

Control Systems - ENGR 33041

Instructor:
Hossein Mirinejad, Ph.D.

Slides prepared based on the following:

CDS 101 Lecture notes, Dr. Richard M. Murray, California Institute of Technology

ECE4510 Lecture notes: Dr. Gregory L. Plett, University of Colorado Colorado Springs

Control Systems Technology, C. Johnson and H. Malki



Background

Hossein Mirinejad, Ph.D.

- Assistant Professor in CAE
- Research Fellow at FDA (2017-2019)
- Postdoctoral Fellow at UM (2016-2017)
- Ph.D. in Electrical Engineering at UofL (2016)



Lab Website <https://sites.google.com/kent.edu/camlab>

Control, Automation, and Mechatronics (CAM) lab



Research Areas

- Modeling and control of autonomous systems
- Automated medical systems
- Design of advanced controllers for robotic platforms

Lab Capabilities



QArm: 4 DOF manipulator
with gripper & camera



Digital pump
with remote control



Patient monitoring
system

Textbook and Software

- **Textbook**

No textbook is required. The course materials were mostly prepared using the following textbooks:

- C. Johnson and H. Malki, *Control Systems Technology*, Prentice Hall, 2002.
- K. Ogata, *Modern Control Engineering, 5th ed.*, Pearson, 2009.

- **Required Software/Online Subscriptions**

MATLAB software is required.

Students need to purchase MATLAB license for this class. The best option is MATLAB and Simulink Student Suite (no additional toolboxes are needed) that can be purchased through this link:

<https://www.mathworks.com/products/matlab/student.html>

INSTRUCTOR POLICIES

- (i) The instructor reserves the right to modify the syllabus as needed to best achieve the course learning objectives. The material covered in class may be adjusted based on student comprehension, unforeseen events, and other factors.
- (ii) **The exams will be held on paper during class times.** It is the student's responsibility to attend the class and take the exams. The exam dates are set in the syllabus; however, the instructor reserves the right to modify the exam dates if needed. In such cases, the instructor will notify students of any changes at least 7 days prior to the new exam date.
- (iii) **Homework** assignments and course **exams** are to be completed **individually**. Additionally, there is a **final project** where students are required to work **collaboratively** as team members and submit their work collectively. Each team will be composed of students assigned by the instructor during the semester. While teams are permitted to seek clarification from their peers, each team must independently draft and submit their own work. Replicating another team's responses, reports, or computer codes is a breach of the university's academic honesty policy and may result in significant penalties.
- (iv) The Use of AI assisted tools, such as ChatGPT, in preparing assignments for grade is NOT permitted.

Cheating and plagiarism will not be tolerated. If you have ANY concerns or questions about cheating, plagiarism, or other academic integrity issues, it is imperative that you consult with the professor before submitting your work.

COURSE LEARNING OBJECTIVES

The following lists course learning objectives and how they support student outcomes as well as meet the university's experiential learning requirement.

1. Students demonstrate an understanding of the fundamentals of feedback control system and control models.
2. Students are capable to analyze static and dynamic responses of a feedback control system.
3. Students would be able to determine the stability of a closed-loop control system.
4. Students would be able to analyze feedback control systems using time and frequency domain techniques.

Assessment

Requirement	Weight
Class Attendance (< 10 min late each time)	10%
Homework	15%
Exam 1	20%
Exam 2	20%
Exam 3	20%
Final Project	15%

*Bonus questions may be asked in class, in homework or during exams.

GRADE SCALE

$92 \leq A < 100$	$88 \leq A^- < 92$	$84 \leq B^+ < 88$	$80 \leq B < 84$	$76 \leq B^- < 80$	
$72 \leq C^+ < 76$	$68 \leq C < 72$	$64 \leq C^- < 68$	$60 \leq D^+ < 64$	$56 \leq D < 60$	$F < 56$

Important Information:

The course needs basic knowledge of calculus, including **complex numbers** and **derivate of fractions** (e.g., quotient rule).

Thus, it is highly recommended to go to tutoring at the beginning of the semester to refresh your mind about math operations such as derivative, integral, and first order differential equations.

Tentative Class Schedule

Week	Date	Topic
0	8/20, 8/22	Introduction to Control Systems
1	8/27, 8/29	Laplace Transform (Part 1)
2	9/3, 9/5	Laplace Transform (Part 2)
3	9/10, 9/12	Control System Models
4	9/19	Exam 1 (No Class on 9/17 - Reading Day)
5	9/24, 9/26	Control System Models - Block Reduction
6	10/1, 10/3	(Fall Break, No Class on 10/1 and 10/3)
7	10/8, 10/10	Static and Dynamic Response (Part 1)
8	10/15, 10/17	Static and Dynamic Response (Part 2)
9	10/22, 10/24	Stability of Control Systems
10	10/31	Exam 2 (No Class on 10/29 - Reading Day)
11	11/5, 11/7	Analysis of Feedback Control Systems (Root Locus)
12	11/12, 11/14	Analysis of Feedback Control Systems (Frequency Response)
13	11/19, 11/21	Design of controller for feedback control systems
14	11/26, 11/28	(Thanksgiving Break, No Class on 11/26 and 11/28)
15	12/5	Exam 3 (No Class on 12/3 - Reading Day)
16	12/10	Final Project Due 12:45 pm

