Control Systems - ENGR 33041

Lecture 5: Control System Models- cont.

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Slides prepared based on

Control Systems Technology, C. Johnson and H. Malki

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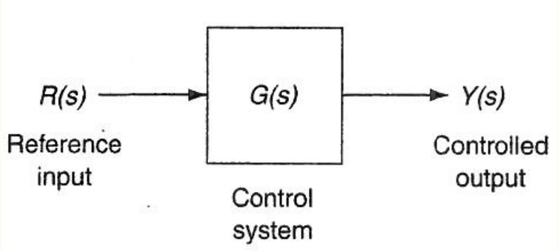
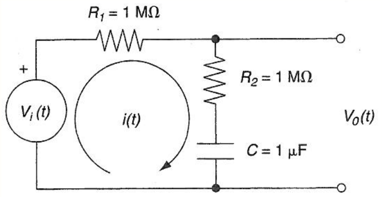
Recap of Previous Lecture

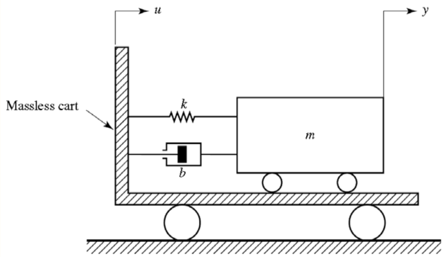


$$G(s) = \frac{Y(s)}{R(s)}$$

In general, the differential equation inside a block is derived from electrical, mechanical, and/or chemical properties of the physical device the block represents.



Electrical System

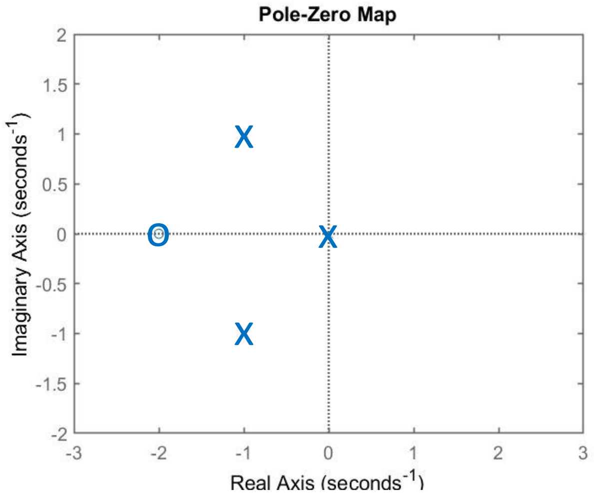


Mechanical System

Zeros= roots of numerator
Poles= roots of denominator

In the s-plane plot, the **poles** are shown by **cross** (x) and **zeros** are shown by **circle** (o).

$$P(s) = \frac{s+2}{s^3+2s^2+2s} = \frac{s+2}{s(s+1+j)(s+1-j)}$$



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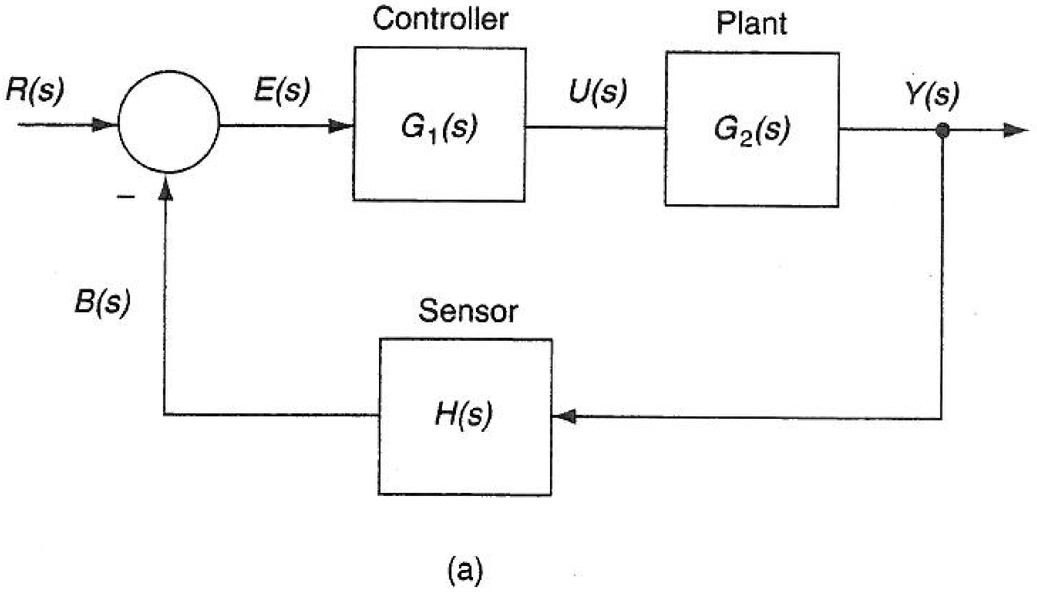
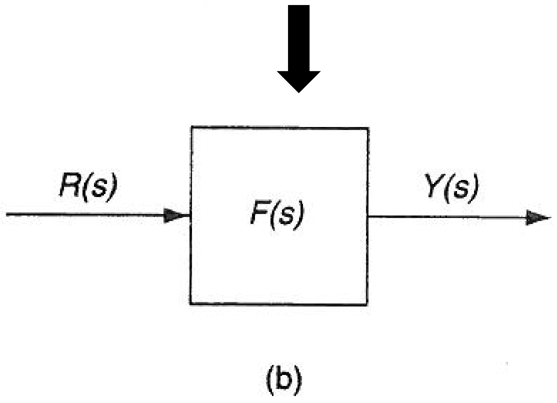
4.3 Block Diagrams

