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Chapter 17

STEADY HEAT CONDUCTION

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Objectives

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries
- Develop an intuitive understanding of thermal contact resistance, and circumstances under which it may be significant

Objectives

- Identify applications in which insulation may actually increase heat transfer
- Analyze finned surfaces, and assess how efficiently and effectively fins enhance heat transfer
- Solve multidimensional practical heat conduction problems using conduction shape factors.

17-1 STEADY HEAT CONDUCTION IN PLANE WALLS

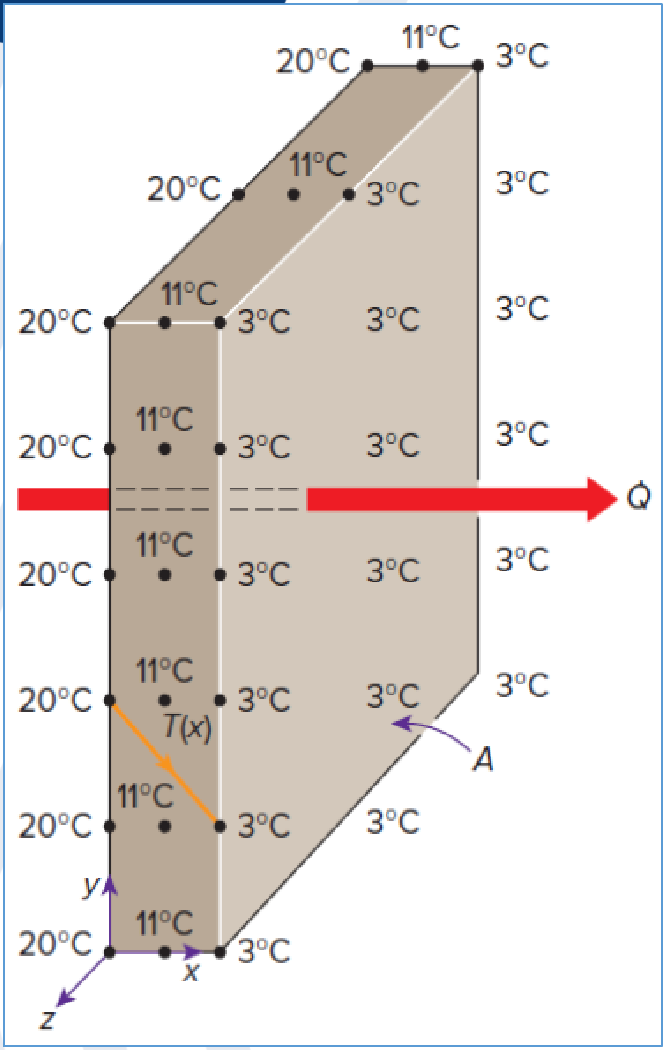


FIGURE 17-1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.*

The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as *T*(*x*).

$$\begin{pmatrix}\text{Rate of}\\ \text{heat tranfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{\text{wall}}}{dt}$$

$$dE_{wall}/dt = 0$$

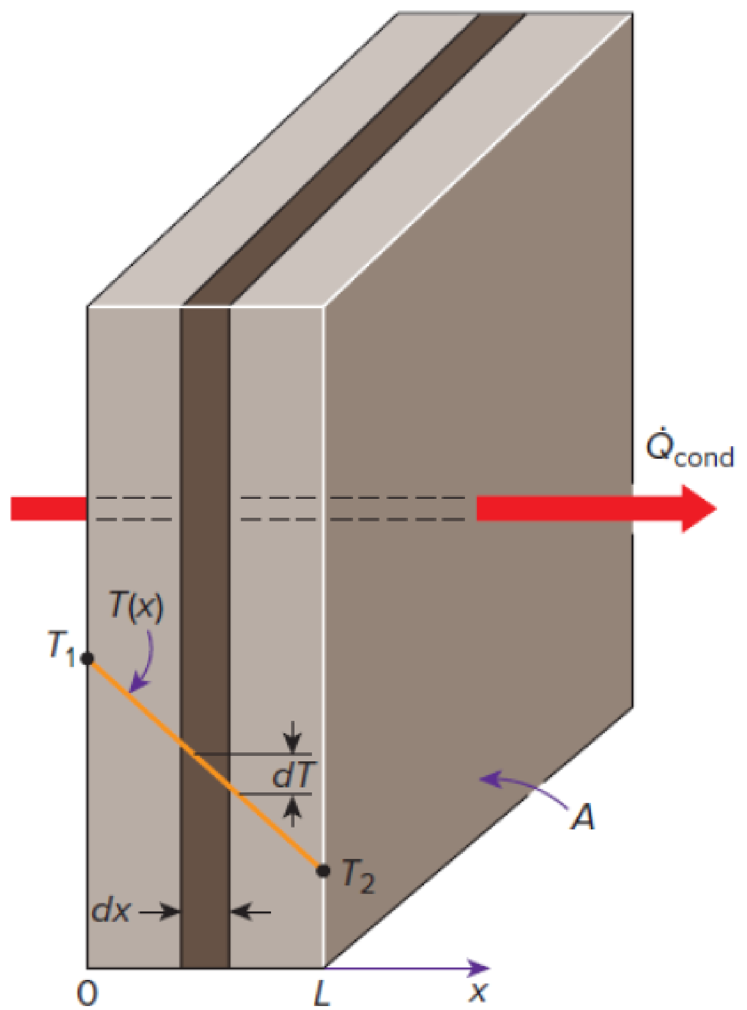
for *steady* operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{\text{cond,wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Fourier's law of heat conduction

17-1 STEADY HEAT CONDUCTION IN PLANE WALLS-1



$$\dot{Q}_{\text{cond,wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond,wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond,wall}} = kA\frac{T_1 - T_2}{L} \quad \text{(W)}$$

The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available, the temperature *T*(*x*) at any location *x* can be determined by replacing T_2 by *T*, and *L* by *x*.

Under steady conditions, the temperature distribution in a plane wall is a straight line: $dT/dx = \text{const}$.

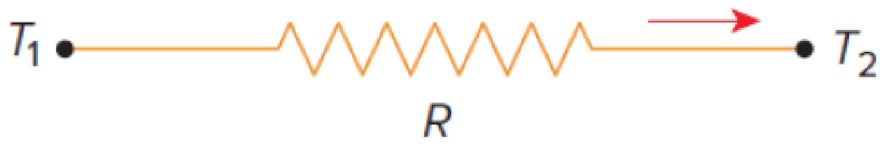
Thermal Resistance Concept

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L}$$

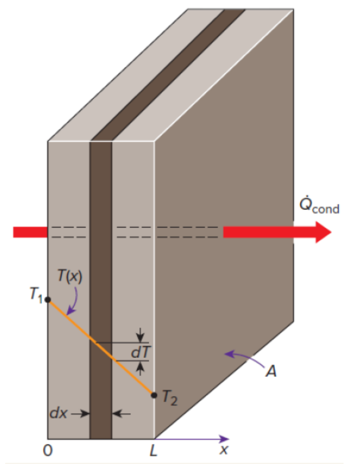
$$\dot{Q}_{\text{cond,wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad (\text{W})$$

$$R_{\text{wall}} = \frac{L}{kA} \quad (°\text{C/W})$$

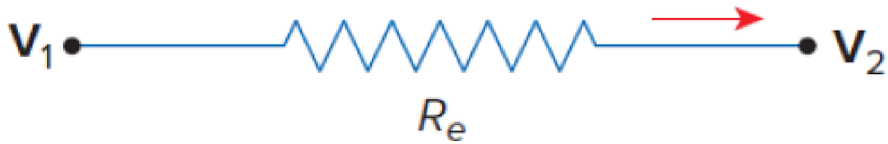
$$\dot{Q} = \frac{T_1 - T_2}{R}$$



(a) Heat flow



$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$



(b) Electric current flow

Conduction resistance of the wall: *Thermal resistance* of the wall against heat conduction.

Thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

Analogy between thermal and electrical resistance concepts.

rate of heat transfer → electric current
thermal resistance → electrical resistance
temperature difference → voltage difference

$$I = \frac{V_1 - V_2}{R_e} \qquad R_e = L/\sigma_e\, A$$

Electrical resistance

Thermal Resistance Concept-1

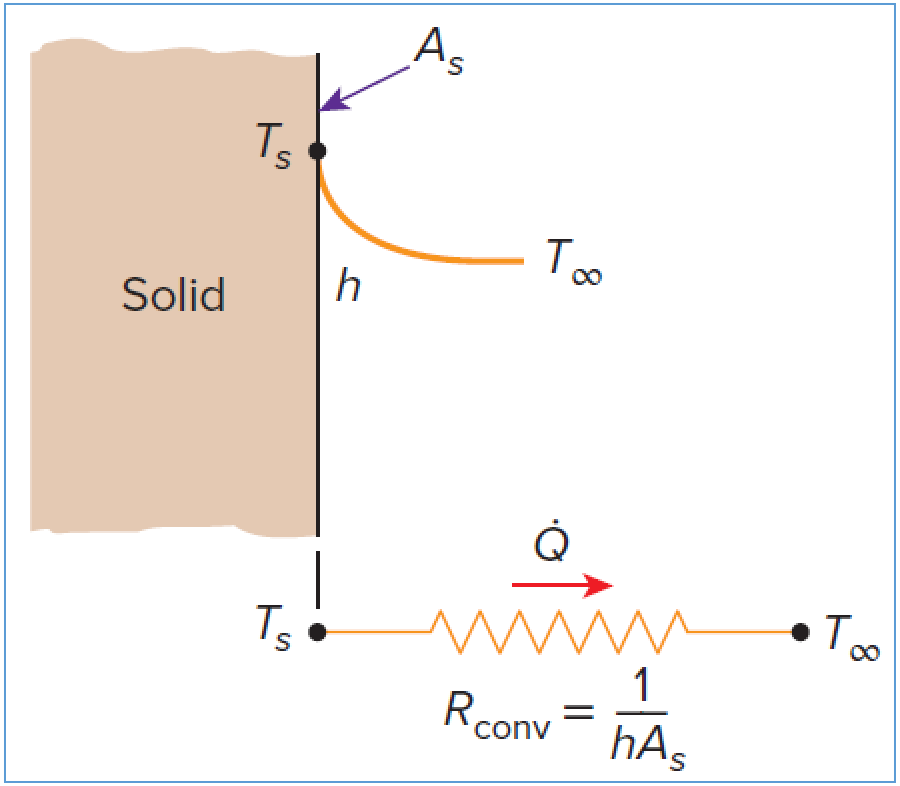
Newton's law of cooling

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \qquad \text{(W)}$$

$$R_{\text{conv}} = \frac{1}{hA_s} \qquad (^\circ\text{C/W})$$

Convection resistance of the surface: ***Thermal resistance*** **of the surface against heat convection.**



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large $(h \to \infty)$, the convection resistance becomes *zero* and $T_s \approx T$.

That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

This situation is approached in practice at surfaces where boiling and condensation occur.

Thermal Resistance Concept-2

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}}A_s} \qquad (\text{K/W})$$

Radiation resistance of the surface: ***Thermal resistance*** **of the surface against radiation.**

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \quad (\text{W/m}^2 \cdot \text{K})$$

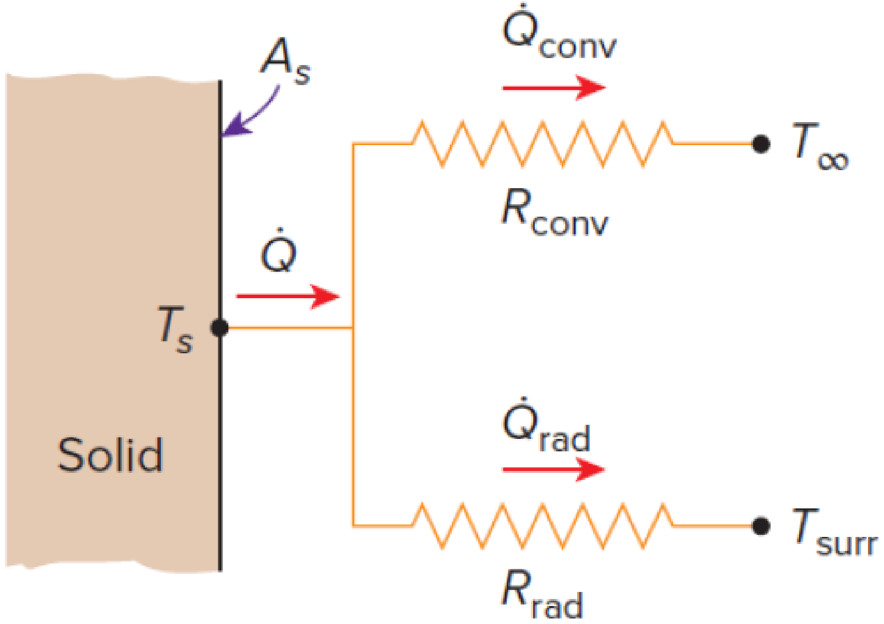
Radiation heat transfer coefficient

When $T_{surr} \approx T_\infty$

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}}$$

Combined heat transfer coefficient

Schematic for convection and radiation resistances at a surface.