Exam 1, **Exam 2**, and **Exam 3** are currently scheduled for **9/19**, **10/31**, and **12/5**, respectively. The exams will be held in person during class time. The course also has a **Final Project** due at the beginning of the final exam period, which is **Tuesday, 12/10, 2024, 12:45 pm**. Further information regarding Exam 1, Exam 2, Exam 3, and Final Project will be later announced by the instructor. Also, if there is any change in the exam and project dates, it will be announced by the instructor in advance.

Introduction to Control Systems

What is feedback control?

- Control-system engineers often face this question (or “What is it that you do, anyway?”) when trying to explain their professional field.
- Loosely speaking, control is the process of getting “something” to do what you want it to do (or “not do,” as the case may be).
- The “something” can be almost anything. Some common examples: aircraft, spacecraft, cars, machines, robots, radars, telescopes, etc. Less common examples: energy systems, economy, biological systems, human body, etc.
- Control is a very common concept, e.g., Human-machine interaction: Driving a car.

Control Systems Categories: Manual vs. Automatic

- ❖ **Manual control**

e.g., Human-machine interaction: Driving a car

- ❖ **Automatic control** (our focus in this COURSE).

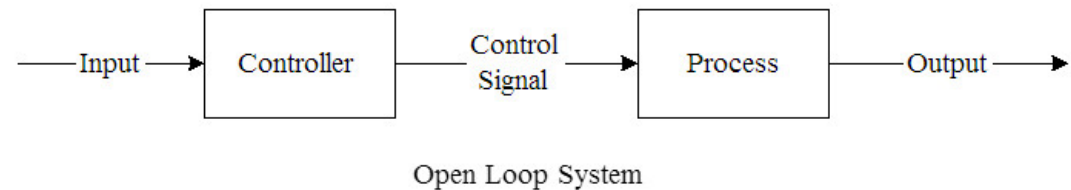
e.g., Independent machine: Room temperature control. Furnace in winter, air conditioner in summer. Both controlled (turned “on”/“off”) by thermostat.

Control Systems Categories: Open loop vs. Closed loop

DEFINITION: Control is the process of causing a system variable (physical variable) to conform to some desired value, called a reference value or setpoint. (e.g., reference value = temperature set point)

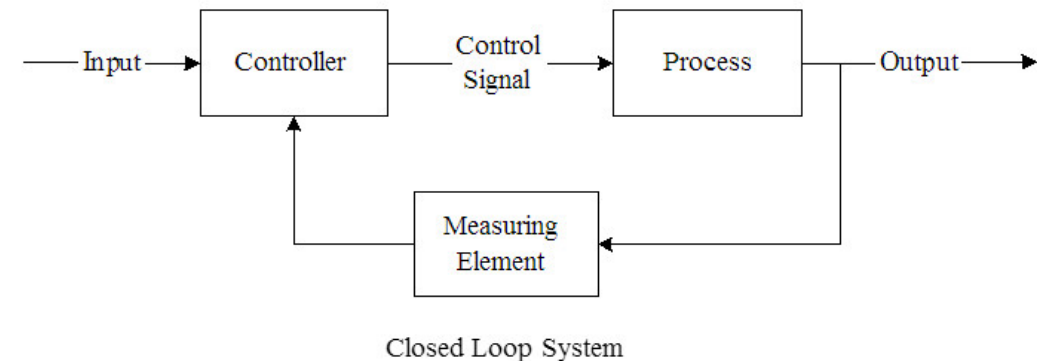
Open Loop

The controlled variable is not measured.



Closed Loop or Feedback (Our Focus in this COURSE)

In a closed-loop system (feedback control system), the controlled/system variable (e.g., temperature) is measured and that information is used to influence the value of the controlled variable. Feedback **IS NOT** necessary for control. Having a closed loop system is necessary to deal with uncertainties. **Feedback is necessary to deal with uncertainty.**



Control Systems Categories: Regulation vs. Servomechanism

- **Process Control (Regulation)**

The objective of a control system is to force a system variable (e.g., temperature, liquid level, flow rate) to remain constant in time and equal to some desired value (setpoint).

- **Servomechanism**

The objective is to force a system variable to change in time in a precisely prescribed manner. That is, the physical variable will be forced to follow or track some target value as it changes in time. Ex. control of industrial robot arm motion (trajectory: position, velocity, acceleration)

Note: We deal with both Regulation and Servomechanism control systems in this course.

