

# Signals and Circuits

**AERN 35500**

## Capacitors

Chapter 5: 5-4(Response of the RC circuit) pp. 230-233

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2<sup>nd</sup> Edition, National Technology and Science Press, 2013.

Chapter 9: 9-5(Capacitor in DC circuits) and 9-6 (Capacitor in AC circuits) pp. 407-423

Floyd, T. L., and Buchla, D. M., *Electronics Fundamentals: Circuits, Devices & Applications*, 8<sup>th</sup> Edition, Pearson, 2009.

Web:

[https://www.electronics-tutorials.ws/rc/rc\\_1.html](https://www.electronics-tutorials.ws/rc/rc_1.html)

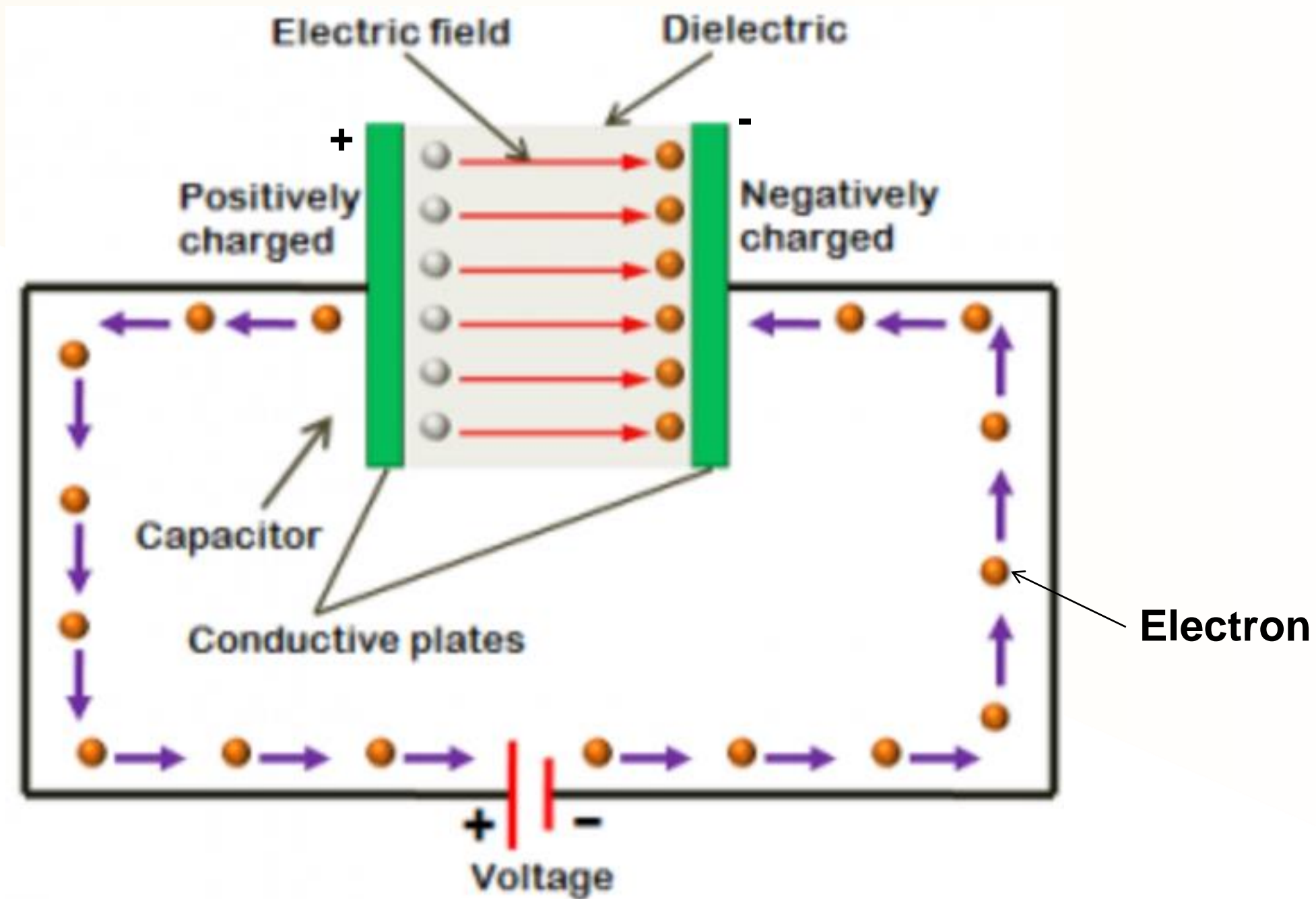
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# Capacitor

How a capacitor stores the charge.



# Capacitor

- The voltage across a capacitor cannot change instantaneously.

$$C = \frac{q}{v}$$

$$i = \frac{dq}{dt} = \frac{dCv}{dt} = C \frac{dv}{dt}$$

- Under dc stable conditions, a capacitor behaves like an open circuit.

