Signals and Circuits

AERN 35500

Capacitors

Chapter 5: 5-1(Capacitor) pp. 211-220;

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

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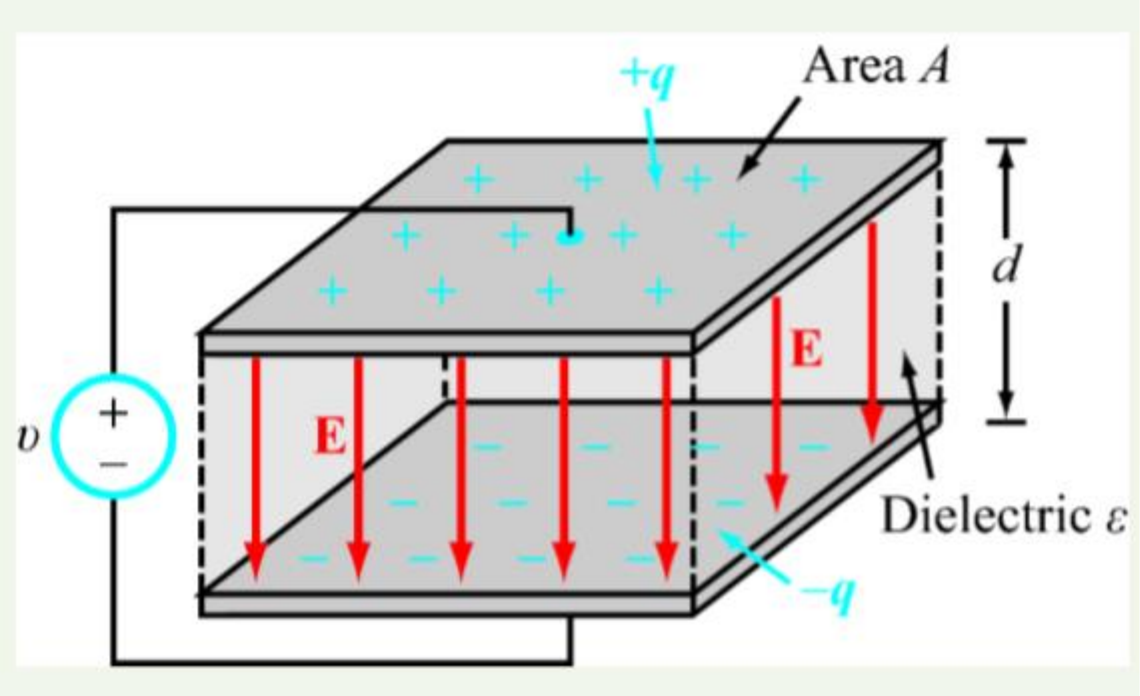
Capacitor

Capacitor is a passive element.

A capacitor is an electrical device constructed of two parallel conductive plates separated by an insulating material called the dielectric.



It can store energy/charge and give it back at a later time.

