Signals and Circuits

ENGR 35500

AC circuit involves capacitors, inductors and resistors.

Chapter 10: 10-2(Impedance and phase angle of series RC); 10-3(Analysis of series RC Circuits) pp. 445-457

Chapter 12: 12-2(Impedance and phase angle of series RC circuits) and 12-3 (Analysis of series RC Circuits) pp. 539-549

Chapter 13: 13-1(Impedance and phase angle of series RCL circuits) and 13-2 (Analysis of series RCL Circuits) pp. 581-588

Floyd, T. L., and Buchla, D. M., *Electroics Fundamentals: Circuits, Devices & Applications*, 8th Edition, Pearson, 2009.

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Impedance

Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied.

Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude.

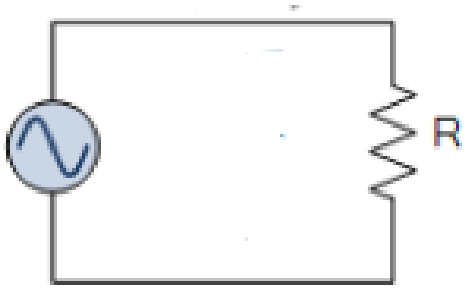
The phase angle is the phase difference between the source voltage and total current in AC; it must indicate voltage leads or lags the current.

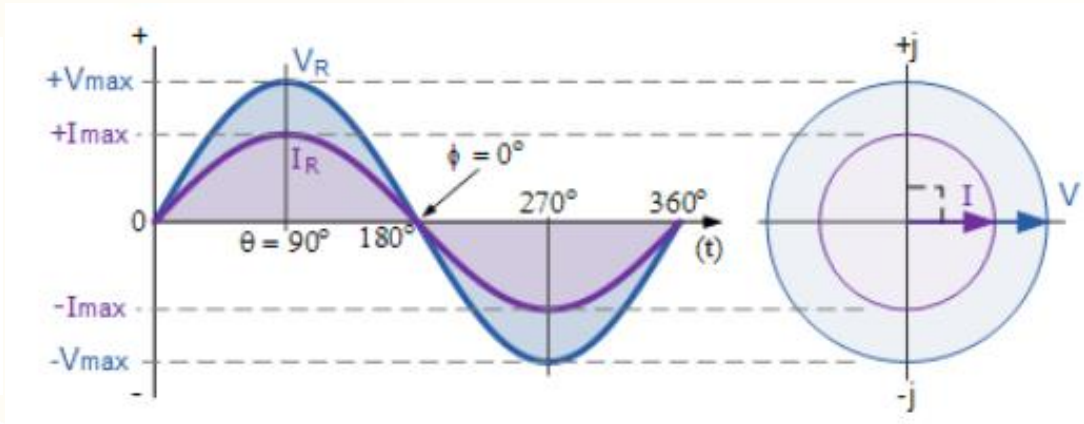
The magnitude of the impedance is usually denoted as Z (ohm); the phase angle is usually denoted as $\emptyset \; or \; \theta \; (\mathbf{degree})$.

Impedance

Ohm's law in AC

$$Z = \frac{V}{I}$$

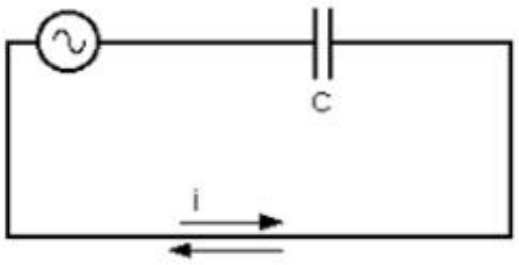


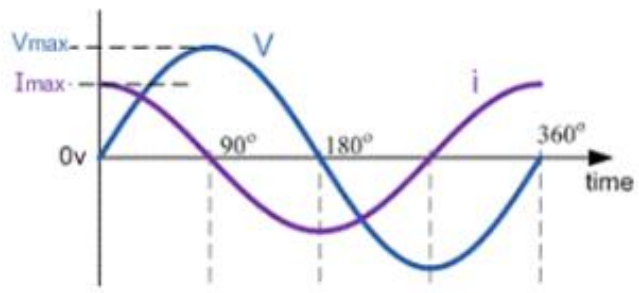


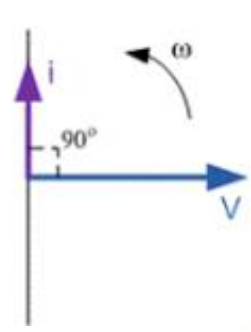
Impedance $Z = R$

Phase angle $\theta = 0^{\circ}$

The source voltage leads/lags the current by 0°



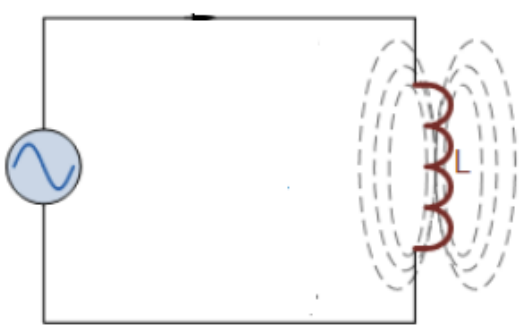


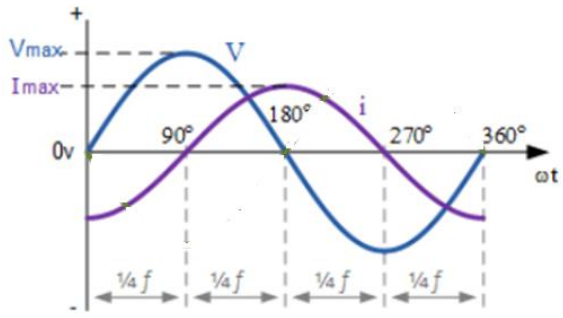


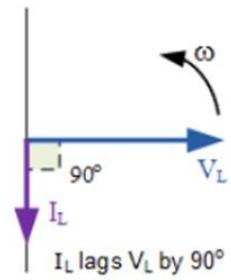
Impedance $Z = X_c = \frac{1}{2\pi f C}$