For DC  $\frac{dv}{dt} = 0$

$$i = C \frac{dv}{dt} = 0$$

- To get voltage

$$\int_{t_0}^t \frac{dv}{dt'} dt' = \frac{1}{C} \int_{t_0}^t i dt'$$

$$v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^t i dt'$$

- To get power

$$p(t) = vi = Cv \frac{dv}{dt}$$

Sign of the power?

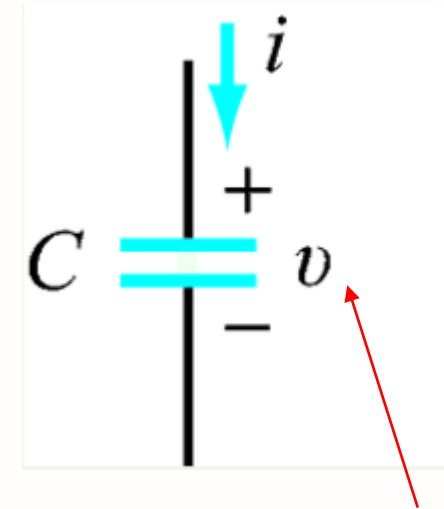
- To get energy change

$$w(t) = \int_{t_0}^t p dt' = C \int_{t_0}^t \left( v \frac{dv}{dt'} \right) dt' = C \int_{t_0}^t \left[ \frac{d}{dt'} \left( \frac{1}{2} v^2 \right) \right] dt' = \frac{1}{2} C (v(t))^2 - \frac{1}{2} C (v(t_0))^2$$

Sign of the ene

- To get energy in a capacitor

$$E(t) = \frac{1}{2} C (v(t))^2$$

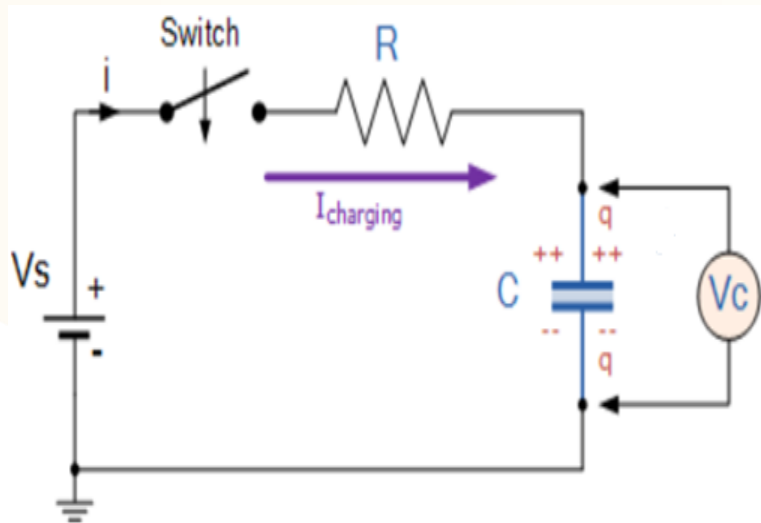


$v$  is the voltage across the capacitor

The direction of  $i$  is entering the positive terminal of the capacitor.

# Capacitor

## RC charging circuit



Assume the switch is open for a long time

These are the initial conditions of the circuit, then  $t = 0$ ,  $i = 0$ ,  $q = 0$ , and  $V_C = 0$

Then close the switch?

