Forced Oscillations

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Abstract

The aims of this experiment were to find the Q-value of a phosphor-bronze bar by investigating varying velocity wave-form amplitudes of the bar as a function of the driving frequency, with a constant drive amplitude. This was done by varying driving frequencies using a function generator and measuring the velocity wave-form amplitude of the bar's oscillations using an oscilloscope. The results of this experiment gave the Q-value of the phosphor-bronze bar to be 207 ± 2.

1. Introduction

The quality factor (Q-factor) is used to describe the motion of a vibrating system as it is a measure of how well a system oscillates. It describes the shape of the peak on the resonance curve. This has many applications from engineering to atomic and nuclear physics as the power resonance curves used here apply to many other physical situations [1]. During this experiment the amplitude was studied as a function of the driving frequency and used the power absorption curve to determine the quality factor of the oscillating system.

2. Theory

The motion of a body acting under forced oscillations can be described by

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega t \quad (1)$$

or

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F\_0/m \cos \omega t \quad (2)$$

where m is the mass, F_0 is the force and γ is the damping factor of the system [1].

Equations 1 and 2 have the solution

$$x = A \cos(\omega t + \phi) \quad (3)$$

Where x is the displacement, A is the amplitude of the oscillation and ϕ is the phase angle [1].

From equation 3 it is clear that the velocity (dx/dt) is

$$v = -A\omega \sin(\omega t + \phi) \quad (4)$$

and from equation 4 we can see that the velocity is proportional to the amplitude of the oscillation.

Oscillating bodies dramatically increase their amplitude of oscillation close to their natural frequency. A system with a high Q factor has a taller and sharper peak at the resonant frequency as the system with a higher Q factor has a greater increase in the amplitude as you get closer to the resonant frequency[2]. This phenomenon is shown in figure 1 below.

The quality factor is defined as

$$Q = \omega_0 / \gamma \qquad (5)$$

where ω_0 is the resonant angular frequency. [1]

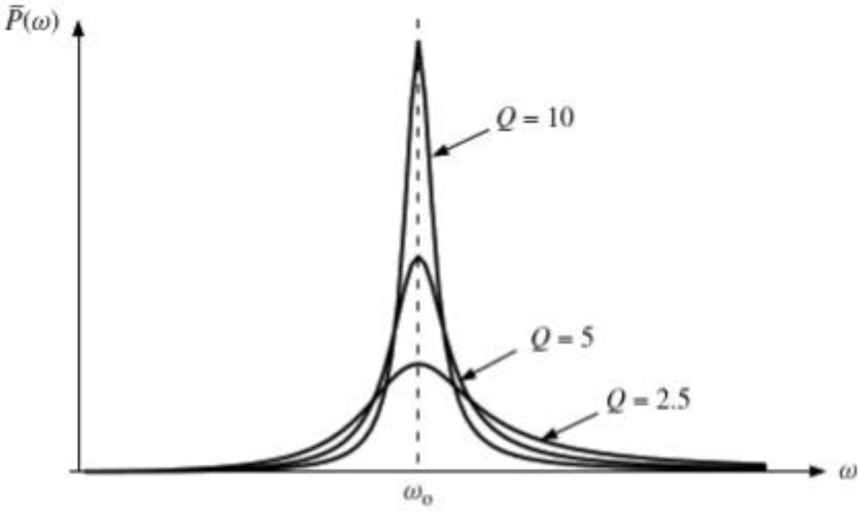

Fig 1 - Plot of the average power against angular frequency for different values of Q [1]

The overall power gained by the system is 0 so the power being absorbed by the system is equal to the power dissipated by the system. The power as a function of time is given by

$$P(t) = b[v_0\,(\omega)]^2 \sin^2(\omega t - \delta) \qquad (6)$$

so

$$P\ BAR\ (\omega) = \frac{1}{T}\int_{t_0}^{t_0+T} P(t)\,dt \qquad (7)$$

so

$$P\ BAR\ (\omega) = (b[v\_0\ (\omega)]^2)/2 \qquad (8)$$

from equation 8 we it is clear that the power is proportional to the square of the velocity. [1] Plotting the power against the angular frequency allows the damping coefficient (γ) to be measured. γis equal to the full width half maximum (ω_FWHM) that is the width of the curve at half the maximum power (illustrated in figure 2).

