Chapter 16

MECHANISMS OF HEAT TRANSFER

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Objectives

- Understand the basic mechanisms of heat transfer, which are conduction, convection, and radiation, and Fourier's law of heat conduction, Newton's law of cooling, and the Stefan–Boltzmann law of radiation
- Identify the mechanisms of heat transfer that occur simultaneously in practice
- Develop an awareness of the cost associated with heat losses
- Solve various heat transfer problems encountered in practice

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What is heat transfer and why is it important ?

Heat transfer occurs when there is temperature difference in a body, this temperature difference is reduced in magnitude in a course of time by heat flow from a region of high temperature to a region of low temperature. the body can be in solid state, gas or liquid state….

Therefore, the subject dealing with the rate at which the heat flow process occurs is called heat transfer.

Distinction between thermodynamics and heat transfer

- In thermodynamics when we have a system that undergoes Heat-work interactions, the system state changes form one equilibrium state to another equilibrium state during that shift

- because of the heat-work interaction, the system attains a certain state which is described by temperature, pressure etc…during this process, the time is not a factor and we do not care how much time goes in that process or at which rate the interaction takes place…

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Distinction between thermodynamics and heat transfer

- In heat transfer because there is a temperature difference, heat flow occurs, then it is vital to know what is the rate at which that heat flow has occurred or how long did it take to attain a certain temperature……
- Therefore, heat transfer is a subject dealing with the rate at which heat flow occurs.
- Knowing these laws and how they are applied, it will help heat transfer engineers design and size equipment where heat transfer occurs.

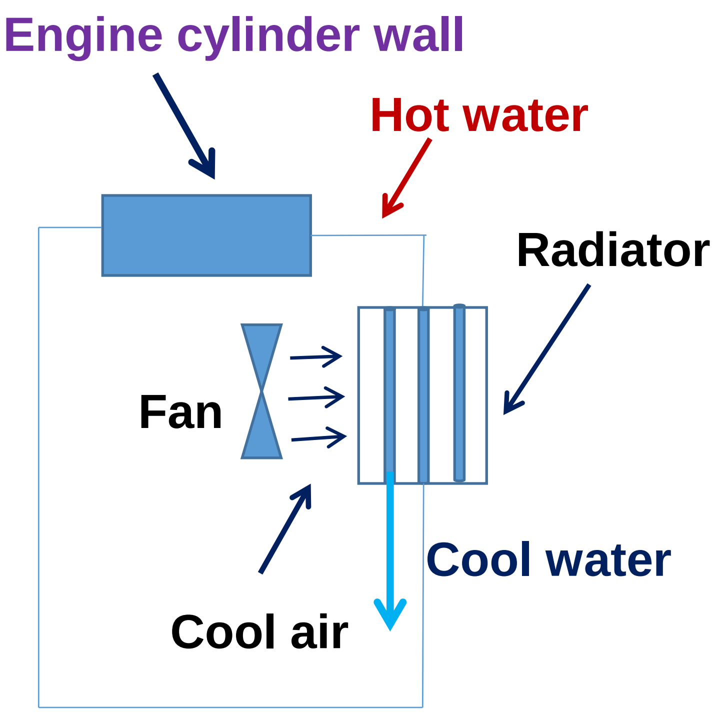
Example of Heat Transfer process

Concept is clearly applied in a car radiator; the following cooling scheme takes place;

Coolant is pumped to the engine cylinder wall; it picks up heat and the hot water is released back to the radiator to be cooled by the fan and pumped back to the cylinder wall…

So the radiator take heat from hot water (conduction) and give it to the cool air provided by the fan (convection) and send it back to the cylinder.

!!!! It is very fundamental to understand the Process of heat transfer and how is applied so we are able to design and size hardware such as radiators and other similar equipment



The law of heat conduction, also known as Fourier's law, states that the rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat flows.

$$\dot{Q}_{\mathrm{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad \text{(W)}$$

The **minus sign** accounts for the fact that heat flows from a higher temperature to a lower temperature.

Joseph **Fourier**, in full Jean-Baptiste-Joseph, Baron **Fourier**, (born March 21, 1768, Auxerre, France—died May 16, 1830, Paris), French mathematician, known also as an Egyptologist and administrator, who exerted strong influence on mathematical physics through his Théorie analytique de la chaleur (1822; The Analytical ...

16-1 INTRODUCTION

- *Heat* as the form of energy that can be transferred from one system to another as a result of temperature difference.
- A thermodynamic analysis is concerned with the *amount* of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the *rates* of such energy transfers is the *heat transfer*.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.

16-1 INTRODUCTION

- **Heat can be transferred in three basic modes:**
  - conduction
  - convection
  - radiation
- **All modes of heat transfer require the existence of a temperature difference.**

16-2 CONDUCTION

Conduction: The transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles.

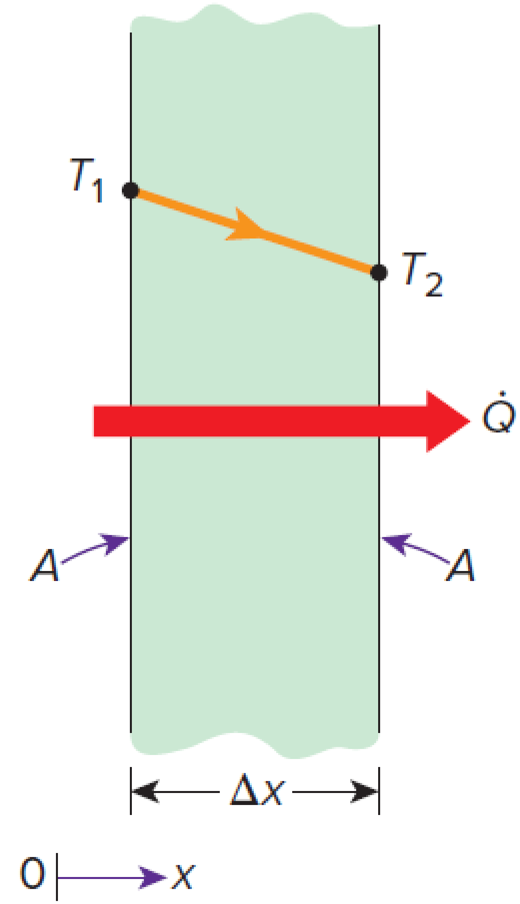
In gases and liquids, conduction is due to the ***collisions*** and ***diffusion*** of the molecules during their random motion.

In solids, it is due to the combination of ***vibrations*** of the molecules in a lattice and the energy transport by ***free electrons***.

The rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area but is inversely proportional to the thickness of the layer.

$$\text{Rate of heat conduction} \propto \frac{(\text{Area})(\text{Temperature difference})}{\text{Thickness}}$$

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad (\text{W})$$



Heat conduction through a large plane wall of thickness Δx and area *A*.

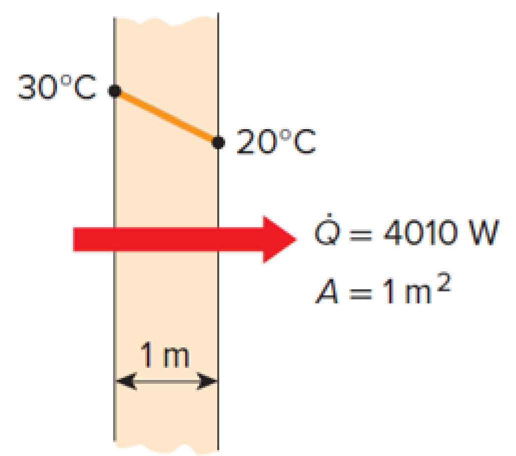
$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

Fourier's law of heat conduction

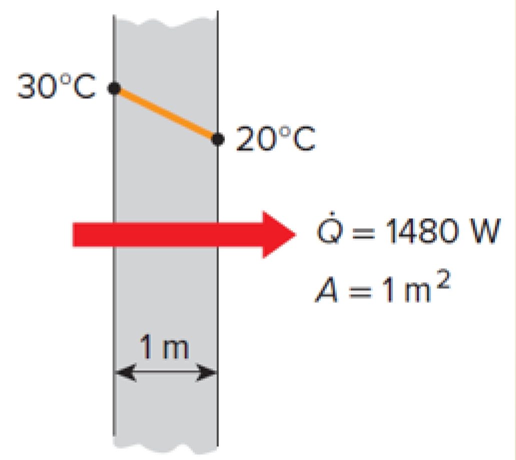
Thermal conductivity, *k*: A measure of the ability of a material to conduct heat.

Temperature gradient dT/dx: The slope of the temperature curve on a *T-x* diagram.

Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing *x*. The *negative sign* in the equation ensures that heat transfer in the positive *x* direction is a positive quantity.

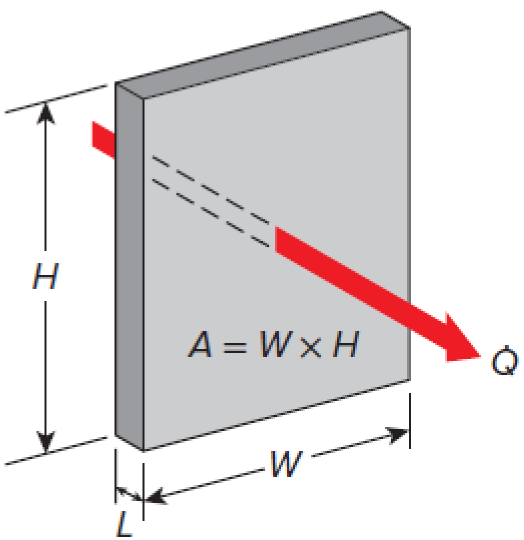


***(a)* Copper** $(k = 401\ W/m.K)$



***(b)* Silicon** $(k = 148\ W/m.K)$

In heat conduction analysis, *A* represents the area *normal* to the direction of heat transfer.



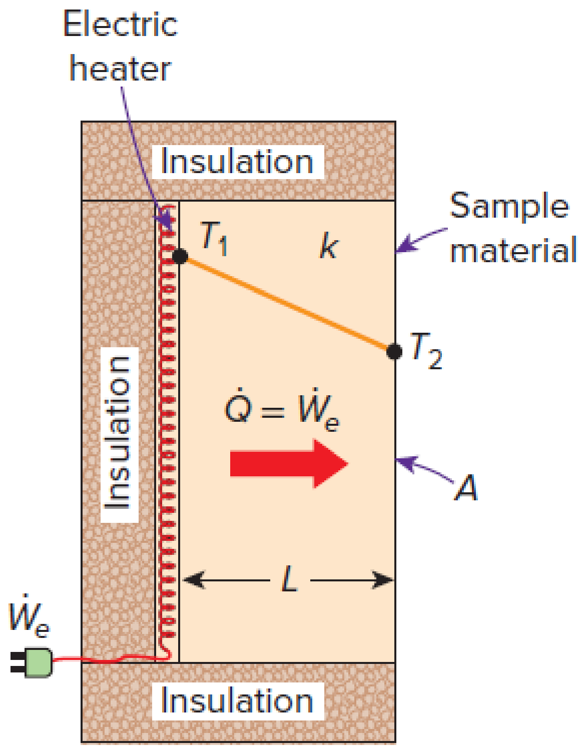
The rate of heat conduction through a solid is directly proportional to its thermal conductivity.