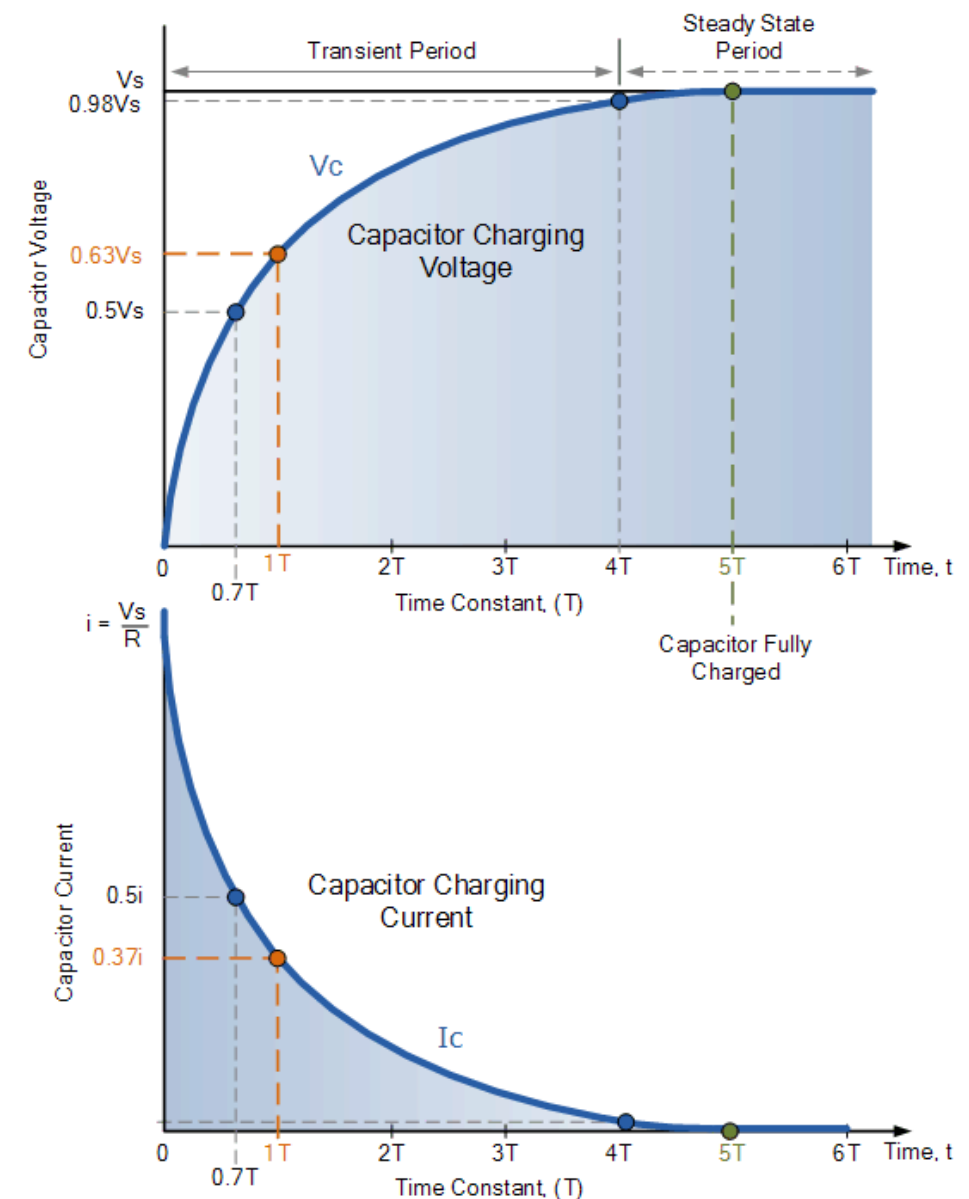
At the moment it closes, then  $t = 0$ ,  $i = \frac{V_s}{R}$ ,  $q = 0$ , and then  $V_C = 0$

KVL

$$V_s - R \times i(t) - V_C(t) = 0$$

A long time after the closing

then  $t = \infty$ ,  $i = 0$ ,  $q = Q$ , and then  $V_C = V_s$



$\tau$  (usually  $\tau$ ) is the time constant or time delay. (63% voltage rises)

$$\tau = R \times C \quad \text{second} = \text{Ohm} \times \text{Farads}$$

$4\tau$  (usually  $\tau$ ) is the transient period; 98% charged

$5\tau$  (usually  $\tau$ ) is the steady state period; almost fully charged

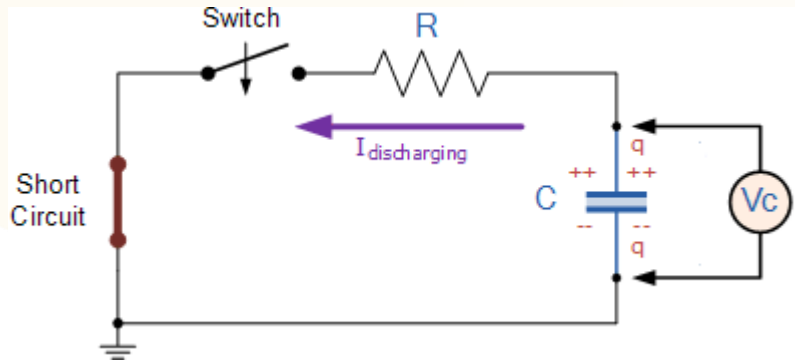
$$V_C = V_s (1 - e^{-t/RC})$$

$$i_C = \frac{V_s}{R} e^{-t/RC}$$



# Capacitor

## RC discharging circuit



Assume the capacitor is fully charged for a long time

These are the initial conditions of the circuit, then  $t = 0$ ,  $i = 0$ ,  $q = Q$ , and then  $V_c = V_s$

Then replace the battery with a short wire, and close the switch?

