Control Systems - ENGR 33041

Lecture 11A: Controller Design

Instructor:

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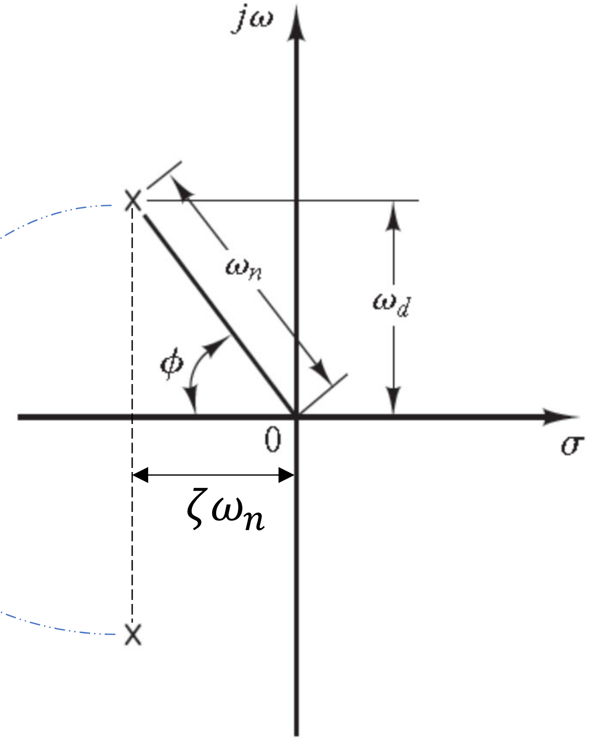
Representation of damping ratio ζ and natural frequency ω_n in the s-plane for underdamped systems

$$\zeta = -\frac{\ln(Mp\%/100)}{\sqrt{\pi^2 + (\ln(Mp\%/100))^2}}$$

$$\cos\phi = \frac{\zeta\omega_n}{\omega_n} = \zeta$$

$$p_{1,2} = -\zeta\omega_n \pm j\omega_d$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$



% Overshoot	$Mp\% = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$
Peak Time	$T_P = \dfrac{\pi}{\omega_d}$
Time Constant	$\tau = \dfrac{1}{\zeta\omega_n}$
Settling Time	$T_s = 4 \times \tau = \dfrac{4}{\zeta\omega_n}$

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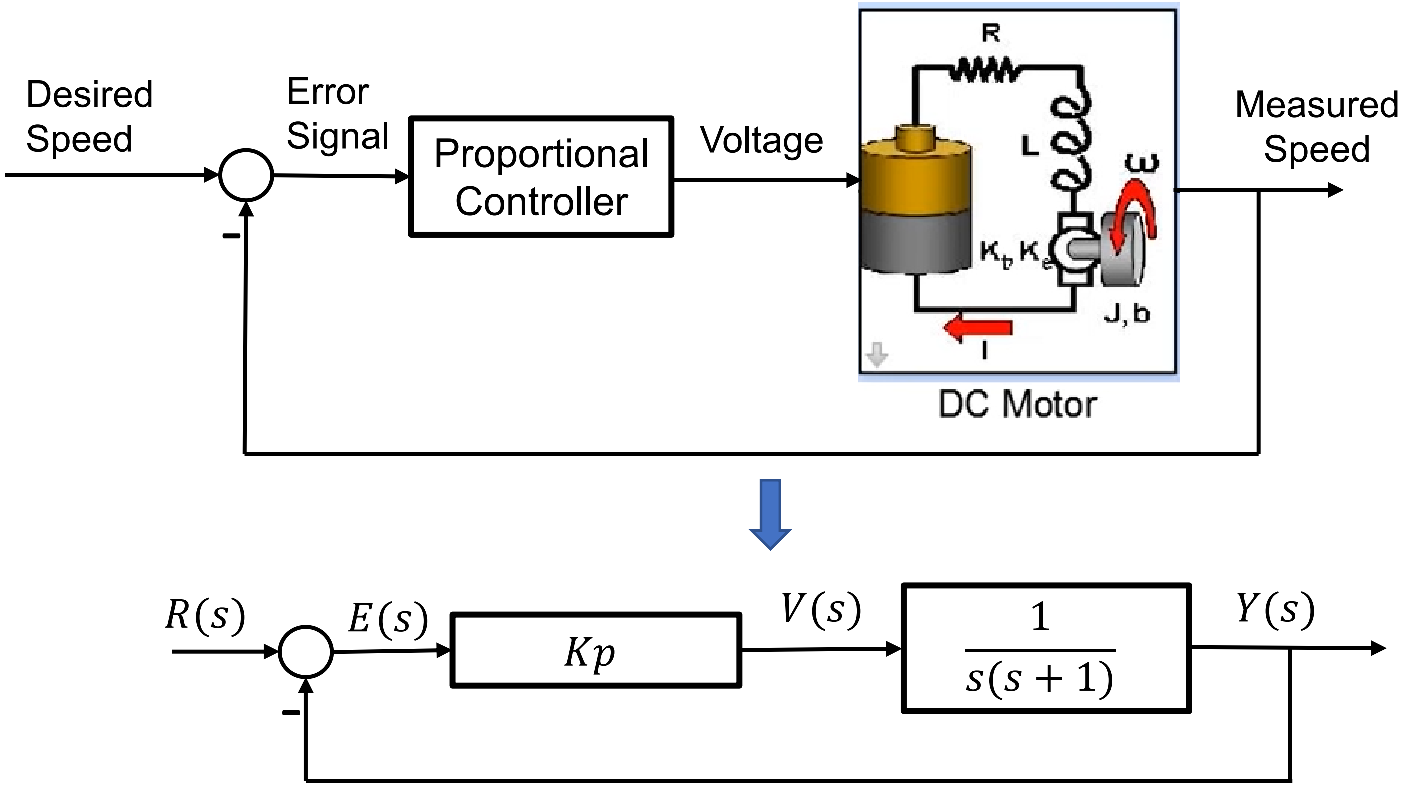
Control of DC Motor Speed