Control theory is a truly multi-disciplinary study that cuts across boundaries of many disciplines, including electrical eng., mechanical eng., math, and physics.

But, if you are to choose only one field of study, which one provides the most critical knowledge (cornerstone) to be a good control engineer?

☒ Mathematics

☐ Electrical Engineering → Signals and systems

☐ Mechanical Engineering → Dynamical system behavior

☐ Physics → Conservation of Energy, thermodynamics

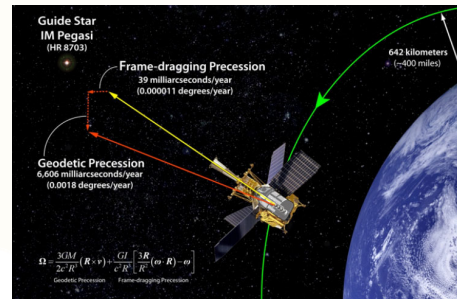
Applications of Control Systems

❑ Aerospace

Automatic Piloting



Spacecraft Control

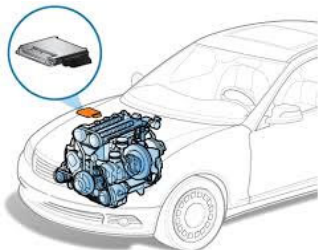


UAV Guidance & Navigation



❑ Automotive

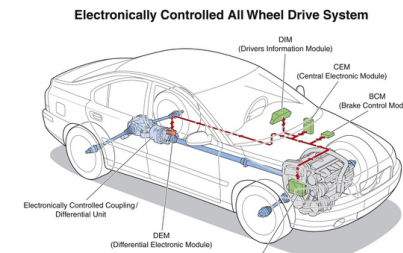
Engine Control



Cruise Control



Transmission Control



Applications of Control Systems

❑ Home Appliances

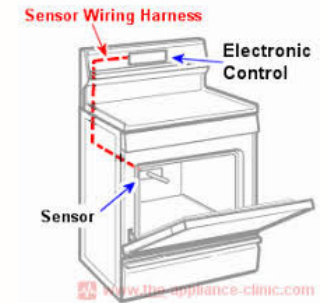
Temperature Control



Humidity Control



Temp Control of Ovens



❑ Medical

Automated Medical Devices



Artificial Limb



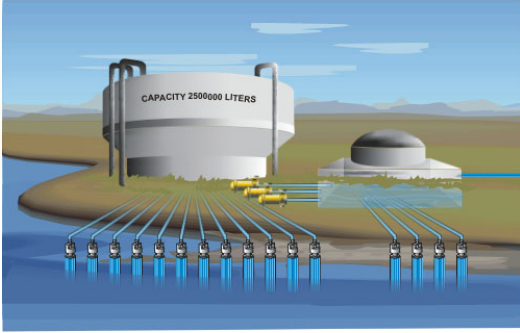
Robotic Surgery



Applications of Control Systems

❑ Ecological

Water Control and Distribution



Flood Control via Dams



Air Pollution Abatement



❑ Power/energy

Optimal control of windmill blade and solar panel surfaces

Power electronics: dc-dc conversion, battery chargers, motor controls

❑ Transportation

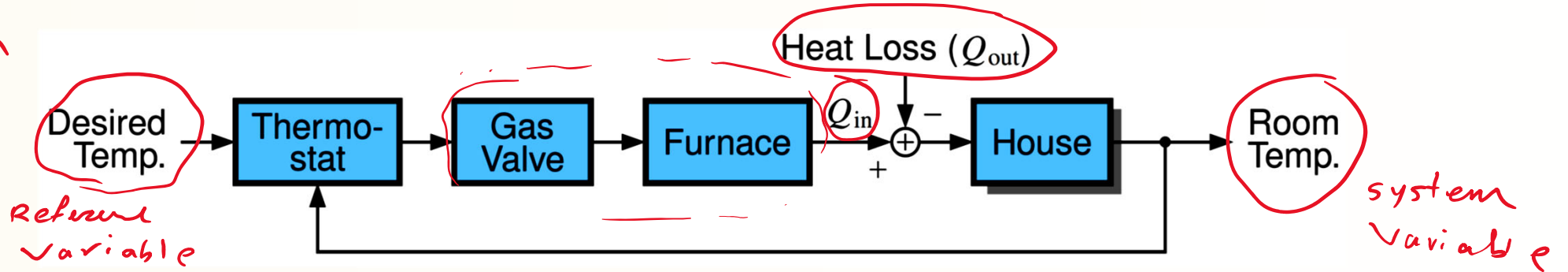
Control of roadway vehicle traffic flows using sensors