Phase angle $\theta = 90^{\circ}$

The source voltage lags the current by 90°







Impedance $Z = X_L = 2\pi f L$

Phase angle $\theta = 90^{\circ}$

The source voltage leads the current by 90°

Figures: **Electronics Tutorials**

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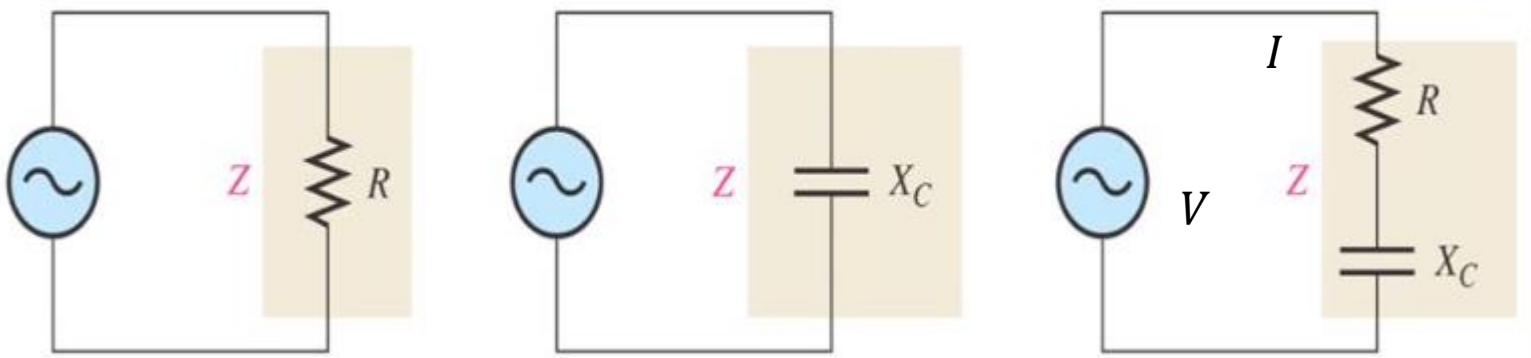
Impedance

Impedance and phase angle in RC Series



(a) $Z = R$ (b) $Z = X_C$ (c) Z includes both R and X_C

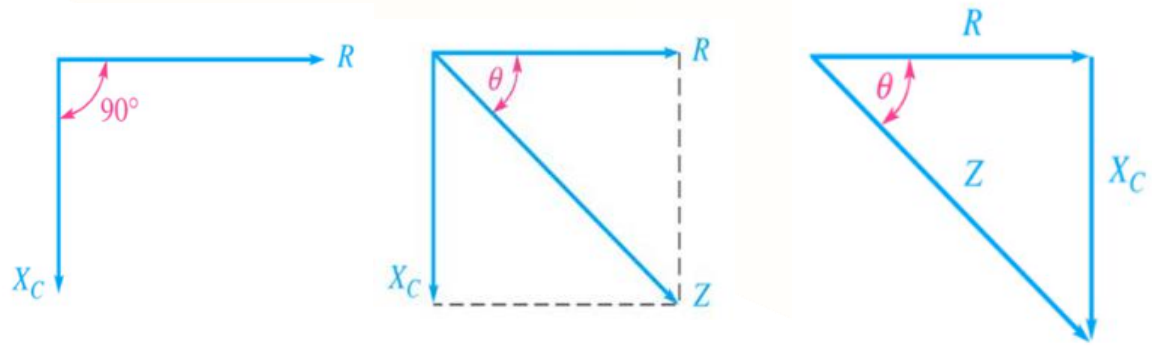
$$I = \frac{V}{Z}$$

Purely resistor makes the phase angle between the source voltage and the total current zero degree;

Purely capacitor makes the voltage source voltage lag the current by 90 degree;

Thus, resistor and capacitor in series can make the voltage source lag the current by a certain angle between 0 and 90 degree.

