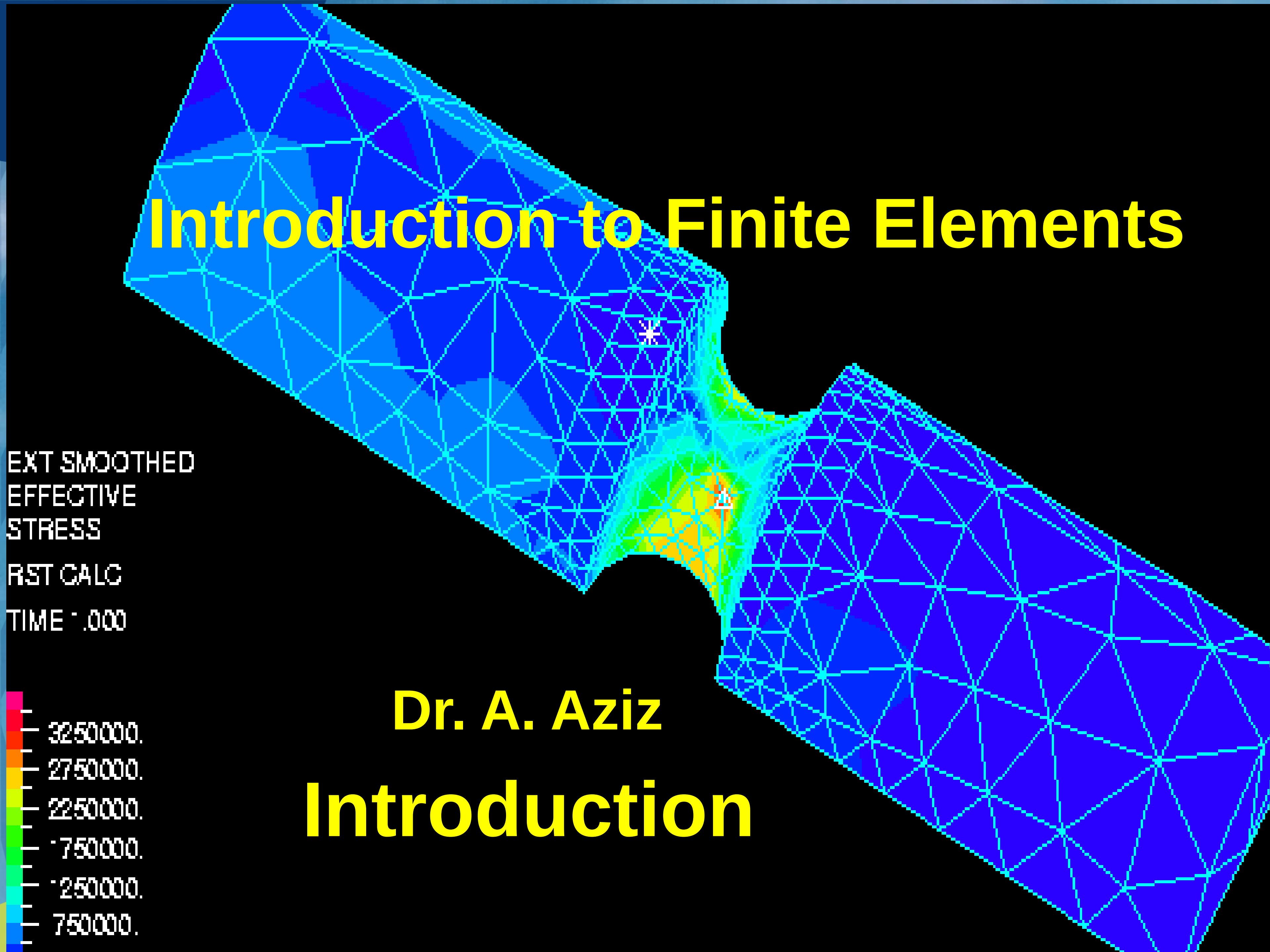
Introduction to Finite Elements



Dr. A. Aziz

Introduction

Info

ENGR 45901/55901/75901: Introduction to Finite Element

Spring 2024

Dr. A. Aziz

COURSE INFORMATION

Class meeting…………………………………………………………………………………………9:15- 10:30 AM M-FR, ATB227
Office location…… ATB 220E
Phone/E-mail………………………………………………………………………… (330) 672-1032 / aabdula3@kent.edu
Office Hours……………………………………………………………………………………………………12:30 – 2:30p M-W
All other times by appointment

Info

Course objective:

- The objective of this course is to teach in a unified manner the fundamentals of the finite element method for the analysis of engineering problems arising in solids and structures.
- The course will emphasize the solution of real-life problems using the finite element method and students will get exposure to commercial finite element software (ANSYS or ABAQUS) and will learn to critically evaluate finite element models.
- Examples will be provided for solid, fluid, and heat transfer applications. Finally, developing an understanding of the computational aspects of the finite-element method and its application in realistic aerospace applications are the core goals of this course.

Course texts and references

- **Textbook and Required Material**:

- **Finite element Modeling and Simulation with Ansys Workbench, 2nd Edition, Xiaolin Chen, Yijun Liu**
- **ISBN-13:** 978-1138486294
- **Course additional information are located at FlashLine Student Folder.**
- **Prerequisites by Topic**: Basics of Linear Algebra; Introductory calculus (differentiation, integration, differential equations); Computer aided design; Engineering statics

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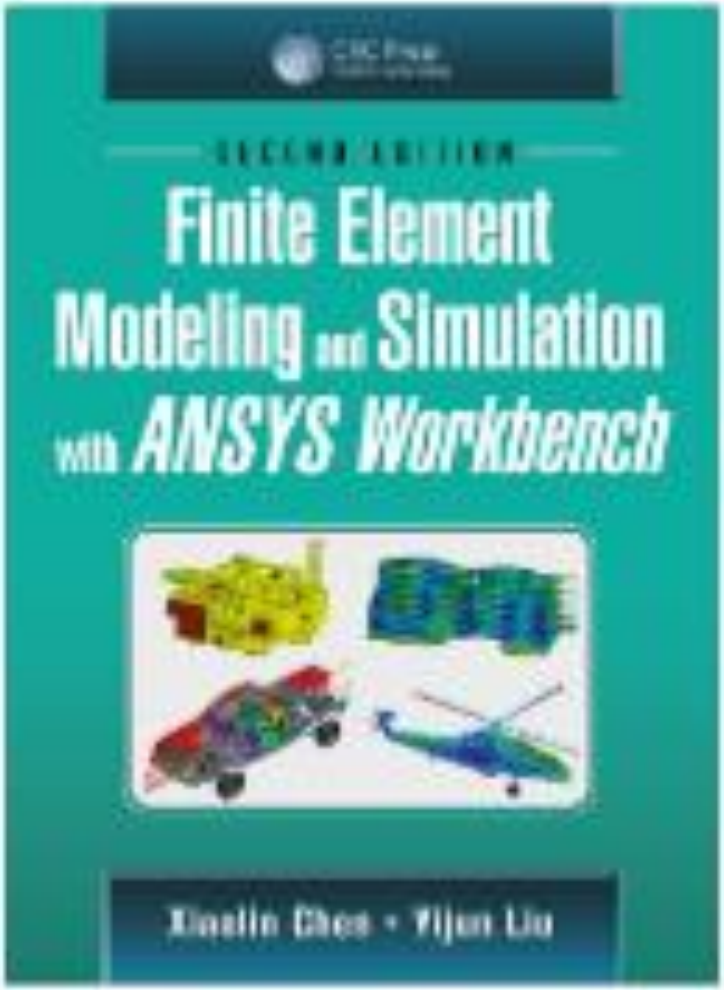
- **Finite element Modeling and Simulation with Ansys Workbench, 2nd Edition, Xiaolin Chen, Yijun Liu**

  ISBN-13: 978-1138486294



Expected outcomes:

Upon completion of this course, the student will:

1. To understand fundamental concepts of finite element methods.
2. Demonstrate an understanding of the fundamental concepts of the Finite Element Method (FEM) as a numerical solution method for engineering problems.
3. Demonstrate an ability to identify, formulate, and solve engineering problems using the finite element method.
4. To be able to assemble the global stiffness matrix and global equivalent load vector.
5. To understand the difference of elements and their application scopes and limitations.
6. To be able to interpret, justify and communicate the numerical results in a professional manner.
7. Have experience with FEM packages **(ANSYS Workbench).**

(3.1): Demonstrate an ability to identify, formulate, and solve complex engineering problems by applying principles of
engineering, science, and mathematics.

(3.6): Demonstrate an ability to develop and conduct appropriate experimentation, analyze, and interpret data, and use
engineering judgment to draw conclusions.

Schedule Type: Lecture

Contact Hours: 3 lecture

Grade Mode: Standard Letter

Info

Topics:

- Introduction to finite element analysis
- Direct stiffness approach: Spring elements
- Bar and truss elements
- Introduction to differential equations and strong formulation
- Principle of minimum potential energy and weak formulation
- Finite element formulation of linear elastostatics
- Constant strain triangle • Quadrilateral element
- Practical considerations in FEM modeling
- Convergence of analysis results
- Higher order elements
- Isoparametric formulation
- Numerical integration

COURSE PROJECT: In this project you will be required to

- choose an engineering system
- develop a mathematical model for the system
- develop the finite element model
- solve the problem using ANSYS Workbench commercial software
- present a convergence plot and discuss whether the mathematical model you chose gives you physically meaningful results.
- refine the model if necessary. Projects that demonstrate intelligent choice of models to solve comparatively difficult problems are encouraged.

Project Logistics:

1. Choose a tractable problem that you can analyze in depth and submit a one-page project proposal.
2. Please proceed to work on the project ONLY if approved by the instructor (zero credits otherwise).
3. Submit a one-page progress report within a week of starting the analysis. This will carry 10% of the project grade.
4. Submit a project report (typed) when you are done. This report should include

a. Introduction

b. Problem statement

c. Analysis

d. Results and discussion

Useful Websites of Interest to Finite Element Applications:

https://courses.ansys.com/?utm_source=google&utm_medium=ppc&utm_campaign=academic&utm_content=digital_academic_learn-more_course-aic_rsa-tutorials_en_global&utm_term=%2Bansys%20%2Btutorials&campaignid=7013g000000HSaLAAW&creative=501033483541&keyword=%2Bansys%20%2Btutorials&matchtype=b&network=g&device=c&gclid=EAIaIQobChMI_s6NmO7k9AIVZciUCR1ymQs7EAAYASAAEgJ8yfD_BwE.

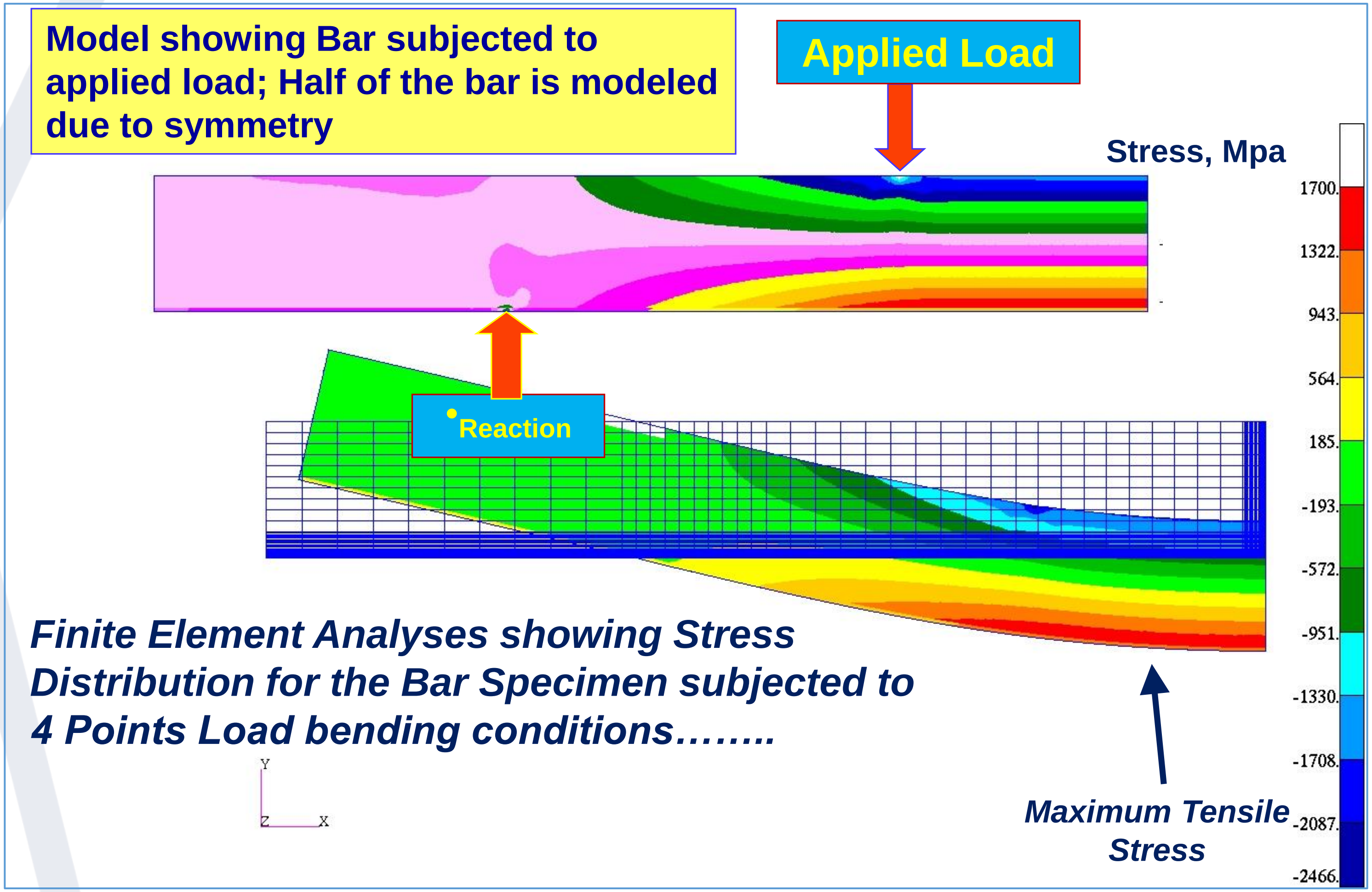
Finite Element Procedures for Solids and Structures | MIT OpenCourseWare