$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad}$$

Thermal Resistance Network

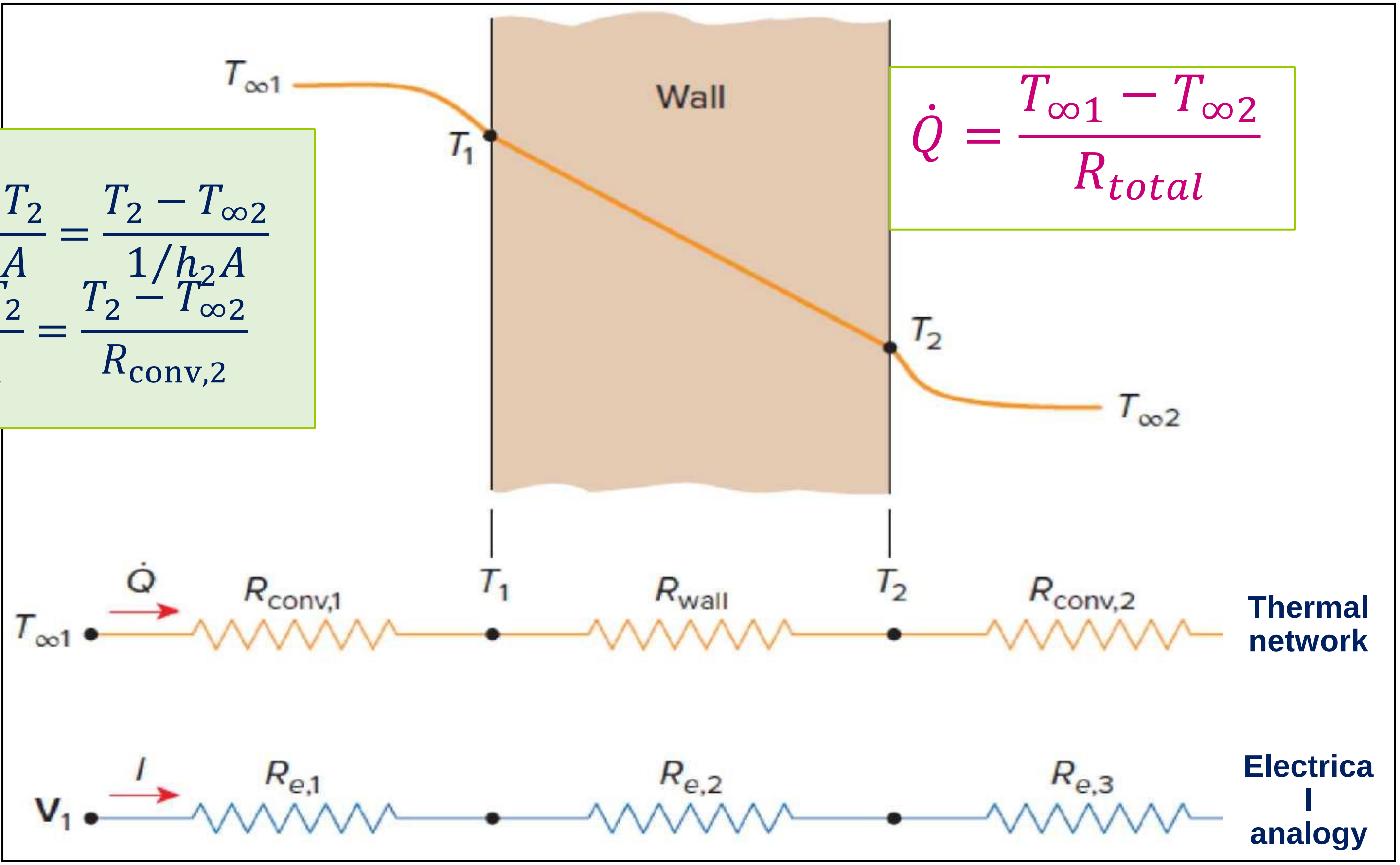
$$\begin{pmatrix}\text{Rate of}\\ \text{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat convection}\\ \text{from the wall}\end{pmatrix}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$
$$= \frac{T_{\infty 1} - T_1}{R_{\mathrm{conv},1}} = \frac{T_1 - T_2}{R_{\mathrm{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\mathrm{conv},2}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{wall} + R_{conv,2}}$$

$$I = \frac{\boldsymbol{V_1 - V_2}}{R_{e,1} + R_{e,2} + R_{e,2}}$$



The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\mathrm{total}} = R_{\mathrm{conv.1}} + R_{\mathrm{wall}} + R_{\mathrm{conv,2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\mathrm{C/W})$$

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Thermal Resistance Network-1

Temperature drop

$$\Delta T = \dot{Q}R \quad (°C)$$

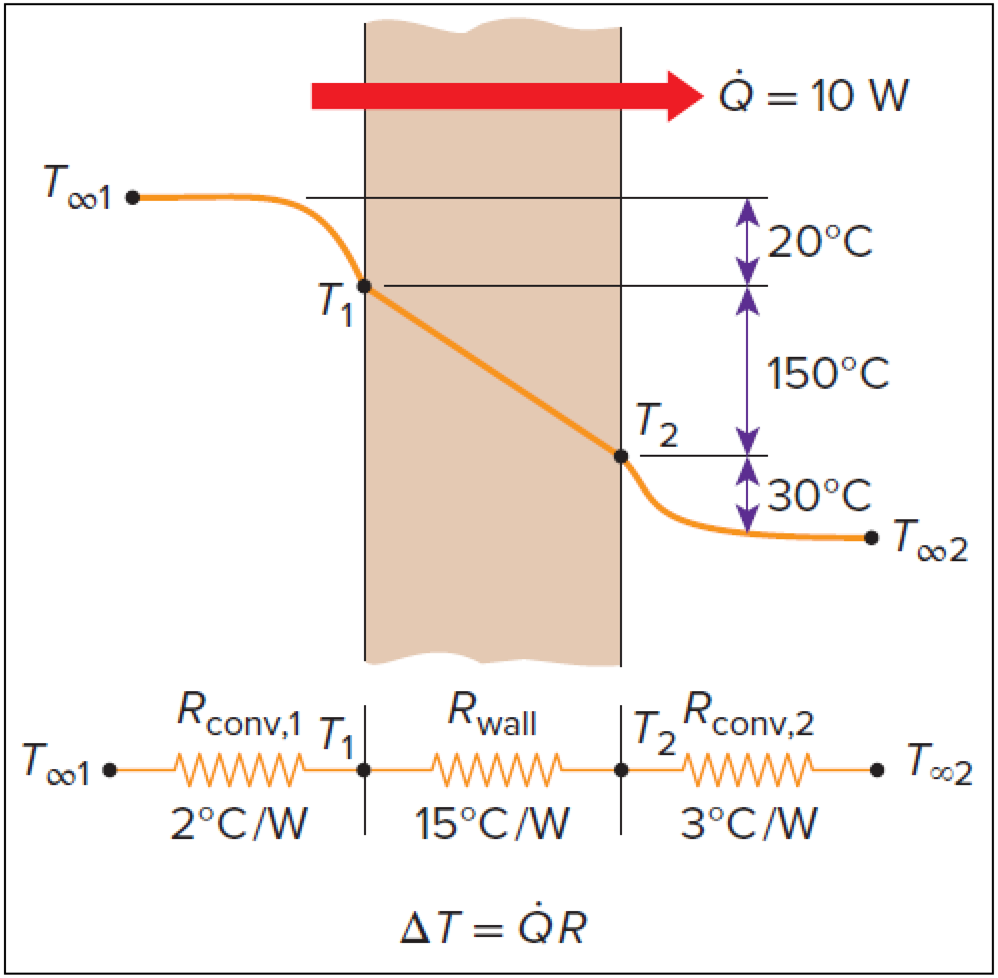
$$\dot{Q} = UA\,\Delta T \qquad \textbf{(W)}$$