Impedance triangle



$$Z = \sqrt{(R^2 + X_C^2)}$$

$$voltage\ lags\ current\ by\ \theta = tan^{-1}\frac{X_C}{R}$$

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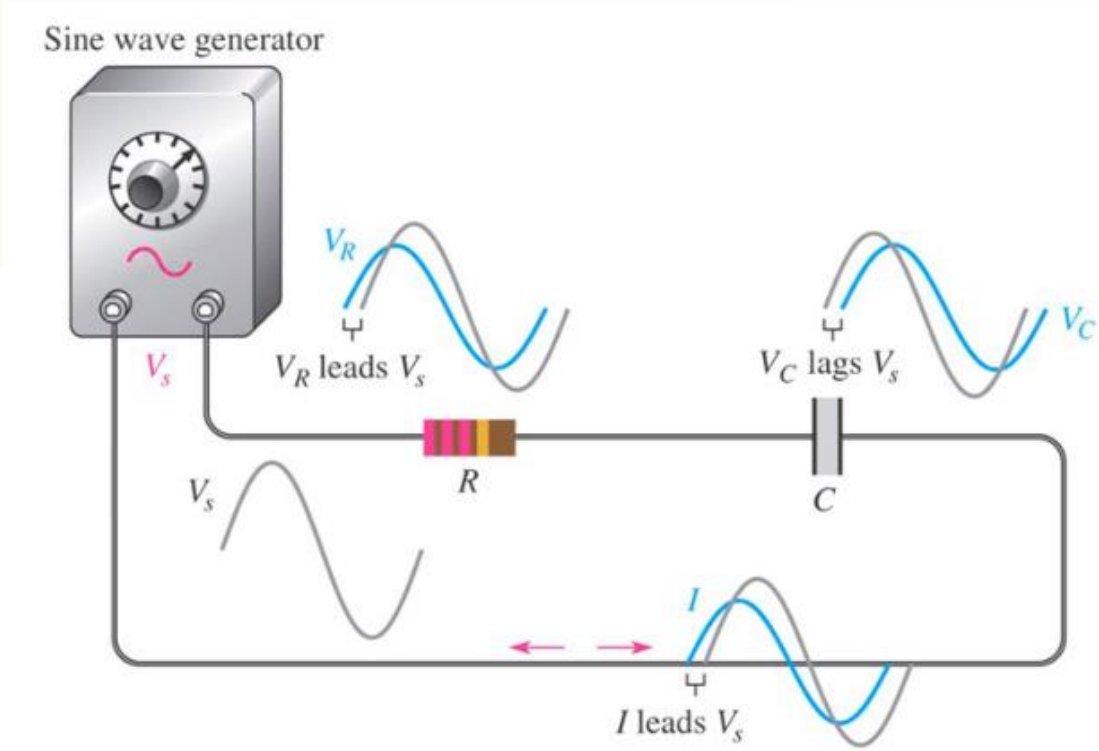
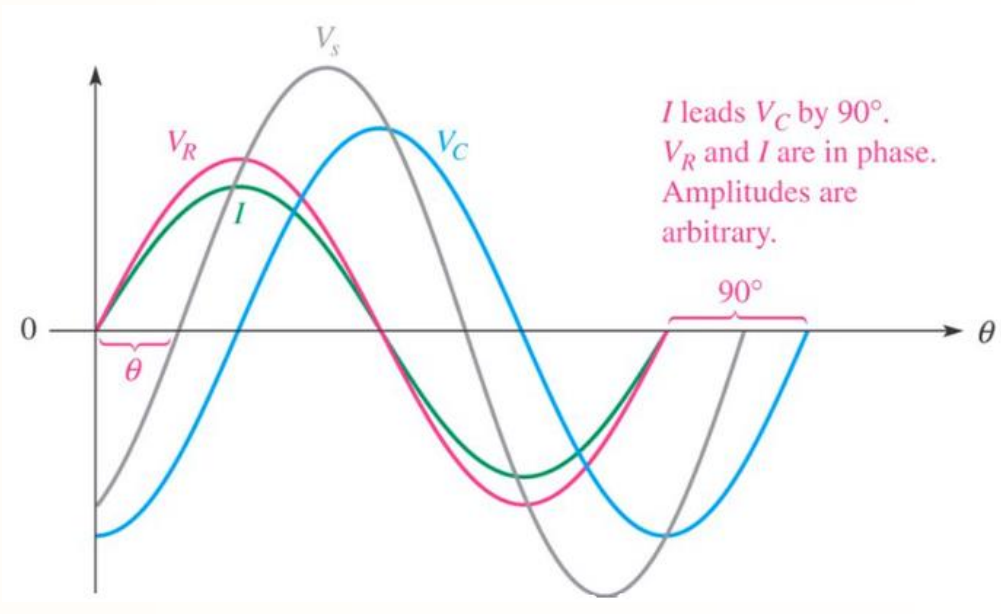
Impedance

RC Series in AC





V_R and ***I*** are in the same phase;

V_C *lags* ***I*** by 90 degree;

V_S *lags* ***I*** by θ;

$$\theta = tan^{-1}\frac{X_C}{R}$$

phase angle between V_S and V_C ?

V_C lags V_S by (90- θ) degree

phase angle between V_R and V_S ?

V_S lags V_R by θ degree

phase angle between V_R and V_C ?

V_C *lags* V_R by 90 degree;

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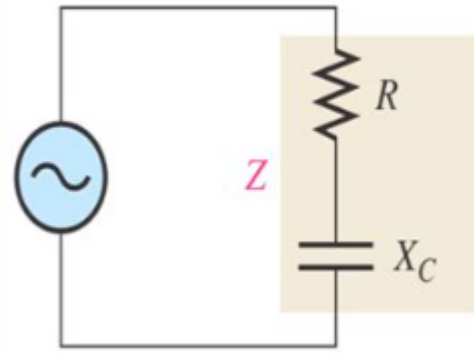
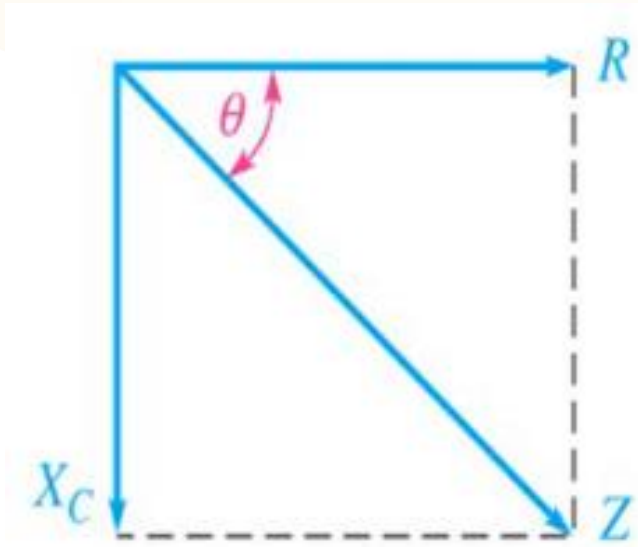
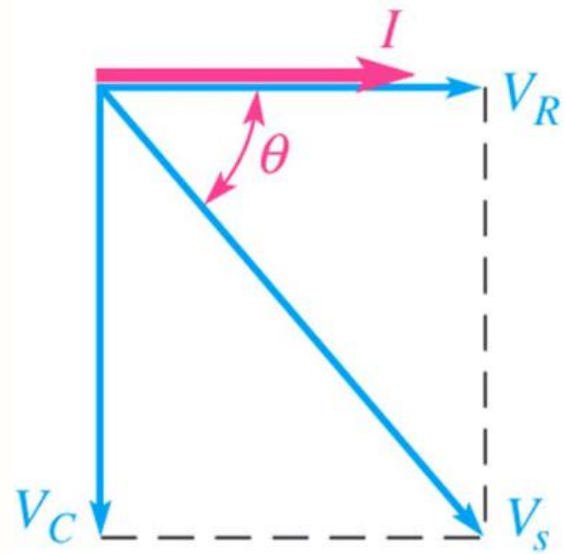
RC Series in AC