SOFTWARE

ANSYS WORKBENCH (REQUIRED)

[HTTPS://WWW.ANSYS.COM/ACADEMIC/STUDENTS/ANSYS-STUDENT](https://www.ansys.com/academic/students/ansys-student)

Excel, Matlab®, etc. (optional)



ACADEMIC HONESTY

1. All students must consent and sign the ethics & integrity contract prior to receiving academic credit for work.
2. If the submitted work is the result of a collaborative effort, names of all members and contributors must be clearly indicated.
3. Aeronautics & Engineering require integrity. Document and cite sources accordingly.
4. The use of electronic devices to capture images of online quizzes and exams in order to pass to other students is cheating.
5. The use of Chegg, Course Hero, or any other online source without citation in preparing assignments for grade is a violation of the university's academic honesty.
6. Copying another student's computer code, spreadsheets, documents, handwritten problems is also a violation of university policy regarding academic honesty.

The policy that governs academic honesty is University Policy 3-01.8: Administrative policy regarding student cheating and plagiarism.

INSTRUCTOR POLICIES.

Laptop Policy

The use of laptops will not be **permitted** in class during lectures. They only time it can be used during projects assignments activities to run Ansys Workbench.

Homework Policy

Homework will be assigned on Mondays and will be due on the following Monday. You can submit your homework in digital form (PDF preferred) via email. Your submission should include a title, your name. All work should be shown, and it must be clearly written. Where appropriate, all dimensional units should be included. Unreadable submissions will not be graded. Late homework will not be accepted, except for extreme circumstances or documented illness.

Ethical Conduct:

Students are expected to conduct themselves in a professional manner and uphold the standards expected of the engineering profession. During your academic studies, unacceptable actions include cheating during in-class quizzes or exams, copying homework and/or lab reports, fudging data, etc. During your employment as an engineer, nothing will end your career faster than falsifying data or forwarding inadequate designs. Unethical conduct during your engineering career will be dealt with by your employer and the legal system.

Unethical conduct during this class will be dealt with according to the Kent State University Code of Student Conduct that can be found at:

http://www.kent.edu/studentconduct/code-student-conduct

Withdrawal Deadline:
Check the link below.
http://www.kent.edu/registrar/spring-important-dates

Experiential Learning Requirement / Writing Intensive Requirement

Does the course meet one of the university's ELR or WUC requirements?

COURSE LEARNING OBJECTIVES (CLOS)

The following table lists the course learning objectives and how they support student outcomes as well as meet the university's experiential learning requirement.

Expected outcomes/Learning objectives.

Upon completion of this course, the student will:

1. Understand fundamental concepts of finite element methods.
2. Demonstrate an understanding of the fundamental concepts of the Finite Element Method (FEM) as a numerical solution method for engineering problems.
3. Demonstrate an ability to identify, formulate, and solve engineering problems using the finite element method.
4. Understand the difference of elements and their application scopes and limitations.
5. Be able to formulate and solve basic problems in heat transfer, solid mechanics and fluid mechanics.
6. Write simple computer code to apply finite element method.
7. Be able to interpret, justify and communicate the numerical results in a professional manner.
8. Have experience with a commercial FEM package.

STUDENT PROGRAM OUTCOMES (SPOs)

	CLOs				
	1	2	3	4	5
ABET Engineering Student Outcomes. Upon completion, graduates will have demonstrated					
(E1) an ability to identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics	•		•		
(E2) an ability to apply engineering design to produce solutions that meet specified needs with consideration of public health, safety, and welfare, as well as global, cultural, social, environmental, and economic factors	•			•	
(E3) an ability to recognize ethical and professional responsibilities in engineering situations and make informed judgments, which must consider the impact of engineering solutions in global, economic, environmental, and societal contexts	•		•	•	•
(E4) an ability to function effectively on a team whose members together provide leadership, create a collaborative and inclusive environment, establish goals, plan tasks, and meet objectives	•	•			•
(E5) an ability to develop and conduct appropriate experimentation, analyze, and interpret data, and use engineering judgment to draw conclusions	•		•	•	

Homework Assignments Summary

Homework has two purposes:

1. To provide the student with timely exercise in the use of the models developed in the lectures.
2. To extend the logic in directions of importance that cannot be covered within the time constraints of the lectures.
3. The problem sets must, therefore, represent your own work. You may discuss the sets with your fellow students, but the engineering should be your own.

Unit of Study Program*

* Additional materials for graduate students

Problems Assignments Summary

Chap 1. Problems; 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7
Chap 2. Problems; 2.1, 2.2, 2.3, 2.4, 2.9
Chap 3. Problems; 3.1, 3.2, 3.3, 3.6, 3.8, 3.10, 3.11
Chap 5. Problems; TBD
Chap 7. Problems; TBD
Chap. 9. Problems; TBD

Course Outline

- The course outlines are subject to change throughout the semester
- **Please note that the data in this syllabus is subject to modifications and changes throughout the semester. This is dependent on the course subjects' coverage, schedule, and progress.**

week	Days	Topic
1	Jan 17, 22	Chap. 1, Introduction, Application of the Finite Element Method FEA with Ansys Workbench, Introductory Linear Algebra Review
2	Jan 24, 29	Chap. 2, Bars and Trusses; Modeling of Trusses, Formulation of the Bar Element
3	Jan 31, Feb 5, 7	Chap. 2, Bars and Trusses; Modeling of Trusses, Formulation of the Bar Element
4	Feb 12, 14	Chapter 3, Beams and Frames, Development of Truss Equations, Development of Beams Equations, Frames and Grid Equations, Plane Stress and Plane Strain7
5	Feb 19, 21	Preprocessing considerations, such as element type, mesh size and numerical methods, for the efficiency and accuracy of the analysis, project Presentations
3.16	Feb 26, Feb 28	Self-Learning, Projects Presentation, Discussions
7	Mach 4, 6	Chapter 5, Structural modeling – Modeling techniques for creating mechanical components with complex geometry.
8	March 11, 13	Self-learning, Project Presentation
9	March 18, 20	Chapter 7, Three-Dimensional Elasticity
10	March 25-Mar 31	Spring Recess, No Classes
11	April 1, 3	Chap. 9, Thermal Analysis, Introduction, Review of basic equations
12	April 8, 10	Chap. 9, Finite Element Formulation for Heat Conduction, Thermal Stress Analysis
13	April 15, 17	Modeling of Thermal Problems, project Presentations
14	April 22, 24	Review and project Presentations
15	May 1	Final Exam/ Final Project Presentations

- **Please note that the data in this syllabus is subject to modifications and changes throughout the semester. This is dependent on the course subjects' coverage, schedule, and progress.**

Final Exam, Tuesday May 10, 10:15-12:30 AM, ATB 212

ASSESSMENT

Homework	10 %
Quizzes	20 %
Projects	40 %
Final project	20 %
Attendance	
Class Participation	
Instructor concession	0-**10%**
Total	**100 %**

Grading Scale:

A	90-100 %
A-	88-89.9 %
B+	86-87.9 %
B	83-85.9 %
B-	80-82.9 %
C+	76-79.9 %
B-	80-82.9 %
C+	76-79.9 %
C	70-75 %
C-	68-69.9 %
D+	66-67.9 %
D	60-65.9 %
F	Below 60%

Note: *Letter grades can be adjusted by the instructor which can include adopting modifications to the above indicated scale if the class performance calls for such action*.

Attendance Policy

Attendance will be taken; it is expected that you attend all lectures and recitations. If you have planned absence, please let Dr. Aziz know.

Recommended Tutorial Videos:

https://www.youtube.com/results?search_query=ansys+workbenh+tutrial

ANSYS Workbench Explicit Dynamics tutorial for beginners
https://www.youtube.com/watch?v=O8dF-sIHdWk
ANSYS Workbench Tutorial - Simply Supported Beam - PART 1
https://www.youtube.com/watch?v=CTi1ru-pfi0
ANSYS Workbench Tutorial - Simply Supported Beam - Center Load - PART 2
https://www.youtube.com/watch?v=XECmd-BPKa8
ANSYS Workbench Tutorial - Introduction to Static Structural
https://www.youtube.com/watch?v=vnpq5zzOS48
How to use Ansys Workbench? | Static structural analysis | Comparison of results
https://www.youtube.com/watch?v=clccZEmyO10
Steady state heat transfer analysis using ANSYS workbench | Tutorial for beginners
https://www.youtube.com/watch?v=UUAqRxChXy4
Basic Truss Analysis using ANSYS workbench | Static Truss Analysis
https://www.youtube.com/watch?v=3lOQBH_bgyQ

You can download all the files from: https://github.com/NotRealEngineering
ANSYS Workbench | Steady State Analysis | Thermal Analysis
https://www.youtube.com/watch?v=EGnobuCdFik
Steady state heat transfer through composite slab -Ansys Tutorial
https://www.youtube.com/watch?v=zM86XDUsMmA
Ansys Tutorial | Ansys Workbench | Modal Analysis
https://www.youtube.com/watch?v=4rQf4oTTEdk
Modal Analysis : Vibration Analysis on Shaft || Ansys Workbench 18.1 || Analysis Tutorial
https://www.youtube.com/watch?v=vXndHleFc-8
Ansys | Modal Analysis | Natural Frequencies
https://www.youtube.com/watch?v=uJ_T98IhAVc

Students with Disabilities

University policy 3-01.3 requires that students with disabilities be provided reasonable accommodations to ensure their equal access to course content. If you have a documented disability and require accommodations, please contact the instructor at the beginning of the semester to make arrangements for necessary classroom adjustments. Please note, you must first verify your eligibility for these through Student Accessibility Services (contact 330-672-3391 or visit www.kent.edu/sas for more information on registration procedures).

Notice of Copyright and Intellectual Property Rights

Any intellectual property (IP) displayed or distributed to students during this course (including but not limited to slides, notes, quizzes, and examinations by the professor remains the (IP) of the same. This means that the student may not distribute, publish or provide such IP to any other person or entity for any reason, commercial or otherwise, without the express written permission of the professor. **Student IP remains with the students with the caveat that reporting, and presentations will be maintained online and accessible by the public.**

Please check the links listed below for Coronavirus Updates

KENT STATE'S SAFETY PRINCIPLES

https://www.kent.edu/coronavirus/flashes-safe-seven#while-on-campus
https://www.kent.edu/coronavirus

1. ALWAYS WEAR YOUR FACE COVERING
2. WASH YOUR HANDS FREQUENTLY
3. CLEAN AND SANITIZE
4. STAY AT LEAST 6 FEET APART
5. MONITOR YOUR HEALTH EVERY DAY
6. HAVE QUESTIONS? REACH OUT
7. For health questions about COVID-19 and safe practices, call **University Health Services** at 330-672-2326, or after hours, contact the Kent State Nurse Line at 330-672-2326. Also, visit our **COVID-19 website** for more information.
8. FLASHES TAKE CARE OF FLASHES
9. As we take care of each other during the COVID-19 pandemic, always demonstrate kindness and respect in all that you do.

Note: This course is being instructed remotely. Please check the guidelines provided by the university concerning more details and any additional information. Office hours posted on this syllabus are not applicable, communication is only available via emails or pre-arranged zoom-team meetings.

Collaboration / academic integrity

1. Students are encouraged to collaborate in the solution of HW problems but submit independent solutions that are NOT copies of each other. Funny solutions (that appear similar/same) will be given zero credit.