The speed of a DC motor is controlled by a proportional controller, where the controller takes the error signal between the measured and desired speed of the motor and adjusts the voltage signal to the motor. The system block diagram is shown below, where the DC motor is modeled by a second order system. Design your controller such that

(a) Closed-loop system is critically damped.
(b) Closed-loop system is underdamped with 15% maximum overshoot.
(c) Draw step responses for parts (a) and (b) in MATLAB and compare their results against each other.



Solution:

Closed-loop TF: $$\frac{Y(s)}{R(s)} = \frac{\frac{K_p}{s(s+1)}}{1+\frac{K_p}{s(s+1)}} = \frac{K_p}{s(s+1)+K_p} = \frac{K_p}{s^2+s+K_p} = \frac{K_p}{s^2+2\zeta\omega_n s+\omega_n^2}$$

(a) Critically damped $\longrightarrow$ $\zeta = 1$

$$2\zeta\omega_n = 1 \longrightarrow \omega_n = \frac{1}{2\zeta} = \frac{1}{2\times 1} = 0.5\ rad/s \longrightarrow K_p = \omega_n^2 = (0.5)^2 = 0.25$$

(b) Underdamped

$$15\% = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\% \longrightarrow \zeta = 0.517$$

$$2\zeta\omega_n = 1 \longrightarrow \omega_n = \frac{1}{2\zeta} = \frac{1}{2\times 0.517} = 0.9671\ rad/s \longrightarrow K_p = \omega_n^2 = 0.9353$$

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c)