FIGURE 4.6
Generic block diagram of a control system with transfer functions.



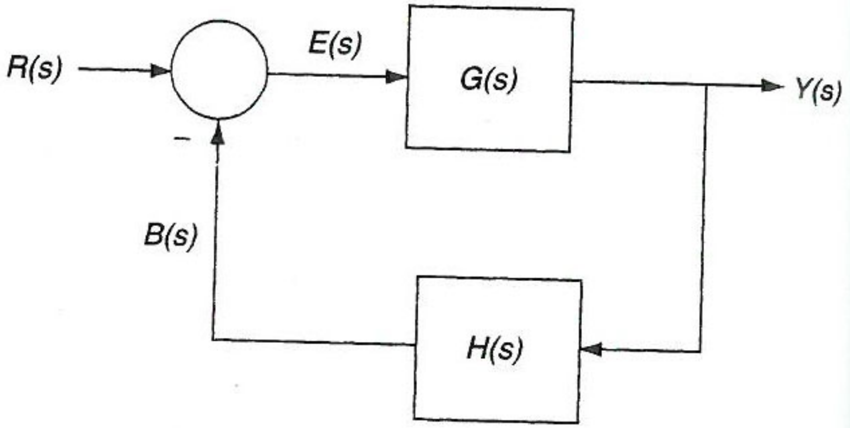


System Transfer Function
Or
Closed-Loop Transfer Function

Single block
Representing the
entire control system →

4.3.1 Canonical Form

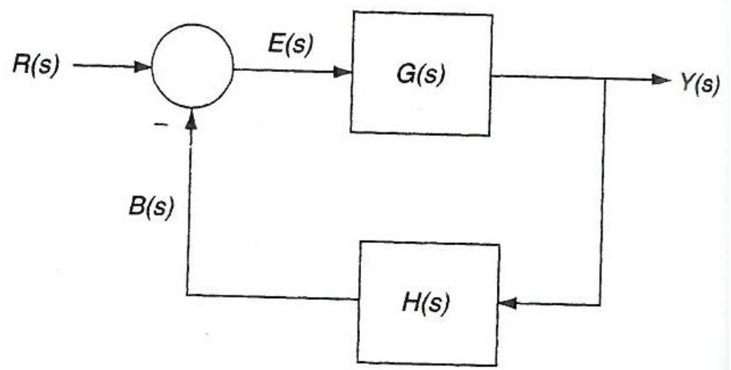
FIGURE 4.15
The canonical form of a control system block diagram.



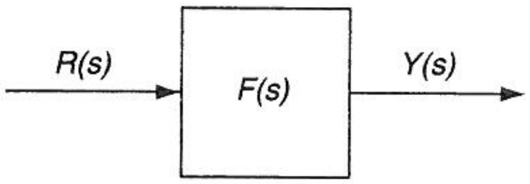
If you compare this to the block diagram shown in Figure 4.6a, you can see that the feedforward blocks have been combined into a single block, G(s), while the feedback block remains the same, H(s). If we depict a control system block diagram as shown in Figure 4.15, with one block in the feedforward path and one block in the feedback path, it is called a **Canonical Block Diagram.**