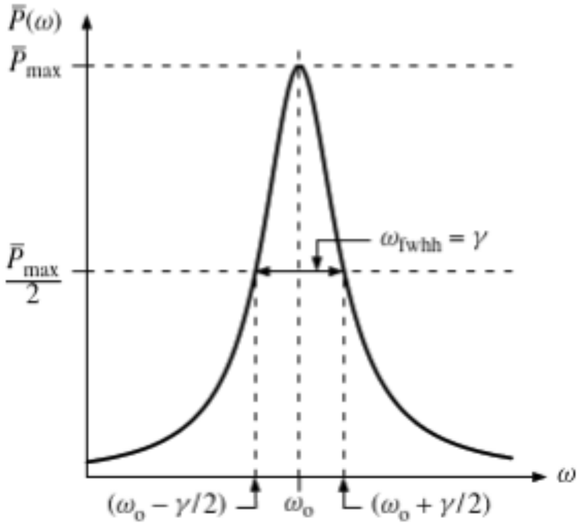

Fig 2 - Plot of power resonance curve illustrating the full width half maximum [1]

Equation 5 now becomes

$$Q=\omega_0 / \omega_{FWHM} \tag{9}$$

This simplifies down to

$$Q= \omega_0/f_0 \tag{10}$$

Which makes it a simple calculation to determine the Q factor experimentally.

3. Method

A function generator was used to create a driving force with constant amplitude by connecting it to an electromechanical transducer, consisting of a loudspeaker coil which drives a rod by transferring energy through an elastic band. The function generator was used to create sine-waves of varying frequencies as the driving force.

Previous to starting the experiment it was checked that the oscillator did not hit the horseshoe magnet at resonant frequency so that amplitude could be accurately measured and that the current through the helmholtz coils was 0A so that no eddy currents would dampen the oscillators motion.

The amplitude of the bar's oscillations at a given frequency was found by using a coil of 20 turns of copper wire on the end of the bar which is free to oscillate, this coil passed through the poles of a horseshoe magnet to generate a voltage proportional to the velocity of the bar, as seen in equation 4 this is proportional to amplitude. The generated voltage was then magnified for viewing on an oscilloscope, a piece of equipment which allows electrical functions to be seen alongside the driving voltage as well as displaying useful measurements such as peak to peak and phase difference. The set up of the equipment described is shown below in figure 3.