1. Software may be used to verify the HW solutions. But submission of software solution will result in zero credit.

2. Groups of 2 for the projects no two projects to be the same/similar). A single grade will be assigned to the group and not to the individuals.

Course Projects (40 %)

In this project you will be required to

- Assign or choose an engineering system
- develop a mathematical model for the system
- develop the finite element model
- solve the problem using commercial software
- present a convergence plot and discuss whether the mathematical model you chose gives you physically meaningful results.
- refine the model if necessary.

Major/Final project (20 %)..contd.

Project report:

1. Must be professional (Text font Times 11pt with single spacing).

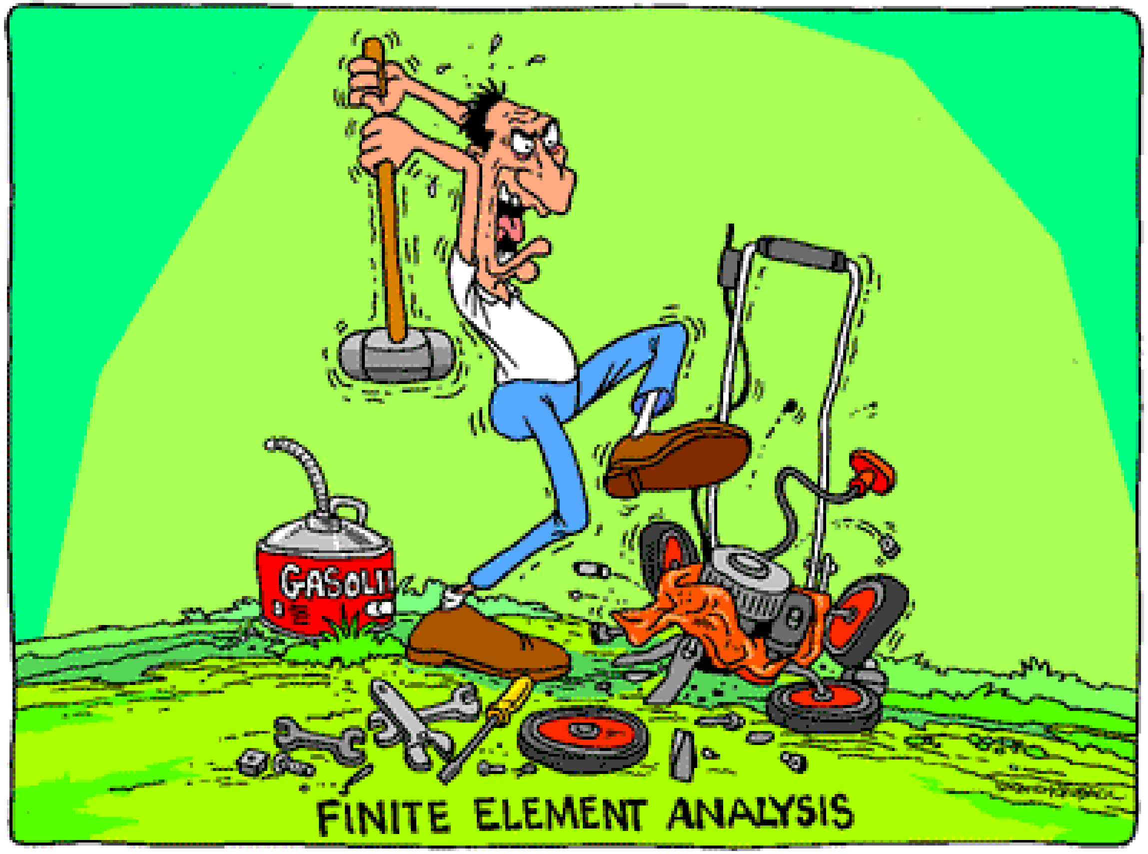
1. Must include the following sections:
   - Introduction
   - Problem statement
   - Analysis
   - Results and Discussions

Major project (20 %)..contd.

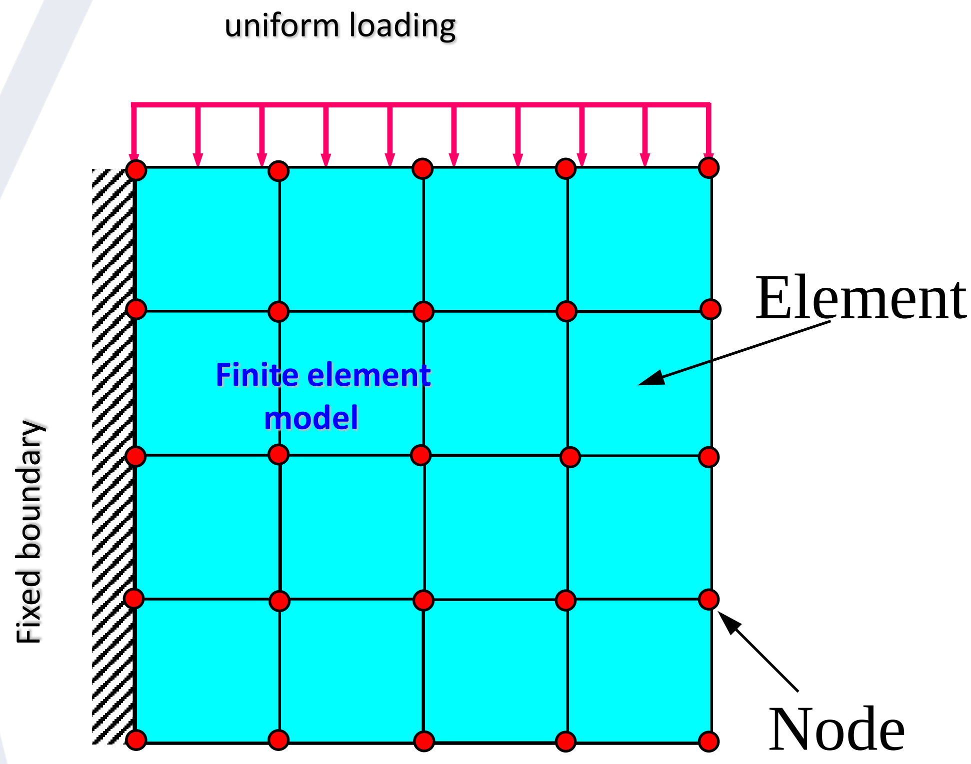
Project grade will depend on

1. Originality of the idea
2. Techniques used
3. Critical discussion

Finite Element Analysis



Finite Element Analysis



Problem: Obtain the stresses/strains in the plate

- Approximate method
- Geometric model
- Node
- Element
- Mesh
- Discretization

Course content

1. "Direct Stiffness" approach for springs
2. Bar elements and truss analysis
3. Introduction to boundary value problems: strong form, principle of minimum potential energy and principle of virtual work.
4. Displacement-based finite element formulation in 1D: formation of stiffness matrix and load vector, numerical integration.
5. Displacement-based finite element formulation in 2D: formation of stiffness matrix and load vector for CST and quadrilateral elements.
6. Discussion on issues in practical FEM modeling
7. Convergence of finite element results
8. Higher order elements
9. Isoparametric formulation
10. Numerical integration in 2D
11. Solution of linear algebraic equations

Linear Algebra Recap

What is a matrix?

A rectangular array of numbers (we will concentrate on real numbers). A n_{xm} matrix has 'n' rows and 'm' columns

$$\mathbf{M}_{\mathbf{3x4}} = \begin{array}{cc} \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{bmatrix} & \begin{array}{l} \text{First row} \\ \text{Second row} \\ \text{Third row} \end{array} \\ \begin{array}{cccc} \text{First column} & \text{Second column} & \text{Third column} & \text{Fourth column} \end{array} & \end{array}$$

M_{12} — Row number, Column number

What is a vector?

A vector is an array of ‘n’ numbers

A **row vector of length ‘n’ is a 1xn matrix**

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$$

A **column vector of length ‘m’ is a mx1 matrix**

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Special matrices

Zero matrix: A matrix all of whose entries are zero

$$\underline{0}_{3x4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Identity matrix: A **square** matrix which has '**1**' s on the diagonal and zeros everywhere else.

$$\underline{\mathbf{I}}_{3x3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix operations

Equality of matrices

If $\underline{A}$ and $\underline{B}$ are two matrices of the same size,

then they are "equal" if each and every entry of one matrix equals the corresponding entry of the other.

$$\underline{A} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\underline{A} = \underline{B} \Leftrightarrow \begin{array}{lll} a = 1, & b = 2, & c = 4, \\ d = -3, & e = 0, & f = 7, \\ g = 9, & h = 1, & i = 5. \end{array}$$

Matrix operations

Addition of two matrices

If A and B are two matrices of the same size,

then the sum of the matrices is a matrix C=A+B whose entries are the sums of the corresponding entries of A and B

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -1 & 3 & 10 \\ -3 & 1 & 0 \\ 1 & 0 & 6 \end{bmatrix}$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & 5 & 14 \\ -6 & 1 & 7 \\ 10 & 1 & 11 \end{bmatrix}$$

Matrix operations

Properties of matrix addition:

1. Matrix addition is **commutative** (order of addition does not matter)

$$\bar{A} + \bar{B} = \bar{B} + \bar{A}$$

2. Matrix addition is **associative**

$$\bar{A} + \left(\bar{B} + \bar{C}\right) = \left(\bar{A} + \bar{B}\right) + \bar{C}$$

3. Addition of the zero matrix

$$\bar{A} + \bar{0} = \bar{0} + \bar{A} = \bar{A}$$

Matrix operations

Multiplication by a scalar

If A is a matrix and c is a scalar, then the product cA is a matrix whose entries are obtained by multiplying each of the entries of A by c

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \quad \mathbf{c} = 3$$

$$\mathbf{cA} = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & 21 \\ 27 & 3 & 15 \end{bmatrix}$$

Multiplication by a scalar

Special case

Matrix operations

If A is a matrix and c =-1 is a scalar, then the product

(-1)A =-A is a matrix whose entries are obtained by multiplying each of the entries of A by -1

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \quad \mathbf{c} = -1$$

$$\mathbf{c}\underline{\mathbf{A}} = \mathbf{-}\underline{\mathbf{A}} = \begin{bmatrix} -1 & -2 & -4 \\ 3 & 0 & -7 \\ -9 & -1 & -5 \end{bmatrix}$$

Matrix operations

Subtraction

If $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ are two square matrices of the same size, then $\underline{\mathbf{A}}$-$\underline{\mathbf{B}}$ is defined as the sum $\underline{\mathbf{A}}$+(-1)$\underline{\mathbf{B}}$

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \qquad \underline{\mathbf{B}} = \begin{bmatrix} -1 & 3 & 10 \\ -3 & 1 & 0 \\ 1 & 0 & 6 \end{bmatrix}$$

$$\underline{\mathbf{C}} = \underline{\mathbf{A}} - \underline{\mathbf{B}} = \begin{bmatrix} 2 & -1 & -6 \\ 0 & -1 & 7 \\ 8 & 1 & -1 \end{bmatrix}$$

Note that $\underline{\mathbf{A}} - \underline{\mathbf{A}} = \underline{\mathbf{0}}$ **and** $\underline{\mathbf{0}} - \underline{\mathbf{A}} = -\underline{\mathbf{A}}$

Special operations

Transpose

If **A** is a $\mathbf{m_{xn}}$ matrix, then the **transpose** of **A** is the $\mathbf{n_{xm}}$ matrix whose first column is the first row of **A**, whose second column is the second column of **A** and so on.

$$\underline{A} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \Leftrightarrow \underline{A}^T = \begin{bmatrix} 1 & -3 & 9 \\ 2 & 0 & 1 \\ 4 & 7 & 5 \end{bmatrix}$$

Special operations

Transpose

If $\underline{A}$ is a square matrix ($\mathbf{m_{xm}}$), it is called **symmetric** if

$$\underline{A} = \underline{A}^T$$

Scalar (dot) product of two vectors

If **a** and **b** are two vectors **of the same size**

$$\underline{\mathbf{a}} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}; \quad \underline{\mathbf{b}} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

The scalar (dot) product of a and b is a scalar obtained by adding the products of corresponding entries of the two vectors

$$\underline{\mathbf{a}}^T \underline{\mathbf{b}} = \left(a_1 b_1 + a_2 b_2 + a_3 b_3 \right)$$

Matrix multiplication

For a product to be defined, the number of columns of **A** must be equal to the number of rows of **B**.

$$\underset{m \times r}{\mathbf{A}} \qquad \underset{r \times n}{\mathbf{B}} \qquad = \qquad \underset{m \times n}{\mathbf{AB}}$$

outside

Basic Operations

- **Addition, Subtraction, Multiplication**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Just add elements

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

Just subtract elements

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Multiply each row by each column

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How to Multiply Matrices

A Matrix is an array of numbers:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 3 Columns)

To multiply a matrix by a single number is easy:

2x4=8

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

These are the calculations:

2×4=8	2×0=0
2×1=2	2×-9=-18

We call the number ("2" in this case) a **scalar**, so this is called "scalar multiplication".