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>> step(.25, [1  1  .25])
>> hold on
>> step(.9353, [1  1  .9353])
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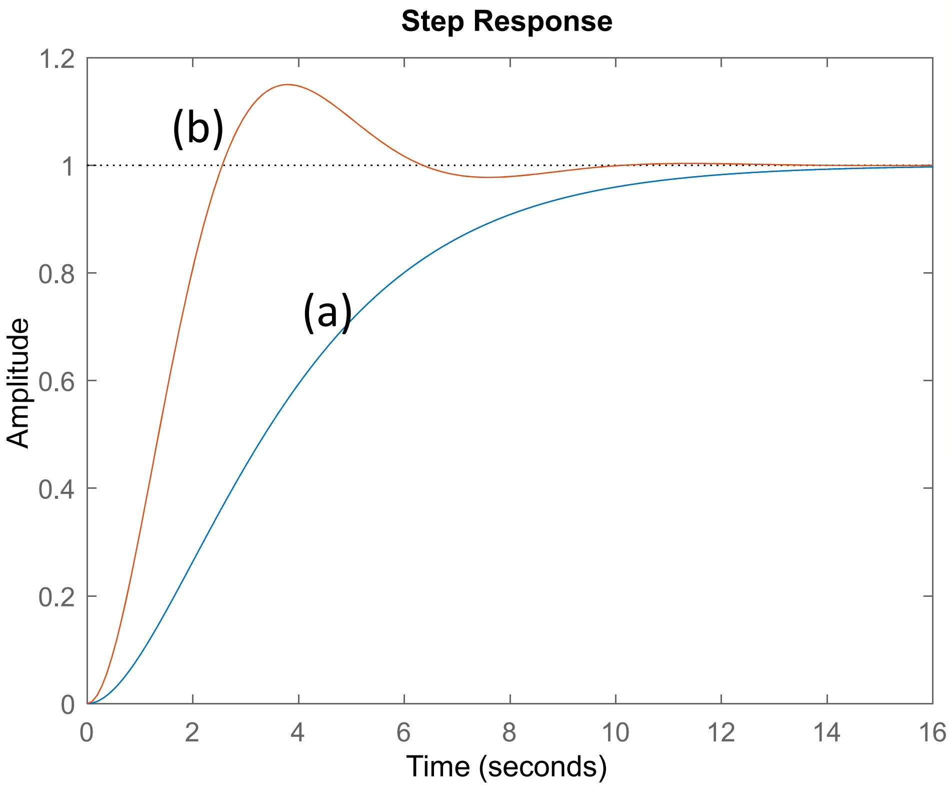

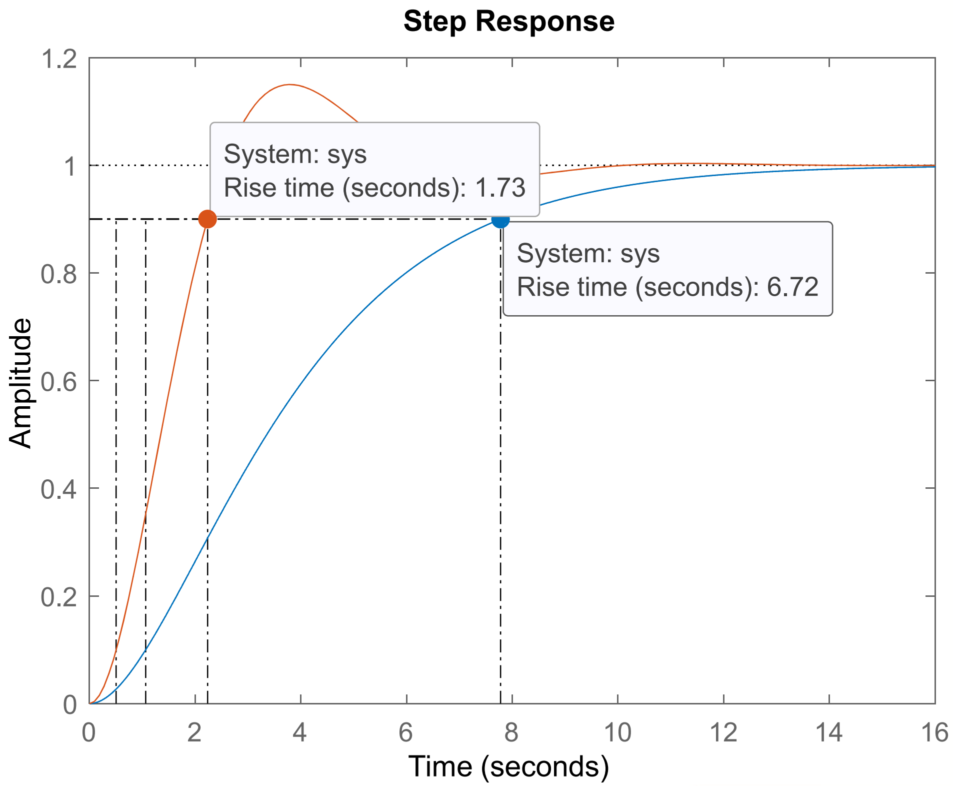

Comparing the step responses of part (a) and (b) in MATLAB shows that by increasing the value of the proportional gain, the response gets faster (rise time decreases) while the overshoot increases.