Thermal Conductivity

Thermal conductivity: The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.*



$$k = \frac{L}{A(T_1 - T_2)}\dot{Q}$$

A simple experimental setup to determine the thermal conductivity of a material.

The thermal conductivities of some materials at room temperature

Material	*k*, W/m.℃*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.607
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fibre	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

EXAMPLE 16–1 The Cost of Heat Loss through a Roof

The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is $k = 0.8$ W/m·K (**Fig. 16–4**). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 hours. Determine (*a*) the rate of heat loss through the roof that night and (*b*) the cost of that heat loss to the home owner if the cost of electricity is \$0.08/kWh.

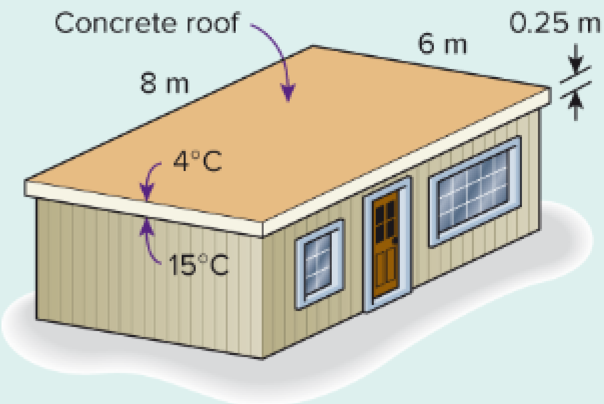


FIGURE 16–4
Schematic for **Example 16–1**.

SOLUTION

The inner and outer surfaces of the flat concrete roof of an electrically heated home are maintained at specified temperatures during a night. The heat loss through the roof and its cost that night are to be determined.

Assumptions

1 Steady operating conditions exist during the entire night since the surface temperatures of the roof remain constant at the specified values.
2 Constant properties can be used for the roof.

Properties

The thermal conductivity of the roof is given to be k = 0.8 W/m·K.

Analysis

(*a*) Noting that heat transfer through the roof is by conduction and the area of the roof is A = 6 m × 8 m = 48 m^2, the steady rate of heat transfer through the roof is

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\text{ K})(48\text{ m}^2)\frac{(15-4)^\circ\text{C}}{0.25\text{ m}} = \mathbf{1690\ W = 1.69\ kW}$$

(*b*) The amount of heat lost through the roof during a 10-hour period and its cost is

$$Q = \dot{Q}\,\Delta t = (1.69\text{ kW})(10\text{ h}) = 16.9\text{ kWh}$$

$$\begin{aligned}\text{Cost} &= (\text{Amount of energy})(\text{Unit cost of energy})\\ &= (16.9\text{ kWh})(\$0.08/\text{kWh}) = \mathbf{\$1.35}\end{aligned}$$

Discussion

The cost to the home owner of the heat loss through the roof that night was \$1.35. The total heating bill of the house will be much larger since the heat losses through the walls are not considered in these calculations.

EXAMPLE 16–2 Measuring the Thermal Conductivity of a Material

A common way of measuring the thermal conductivity of a material is to sandwich an electric thermofoil heater between two identical samples of the material, as shown in Fig. 16–9. The thickness of the resistance heater, including its cover, which is made of thin silicon rubber, is usually less than 0.5 mm. A circulating fluid such as tap water keeps the exposed ends of the samples at constant temperature. The lateral surfaces of the samples are well insulated to ensure that heat transfer through the samples is one-dimensional. Two thermocouples are embedded into each sample some distance L apart, and a differential thermometer reads the temperature drop ΔT across this distance along each sample. When steady operating conditions are reached, the total rate of heat transfer through both samples becomes equal to the electric power drawn by the heater.

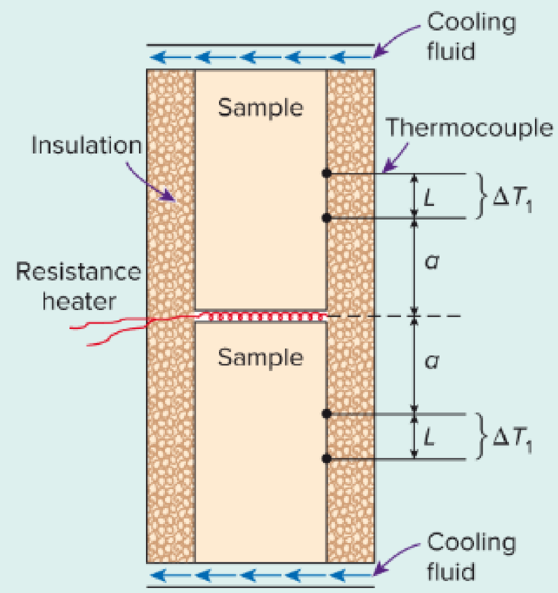


FIGURE 16–9

FIGURE 16–9
Apparatus to measure the thermal conductivity of a material using two identical samples and a thin resistance heater (**Example 16–2**).

In a certain experiment, cylindrical samples of diameter 5 cm and length 10 cm are used. The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.4 A at 110 V, and both differential thermometers read a temperature difference of 15°C. Determine the thermal conductivity of the sample.

SOLUTION

The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

Assumptions

1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis

The electrical power consumed by the resistance heater and converted to heat is

$$\dot{W}_e = \mathbf{V}I = (110\ \text{V})(0.4\ \text{A}) = 44\ \text{W}$$

The rate of heat flow through each sample is

$$\dot{Q} = \tfrac{1}{2}\dot{W}_e = \tfrac{1}{2} \times (44\ \text{W}) = 22\ \text{W}$$

since only half of the heat generated flows through each sample because of symmetry. Reading the same temperature difference across the same distance in each sample also confirms that the apparatus possesses thermal symmetry. The heat transfer area is the area normal to the direction of heat transfer, which is the cross-sectional area of the cylinder in this case:

$$A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.05\text{ m})^2 = 0.001963\text{ m}^2$$

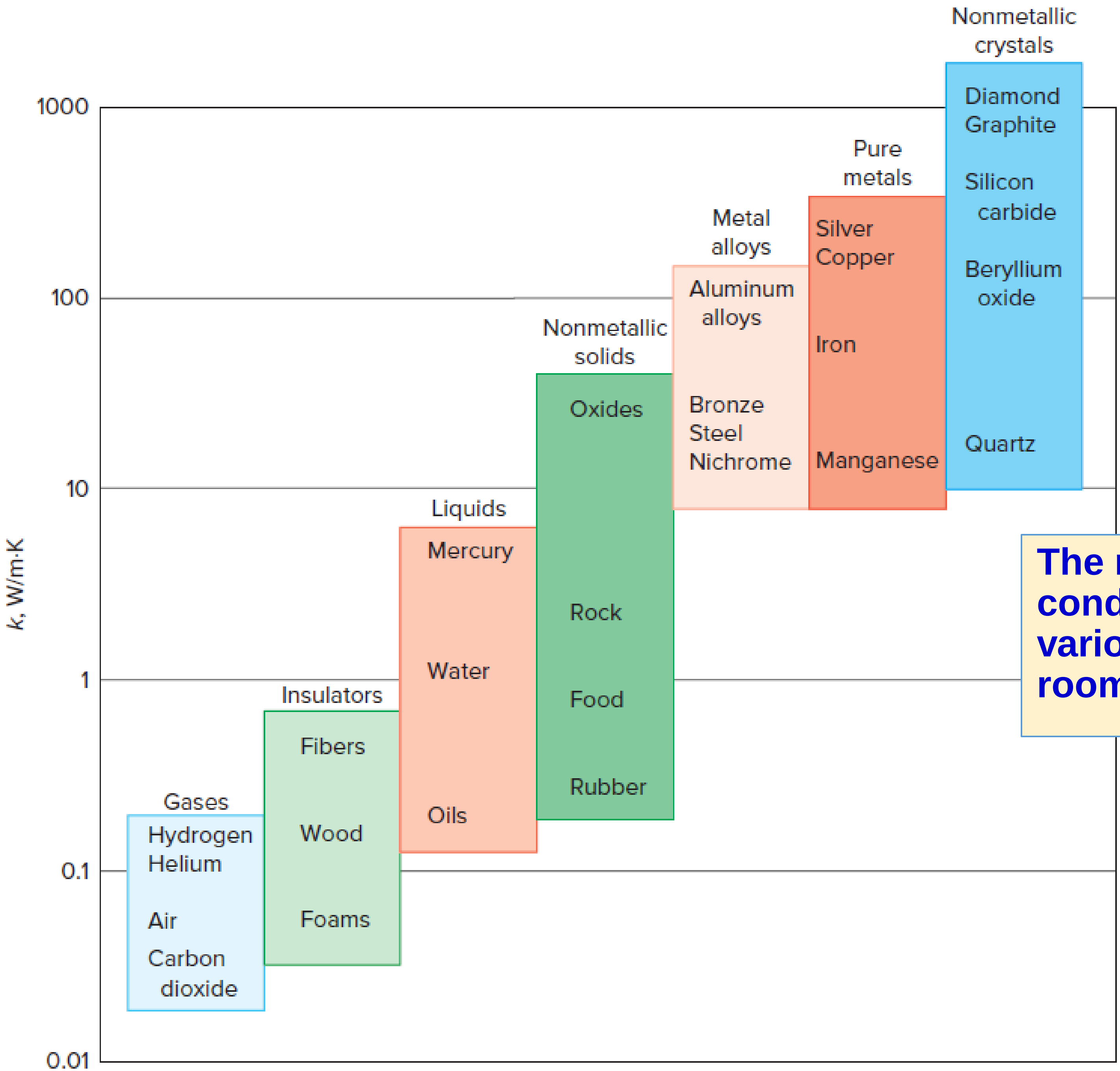
Noting that the temperature drops by 15°C within 3 cm in the direction of heat flow, the thermal conductivity of the sample is determined to be

$$\dot{Q} = kA\frac{\Delta T}{L} \quad \rightarrow \quad k = \frac{\dot{Q}L}{A\,\Delta T} = \frac{(22\text{ W})(0.03\text{ m})}{(0.001963\text{ m}^2)(15°\text{C})} = \mathbf{22.4\ W/m{\cdot}K}$$