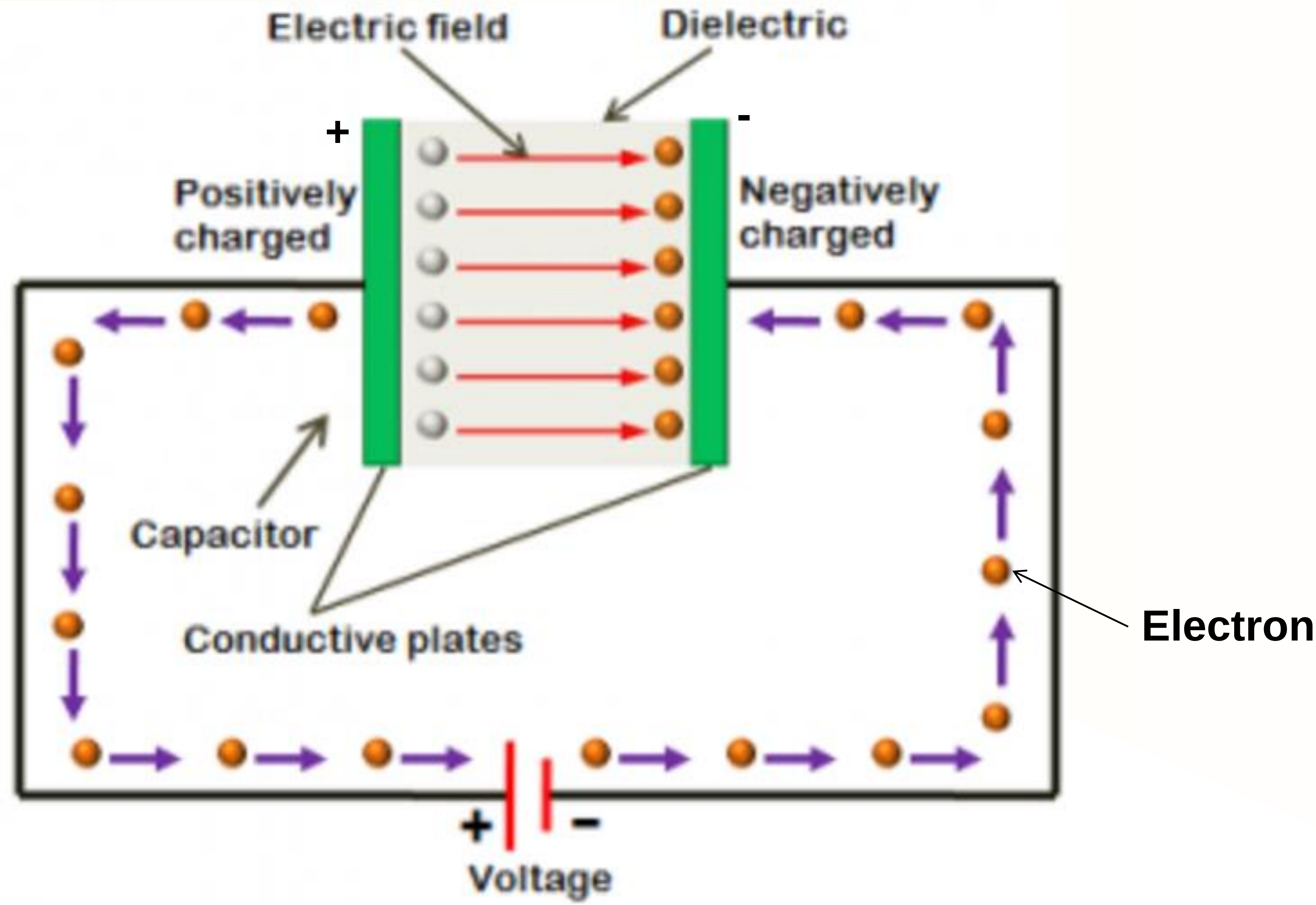
Capacitor

Application

- Store energy
- A filter
- Decoupling and coupling signals
- Circuit protection

Capacitor

How a capacitor stores the charge.



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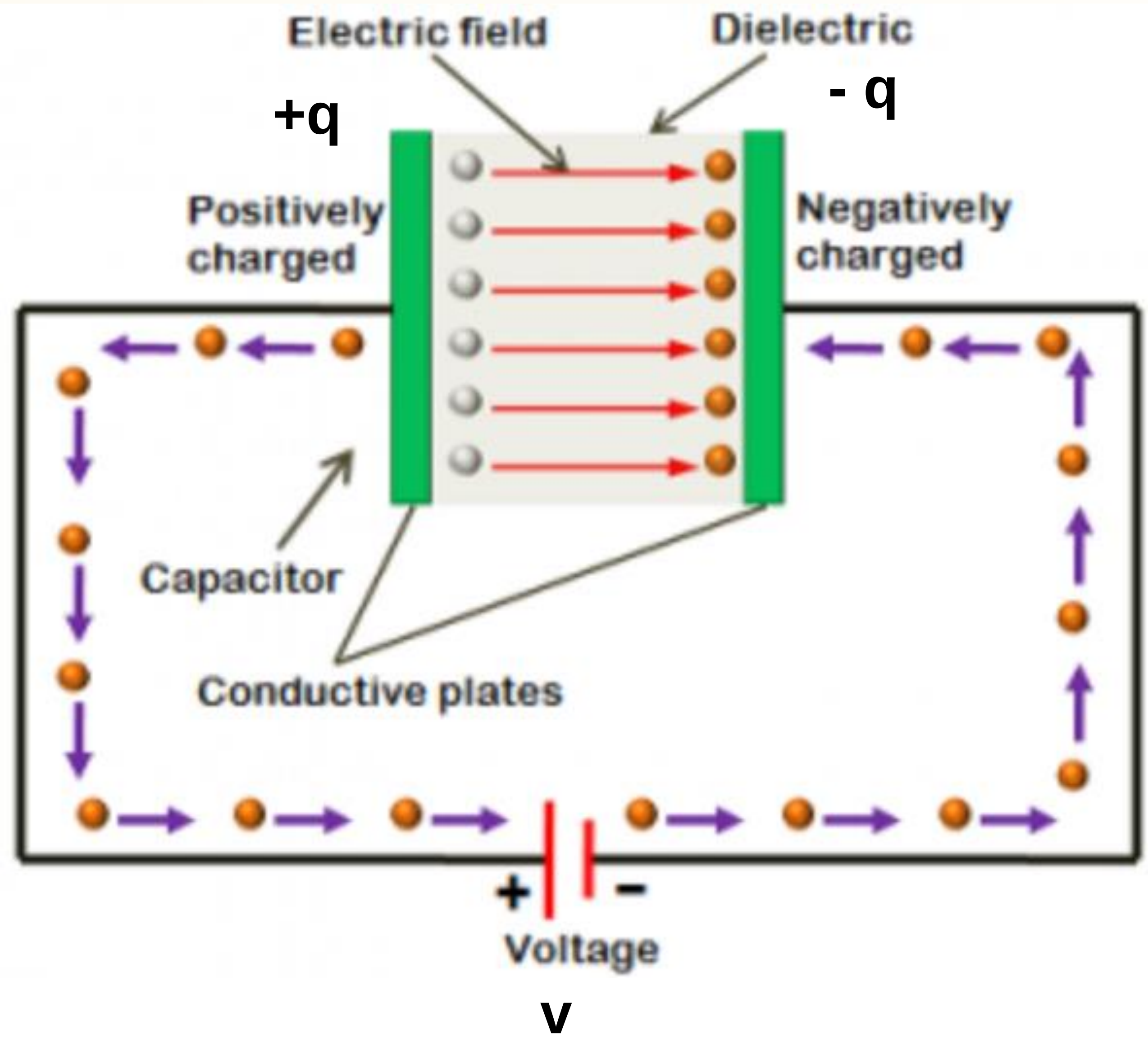
Capacitor

