Chapter 12

BERNOULLI AND ENERGY EQUATIONS

Objectives

- **Understand the use and limitations of the Bernoulli equation and apply it to solve a variety of fluid flow problems.**

- **Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.**

Bernoulli's Principle

"In fluid dynamics, Bernoulli's principle states that for an inviscid flow, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy." Devices can be made based on this principle to create low pressure areas, to draw a fluid to a chosen area. Pilot tubes are used on aircraft to determine the flight speed. The equation is also used to determine the flow of water from a draining tank. With many more uses applied in flight, sailing, an yet more to be discovered!

Energy per unit volume before = Energy per unit volume after

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

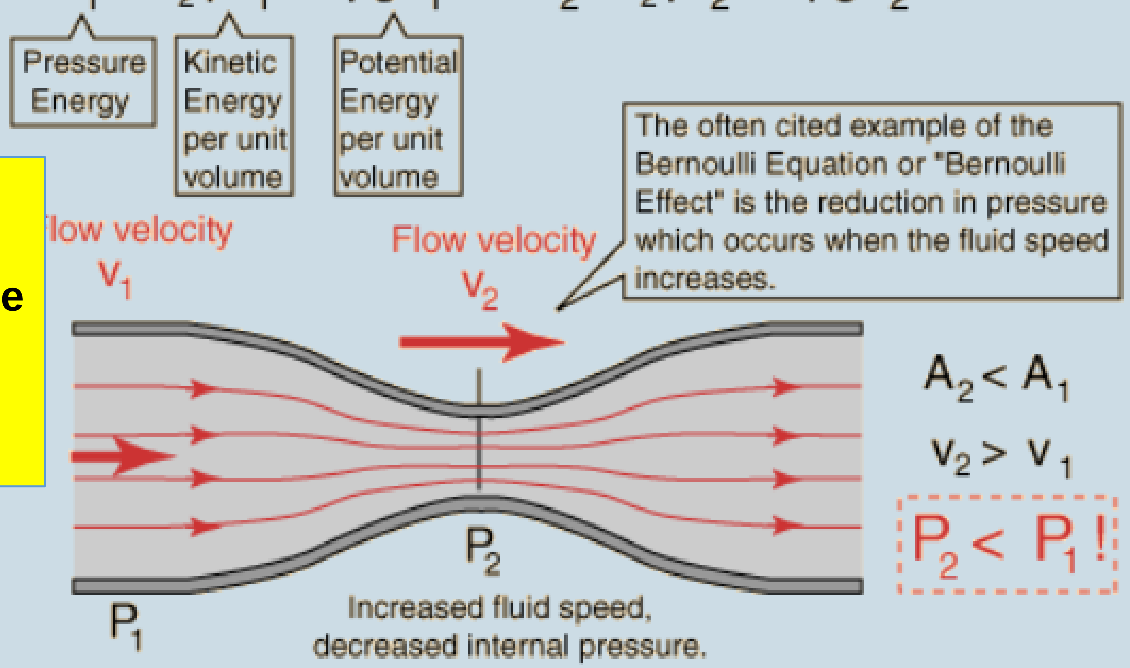


image from http://hyperphysics.phy-astr.gsu.edu/hbase/pber.html

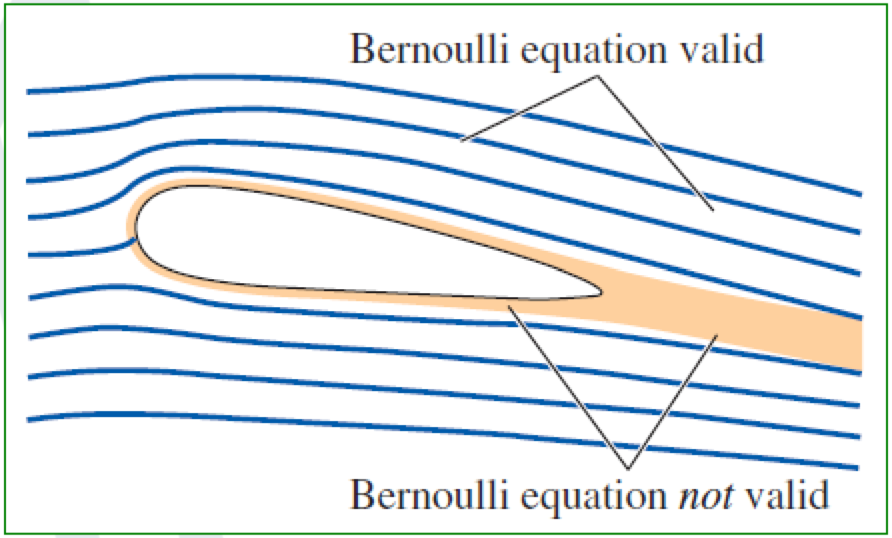
Increase in fluid speed
Will occur simultaneously
With a decrease in pressure
or
A decrease in fluid's potential energy

12-1 THE BERNOULLI EQUATION

Bernoulli equation: An approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible.

Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics.

The Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The *Bernoulli equation* is an *approximate* equation that is valid only in *inviscid regions of flow* where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of *boundary layers* and *wakes*.

(Viscosity=0)

Acceleration of a Fluid Particle

In two-dimensional flow, the acceleration can be decomposed into two components:

streamwise acceleration a_s along the streamline and

normal acceleration a_n in the direction normal to the streamline, which is given as $a_n = V^2/R$.

streamwise acceleration is due to a change in speed along a streamline, and normal acceleration is due to a change in direction.

For particles that move along a ***straight path***, $a_n = 0$ since the radius of curvature is infinity and thus there is no change in direction. The Bernoulli equation results from a force balance along a streamline.

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \qquad \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

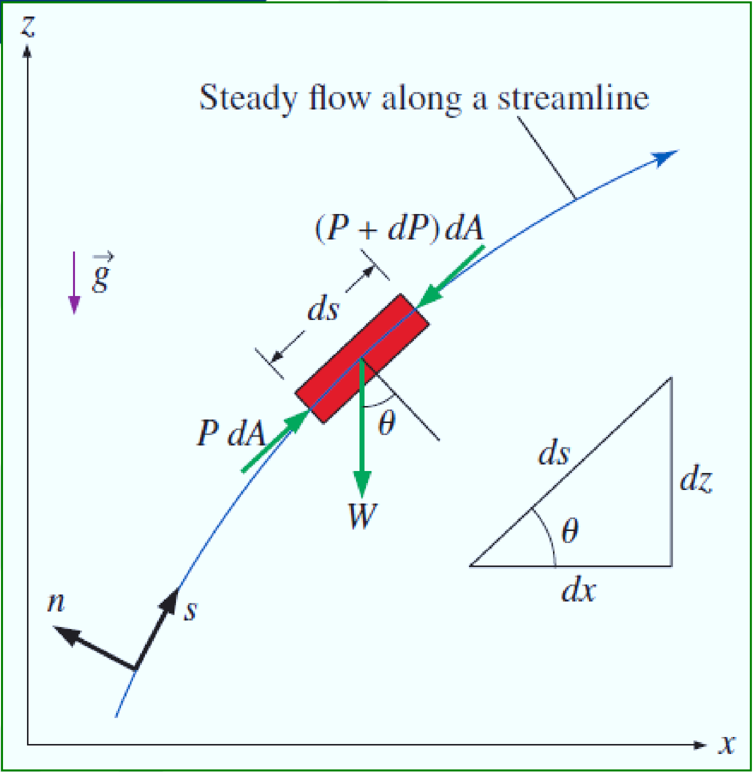
$$\partial V/\partial t = 0 \qquad V = V(s)$$

$$a_s = \frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} = \frac{\partial V}{\partial s}V = V\frac{dV}{ds}$$

$$V = ds/dt$$

Aacceleration in steady flow is due to the change of velocity with position.

Derivation of the Bernoulli Equation



The forces acting on a fluid particle along a streamline.

$$\sum F_s = ma_s \qquad P\,dA - (P+dP)dA = W\sin\theta = mV\frac{dV}{ds}$$

$$m = \rho V = \rho\, dA\, ds \qquad W = mg = \rho g\, dA\, ds$$

$$\sin\theta = dz/ds\,. \qquad -dP\,dA - \rho g\, dA\, ds\frac{dz}{ds} = \rho dA\, ds\, V\frac{dV}{ds}$$

$$-dP - \rho g\, dz = \rho V\, dV \qquad V dV = \frac{1}{2}d(V^2)$$

$$\frac{dP}{\rho} + \frac{1}{2}d(V^2) + g\,dz = 0$$

***Steady flow*:**

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

***Steady, incompressible flow*:** **Bernoulli equation**

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant (along a streamline)}$$

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.

The Bernoulli equation between any two points on the same streamline:

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Derivation of the Bernoulli Equation-1

(Steady flow along a streamline)

General:

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Incompressible flow (ρ = constant):

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

The Bernoulli equation is derived assuming incompressible flow, and thus it should not be used for flows with significant compressibility effects.

Derivation of the Bernoulli Equation-2

$$\overbrace{\frac{P}{\rho}}^{\text{Flow energy}} + \underbrace{\frac{V^2}{2}}_{\text{Kinetic energy}} + \overbrace{gz}^{\text{Potential energy}} = \text{constant}$$

The Bernoulli equation states that the sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow.

- The Bernoulli equation can be viewed as the "*conservation of mechanical energy principle*."
- This is equivalent to the general conservation of energy principle for systems that do not involve any conversion of mechanical energy and thermal energy to each other, and thus the mechanical energy and thermal energy are conserved separately.

Derivation of the Bernoulli Equation-2

$$\overbrace{\frac{P}{\rho}}^{\text{Flow energy}} + \underbrace{\frac{V^2}{2}}_{\text{Kinetic energy}} + \overbrace{gz}^{\text{Potential energy}} = \text{constant}$$

- The Bernoulli equation states that during steady, incompressible flow with negligible friction, the various forms of mechanical energy are converted to each other, but their sum remains constant.