Discussion

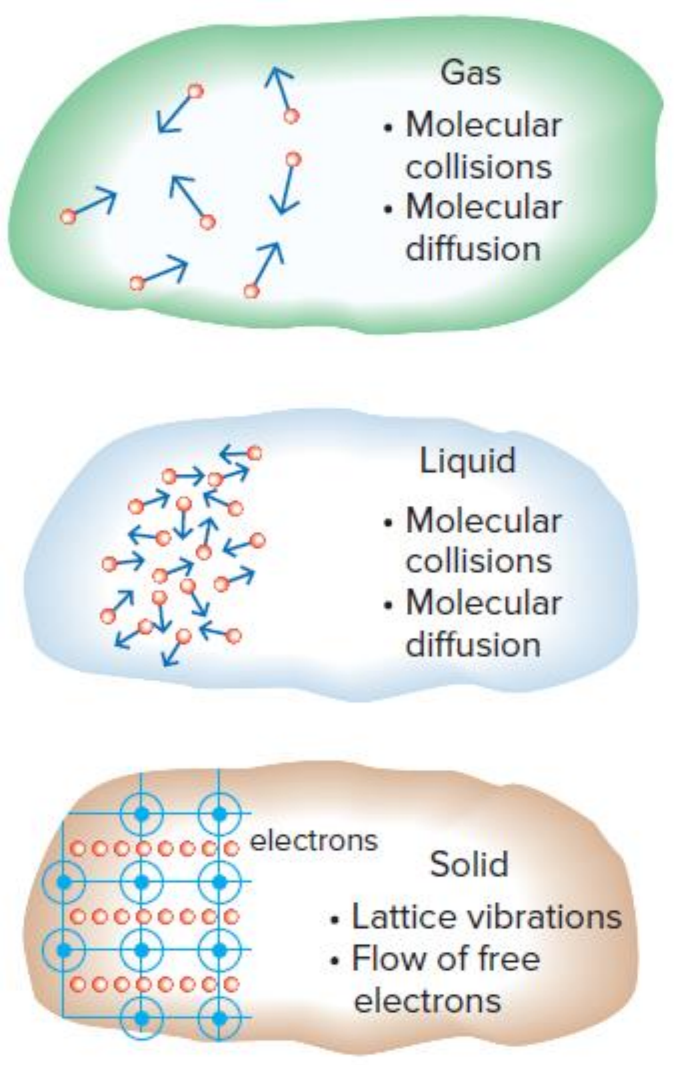
Perhaps you are wondering if we really need to use two samples in the apparatus, since the measurements on the second sample do not give any additional information. It seems like we can replace the second sample by insulation. Indeed, we do not need the second sample; however, it enables us to verify the temperature measurements on the first sample and provides thermal symmetry, which reduces experimental error.

Thermal Conductivity-1



The range of thermal conductivity of various materials at room temperature.

Thermal Conductivity-2



The mechanisms of heat conduction in different phases of a substance.

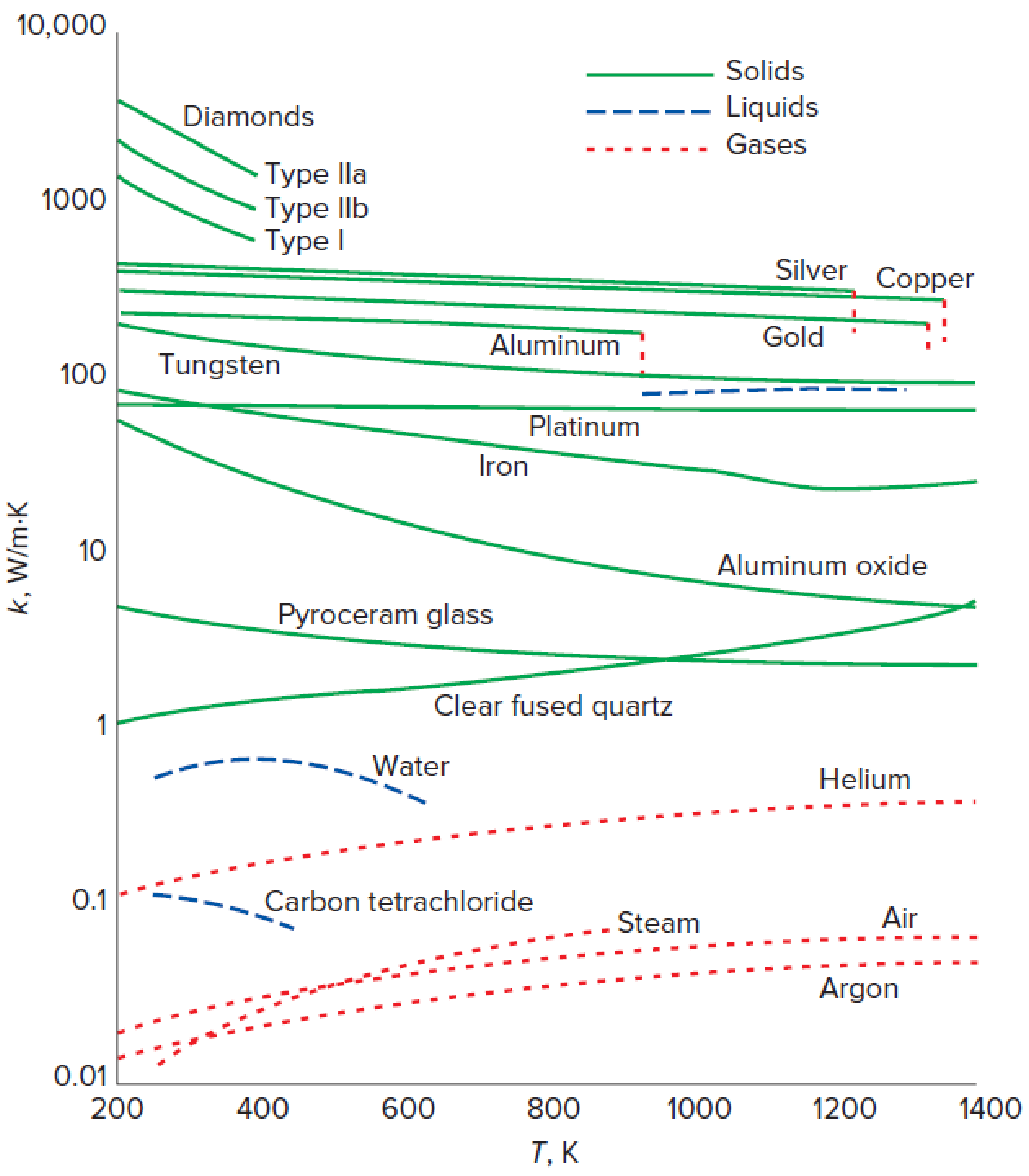
The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper.

Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or Alloy	k, W/m · K, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

Thermal Conductivity-3



Thermal conductivities of materials vary with temperature

T, K	Copper k, W/m · K	Aluminum k, W/m · K
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

The variation of the thermal conductivity of various solids, liquids, and gases with temperature.

Thermal Diffusivity

c_p Specific heat, $\mathrm{J/kg \cdot {}^\circ C}$: **Heat capacity per unit mass**

ρc_p Heat capacity, $\mathrm{J/m^3 \cdot {}^\circ C}$: **Heat capacity per unit volume**

α Thermal diffusivity, $\mathrm{m^2/s}$: **Represents how fast heat diffuses through a material**

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \qquad (\mathrm{m^2/s})$$

- Thermal conductivity, κ, is how well the material passes on heat (thermal energy),
- Thermal diffusivity, α, is how well the material passes on a temperature change.

The thermal conductivities of some materials at room temperature

Material	$\alpha, \mathrm{m^2/s}^*$
Silver	149×10^{-6}
Gold	127×10^{-6}
Copper	113×10^{-6}
Aluminium	97.5×10^{-6}
Iron	22.8×10^{-6}
Mercury(l)	4.7×10^{-6}
Marble	1.2×10^{-6}
Ice	1.2×10^{-6}
Concrete	0.75×10^{-6}
Brick	0.52×10^{-6}
Heavy soil (dry)	0.52×10^{-6}
Glass	0.34×10^{-6}
Glass wool	0.23×10^{-6}
Water(l)	0.14×10^{-6}
Beef	0.14×10^{-6}
Wood (oak)	0.13×10^{-6}

Thermal Diffusivity

- A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity.

- The larger the thermal diffusivity, the faster the propagation of heat into the medium.

- A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.

$$\alpha = \frac{\text{Heat conduction}}{\text{Heat storage}} = \frac{k}{\rho c_p} \qquad (\text{m}^2/\text{s})$$

It tells us how fast the transfer of heat occurs within a material from the hotter side to the cooler side. Thermal diffusivity is a measure of how quickly a material reacts to temperature changes. It is measured in m^2/s.

EXAMPLE 16–3 Conversion between SI and English Units

An engineer who is working on the heat transfer analysis of a brick building in English units needs the thermal conductivity of brick. But the only value he can find from his handbooks is 0.72 W/m·°C, which is in SI units. To make matters worse, the engineer does not have a direct conversion factor between the two unit systems for thermal conductivity. Can you help him out?

SOLUTION

The situation this engineer is facing is not unique, and most engineers often find themselves in a similar position. A person must be very careful during unit conversion not to fall into some common pitfalls and to avoid some costly mistakes. Although unit conversion is a simple process, it requires utmost care and careful reasoning.

The conversion factors for W and m are straightforward and are given in conversion tables to be

$$1 \text{ W} = 3.41214 \text{ Btu/h}$$
$$1 \text{ m} = 3.2808 \text{ ft}$$

But the conversion of °C into °F is not so simple, and it can be a source of error if one is not careful. Perhaps the first thought that comes to mind is to replace °C by (°F − 32)/1.8 since T(°C) = [T(°F) − 32]/1.8. But this will be wrong since the °C in the unit W/m·°C represents *per °C change in temperature*. Noting that 1°C change in temperature corresponds to 1.8°F, the proper conversion factor to be used is

$$1°\text{C} = 1.8°\text{F}$$

Substituting, we get

$$1\ \text{W/m}\cdot{}^\circ\text{C} = \frac{3.41214\ \text{Btu/h}}{(3.2808\ \text{ft})(1.8^\circ\text{F})} = 0.5778\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

which is the desired conversion factor. Therefore, the thermal conductivity of the brick in English units is

$$\begin{aligned} k_{\text{brick}} &= 0.72\ \text{W/m}\cdot{}^\circ\text{C} \\ &= 0.72 \times (0.5778\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}) \\ &= \mathbf{0.42\ Btu/h\cdot ft\cdot{}^\circ F} \end{aligned}$$

Discussion

Note that the thermal conductivity value of a material in English units is about half that in SI units (**Fig. 16–10**). Also note that we rounded the result to two significant digits (the same number in the original value) since expressing the result in more significant digits (such as 0.4160 instead of 0.42) would falsely imply a more accurate value than the original one.