$$V = IZ$$

Voltage phasor triangle



$$V_R = \frac{RV_s}{\sqrt{R^2 + X_C^2}}$$

$$V_C = \frac{X_C V_s}{\sqrt{R^2 + X_C^2}}$$

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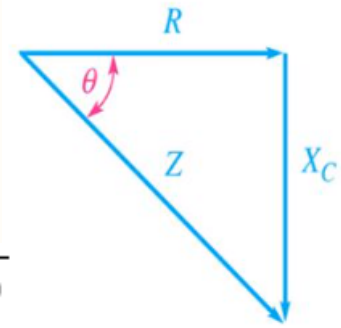
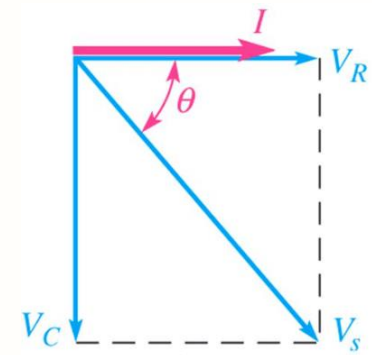
RC Series in AC



$$Z = \sqrt{(R^2 + X_c^2)}$$

$$voltage\ lags\ current\ by\ \theta = tan^{-1}\frac{X_C}{R}$$



$$V_R = \frac{RV_s}{\sqrt{R^2 + X_C^2}}$$

$$V_C = \frac{X_C V_s}{\sqrt{R^2 + X_C^2}}$$

E. g. If an RC circuit has a $50\,\Omega$ resistor in series with a $1\,\mu$F capacitor, what will its impedance be at 500 Hz?

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{50^2 + X_C^2} = \sqrt{50^2 + (\frac{1}{2\pi\times500\times1\times10^{-6}})^2} =$$

$$\theta = tan^{-1}\frac{X_C}{R} = tan^{-1}\frac{\frac{1}{2\pi\times500\times1\times10^{-6}}}{50} = ?$$

$$voltage\ lags\ current\ by\ \theta$$

An RC circuit has a $5\,\text{k}\Omega$ resistor and a $1\,\mu$F capacitor. At what frequency will the current lead the voltage by $\pi/4$?

$$\theta = tan^{-1}\frac{X_C}{R} = tan^{-1}\frac{\frac{1}{2\pi\times f\times1\times10^{-6}}}{5000} = \pi/4$$

So, f=31.83Hz

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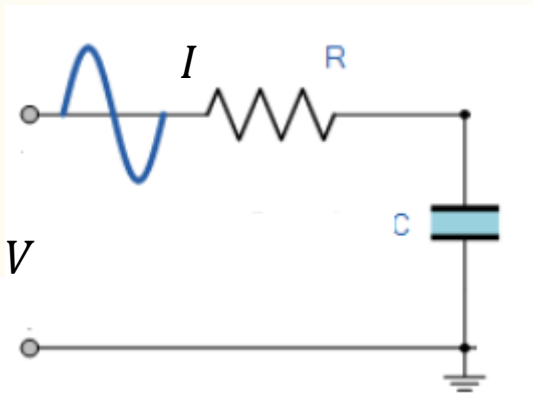
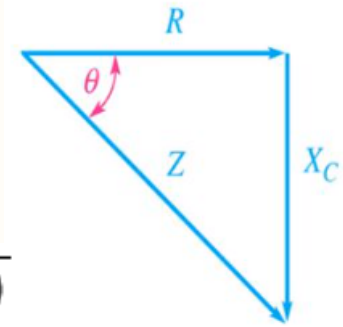
RC Series in AC

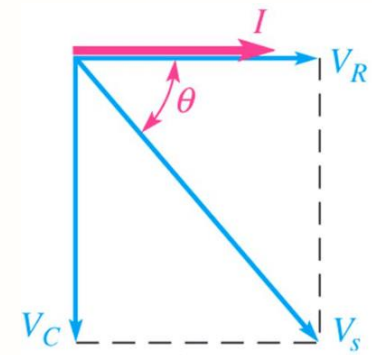
E. g.





$$Z = \sqrt{(R^2 + X_c^2)}$$

$$voltage\ lags\ current\ by\ \theta = tan^{-1}\frac{X_C}{R}$$



$$V_R = \frac{RV_s}{\sqrt{R^2 + X_C^2}}$$

$$V_C = \frac{X_C V_s}{\sqrt{R^2 + X_C^2}}$$

The current in the figure is 0.2 mA. The source frequency is 1kHz, the R is 10kohm, and the C is 0.01 uF. Determine the source voltage and the phase angle. Draw the impedance triangle. Determine the voltage of the resistor and the capacitor. Draw the voltage phasor diagram.