The amount of charge that a capacitor can store per unit of voltage across its plates is its capacitance, designated *C*.

$$C = \frac{q}{v} \quad (F)$$



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q is the charge of the capacitor
v is the voltage applied between its terminals

after Michael Faraday

Coulomb/Volt ≡ Farad

- A farad is the amount of capacitance that can store 1 coulomb (C) of charge when the capacitor is charged to 1 volt.
- 1 microfarad (μF) = 10^{-6} farad
- 1 nanofarad(nF)= 10^{-9} farad
- 1 picofarad (pF) = 10^{-12} farad

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Capacitor

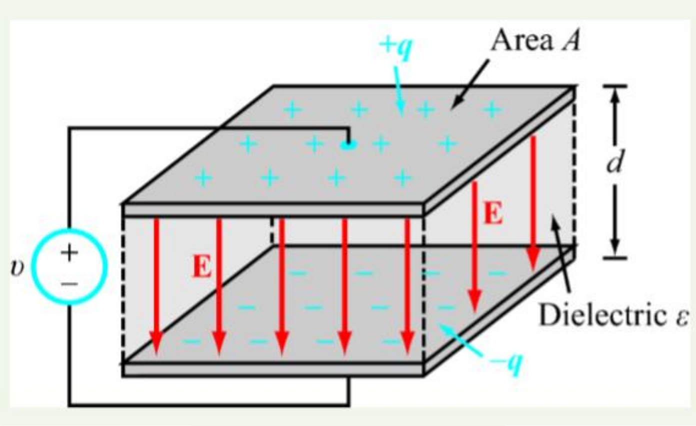
The amount of charge that a capacitor can store per unit of voltage across its plates is its capacitance, designated *C*.

$$C = \frac{q}{v} \quad (F)$$

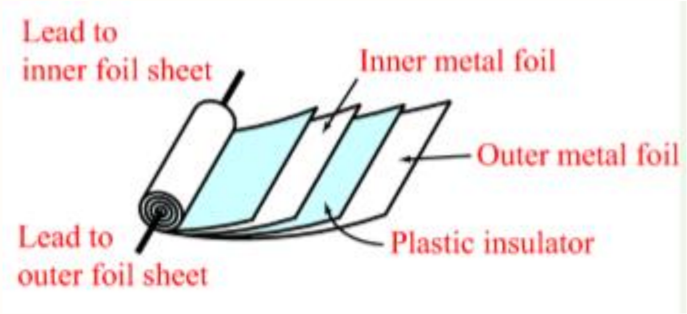
E. g.

A 2.2 *μF* capacitor has 100 v across its plates. How much charge does it store?

