OPKID: Optimal Kinematic Design of Robots
Kinematic Analysis and Design of a 3-$\underline{\text{R}}$RR planar parallel robot

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Experimental setup

The simulation for the 3 DOF 3-RRR planar parallel robot runs on a Java applet available at ParalleMIC. The default setup is shown in table 1.

Table 1: Default parameters of the 3-RRR parallel robot.

r	l_1	l_2	x	y	ϕ
50	100	80	0	0	0

Exercises

The following exercises have been performed:

1. The mobile platform is set with the end effector on a type 2 singularity locus. What can be observed for visualized solutions 1 and 2?
2. Default design parameters portrayed in table 1 were used and the end effector set at coordinates $x = -60$, $y = -40$ and $\phi = -40°$. After visualizing 6 real solutions to the direct kinematic problem, is it possible to have only 1 real solution?
3. The Java applet uses the Newton-Raphson iterative algorithm for the active joint variables modes. Why is crossing of the type 2 singularity in active-joint variables mode not possible?

Following observations have been made:

1. The assembly modes 1 and 2 for a given set of revolute joint positions, result in the same working mode, which is related to the inverse geometric model. In certain cases the solution for both assembly modes is virtually the same, ergo the end effector has the same position x, y and orientation ϕ.
2. No, it is not possible to get only 1 real solution, there will be a minimum of 2 real solutions. For the given set of coordinates of the moving platform the revolute joint positions were constant throughout 4 different working modes with values: $\theta_1 = 30$, $\theta_2 = 163$, $\theta_3 = 224$, when the assembly modes were calculated with default working modes setting at all limbs positive.
3. The Newton-Raphson (NR) algorithm iterates towards a solution using previous solution and it's derivative as portrayed in equation (1). For the type 2 singularity the determinant of matrix A derived from (2) is equal to zero, $det(A) = 0$. This violates the assumption of the NR algorithm.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{1}$$

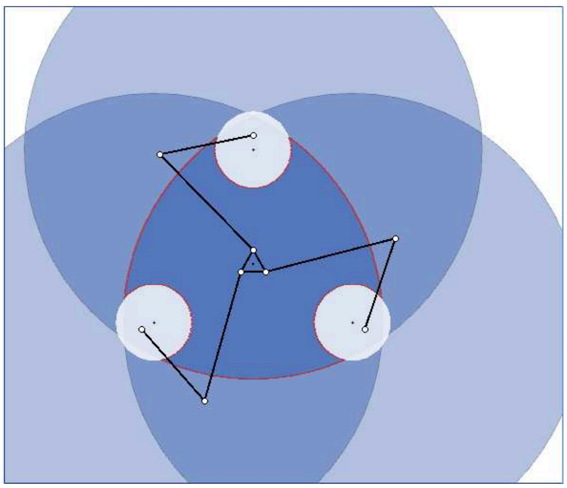
$$A\underline{\dot{\mathbf{x}}} + B\underline{\dot{\mathbf{q}}_{\mathbf{a}}} = 0 \tag{2}$$

Design rules

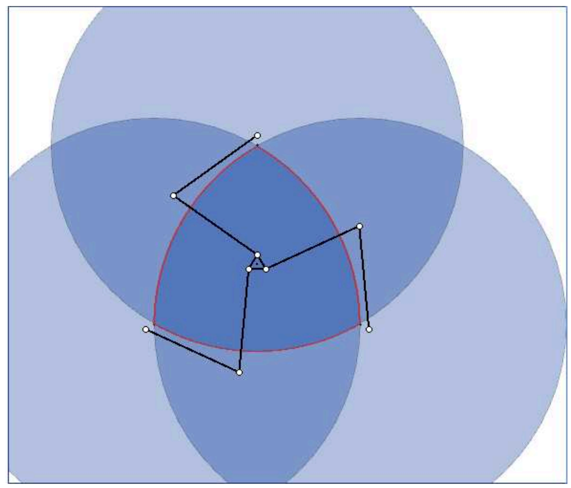
The following design rules have been determined:

- For a workspace without additional type 1 singularities than the boundaries, the link lengths need to be equal, $l_1 = l_2$.