$$UA = \frac{1}{R_{\text{total}}} \quad (°C/K)$$

U **overall heat transfer coefficient**

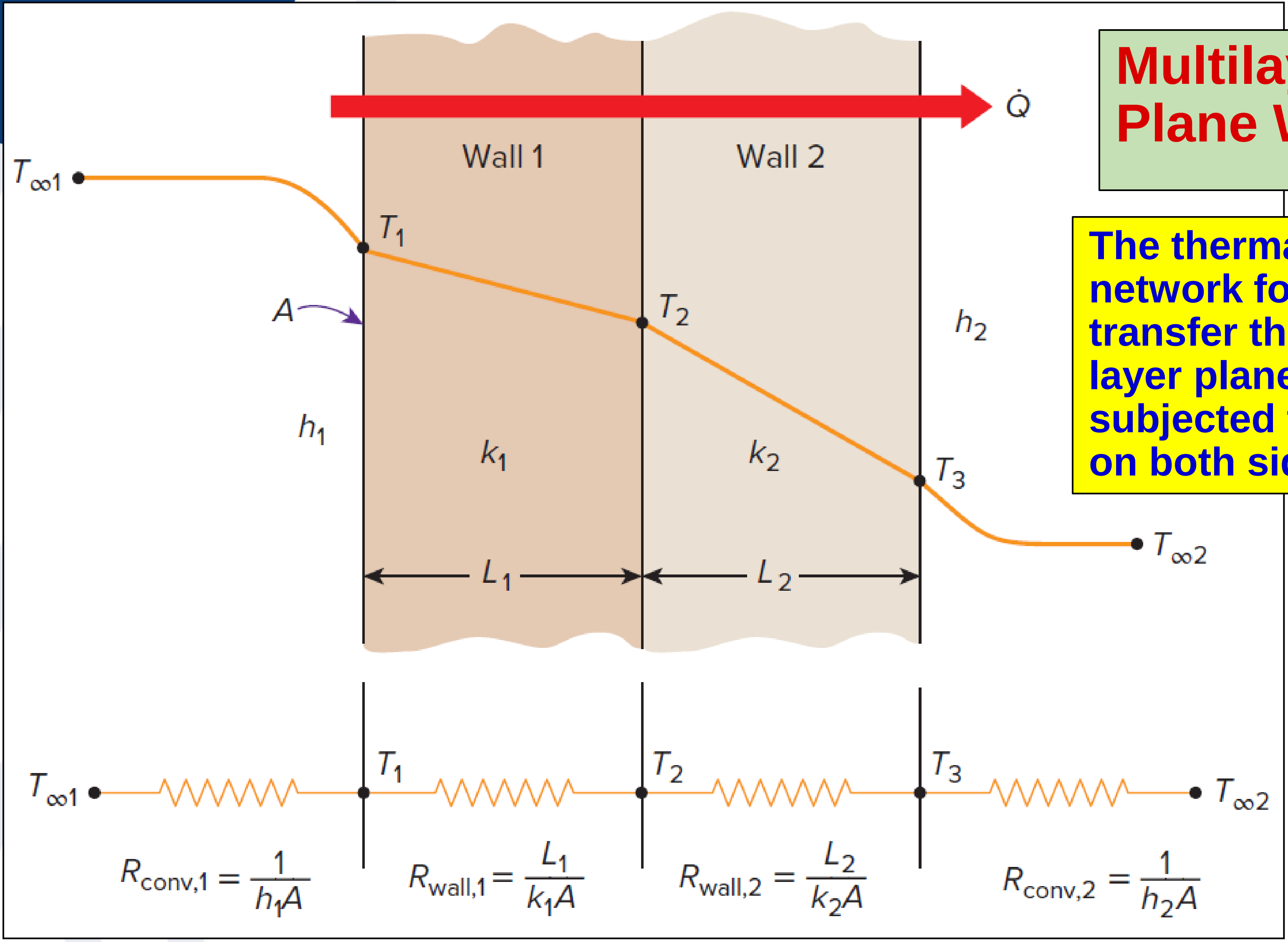
Once Q is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv.1}}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$



The temperature drop across a layer is proportional to its thermal resistance.

Multilayer Plane Walls



The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

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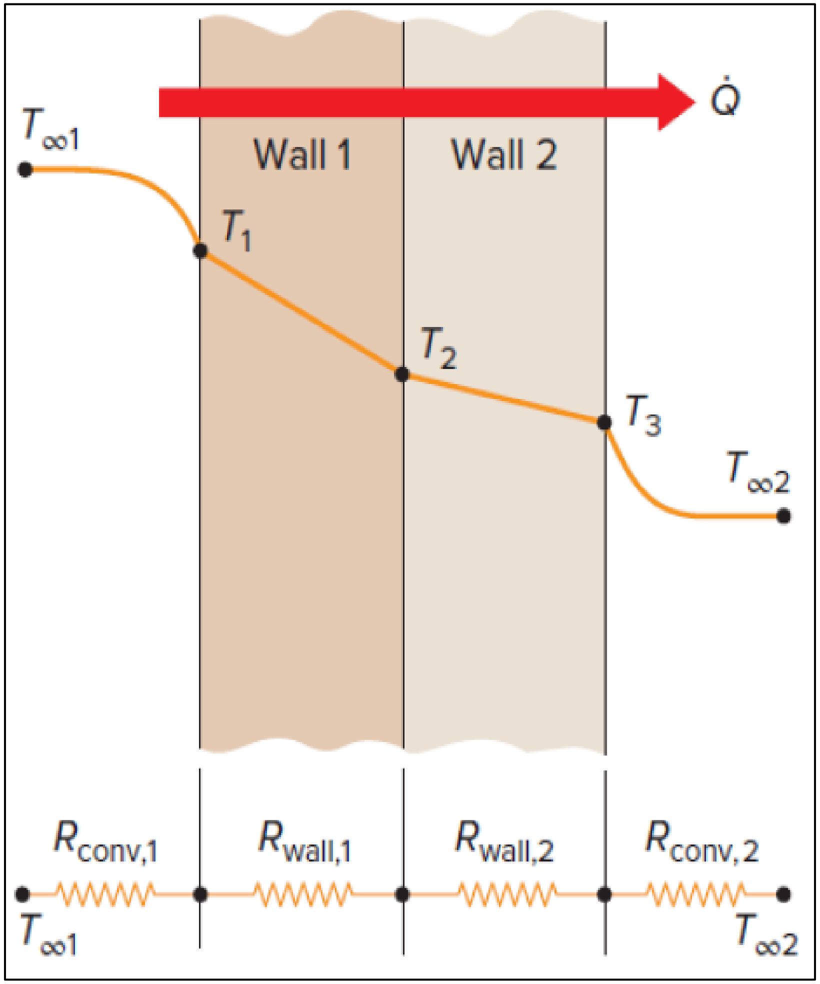
Multilayer Plane Walls-1

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{K_1 A}}$$



The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

To find T_1: $\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$

To find T_2: $\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{wall,1}}$

To find T_3: $\dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{conv,2}}$

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EXAMPLE 17–1 Heat Loss through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m·K (**Fig. 17–11**). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

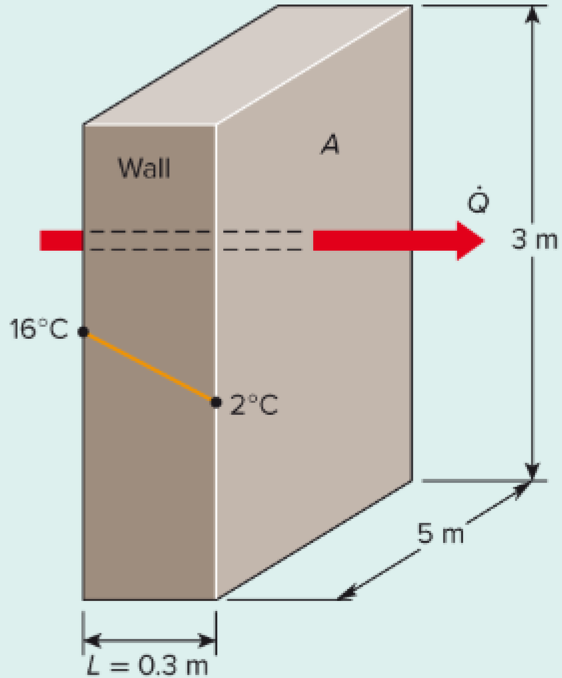

FIGURE 17–11
Schematic for **Example 17–1**.