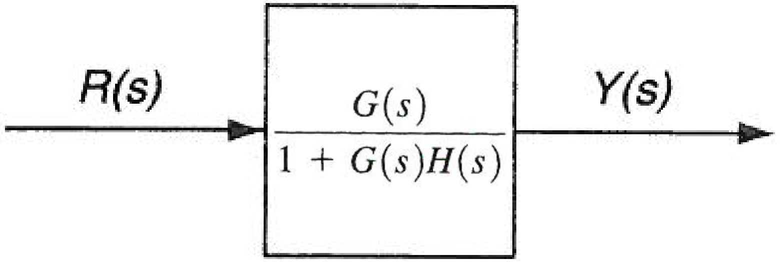
Now, we are interested in finding the closed-loop transfer function (system transfer function) for this Canonical Block Diagram. In other words, we are interested in finding a single block that represents this canonical block diagram, which is the ratio $\frac{Y(s)}{R(s)}$

FIGURE 4.15
The canonical form of a control system block diagram.



?



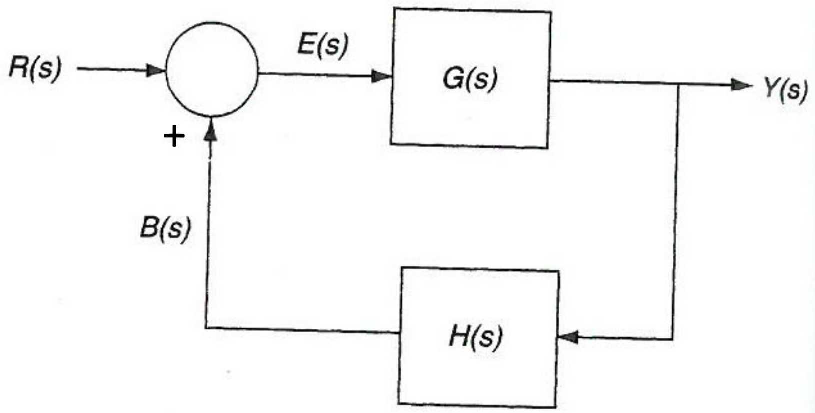


$$\frac{G(s)}{1+G(s)H(s)}$$ **is the closed-loop transfer function.**

$G(s)H(s)$ **is the open-loop transfer function.**

Positive Feedback

FIGURE 4.15
The canonical form of a control system block diagram.
(positive feedback)



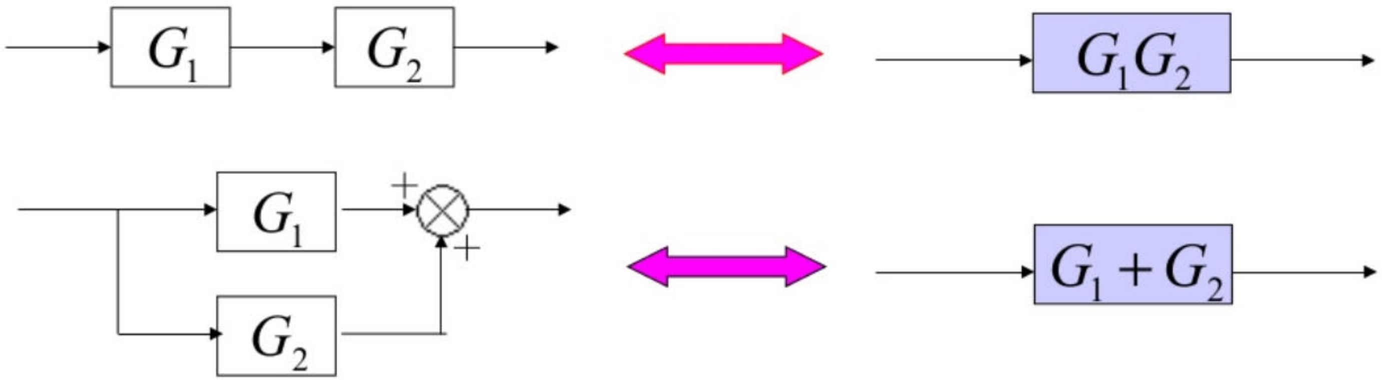
$$E(s) = R(s) + B(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}}$$