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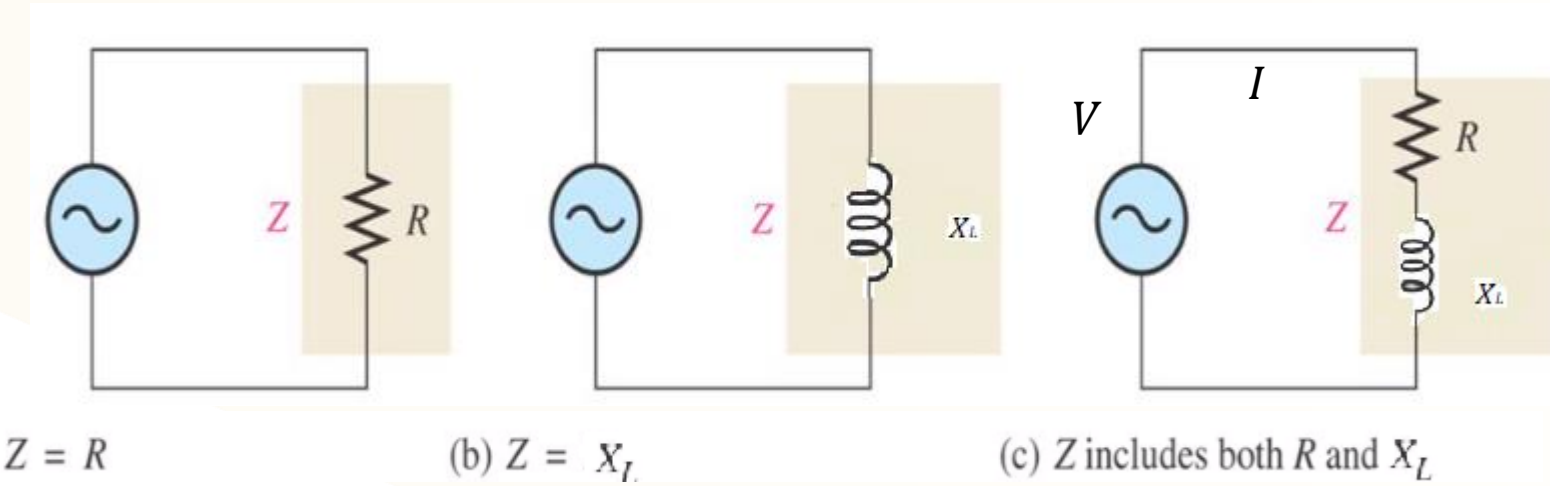
Impedance

Impedance and phase angle in RL Series



(a) $Z = R$ (b) $Z = X_L$ (c) Z includes both R and X_L

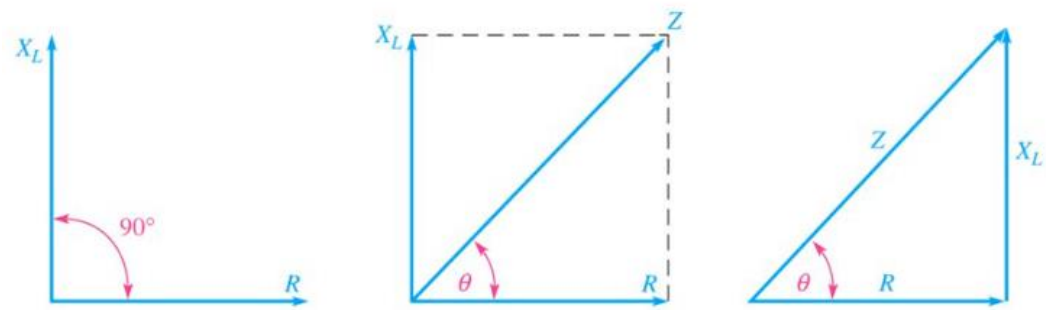
$$I = \frac{V}{Z}$$

Purely resistor makes the phase angle between the source voltage and the total current zero degree;

Purely inductor makes the voltage source voltage leads the current by 90 degree;

Thus, resistor and inductor in series can make the voltage source leads the current by a certain angle between 0 and 90 degree.