SOLUTION

The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions

1 Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties

The thermal conductivity is given to be $k = 0.9$ W/m·K.

Analysis

Noting that heat transfer through the wall is by conduction and the area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$, the steady rate of heat transfer through the wall can be determined from **Eq. 17–3** to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9 \text{ W/m·°C})(15 \text{ m}^2)\frac{(16 - 2)\text{°C}}{0.3 \text{ m}} = \mathbf{630 \text{ W}}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m·°C})(15 \text{ m}^2)} = 0.02222\text{°C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)\text{°C}}{0.02222\text{°C/W}} = 630 \text{ W}$$

Discussion

This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples. Also, the units W/m·°C and W/m·K for thermal conductivity are equivalent, and thus interchangeable. This is also the case for °C and K for temperature differences.

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EXAMPLE 17–2 Heat Loss through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2·K and $h_2 = 40$ W/m^2·K, which includes the effects of radiation.

SOLUTION

Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Assumptions

1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties

The thermal conductivity is given to be $k = 0.78$ W/m·K.

Analysis

This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 17–12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00855°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

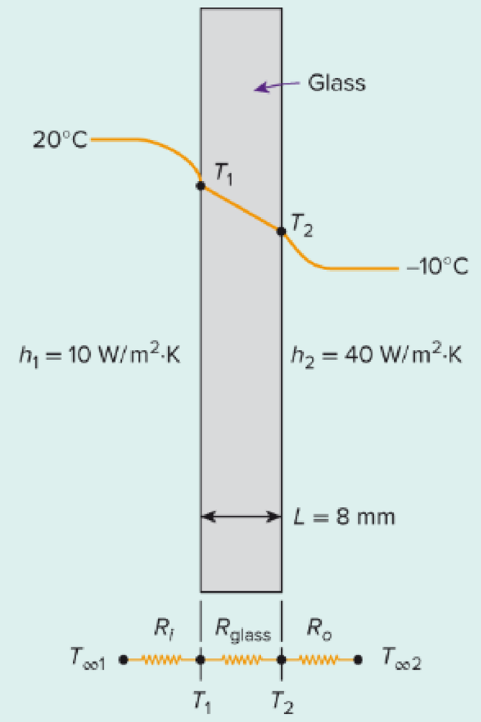

FIGURE 17–12
Schematic for Example 17–2.

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FIGURE 17–12
Schematic for Example 17–2.

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv},\,1} + R_{\text{glass}} + R_{\text{conv},\,2} = 0.08333 + 0.00855 + 0.02083$$
$$= 0.1127°\text{C/W}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.1127°\text{C/W}} = \mathbf{266\ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},\,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv},\,1}$$
$$= 20°\text{C} - (266\ \text{W})(0.08333°\text{C/W})$$
$$= \mathbf{-2.2°C}$$

Discussion

Note that the inner surface temperature of the window glass is −2.2°C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

EXAMPLE 17–3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass (k = 0.78 W/m·K) separated by a 10-mm-wide stagnant air space (k = 0.026 W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be h_1 = 10 W/m^2·K and h_2 = 40 W/m^2·K, which includes the effects of radiation.

SOLUTION

A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis

This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 17–13. Noting that the area of the window is again A = 0.8 m × 1.5 m = 1.2 m^2, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(1.2\ \text{m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004\ \text{m}}{(0.78\ \text{W/m}\cdot\text{K})(1.2\ \text{m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(1.2\ \text{m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2\cdot\text{K})(1.2\ \text{m}^2)} = 0.02083°\text{C/W}$$

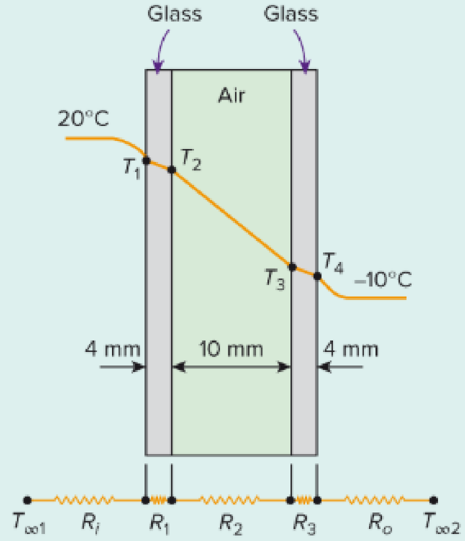

FIGURE 17–13
Schematic for Example 17–3.

FIGURE 17–13
Schematic for **Example 17–3**.

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}}$$

$$= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083$$

$$= 0.4332°\text{C/ W}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{0.4332°\text{C/W}} = \mathbf{69.2\ W}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv, 1}} = 20°\text{C} - (69.2\ \text{W})(0.08333°\text{C/ W}) = \mathbf{14.2°C}$$

which is considerably higher than the −2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

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EXAMPLE 17–4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m^2·K. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (**Fig. 17–17**).

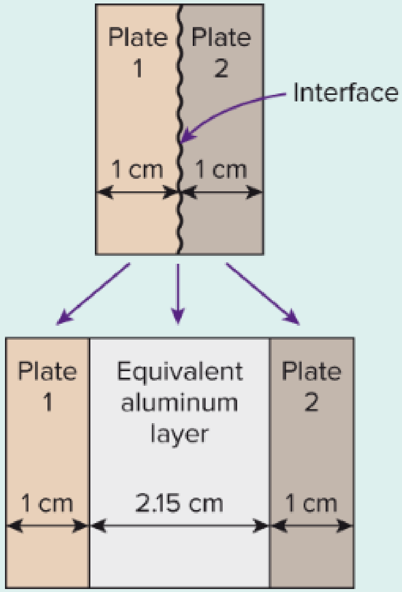


FIGURE 17–17
Schematic for **Example 17–4**.

SOLUTION

The thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance is to be determined.

SOLUTION

The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties

The thermal conductivity of aluminum at room temperature is k = 237 W/m·K (**Table A–24**).

