

A flexible integer linear programming formulation for scheduling clinician on-call service in hospitals

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Abstract

Scheduling of personnel in a hospital environment is vital to improving the service provided to patients and balancing the workload assigned to clinicians. Many approaches have been tried and successfully applied to generate efficient schedules in such settings. However, due to the computational complexity of the scheduling problem in general, most approaches resort to heuristics to find a non-optimal solution in a reasonable amount of time. We designed an integer linear programming formulation to find an optimal schedule in a clinical division of a hospital. Our formulation mitigates computational complexity issues by maintaining a minimal set of constraints, yet still provides the flexibility necessary to adapt it to a variety of clinical divisions.

We then conducted a case study for our approach using data from the Infectious Diseases division at St. Michael's Hospital in Toronto, Canada. We analyzed and compared the results of our approach to manually-created schedules at the hospital, and found improved adherence to departmental constraints and clinician preferences. We used simulated data to examine the sensitivity of the runtime of our linear program for various parameters and observed reassuring results, signifying the practicality and generalizability of our approach in different real-world scenarios.

1 Introduction

Hospital departments provide services where patient needs, and thus the system's demands, often exceed the available supply. In particular, on-call schedules for a fixed number of health-care providers are central to the efficient running of hospitals. Carefully allocated schedules should balance sufficient staff with clinician workload, to maximize quality of care. It is common for on-call schedules in hospitals to be created manually. However, manually-created schedules are subject to three problems. First, when there are a large number of clinicians, or the constraints that need to be satisfied by the schedule are complex, a manual method may not provide a schedule that satisfies all constraints. Second,

manual methods may overlook certain constraints that must be maintained to have an operational department, such as preventing many consecutive work blocks from being assigned or ensuring clinicians are allocated a specific amount of work blocks throughout the year. Third, manual scheduling is often time-consuming. For these reasons, it is important to develop automated methods that can efficiently generate schedules that satisfy the given constraints.

Automated methods to generate schedules have been studied and applied in many industries, including transportation [1, 2, 3], manufacturing [4, 5, 6], retail [7, 8], and military [9, 10]. Of special interest to a clinician on-call scheduling problem are the approaches to scheduling nurses, who often work in shifts. In the nurse scheduling problem, the goal is to find an optimal assignment of nurses to shifts that satisfies all of the hard constraints, such as hospital regulations, and as many soft constraints as possible, which may include nurse preferences. A wide variety of approaches, including exact and heuristic approaches, have been used to solve the nurse scheduling problem: integer linear programming [11, 12, 13], network flows [14], genetic algorithms [15, 16, 17], simulated annealing [18], and artificial intelligence [19, 20]. A comprehensive literature review of these and other methods applied to nurse scheduling is presented in [21].

Many of the approaches to nurse scheduling were designed to satisfy the requirements of a specific hospital department, which results in a large number of variables and constraints to be incorporated into the problem formulation. While these department-specific approaches allow end-users to find precise schedules that satisfy the needs of that department and the preferences of nurses and clinicians in that department, they are difficult to adapt to other departments. Moreover, the large number of variables and constraints also leads to computational complexity issues [22], especially when trying to find the most optimal solution. In this paper, we tackle a version of the nurse scheduling problem arising from a case study of one clinical division, providing two different services (general infectious disease (ID) consults; and HIV consults) at St. Michael’s Hospital in Toronto, Canada. Our goals are to (1) present a integer linear programming (ILP) formulation for the scheduling problem, and describe the flexibility of this formulation for solving similar problems; (2) compare the performance of an ILP-based scheduling tool to the results of a manual approach; and (3) analyze the robustness of this approach in difficult instances of the problem.

We begin by describing the details of the problem in Section 2, and presenting our ILP formulation in Section 3. Next, we compare the results of our formulation to manually-created schedules, and evaluate the performance of the algorithm on simulated data in Section 4. Finally, we discuss and interpret the results in Section 5.