Derivation of the Bernoulli Equation-2

$$\overbrace{\frac{P}{\rho}}^{\text{Flow energy}} + \underbrace{\frac{V^2}{2}}_{\text{Kinetic energy}} + \overbrace{gz}^{\text{Potential energy}} = \text{constant}$$

- There is no dissipation of mechanical energy during such flows since there is no friction that converts mechanical energy to sensible thermal (internal) energy.
- The Bernoulli equation is commonly used in practice since a variety of practical fluid flow problems can be analyzed to reasonable accuracy with it.

Unsteady, Compressible Flow

The Bernoulli equation for *unsteady, compressible flow:*

$$Unsteady, compressible\ flow: \int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{constant}$$

Static, Dynamic, and Stagnation Pressures

The kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. Multiplying the Bernoulli equation by the density gives

$$P + \rho \frac{V^2}{2} + \rho g z = \text{constant} \quad \text{(along a streamline)}$$

P **is the static pressure:** It does not incorporate any dynamic effects; it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

$\rho V^2/2$ **is the dynamic pressure:** It represents the pressure rise when the fluid

- ρgz **is the hydrostatic pressure:** **It is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., fluid weight on pressure.**
- **Be careful of the sign—unlike hydrostatic pressure ρgh which *increases* with fluid depth h, the hydrostatic pressure term ρgz *decreases* with fluid depth.)**

Total pressure: **The sum of the static, dynamic, and hydrostatic pressures. Therefore, the Bernoulli equation states that *the total pressure along a streamline is constant*.**

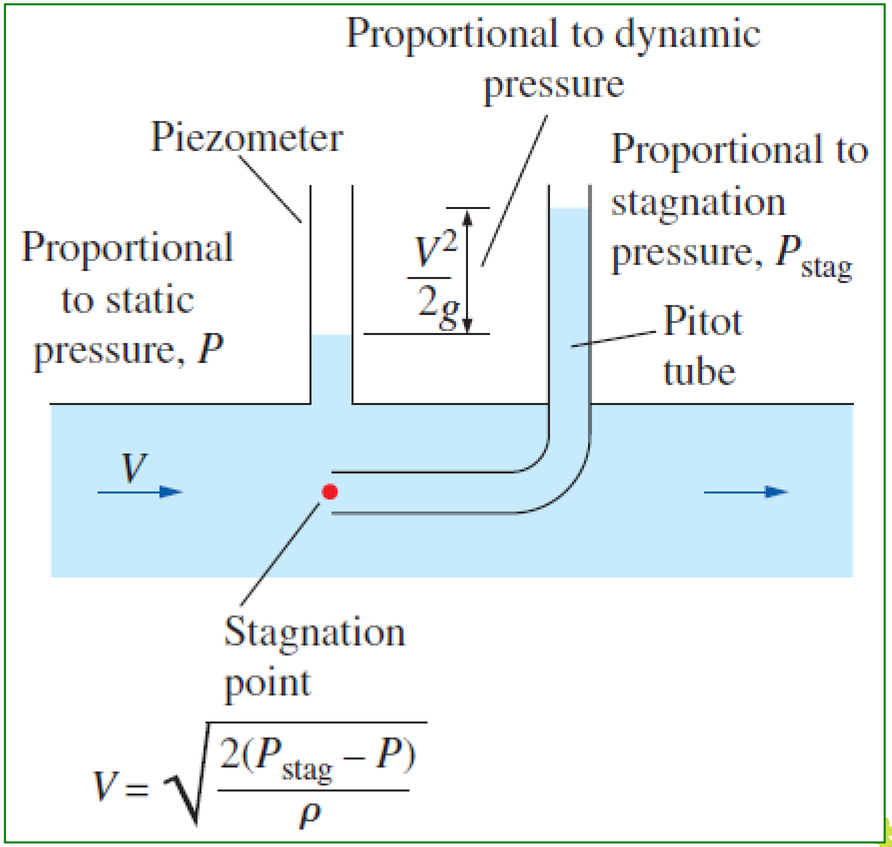
Static, Dynamic, and Stagnation Pressures-1

Stagnation pressure: **The sum of the static and dynamic pressures. It represents the pressure at a point where the fluid is brought to a complete stop isentropically.**

$$P_{stag} = P + \rho \frac{V^2}{2} \qquad \text{(kPa)}$$

$$V = \sqrt{\frac{2(P_{stag} - P)}{\rho}}$$

The static, dynamic, and stagnation pressures measured using piezometer tubes.



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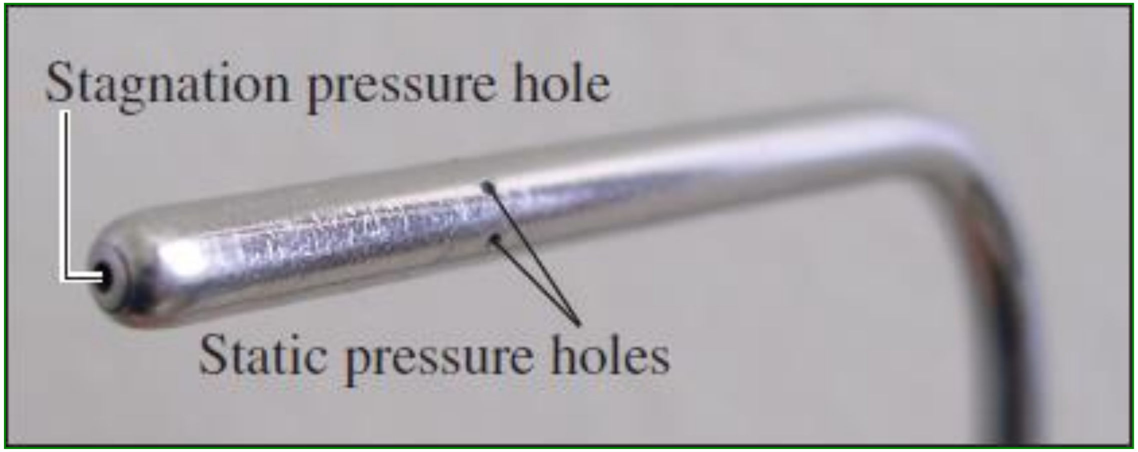
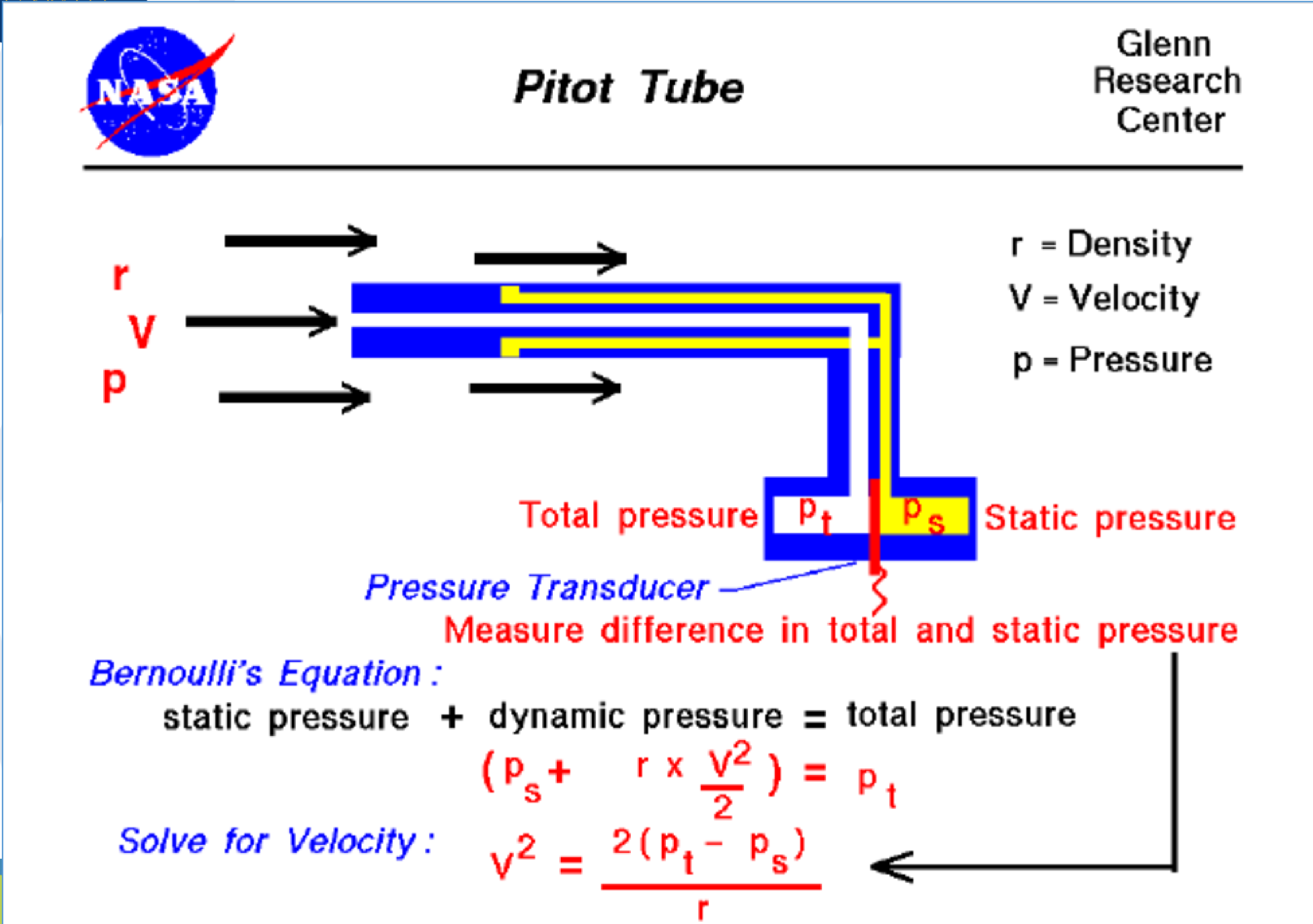
Static, Dynamic, and Stagnation Pressures-1