Propulsion control in rail transit systems

Building elevators and escalators

Block
Diagram

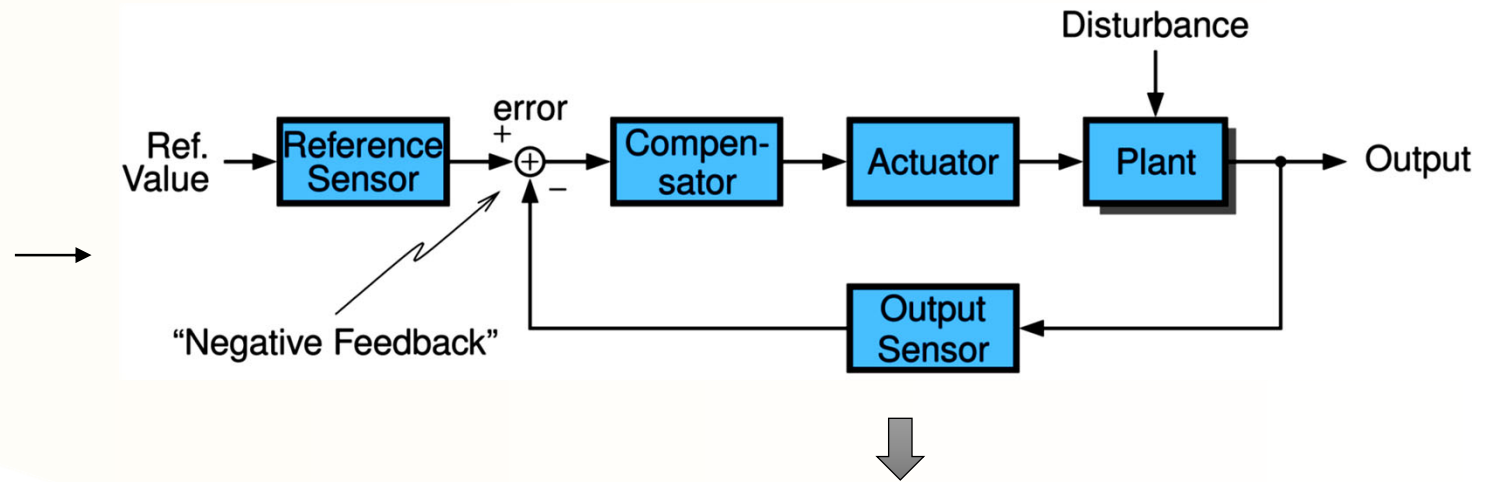
Illustrative Example of a Feedback Control System



- **Process** or **Plant**, one of whose variables we want to control. e.g., Plant = HOUSE ; Variable = Room Temp.
- **Disturbance (External Influence)**= some system input that we do not control. e.g., Disturbance = Heat loss
- **Actuator**= device that influences controlled variable. e.g., Actuator = Gas Valve. + Furnace
- **Reference sensor** measures desired system output. *(ref variable)*
- **measurement (Output sensor)** measures actual system output.
- **Compensator** or **Controller** = device that computes the control effort to apply the actuator, based on sensor readings. e.g. Thermostat combines the last three functions.

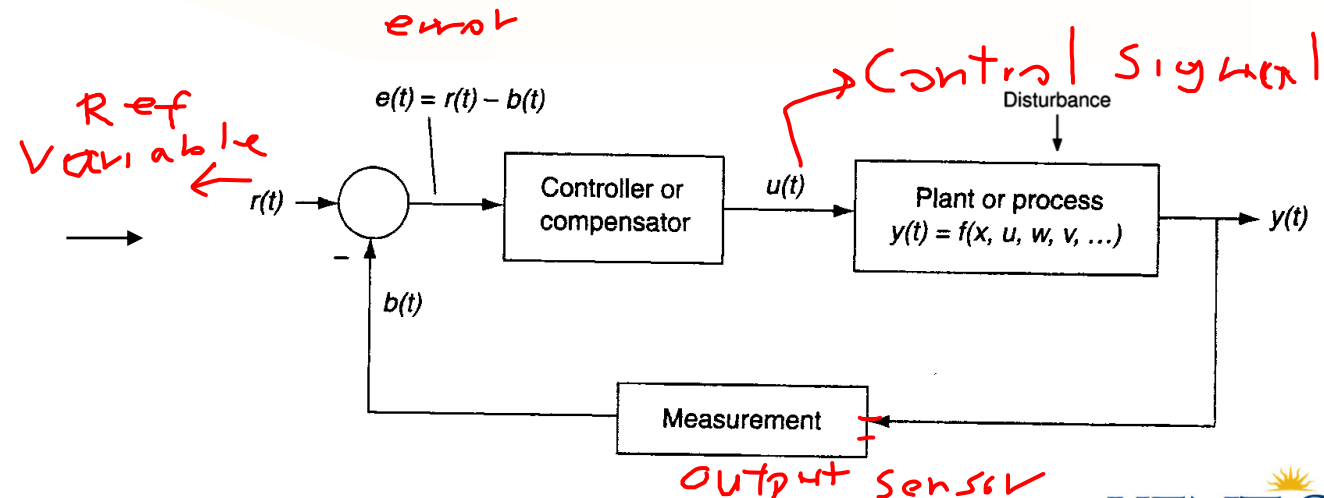
General Block Diagram of a Control System