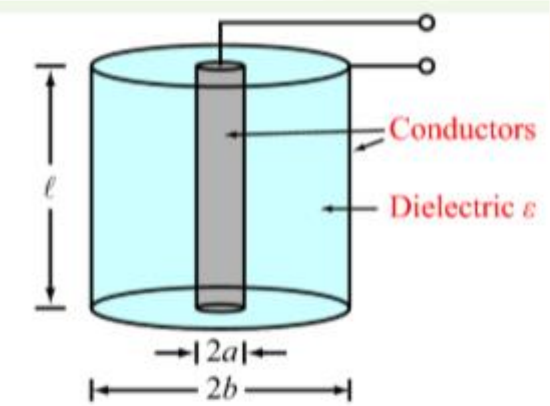
Capacitor



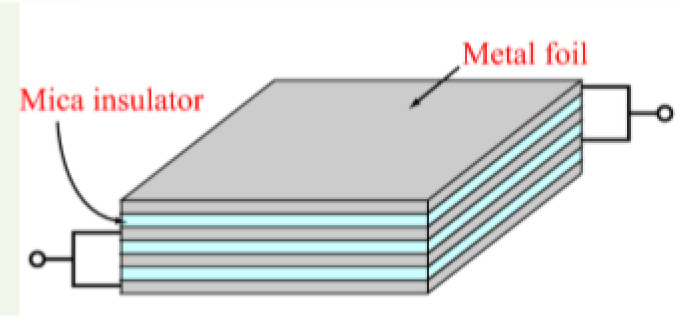
Parallel-plate capacitor



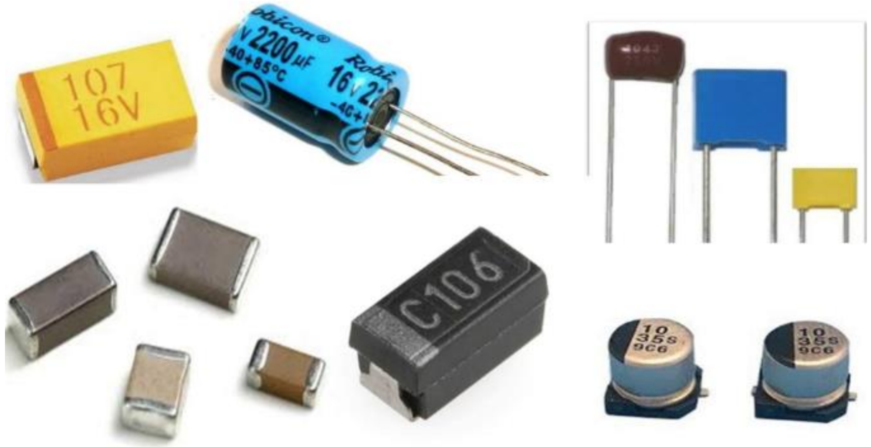
Coaxial capacitor

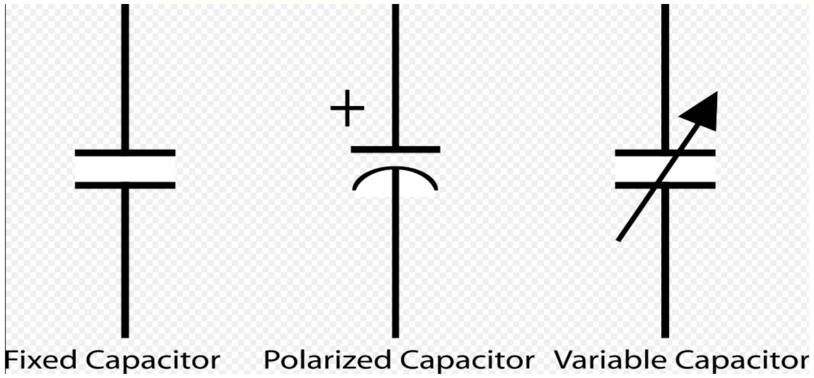


Coaxial capacitor



Mica capacitor





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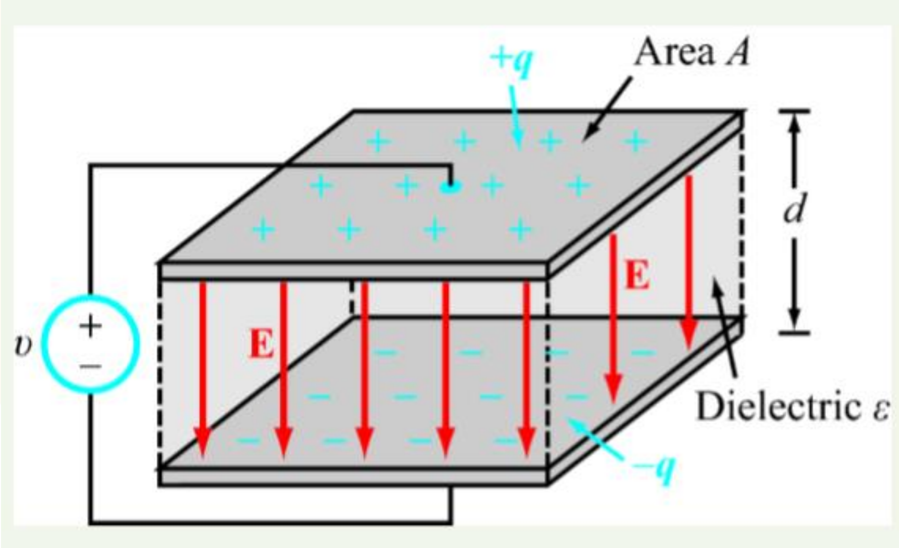
Capacitor