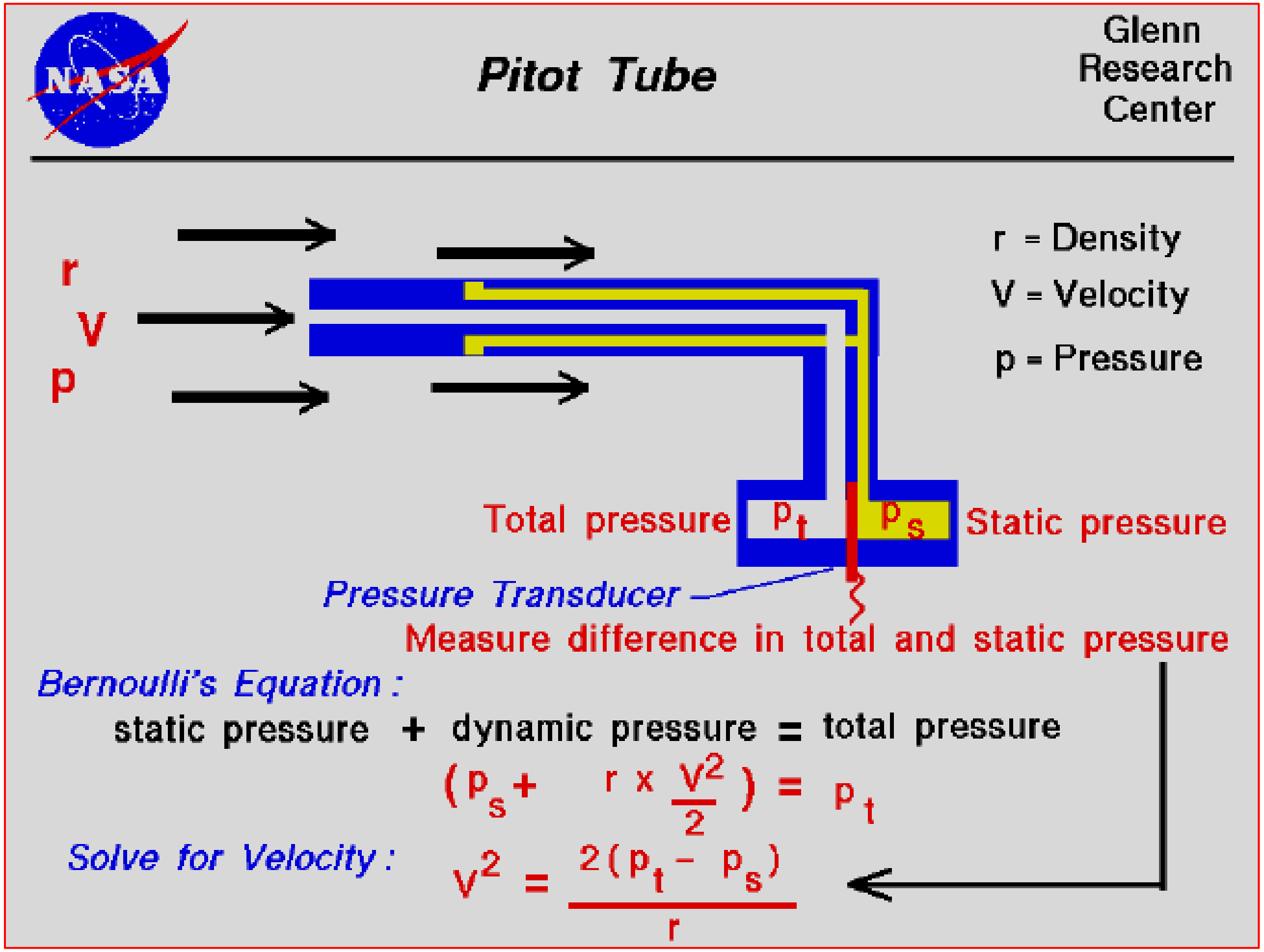


Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

A schematic drawing of a pitot tube.



- The pitot-static tube has two openings, one in the front and one along the sides, used to measure air, gas or liquid flow.
- This system involves placing a pitot tube inside another tube with static ports. The front hole measures the stagnation pressure, while the side openings (static ports) gauge static pressure.
- The difference between these two measurements is called dynamic pressure – this is what is used to calculate airspeed.



How it Works Pitot-Static System

https://www.youtube.com/watch?v=sYPIJ8Vz-FI

- **Pitot tubes** are used on aircraft as speedometers. The actual tube on the aircraft is around 10 inches (25 centimeters) long with a 1/2-inch (1 centimeter) diameter. Several small holes are drilled around the outside of the tube and a center hole is drilled down the axis of the tube.

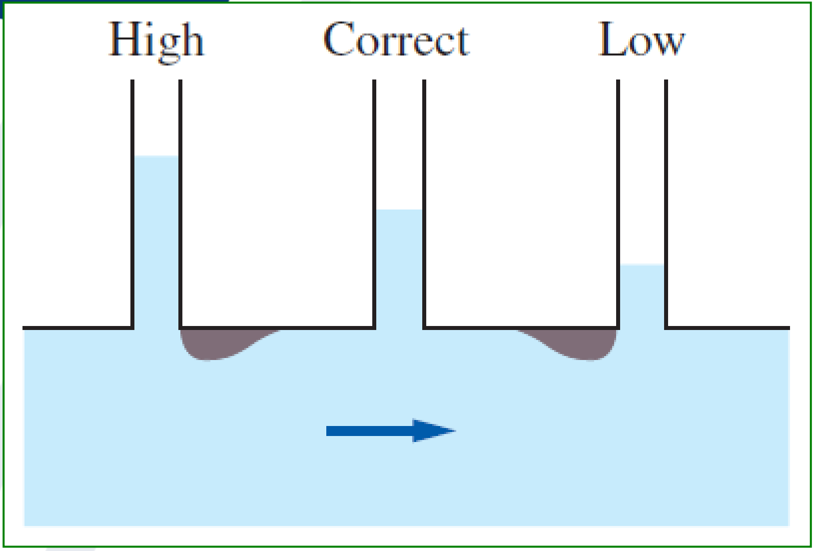
- The outside holes are connected to one side of a device called a **pressure transducer**. The center hole in the tube is kept separate from the outside holes and is connected to the other side of the transducer.

- The transducer measures the difference in pressure in the two groups of tubes by measuring the strain in a thin element using an electronic strain gauge.

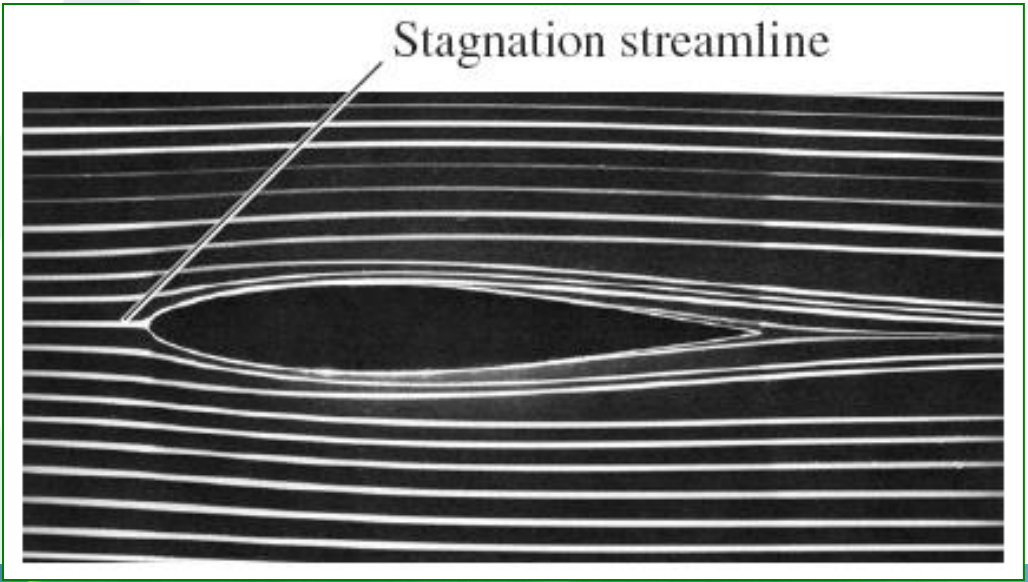
- The pitot tube is mounted on the aircraft so that the center tube is always pointed in the direction of travel and the outside holes are perpendicular to the center tube. (**On some airplanes the pitot tube is put on a longer boom sticking out of the nose of the plane or the wing.**)

https://studentpilotnews.com/2021/08/video-flying-the-waco-biplane/

Static, Dynamic, and Stagnation Pressures-2



Careless drilling of the static pressure tap may result in an erroneous reading of the static pressure head.



Streak lines produced by colored fluid introduced upstream of an airfoil; since the flow is steady, the streaklines are the same as streamlines and pathlines. The stagnation streamline is marked.

Limitations on the Use of the Bernoulli Equation

1. Steady flow The Bernoulli equation is applicable to *steady flow*.
2. Frictionless flow Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.
3. No shaft work The Bernoulli equation is not applicable in a flow section that involves a pump, turbine, fan, or any other machine or impeller since such devices destroy the streamlines and carry out energy interactions with the fluid particles. When these devices exist, the energy equation should be used instead.
4. Incompressible flow Density is taken constant in the derivation of the Bernoulli equation. The flow is incompressible for liquids and also by gases at Mach numbers less than about 0.3.

Limitations on the Use of the Bernoulli Equation

1. No heat transfer The density of a gas is inversely proportional to temperature, and thus the Bernoulli equation should not be used for flow sections that involve significant temperature change such as heating or cooling sections.