2 Problem

At St. Michael’s Hospital, the division of infectious diseases (ID) offers separate but concurrent services for general ID and HIV consultation. Each service provides clinical care throughout the year, during regular working hours and on weekends and holidays. The schedule is created in advance, outlining all work-week and weekend shifts for the full year. In the yearly schedule clinicians are assigned to “blocks” of two consecutive regular work weeks and individual weekends. Apart from long (holiday) weekends, a work week starts on Monday at 8 A.M. and ends on Friday at 5 P.M. Accordingly, weekend service starts on Friday at 5 P.M., and ends on Monday at 8 A.M. During the weekend, ID and HIV consultation services are combined and provided by one clinician. During the regular work week the ID and HIV services are led by one clinician each.

Two types of constraints are placed on the clinician assignments. First, each clinician must work between a minimum and maximum number of blocks of each service during the year. For instance,

one clinician might have to provide 3-5 blocks of general ID service and 2-3 blocks of HIV service throughout the year. These limits may be different for each clinician, and they may change from year to year as the number of clinicians in the department changes. Second, the schedule should not assign a clinician to work for two consecutive blocks or two consecutive weekends. The schedule should also distribute regular weekends and holiday weekends each equally among all clinicians.

In addition to balancing the work load among clinicians, the schedule should accommodate their preferences. Clinicians provide their requests for time off before schedule generation so that the requests may be integrated into the schedule. Clinician may specify days, weeks or weekends off, with the understanding that any blocks overlapping with their request will be assigned to a different clinician where possible. For example, if a clinician only requests a given Monday and Tuesday off, the schedule should avoid assigning the entire block to that clinician. Clinicians also typically prefer to have their weekend and block assignments side by side, so the schedule should accommodate this where possible. A summary of the outlined constraints is given in Table 1.

Table 1: Summary of the constraints for the clinician scheduling problem

Constraint Name	Description	Type
Block Coverage (BC)	each service needs to have exactly one clinician that covers any given block	Hard
Weekend Coverage (WC)	every weekend needs to have exactly one clinician that covers it	Hard
Min/Max (MM)	for a given service, each clinician can only work between the minimum and maximum number of allowed blocks	Hard
No Consecutive Blocks (NCB)	any clinician should not work two consecutive blocks, across all services	Hard
No Consecutive Weekends (NCW)	any clinician should not work two consecutive weekends	Hard
Equal Weekends (EW)	weekends should be equally distributed between clinicians	Hard
Equal Holidays (EH)	long weekends should be equally distributed between clinicians	Hard
Block Requests (BR)	each clinician can request to be off service during certain blocks throughout the year	Soft
Weekend Requests (WR)	each clinician can request to be off service during certain weekends throughout the year	Soft
Block-Weekend Adjacency (BWA)	the block and weekend assignments of a given clinician should be adjacent	Soft

In most scheduling problems, the constraints can be divided into hard and soft constraints. Hard constraints must be satisfied by any candidate solution, while soft constraints can be used to select a more favourable solution from a set of candidate solutions. Typically, soft constraints are therefore encoded as objective functions which are maximized when finding a solution. In the case of the clinician scheduling problem described above, we chose Block Requests, Weekend Requests and Block-Weekend Adjacency as our soft constraints, while the rest of the constraints are hard. While it is important to take clinician requests into account when constructing the schedule, it is crucial that the work-load of the schedule is balanced among all clinicians, and all services are assigned. After the schedule is generated, clinicians may exchange certain weeks or days throughout the year. Our approach to schedule generation does not consider such future exchanges, and instead generates on one full year at a time.

3 Methods

In this section, we present an application of integer linear programming to solve the clinician scheduling problem presented in Section 2. First, we describe the sets, indices and variables necessary for the formulation of the problem. We then write the constraints given in Table 1 as mathematical equations, and define the objective function of the linear program.