Impedance triangle



$$Z = \sqrt{(R^2 + X_L{}^2)}$$

$$voltage\ leads\ current\ by\ \theta = tan^{-1}\frac{X_L}{R}$$

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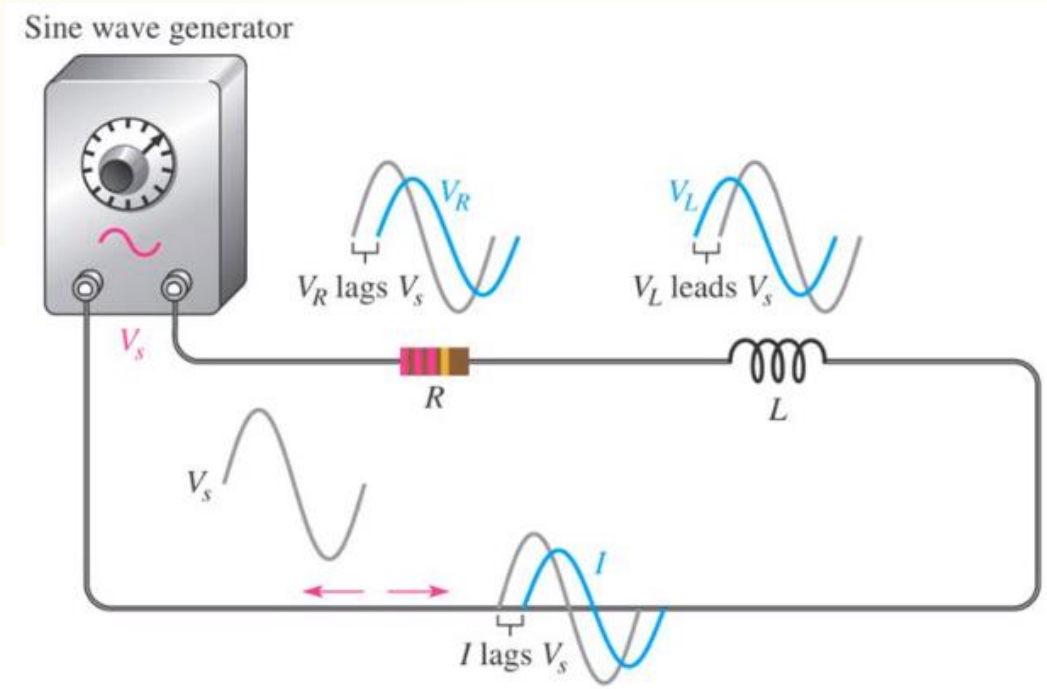
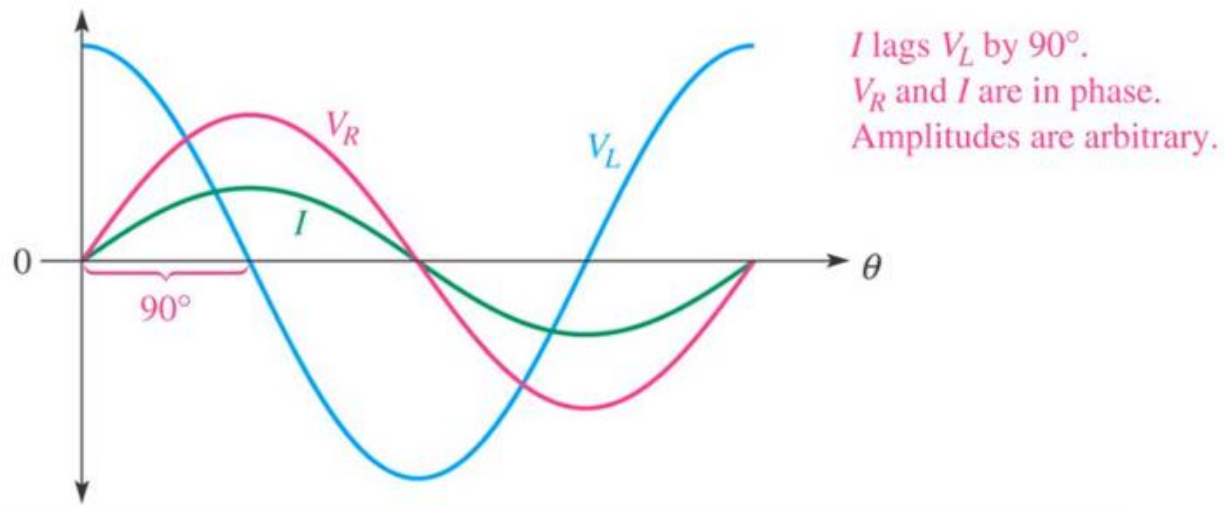
Impedance

RL Series in AC





V_R and *I* are in the same phase;

V_{L} *leads I* by 90 degree;

$$\theta = tan^{-1}\frac{X_L}{R}$$

V_{S} *lead I* by θ;

phase angle between V_{S} and V_{L} ?

V_{L} leads V_{S} by (90- θ) degree

phase angle between V_{R} and V_{S} ?

V_{S} leads V_{R} by θ degree

phase angle between V_{R} and V_{L} ?

V_{L} *leads* V_{R} by 90 degree;

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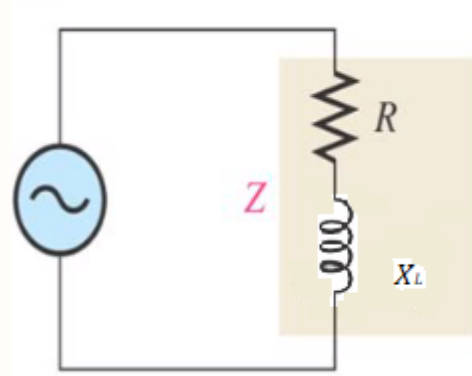
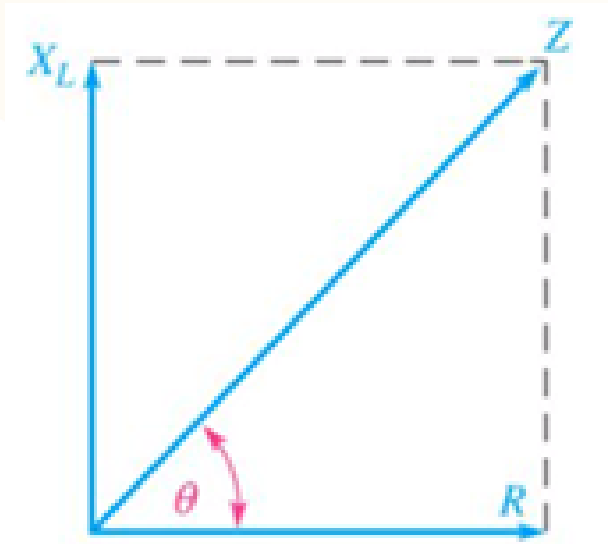
RL Series in AC