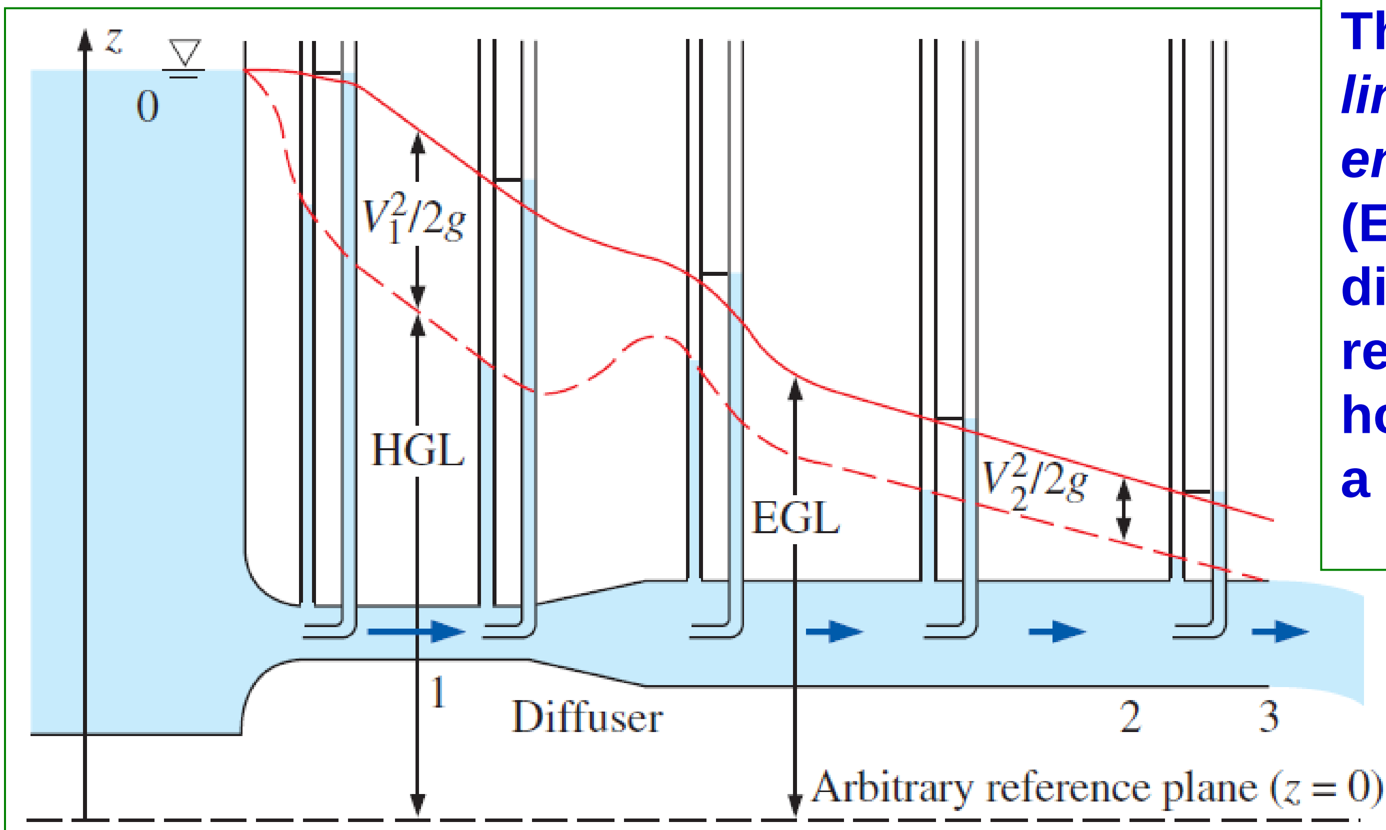
2. Flow along a streamline Strictly speaking, the Bernoulli equation is applicable along a streamline. However, when a region of the flow is *irrotational* and there is negligibly small *vorticity* in the flow field, the Bernoulli equation becomes applicable *across* streamlines as well.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)-1

Hydraulic grade line (HGL), $P/\rho g + z$ The line that represents the sum of the static pressure and the elevation heads.

Energy grade line (EGL), $P/\rho g + V^2/2g + z$ The line that represents the total head of the fluid.

Dynamic head, $V^2/2g$ The difference between the heights of EGL and HGL.



The *hydraulic grade line* (HGL) and the *energy grade line* (EGL) for free discharge from a reservoir through a horizontal pipe with a diffuser.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of the various terms of the Bernoulli equation. Dividing each term of the Bernoulli equation by *g* gives

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant} \qquad \text{(along a streamline)}$$

$P/\rho g$ is the pressure head; it represents the height of a fluid column that produces the static pressure *P*.

$V^2/2g$ is the velocity head; it represents the elevation needed for a fluid to reach the velocity *V* during frictionless free fall.

z is the elevation head; it represents the potential energy of the fluid.

Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)

$$\overbrace{\frac{P}{\rho g}}^{\text{Pressure head}} + \underbrace{\frac{V^2}{2g}}_{\text{Velocity head}} + \overbrace{z}^{\text{Elevation head}} = \underbrace{H}_{\text{Total head}} = \text{constant}$$

An alternative form of the Bernoulli equation is expressed in terms of heads as: *The sum of the pressure, velocity, and elevation heads is constant along a streamline*.

Notes on HGL and EGL

- For *stationary bodies* such as reservoirs or lakes, the EGL and HGL coincide with the free surface of the liquid.
- The EGL is always a distance $V^2/2g$ above the HGL. These two curves approach each other as the velocity decreases, and they diverge as the velocity increases.
- In an *idealized Bernoulli-type flow*, EGL is horizontal, and its height remains constant.
- For *open-channel flow*, the HGL coincides with the free surface of the liquid, and the EGL is a distance $V^2/2g$ above the free surface.
- At a *pipe exit*, the pressure head is zero (atmospheric pressure) and thus the HGL coincides with the pipe outlet.

Notes on HGL and EGL

- The *mechanical energy loss* due to frictional effects (conversion to thermal energy) causes the EGL and HGL to slope downward in the direction of flow.
- The slope is a measure of the head loss in the pipe. A component, such as a valve, that generates significant frictional effects causes a sudden drop in both EGL and HGL at that location.

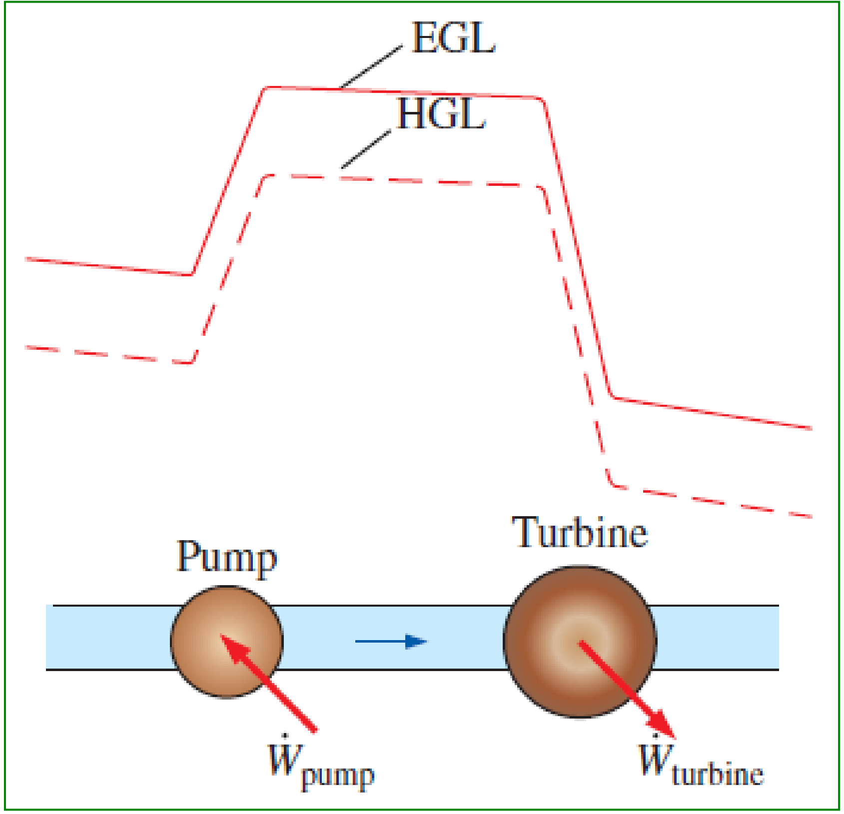
Notes on HGL and EGL

- A *steep jump/drop* occurs in EGL and HGL whenever mechanical energy is added or removed to or from the fluid (pump, turbine).
- The (gage) pressure of a fluid is zero at locations where the HGL *intersects* the fluid. The pressure in a flow section that lies above the HGL is negative, and the pressure in a section that lies below the HGL is positive.

Notes on HGL and EGL-1

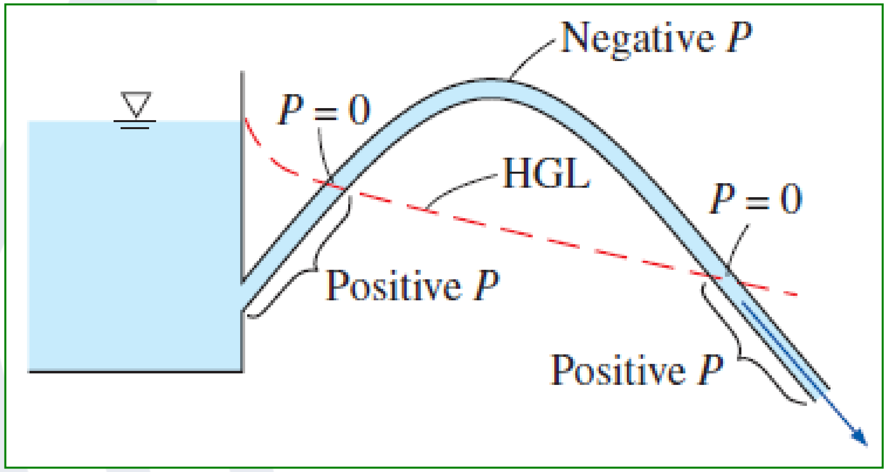
A *steep jump* occurs in EGL and HGL whenever mechanical energy is added to the fluid by a pump, and a *steep drop* occurs whenever mechanical energy is removed from the fluid by a turbine.