Analysis

Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11{,}000\ \text{W/m}^2\cdot\text{K}} = 0.909 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

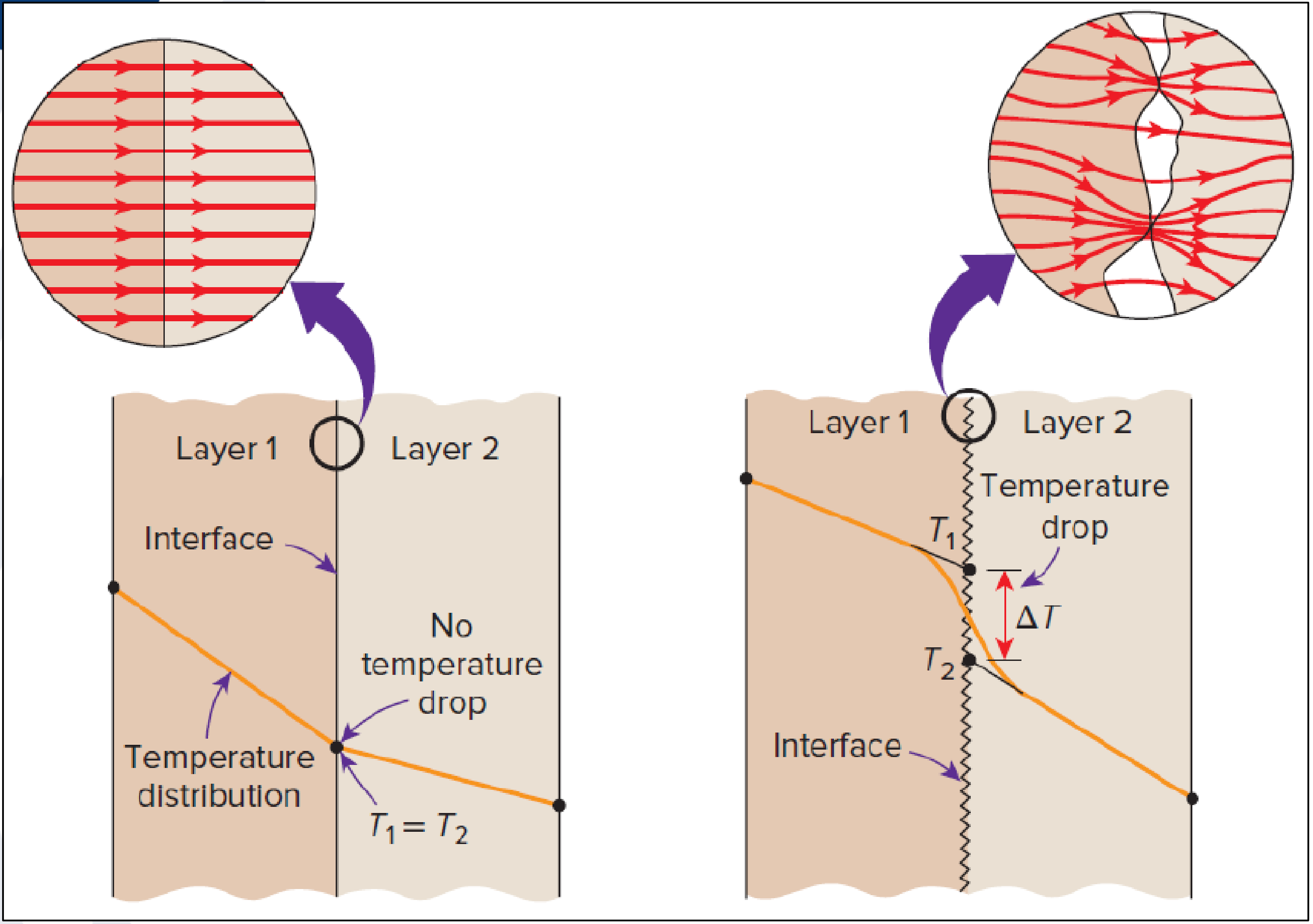
where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237\ \text{W/m}\cdot\text{K})(0.909 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}) = 0.0215\ \text{m} = \mathbf{2.15\ cm}$$

Discussion

Note that the interface between the two plates offers as much resistance to heat transfer as a 2.15-cm-thick aluminum plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

17-2 Thermal Contact Resistance



(*d*) Ideal (perfect) thermal contact

(*b*) Actual (imperfect) thermal contact

Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

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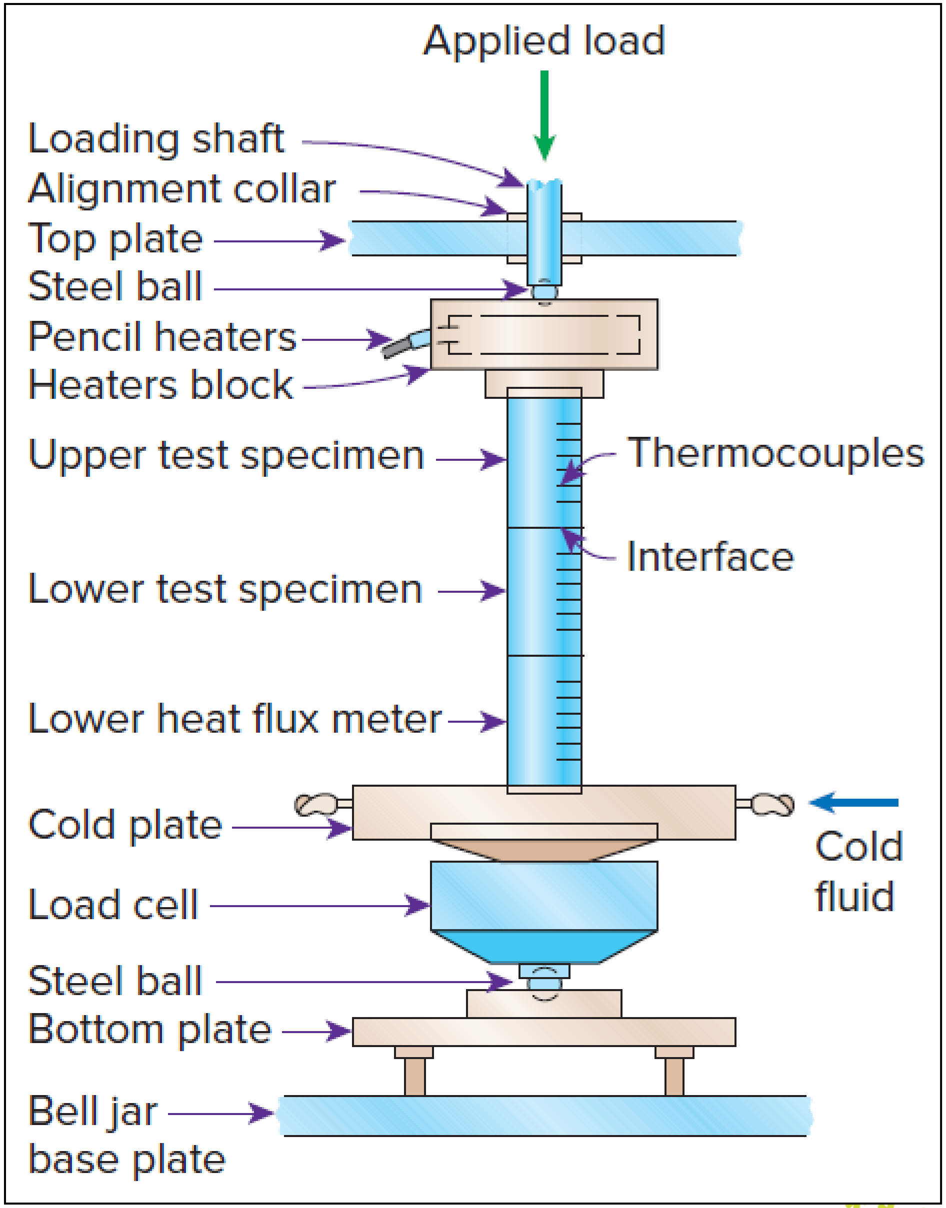
17-2 THERMAL CONTACT RESISTANCE-1

- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air.
- These numerous *air gaps* of varying sizes act as *insulation* because of the low thermal conductivity of air.
- Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the thermal contact resistance, R_c.