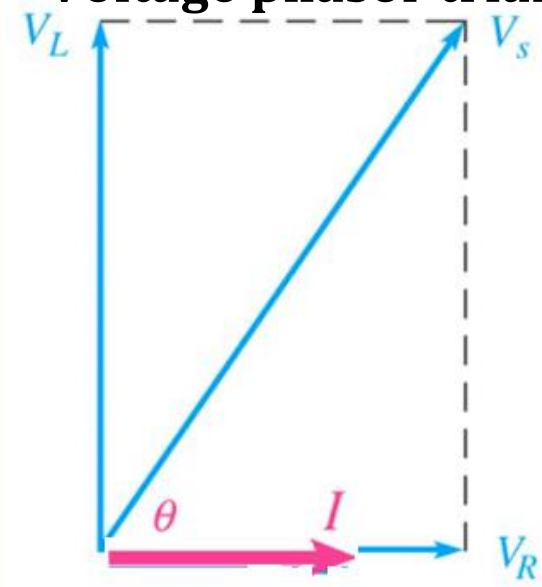




$$voltage\ leads\ current\ by\ \theta = tan^{-1}\frac{X_L}{R}$$

$$V = IZ$$

Voltage phasor triangle



$$V_R = \frac{RV_s}{\sqrt{R^2 + X_L^2}}$$

$$V_L = \frac{X_L V_s}{\sqrt{R^2 + X_L^2}}$$

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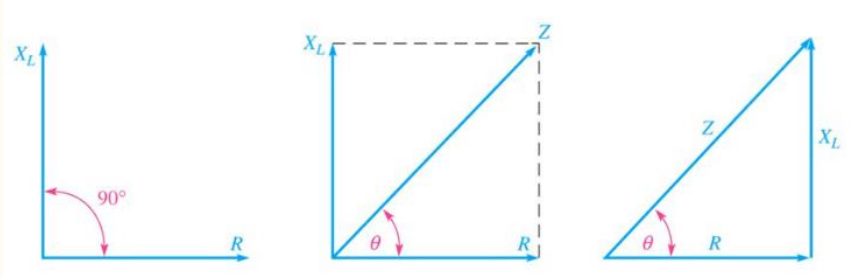
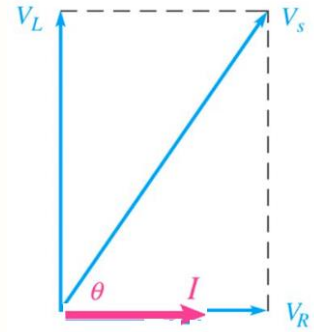
RL Series in AC



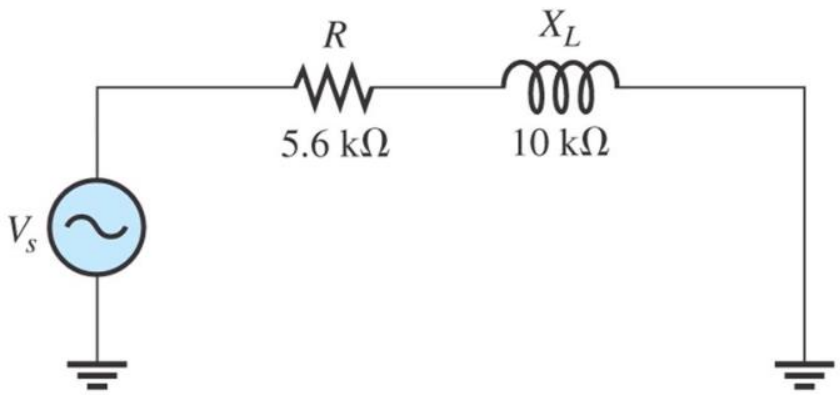
$$Z = \sqrt{(R^2 + X_L{}^2)}$$

$$voltage\ leads\ current\ by\ \theta = tan^{-1}\frac{X_L}{R}$$



$$V_R = \frac{RV_s}{\sqrt{R^2 + X_L^2}} \qquad V_L = \frac{X_L V_s}{\sqrt{R^2 + X_L^2}}$$

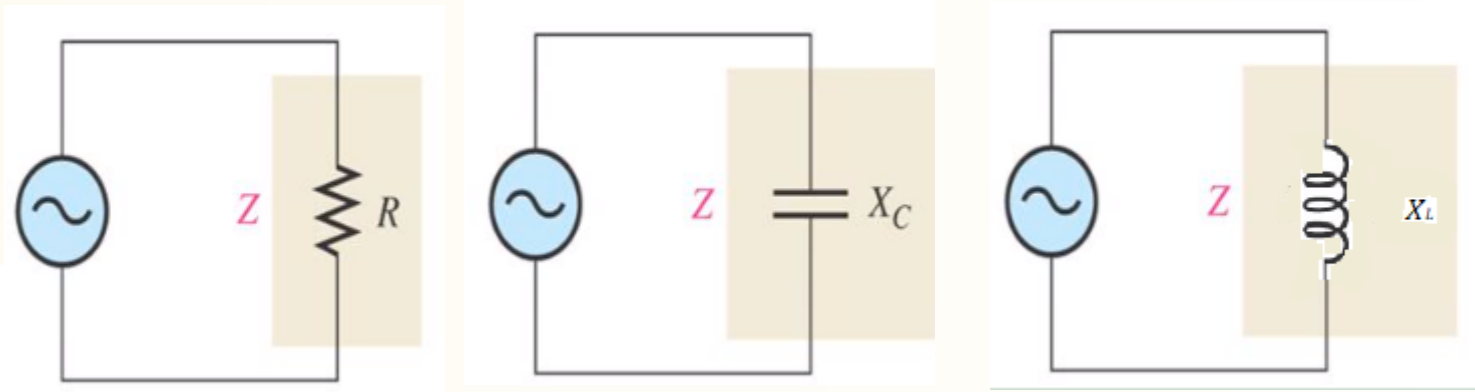
E. g.