- Getting the ideas from the prior diagram to arrive at a more general block diagram that is applicable to most feedback control systems:

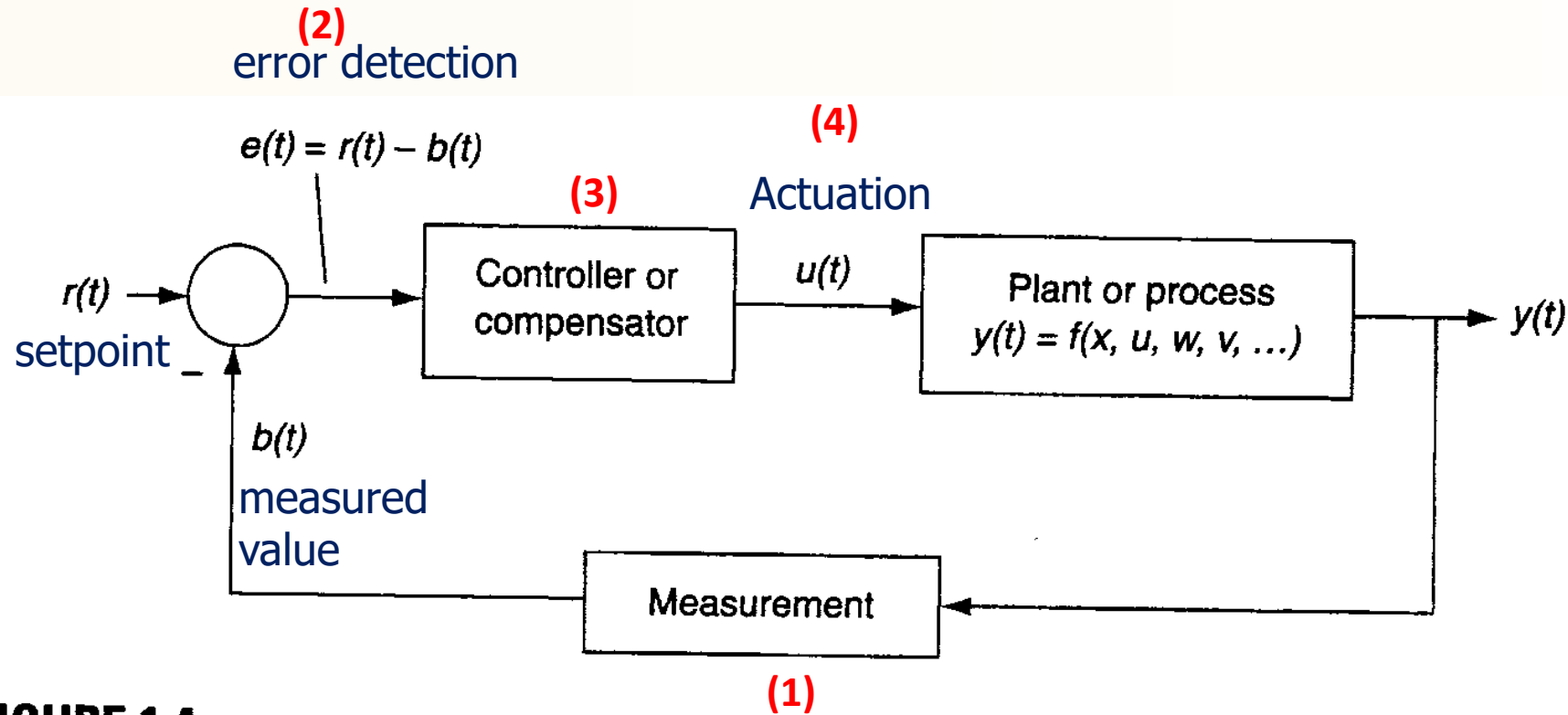


More Abstract Block Diagram

- Assuming the presence of perfect Reference Sensors and actuators (absorbed into the plant), we can make a more abstract block diagram:



1.2.1 Control System Strategy



1. **Measurement:** determines the present value of the system variable through feedback
2. **Error detection:** compares the measured value with the desired value (setpoint)
3. **Controller or Compensator:** does the calculation and determines a corrective action to compensate the error.
4. **Actuation:** the corrective action to the system that influences the system variable and drives the error toward zero.