The gage pressure of a fluid is zero at locations where the HGL *intersects* the fluid, and the pressure is negative (vacuum) in a flow section that lies above the HGL.

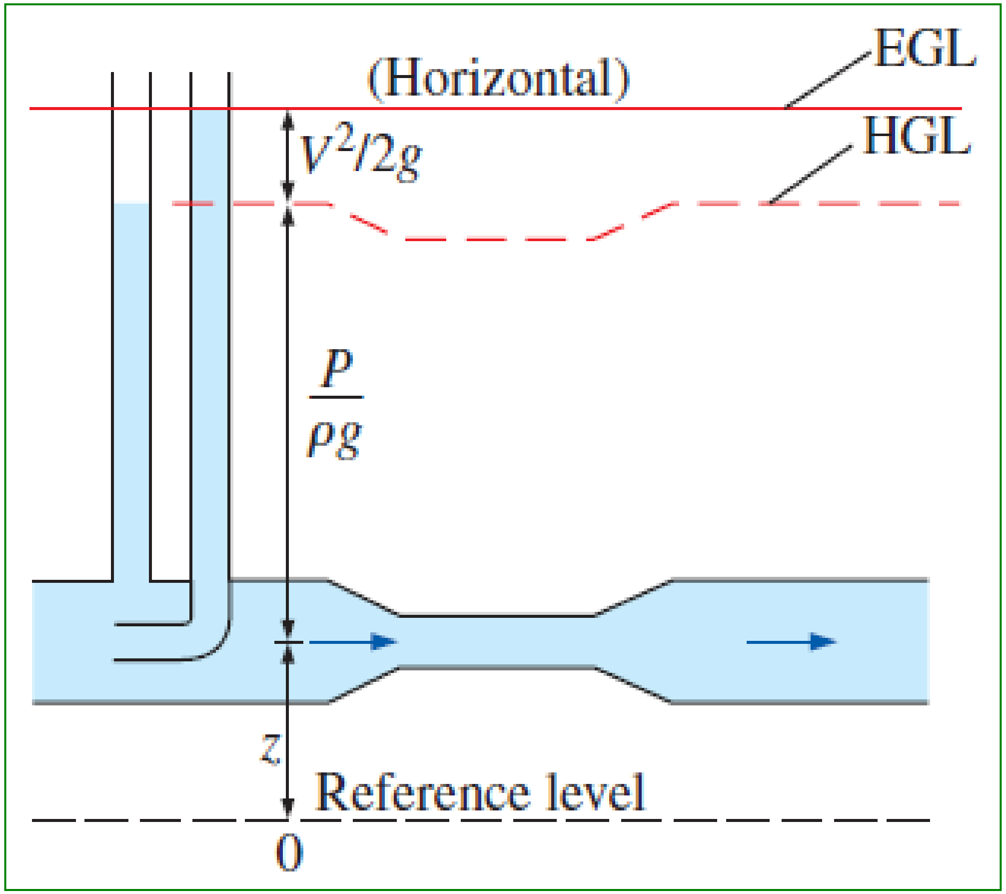


Notes on HGL and EGL-1

In an idealized Bernoulli-type flow, EGL is horizontal and its height remains constant. But this is not the case for HGL when the flow velocity varies along the flow.



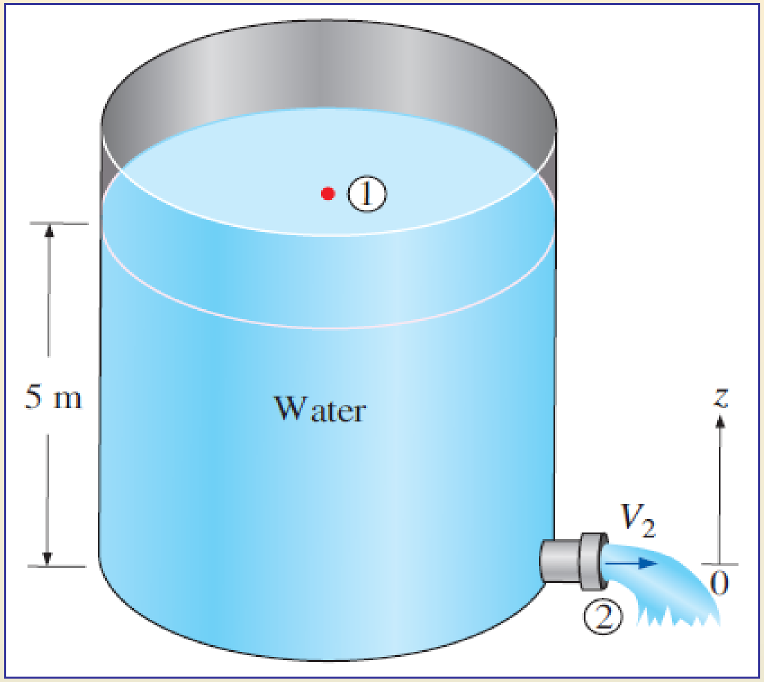
Hydraulic grade line (HGL),



Energy grade line (EGL

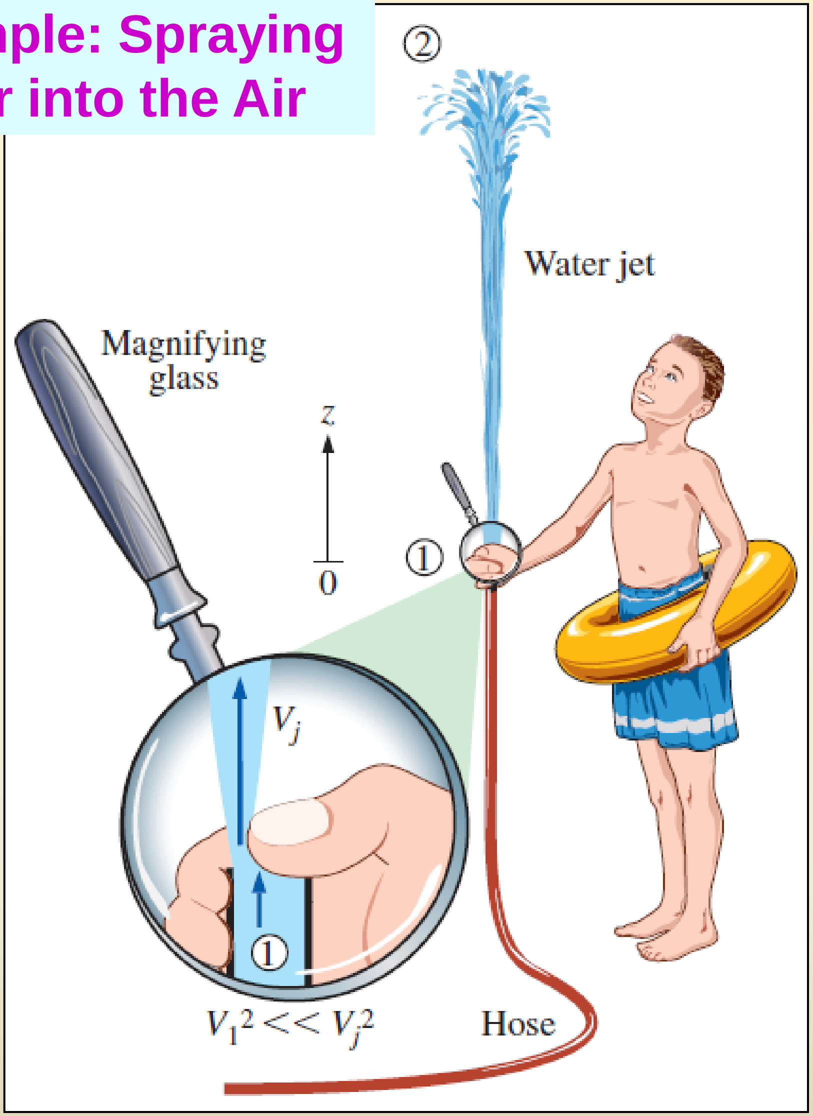
Notes on HGL and EGL

Example: Water Discharge from a Large Tank



$$\frac{\cancel{P_1}}{\cancel{\rho g}} + \frac{V_1^2}{2g} \nearrow^{\approx 0} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho g}} + \frac{V_2^2}{2g} + z_2 \nearrow^{0} \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Example: Spraying Water into the Air