A typical experimental setup for the determination of thermal contact resistance



17-2 THERMAL CONTACT RESISTANCE-2

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

h_c **thermal contact conductance**

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \quad (\mathrm{W/m^2 \cdot {}^\circ C})$$

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \quad (\mathrm{m^2 \cdot {}^\circ C/W})$$

The value of thermal contact resistance depends on:

- ***surface roughness,***
- ***material properties,***
- ***temperature* and *pressure* at the interface**
- ***type of fluid* trapped at the interface.**

$$R_{c,\text{insulation}} = \frac{L}{k} = \frac{0.01\ \mathrm{m}}{0.04\ \mathrm{W/m \cdot {}^\circ C}} = 0.25\ \mathrm{m^2 \cdot {}^\circ C/W}$$

$$R_{c,\text{copper}} = \frac{L}{k} = \frac{0.01\ \mathrm{m}}{386\ \mathrm{W/m \cdot {}^\circ C}} = 0.000026\ \mathrm{m^2 \cdot {}^\circ C/W}$$

Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals but can be disregarded for poor heat conductors such as insulations.

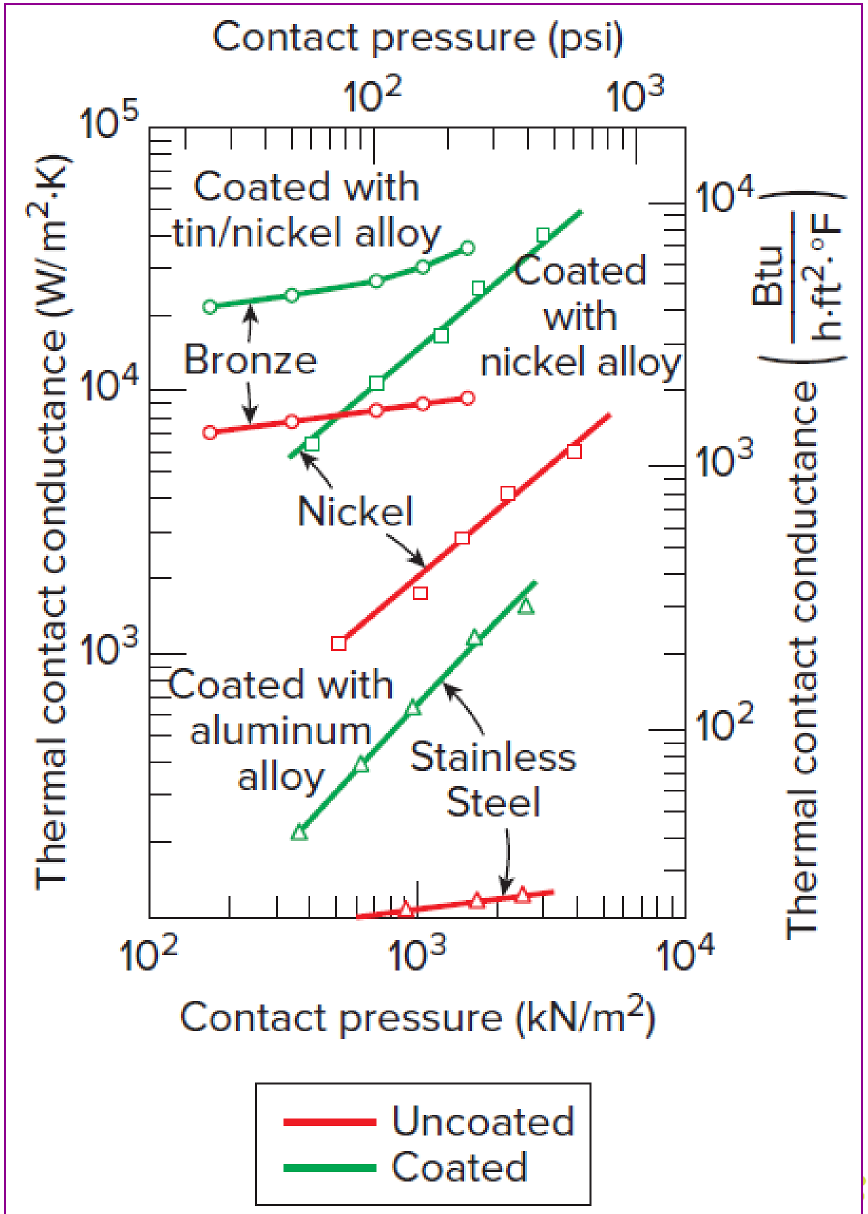
17-2 THERMAL CONTACT RESISTANCE-3

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, 1969).

Fluid at the Interface	Contact conductance, $h_c, \mathrm{W/m^2.K}$
Air	3640
Helium	9520
Hydrogen	13.900
Silicone Oil	19,000
Glycerin	37,700

The thermal contact resistance can be minimized by applying

- **a *thermal grease* such as silicon oil**
- **a *better conducting gas* such as helium or hydrogen**
- **a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum**



Effect of metallic coatings on thermal contact conductance

17-2 THERMAL CONTACT RESISTANCE-4

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, ^oC	Pressure, MPa	h_c, * W/m^2-K
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90-200	0.17-2.5	3800
304 Stainless steel	Ground	1.14	20	4-7	1900
Aluminum	Ground	2.54	150	1.2-2.5	11,400
Copper	Ground	1.27	20	1.2-20	143,000
Copper	Milled	3.81	20	1-5	55,500
Copper (vacuum)	Milled	0.25	30	0.17-7	11,400
Dissimilar Metal Pairs					
Stainless steel-				10	2900
Aluminum		20-30	20	20	3600
Stainless steel-				10	16,400
Aluminum		1.0-2.0	20	20	20,800
Steel Ct-30-				10	50,000
Aluminum	Ground	1.4-2.0	20	15-35	59,000
Steel Ct-30-				10	4800
Aluminum	Milled	4.5-7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.17-1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4-4.5	20	20-35	22,000

The *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure.*

EXAMPLE 17–5 Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate (k = 386 W/m·K) by screws that exert an average pressure of 6 MPa (**Fig. 17–18**). The base area of each transistor is 8 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m^2·K. If the case temperature of the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case–plate interface.

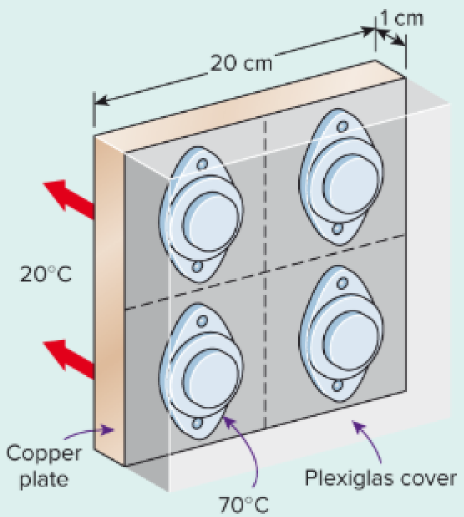


FIGURE 17–18
Schematic for **Example 17–5**.

FIGURE 17–18
Schematic for **Example 17–5**.