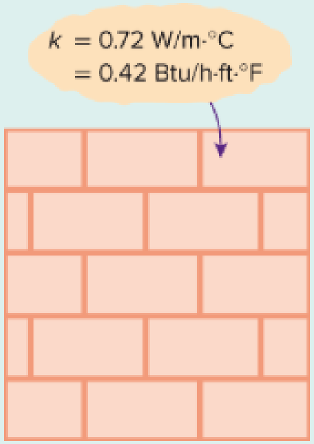


FIGURE 16–10

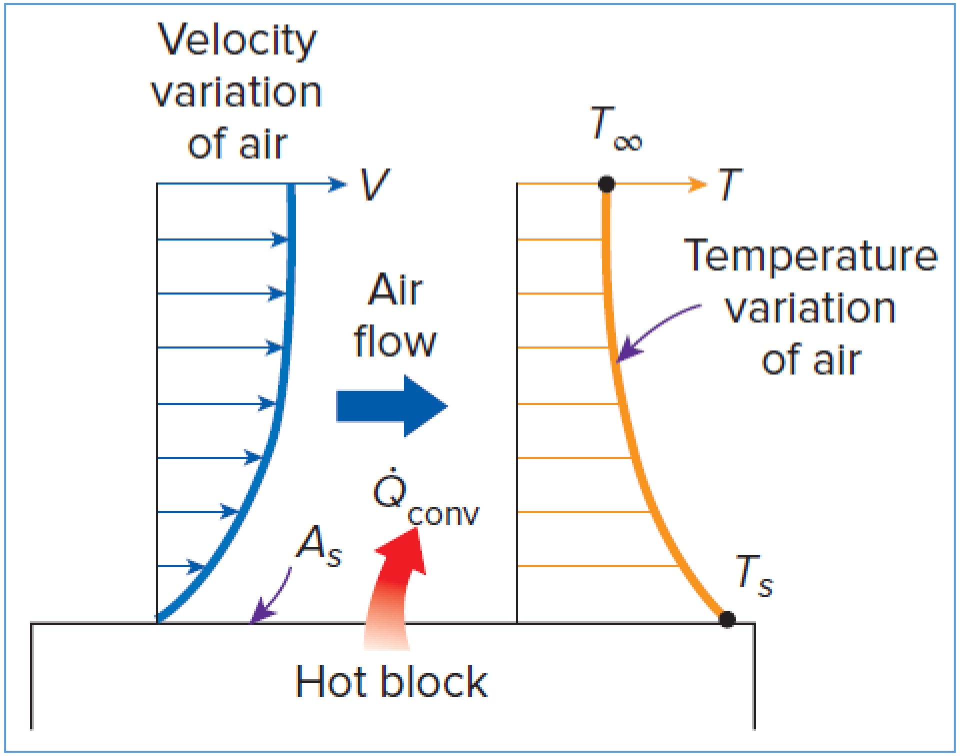
The thermal conductivity value in English units is obtained by multiplying the value in SI units by 0.5778.

16-3 CONVECTION

Convection: **The mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion*.**

The faster the fluid motion, the greater the convection heat transfer.

In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

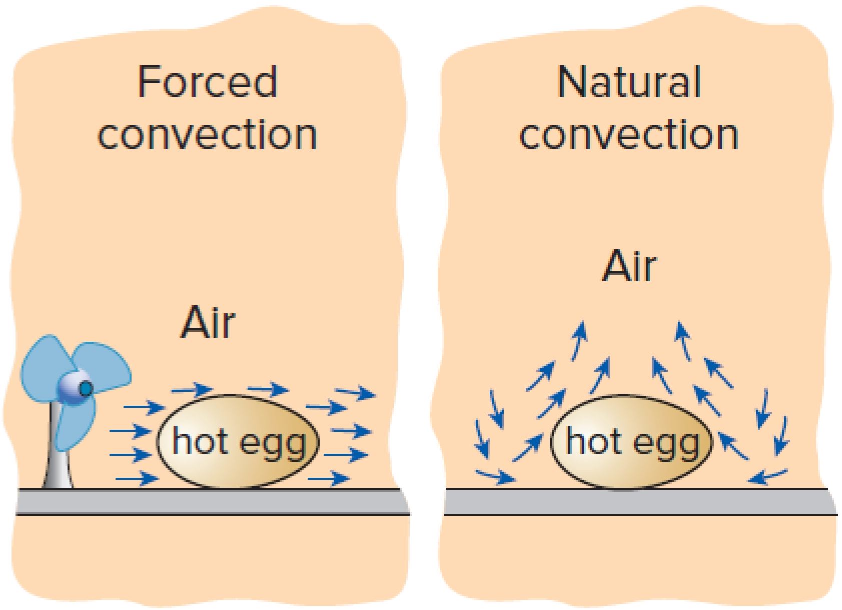


Heat transfer from a hot surface to air by convection.

16-3 CONVECTION-1

Forced convection: **If the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.**

Natural (or free) convection: **If the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.**



The cooling of a boiled egg by forced and natural convection.

Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

16-3 CONVECTION-2

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W)$$

Newton's law of cooling

h	convection heat transfer coefficient, $W/m^2 \cdot °C$
A_s	the surface area through which convection heat transfer takes place
T_s	the surface temperature
T_∞	the temperature of the fluid sufficiently far from the surface.

The convection heat transfer coefficient *h* is not a property of the fluid.

It is an experimentally determined parameter whose value depends on all the variables influencing convection such as

- the surface geometry
- the nature of fluid motion
- the properties of the fluid
- the bulk fluid velocity

Typical values of convection heat transfer coefficient

Type of convection	$h, W/m^2 \cdot °C$*
Free convection of gases	2-25
Free convection of liquids	10-1000
Forced convection of gases	25-250
Forced convection of liquids	50-20,000
Boiling and condensation	2500-100,000

EXAMPLE 16–4 **Measuring Convection Heat Transfer Coefficient**

A 2-m-long, 0.3-cm-diameter electrical wire extends across a room at 15°C, as shown in Fig. 16–13. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

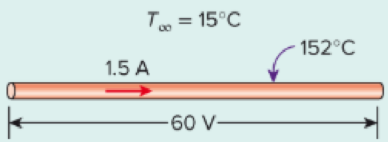


FIGURE 16–13
Schematic for Example 16–4.

SOLUTION

The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions

1 Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis

When steady operating conditions are reached, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is

$$\dot{Q} = \dot{E}_{\text{generated}} = \mathbf{V}I = (60\ \text{V})(1.5\ \text{A}) = 90\ \text{W}$$

The surface area of the wire is

$$A_s = \pi DL = \pi(0.003\ \text{m})(2\ \text{m}) = 0.01885\ \text{m}^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q}_{\text{conv}} = hA_s\,(T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{conv}}}{A_s(T_s - T_\infty)} = \frac{90\ \text{W}}{(0.01885\ \text{m}^2)(152 - 15)^\circ\text{C}} = \mathbf{34.9\ W/m^2{\cdot}K}$$

Discussion

Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

16-4 RADIATION

- Radiation: The energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.

- Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an *intervening medium.*

- In fact, heat transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.

16-4 RADIATION

- In heat transfer studies we are interested in *thermal radiation*, which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
- However, radiation is usually considered to be a *surface phenomenon* for solids.

What is volumetric phenomenon?

In general, radiation is a **volumetric phenomenon**. This is because the electrons, atoms and molecules of all solids, liquids and gases above absolute zero temperature are in constant motion and hence energy is constantly emitted, absorbed and transmitted throughout the entire volume of the matter.

Black-body radiation is the thermal electromagnetic radiation within or surrounding a body in thermodynamic equilibrium with its environment, emitted by a black body (an idealized opaque, non-reflective body). It has a specific spectrum of wavelengths, inversely related to intensity that depend only on the body's temperature, which is assumed for the sake of calculations and theory to be uniform and constant

16-4 RADIATION-1

Stefan–Boltzmann law

$$\dot{Q}_{\text{emit,max}} = \sigma A_s T_s^4 \qquad \text{(W)}$$

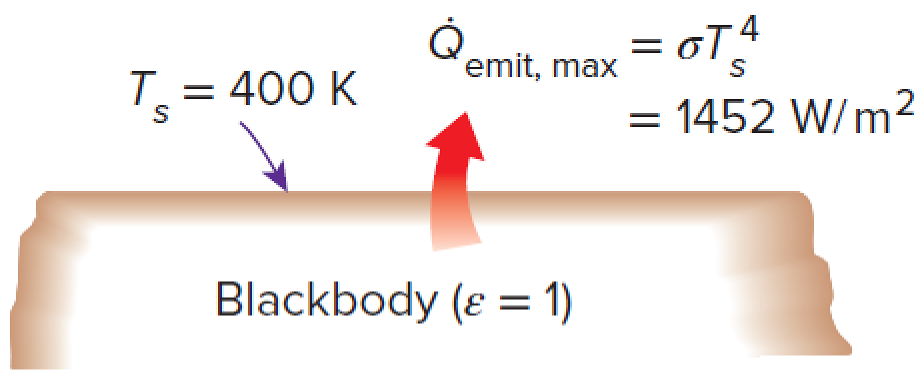
$\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$ ***Stefan–Boltzmann constant***

Blackbody: The idealized surface that emits radiation at the maximum rate.

$$\dot{Q}_{\text{emit}} = \varepsilon \sigma A_s T_s^4 \qquad \text{(W)}$$

Radiation emitted by real surfaces

Emissivity ε: A measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$ of the surface. $0 \le \varepsilon \le 1$.



Blackbody radiation represents the maximum amount of radiation that can be emitted from a surface at a specified temperature.

Emissivities of some materials at 300 k

Material	Emissivity
Aluminum foil	0.07
Anodized aluminum	0.82
Polished copper	0.03
Polished gold	0.03
Polished silver	0.02
Polished stainless steel	0.17
Black paint	0.98
White paint	0.90
White paper	0.92-0.97
Asphalt pavement	0.85-0.93
Red brick	0.93-0.96
Human skin	0.95
Wood	0.82-0.92
Soil	0.93-0.96
Water	0.96
Vegetation	0.92-0.96

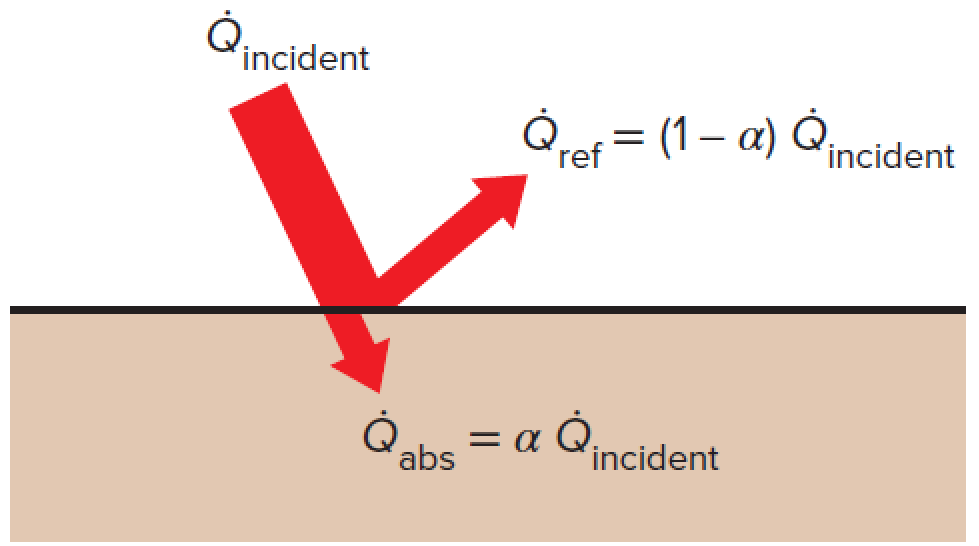
16-4 RADIATION-2

Absorptivity α The fraction of the radiation energy incident on a surface that is absorbed by the surface. $0 \leq \alpha \leq 1$

A blackbody absorbs the entire radiation incident on it ($\alpha = 1$).

Kirchhoff's law: The emissivity and the absorptivity of a surface at a given temperature and wavelength are equal.

$$\dot{Q}_{\text{absorbed}} = \alpha \dot{Q}_{\text{incident}} \qquad \text{(W)}$$



The absorption of radiation incident on an opaque surface of absorptivity α.

16-4 RADIATION-3

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Net radiation heat transfer: The difference between the rates of radiation emitted by the surface and the radiation absorbed.