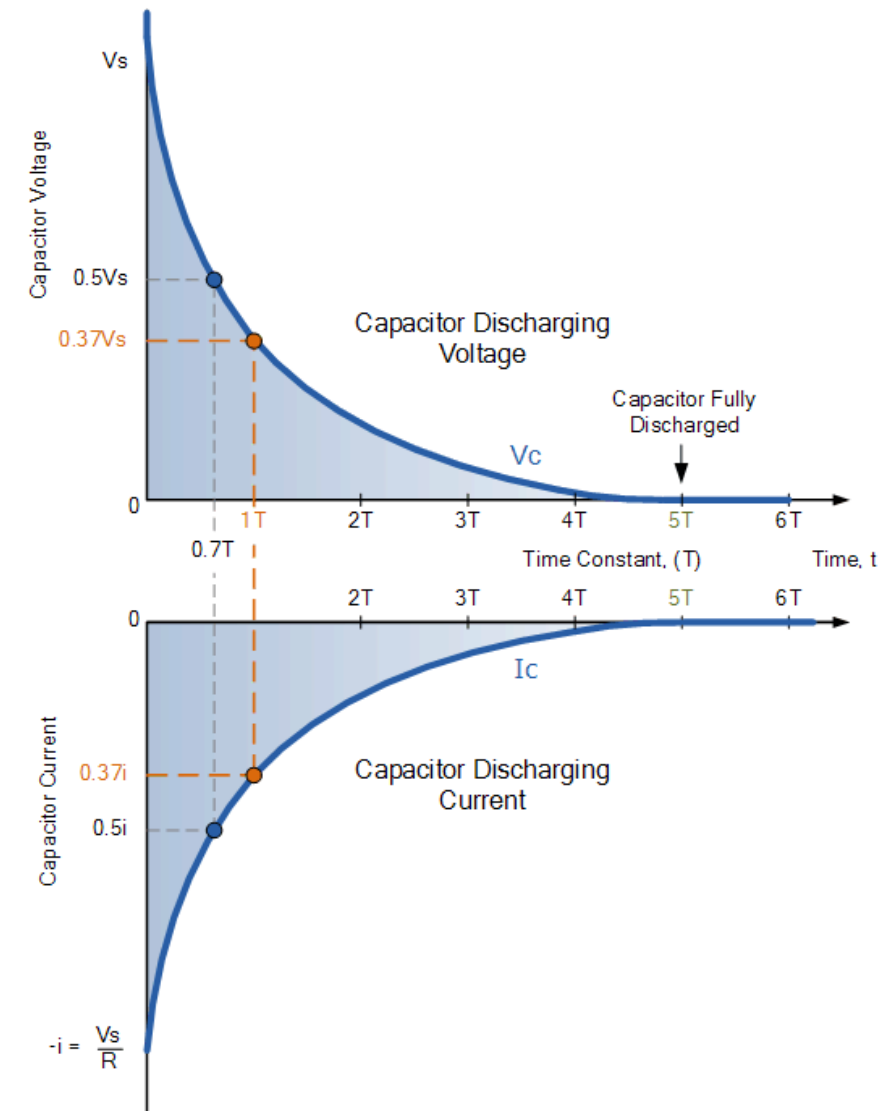
At the moment it closes, then  $t = 0$ ,  $i = -\frac{V_s}{R}$ ,  $q = Q$ , and  $V_c = V_s$

KVL

$$R \times i(t) + V_c(t) = 0$$

A long time after the closing

then  $t = \infty$ ,  $i = 0$ ,  $q = 0$ , and  $V_c = 0$



$\tau$  (usually  $\tau$ ) is the time constant or time delay. (63% voltage drop)

$$\tau = R \times C$$

$4\tau$  (usually  $\tau$ ) is the transient period; 98% discharged

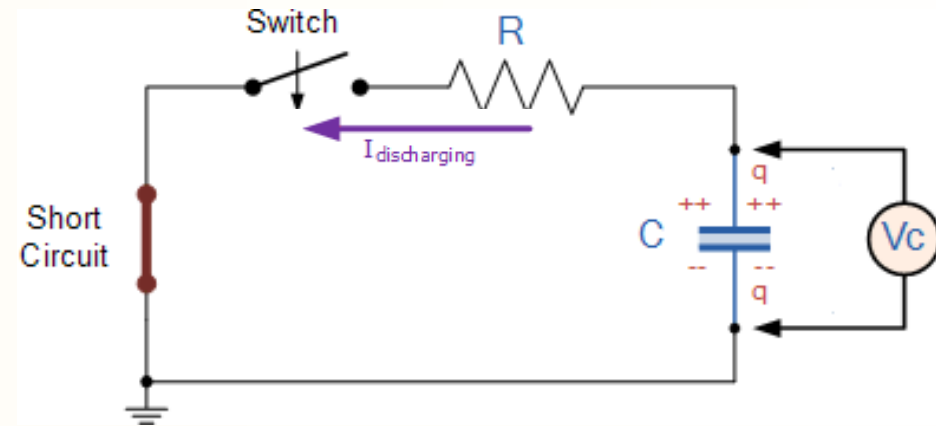
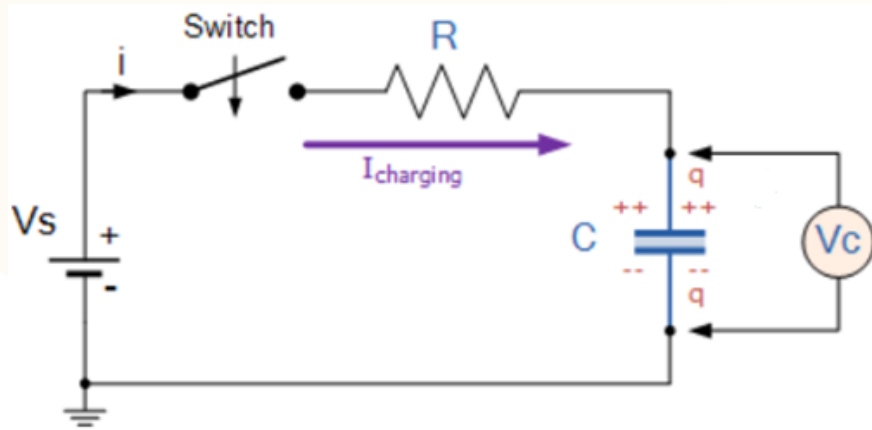
$5\tau$  (usually  $\tau$ ) is the steady state period; almost fully discharged