FIGURE 1.4

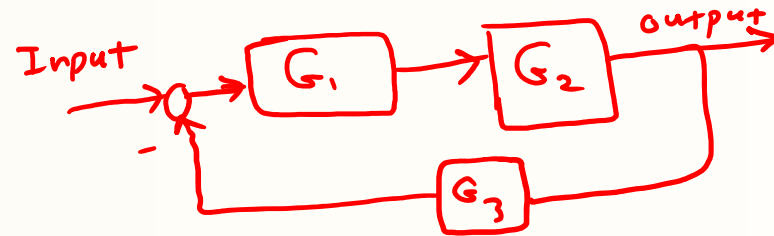
Basic control strategy demonstrated by a block diagram.

Class Discussion: Identify the 4-step control strategy for the following control systems:

- a. A common home refrigerator
- b. A home air conditioner
- c. The steering of an automobile
- d. An automobile's cruise control

System	Measurement	Error Detection	Compensation	Actuation
(a) Refrigerator	Temperature sensor	Part of the Thermostat	Part of the Thermostat	Compressor
(b) Air conditioner	Temperature sensor	Part of the thermostat	Part of the thermostat	Compressor
(c) Steering	Human eyes following the road	Human brain seeing the difference between the road and the auto path	Human brain deciding on the type of steering correction needed	Steering column and linkage to tires
(d) Cruise control	Speed sensor	Electronic detection of error	An electronic or computer control unit	A solenoid connected to the fuel feed system

- **Block Diagram** identifies the major components of a control system as block and omits unnecessary details. It also highlights the information/energy flows in the system by arrows.



- **Transfer Function** each block has a single input and a single output. The block represents some sort of “operation” that is performed on the input in order to produce the output. The **ratio of block output to block input** is called the **transfer function**.
- **For example, a multiplication is a transfer function:**

$$y(t) = Kx(t)$$



Linear Functions

A function $f(x)$ is a linear function of the independent variable x if, and only if, it satisfies two properties:

1. Additivity

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

for all x_1 and x_2 in the domain of $f(x)$.

2. Homogeneity

$$f(\alpha x) = \alpha f(x)$$

for all x in the domain of $f(x)$ and all scalars α .

Note:

The aforementioned properties together are sometimes called the Superposition Principle.

Linear operations: There are only 6 linear operations:

(1) addition, (2) subtraction, (3) integration, (4) differentiation, (5) multiplication by constant, (6) division by nonzero constant

$$\textcircled{1} \quad \mathcal{O}(x_1 + x_2) = \mathcal{O}(x_1) + \mathcal{O}(x_2) \quad \checkmark \quad (1.4)$$

$$\mathcal{O}(\alpha x) = \alpha \mathcal{O}(x) \quad \checkmark$$

$$\textcircled{2} \quad f(x) = \int x \, dt$$

$$f(x_1 + x_2) = \int (x_1 + x_2) \, dt = \int x_1 \, dt + \int x_2 \, dt = f(x_1) + f(x_2) \quad \checkmark \quad (1.5)$$

$$f(\alpha x) = \int \alpha x \, dt = \alpha \int x \, dt = \alpha f(x)$$

$\textcircled{3}$

$$f(x_1 + x_2) = \frac{d}{dt}(x_1 + x_2) = \frac{d}{dt}(x_1) + \frac{d}{dt}(x_2) = f(x_1) + f(x_2) \quad \checkmark \quad (1.6)$$

$$\frac{d\alpha x}{dt} = \alpha \frac{dx}{dt} = \alpha f(x) \quad \checkmark$$

- **Nonlinear Functions:**

$$y = 5x^2$$

Additivity Check:

$$5(x_1 + x_2)^2 = 5(x_1^2 + 2x_1x_2 + x_2^2) \neq 5x_1^2 + 5x_2^2$$

$f(x_1)$

$f(x_2)$

$f(x_1 + x_2)$

$$y(t) = x(t) \frac{d}{dt}(x(t))$$

Additivity Check:

$$[x_1 + x_2] \frac{d}{dt}[x_1 + x_2] = x_1 \frac{dx_1}{dt} + x_1 \frac{dx_2}{dt} + x_2 \frac{dx_1}{dt} + x_2 \frac{dx_2}{dt}$$

$f(x_1 + x_2)$

$$[x_1 + x_2] \left[\frac{dx_1}{dt} + \frac{dx_2}{dt} \right] \neq x_1 \frac{dx_1}{dt} + x_2 \frac{dx_2}{dt}$$

$f(x_1)$

$f(x_2)$

Multiplication or division of linear functions does not necessarily maintain linearity.