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>> step(100, [1  1  100])
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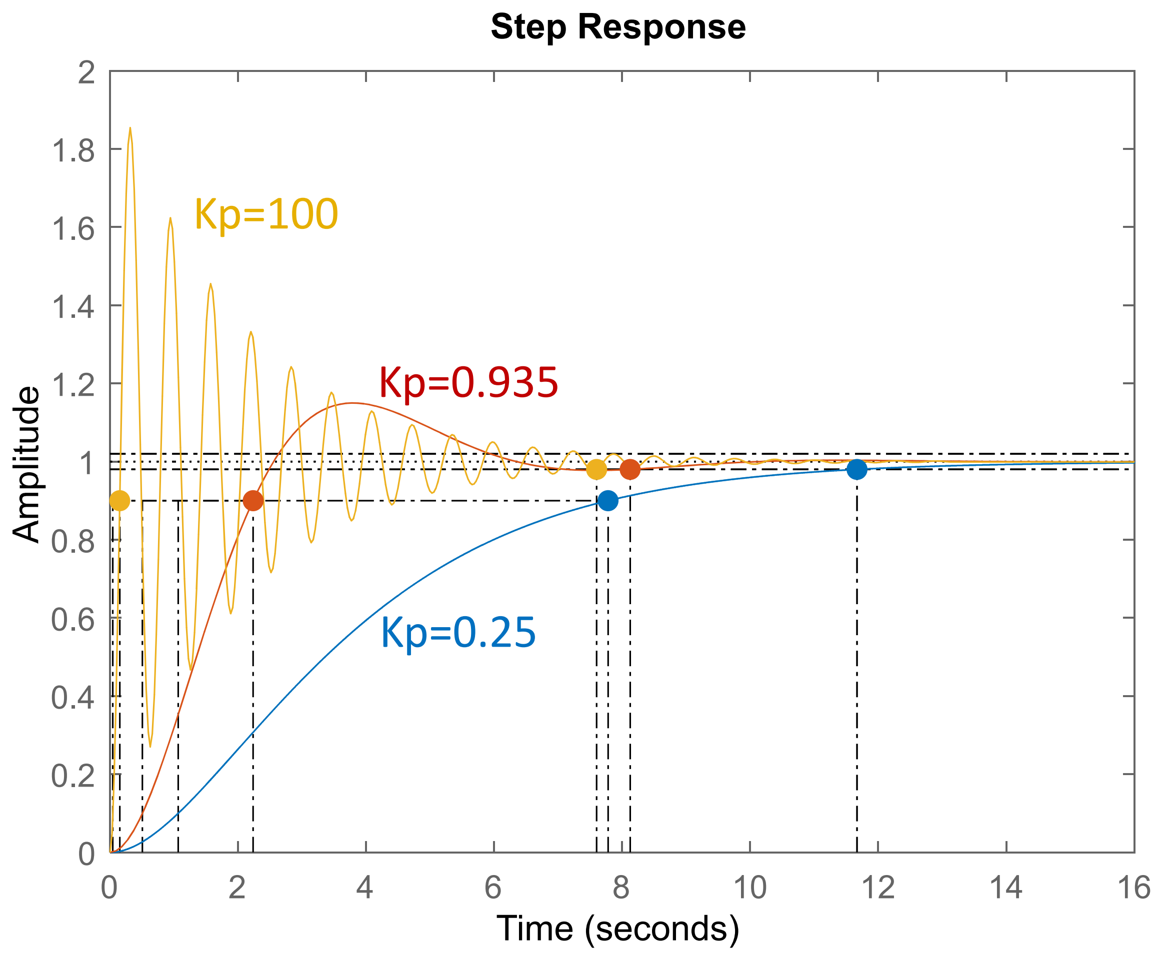
Now, let's increase the controller gain to 100:

As seen in the figure, the response gets faster at the expense of increased overshoot (85.4%).

In practice, such a high value of overshoot is not usually tolerated, but a low percentage (10% to 20%) might be OK depending on the application.