$$V_c = V_s e^{-t/\tau}$$

$$i_c = -\frac{V_s}{R} e^{-t/\tau}$$

# Capacitor

## RC charging/discharging circuit



$$V(t) = V_F + (V_i - V_F) e^{-t/\tau}$$

$$i(t) = I_F + (I_i - I_F) e^{-t/\tau}$$

$$\tau = RC$$

where  $V(t)$  and  $i(t)$  are the instantaneous voltage and current;  
 $V_i$  and  $I_i$  is the initial voltage and current;  
 $V_F$  and  $I_F$  are the final voltage and current.

# Capacitor

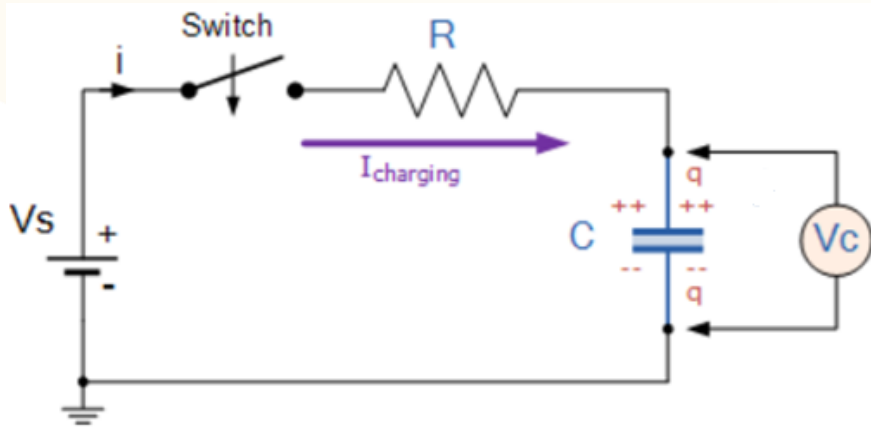
E.g.

Determine the capacitor voltage after the switch is closed if the capacitor initially is uncharged. Draw the charging curving.  $C=50\text{ }\mu\text{F}$ ;  $R=8.2\text{ }\Omega$ ;  $V_s= 50\text{V}$ .

$$V(t) = V_F + (V_i - V_F) e^{-t/\tau}$$

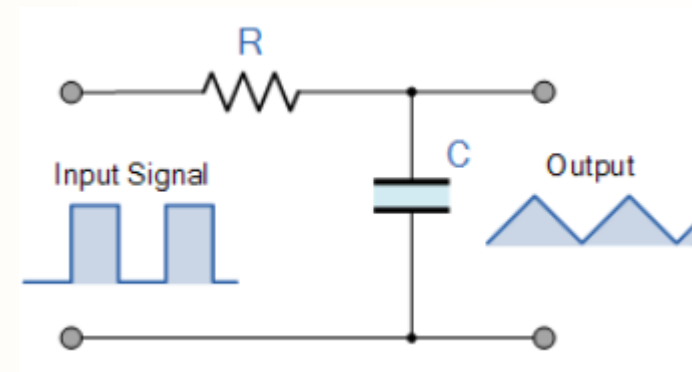
$$i(t) = I_F + (I_i - I_F) e^{-t/\tau}$$