Multiplying a Matrix by Another Matrix

But to multiply a matrix **by another matrix** we need to do the "dot product" of rows and columns ... what does that mean? Let us see with an example:

To work out the answer for the **1st row** and **1st column**:

"Dot Product"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

The "Dot Product" is where we **multiply matching members**, then sum up:

$$(1, 2, 3) \bullet (7, 9, 11) = 1\times7 + 2\times9 + 3\times11$$
$$= 58$$

We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.

Want to see another example? Here it is for the 1st row and **2nd column**:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \end{bmatrix}$$

$$(1, 2, 3) \bullet (8, 10, 12) = 1\times8 + 2\times10 + 3\times12$$
$$= 64$$

We can do the same thing for the **2nd row** and **1st column**:

$$(4, 5, 6) \bullet (7, 9, 11) = 4\times7 + 5\times9 + 6\times11$$
$$= 139$$

And for the **2nd row** and **2nd column**:

$$(4, 5, 6) \bullet (8, 10, 12) = 4\times8 + 5\times10 + 6\times12$$
$$= 154$$

And we get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

DONE!

Matrix operations

Matrix multiplication

If **A** is a m_{xr} matrix and **B** is a r_{xn} matrix, then the product **C**=**AB** is a m_{xn} matrix whose entries are obtained as follows. The entry corresponding to row 'i' and column 'j' of **C** is the dot product of the vectors formed by the row 'i' of **A** and column 'j' of **B**

$$\underline{A}_{3x3} = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & 7 \\ 9 & 1 & 5 \end{bmatrix} \qquad \underline{B}_{3x2} = \begin{bmatrix} -1 & 3 \\ -3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\underline{\underline{C}}_{3x2} = AB = \begin{bmatrix} -3 & 5 \\ 10 & -9 \\ -7 & 28 \end{bmatrix} \quad \text{notice} \quad \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} = -3$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Multiply each row by each column

Matrix operations

Multiplication of matrices

Properties

Properties of matrix multiplication:

1. Matrix multiplication is **noncommutative** (order of addition **does** matter)

$$\underline{A}\,\underline{B} \neq \underline{B}\,\underline{A} \text{ in general}$$

- **It may be that the product <u>AB</u> exists but <u>BA</u> does not (e.g. in the previous example <u>C</u>=<u>AB</u> is a 3x2 matrix, but <u>BA</u> does not exist)**
- **Even if the product exists, the products <u>AB</u> and <u>BA</u> are not generally the same**

Matrix operations

Multiplication of matrices

Properties

2. Matrix multiplication is **associative**

$$\underline{A}\left(\underline{B}\,\underline{C}\right) = \left(\underline{A}\,\underline{B}\right)\underline{C}$$

3. Distributive law

$$\underline{A}\left(\underline{B} + \underline{C}\right) = \underline{A}\,\underline{B} + \underline{A}\,\underline{C}$$

$$\left(\underline{B} + \underline{C}\right)\underline{A} = \underline{B}\,\underline{A} + \underline{C}\,\underline{A}$$

4. Multiplication by identity matrix

$$\underline{A\,I} = \underline{A};\ \underline{I\,A} = \underline{A}$$

5. Multiplication by zero matrix $\underline{A}\,\underline{0} = \underline{0}\,;\underline{0}\,\underline{A} = \underline{0}$

6. $\left(\underline{A}\,\underline{B}\right)^T = \underline{B}^T\,\underline{A}^T$

What is a determinant?

The **determinant** of a **square matrix** is a **number** obtained in a specific manner from the matrix.

For a 1x1 matrix:

$$\underline{A} = \begin{bmatrix} a_{11} \end{bmatrix}; \quad \det(\underline{A}) = a_{11}$$

For a 2x2 matrix:

$$\underline{\mathbf{A}} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{bmatrix}; \quad \det(\underline{\mathbf{A}}) = \mathbf{a}_{11}\mathbf{a}_{22} - \mathbf{a}_{12}\mathbf{a}_{21}$$

Product along red arrow minus product along blue arrow

Example 1

Consider the matrix $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

Notice (1) A matrix is an array of numbers

(2) A matrix is enclosed by square brackets

$$\det(A) = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} = 1 \times 7 - 3 \times 5 = -8$$

Notice (1) The determinant of a matrix is a number

(2) The symbol for the determinant of a matrix is a pair of parallel lines

Computation of larger matrices is more difficult

Duplicate column method for 3x3 matrix

For **ONLY a 3x3 matrix** write down the first two columns after the third column

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \mathbf{a}_{13} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \mathbf{a}_{23} \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} \end{bmatrix} \begin{matrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \\ \mathbf{a}_{31} & \mathbf{a}_{32} \end{matrix}$$

Sum of products along red arrow
minus sum of products along blue arrow

$$\det(A) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

This technique works only for 3x3 matrices

Example

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|cc} 2 & 4 & -3 & 2 & 4 \\ 1 & 0 & 4 & 1 & 0 \\ 2 & -1 & 2 & 2 & -1 \end{array}\right]$$

0 -8 8 0 32 3

Sum of red terms = 0 + 32 + 3 = 35

Sum of blue terms = 0 – 8 + 8 = 0

Determinant of matrix A= det(A) = 35 – 0 = 35

Finding determinant using inspection

Special case. If **two rows** or **two columns** are **proportional** (i.e. multiples of each other), then the determinant of the matrix is zero

$$\begin{vmatrix} 2 & 7 & 8 \\ 3 & 2 & 4 \\ -2 & -7 & -8 \end{vmatrix} = 0$$

because rows 1 and 3 are proportional to each other

<u>If the **determinant of a matrix is zero**, it is called a **singular matrix**</u>

What is a cofactor?
Cofactor method

If A is a square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The **minor**, $\mathbf{M_{ij}}$, of entry a_{ij} is the determinant of the submatrix that remains after the i^{th} row and j^{th} column are deleted from A.
The **cofactor** of entry a_{ij} is $C_{ij}=(-1)^{(i+j)}\,\mathbf{M_{ij}}$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21}a_{33} - a_{23}a_{31} \quad C_{12} = -M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

What is a cofactor?

Sign of cofactor

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Find the minor and cofactor of a_{33}

$$A = \begin{bmatrix} 2 & 4 & -3 \\ 1 & 0 & 4 \\ 2 & -1 & 2 \end{bmatrix}$$

Minor

$$M_{33} = \begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} = 2 \times 0 - 4 \times 1 = -4$$

Cofactor

$$C_{33} = (-1)^{(3+3)} M_{33} = M_{33} = -4$$

Cofactor method of obtaining the determinant of a matrix

The determinant of a n x n matrix A can be computed by multiplying **ALL the entries in ANY row (or column)** by their **cofactors** and **adding** the resulting products. That is, for each $1 \le i \le n$ and $1 \le j \le n$

Cofactor expansion along the j^{th} column

$$\det(A) = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj}$$

Cofactor expansion along the i^{th} row

$$\det(A) = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}$$

Example: evaluate det(A) for:

$$A=\begin{bmatrix} 1 & 0 & 2 & -3 \\ 3 & 4 & 0 & 1 \\ -1 & 5 & 2 & -2 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}$$

$$\det(A)=(1)\begin{vmatrix} 4 & 0 & 1 \\ 5 & 2 & -2 \\ 1 & 1 & 3 \end{vmatrix} - (0)\begin{vmatrix} 3 & 0 & 1 \\ -1 & 2 & -2 \\ 0 & 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 4 & 1 \\ -1 & 5 & -2 \\ 0 & 1 & 3 \end{vmatrix}$$

$$- (-3)\begin{vmatrix} 3 & 4 & 0 \\ -1 & 5 & 2 \\ 0 & 1 & 1 \end{vmatrix} = (1)(35)-0+(2)(62)-(-3)(13)=198$$

Example : evaluate

$$\det(A)=\begin{vmatrix} 1 & 5 & -3 \\ 1 & 0 & 2 \\ 3 & -1 & 2 \end{vmatrix}$$

By a cofactor along the third column

$$\det(A)=a_{13}C_{13}+a_{23}C_{23}+a_{33}C_{33}$$

$$\det(A)= -3*\left\{(-1)^4\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix}\right\}+2*\left\{(-1)^5\begin{vmatrix} 1 & 5 \\ 3 & -1 \end{vmatrix}\right\}+2*\left\{(-1)^6\begin{vmatrix} 1 & 5 \\ 1 & 0 \end{vmatrix}\right\}$$

$$= \det(A)= -3(-1-0)+2(-1)^5(-1-15)+2(0-5)=25$$

Outline

Role of FEM simulation in Engineering Design

Role of simulation in design: Boeing 777

Source: Boeing Web site (http://www.boeing.com/companyoffices/gallery/images/commercial/).

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Another success ..in failure: Airbus A380

http://www.airbus.com/en/aircraftfamilies/a380/

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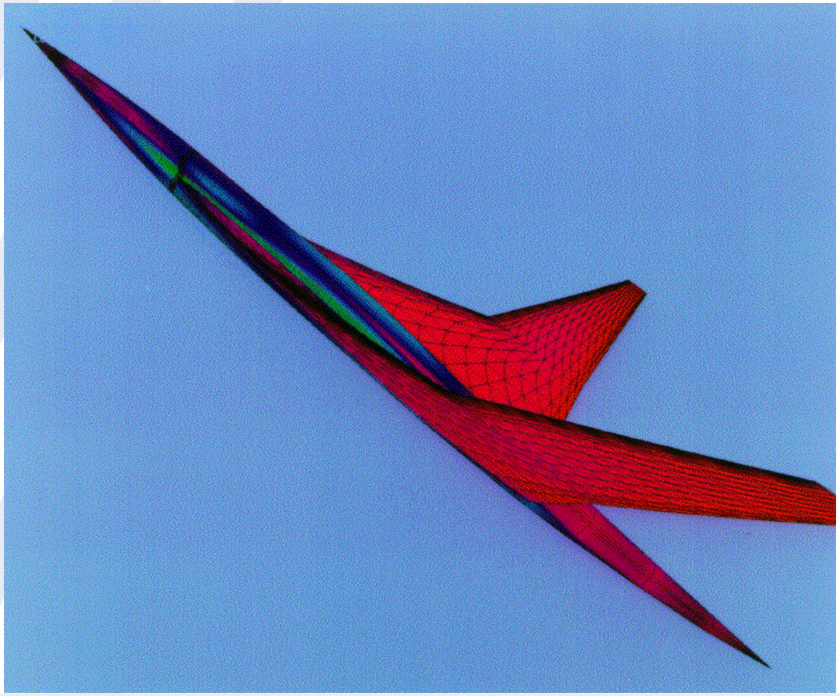
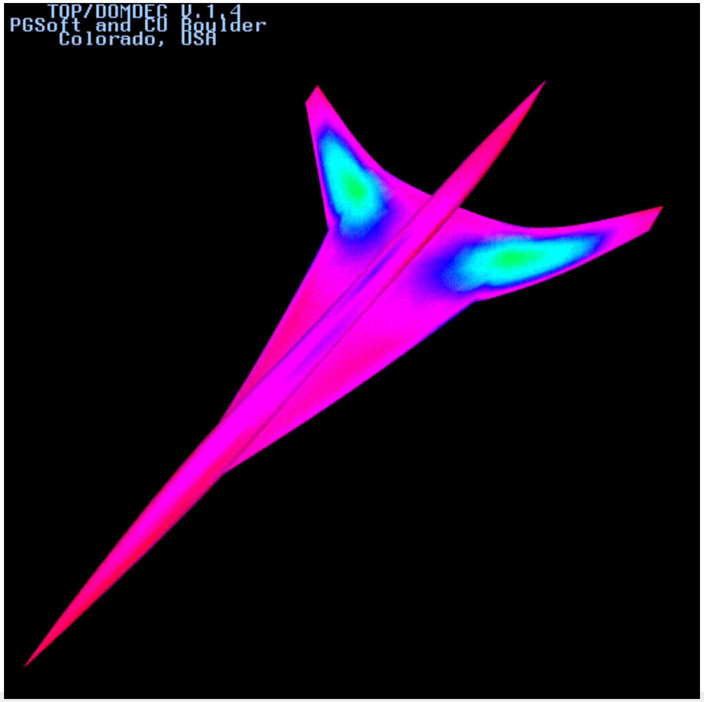
Drag Force Analysis of Aircraft





- Question

  What is the drag force distribution on the aircraft?