Determine if the following equations are linear or nonlinear:

(a) $f(x) = 2x + 9$

Not Linear

Additivity:

$$f(x_1 + x_2) = 2(x_1 + x_2) + 9 = 2x_1 + 2x_2 + 9$$

$$f(x_1) = 2x_1 + 9$$

$$f(x_2) = 2x_2 + 9$$

$$f(x_1 + x_2) = (2x_1 + 9) + (2x_2 + 9)$$

$$= 2x_1 + 2x_2 + 18$$

$$f(x_1 + x_2) \neq f(x_1) + f(x_2)$$

(b) $y = x + \sin x$

Additivity:

$$f(x_1 + x_2) = (x_1 + x_2) + \sin(x_1 + x_2)$$

$$\left. \begin{array}{l} f(x_1) = x_1 + \sin x_1 \\ f(x_2) = x_2 + \sin x_2 \end{array} \right\} \Rightarrow f(x_1) + f(x_2) = x_1 + \sin x_1 + x_2 + \sin x_2$$
$$= (x_1 + x_2) + (\sin x_1 + \sin x_2)$$

since $\sin(x_1 + x_2) \neq \sin x_1 + \sin x_2$

$$\Rightarrow f(x_1 + x_2) \neq f(x_1) + f(x_2)$$

Non linear

$$(c) \quad y = x + \frac{dx}{dt} - 3 \frac{d^2x}{dt^2}$$

Additivity: $f(x_1 + x_2) = (x_1 + x_2) + \frac{d(x_1 + x_2)}{dt} - 3 \frac{d^2(x_1 + x_2)}{dt^2}$

$$= x_1 + x_2 + \frac{dx_1}{dt} + \frac{dx_2}{dt} - 3 \frac{d^2x_1}{dt^2} - 3 \frac{d^2x_2}{dt^2}$$

$$f(x_1) = x_1 + \frac{dx_1}{dt} - 3 \frac{d^2x_1}{dt^2}, \quad f(x_2) = x_2 + \frac{dx_2}{dt} - 3 \frac{d^2x_2}{dt^2}$$

$$f(x_1) + f(x_2) = x_1 + \frac{dx_1}{dt} - 3 \frac{d^2x_1}{dt^2} + x_2 + \frac{dx_2}{dt} - 3 \frac{d^2x_2}{dt^2}$$

$$= x_1 + x_2 + \frac{dx_1}{dt} + \frac{dx_2}{dt} - 3 \frac{d^2x_1}{dt^2} - 3 \frac{d^2x_2}{dt^2} = f(x_1 + x_2) \quad \checkmark$$

Homogeneity: $f(\alpha x) = (\alpha x) + \frac{d(\alpha x)}{dt} - 3 \frac{d^2(\alpha x)}{dt^2} = \alpha x + \alpha \frac{dx}{dt} - 3 \alpha \frac{d^2x}{dt^2}$

$$\alpha f(x) = \alpha \left(x + \frac{dx}{dt} - 3 \frac{d^2x}{dt^2} \right) = \alpha x + \alpha \frac{dx}{dt} - 3 \alpha \frac{d^2x}{dt^2} = f(\alpha x) \quad \checkmark$$