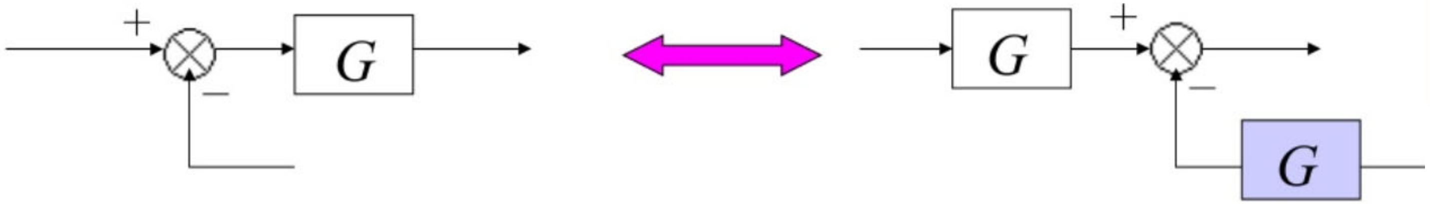
Prove it yourself.

4.3.2 Block Diagram Reduction

1. Combining blocks which are in cascade or in parallel

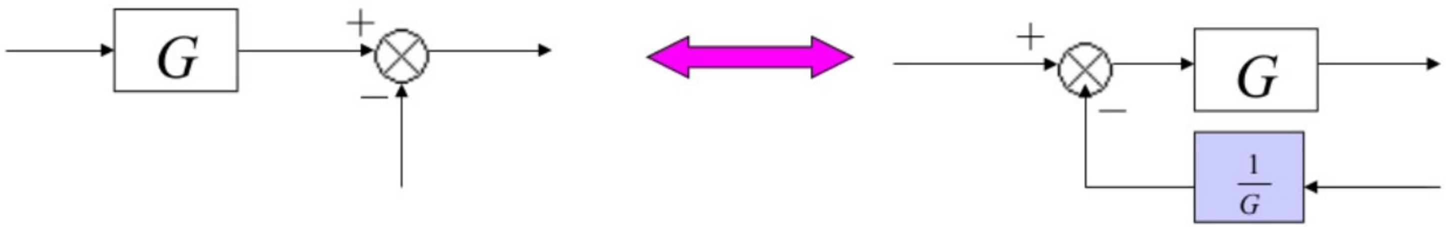


2. Moving a summing point behind a block

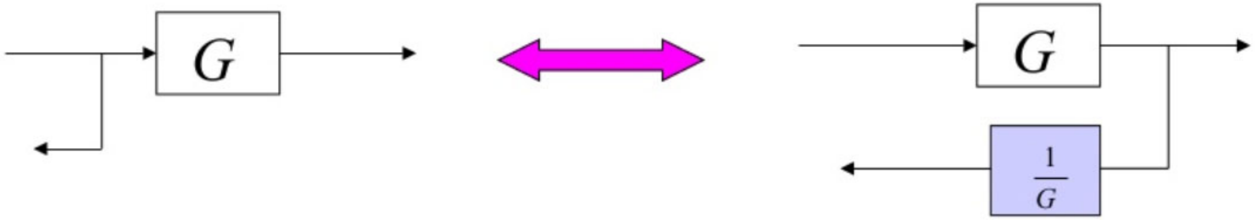


4.3.2 Block Diagram Reduction-cont.

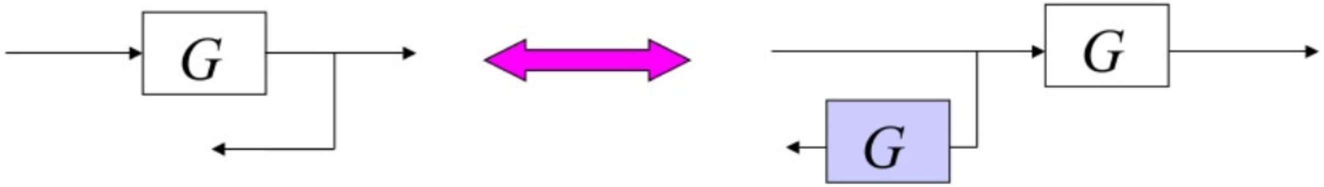
3. Moving a summing point ahead of a block



4. Moving a pickoff point behind a block



5. Moving a pickoff point ahead of a block



Ex. 4.15

Reduce the control system block diagram of figure 4.17 to canonical form and then write out the closed-loop transfer function of the system in terms of the block transfer functions. What is the open-loop transfer function?