- Solve
  - Navier-Stokes Partial Differential Equations.
- Recent Developments
  - Multigrid Methods for Unstructured Grids

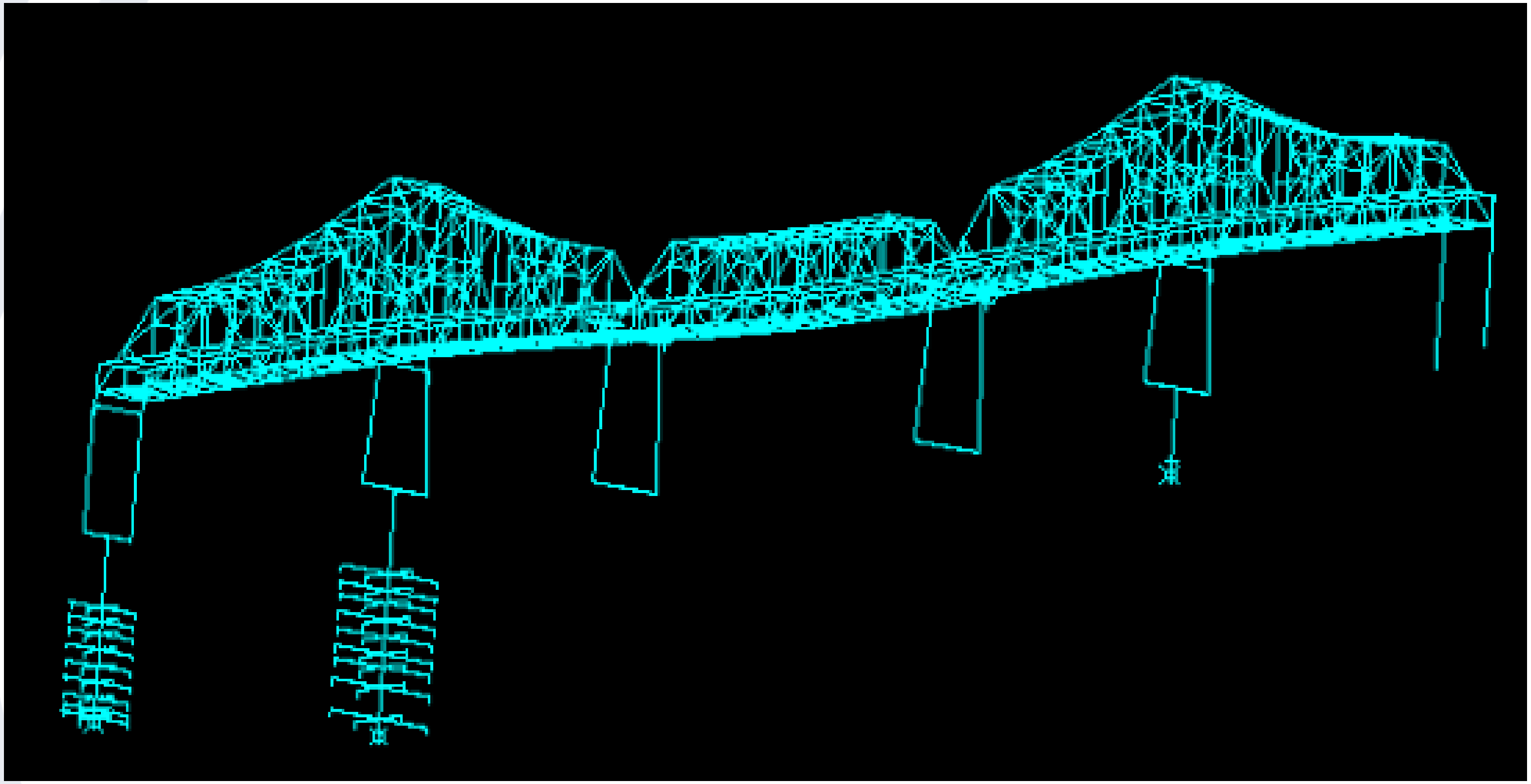
San Francisco Oakland Bay Bridge

Before the 1989 Loma Prieta earthquake

San Francisco Oakland Bay Bridge

After the earthquake

San Francisco Oakland Bay Bridge

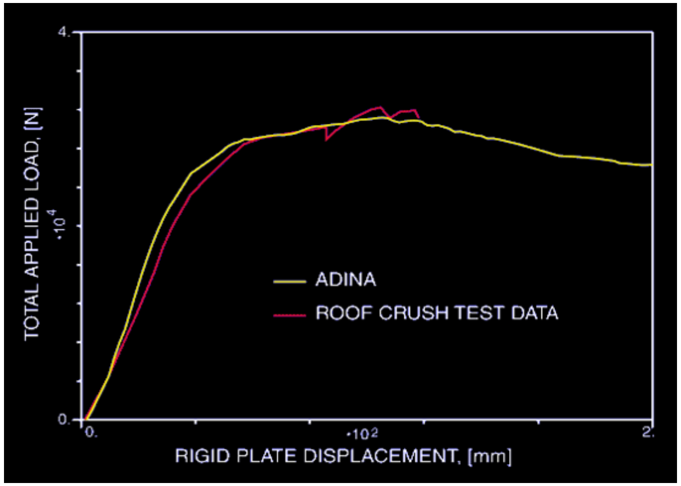


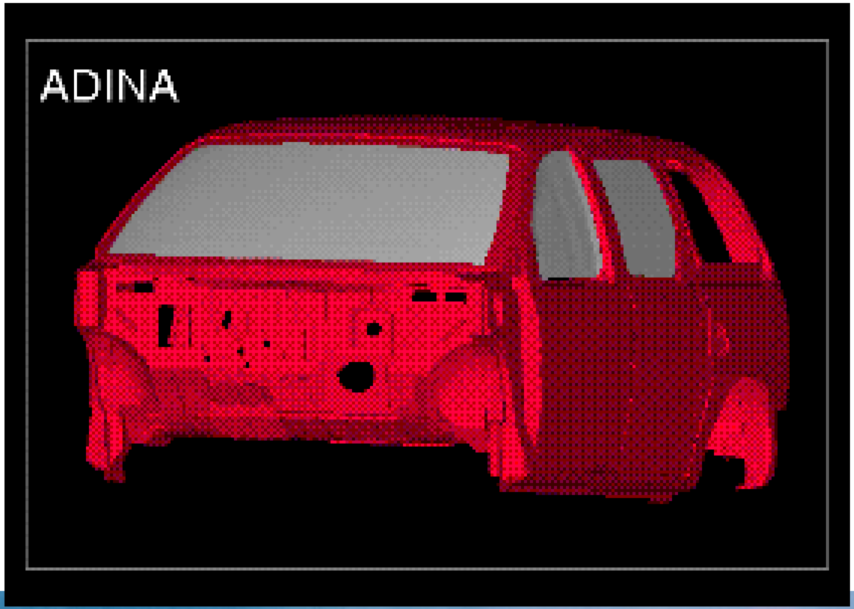
A finite element model to analyze the bridge under seismic loads

Courtesy: ADINA R&D

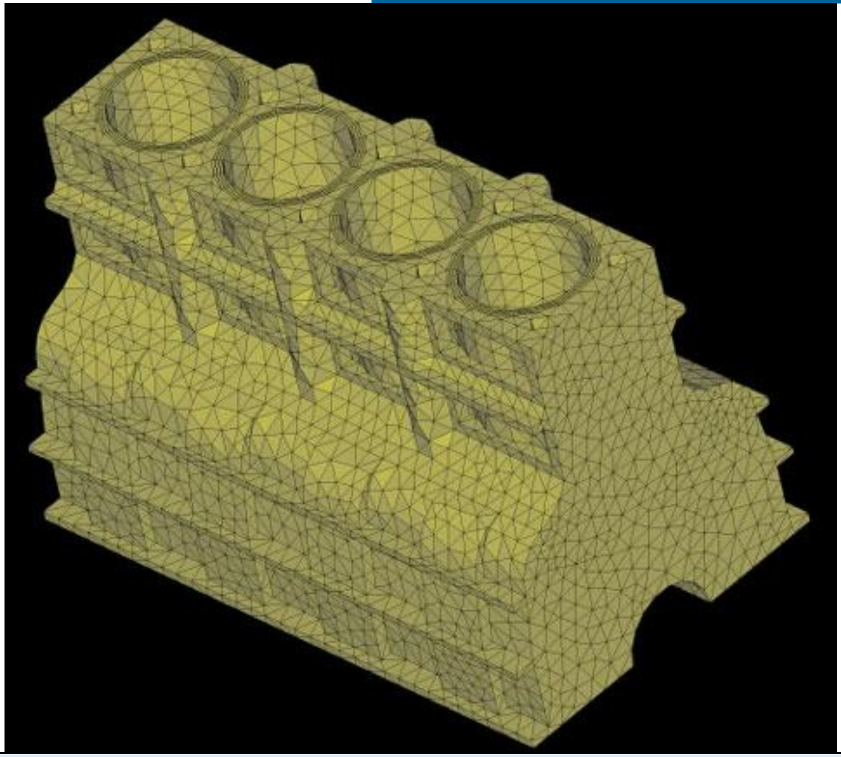
Crush Analysis of Ford Windstar

- **Question**
  - What is the load-deformation relation?
- **Solve**
  - Partial Differential Equations of Continuum Mechanics
- **Recent Developments**
  - Meshless Methods, Iterative methods, Automatic Error Control





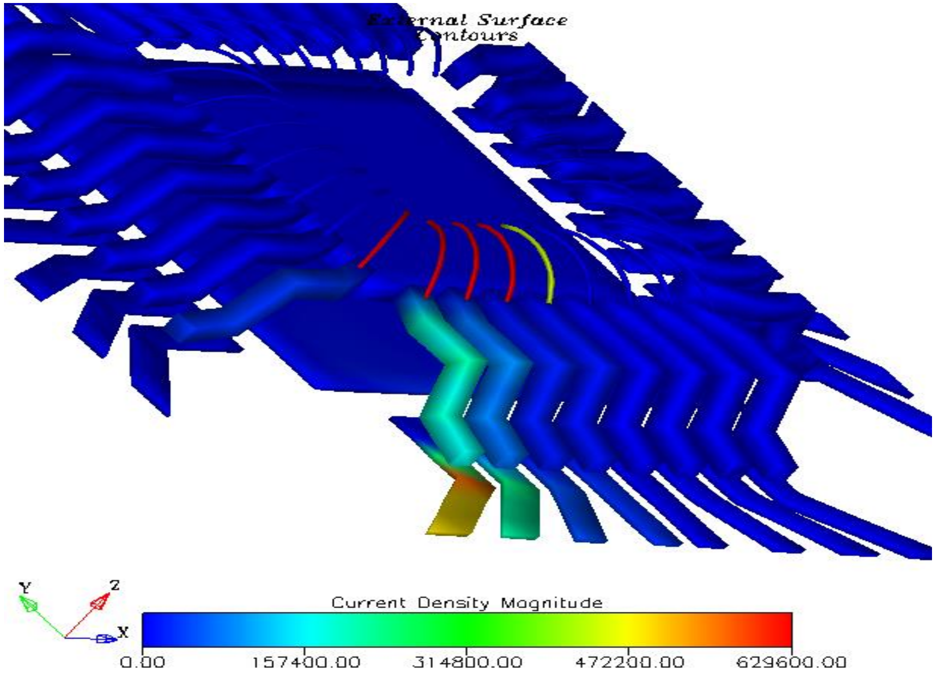
Engine Thermal Analysis



Picture from
http://www.adina.com

- Question
  - What is the temperature distribution in the engine block?
- Solve
  - Poisson Partial Differential Equation.
- Recent Developments
  - Fast Integral Equation Solvers, Monte-Carlo Methods

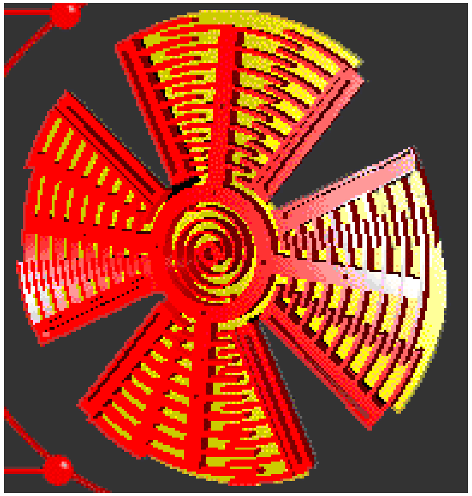
Electromagnetic Analysis of Packages



Thanks to Coventor http://www.coventor.com

- Solve
  - Maxwell's Partial Differential Equations
- Recent Developments
  - Fast Solvers for Integral Formulations