- Decreasing the radius of the platform to link length ratio $\frac{r}{l}$ with links of the same length $l_1 = l_2$, increases the workspace.
- For link lengths $l_2 > l_1$ the span of the workspace increases, however because of additional singularities of type 1 inside (analogous to obstruction), the workspace area becomes more irregular as shown in (1a). It appears to be larger than in case of equal link lengths, however since there is no calculation of the area in the applet, a definitive statement cannot be made.
- From observations the radius r should be less than 60 for this 3-RRR robot.



(a) Suboptimal design: $r = 15$, $l_1 = 100$ and $l_2 = 140$.



(b) Optimal design: $r = 10$, $l_1 = 108$ and $l_2 = 108$.

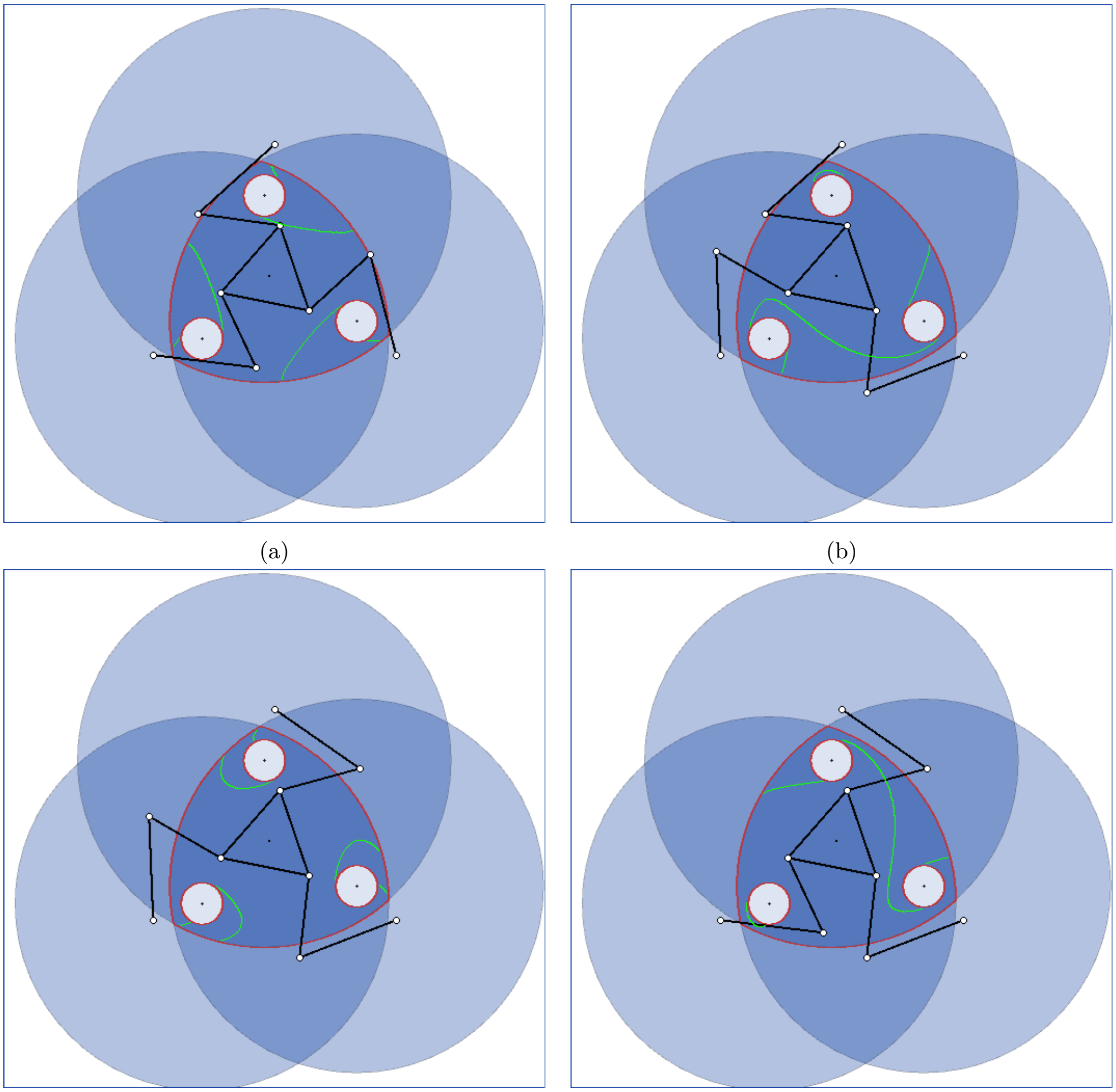
Figure 1: Observations of different parameter designs according to presented design rules.