3.1 Sets and Indices

We denote the set of all services that clinicians in a single department can provide as \mathcal{S} . The following formulation assumes that all clinicians are able to provide all services. The set of all clinicians in the department is denoted as \mathcal{C} . The sets of blocks and weekends are denoted as \mathcal{B} and \mathcal{W} respectively. The block size used in our experiments is 2 weeks, but the following LP formulation does not require a particular size for blocks, and so it can be adapted for other cases. A subset of weekends are denoted \mathcal{L} , corresponding to established long/holiday weekends such as the Canadian Civic Day weekend in August. Lastly, the block and weekend request-off dates of clinicians as are denoted as the following subsets of all blocks and weekends: \mathcal{BR}_c and \mathcal{WR}_c , respectively. For instance, if clinician c 's request-off dates intersect with blocks 1 and 2, and weekend 1, then $\mathcal{BR}_c = \{1, 2\}$ and $\mathcal{WR}_c = \{1\}$. Table 2 presents a summary of the sets and indices described.

Table 2: Description of sets and indices in the problem

Set	Index	Description
$\mathcal{S} = \{1, \dots, S\}$	s	services
$\mathcal{C} = \{1, \dots, C\}$	c	clinicians
$\mathcal{B} = \{1, \dots, B\}$	b	blocks
$\mathcal{W} = \{1, \dots, W\}$	w	weekends
$\mathcal{L} \subset \mathcal{W}$		long weekends
$\mathcal{BR}_c \subset \mathcal{B}$		block requests of clinician c
$\mathcal{WR}_c \subset \mathcal{W}$		weekend requests of clinician c

3.2 Variables

Since each clinician may be assigned to work on any service, during any block of the year, we denote such an assignment as a binary variable $X_{c,b,s} \in \{0, 1\}$. A value of 1 indicates that clinician c is assigned to service s during block b , while a value of 0 indicates they are not assigned. Weekend assignments are similarly defined using a binary variable $Y_{c,w} \in \{0, 1\}$, but without a service index, as clinicians are expected to provide all services during the weekends. We then define $m_{c,s}$ and $M_{c,s}$ to represent the minimal and maximal number of blocks of service s that clinician c is required to work during the year. Table 3 presents a summary of the constants and variables in the problem.

3.3 Constraints

We now formalize the hard constraints in Table 1 using the variables defined above.

Table 3: Description of variables and constants in the problem

Name	Description
$X_{c,b,s} \in \{0, 1\}$	assignment of clinician c to service s on block b
$Y_{c,w} \in \{0, 1\}$	assignment of clinician c on weekend w
$m_{c,s}$	minimum number of blocks clinician c should cover on service s
$M_{c,s}$	maximum number of blocks clinician c should cover on service s

$$\sum_{c=1}^C X_{c,b,s} = 1 \quad \forall b \in \mathcal{B}, s \in \mathcal{S} \quad (\text{BC})$$

$$\sum_{c=1}^C Y_{c,w} = 1 \quad \forall w \in \mathcal{W} \quad (\text{WC})$$

$$m_{c,s} \leq \sum_{b=1}^B X_{c,b,s} \leq M_{c,s} \quad \forall c \in \mathcal{C}, s \in \mathcal{S} \quad (\text{MM})$$

$$X_{c,b,s} + X_{c,b+1,s} \leq 1 \quad \forall c \in \mathcal{C}, b \leq B-1, s \in \mathcal{S} \quad (\text{NCB})$$

$$Y_{c,w} + Y_{c,w+1} \leq 1 \quad \forall c \in \mathcal{C}, w \leq W-1 \quad (\text{NCW})$$

$$\left\lfloor \frac{W}{C} \right\rfloor \leq \sum_{w=1}^W Y_{c,w} \leq \left\lceil \frac{W}{C} \right\rceil \quad \forall c \in \mathcal{C} \quad (\text{EW})$$

$$\left\lfloor \frac{|\mathcal{L}|}{C} \right\rfloor \leq \sum_{w \in \mathcal{L}} Y_{c,w} \leq \left\lceil \frac{|\mathcal{L}|}{C} \right\rceil \quad \forall c \in \mathcal{C} \quad (\text{EH})$$