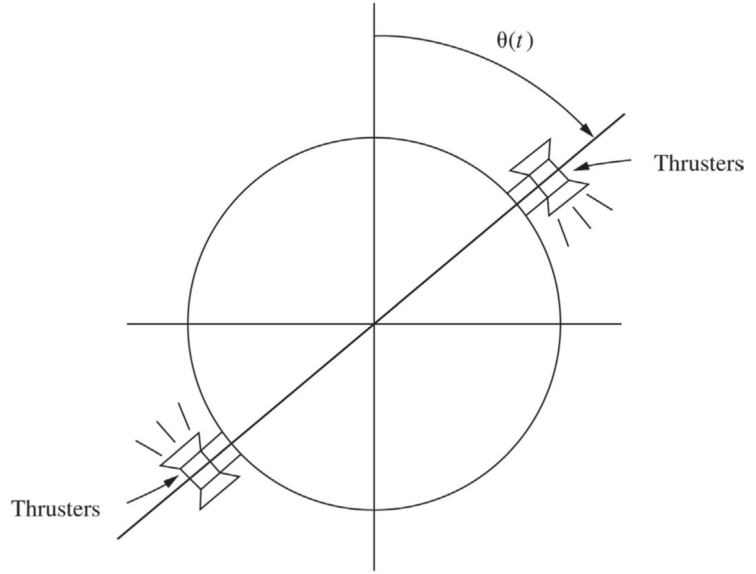


Attitude Control of Satellite

Consider the attitude (position) control system of a satellite. Assume that the satellite is spherical and has the thruster configuration as shown below. The thrusters, when active, apply a torque $m(t)$. The torque of the two active thrusters tends to reduce the yaw angle θ(t). The transfer function of the satellite can be represented as

$$\frac{\Theta(s)}{M(s)} = G_p(s) = \frac{1}{Js^2}$$

where J is the satellite's moment of inertia about the yaw axis.

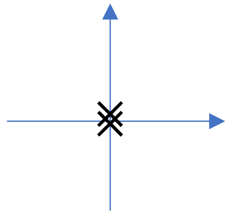


(a) Verify that the satellite without any type of closed-loop control is unstable.

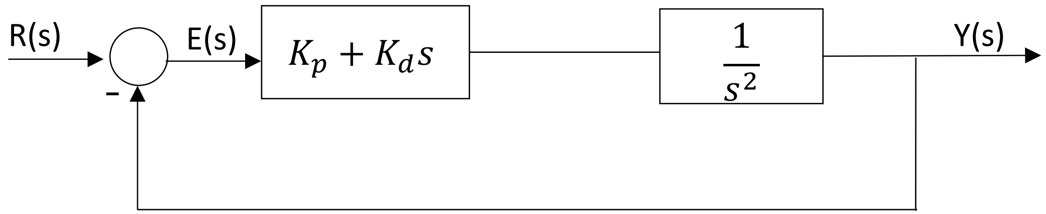
Solution:

$$Js^2 = 0 \longrightarrow s^2 = 0 \begin{cases} s_1 = 0 \\ s_2 = 0 \end{cases}$$

Repeated poles at $j\omega$ axis (origin), so the system is <u>unstable</u>. $\Rightarrow$



(b) Assuming $J=1$, wrap a feedback around the satellite attitude control system with a PD controller to relocate the closed-loop poles to $s=-1\pm j$. What are the gains of the PD controller. Draw the step response in MATLAB and report the rise time and overshoot %.



Solution:

Closed-loop TF:

$$\frac{Y(s)}{R(s)} = \frac{\frac{K_p + K_d s}{s^2}}{1+\frac{K_p + K_d s}{s^2}} = \frac{K_d s + K_p}{s^2 + K_d s + K_p} = \frac{P(s)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Desired closed-loop poles: $s=-1\pm j$ $\longrightarrow$ $(s+1+j)(s+1-j) = (s+1)^2 - j^2 = s^2 + 2s + 2$

$\longrightarrow$ $s^2 + K_d s + K_p = s^2 + 2s + 2$ $\longrightarrow$ $K_p = 2$, $K_d = 2$

$$\frac{Y(s)}{R(s)} = \frac{K_d s + K_p}{s^2 + K_d s + K_p} = \frac{2s + 2}{s^2 + 2s + 2}$$

