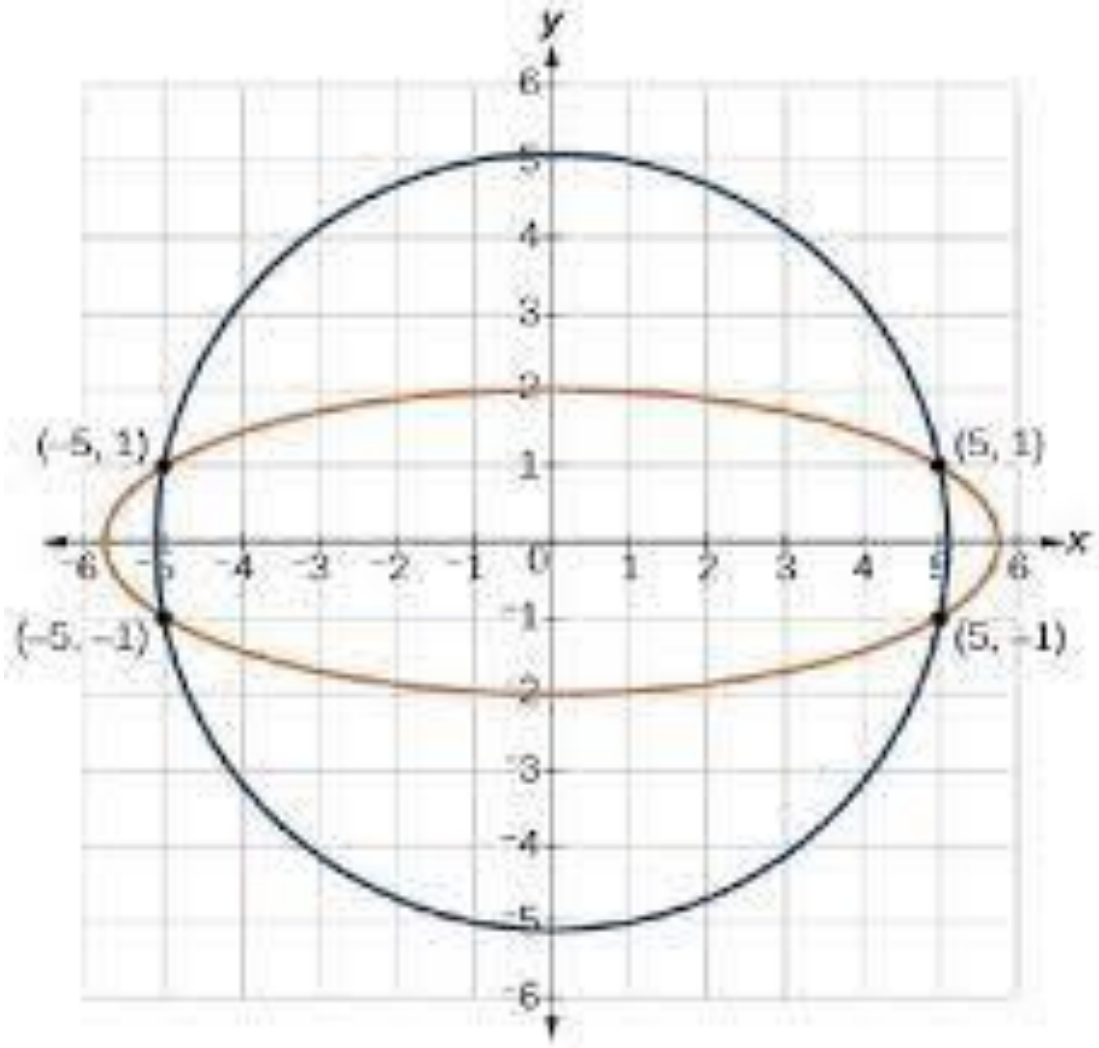
Solution 1: $\left(\frac{3+\sqrt{39}}{5}, \frac{1-3\sqrt{39}}{5}\right)$ (x, y)

Solution 2: $\left(\frac{3-\sqrt{39}}{5}, \frac{1+3\sqrt{39}}{5}\right)$ (x, y)