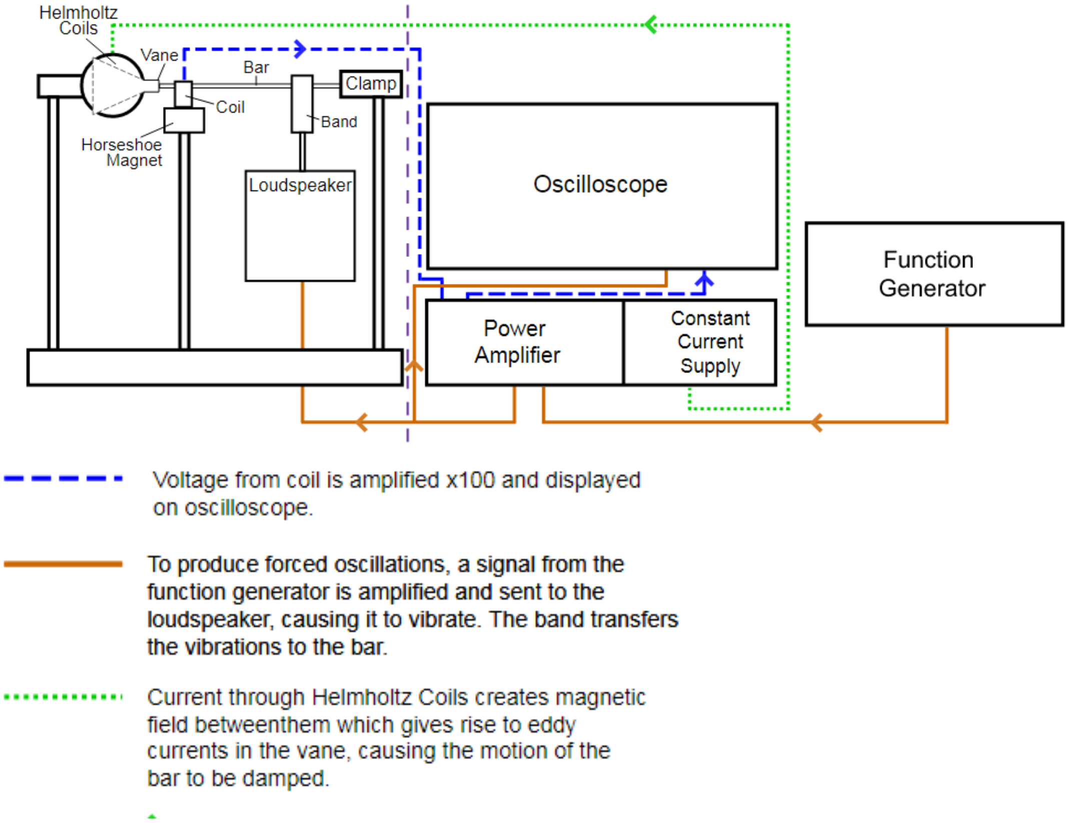

Fig 3 – A visual representation of the equipment used and how it was set up [4].

An initial driving frequency of 15Hz was used with 1Hz increment from 15Hz-16Hz and from 18Hz-20Hz, and 0. 1Hz increments from 16. 5Hz to 17. 5Hz, this was done to increase the data points near resonant frequency. For each driving frequency the corresponding amplified voltage was found by measuring 5 separate peak to peak values of the velocity waveform from the oscilloscope, averaging these values for each frequency and recording the results in a table, shown in table 1 in the appendix. Uncertainties for these amplified voltages were calculated from these measurements and recorded in the same table [Table 1] to be used for later data analysis.

Once all of the data had been correctly recorded, we plotted a graph of voltage squared against frequency to a lorentzian fit using origin.

4. Results and analysis

The data we obtained to find the Q-factor of the bronze phosphor bar is represented in the graph of voltage squared against driving frequency [Fig. 4] below and exact values can be found in [Table 1] in the appendix. Uncertainties for velocity squared values were found by finding the standard deviation of the peak-to-peak values, and then multiplying this by twice the velocity squared value divided by the peak to peak value.

This graph was plotted using origin software, a program used to create graphs and plots, by setting the program to give a graph of lorentzian fit.

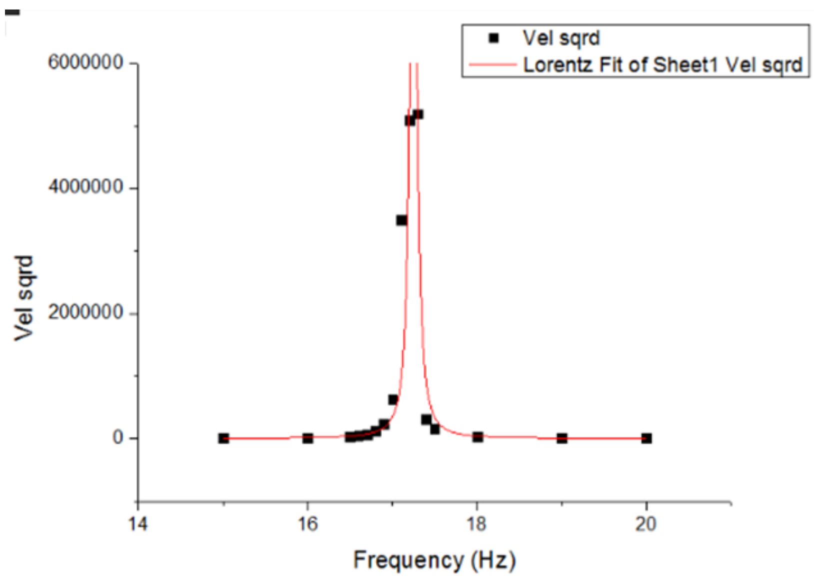

Fig 4 – Summary of our data as a plot of velocity squared against frequency. Error bars are present but aren't visible. Unfortunately Origin cut off the top of the graph.