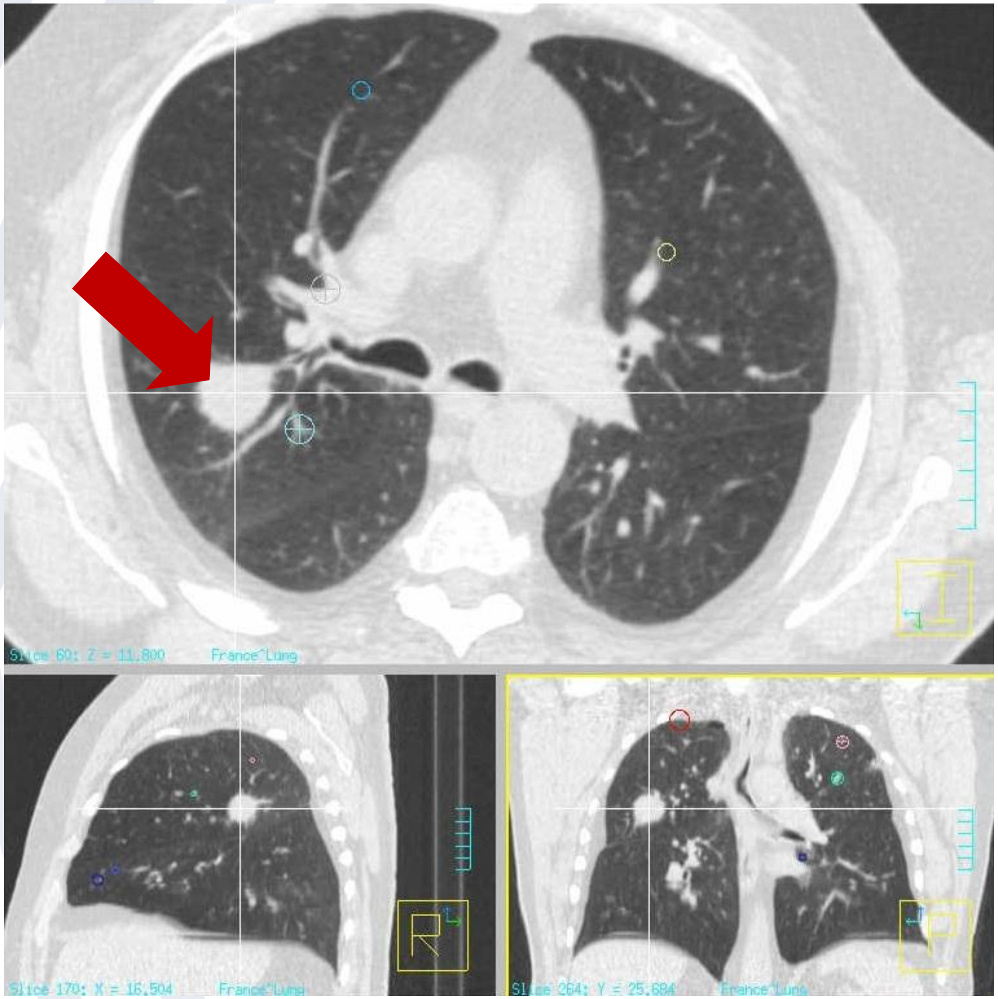
Micromachine Device Performance Analysis

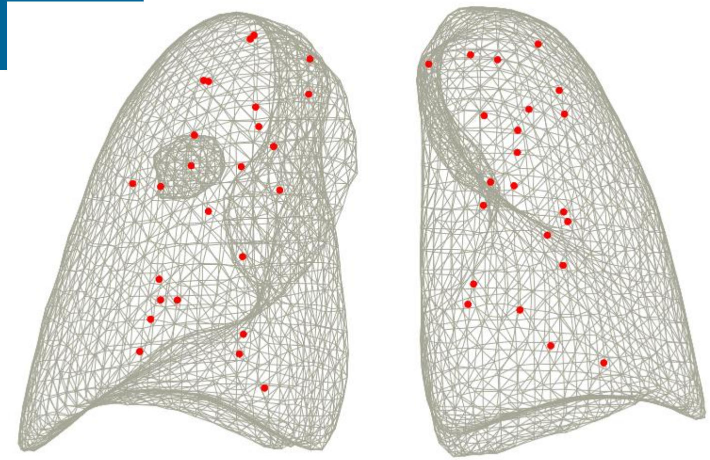


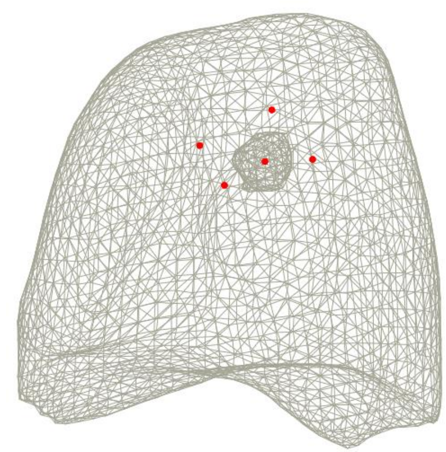
From www.memscap.com

- Equations
  - Elastomechanics, Electrostatics, Stokes Flow.
- Recent Developments
  - Fast Integral Equation Solvers, Matrix-Implicit Multi-level Newton Methods for coupled domain problems.

Radiation Therapy of Lung Cancer







http://www.simulia.com/academics/research_lung.html

Engineering design

General scenario..

Physical Problem

Question regarding the problem
...how large are the deformations?
...how much is the heat transfer?

Mathematical model

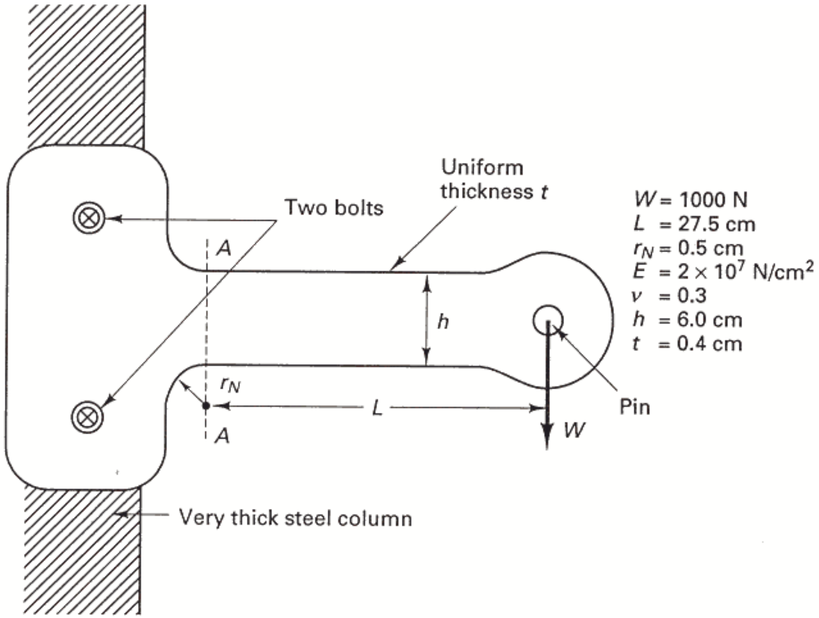
Governed by differential equations

Assumptions regarding
Geometry
Kinematics
Material law
Loading
Boundary conditions
Etc.

Example: A bracket

Physical problem

Engineering design



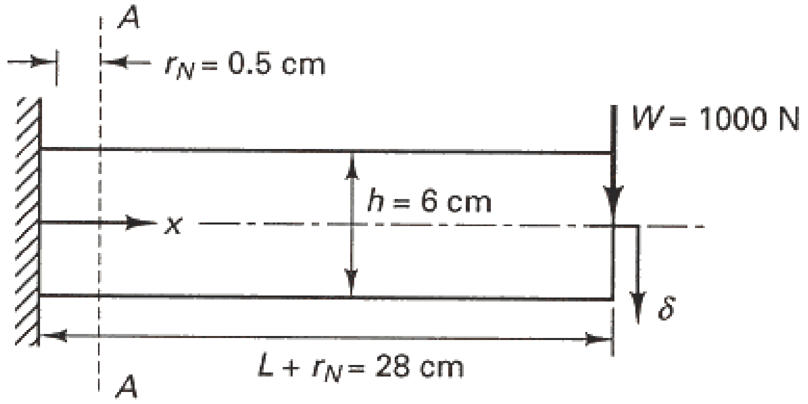
Questions:

1. What is the bending moment at section AA?
2. What is the deflection at the pin?

Finite Element Procedures, K J Bathe

Engineering design

Example: A bracket



Mathematical model 1: beam

Moment at section AA

$$M = WL$$
$$= 27{,}500 \text{ N cm}$$

Deflection at load

$$\delta_{\text{at load W}} = \frac{1}{3}\frac{W(L+r_N)^3}{EI} + \frac{W(L+r_N)}{\frac{5}{6}AG}$$

$$= 0.053 \text{ cm}$$

How **reliable** is this model?

How **effective** is this model?

Example: A bracket

Engineering design

Mathematical model 2: plane stress

FEM solution to mathematical model 2 (plane stress)

Moment at section AA $M = 27{,}500 \text{ N cm}$

Deflection at load $\delta_{\text{at load } W} = 0.064 \text{ cm}$

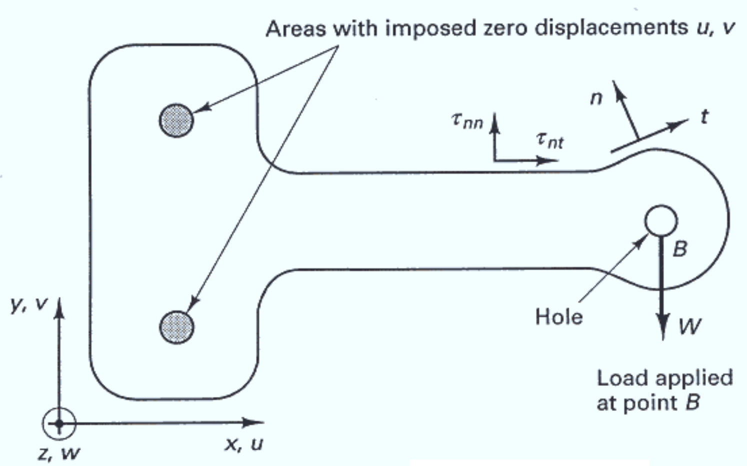
Conclusion: With respect to the questions we posed, the beam model is ***reliable*** if the required bending moment is to be predicted within 1% and the deflection is to be predicted within 20%. The beam model is also highly ***effective*** since it can be solved easily (by hand).

What if we asked: what is the maximum stress in the bracket?

would the beam model be of any use?

Engineering design

Example: A bracket



Equilibrium equations

$$\left.\begin{aligned} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} &= 0 \end{aligned}\right\} \text{ in domain of bracket}$$

$\tau_{nn} = 0$, $\tau_{nt} = 0$ on surfaces except at point B and at imposed zero displacements

Stress-strain relation

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix}$$

E = Young's modulus, ν = Poisson's ratio

Strain-displacement relations

$$\epsilon_{xx} = \frac{\partial u}{\partial x}; \qquad \epsilon_{yy} = \frac{\partial v}{\partial y}; \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Mathematical model 2: plane stress

Difficult to solve by hand!

Engineering design

..General scenario..

Physical Problem

↓

Mathematical model

Governed by differential equations

↓

Numerical model

e.g., finite element model

..General scenario..