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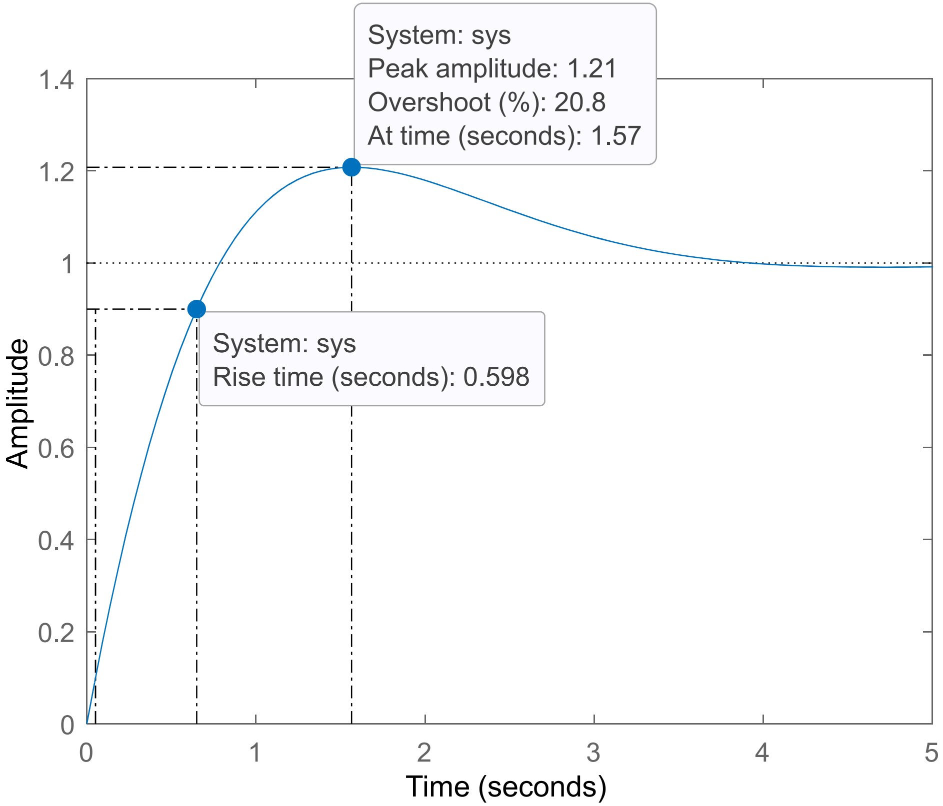
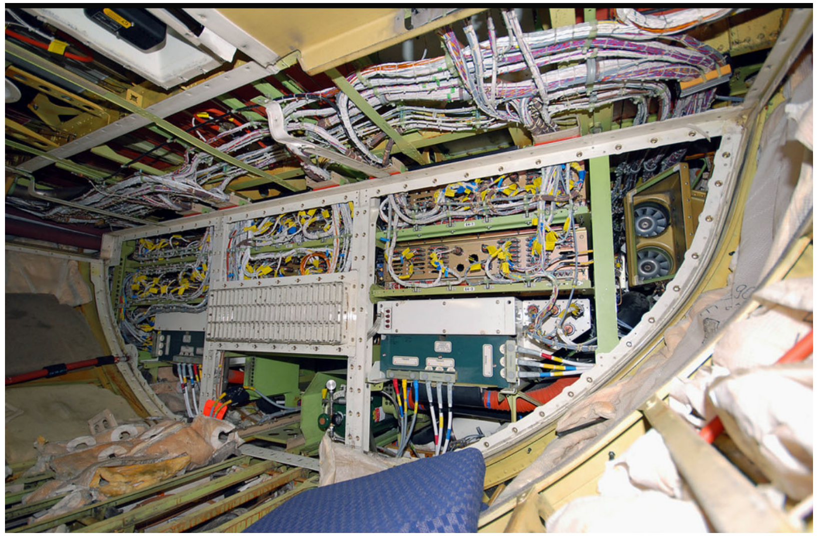
>> step([2 2],[1 2 2])
```

Rise time = 0.598 s
Overshoot % = 20.8%



Thermal Control of Avionics Bay

Avionics bay, also known as E&E bay, is a compartment in an aircraft that houses the avionics and other electronic equipment, essential for the operation. We would like to design A thermal control system where a heater controls the temperature inside the avionics bay.



The dynamics of the temperature control system (heat transfer) can be modeled as

$$\tau \frac{dT(t)}{dt} + T(t) = KQ(t)$$

$T(t)$: Bay Temperature (output)

$Q(t)$: Heater input power (input)

K: System gain (constant value)

τ: Thermal time constant (constant value)

(a) Assuming $K = \tau = 1$, find the transfer function of this system.

$$\tau \frac{dT(t)}{dt} + T(t) = KQ(t) \xrightarrow{K = \tau = 1} \frac{dT(t)}{dt} + T(t) = Q(t)$$

Taking Laplace transform of both sides:

$$sT(s) + T(s) = Q(s) \longrightarrow T(s)(s+1) = Q(s) \longrightarrow \boxed{\frac{T(s)}{Q(s)} = \frac{1}{s+1}}$$

(b) What is the system type for this thermal control system?

The system type depends on the number of integrators ($\frac{1}{s^n}$) in the open-loop transfer function (number of poles at the origin): $\frac{T(s)}{Q(s)} = \frac{1}{s+1}$ $\longrightarrow$ Type 0 System

(c) We would like to design a controller to have a zero steady-state error to a constant reference input (perfect track of the reference temperature). What type of controller should I choose?

According to Table 5.1 in the Lecture 6, the steady-state error of the type 0 system to a constant input is 1/(1+Kp), which is non-zero. To have a zero steady-state error, the system type needs to be increased to type 1, which happens by adding the integral gain. Therefore, the PI controller is the right choice in this case.

