Signals and Circuits

ENGR 35500

Inductors

Chapter 5: 5-3(inductors) pp. 224-230;

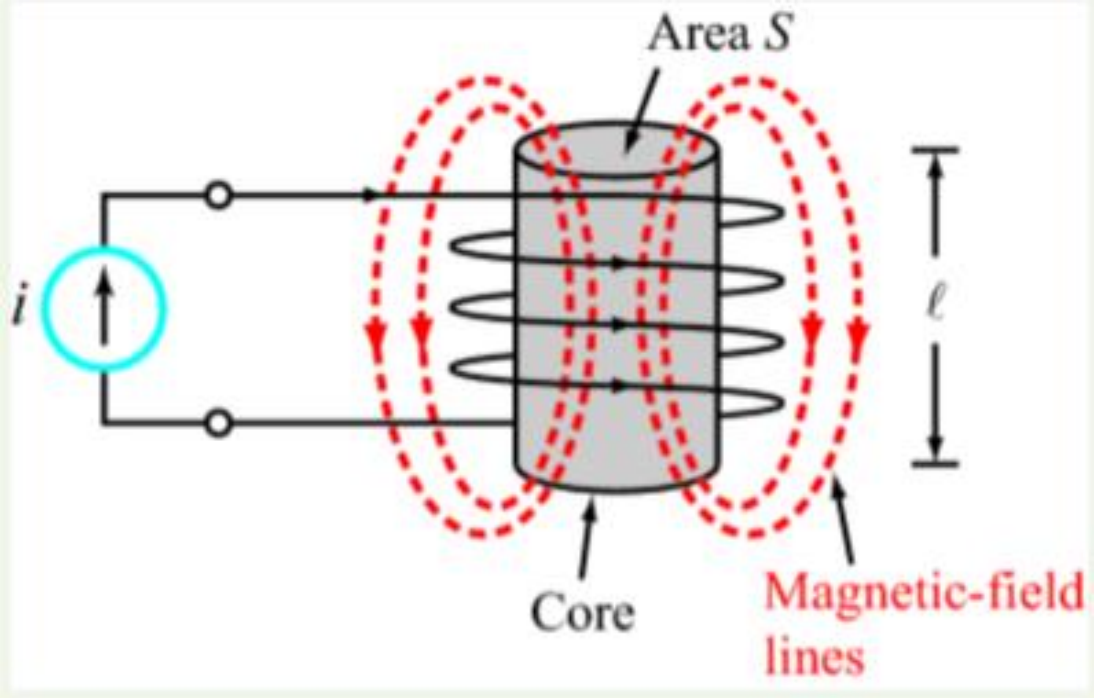
Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

KENT STATE UNIVERSITY

Inductor

An inductor is a passive element.

An inductor is an electrical device composed of a coil of resistance-less wire wound around a supporting core whose material may be magnetic or non-magnetic. (e. g. solenoid)



It can store energy/magnetic-flux linkage and give it back at a later time.