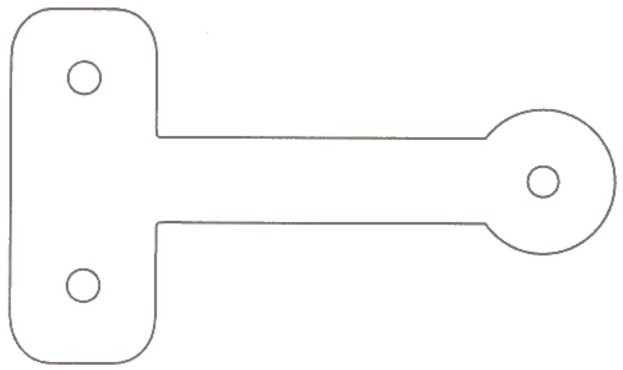
Engineering design

Finite element analysis

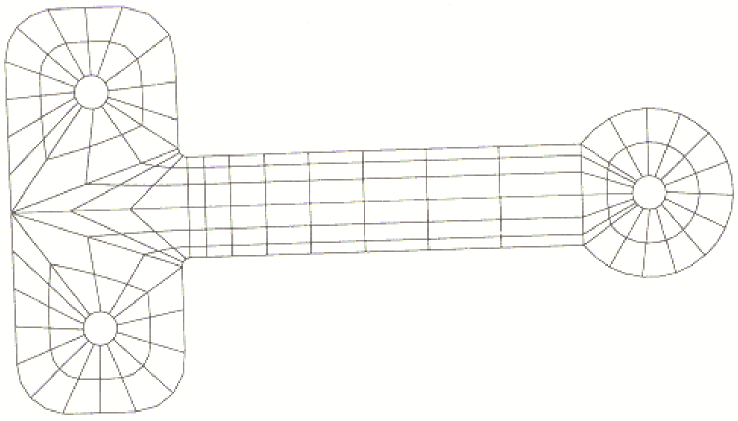
PREPROCESSING

1. Create a geometric model

2. Develop the finite element model



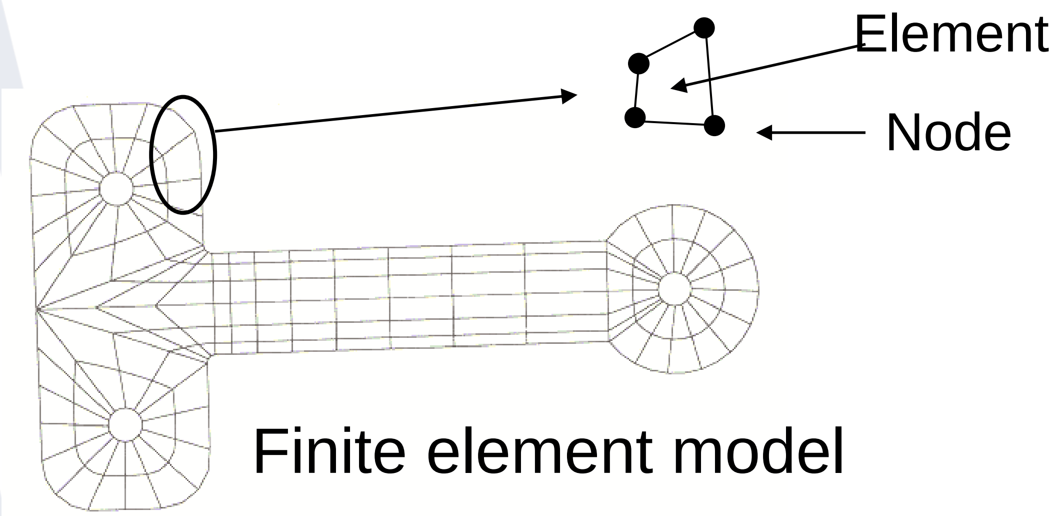
Solid model



Finite element model

FEM analysis scheme

Step 1: Divide the problem domain into non overlapping regions ("**elements**") connected to each other through special points ("**nodes**")



Finite element model

FEM analysis scheme

Step 2: Describe the behavior of each element

Step 3: Describe the behavior of the entire body by putting together the behavior of each of the elements (this is a process known as "**assembly**")

..General scenario..

Engineering design

Finite element analysis

POSTPROCESSING

Compute moment at section AA

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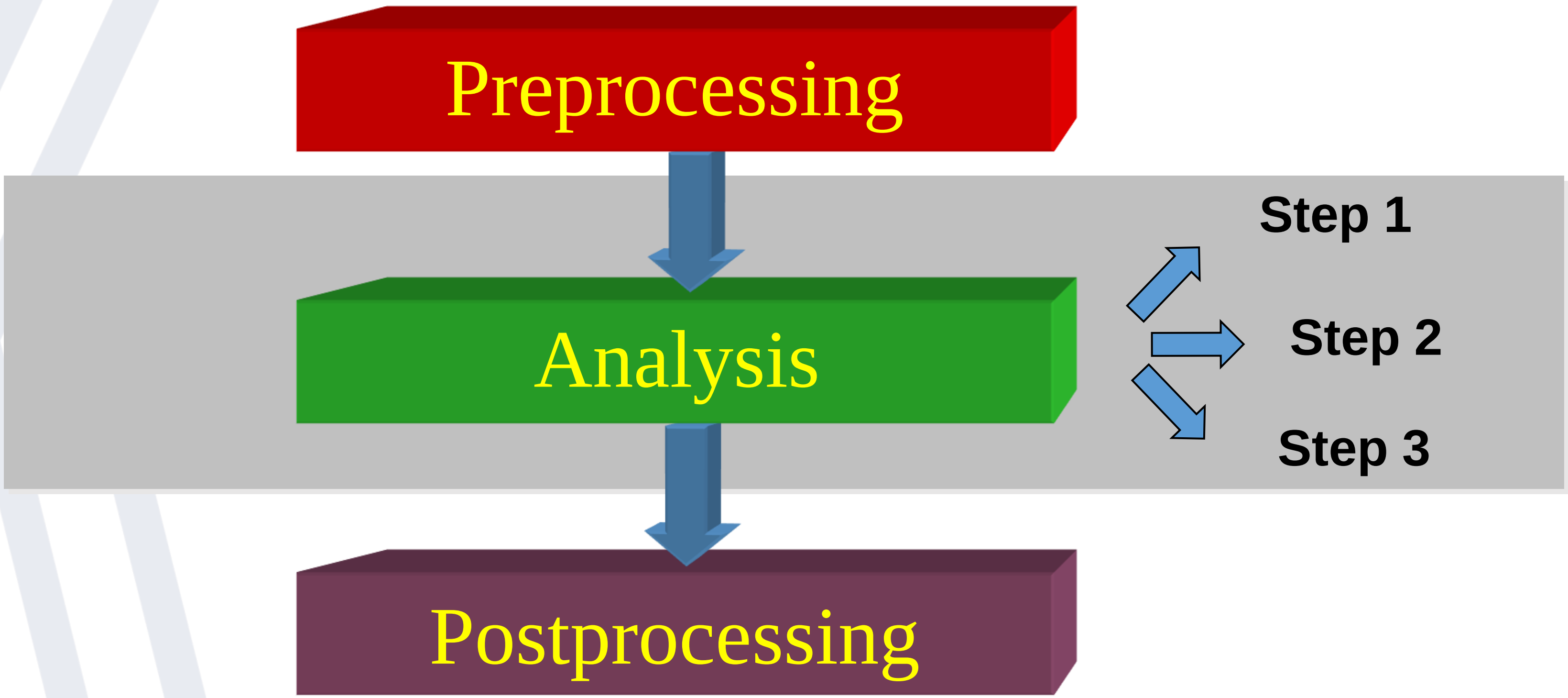
Engineering design

..General scenario..

Finite element analysis



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Engineering design

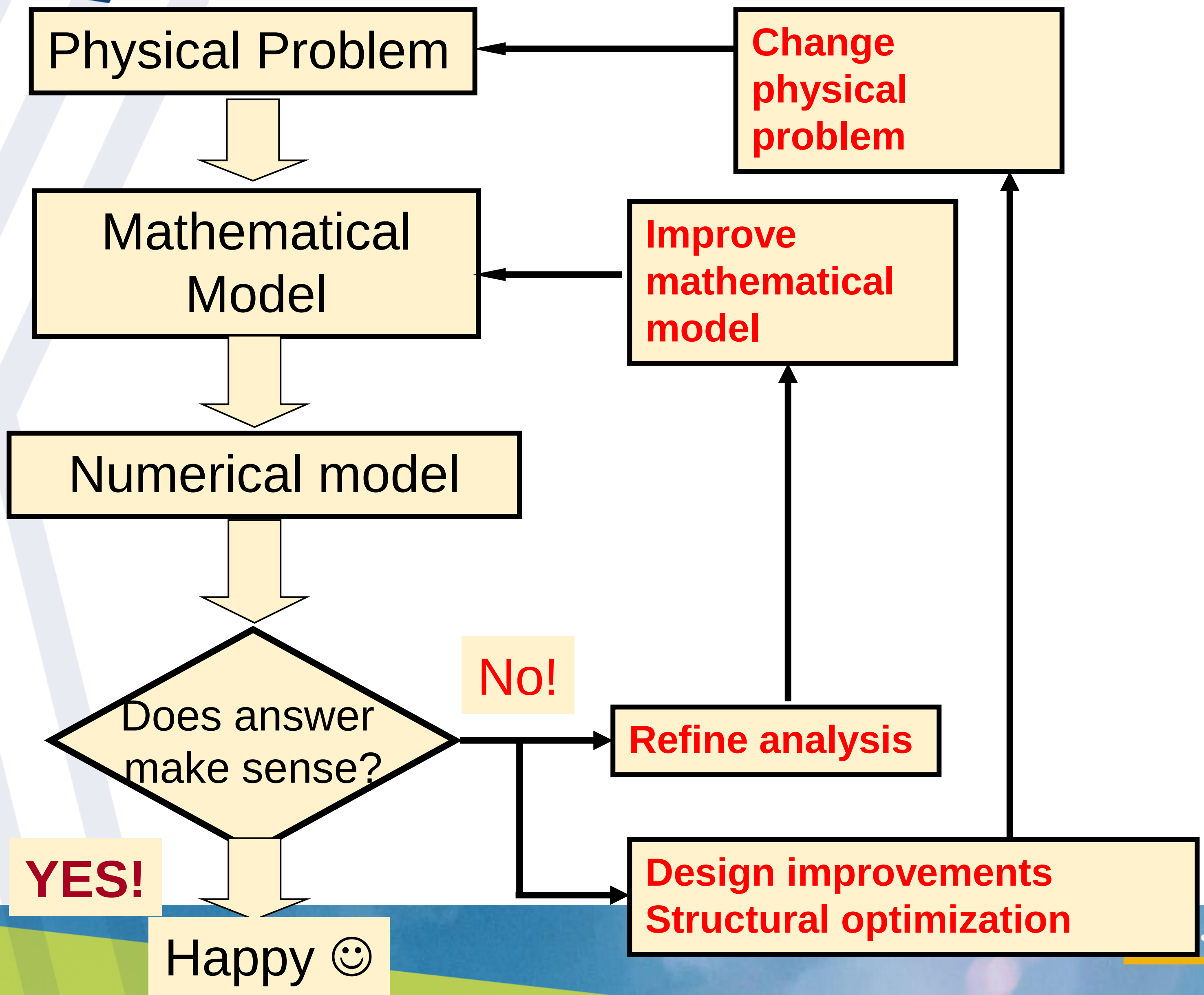
Example: A bracket

Summary

1. The **selection** of the mathematical model depends on the response to be predicted.
2. The **most effective** mathematical model is the one that delivers the answers to the questions in reliable manner with least effort.
3. **The numerical solution is only as accurate as the mathematical model.**

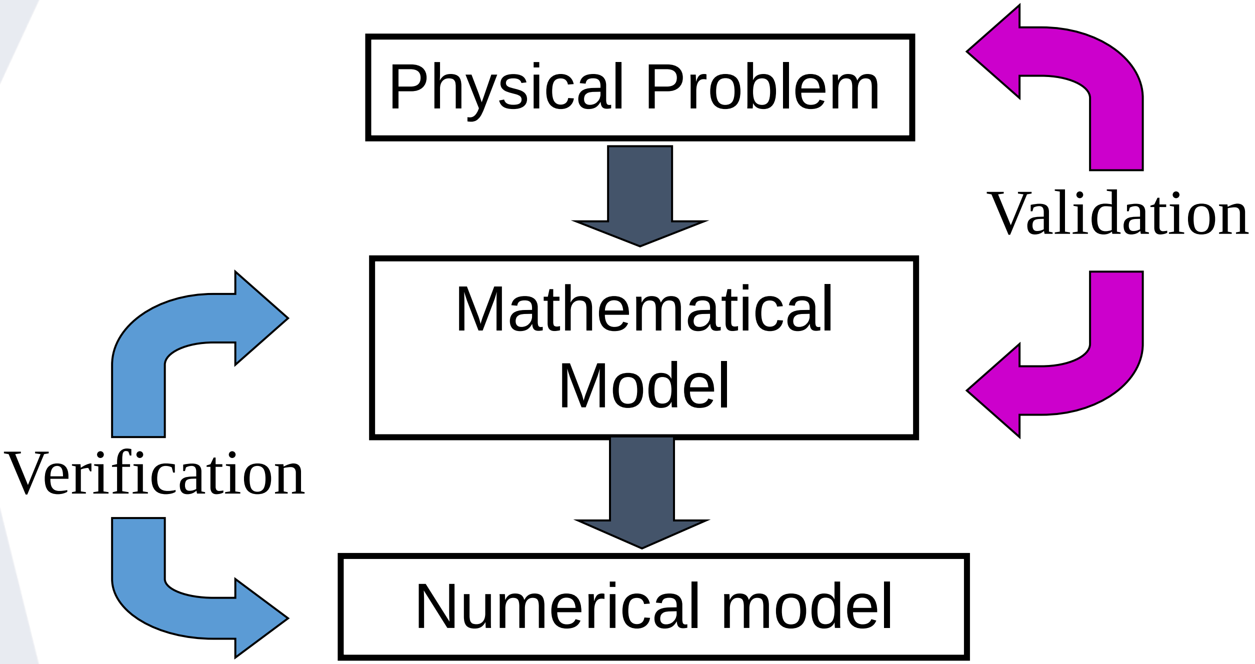
Modeling a physical problem

...General scenario



Modeling a physical problem

Verification and validation



Critical assessment of the FEM

Reliability:

For a well-posed mathematical problem, the numerical technique should ***always***, for a reasonable discretization, give a reasonable solution which ***must*** converge to the accurate solution as the discretization is refined.

e.g., use of reduced integration in FEM results in an unreliable analysis procedure.