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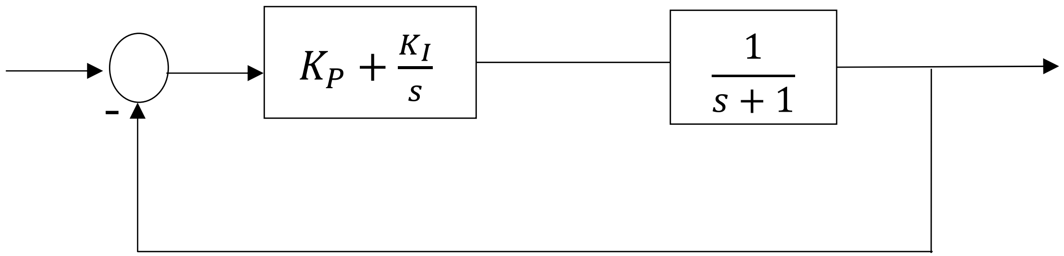
(d) Wrap a feedback around the controller and the plant and sketch the block diagram of the system.



(e) Design your controller parameters such that the settling time and maximum overshoot are 0.8 seconds and 10%, respectively. Draw the step response in MATLAB and verify the overshoot and settling time.

$$\text{Closed-loop TF} = \frac{G(s)}{1+G(s)}, \quad G(s) = \frac{s k_p + k_I}{s(s+1)}$$

$$\Rightarrow \text{Closed-loop TF} = \frac{k_p s + k_I}{s(s+1) + k_p s + k_I} = \frac{k_p s + k_I}{s^2 + (k_p+1)s + k_I}$$

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Closed-loop TF: $$\frac{Y(s)}{R(s)} = \frac{K_p s + K_I}{s^2 + (K_p + 1)s + K_I} = \frac{K_p s + K_I}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

10% overshoot: → $10\% = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100\%$ → $\zeta = 0.591$

Settling time of 0.8 s → $T_s = \frac{4}{\zeta\omega_n} = 0.8$ → $\omega_n = \frac{4}{0.8\,\zeta} = \frac{4}{0.8 \times 0.591} = 8.46\, rad/s$

$K_p + 1 = 2\zeta\omega_n = 10$ → $K_p = 9$ $K_I = \omega_n^2 = 71.576$

When drawing the step response in MATLAB, while the settling time is 0.8 s, the % overshoot is 21.9% rather than 10%. This discrepancy is related to the contribution of the zero in the closed-loop transfer function. In other words, while the denominator is quadratic (second-order), the numerator's contribution, $K_p s + K_I$, can lead to discrepancies in overshoot and transient response compared to the idealized second-order approximation , when there is no zero in the system.

You can write a custom script in MATLAB to minimize the difference between the simulated response and the desired 10% overshoot, if interested.