Factors affecting capacitance

– Area of the plate.
– Distance between the plates.
– Type of dielectric material.

$$C = \frac{\varepsilon A}{d}$$

Where ε represents the permittivity of the material between the plates, *A* the area of the plates and *d* their separation distance

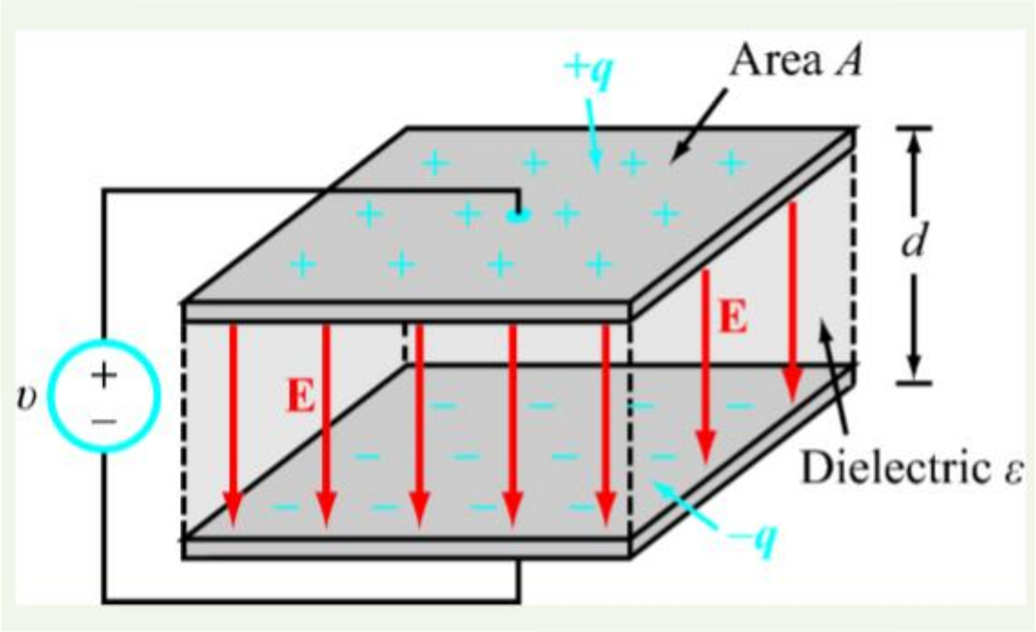
C : Capacitance in Farads (F)

d : distance between the conducting plates (m)

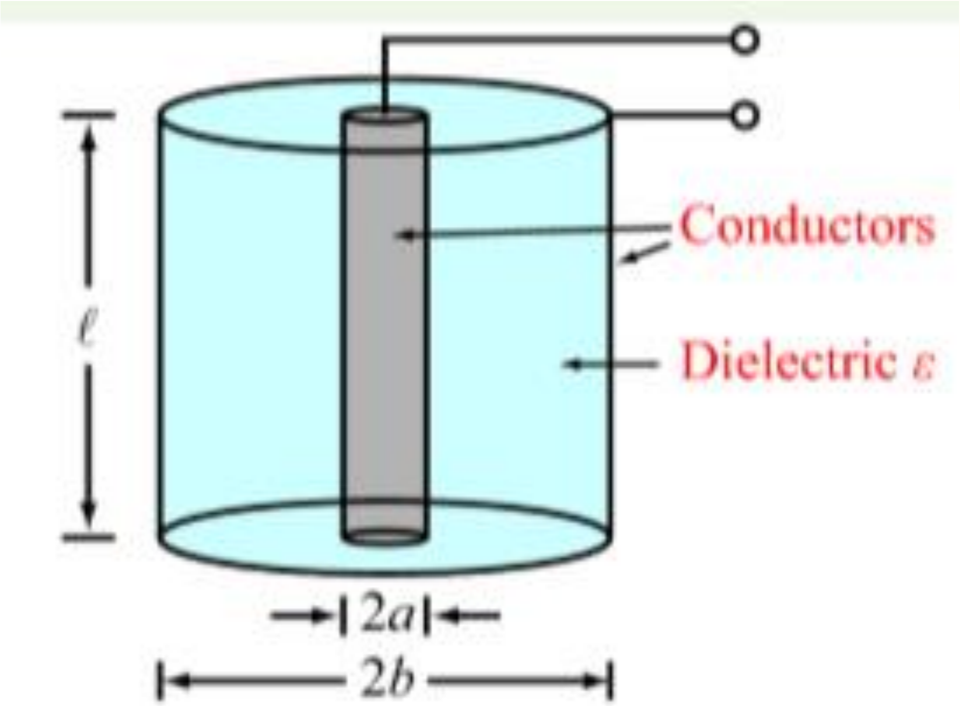
ε: permittivity (F/m)