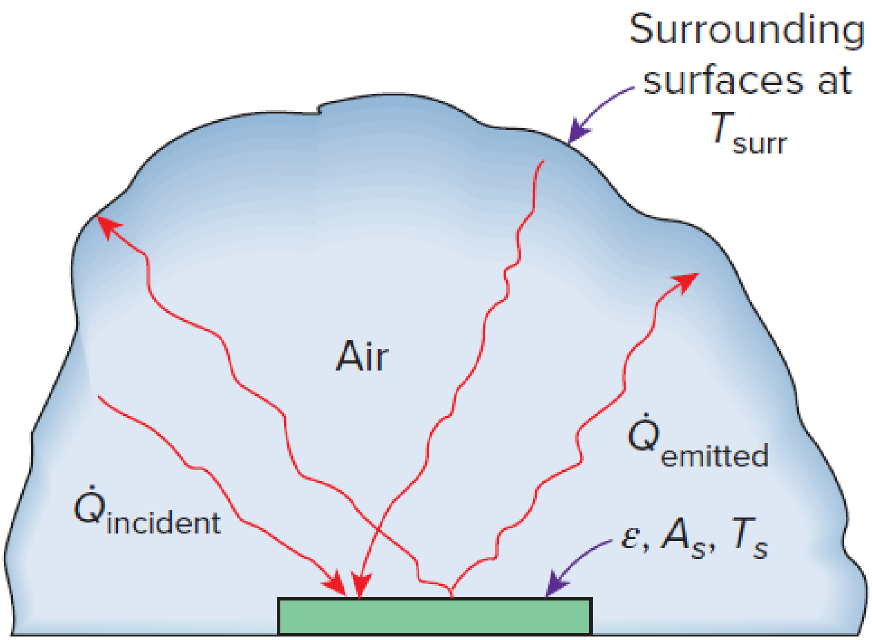
The determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on

- the properties of the surfaces
- their orientation relative to each other
- the interaction of the medium between the surfaces with radiation

Radiation is usually significant relative to conduction or natural convection, but negligible relative to forced convection.

When a surface is *completely enclosed* by a much larger (or black) surface at temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \qquad \text{(W)}$$



$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4)$

Radiation heat transfer between a surface and the surfaces surrounding it.

EXAMPLE 16–5 Radiation Effect on Thermal Comfort

It is a common experience to feel "chilly" in winter and "warm" in summer in our homes even when the thermostat setting is kept the same. This is due to the so-called "radiation effect" resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22°C at all times. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 10°C in winter and 25°C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average outer surface temperature of the person are 1.4 m^2 and 30°C, respectively (**Fig. 16–17**).

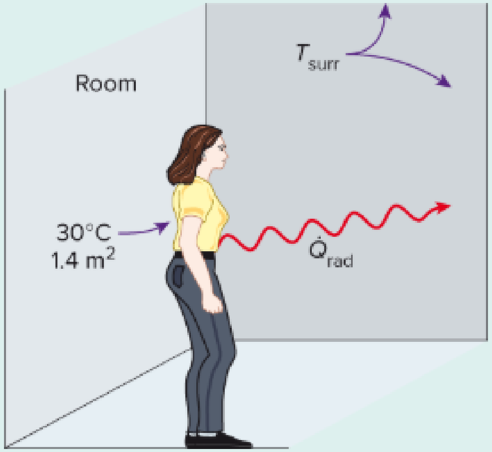


FIGURE 16–17
Schematic for **Example 16–5**.

SOLUTION

The rates of radiation heat transfer between a person and the surrounding surfaces at specified temperatures are to be determined in summer and winter.

FIGURE 16–17
Schematic for **Example 16–5**.

SOLUTION

The rates of radiation heat transfer between a person and the surrounding surfaces at specified temperatures are to be determined in summer and winter.

Assumptions

1 Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties

The emissivity of a person is $\varepsilon = 0.95$ (**Table 16–6**).

Analysis

The net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and floor in winter and summer are

$$\begin{aligned}\dot{Q}_{\text{rad, winter}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, winter}}^4) \\ &= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2) \\ &\quad \times [(30 + 273)^4 - (10 + 273)^4]\ \text{K}^4 \\ &= \mathbf{152\ W}\end{aligned}$$

and

$$\begin{aligned}\dot{Q}_{\text{rad, summer}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr, summer}}^4) \\ &= (0.95)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.4\ \text{m}^2) \\ &\quad \times [(30 + 273)^4 - (25 + 273)^4]\ \text{K}^4 \\ &= \mathbf{40.9\ W}\end{aligned}$$

Discussion

Note that we must use *thermodynamic (i.e., absolute) temperatures* in radiation calculations. Also note that the rate of heat loss from the person by radiation is almost four times as large in winter than it is in summer, which explains the "chill" we feel in winter even if the thermostat setting is kept the same.

16-4 RADIATION-4

When radiation and convection occur simultaneously between a surface and a gas:

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s \left(T_s - T_\infty\right) \qquad \text{(W)}$$

Combined heat transfer coefficient h_{combined}

Includes the effects of both convection and radiation

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = h_{\text{conv}} A_s \left(T_s - T_{\text{surr}}\right) + \varepsilon\sigma A_s \left(T_s^4 - T_{\text{surr}}^4\right)$$

$$\dot{Q}_{\text{total}} = h_{\text{combined}} A_s \left(T_s - T_\infty\right) \quad \text{(W)}$$

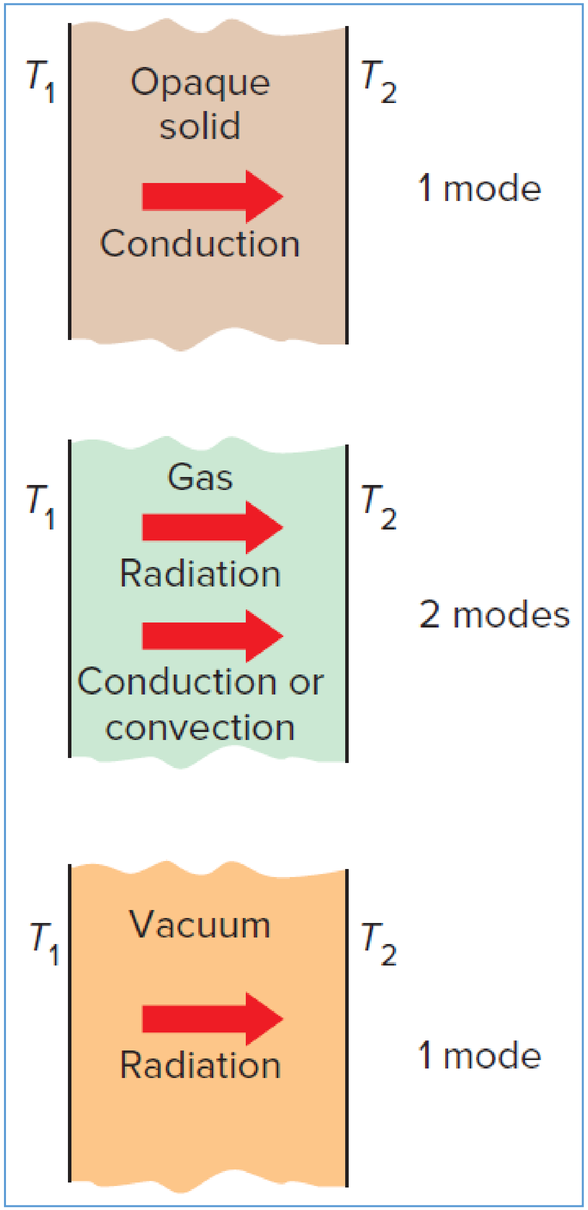
$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} = h_{\text{conv}} + \varepsilon\sigma\left(T_s + T_{\text{surr}}\right)\left(T_s^2 + T_{\text{surr}}^2\right)$$

16-5 SIMULTANEOUS HEAT TRANSFER MECHANISMS

Heat transfer is only by conduction in *opaque solids,* but by conduction and radiation in *semitransparent solids*.

A solid may involve conduction and radiation but not convection. A solid may involve convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid.*



Most gases between two solid surfaces do not interfere with radiation.

Liquids are usually strong absorbers of radiation.

Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

16-5 SIMULTANEOUS HEAT TRANSFER MECHANISMS

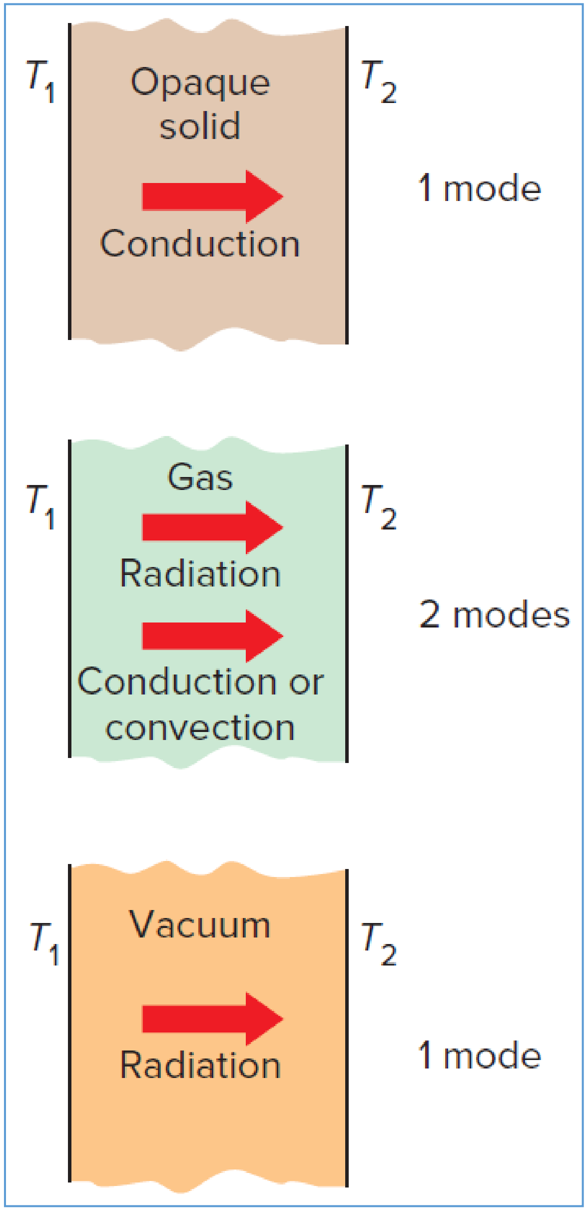
In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion.

Convection = Conduction + Fluid motion

Heat transfer through a *vacuum* is by radiation.