No solutions
y
x
6
4
−2
−4
−6
−8
−6
−4
−2
0
2
4
6
(a)
One solution
y
x
1
−1
−2
−3
−4
−5
−6
−6
−4
−2
0
2
4
6
8
(b)
Two solutions
y
x
6
5
4
3
2
1
−1
−3
−2
−1
0
1
2
3
4
(c)


4
2
0
-2
-4
-4
-2
0
2
4
$xy = 2$
$(2, 1)$
$(-2, -1)$
$x^2 - 2y^2 = 2$


2
1
-1
-2
-2
-1
0
1
2


y
x
(-5, 1)
(5, 1)
(-5, -1)
(5, -1)

Solving a system of nonlinear equations using elimination

a) $\begin{cases} x^2+y^2=27 \\ x^2-2y=3 \end{cases} \Rightarrow$

$x^2+y^2=27$

$-x^2+2y=-3$

$y^2+2y=24$

$y^2+2y-24=0$

$(y+6)(y-4)=0$

$y+6=0$ $\quad$ $y-4=0$

$y=-6$ $\quad$ $y=4$

$x^2-2(-6)=3$ $\quad$ $x^2-2(4)$

$x^2+12=3$

$x^2=-9$

Solving systems of non-linear equations using substitution

a) $\begin{cases} 3x+y=9 \\ x^2-y=-5 \end{cases}$

Solve for one variable

$3x+y=9$

$y=9-3x$ ←

$x^2-y=-5$

$x^2-\underline{y}=-5$

$x^2-(9-3x)=-5$

$x^2-9+3x=-5$

$x^2+3x-9+5=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x+4=0$ | $x-1=0$

$x=-4$ | $x=1$

$y=9-3(-4)$ | $y=9-3(1)$

$=9+12$ | $y=6$

$y=21$

$(-4,21)$ $(1,6)$

Non-Homogeneous linear Equations

$$a_1x + b_1y + c_1z = k_1$$
$$a_2x + b_2y + c_2z = k_2$$
$$a_3x + b_3y + c_3z = k_3$$

is called a system of non − homogeneous linear equations

in the three variables x, y and z

And constant terms k_1, k_2 and k_3 are not all zero.