A : surface area of plates (m^2)

Capacitor



$$C = \varepsilon A/d$$



$$C = \frac{2\pi\varepsilon l}{\ln(b/a)}$$

Capacitor

Dielectric constant ε_r

A measure of the effectiveness of a material as an insulator.

$$\varepsilon_r = \varepsilon / \varepsilon_o$$

Where ε_r is the dielectric constant of the material, ε its permittivity value, and ε_o the permittivity of air

Some examples of dielectric constants:

- Paper = 2 - 3
- Mica = 5 - 6
- Titanium = 90 - 170

Capacitor

- The voltage across a capacitor cannot change instantaneously.

$$c = \frac{q}{v}$$

$$i = \frac{dq}{dt} = \frac{dcv}{dt} = c\frac{dv}{dt}$$

- Under a stable dc condition, a capacitor behaves like an open circuit.

For stable DC $\frac{dv}{dt} = 0$

$$i = c\frac{dv}{dt} = 0$$

- To get voltage

$$\int_{t_0}^{t} \frac{dv}{dt'}\,dt' = \frac{1}{c}\int_{t_0}^{t} i\,dt'$$

$$v(t) = v(t_0) + \frac{1}{c}\int_{t_0}^{t} i\,dt'$$

- To get power

$$p(t) = vi = Cv\frac{dv}{dt}$$

Sign of the power?

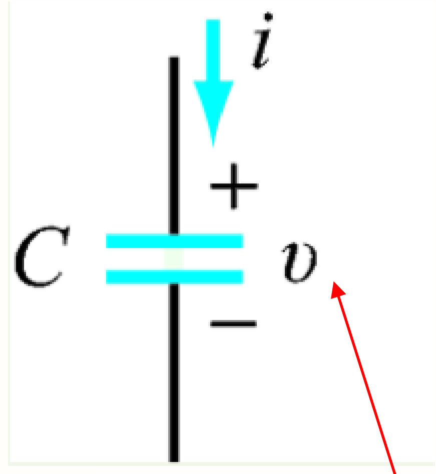
- To get energy change

$$w(t) = \int_{t_0}^{t} p\,dt' = C\int_{t_0}^{t} (v\frac{dv}{dt'})\,dt' = C\int_{t_0}^{t} [\frac{d}{dt'}\left(\frac{1}{2}v^2\right)]\,dt' = \frac{1}{2}C(v(t))^2 - \frac{1}{2}C(v(t_0))^2$$

Sign of the energy change?

- To get energy in a capacitor

$$E(t) = \frac{1}{2}C(v(t))^2$$



v is the voltage across the capacitor

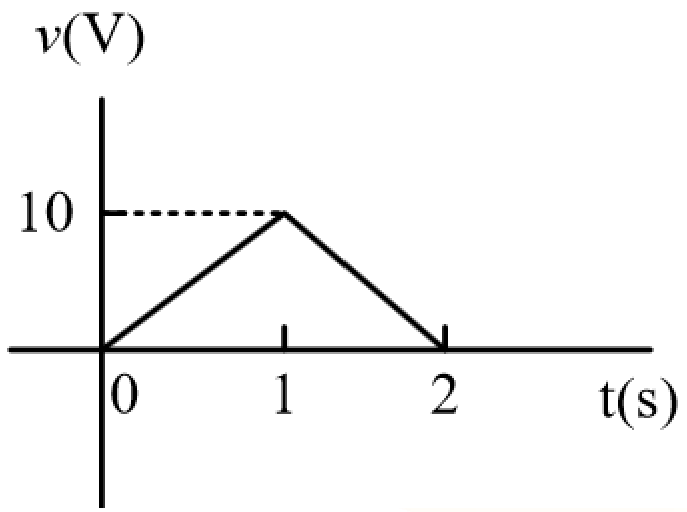
The direction of *i* is entering the positive terminal of the capacitor.