LINEAR

1.2.2 Examples of Control Systems

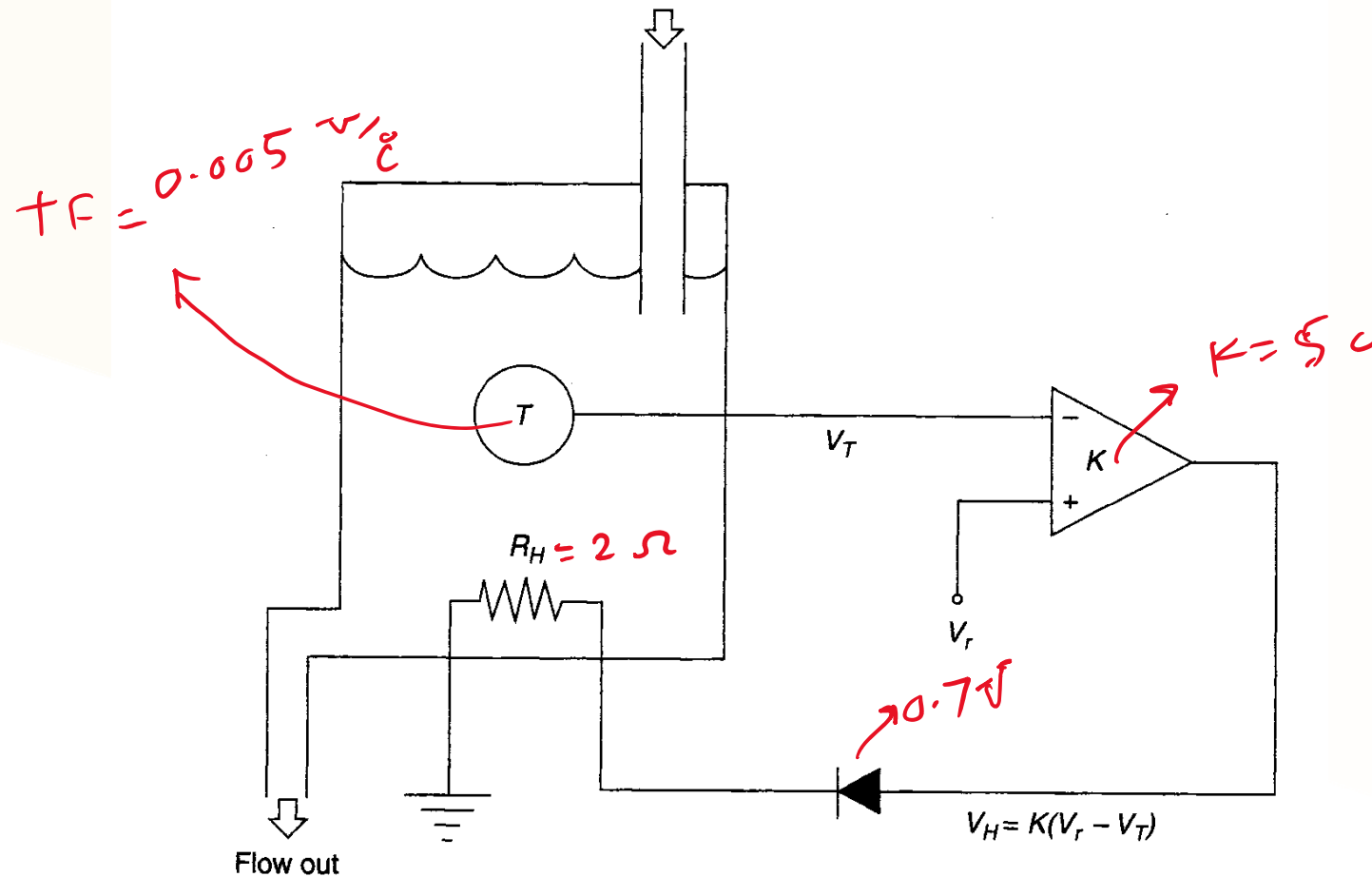


FIGURE 1.2
Electronic water temperature control system.

Water Temperature Control:

In this system, the temperature measurement is made by a “sensor” that provides a voltage proportional to temperature. The “error detector” and “the “controller” are incorporated in a differential amplifier. This amplifier outputs an error voltage, V_H , proportional to the difference between the desired temperature (V_r) and the measured temperature (V_T). Again, both desired and measured temperature are in voltage domain, which means that the voltage is proportional to the temperature. The controller function occurs when the difference is multiplied by a gain K , which determines how much voltage V_H to apply to the heater resistor R_H . The heater resistor is the “actuation” element that provides the corrective action.

- 1.5** Consider the water heater example of a proportional control system as presented in figure 1.2. The system should maintain the tank temperature at 80°C and has the following characteristics: (1) the sensor has a transfer function of 5 mV/°C, (2) the amplifier gain is 50, (3) the heater resistor is 2.0 Ω , (4) the tank loses temperature at 0.2°C per minute, (5) the heater resistor causes the tank temperature to increase 0.05°C per minute for every watt of dissipated power, and (6) the diode forward voltage drop is 0.7 volts. Assuming the starting tank temperature is 20°C, answer the following questions:
- a. What voltage should be used for the amplifier reference?
 - b. What is the initial tank heating rate in °C per minute?
 - c. What is the tank heating rate for temperatures of 40°, 60°, and 80°C?

Ans.

(a) Since the sensor outputs 5 mV/oC and we want 80oC, we need a voltage of $(5 \text{ mV/oC})(80\text{oC}) = 0.4 \text{ V}$.

(b) At 20°C the sensor outputs $(5 \text{ mV/}^\circ\text{C})(20^\circ\text{C}) = 0.1 \text{ V}$. Therefore the amplifier outputs $50(0.4 - 0.1) \text{ V} = 15 \text{ V}$. There is a drop of 0.7 volts across the diode so the voltage applied to the heater resistor is 14.3 V. Since the heater resistance is two ohms the power dissipated by the resistor is

$$P = \frac{V^2}{R} = \frac{14.3^2}{2} = 102 \text{ W}$$

Thus the heater temperature rise of the tank is given by:
 $(0.05^\circ\text{C/min-W})(102 \text{ W}) = 5.1^\circ\text{C/min}$. But, the tank is losing 0.2°C/min so the net increase in tank heating rate is 4.9°C/min .

(c) Following the same logic as part (b) we can construct the following responses:

Tank Temperature (°C)	Amplifier Output (V)	Heater Power (W)	Heating Rate (°C/min)
40	10	43	1.96
60	5	9.2	0.26
80	0	0	-0.2

This shows that at the setpoint of 80°C, the system is actually losing temperature at a rate of -0.2 °C/min.

Homework 1 was posted on Canvas.

- HW 1 is due August 29, 11:00 AM and needs to be submitted on Canvas.