3.4 Objectives

As described in Section 2, the soft constraints of the clinician scheduling problem include: satisfying clinician block off requests (BR), satisfying clinician weekend off requests (WR), and assigning weekends closer to blocks (BWA). We convert these soft constraints into linear objective functions of the binary variables defined in Section 3.2. Objectives (BR) and (WR) can be written as the following linear functions:

$$Q_1(X) = \sum_{c=1}^C \sum_{b=1}^B \sum_{s=1}^S (-1)^{\mathbf{1}(b \in \mathcal{BR}_c)} \cdot X_{c,b,s} \quad (\text{BR})$$

$$Q_2(Y) = \sum_{c=1}^C \sum_{w=1}^W (-1)^{\mathbf{1}(w \in \mathcal{WR}_c)} \cdot Y_{c,w} \quad (\text{WR})$$

where $\mathbf{1}(P)$ is the indicator function that has value 1 when predicate P holds and 0 otherwise. In the above two objectives, we penalize any assignments that conflict with a block or weekend request, and aim to maximize the non-conflicting assignments.

The Block-Weekend Adjacency is optimized by considering the product $X_{c,b,s} \cdot Y_{c,w}$ for values of w “adjacent” to the value of b . This leads to the maximization objective

$$Q_3(X, Y) = \sum_{c=1}^C \sum_{b=1}^B \sum_{s=1}^S X_{c,b,s} \cdot Y_{c,w=\varphi(b)} \quad (\text{BWA})$$

where $\varphi(b)$ is a one-to-one mapping of a block to an adjacent weekend, by some appropriate definition of adjacency. For instance, clinicians might want to be assigned during a weekend that falls within an assigned block. In this case, we will have $\varphi(b) = 2b - 1$.

However, as it is, Q_3 is not a linear function of its variables and cannot be optimized in a linear programming framework. An approach used to convert such functions into linear objectives involves introducing a helper variable and additional constraints [23]. In our case, introducing a variable $Z_{c,b,s}$ for every product $X_{c,b,s} \cdot Y_{c,w}$ with $w = \varphi(b)$, and constraining Z such that

$$Z_{c,b,s} \leq X_{c,b,s} \tag{1}$$

$$Z_{c,b,s} \leq Y_{c,w=\varphi(b)} \quad \forall s \in \mathcal{S} \tag{2}$$

allows us to rewrite Q_3 as a linear function of Z ,

$$Q_3(Z) = \sum_{c=1}^C \sum_{b=1}^B \sum_{s=1}^S Z_{c,b,s} \tag{3}$$

Indeed from Eqns. (1) and (2), whenever $X_{c,b,s} \cdot Y_{c,w} = 1$ (respectively, 0), $Z_{c,b,s}$ can attain a maximum value of 1 (respectively, 0), giving us the correct adjacency maximization objective.

In order to optimize all objectives simultaneously, we optimize a weighted sum of the normalized objective functions,

$$\max_{X,Y,Z} \alpha \bar{Q}_1(X) + \beta \bar{Q}_2(Y) + (1 - \alpha - \beta) \bar{Q}_3(Z) \tag{4}$$

where \bar{Q}_i is the normalization of objective Q_i , and $0 \leq \alpha, \beta \leq 1$. This method guarantees an optimal solution to be Pareto optimal [24].

4 Results

4.1 Implementation

We developed a software package with a user interface that implements the above linear program and allows configuration of clinicians at [ref ??], to be used by the ID division at St. Michael's Hospital. The software was used to generate the results in the following sections, using real data as well as simulated data as input. All the following experiments were conducted on an Intel Core i7-4770k CPU @ 3.50 GHz and 16 GB of RAM running 64-bit Windows 10. The software was implemented using the COIN-OR Branch-and-Cut open source solver version 2.9.9 [25].

4.2 Comparison with Manually Generated Schedules

We used clinician time-off requests and minimum/maximum requirements from 2015-2018 as input data for the ILP problem. Table 4 compares the optimal schedule generated using the software with the manually-created schedule for data from 2018. The schedule is color-coded to distinguish between the different clinicians.