Capacitor

$$i = \frac{dq}{dt} = \frac{dcv}{dt} = c\frac{dv}{dt}$$

E. g.

Consider the following voltage waveform across a capacitor C = 1mF, Find the current through the capacitor.



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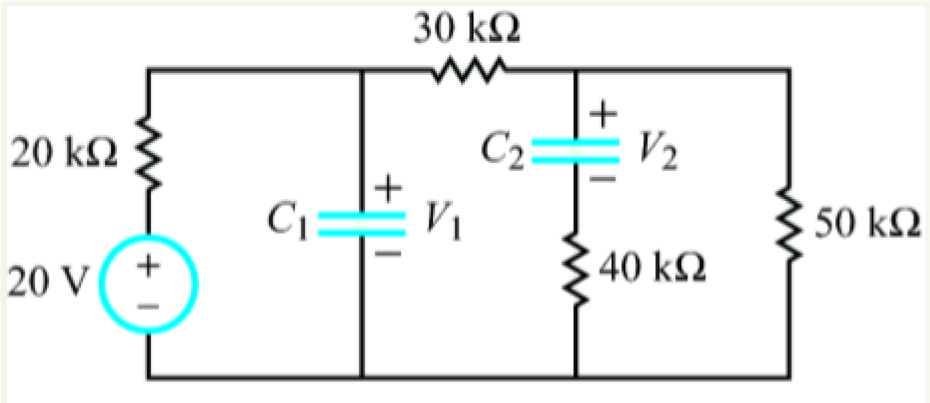
Capacitor

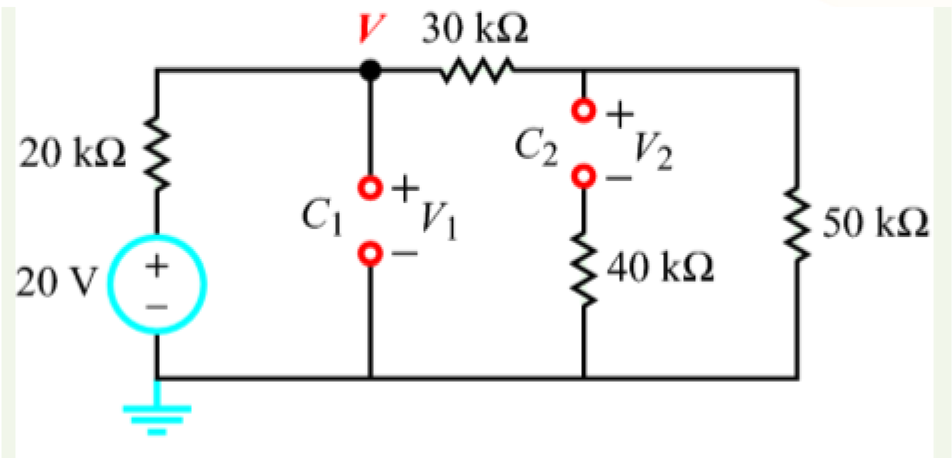
- Under a stable dc condition, a capacitor behaves like an open circuit.

E. g.

Determine voltages across the capacitors in the circuit. Assume that the circuit has been in its present condition for a long time.



Original circuit

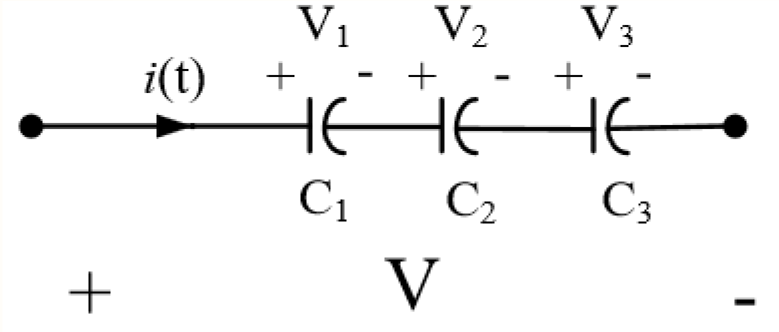


Equivalent circuit

Capacitor

- Capacitors in series

$$V(t) = V_1 + V_2 + V_3$$



$$\frac{1}{C_{eq}}\int_0^t i(t)dt = \frac{1}{C_1}\int_0^t i(t)dt + \frac{1}{C_2}\int_0^t i(t)dt + \frac{1}{C_3}\int_0^t i(t)dt$$

$$\rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The equivalence relationship for capacitors connected in series is similar in form to the relationship for resistors connected in parallel.

$$q_1 = q_2 = q_3 = q$$

$$V_1 = \frac{q_1}{C_1} \qquad V_2 = \frac{q_2}{C_2} \qquad V_3 = \frac{q_3}{C_3} \qquad V = \frac{q}{C_{eq}}$$

$$V_x = \frac{C_{eq}}{C_x}V$$

The largest-value capacitor in a series connection will have the smallest voltage across it. The smallest-value capacitor will have the largest voltage across it.