The voltage source is 1kHz and 20V sinusoidal AC. Determine the impedance and the phase angle for the RL circuit. Draw the impedance triangle. Determine the voltage of the resistor and the inductor. Draw the voltage phasor diagram.

Impedance

Impedance and phase angle in RLC Series

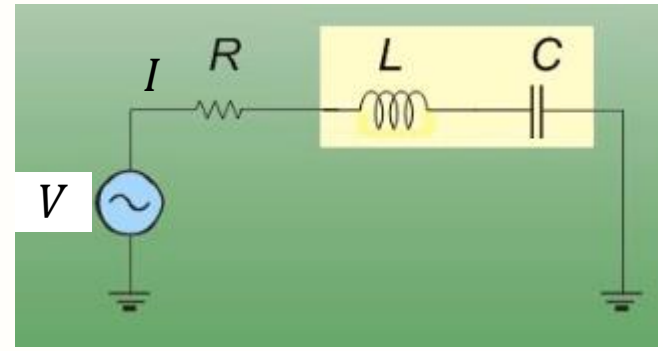


Purely resistor makes the phase angle between the source voltage and the total current zero degree;

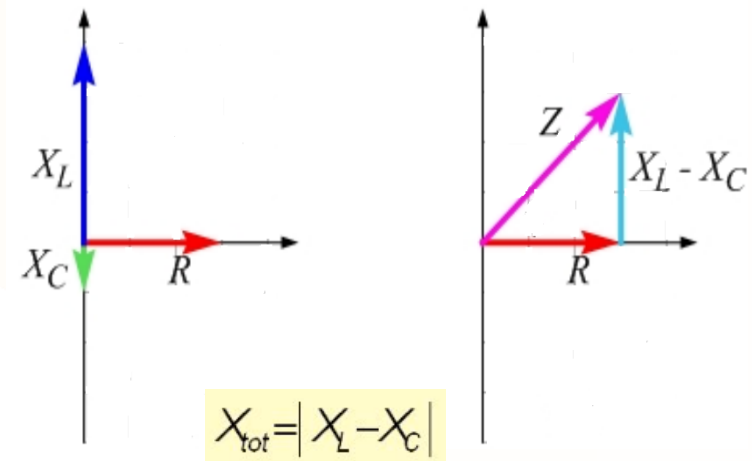
Purely capacitor makes the voltage source voltage lags the current by 90 degree;

Purely inductor makes the voltage source voltage leads the current by 90 degree;

What will happen if resistor, inductor and capacitor are connected together in series?



Impedance triangle



$$X_{tot} = |X_L - X_C|$$

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2}$$

$$\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$voltage\ leads\ current\ by\ \theta\ if X_L > X_C$

$voltage\ lags\ current\ by\ \theta\ if X_L < X_C$

Figures: Text books;
https://slideplayer.com/slide/6379082/

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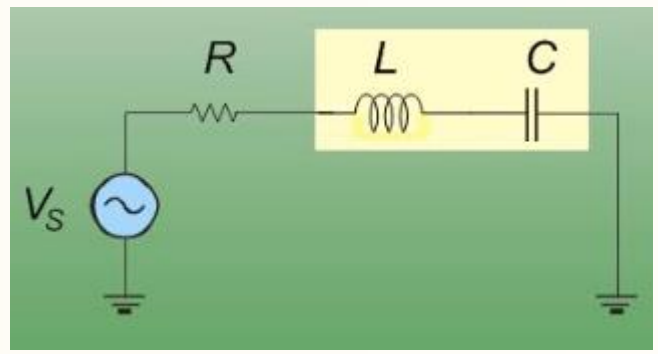
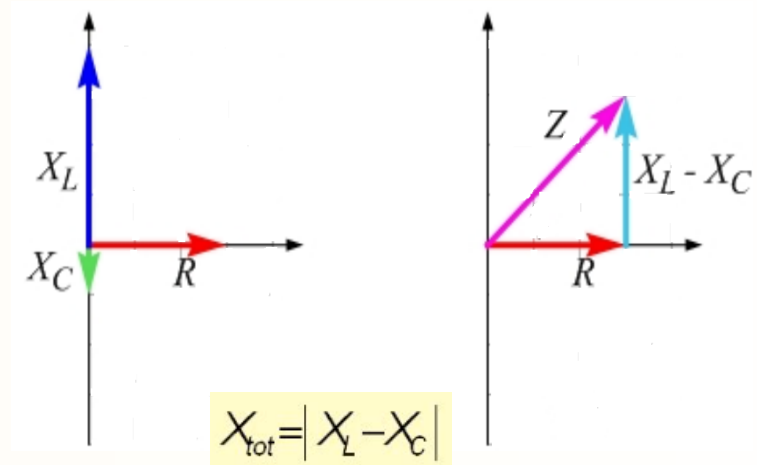
RLC Series in AC

E. g.





$$X_{tot} = |X_L - X_C|$$

$$Z_{tot} = \sqrt{R^2 + X_{tot}^2}$$

$$\theta = \tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

R=560 ohm, L= 100mH, C=0.56uF, frequency f= 1kHz. Determine the impedance and the phase angle. Draw the impedance triangle.

Figures: https://slideplayer.com/slide/6379082/