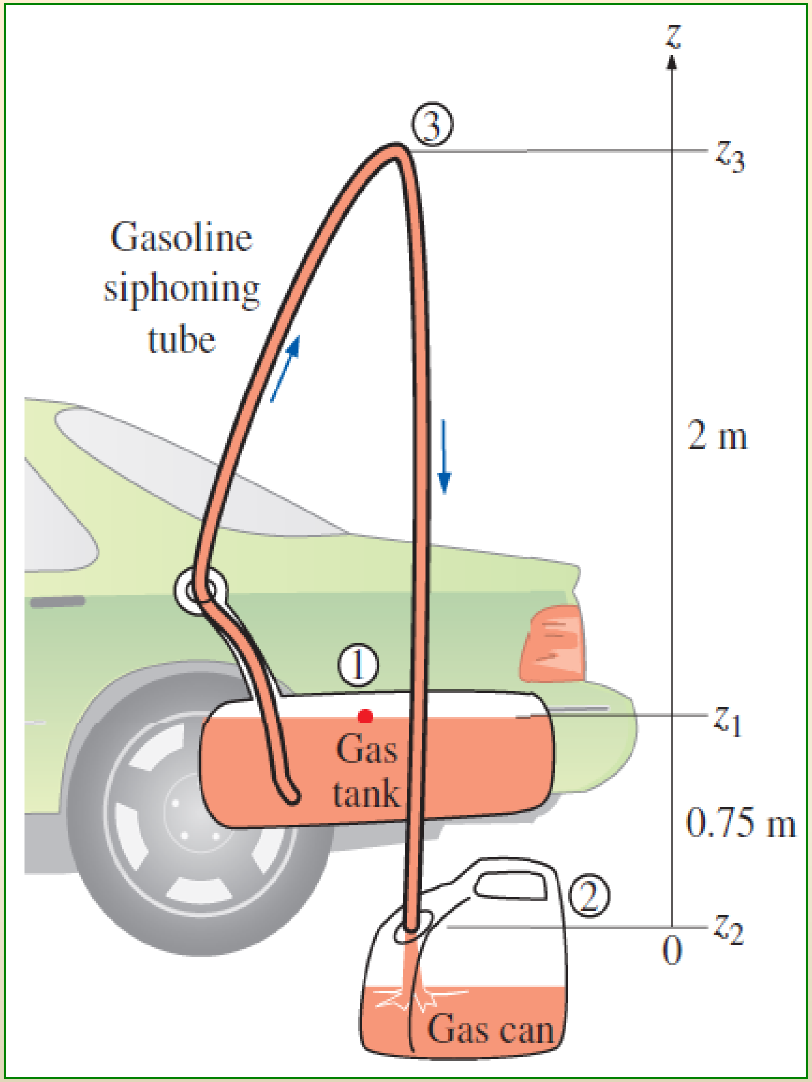


$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} \nearrow^{\approx 0} + z_1 \nearrow^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \nearrow^{0} + z_2 \quad \rightarrow \qquad \frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + z_2$$

Notes on HGL and EGL-3

$$\frac{\cancel{P_1}}{\cancel{\rho g}} + \frac{V_1^2}{2g}\nearrow^{\approx 0} + z_1 = \frac{\cancel{P_2}}{\cancel{\rho g}} + \frac{V_2^2}{2g} + z_2 \nearrow^{0} \rightarrow \; z_1 = \frac{V_2^2}{2g}$$

Example: Siphoning Out Gasoline from a Fuel Tank



$$\frac{P_2}{\rho g} + \frac{\cancel{V_2^2}}{\cancel{2g}} + \cancel{z_2}\nearrow^{0} = \frac{P_3}{\rho g} + \frac{\cancel{V_3^2}}{\cancel{2g}} + z_3 \quad \rightarrow \quad \frac{P_{atm}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

EXAMPLE 12–1 Spraying Water into the Air

Water is flowing from a garden hose (**Fig. 12–18**). A child places his thumb to cover most of the hose outlet, causing a thin jet of high-speed water to emerge. The pressure in the hose just upstream of his thumb is 400 kPa. If the hose is held upward, what is the maximum height that the jet could achieve?

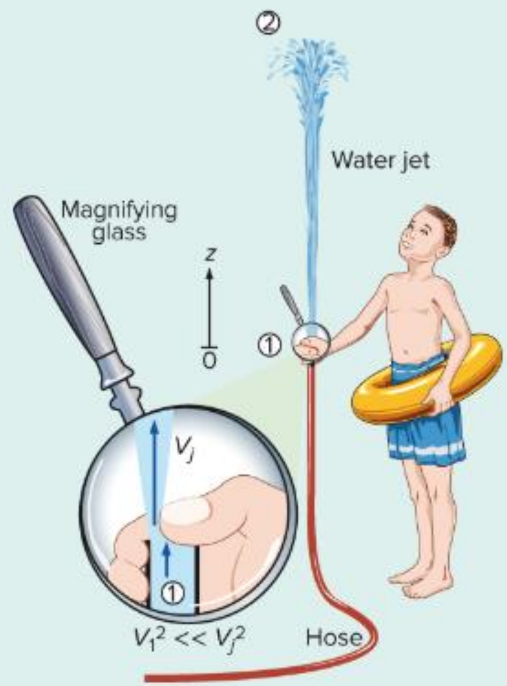


FIGURE 12–18
Schematic for **Example 12–1**. Inset shows a magnified view of the hose outlet region.

SOLUTION

Water from a hose attached to the water main is sprayed into the air. The maximum height the water jet can rise is to be determined.

Assumptions

1 The flow exiting into the air is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The surface tension effects are negligible. **3** The friction between the water and air is negligible. **4** The irreversibilities that occur at the outlet of the hose due to abrupt contraction are not taken into account.

Properties

We take the density of water to be 1000 kg/m^3.

Analysis

This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1^2 << V_j^2$, and thus $V_1 \cong 0$ compared to V_j) and we take the elevation just below the hose outlet as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation along a streamline from 1 to 2 simplifies to

$$\frac{P_1}{\rho g} + \overbrace{\cancel{\frac{V_1^2}{2g}}}^{\approx 0} + \cancel{z_1} = \frac{P_2}{\rho g} + \overbrace{\cancel{\frac{V_2^2}{2g}}}^{0} + z_2 \quad \rightarrow \quad \frac{P_1}{\rho g} = \frac{P_{\text{atm}}}{\rho g} + z_2$$

Solving for z_2 and substituting,

$$z_2 = \frac{P_1 - P_{\text{atm}}}{\rho g} = \frac{P_{1,\text{ gage}}}{\rho g} = \frac{400 \text{ kPa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$
$$= \mathbf{40.8\ m}$$

Therefore, the water jet can rise as high as 40.8 m into the sky in this case.

Discussion

The result obtained by the Bernoulli equation represents the upper limit and should be interpreted accordingly. It tells us that the water cannot possibly rise more than 40.8 m, and, in all likelihood, the rise will be much less than 40.8 m due to irreversible losses that we neglected.

EXAMPLE 12–2 Water Discharge from a Large Tank

A large tank open to the atmosphere is filled with water to a height of 5 m from the outlet tap (**Fig. 12–19**). A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the maximum water velocity at the outlet.

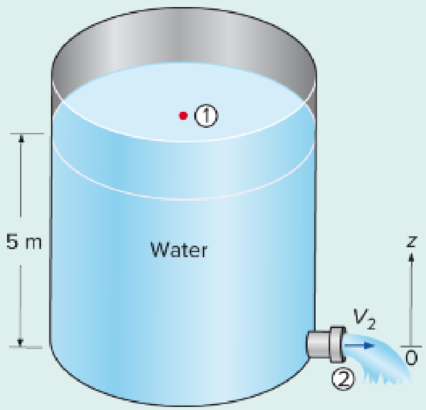


FIGURE 12–19
Schematic for **Example 12–2**.

SOLUTION

A tap near the bottom of a tank is opened. The maximum exit velocity of water from the tank is to be determined.

Assumptions

1 The flow is incompressible and irrotational (except very close to the walls). **2** The water drains slowly enough that the flow can be approximated as steady (actually quasi-steady when the tank begins to drain). **3** Irreversible losses in the tap region are neglected.

Analysis

This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of water so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1^2 << V_2^2$ and thus $V_1 \cong 0$ compared to V_2 (the tank is very large relative to the outlet), $z_1 = 5$ m and $z_2 = 0$ (we take the reference level at the center of the outlet). Also, $P_2 = P_{atm}$ (water discharges into the atmosphere). For flow along a streamline from 1 to 2, the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \overbrace{\cancel{\frac{V_1^2}{2g}}}^{\approx 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \overbrace{\cancel{z_2}}^{0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(5 \text{ m})} = \mathbf{9.9 \text{ m/s}}$$

The relation $V = \sqrt{2gz}$ is called the **Torricelli equation.**

Therefore, the water leaves the tank with an initial maximum velocity of 9.9 m/s. This is the same velocity that would manifest if a solid were dropped a distance of 5 m in the absence of air friction drag. (What would the velocity be if the tap were at the bottom of the tank instead of on the side?)

Discussion

If the orifice were sharp-edged instead of rounded, then the flow would be disturbed, and the average exit velocity would be less than 9.9 m/s. Care must be exercised when attempting to apply the Bernoulli equation to situations where abrupt expansions or contractions occur since the friction and flow disturbance in such cases may not be negligible. From conservation of mass, $(V_1/V_2)^2 = (D_2/D_1)^4$. So, for example, if $D_2/D_1 = 0.1$, then $(V_1/V_2)^2 = 0.0001$, and our approximation that $V_1^2 << V_2^2$ is justified.

EXAMPLE 12–3 Siphoning Out Gasoline from a Fuel Tank

During a trip to the beach (P_{atm} = 1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan (**Fig. 12–20**). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 5 mm, and frictional losses in the siphon are to be disregarded. Determine (*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and (*b*) the pressure at point 3. The density of gasoline is 750 kg/m^3.

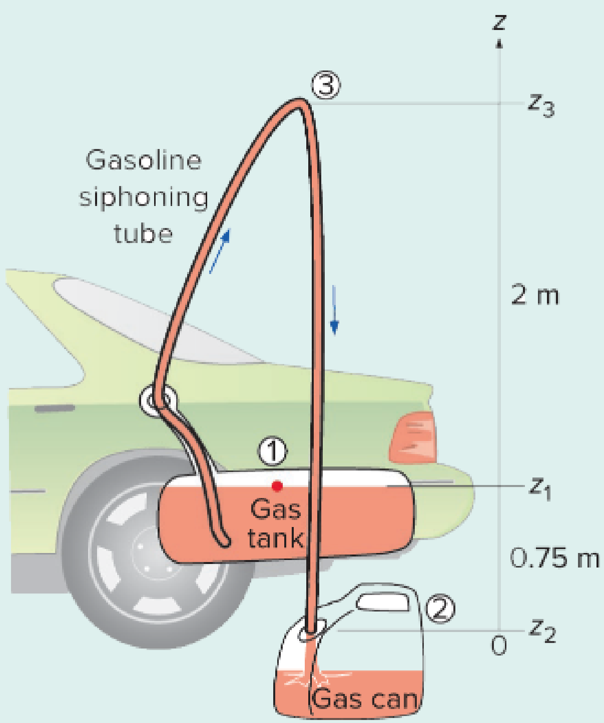


FIGURE 12–20
Schematic for **Example 12–3**.