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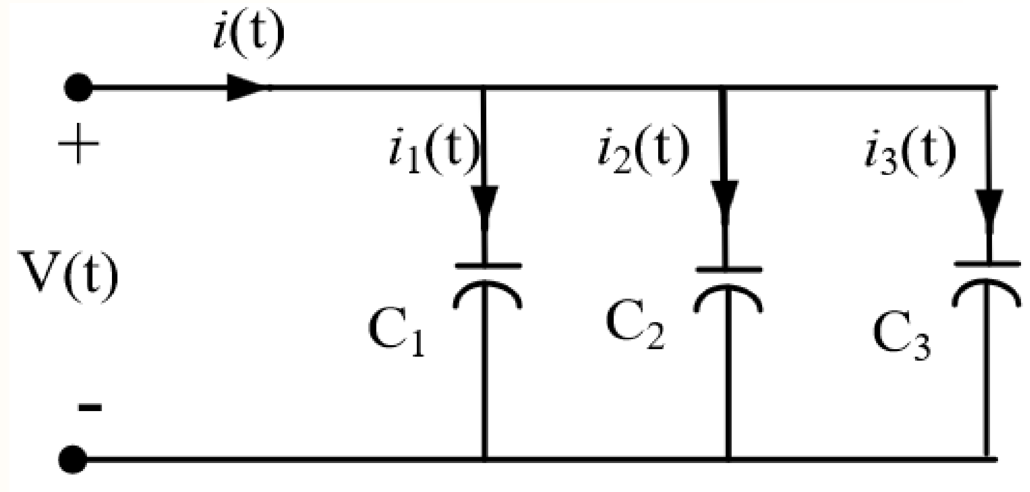
Capacitor

➢ Capacitors in parallel:



$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

$$C_{eq}\frac{dV(t)}{dt} = C_1\frac{dV(t)}{dt} + C_2\frac{dV(t)}{dt} + C_3\frac{dV(t)}{dt}$$

$$\rightarrow \quad C_{eq} = C_1 + C_2 + C_3$$

The equivalence relationship for capacitors connected in parallel is similar in form to the relationship for resistors connected in series.

Capacitor

E. g.

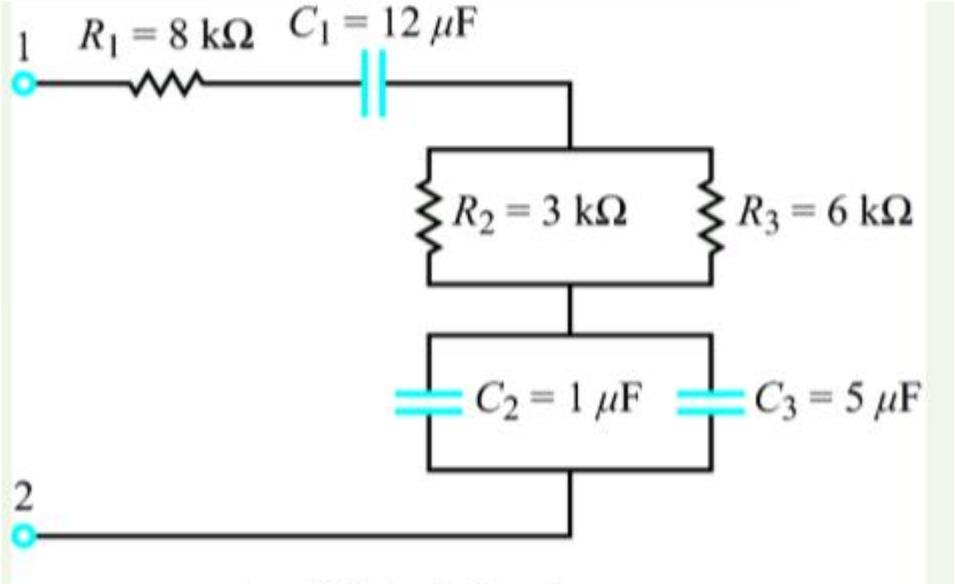
➢ Capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

➢ Capacitors in parallel:

$$C_{eq} = C_1 + C_2 + C_3$$

Find the equivalent circuit for the following circuit.



Original circuit

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