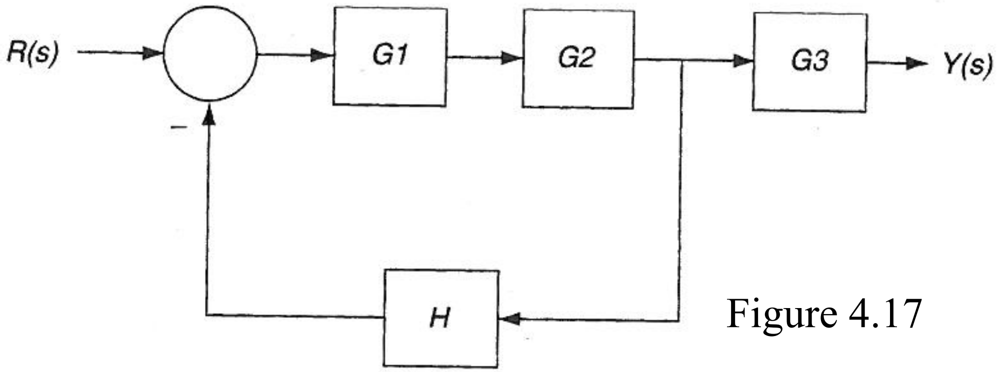


Figure 4.17

Solution

Ex. 4.16

a) Reduce the block diagram below to a canonical form.

b) Specify the open-loop transfer function.

c) Specify the closed-loop transfer function.

d) Specify the characteristic equation.

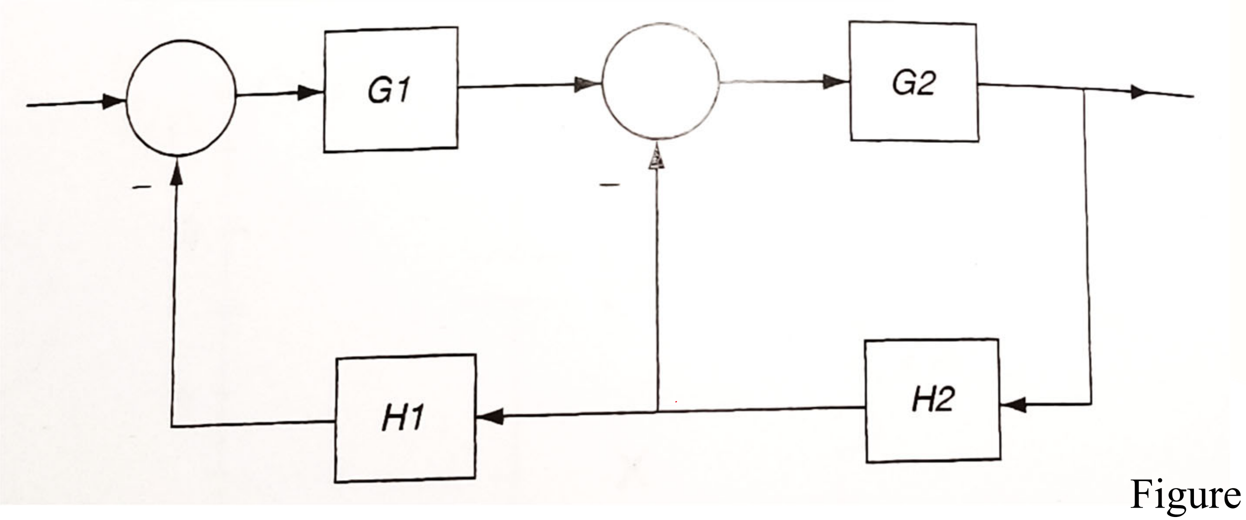


Figure 4.18

Solution

Ex. 4.17

Reduce the control system shown in figure 4.19 to canonical form and then specify the closed-loop and open-loop transfer functions.