Task 3

How to transpose Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Input

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Output

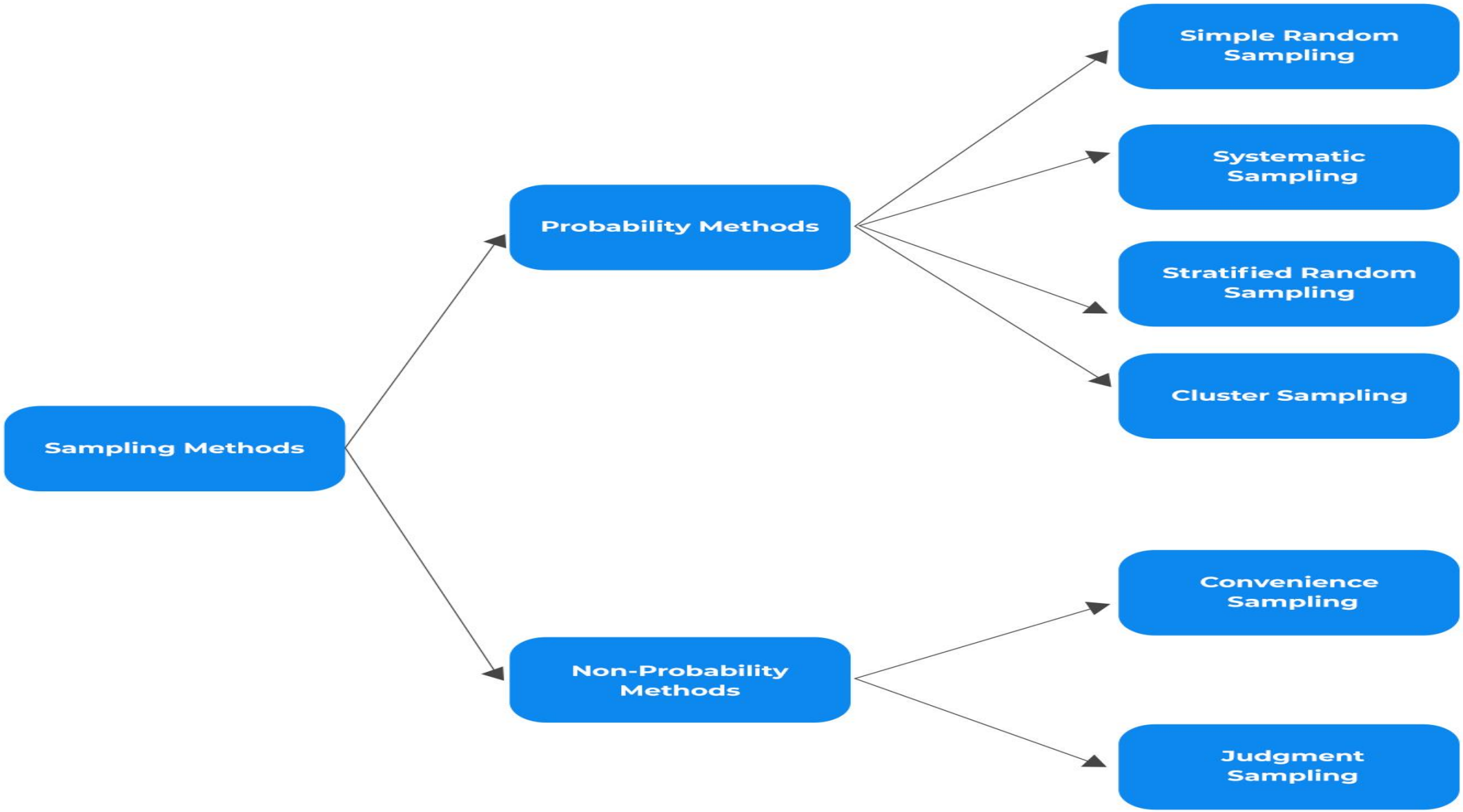
2	4	-1
-10	5	11
18	-7	6

2	-10	18
4	5	-7
-1	11	6

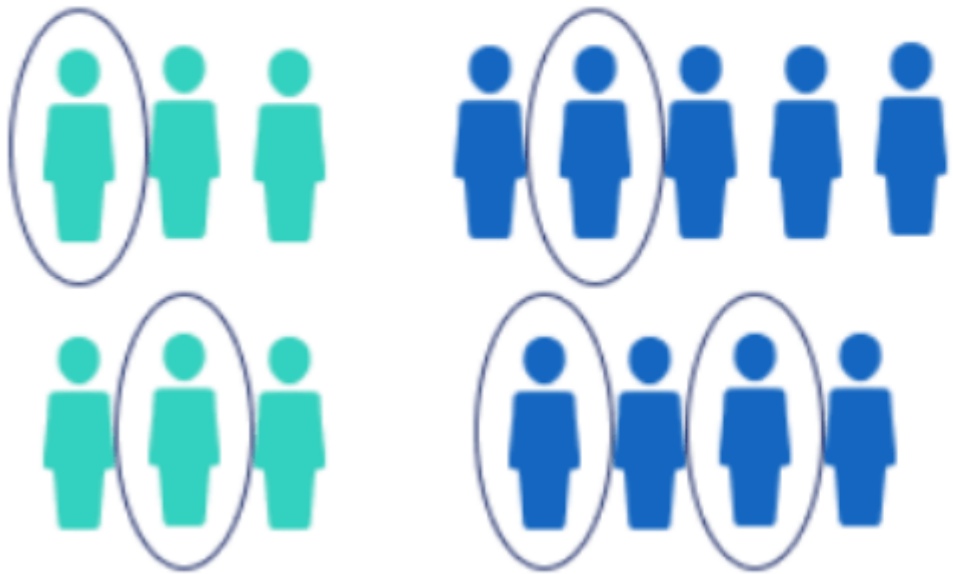
Task 4

Probability and non probability Sampling

Probability and Non-Probability Sampling

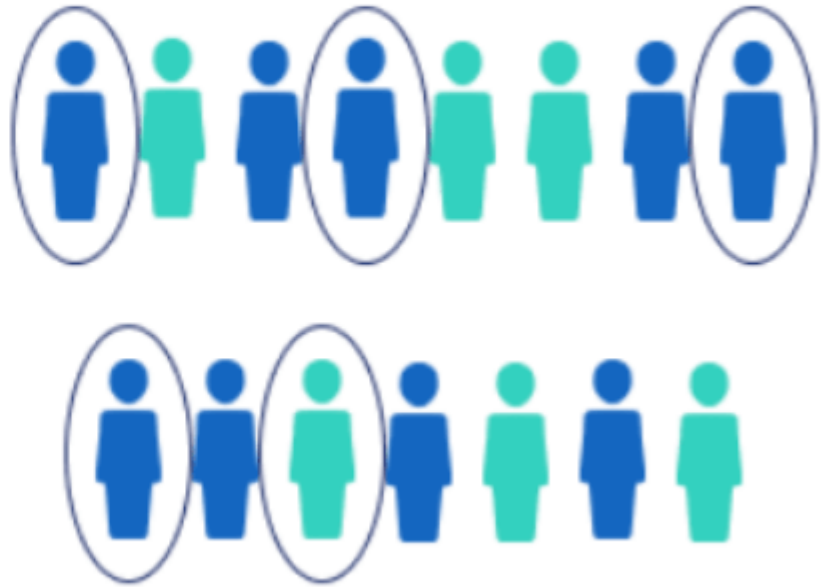


Stratified sample



VS

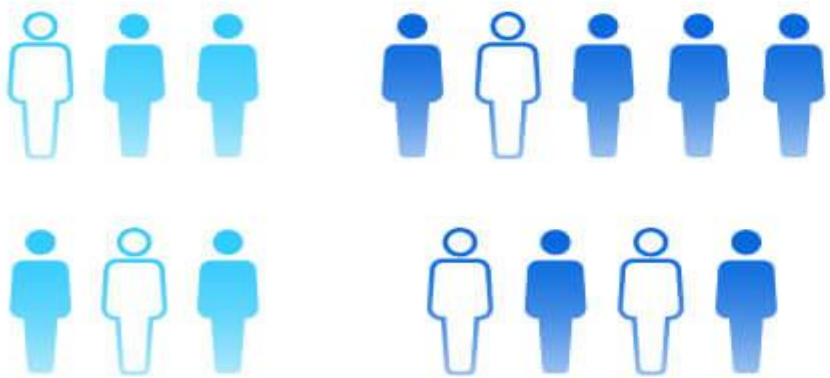
Simple random sample

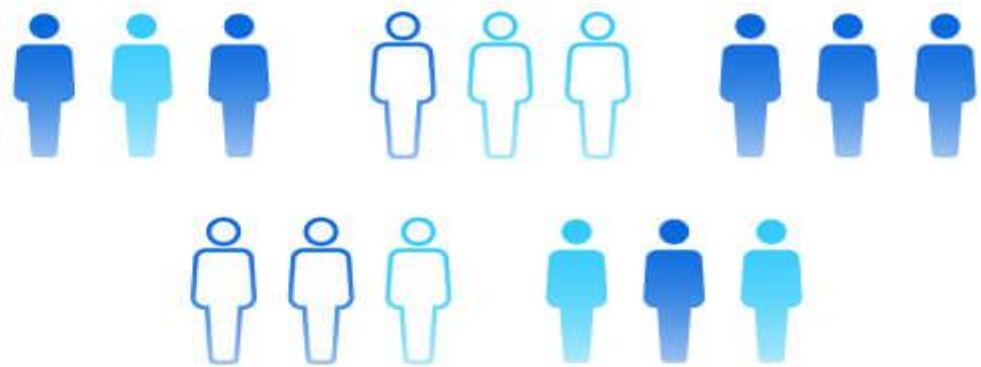


Stratified Random Sampling

VS

Cluster Sampling





QuestionPro

Task 5

Factors affect choosing Samples

Watch this:
https://www.youtube.com/watch?v=OITSHL9-OS0

Factors affect choice of sample size and sample technique:

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1. Size of universe
2. Nature of problem
3. Availability of resources
4. Level of accuracy required
5. Homogeneity and heterogeneity of the universe
6. Nature of study
7. Selection of sampling technique
8. Sample size
9. Attitude of respondents
10. Degree of variability
11. Time