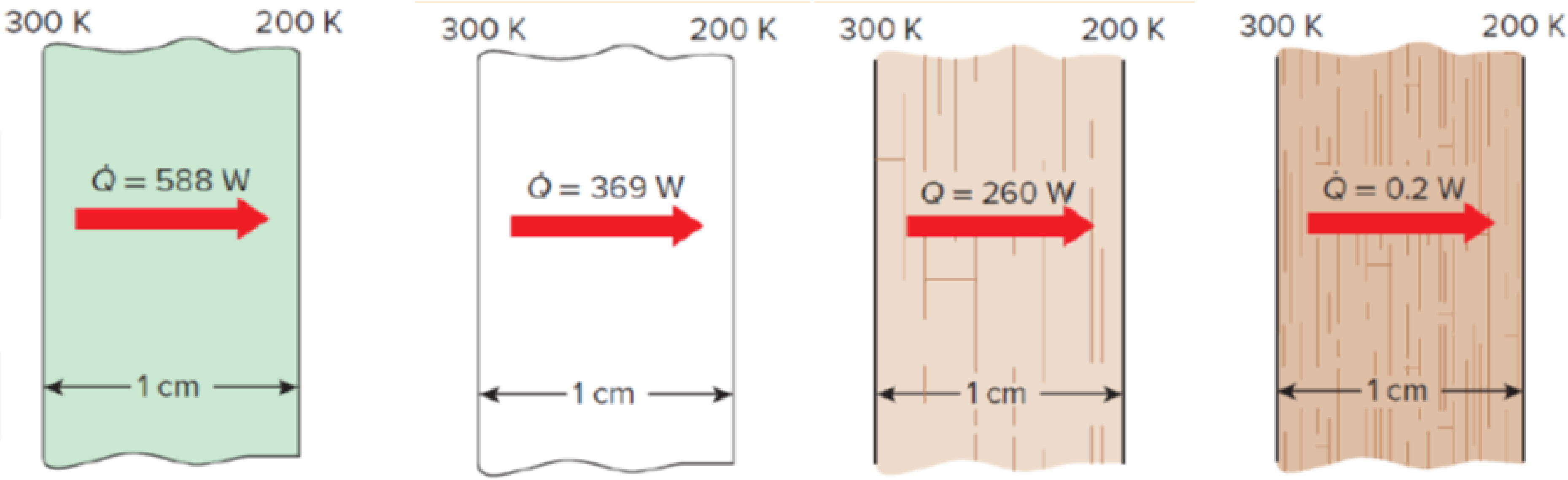
Most gases between two solid surfaces do not interfere with radiation.

Liquids are usually strong absorbers of radiation.



Although there are three mechanisms of heat transfer, a medium may involve only two of them simultaneously.

16-5 SIMULTANEOUS HEAT TRANSFER MECHANISMS-1



(a) **Air space** ***(b)*** **vacuum** ***(c)*** **Insulation** ***(d)*** **Superinsulation**

Figure 16-21

Different ways of reducing heat transfer between two isothermal plates, and their effectiveness.

EXAMPLE 16–6 Heat Loss from a Person

Consider a person standing in a breezy room at 20°C. Determine the total rate of heat transfer from this person if the exposed surface area and the average outer surface temperature of the person are 1.6 m^2 and 29°C, respectively, and the convection heat transfer coefficient is 6 $W/m^2 \cdot K$ (**Fig. 16–19**).

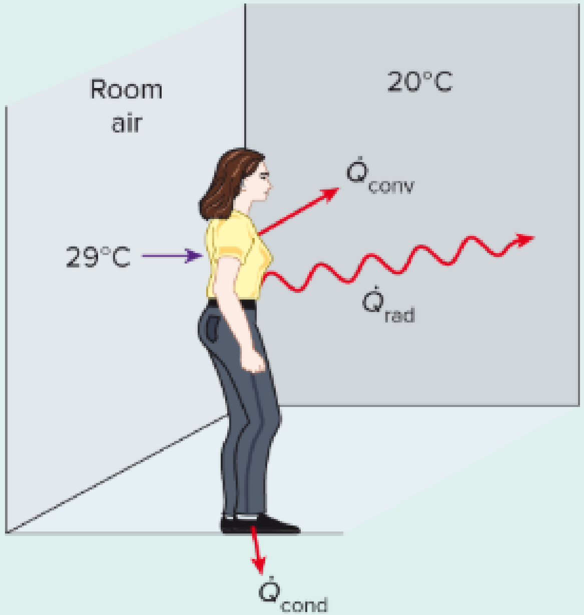


FIGURE 16–19

Heat transfer from the person described in **Example 16–6**.

SOLUTION

The total rate of heat transfer from a person by both convection and radiation to the surrounding air and surfaces at specified temperatures is to be determined.

Assumptions

1 Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible.

Properties

The emissivity of a person is $\varepsilon = 0.95$ (Table 16–6).

Analysis

The heat transfer between the person and the air in the room is by convection (instead of conduction) since it is conceivable that the air in the vicinity of the skin or clothing warms up and rises as a result of heat transfer from the body, initiating natural convection currents. It appears that the experimentally determined value for the rate of convection heat transfer in this case is 6 W per unit surface area (m^2) per unit temperature difference (in K or °C) between the person and the air away from the person. Thus, the rate of convection heat transfer from the person to the air in the room is

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty)\\ &= (6\ \text{W/m}^2\cdot\text{K})(1.6\ \text{m}^2)(29 - 20)°\text{C}\\ &= 86.4\ \text{W}\end{aligned}$$

The person also loses heat by radiation to the surrounding wall surfaces. We take the temperature of the surfaces of the walls, ceiling, and floor to be equal to the air temperature in this case for simplicity, but we recognize that this does not need to be the case. These surfaces may be at a higher or lower temperature than the average temperature of the room air, depending on the outdoor conditions and the structure of the walls. Considering that air does not intervene with radiation and the person is completely enclosed by the surrounding surfaces, the net rate of radiation heat transfer from the person to the surrounding walls, ceiling, and floor is

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)\\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.6\ \text{m}^2)\\ &\quad\times[(29+273)^4 - (20+273)^4]\ \text{K}^4\\ &= 81.7\ \text{W}\end{aligned}$$

Note that we must use *thermodynamic* temperatures in radiation calculations. Also note that we used the emissivity value for the skin and clothing at room temperature since the emissivity is not expected to change significantly at a slightly higher temperature.

Then the rate of total heat transfer from the body is determined by adding these two quantities:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = (86.4 + 81.7)\ \text{W} \cong \mathbf{168\ W}$$

Discussion

The heat transfer would be much higher if the person were not dressed since the exposed surface temperature would be higher. Thus, an important function of the clothes is to serve as a barrier against heat transfer.

In these calculations, heat transfer through the feet to the floor by conduction, which is usually very small, is neglected. Heat transfer from the skin by perspiration, which is the dominant mode of heat transfer in hot environments, is not considered here.

Also, the units $W/m^2\cdot°C$ and $W/m^2\cdot K$ for heat transfer coefficient are equivalent, and can be interchanged.

EXAMPLE 16–7 Heat Transfer between Two Isothermal Plates

Consider steady heat transfer between two large parallel plates at constant temperatures of T_1 = 300 K and T_2 = 200 K that are L = 1 cm apart, as shown in **Fig. 16–20**. Assuming the surfaces to be black (emissivity ε = 1), determine the rate of heat transfer between the plates per unit surface area assuming the gap between the plates is (*a*) filled with atmospheric air, (*b*) evacuated, (*c*) filled with urethane insulation, and (*d*) filled with superinsulation that has an apparent thermal conductivity of 0.00002 W/m·K.

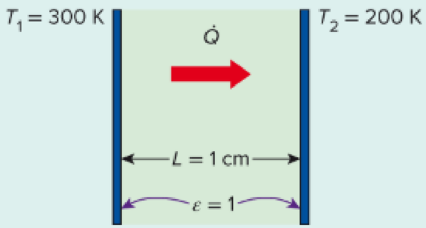


FIGURE 16–20
Schematic for Example 1–11.

SOLUTION

The total rate of heat transfer between two large parallel plates at specified temperatures is to be determined for four different cases.

Assumptions

1 Steady operating conditions exist. **2** There are no natural convection currents in the air between the plates. **3** The surfaces are black and thus ε = 1.

Properties

The thermal conductivity at the average temperature of 250 K is k = 0.0219 W/m·K for air (**Table A–22**), 0.026 W/m·K for urethane insulation, and 0.00002 W/m·K for the superinsulation.

Analysis

(*a*) The rates of conduction and radiation heat transfer between the plates through the air layer are

$$\dot{Q}_{\text{cond}} = kA\,\frac{T_1 - T_2}{L} = (0.0219 \text{ W/m·K})(1 \text{ m}^2)\,\frac{(310 - 200)\text{K}}{0.01 \text{ m}} = 219 \text{ W}$$

and

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A(T_1^4 - T_2^4)$$
$$= (1)(5.67 \times 10^{-8} \text{ W/m}^2\text{·K}^4)(1 \text{ m}^2)[(300 \text{ K})^4 - (200 \text{ K})^4] = 369 \text{ W}$$

Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 219 + 369 = \mathbf{588\ W}$$

The heat transfer rate in reality will be higher because of the natural convection currents that are likely to occur in the air space between the plates.

(*b*) When the air space between the plates is evacuated, there will be no conduction or convection, and the only heat transfer between the plates will be by radiation. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{369\ W}$$

(*c*) An opaque solid material placed between two plates blocks direct radiation heat transfer between the plates. Also, the thermal conductivity of an insulating material accounts for the radiation heat transfer that may be occurring through the voids in the insulating material. The rate of heat transfer through the urethane insulation is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\,\frac{T_1 - T_2}{L} = (0.026 \text{ W/m·K})(1 \text{ m}^2)\,\frac{(300 - 200)\text{K}}{0.01 \text{ m}} = \mathbf{260\ W}$$

Note that heat transfer through the urethane material is less than the heat transfer through the air determined in (*a*), although the thermal conductivity of the insulation is higher than that of air. This is because the insulation blocks the radiation whereas air transmits it.

(*d*) The layers of the superinsulation prevent any direct radiation heat transfer between the plates. However, radiation heat transfer between the sheets of superinsulation does occur, and the apparent thermal conductivity of the superinsulation accounts for this effect. Therefore,

$$\dot{Q}_{\text{total}} = kA\frac{T_1 - T_2}{L} = (0.00002\ \text{W/m·K})(1\ \text{m}^2)\frac{(300 - 200)\text{K}}{0.01\ \text{m}} = \mathbf{0.2\ W}$$

which is $\frac{1}{1845}$ of the heat transfer through the vacuum. The results of this example are summarized in **Fig. 16–21** to put them into perspective.

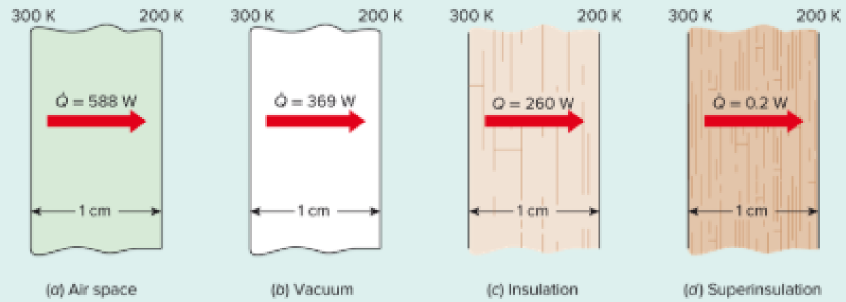


FIGURE 16–21
Different ways of reducing heat transfer between two isothermal plates, and their effectiveness.