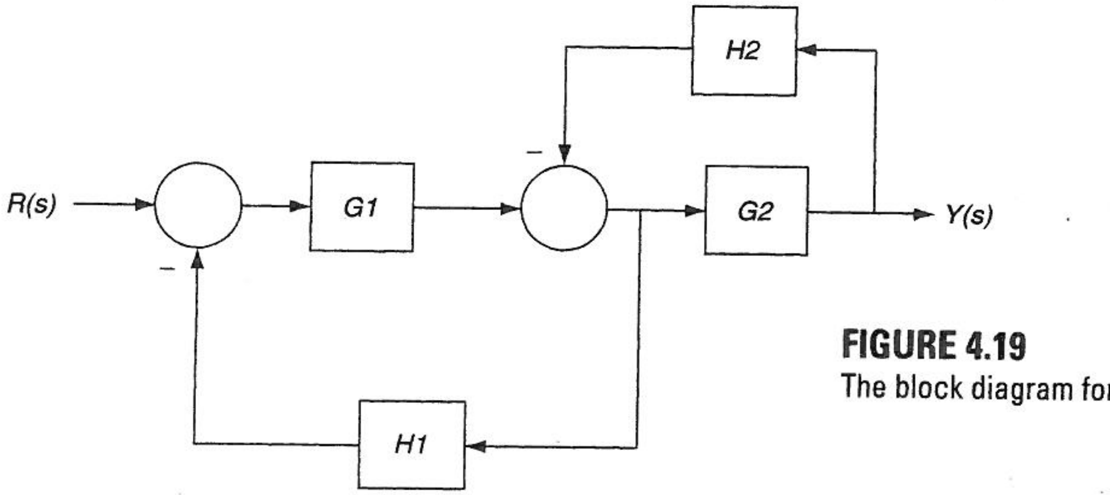


FIGURE 4.19
The block diagram for example 4.16.

Ex. 4.17

Reduce the control system shown in figure 4.19 to canonical form and then specify the closed-loop and open-loop transfer functions.

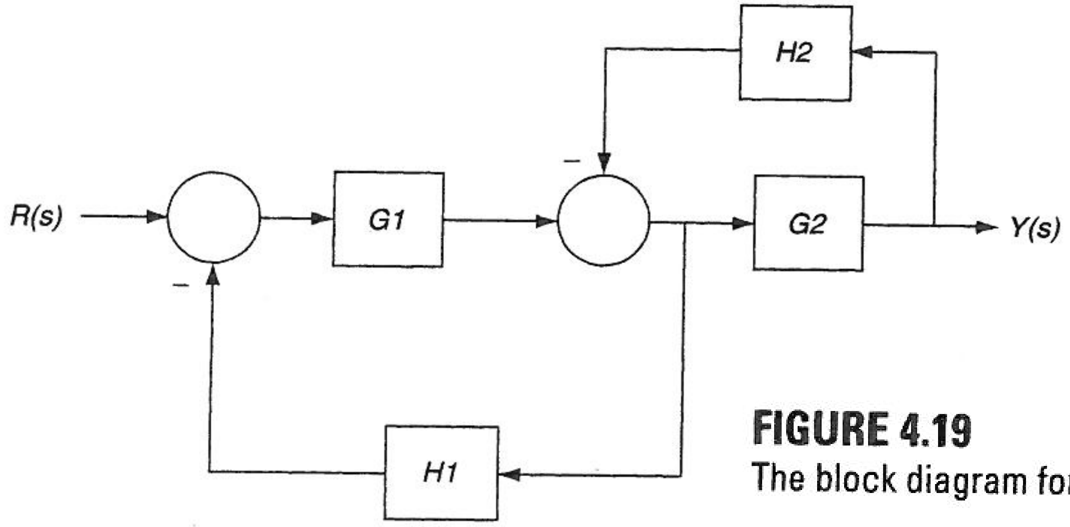
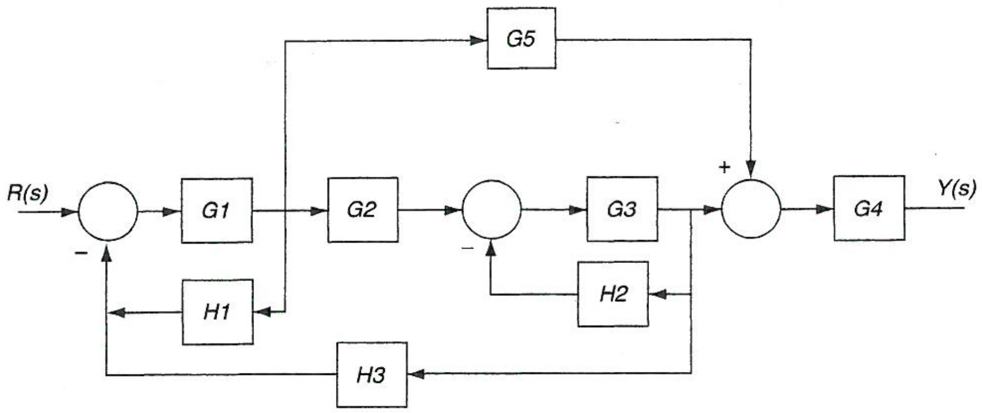


FIGURE 4.19
The block diagram for example 4.16.

Solution:

4.4 MASON'S GAIN FORMULA



Forward path: A **forward path** is one for which the signal flow starts from the input and terminates at the output. There could be more than one forward path in a block diagram. A forward path cannot contain any feedback traversing from input node to the output node.