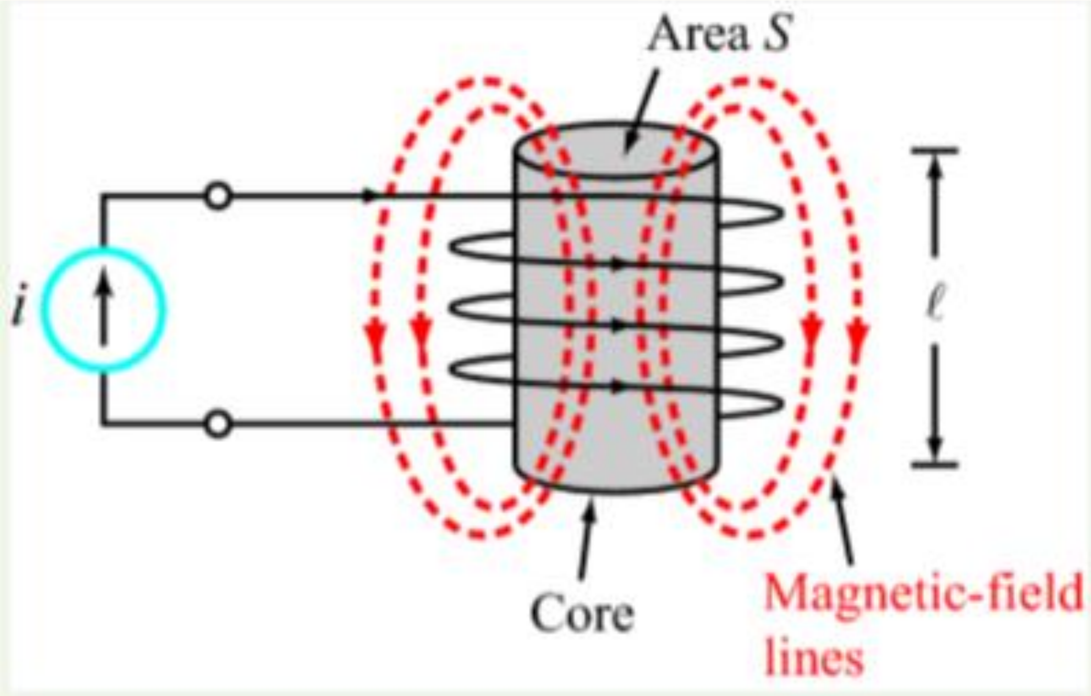
Inductors

Application

- Magnetic characteristics
- RF chokes
- Tuned circuit

Inductors

How to form magnetic-flux linkage ʌ



Henry, after Joseph Henry (1797-1878)

$$L = \frac{\Lambda}{i} = \frac{\mu N^2 S}{l} \ (H)$$

L is the inductance.

after Wilhelm Weber (1804-1891)

$$\Lambda = \left(\frac{\mu N^2 S}{l}\right) i \qquad (Wb)$$

S is the cross-sectional area of the core (m^2)

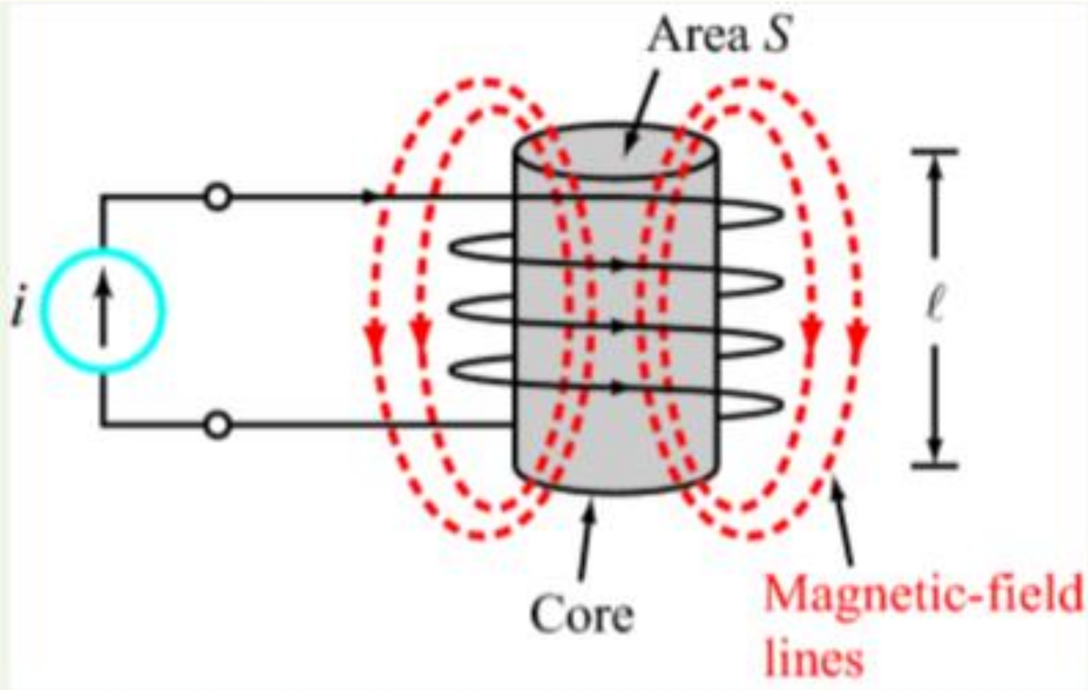
l is the length of the core (m)

i is the current entering the coil (A)

N is the number of turns

μ is the permeability of the core material

Inductors



Henry, after Joseph Henry (1797-1878)

$$L = \frac{\Lambda}{i} = \frac{\mu N^2 S}{l} \ (H)$$

S is the cross-sectional area of the core (m^2)
l is the length of the core (m)
i is the current entering the coil (A)
N is the number of turns
μ is the permeability of the core material

The permeability is a property of a material that relates to how easy it is for the material to get magnetized.
Usually permeabilities are measured with respect to the permeability of vacuum. Its value is:

$$\mu_o = 4\pi x10^{-7} \text{ H/m}$$

The relative permeability is defined as:

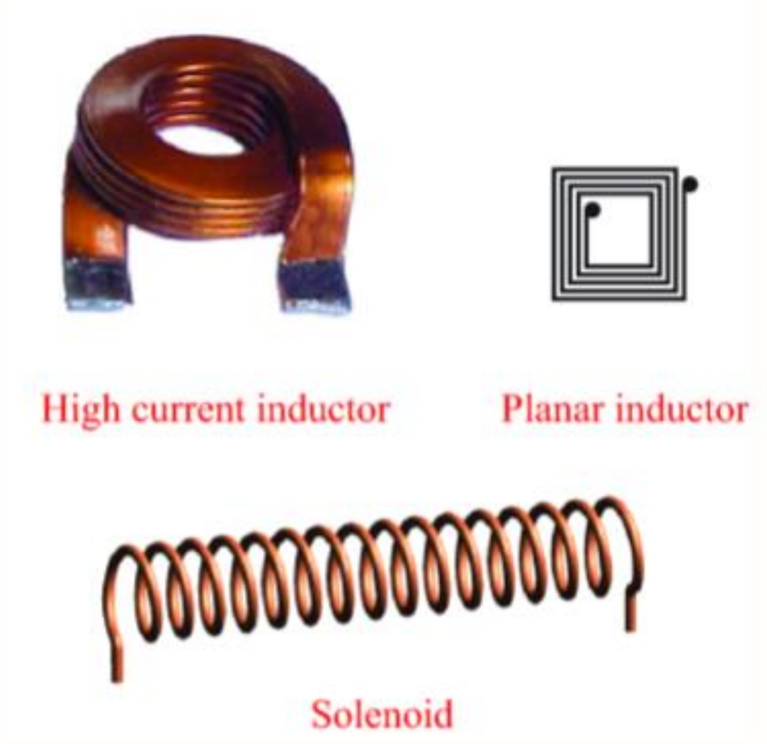
$$\mu_r = \frac{\mu}{\mu_o}$$

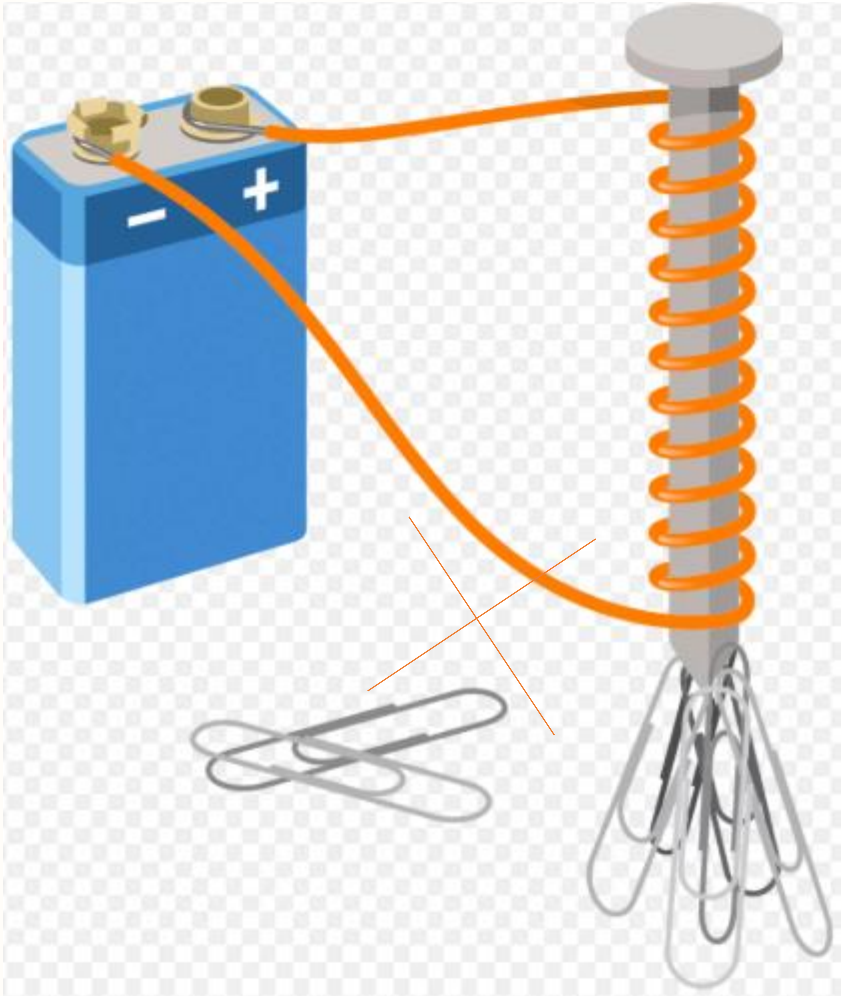
Conductive metal usually has high relative permeability:

Cobalt	250
Nickel	600
Steel	2000

KENT STATE UNIVERSITY

Inductors





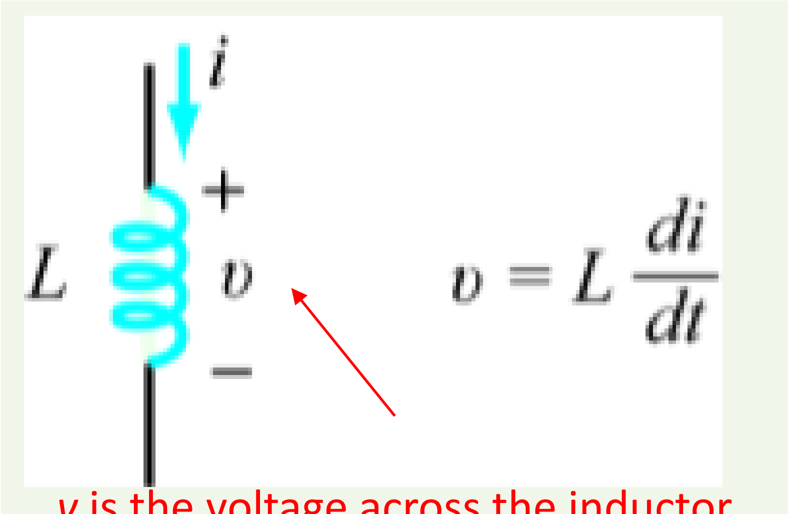
https://www.dkfindout.com/us/science/magnets/solenoids/

Inductors

Electrical properties $$L = \frac{\Lambda}{i}$$

- According to Faradays law, if the magnetic-flux linkage in an inductor (or a circuit) changes with time, it induces a voltage v across the inductor's terminal.

$$v = \frac{d\Lambda}{dt} = \frac{diL}{dt} = L\frac{di}{dt}$$



v is the voltage across the inductor

The direction of *i* is entering the positive terminal of the inductor.

- Under a stable DC condition, an inductor acts like a short circuit.

For stable DC $\begin{cases} \frac{di}{dt} = 0 \\ v = L\frac{di}{dt} = 0 \end{cases}$

- Get the current

$$\int_{t_0}^{t} \frac{di}{dt'} dt' = \frac{1}{L}\int_{t_0}^{t} v\, dt' \qquad i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\, dt'$$

- To get power $p(t) = vi = Li\frac{di}{dt}$ Sign of the power?

- To get energy change

$$w(t) = \int_{t_0}^{t} p\, dt' = L\int_{t_0}^{t} (i\frac{di}{dt'})\, dt' = L\int_{t_0}^{t} [\frac{d}{dt'}\left(\frac{1}{2}i^2\right)]\, dt' = \frac{1}{2}L(i(t))^2 - \frac{1}{2}L(i(t_0))^2$$

Sign of the energy?

- To get the stored energy

$E(t)=\frac{1}{2}L(i(t))^2$

KENT STATE UNIVERSITY

inductors

- E. G.

Given $$i_L(t) = \begin{cases} 0, & t < 0 \\ 20t, & 0 < t < 1 \\ 20, & t > 1 \end{cases} \quad \text{A}$$

through 0.1H inductor, Find $V_L(t)$, $P_L(t)$, and $W_L(t)$.

Note: Under a stable DC condition, an inductor acts like a short circuit.

inductors

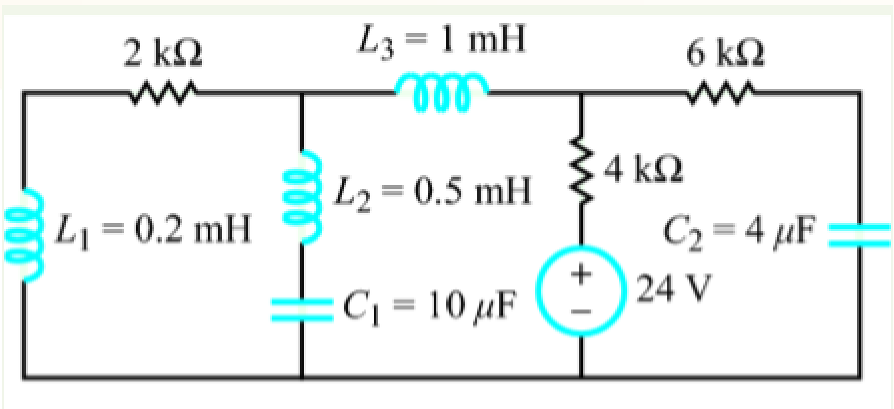
- E. G.

For stable DC $\begin{cases} \dfrac{di}{dt} = 0 \\ v = L\dfrac{di}{dt} = 0 \end{cases}$

To get energy change

$$w(t) = = \frac{1}{2}Li(t)^2 - \frac{1}{2}Li(t_0)^2$$

Determine the currents in the circuit. Assume that the circuit has been in its present condition for a long time.



Original circuit