$$\tau=RC$$

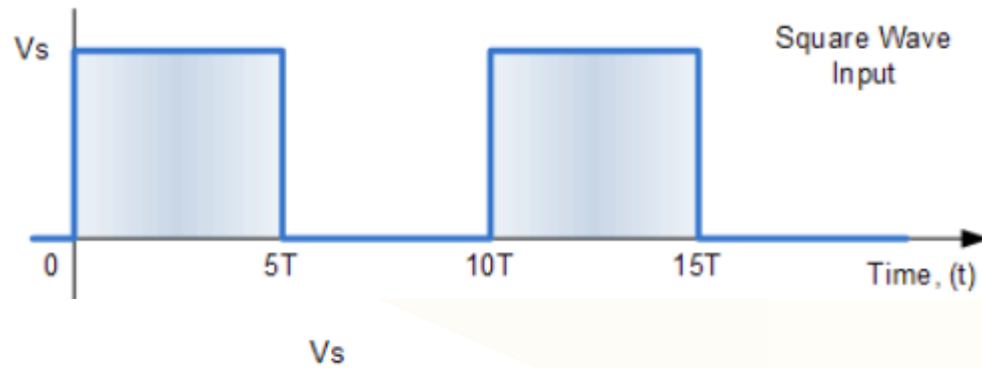


# Capacitor

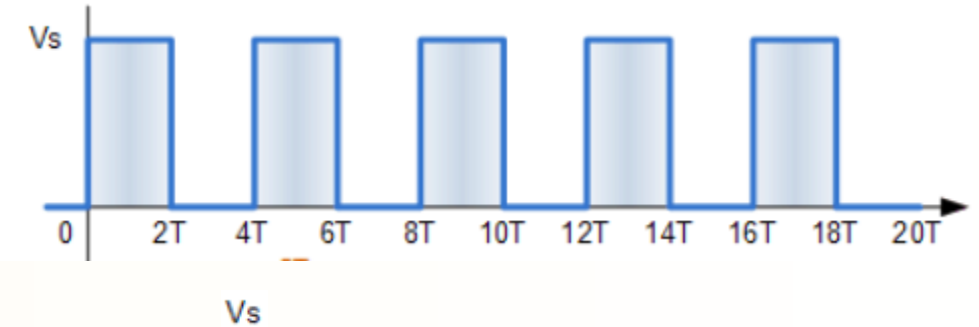
## Response to square wave



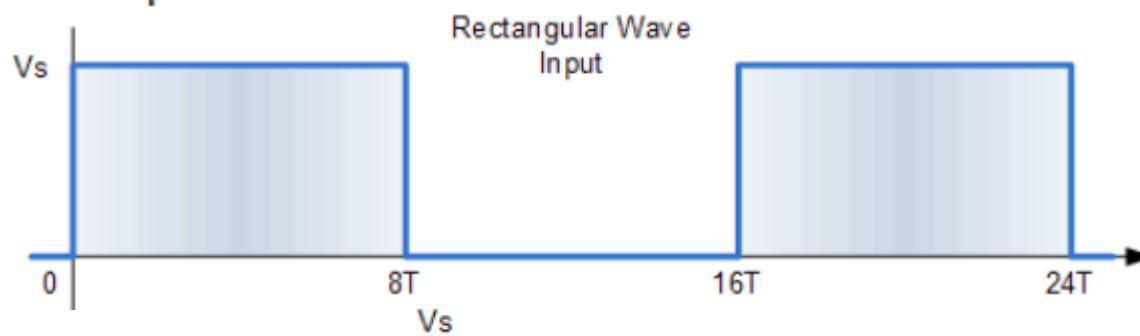
10 RC Input Waveform



4RC Input Waveform



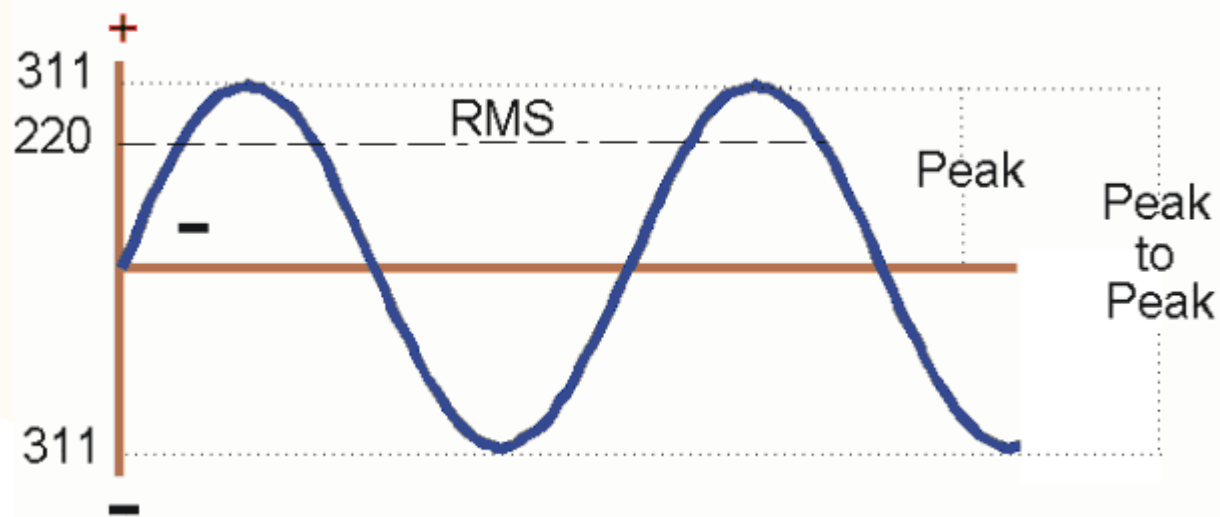
16 RC Input Waveform



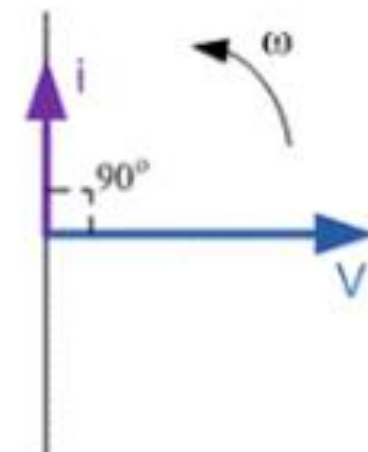
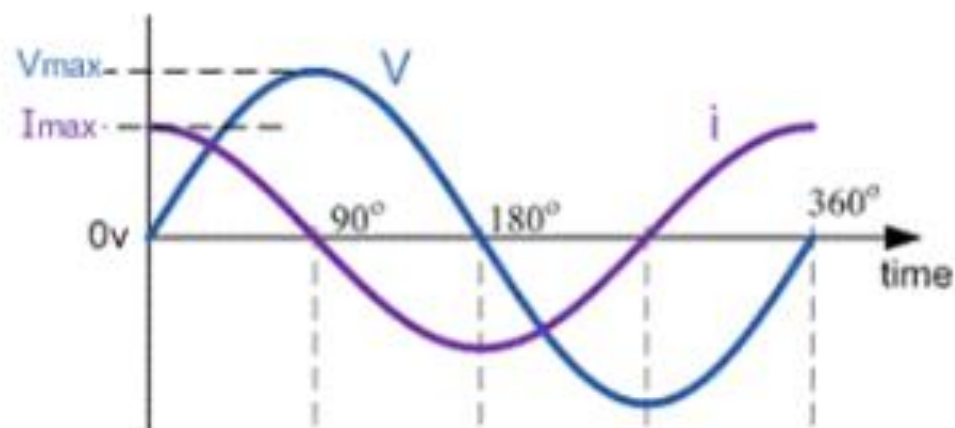
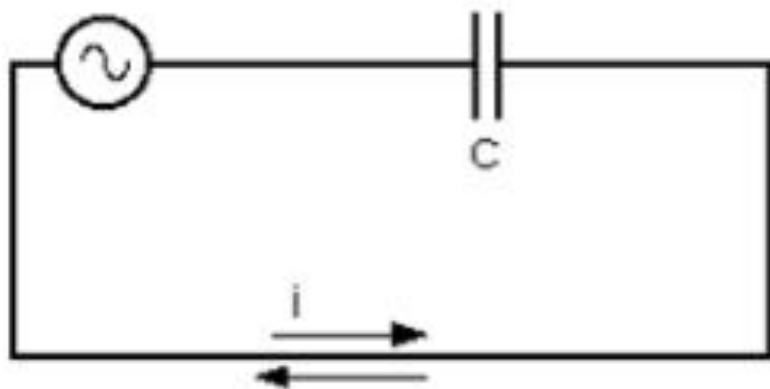


# Capacitor in AC

220 V AC

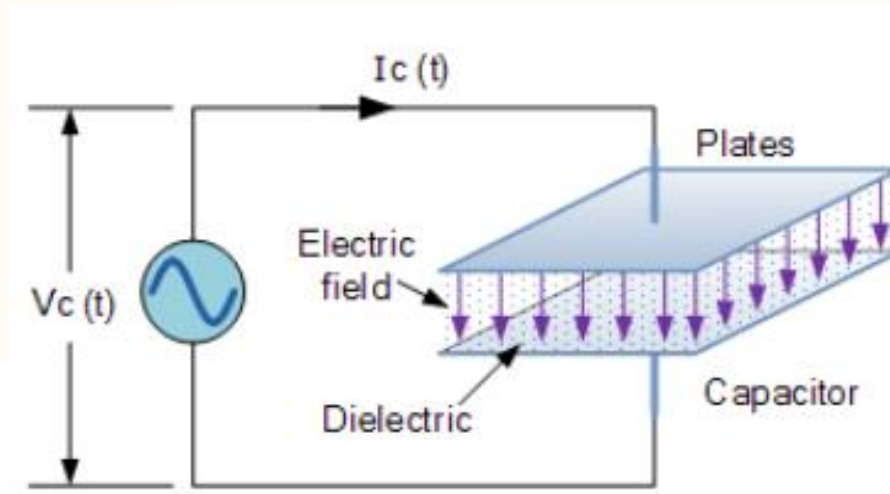


AC with a capacitor



# Capacitor in AC

## Capacitive Reactance



With the same AC source, we used DMM and found the current **increases** as the **capacitance increases**;

With the same capacitance and AC voltage amplitude, we used DMM and found the current **increases** as the AC source **frequency increases**.

The **opposition** to sinusoidal current in a capacitor is called **capacitive reactance**.

$$X_c = \frac{1}{2\pi fC}$$

Where:

$X_c$  = Capacitive Reactance in Ohms, ( $\Omega$ )

$\pi$  (pi) = 3.142 (decimal) or as  $22 \div 7$  (fraction)

$f$  = Frequency in Hertz, (Hz)