Loop: A **feedback loop** is a path that starts and ends at the same point. The point cannot be traversed more than once in a loop.

Loop gain: The product of block transfer functions in a loop constitutes the **loop gain.** Note that the negative sign of the feedback will be multiplied.

Forward-path gain: A **forward-path gain** is the product of each block transfer function in the path.

Mason's Gain Formula

$$\mathrm{TF} = \frac{\sum_{k=1}^{N} M_k(s)\Delta_k(s)}{\Delta(s)}$$

Eq. (4.11)

TF is the overall transfer function

N = Number of *independent* forward path from input to output

M_k is the k^{th} forward path gain (transfer function product of the k^{th} forward path)

$$\begin{aligned}\Delta = 1 &- (\text{sum of all individual loop gains}) \\ &+ (\text{sum of gain products of all two nontouching loops}) \\ &- (\text{sum of gain products of all three nontouching loops}) \\ &+ \dots\end{aligned}$$

Δ_k is the value of that part of Δ that does not touch M_k

(Part of Δ that does not have common elements with M_k)

There are four steps in reducing the block diagrams using Mason's Gain formula:

Step 1: Determine the number of forward paths, N, and show the product of each forward path block transfer function.

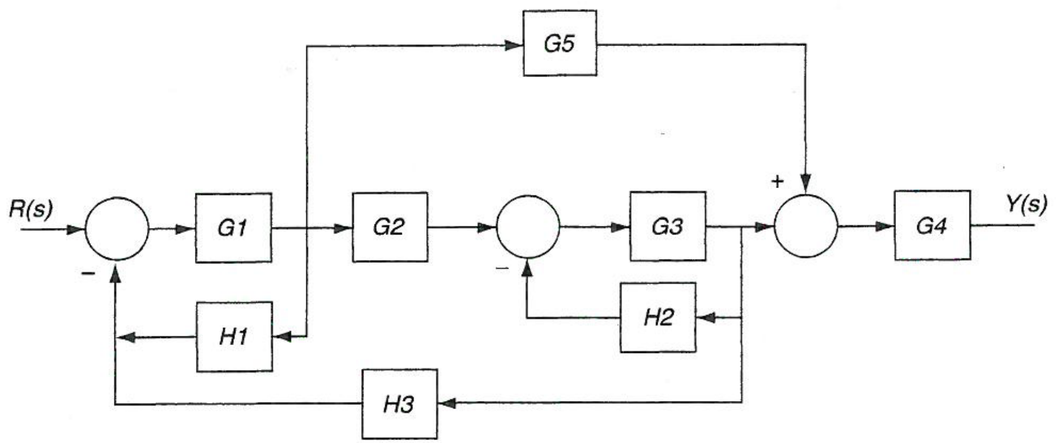
Step 2: Identify all of the individual loops, deduce the loop gain of each, and then find $\Delta(s)$ using equation 4.11.

Step 3: Determine all of the $\Delta_k(s)$ by eliminating any touching terms in $\Delta(s)$ for the kth path.

Step 4: Substitute algebraic equations obtained in steps 1–3 into equation 4.11.

Ex. 4.21

Reduce the control system block diagram of figure 4.27 using Mason's gain formula and then obtain the closed-loop transfer function.



EXAMPLE 4.22 Reduce the control system block diagram of figure 4.28 using Mason's gain formula and then obtain the closed-loop transfer function.