FIGURE 12–20
Schematic for **Example 12–3**.

SOLUTION

Gasoline is to be siphoned from a tank. The minimum time it takes to withdraw 4 L of gasoline and the pressure at the highest point in the system are to be determined.

Assumptions

1 The flow is steady and incompressible. **2** Even though the Bernoulli equation is not valid through the pipe because of frictional losses, we employ the Bernoulli equation anyway in order to obtain a *best-case estimate*. **3** The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties

The density of gasoline is given to be 750 kg/m^3.

Analysis

(*a*) We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere), $V_1 \cong 0$ (the tank is large relative to the tube diameter), and $z_2 = 0$ (point 2 is taken as the reference level). Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\cancel{\frac{P_1}{\rho g}} + \overbrace{\frac{V_1^2}{2g}}^{\approx 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \overbrace{z_2}^{0} \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

Example 12-3 solution

$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(0.75 \text{ m})} = 3.84 \text{ m/s}$$

The cross-sectional area of the tube and the flow rate of gasoline are

$$A = \pi D^2/4 = \pi(5\times10^{-3} \text{ m})^2/4 = 1.96\times10^{-5} \text{ m}^2$$

$$\dot{V} = V_2 A = (3.84 \text{ m/s})(1.96\times10^{-5} \text{ m}^2) = 7.53\times10^{-5} \text{ m}^3\text{/s} = 0.0753 \text{ L/s}$$

Then the time needed to siphon 4 L of gasoline becomes

$$\Delta t = \frac{V}{\dot{V}} = \frac{4 \text{ L}}{0.0753 \text{ L/s}} = \mathbf{53.1 \text{ s}}$$

(*b*) The pressure at point 3 is determined by writing the Bernoulli equation along a streamline between points 3 and 2. Noting that $V_2 = V_3$ (conservation of mass), $z_2 = 0$, and $P_2 = P_{atm}$,

$$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{\,0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3 \quad \rightarrow \quad \frac{P_{\text{atm}}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

Solving for P_3 and substituting,

$$\begin{aligned} P_3 &= P_{\text{atm}} - \rho g z_3 \\ &= 101.3 \text{ kPa} - (750 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.75 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= \mathbf{81.1 \text{ kPa}} \end{aligned}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

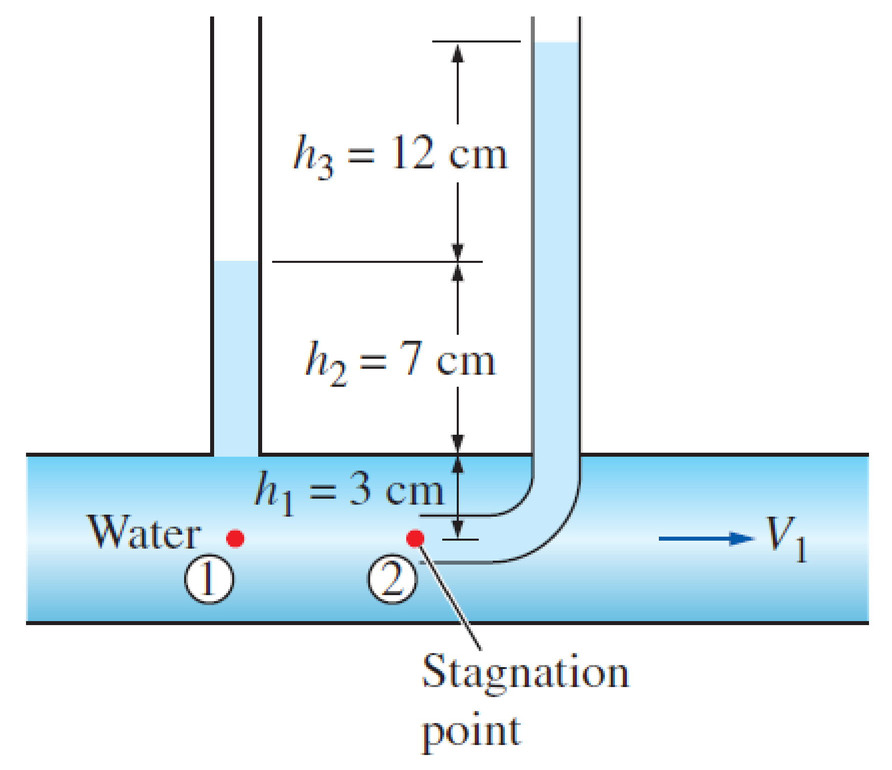
Solving for V_1 and substituting,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81 \text{ m/s}^2)(0.12 \text{ m})} = \mathbf{1.53 \text{ m/s}}$$

Discussion

Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube compared to that in the piezometer tube.

Example: Velocity Measurement by a Pitot Tube



Notes on HGL and EGL-4

$$P_1 = \rho g(h_1 + h_2)$$
$$P_2 = \rho g(h_1 + h_2 + h_3)$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancelto{0}{\frac{V_2^2}{2g}} + \cancel{z_2} \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

EXAMPLE 12–4 Velocity Measurement by a Pitot Tube

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. 12–21, to measure static and stagnation (static 1 dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe.

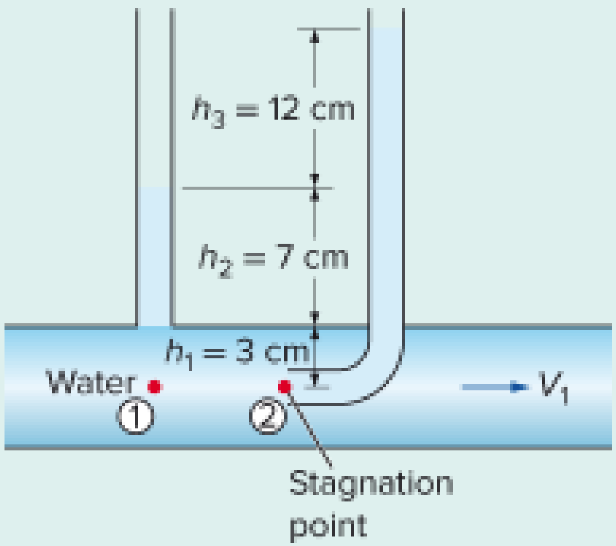


FIGURE 12–21
Schematic for Example 12–4.

SOLUTION

The static and stagnation pressures in a horizontal pipe are measured. The velocity at the center of the pipe is to be determined.

Assumptions

1 The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible, and thus we can use the Bernoulli equation.

Analysis

We take points 1 and 2 along the streamline at the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$$P_1 = \rho g(h_1 + h_2)$$

$$P_2 = \rho g(h_1 + h_2 + h_3)$$

Noting that $z_1 = z_2$, and point 2 is a stagnation point and thus $V_2 = 0$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{\cancelto{0}{V_2^2}}{2g} + \cancel{z_2} \rightarrow \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

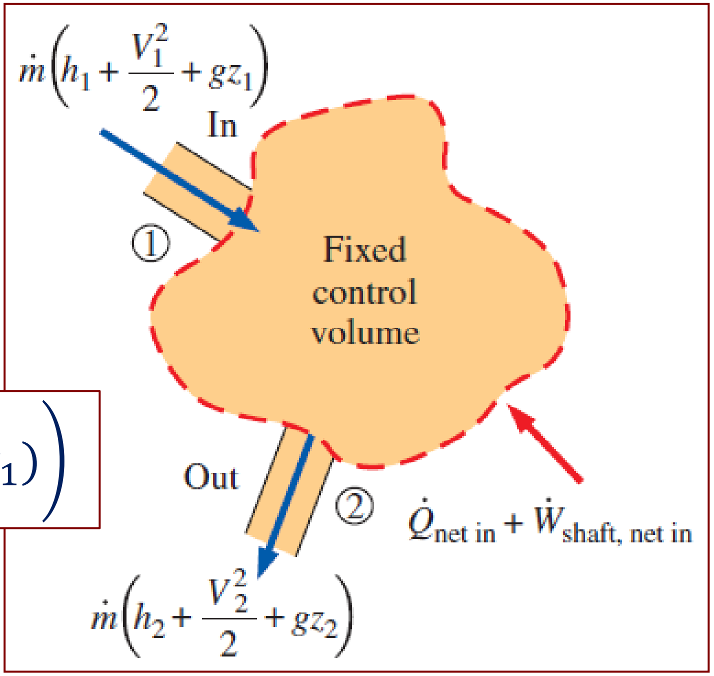
- On commercial airliners, **pitot tubes** bear the burden of measuring airspeed. The devices get their name from Henri Pitot, a Frenchman who needed a tool to measure the speed of water flowing in rivers and canals.

- His solution was a slender tube with two holes -- one in front and one on the side. Pitot oriented his device so that the front hole faced upstream, allowing water to flow through the tube. By measuring pressure differential at the front and side holes, he could calculate the speed of the moving water.

12-2 ENERGY ANALYSIS OF STEADY FLOWS

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft,net in}} = \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

The net rate of energy transfer to a control volume by heat transfer and work during steady flow is equal to the difference between the rates of outgoing and incoming energy flows by mass flow.