First, we evaluated the automatically-generated schedule by comparing it with the manually-created schedule. Specifically, we examined the adherence of each schedule to the constraints presented in Table 1. As shown in Table 5, the optimal schedule generated using the software satisfied all mandatory

Table 4: Comparison of automatically generated (LP Solution) and manually generated (Historical Data) schedules for 2018

Week #	LP Solution			Historical Data		
	HIV	ID	Weekend	HIV	ID	Weekend
1	A	E	E	A	E	H
2	A	E	H	A	E	A
3	B	F	F	B	H	G
4	B	F	H	B	H	I
5	A	G	A	A	G	F
6	A	G	E	A	G	C
7	B	C	C	A	F	B
8	B	C	G	D	C	G
9	D	I	D	B	C	D
10	D	I	H	B	D	H
11	A	B	B	A	B	F
12	A	B	I	A	B	A
13	C	F	F	C	H	H
14	C	F	I	C	H	I
15	A	H	H	B	I	C
16	A	H	D	B	I	E
17	B	E	B	A	E	D
18	B	E	H	A	E	E
19	A	I	I	A	C	F
20	A	I	D	A	C	C
21	C	D	C	B	G	A
22	C	D	I	B	G	C
23	B	F	F	C	F	D
24	B	F	G	C	F	C
25	A	H	A	C	C	G
26	A	H	H	D	I	D
27	C	D	D	A	B	E
28	C	D	E	A	B	I
29	A	B	A	B	D	D
30	A	B	B	B	D	A
31	C	E	C	C	F	E
32	C	E	A	C	F	F
33	B	D	D	B	F	I
34	B	D	E	B	I	C
35	A	I	A	A	G	G
36	A	I	G	A	G	I
37	D	C	C	D	C	A
38	D	C	F	D	D	E
39	B	E	E	A	B	D
40	B	E	B	B	I	I
41	A	G	G	B	I	G
42	A	G	E	D	F	F
43	C	I	C	C	F	C
44	C	I	I	D	E	I
45	A	F	A	D	E	E
46	A	F	F	A	B	A
47	B	G	B	A	B	D
48	B	G	I	A	D	G
49	D	C	D	A	D	F
50	D	C	B	B	G	G
51	B	G	G	B	G	E

constraints. In contrast, the manually-created schedule did not satisfy all mandatory constraints. In particular, we see that the manual schedule assigned clinicians to multiple consecutive blocks in all four years. Moreover, the manual schedule did not have an equal distribution of weekends and holidays for all four years of data. Considering all objectives, we see that LP outperforms the manually created schedule in all four years, by accommodating almost all time-off requests and ensuring that weekends are always assigned close to blocks.

4.3 Influence of Problem Complexity of Runtime

Next, we examined the influence of the following four parameters on the runtime of the ILP solver using simulated data: number of clinicians; number of services offered; number of time-off requests per clinician per year; time-horizon of the schedule.

We simulated a department with 10 clinicians offering two services, similar to the department at St. Michael’s Hospital. Each clinician was configured with $[0, 5, 10, 15]$ non-overlapping block requests. The effect of an increasing number of requests per clinician on the runtime of the ILP solver is shown in Figure 1. The increase in the number of requests did not affect the runtime of the ILP solver, indicating that it can accommodate a lot of flexibility in clinician requests. Moreover, we see that all four runs were completed in under 1 second.