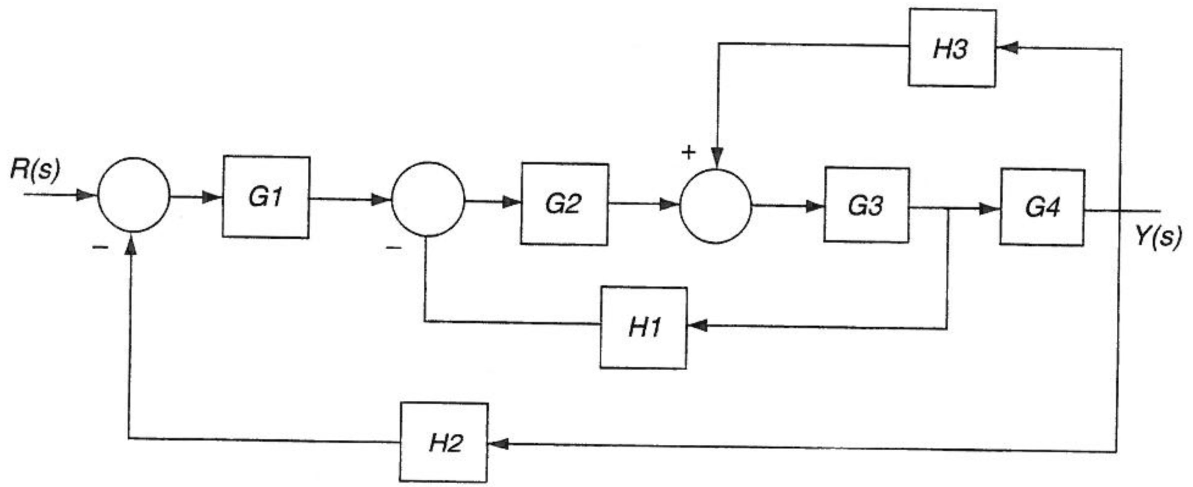


FIGURE 4.28
Block diagram for example 4.22.

Solution:

Step 1: There is one forward path: $G1G2G3G4$.

Step 2: There are three individual loops: $-G2G3H1$, $-G1G2G3G4H2$, and $G3G4H3$.

Step 3: $\Delta_1(s) = 1$ and all loops are touching.

Step 4:

$$\frac{Y(s)}{R(s)} = \frac{G1G2G3G4}{1 + G1G2G3G4H2 + G2G3H1 - G3G4H3}$$

Unity Feedback Loop

Unity feedback loop is a special case of canonical form, where there is no block on the feedback loop, i.e. *H(s)*=1.

Using equation for canonical form in slide 5:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad , H(s) = 1$$

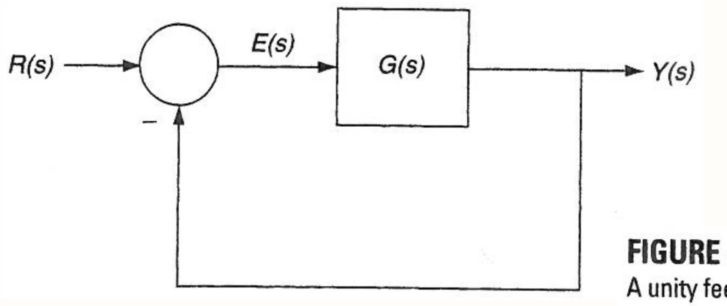


FIGURE 4.21
A unity feedback canonical block diagram.

Open-loop TF: $G(s) \times H(s) = G(s) \times 1 = G(s)$

Closed-loop TF:

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)}}$$

Advantage of a unity feedback control system

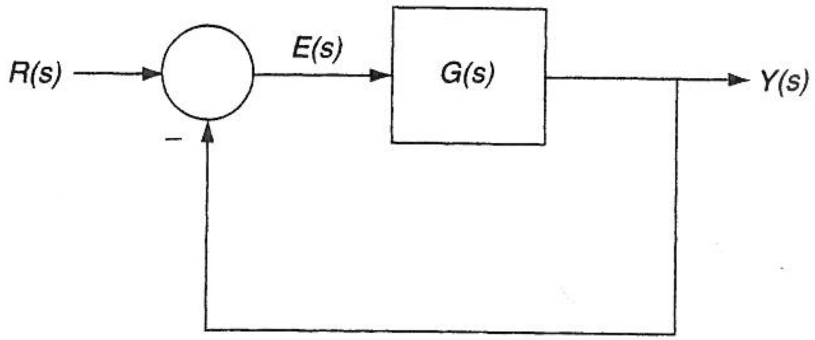
The reference input *r(t)* and the output *y(t)* represent the same quantity in a unity feedback system, so they can be compared directly, i.e., the error *e(t)* is the difference between the input and output: e(t)=r(t) – y(t).

On the other hand, systems without unity feedback, which is the characteristic of most systems, have the system output, *y(t),* measured and expressed as some other quantity, *b(t)*. Then *r(t)* has to be expressed in the same fashion. For example, *y(t)* may represent a temperature, whereas *b(t)* may be some voltage resulting from the sensing of the temperature. Likewise, then, *r(t)* will be the voltage in the same fashion as *b(t)*.

Ex. 4.23
MATLAB

Assume a unity-feedback system with

$$G(s) = \frac{1}{s^3+3s^2+91s+504}$$



a) Find the characteristic equation for this system.

b) Use MATLAB to determine the roots of the characteristic equation for this system. Draw pole-zero plot in s-domain.

c) Use MATLAB to find the time response of this system to a unit-step input and plot the response. Is the system stable?

Solution

- Homework 5 is due Oct 10, 11 AM and must be submitted on Canvas.