Discussion

This example demonstrates the effectiveness of superinsulations and explains why they are the insulation of choice in critical applications despite their high cost.

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EXAMPLE 16–9 Heating of a Plate by Solar Energy

A thin metal plate is insulated on the back and exposed to solar radiation at the front surface (Fig. 16–23). The exposed surface of the plate has an absorptivity of 0.6 for solar radiation. If solar radiation is incident on the plate at a rate of 700 W/m^2 and the surrounding air temperature is 25°C, determine the surface temperature of the plate when the heat loss by convection and radiation equals the solar energy absorbed by the plate. Assume the combined convection and radiation heat transfer coefficient to be 50 W/m^2·K.

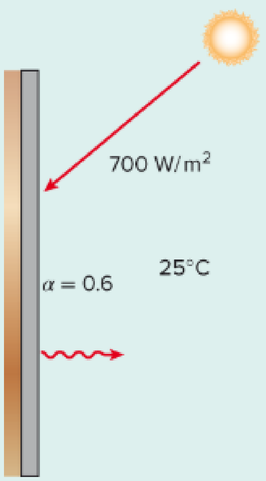

FIGURE 16–23
Schematic for Example 16–9.

SOLUTION

The back side of the thin metal plate is insulated and the front side is exposed to solar radiation. The surface temperature of the plate is to be determined when it stabilizes.

Assumptions

1 Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient remains constant.

Properties

The solar absorptivity of the plate is given to be α = 0.6.

Analysis

The absorptivity of the plate is 0.6, and thus 60 percent of the solar radiation incident on the plate is absorbed continuously. As a result, the temperature of the plate rises, and the temperature difference between the plate and the surroundings increases. This increasing temperature difference causes the rate of heat loss from the plate to the surroundings to increase. At some point, the rate of heat loss from the plate equals the rate of solar energy absorbed, and the temperature of the plate no longer changes. The temperature of the plate when steady operation is established is determined from

$$\dot{E}_{\text{gained}} = \dot{E}_{\text{lost}} \quad \text{or} \quad \alpha A_s\, \dot{q}_{\text{incident, solar}} = h_{\text{combined}} A_s (T_s - T_\infty)$$

Solving for T_s and substituting, the plate surface temperature is determined to be

$$T_s = T_\infty + \alpha \frac{\dot{q}_{\text{incident, solar}}}{h_{\text{combined}}} = 25°\text{C} + \frac{0.6 \times (700\ \text{W/m}^2)}{50\ \text{W/m}^2\cdot\text{K}} = 33.4°\text{C}$$

Discussion

Note that the heat losses prevent the plate temperature from rising above 33.4°C. Also, the combined heat transfer coefficient accounts for the effects of both convection and radiation, and thus it is very convenient to use in heat transfer calculations when its value is known with reasonable accuracy.

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SUMMARY

Heat can be transferred in three different modes: conduction, convection, and radiation. *Conduction* is the transfer of heat from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by *Fourier's law of heat conduction* as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient*. The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling as*

$$\dot{Q}_{\text{convection}} = hA_s(T_s - T_\infty)$$

where h is the *convection heat transfer coefficient* in $W/m^2 \cdot K$ or $Btu/h \cdot ft^2 \cdot R$, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature*, and T_∞ is the *temperature of the fluid* sufficiently far from the surface.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s is given by the *Stefan–Boltzmann law* as $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$ or $0.1714 \times 10^{-8}\ Btu/h \cdot ft^2 \cdot R^4$ is the *Stefan–Boltzmann constant.*

When a surface of emissivity ε and surface area A_s at a temperature T_s is completely enclosed by a much larger (or black) surface at a temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{rad} = \varepsilon \sigma A_s \left(T_s^4 - T_{surr}^4\right)$$

In this case, the emissivity and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

The rate at which a surface absorbs radiation is determined from $\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident}$ where $\dot{Q}_{incident}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface.

Summary

- **Introduction**
- **Conduction**
  - Fourier's law of heat conduction
  - Thermal Conductivity
  - Thermal Diffusivity
- **Convection**
  - Newton's law of cooling
- **Radiation**
  - Stefan–Boltzmann law
- **Simultaneous Heat Transfer Mechanisms**

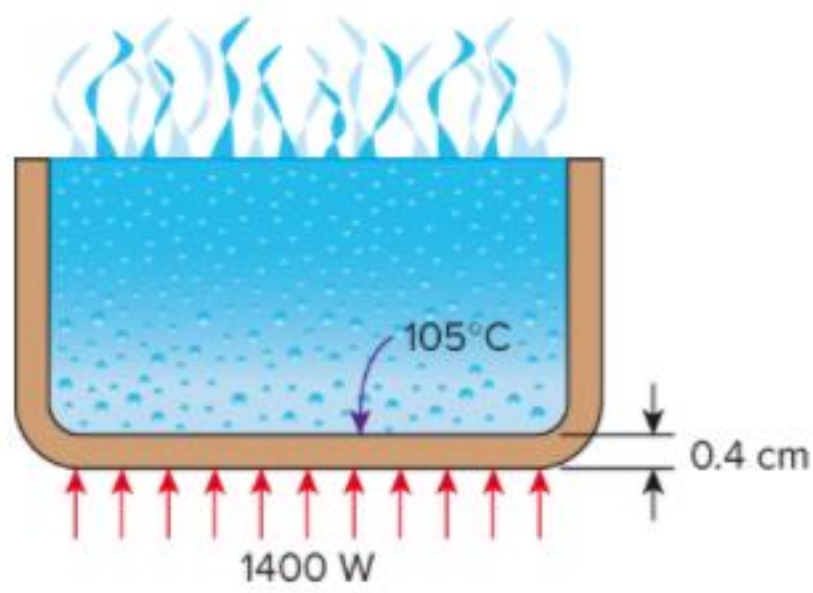
16–24 An aluminum pan whose thermal conductivity is 237 W/m·K has a flat bottom with diameter 15 cm and thickness 0.4 cm. Heat is transferred steadily to boiling water in the pan through its bottom at a rate of 1400 W. If the inner surface of the bottom of the pan is at 105°C, determine the temperature of the outer surface of the bottom of the pan.

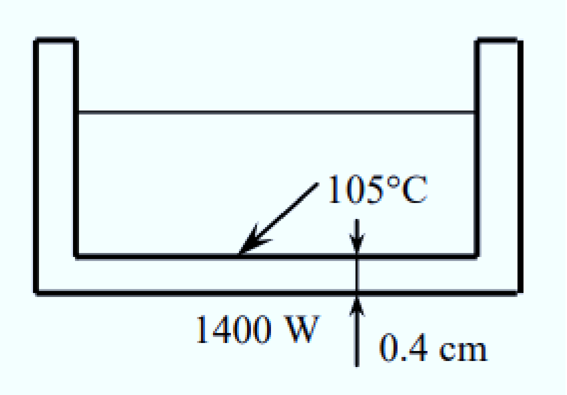
Assumptions

1 Steady operating conditions exist since the surface temperatures of the pan remain constant at the specified values.

2 Thermal properties of the aluminum pan are constant.

Properties The thermal conductivity of the aluminum is given to be $k = 237\ W/m^2\ {}^{o}C$.





$$A = \pi r^2 = \pi(0.075\text{ m})^2 = 0.0177\text{ m}^2$$

Under steady conditions, the rate of heat transfer through the bottom of the pan by conduction is

$$\dot{Q} = kA\frac{\Delta T}{L} = kA\frac{T_2 - T_1}{L}$$

Substituting,

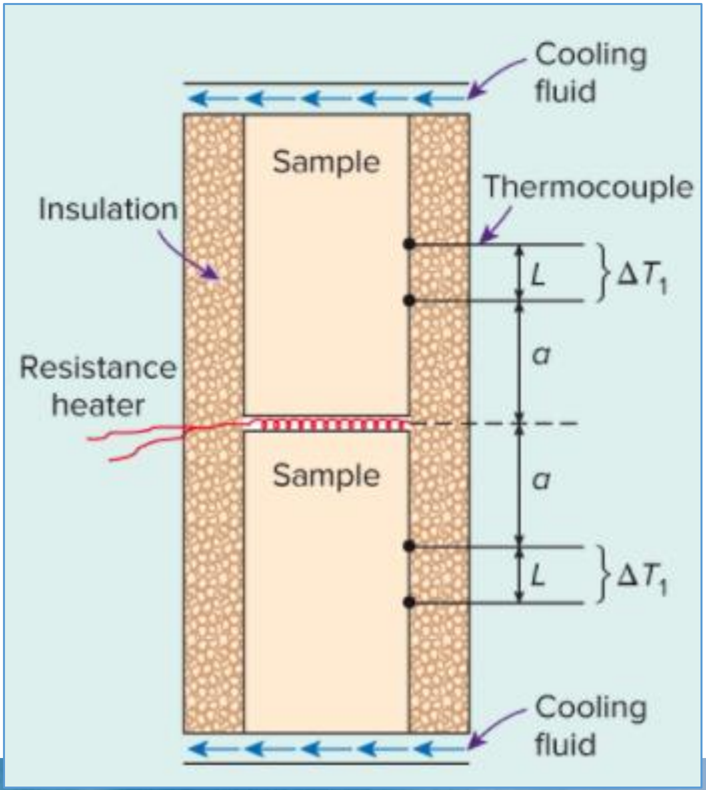
$$1400\text{ W} = (237\text{ W/m}\cdot{}^\circ\text{C})(0.0177\text{ m}^2)\frac{T_2 - 105^\circ\text{C}}{0.004\text{ m}}$$

which gives

$$T_2 = \mathbf{106.33^\circ C}$$

16–26 In a certain experiment, cylindrical samples of diameter 4 cm and length 7 cm are used (see **Fig. 16–9**). The two thermocouples in each sample are placed 3 cm apart. After initial transients, the electric heater is observed to draw 0.6 A at 110 V, and both differential thermometers read a temperature difference of 8°C. Determine the thermal conductivity of the sample.

Fig. 16–9



Assumptions

- Steady operating conditions exist since the temperature readings do not change with time.
- Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples.
- The apparatus possesses thermal symmetry.