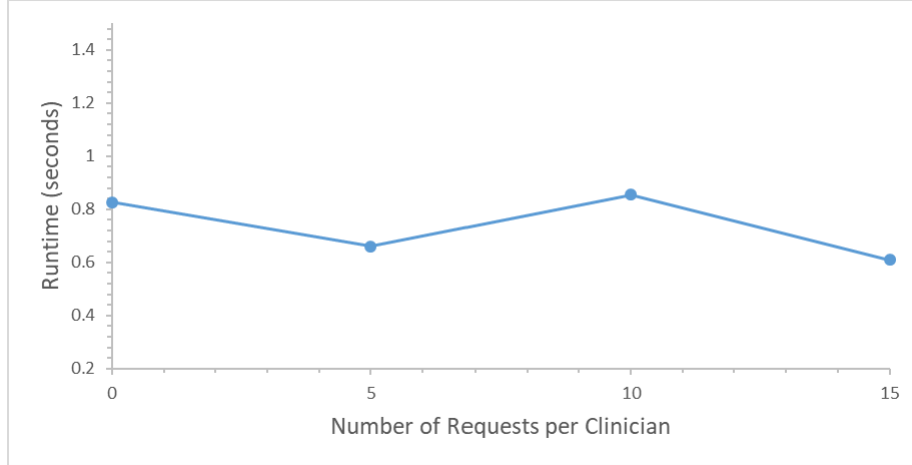


Figure 1: Runtime of ILP solver with an increasing number of requests per clinician

Figure 2 presents the change in runtime when increasing the number of 2-week blocks $B = \{26, 52, 78, 104\}$ in a department with 10 clinicians offering two services. There does not seem to be a strong effect on runtime when we lengthen the time-horizon of the schedule, and in fact the solver was able to find an optimal schedule for all time horizons within 3 seconds.

The effect of increasing the number of services on the runtime of the program is shown in Table 6. We executed the algorithm for $S = \{1, 2, 3\}$ total services and $C = \{10, 20, 30, 50\}$ clinicians in total across all services. For 2 concurrent services, a roster of 30 or more clinicians becomes impractical to schedule, as searching for a solution required over 24 hours. We saw similar issues for a roster of 20 or more clinicians assigned to a division with 3 concurrent services. However, when relaxing the NCB constraint, we saw a great improvement in runtime for divisions with 2 and 3 services, and we were able to generate a schedule with upwards of 50 clinicians in under 1.5 seconds.

Table 5: Comparison of constraint satisfaction and objectives values in LP-generated and historical schedules.

	2015		2016		2017		2018	
	LP	Historical	LP	Historical	LP	Historical	LP	Historical
Constraint								
Block Coverage	✓	✓	✓	✓	✓	✓	✓	✓
Weekend Coverage	✓	✓	✓	✓	✓	✓	✓	✓
Min/Max	✓	✓	✓	✓	✓	✓	✓	✓
No Consecutive Blocks	✓		✓		✓		✓	
No Consecutive Weekends	✓		✓		✓	✓	✓	✓
Equal Weekends	✓		✓		✓		✓	
Equal Holidays	✓		✓	✓	✓		✓	✓
Objective								
Satisfied Block Requests	123/129	121/129	120/126	116/126	99/99	95/99	124/128	121/128
Satisfied Weekend Requests	113/113	111/113	119/119	112/119	75/75	75/75	115/115	113/115
Adjacent Block-Weekend Assignments	26/26	9/26	26/26	6/26	26/26	7/26	26/26	5/26

The objective denominator represents: the total number of requests submitted by clinicians for request objectives; and the total number of possible adjacencies for adjacency objectives.