The origin program also outputs useful values shown below [Table 1].

Model	Lorentz		
Equation	y = y0 + (2*A/PI)*(w/(4*(x-xc)^2 + w^2))		
Reduced Chi-Sqr	1.52249E9		
Adj. R-Square	0.93917		
		Value	Standard Error
Vel sqrd	y0	1001.39627	31333.82109
	xc	17.2495	0.00322
	w	0.08352	0.03998
	A	1.66034E6	205778.90879
	H	1.26552E7	

Table 1 – Showing values for resonant frequency (xc), Reduced Chi-Squared and angular velocity (w) and their respective standard deviations.

The χ_R^2 value of 1. 52x10^9 here is unacceptable and suggest a very poor lorentzian fit. The uncertainty of the Q-factor is found by adding the partial differentials of equation__ with respect to w and f0 multiplied by their respective standard deviations and added in quadrature.

From equation __ the value for the Q factor is found from values for angular velocity and resonant frequency. Using all of this we obtain a final result of Q= 207 ± 2.

5. Discussion

The uncertainty in our value is quite low, <1%. Human error is not a large factor in this experiment as all readings could be taken slowly from the oscilloscope and were accurate to a fairly high number of decimal places. Problems in the uncertainties of the peak to peak values arise when 5 of the same result is output giving an uncertainty of 0 which is not possible. Due to the oscilloscope being able to give a range of precision the uncertainties for these values must be considered separately and not all as half of the same smallest deviation. This can be seen in our table of results [table 1] in the appendix.

The large χR2 value was due to the poor quality of our data as we did not have enough data points around the resonant frequency to get an accurate fit. For future experiments testing the Q-factor of this material we would recommend smaller increments near the resonant frequency, perhaps 0. 01Hz from 17. 1Hz to 17. 3Hz .

As the Q-factor is not a property of a material and depends on thickness, length and so are unique for each different object being tested, this it is difficult to compare the experimental value with a standard value.

6. Summary

In conclusion the value for the quality factor of the system determined by this experiment is useless as the lorentz distribution did not fit the data so it was not a valid method of determining the ω_(FWHM.)The issue with the experiment is that the data was not of a high enough quality to enable a good calculation of the ω_FWHM. To improve this experiment more data should have been taken about the resonant frequency so that a better fit of the data could be achieved. It is not possible to compare the value of the Q-factor obtained by this experiment as the Q-factor depends on the material and the dimensions so it cannot be compared to a true value.

7. References

[1] G. C. King, 2009, Vibrations and Waves, Manchester Physics Series,Wiley, chapter3

[2] First year laboratory Forced oscillations Dr. F. K. Loebinger, January 1994

[3] The pre lab theory, accessed November 2019

[4] Blackboard Forced oscillations folder, accessed November 2019

8. Appendix

Driving frequency/Hz	Peak to Peak/ mV					Average Peak to Peak/ mV
	1	2	3	4	5	
15	108	124	124	108	108	116
16	208	212	204	204	204	206. 4
16. 5	372	372	364	368	376	370. 4
16. 6	448	428	440	436	440	438. 4
16. 7	536	536	540	540	536	537. 6
16. 8	696	696	704	704	704	700. 8
16. 9	968	960	968	968	968	966. 4
17	1560	1560	1580	1600	1620	1584
17. 1	3760	3720	3720	3760	3760	3744
17. 2	4520	4520	4520	4520	4480	4512
17. 3	4560	4560	4560	4560	4560	4560
17. 4	1120	1120	1120	1120	1120	1120
17. 5	800	800	800	800	800	800
18	400	324	326	326	328	340. 8
19	164	164	164	164	162	163. 6
20	124	116	115	115	115	117

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