- Analysis The electrical power consumed by the heater and converted to heat is

$$\dot{W}_e = \mathbf{V}I = (110\,\mathrm{V})(0.6\,\mathrm{A}) = 66\,\mathrm{W}$$

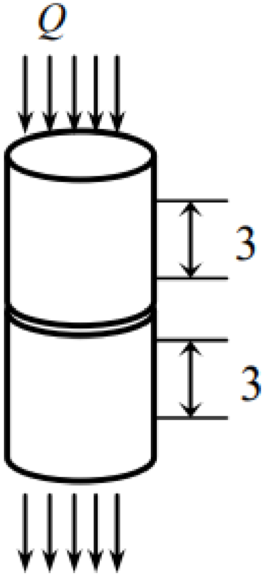
The rate of heat flow through each sample is

$$\dot{Q} = \frac{\dot{W}_e}{2} = \frac{66\,\mathrm{W}}{2} = 33\,\mathrm{W}$$

Then the thermal conductivity of the sample becomes

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.04\,\mathrm{m})^2}{4} = 0.001257\,\mathrm{m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(33\,\mathrm{W})(0.03\,\mathrm{m})}{(0.001257\,\mathrm{m}^2)(8^\circ\mathrm{C})} = \mathbf{98.5\,W/m.^\circ C}$$

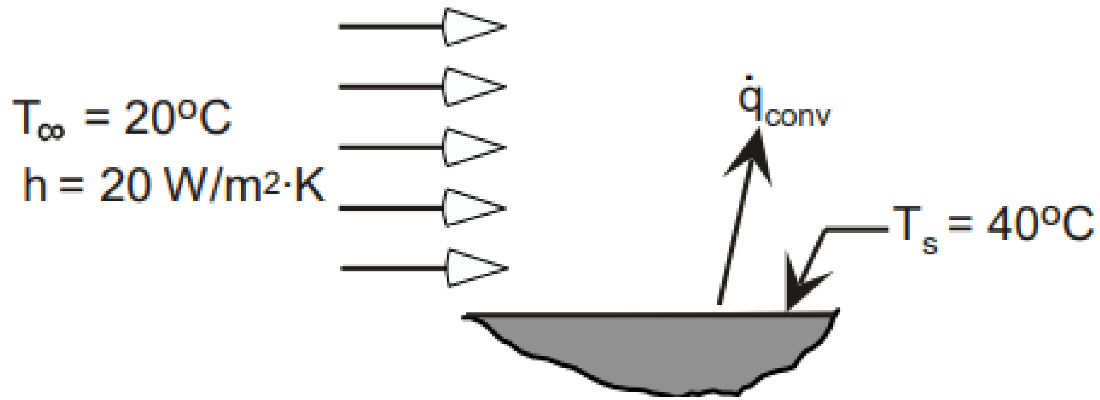


16–33 Air at 20°C with a convection heat transfer coefficient of 20 W/m^2·K blows over a pond. The surface temperature of the pond is at 40°C. Determine the heat flux between the surface of the pond and the air.

Assumptions

1. Steady operating conditions exist.
2. Convection heat transfer coefficient is uniform.
3. Heat transfer by radiation is negligible.
4. Air temperature and the surface temperature of the pond remain constant.

Analysis From Newton's law of cooling, the heat flux is given as



$$\dot{q}_{conv} = h\,(T_s - T_\infty)$$

$$\dot{q}_{conv} = 20\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}\ (40 - 20)^\circ\mathrm{C}\ = \mathbf{400\ W/m^2}$$

Discussion

(1) Note the direction of heat flow is out of the surface since $T_s > T_\infty$;

(2) Recognize why units of K in h and units of ºC in $(T_s - T_\infty)$ cancel.

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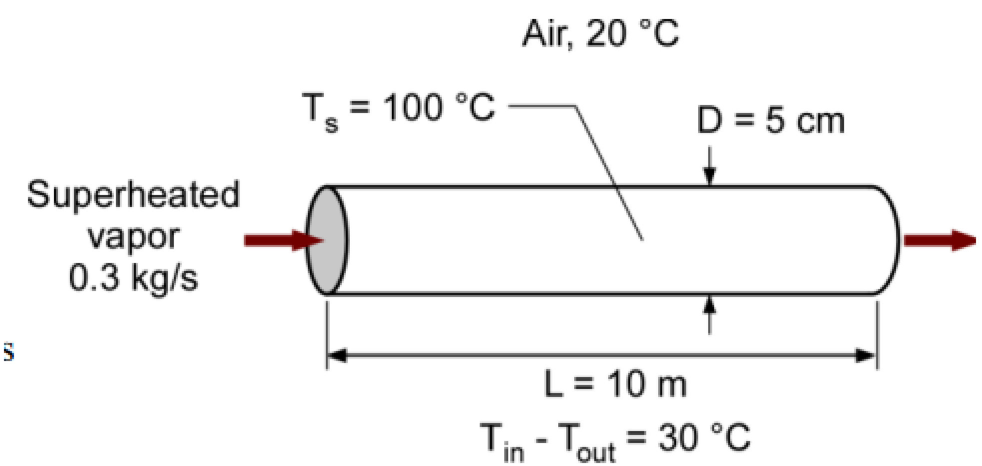
16–35 In a power plant, pipes transporting superheated vapor are very common. Superheated vapor is flowing at a rate of 0.3 kg/s inside a pipe with 5 cm in diameter and 10 m in length.

The pipe is located in a power plant at 20°C, and has a uniform pipe surface temperature of 100°C. If the temperature drop between the inlet and exit of the pipe is 30°C, and the specific heat of the vapor is 2190 J/Kg·K.

Determine the heat transfer coefficient as a result of convection between the pipe surface and the surrounding.

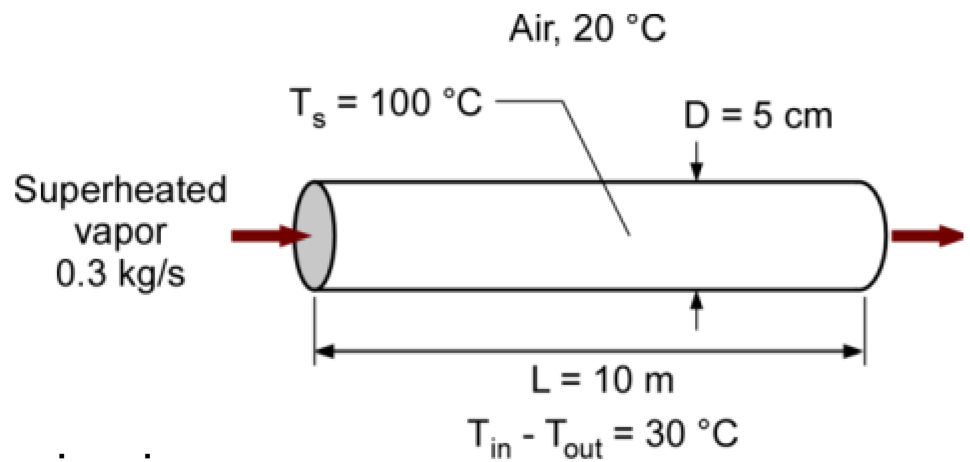


s

Assumptions

1. Steady operating conditions exist.
2. Heat transfer by radiation is not considered.
3. Rate of heat loss from the vapor in the pipe is equal to the heat transfer rate by convection between pipe surface and the surrounding.

Properties The specific heat of vapor is given to be 2190 J/kg · °C.



Analysis The surface area of the pipe is

The rate of heat loss from the vapor in the pipe can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}})$$
$$= (0.3\ \text{kg/s})(2190\ \text{J/kg}\cdot{}^\circ\text{C})(30)\ {}^\circ\text{C} = 19710\ \text{J/s}$$
$$= 19710\ \text{W}$$

With the rate of heat loss from the vapor in the pipe assumed equal to the heat transfer rate by convection, the heat transfer coefficient can be determined using the Newton's law of cooling:

$$\dot{Q}_{\text{loss}} = \dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

Rearranging, the heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{loss}}}{A_s(T_s - T_\infty)} = \frac{19710\,\text{W}}{(1.571\,\text{m}^2)(100-20)\,°\text{C}} = \mathbf{157\ W/m^2 \cdot °C}$$

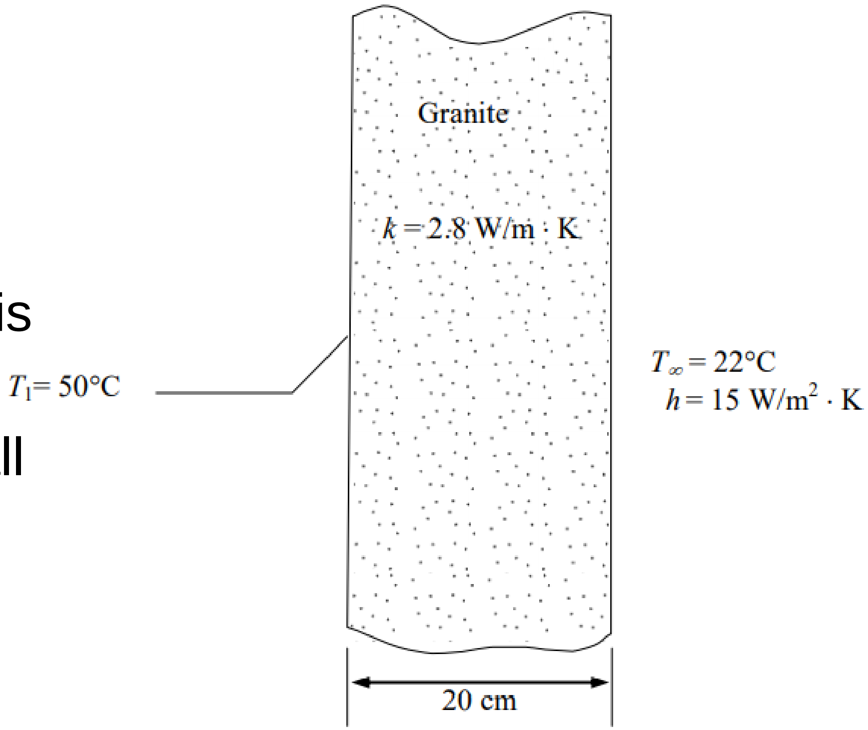
Discussion

By insulating the pipe surface, heat loss from the vapor in the pipe can be reduced.

16–60 Consider a 20 cm thick granite wall with a thermal conductivity of 2.79 W/m·K. The temperature of the left surface is held constant at 50°C, whereas the right face is exposed to a flow of 22°C air with a convection heat transfer coefficient of 15 W/m^2·K. Neglecting heat transfer by radiation, find the right wall surface temperature and the heat flux through the wall.

Assumptions

1. Steady operating conditions exist.
2. Heat transfer through the granite wall is one dimensional.
3. Thermal conductivity of the granite wall is constant.
4. Radiation heat transfer is negligible.



Analysis The heat transfer through the wall by conduction is equal to heat transfer to the outer wall surface by convection:

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}}$$

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty)$$

$$T_2 = \frac{(kT_1 / L) + hT_\infty}{(k / L) + h}$$

$$T_2 = \frac{(2.8\,\text{W/m}\cdot\text{K})(50°\text{C})/(0.20\,\text{m}) + (15\,\text{W/m}^2\cdot\text{K})(22°\text{C})}{(2.8\,\text{W/m}\cdot\text{K})/(0.20\,\text{m}) + 15\,\text{W/m}^2\cdot\text{K}}$$

$$T_2 = \mathbf{35.5°C}$$

Now that T_2 is known, we can calculate the heat flux. Since the heat transfer through the wall by conduction is equal to heat transfer to the outer wall surface by convection, we may use either the Fourier's law of heat conduction or the Newton's law of cooling to find the heat flux. Using Fourier's law of heat conduction:

$$\dot{q}_{\text{cond}} = k\frac{T_1 - T_2}{L} = (2.8\,\text{W/m}\cdot\text{K})\frac{(50 - 35.5)\,°\text{C}}{0.20\,\text{m}} = \mathbf{203\,W/m^2}$$

Discussion Alternatively using Newton's law of cooling to find the heat flux, we obtain the same result:

$$\dot{q}_{conv} = h(T_2 - T_\infty) = (15\,\text{W/m}^2\cdot\text{K})(35.5 - 22) = 203\,\text{W/m}^2$$