$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft,net in}} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

single-stream devices

$$q_{\text{net in}} + w_{\text{shaft,net in}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h = u + PV = u + P/\rho$$

A control volume with only one inlet and one outlet and energy interactions.

$$w_{\text{shaft,net in}} + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + (u_2 - u_1 - q_{\text{net in}})$$

EXAMPLE 12–5 Pumping Power and Frictional Heating in a Pump

The pump of a water distribution system is powered by a 15-kW electric motor whose efficiency is 90 percent (**Fig. 12–27**). The water flow rate through the pump is 50 L/s. The diameters of the inlet and outlet pipes are the same, and the elevation difference across the pump is negligible. If the absolute pressures at the inlet and outlet of the pump are measured to be 100 kPa and 300 kPa, respectively, determine (*a*) the mechanical efficiency of the pump and (*b*) the temperature rise of water as it flows through the pump due to mechanical inefficiencies.

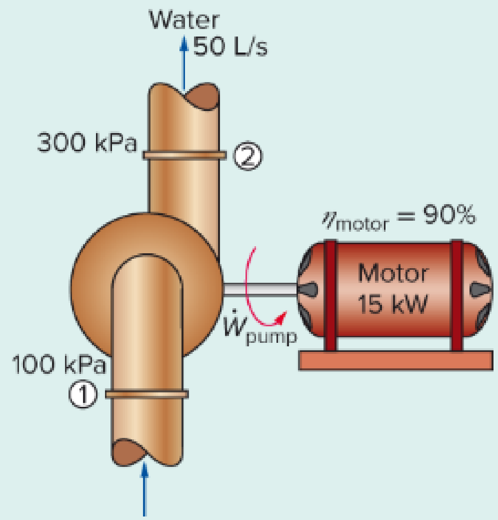


FIGURE 12–27
Schematic for **Example 12–5**.

SOLUTION

The pressures across a pump are measured. The mechanical efficiency of the pump and the temperature rise of water are to be determined.

Assumptions

1 The flow is steady and incompressible. **2** The pump is driven by an external motor so that the heat generated by the motor is dissipated to the atmosphere. **3** The elevation difference between the inlet and outlet of the pump is negligible, $z_1 \cong z_2$. **4** The inlet and outlet diameters are the same and thus the average inlet and outlet velocities are equal, $V_1 = V_2$. **5** The kinetic energy correction factors are equal, $\alpha_1 = \alpha_2$.

Properties

We take the density of water to be 1 kg/L = 1000 kg/m^3 and its specific heat to be 4.18 kJ/kg·°C.

Analysis

(*a*) The mass flow rate of water through the pump is

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(50 \text{ L/s}) = 50 \text{ kg/s}$$

The motor draws 15 kW of power and is 90 percent efficient. Thus the mechanical (shaft) power it delivers to the pump is

$$\dot{W}_{\text{pump, shaft}} = \eta_{\text{motor}} \dot{W}_{\text{electric}} = (0.90)(15 \text{ kW}) = 13.5 \text{ kW}$$

To determine the mechanical efficiency of the pump, we need to know the increase in the mechanical energy of the fluid as it flows through the pump, which is

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{E}_{\text{mech, out}} - \dot{E}_{\text{mech, in}} = \dot{m}\left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + g z_2\right) - \dot{m}\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1\right)$$

Simplifying it for this case and substituting the given values,

$$\Delta \dot{E}_{\text{mech, fluid}} = \dot{m}\left(\frac{P_2 - P_1}{\rho}\right) = (50 \text{ kg/s})\left(\frac{(300 - 100) \text{ kPa}}{1000 \text{ kg/m}^3}\right)\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) = 10.0 \text{ kW}$$

Then the mechanical efficiency of the pump becomes

This error can be corrected by replacing the kinetic energy terms V 2 /2 in the energy equation by αV^2_{avg} /2, where α is the kinetic energy correction factor

$$\eta_{\text{pump}} = \frac{\dot{W}_{\text{pump, }u}}{\dot{W}_{\text{pump, shaft}}} = \frac{\Delta \dot{E}_{\text{mech, fluid}}}{\dot{W}_{\text{pump, shaft}}} = \frac{10.0 \text{ kW}}{13.5 \text{ kW}} = \mathbf{0.741} \text{ or } \mathbf{74.1\%}$$

(*b*) Of the 13.5-kW mechanical power supplied by the pump, only 10.0 kW is imparted to the fluid as mechanical energy. The remaining 3.5 kW is converted to thermal energy due to frictional effects, and this "lost" mechanical energy manifests itself as a heating effect in the fluid,

$$\dot{E}_{\text{mech, loss}} = \dot{W}_{\text{pump,shaft}} - \Delta \dot{E}_{\text{mech, fluid}} = 13.5 - 10.0 = 3.5\text{kW}$$

The temperature rise of water due to this mechanical inefficiency is determined from the thermal energy balance,
$E_{\text{mech, loss}} = \dot{m}(u_2 - u_1) = \dot{m}c\Delta T$. Solving for ΔT,

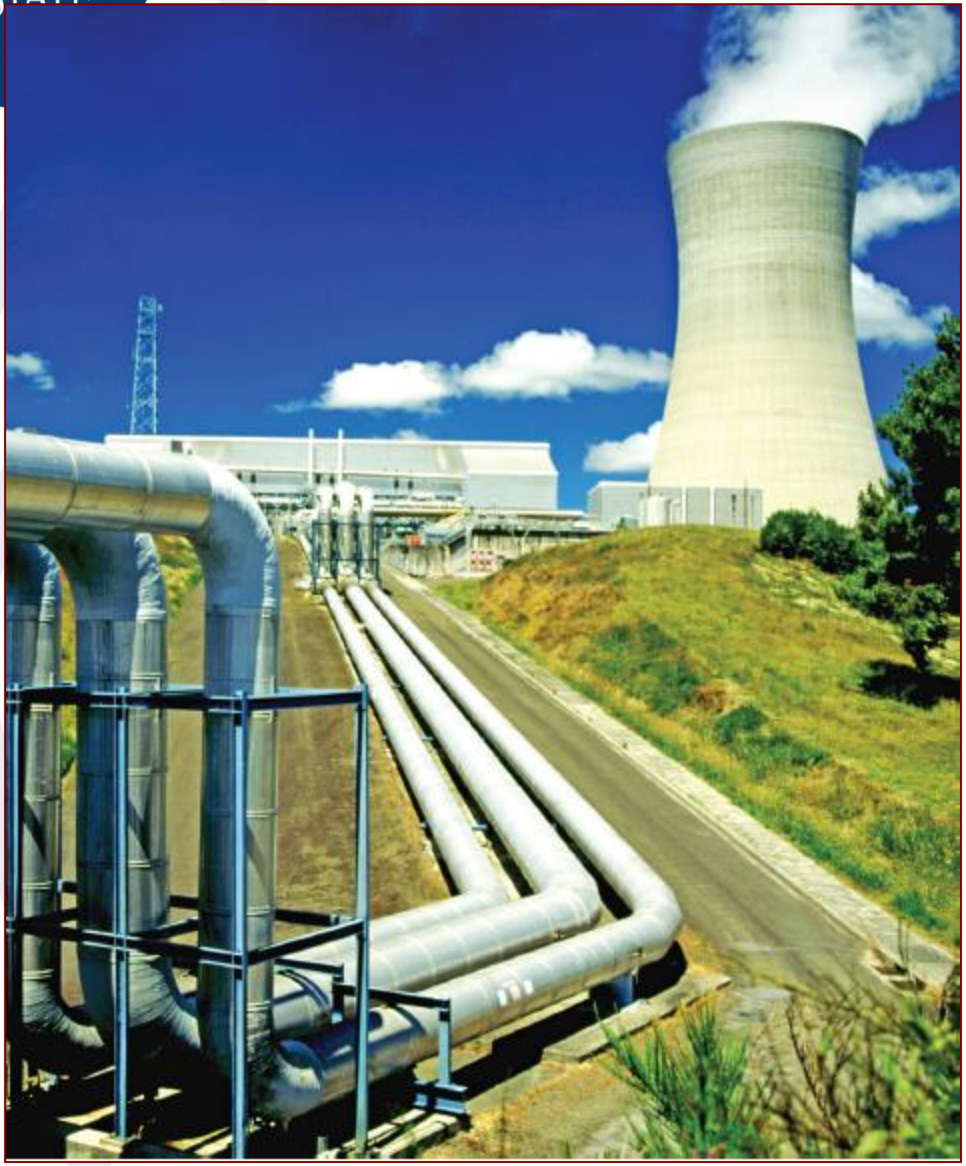
$$\Delta T = \frac{\dot{E}_{\text{mech, loss}}}{\dot{m}c} = \frac{3.5 \text{ kW}}{(50 \text{ kg/s})(4.18 \text{ kJ/kg·°C})} = \mathbf{0.017°C}$$

Therefore, the water experiences a temperature rise of 0.017°C which is very small, due to mechanical inefficiency, as it flows through the pump.

Discussion

In an actual application, the temperature rise of water would probably be less since part of the heat generated would be transferred to the casing of the pump and from the casing to the surrounding air. If the entire pump and motor were submerged in water, then the 1.5 kW dissipated due to motor inefficiency would also be transferred to the surrounding water as heat.

12-2 ENERGY ANALYSIS OF STEADY FLOWS-2



A typical power plant has numerous pipes, elbows, valves, pumps, and turbines, all of which have irreversible losses.

Summary

- **The Bernoulli Equation**
  - Acceleration of a Fluid Particle
  - Derivation of the Bernoulli Equation
  - Force Balance across Streamlines
  - Unsteady, compressible flow
  - Static, Dynamic, and Stagnation Pressures
  - Limitations on the Use of the Bernoulli Equation
  - Hydraulic Grade Line (HGL) and Energy Grade Line (EGL)
  - Applications of the Bernoulli's Equation

1. **Energy Analysis of Steady Flows**
   - Special Case: Incompressible Flow with No Mechanical Work Devices and Negligible Friction
   - Kinetic Energy Correction Factor, α