According to the design rules and based on observations from , the design shown in (1b) appears to be optimal according to desired criteria, that is the highest workspace area without any additional singularities.

Working modes

As per definition the working modes do not change the position and orientation of the platform, but instead change the configuration of robot to another possible inverse kinematic solution. A limb changes aspect when its working mode is changed, except when the platform is in type 1 singularity configuration on the boundary of the working space. Theoretically the aspect do change in type 1 singularity on the boundary of the workspace, however since the two solutions result in the same configuration, there is no distinction. The type 1 singularities inside the workspace introduced by different link lengths do results in changing of aspects too when working mode is changed.

Since the analysed robot is fully parallel, ergo it has identical serial chains with one actuator, symmetry can be observed also in singularities. Where different working modes in (2a) and (2c) result in symmetrical singularity type 2 distribution across the workspace, the loci of the singularities type 2 in cases (2b) and (2d) are asymmetrical in their distribution across the workspace, but clearly symmetrical globally to each other. The same working modes for all 3 legs result in symmetrical distribution inside the workspace, usually along its border. Different working modes for each limb on the other hand result in type 2 singularities running across the middle of the workspace for the desired platform placement $x = 0$, $y = 0$ and $\phi = 0$.



(a) (b) (c) (d)

Figure 2: Different working modes for the default design parameters for the platform set at coordinates $x = -5$, $y = 8$ and $\phi = -11°$.

Additional observations

A reoccurring aspect led to experimentation with the *golden ratio* defined in (3) and expressed numerically in (4), where $a > b > 0$.

$$\phi_{GR} = \frac{a+b}{a} = \frac{a}{b} \tag{3}$$

$$\phi_{GR} = \frac{1+\sqrt{5}}{2} \approx 1.618 \tag{4}$$

All three design parameters r, l_1 and l_2 were parametrized by multiplication or division by the *golden ratio*.

Table 2: Experimentally determined design parameters of the 3-RRR parallel robot for the configuration $x = 0$, $y = 0$ and $\phi = 0$.

r	l_1	l_2	Workspace	Singularities
61	100	100	Area relatively high	Type 1 in W
61	61	100	Area relatively low and irregular	Type 2 inside W
61	100	161	Area relatively high, but irregular	Type 1 and 2 inside W
15	100	100	Area relatively high	None
10	108	108	Area relatively highest	None
10	108	156	Area relatively high	Type 1 in W

Since a smaller radius of the platform led to a larger workspace and the minimal value allowed by the Java applet was limited at $r = 10$, this value was multiplied by the *golden ratio* in an iterative manner to set other possible design values.

Using the *golden ratio* led to designs, where a *degree of freedom* for the moving platform around the z axis could be observed, resulting in a virtually infinite rotation angle of the platform ϕ. Practical realization problems of this type of design and numerical errors that should be taken into consideration however, since the 3 links would have to cross each other after a full rotation and the calculation errors might provide impractical solutions due to numerical aspects.

Conclusions

The Java applet relies on numerical methods for calculations and limits parameters to integers, therefore the analysis was limited. Fractional designs using relationships like the golden ratio should be taken into account, since they might introduce additional capabilities and significantly change performances of a robot. From observation during experiments it can be concluded that the radius of the platform r and it's ratio with respect to the links length l_1 and l_2 seem to be the most sensitive design parameters. Optimal design for the platform located at $x = 0$, $y = 0$ and $\phi = 0$ is the design $r = 10$, which is the minimal value for the radius and $l_1 = l_2 = 108$, where 3 type 1 singularities are just on the 3 vertices of the workspace W about to enter it, thus ensuring optimal design for this 3-RRR. Lastly it should be noted that designs, where the working modes of all limbs can be the same should be preferred, since it ensures that type 2 singularities will be along the borders of the workspace and can possibly be omitted for a given task.