Robustness:

The performance of the numerical method should not be excessively sensitive to the material data, the boundary conditions, and the loading conditions used.

e.g., displacement-based formulation for incompressible problems in elasticity

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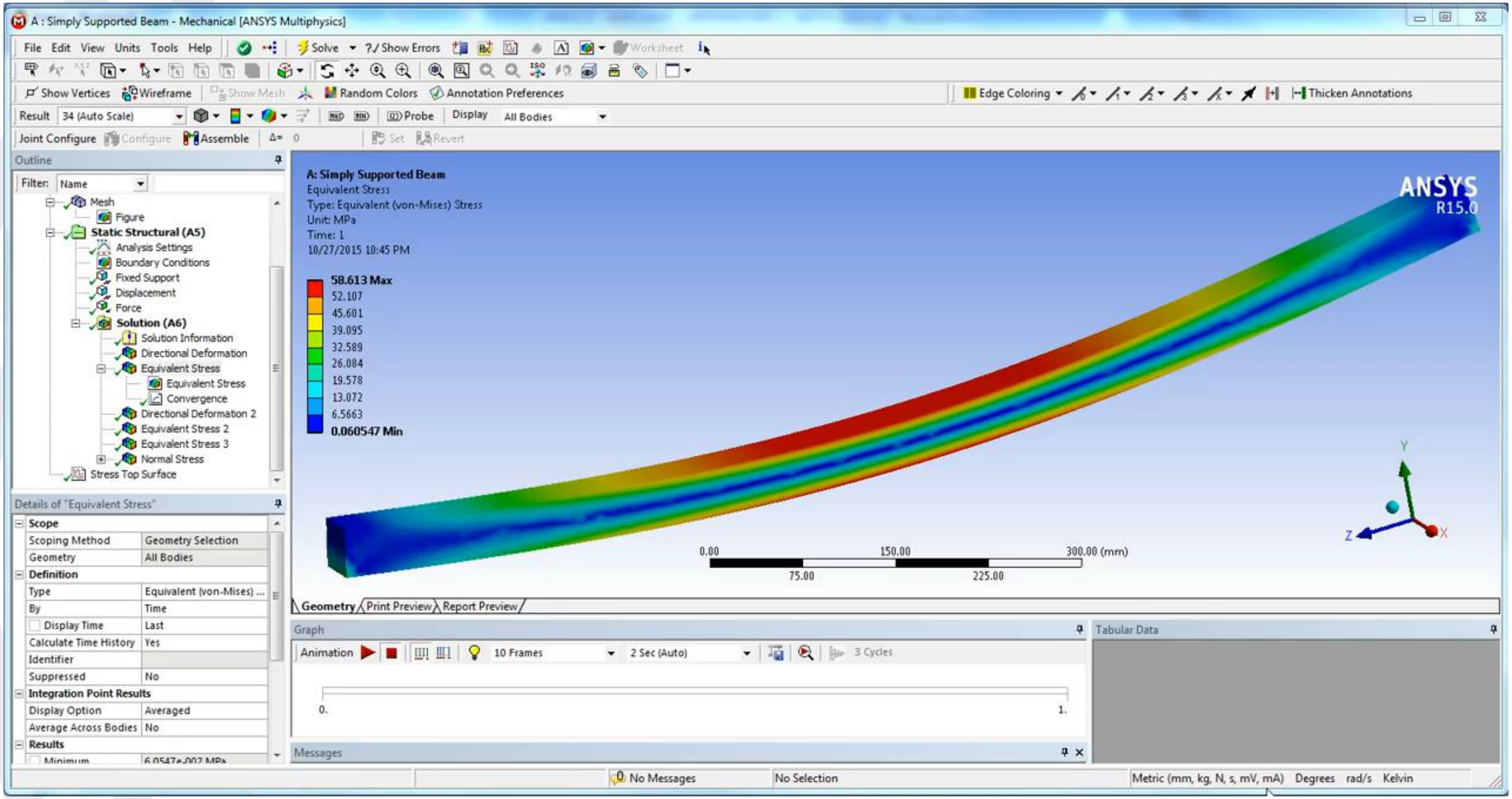
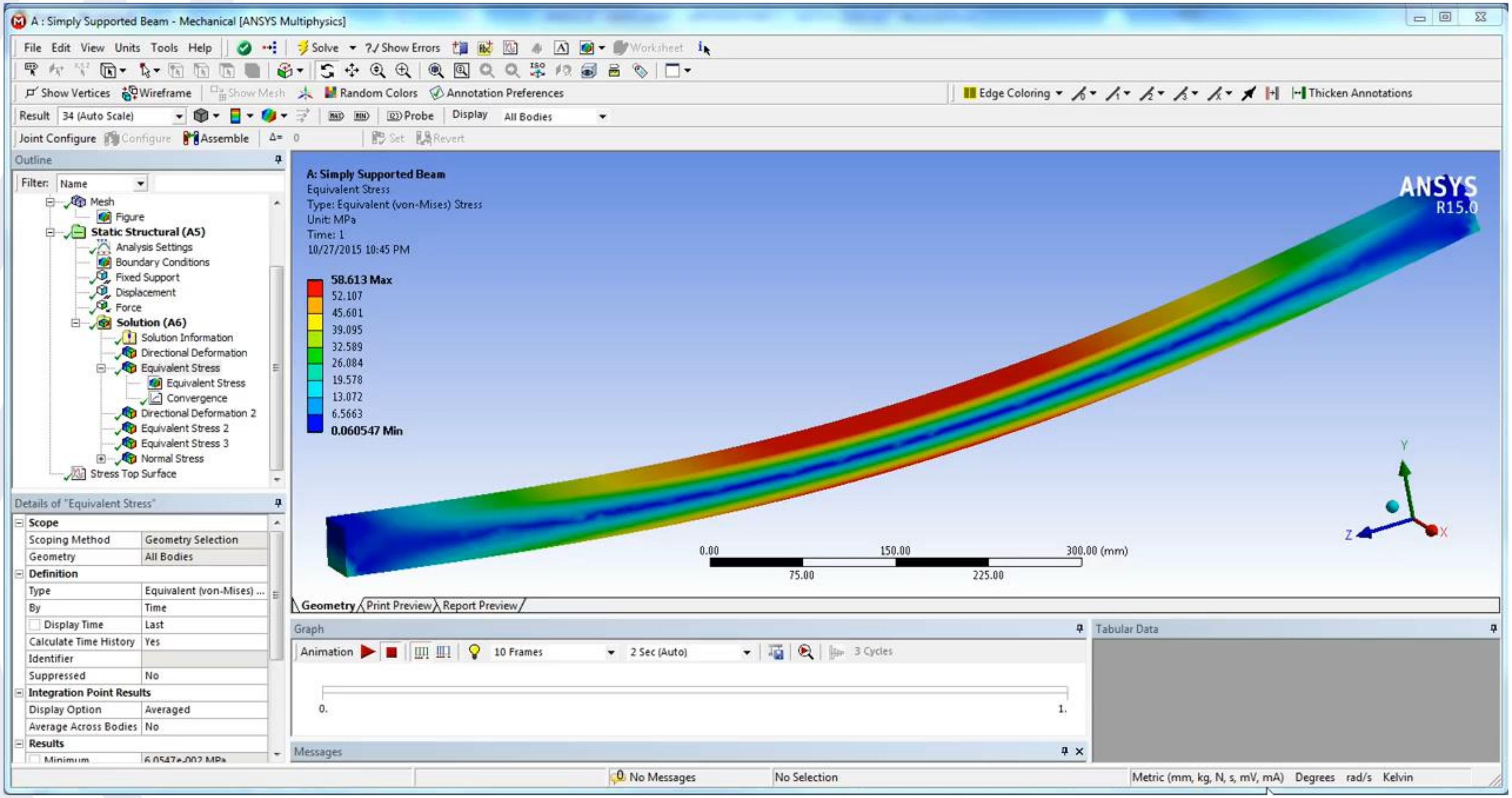
ANSYS Workbench Tutorial - Simply Supported Beam - PART 1

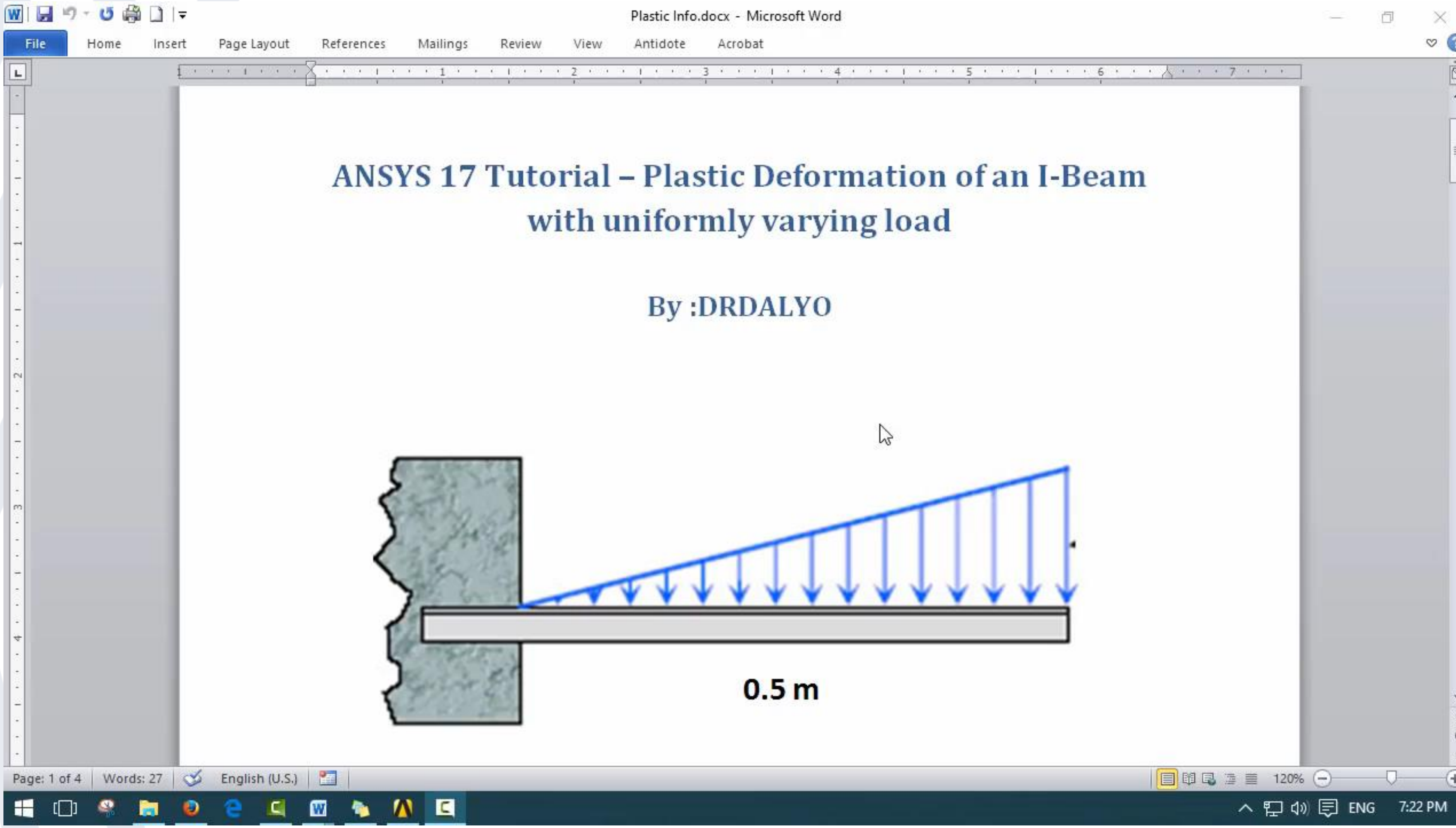


https://www.youtube.com/watch?v=XECmd-BPKa8

NSYS Workbench Tutorial - Simply Supported Beam - Center Load - PART 2



https://www.youtube.com/watch?v=XECmd-BPKa8



https://www.youtube.com/watch?v=XECmd-BPKa8

Tutorial No. 1

ANSYS Workbench Tutorial - Simply Supported Beam - PART 1

https://www.youtube.com/watch?v=CTi1ru-pfi0

Tutorial No. 2

ANSYS 2020 Workbench Tutorial

https://www.youtube.com/watch?v=f3N0yM_Wt7w&list=PLICzjIuc4UqH3hllQc1kEF19YoBfAkEBQ

HOW TO DOWNLOAD & INSTALL FREE ANSYS SOFTWARE?

The links below can show you how to install the student version of ANSYS workbench

HOW TO DOWNLOAD & INSTALL FREE ANSYS SOFTWARE? (youtube.com)

Ansys 2023 R2 student version

Installing Ansys 2023 R1 student version (youtube.com)