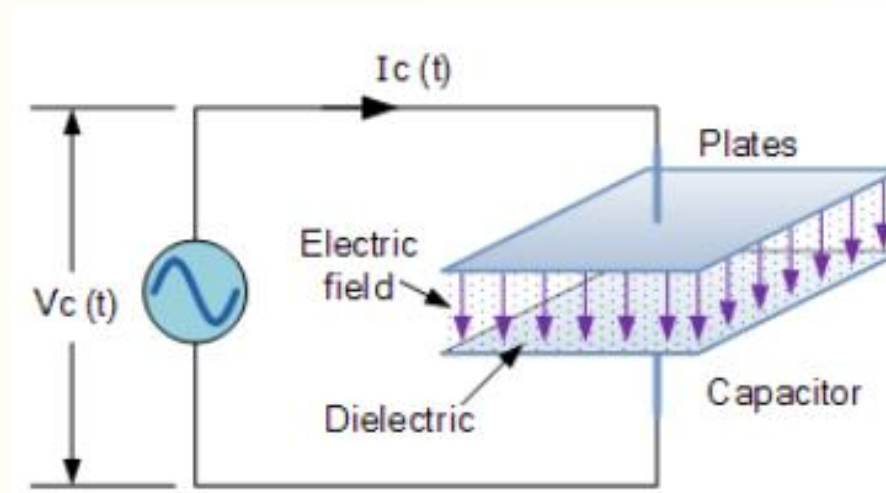
$C$  = Capacitance in Farads, (F)

# Capacitor in AC

## Capacitive Reactance

E. g.

Calculate the capacitive reactance value of a 220nF capacitor at a frequency of 1kHz and again at a frequency of 20kHz.

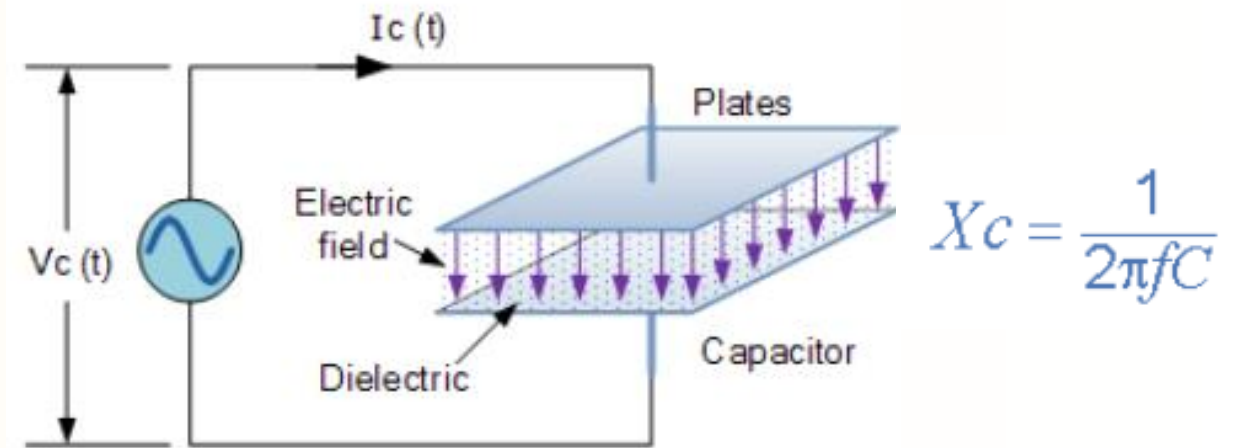


$$X_c = \frac{1}{2\pi fC}$$

# Capacitor in AC

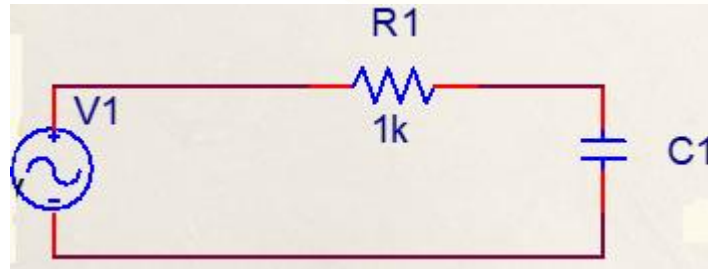
## Ohm's law in AC

$$I = \frac{V}{X_C}$$



When applying Ohm's law in ac circuits, you must express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on (**This is not in time domain**).

E. g.



$$V_{1\text{rms}} = 10 \text{ V}$$

$$f = 10 \text{ kHz}$$

$$C_1 = 0.0056 \mu\text{F}$$

$$R_1 = 1 \text{ k}\Omega$$

$$I_{\text{rms}} = ?$$



$$V_{1\text{rms}} = 10 \text{ V}$$

$$f = 10 \text{ kHz}$$

$$I_{\text{rms}} = ?$$

# Capacitor in AC

Capacitive reactance for capacitors in series in AC

$$X_{CT} = X_{C_1} + X_{C_2} + \cdots + X_{C_n}$$

Capacitive reactance for capacitors in parallel in AC

$$\frac{1}{X_{CT}} = \frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} + \cdots + \frac{1}{X_{C_n}}$$

Voltage divider in AC

$$V_x = \frac{X_{cx}}{X_{c1} + X_{c2} \cdots X_{cn}} V$$

Current divider in AC?

# Capacitor in AC

## Instantaneous power

$$p(t) = v(t)i(t)$$

## True power (one cycle power)

$$p_{ture} = 0$$

## Reactive power (the rate at which a capacitor stores or returns energy)

$$p_r = V_{rms}I_{rms} = \frac{V_{rms}^2}{X_c} = I_{rms}^2 X_c$$

## AC power waveforms for a pure capacitor