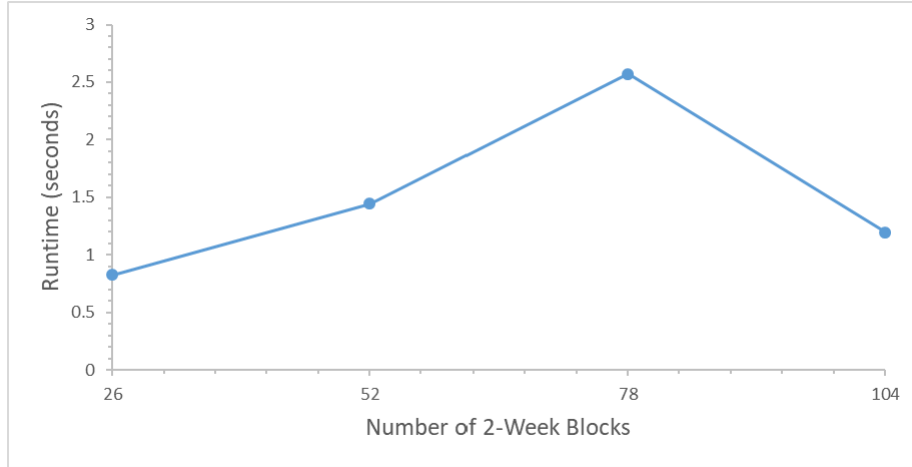


Figure 2: Runtime of ILP solver with an increasing number of 2-week blocks

Table 6: Comparison of program runtime (in seconds) for different numbers of services and total clinicians in the division.

	Number of Services				
Number of Clinicians	1	2	2 (without NCB)	3	3 (without NCB)
10	0.16	0.74	0.16	1.40	0.23
20	0.25	7468.86	0.32	–	0.43
30	0.42	–	0.49	–	0.66
50	0.62	–	0.82	–	1.14

Notes: “–” indicates that no solution was found within 24 hours; “without NCB” indicates that the No Consecutive Blocks (NCB) constraint was relaxed.

5 Discussion

In this paper, we present a simple, yet flexible, integer linear programming formulation to generate schedules for clinical departments at hospitals. The challenge in applying ILP to the task of scheduling clinicians lies in the computational complexity of finding an optimal solution. As the size of the scheduling problem grows, due to a larger roster of clinicians or more complicated constraints, the time it takes to generate an optimal schedule may grow exponentially. As a result, the majority of approaches taken to create schedules for similar scenarios tend to use heuristics in order to find an approximately optimal solution in a shorter time [21].

We presented a formulation that includes both hard constraints to ensure the schedule satisfies hospital and logistics requirements, and a multi-goal objective function to satisfy soft constraints (work preferences of clinicians). Although we restricted our application of the formulation to a set of constraints for the particular needs of the case study (St. Michael’s Hospital Division of Infectious Diseases), our formulation can be adapted to various clinical departments at different hospitals. The flexibility of our ILP allows changing the number of services provided in a division, the length of a work block, clinicians’ preference for block to weekend adjacency as well as clinicians’ requests for time off.

When comparing the optimal schedule generated by our tool to the manually-created schedules at St. Michael’s Hospital, we found that the ILP formulation was always able to find an optimal schedule satisfying all required hard constraints, unlike the manual schedule, which often did not satisfy all constraints. Moreover, due to the multi-goal objective function in the ILP, the algorithm was able to fulfill the majority of clinician preferences and requests, more so than the manually-created schedule. These observations reinforce the benefits of automated tools when generating schedules in hospital departments to balance the work-load of clinicians and improve the service provided to patients. The use of automated tools alleviates the time spent on designing the schedule by hand, and provides clinical departments with a more fair distribution of work that helps improve the overall satisfaction of both employees and patients [26].

In our simulations, we also found that increasing the number of requests per clinician did not affect the runtime of the algorithm, highlighting the flexibility of the tool to incorporate clinician preferences. Further, we saw that the algorithm can accommodate an increase in time-horizon up to four years with little impact on runtime, suggesting the algorithm can be used generate schedules far in advance. One key limitation we identified was the sensitivity of the runtime to larger numbers of services offered by a single department. Such cases are unlikely to be encountered in the real-world because most clinical departments tend to provide a single service or at most two services by the same roster of clinicians [SM/Kevin, could you help with reference here?]. One solution to mitigate the runtime issues created by a larger number of services would be relaxing the constraints that prevent assignment of consecutive blocks, followed by manual readjustment from the generated schedule. Overall, our sensitivity analyses using simulated data provided reassurance that the ILP formulation can be applied to schedule clinicians across real-world variability between clinical departments. Next steps include expanding the generalizability of the tool beyond smaller clinical departments to larger departments within and outside of health-care – especially those that provide multiple services in parallel for patients and other clients.

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