SOLUTION

Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions

1 Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. **4** Thermal conductivities are constant.

Properties

The thermal conductivity of copper is given to be k = 386 W/m·K. The contact conductance is obtained from **Table 17–2** to be h_c = 42,000 W/m^2·K, which corresponds to copper–aluminum interface for the case of 1.17–1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis

The contact area between the case and the plate is given to be 8 cm^2, and the plate area for each transistor is 100 cm^2. The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42{,}000\ \text{W/m}^2\cdot\text{K})(8 \times 10^{-4}\ \text{m}^2)} = 0.030°\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m}\cdot\text{K})(0.01\ \text{m}^2)} = 0.0026°\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25\ \text{W/m}^2\cdot\text{K})(0.01\ \text{m}^2)} = 4.0°\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326°\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70 - 20)°\text{C}}{4.0326°\text{C/W}} = \mathbf{12.4\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}} = (12.4\ \text{W})(0.030°\text{C/W}) = \mathbf{0.37°C}$$

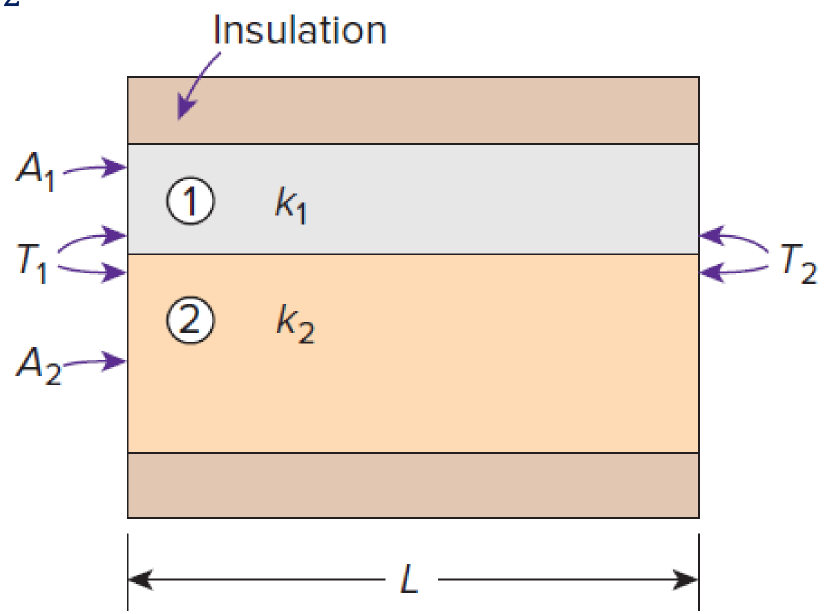
which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we lower the operating temperature of the transistor in this case by less than 0.4°C.

17-3 Generalized Thermal Resistance Networks

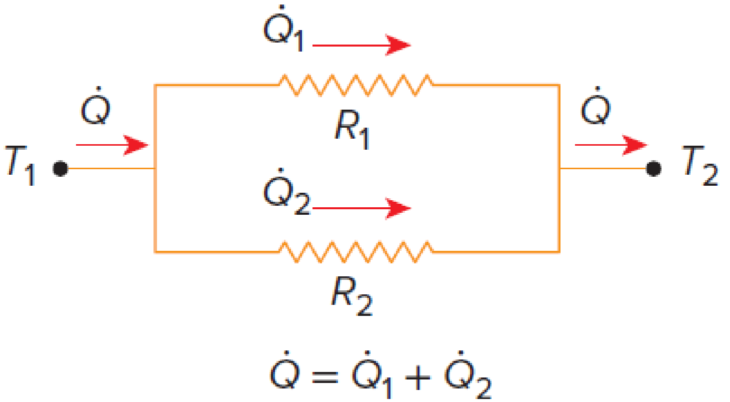
$$\dot{Q}=\dot{Q}_1+\dot{Q}_2=\frac{T_1-T_2}{R_1}+\frac{T_1-T_2}{R_2}=(T_1-T_2)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\dot{Q}=\frac{T_1-T_2}{R_{total}}$$

$$\frac{1}{R_{total}}=\frac{1}{R_1}+\frac{1}{R_2}\rightarrow R_{total}=\frac{R_1R_2}{R_1+R_2}$$



Thermal resistance network for two parallel layers.



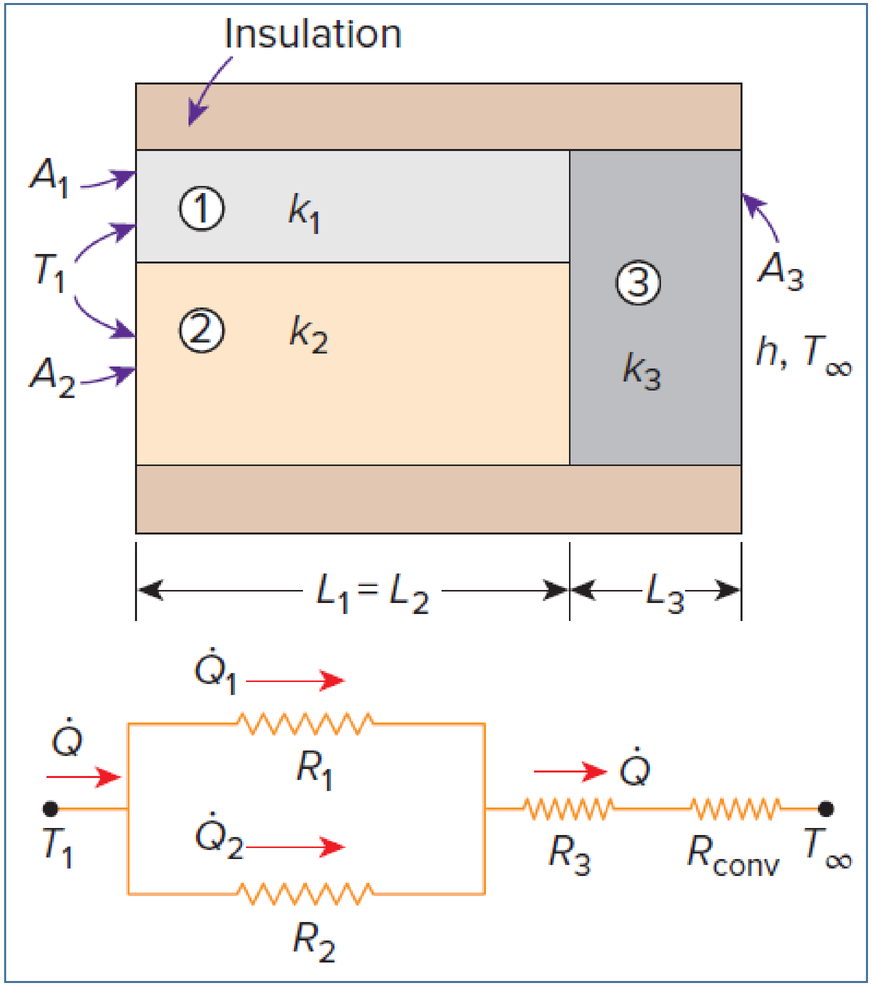
17-3 GENERALIZED Thermal RESISTANCE NETWORKS-1

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{\text{conv}} = \frac{1}{h A_3}$$



Thermal resistance network for combined series-parallel arrangement.

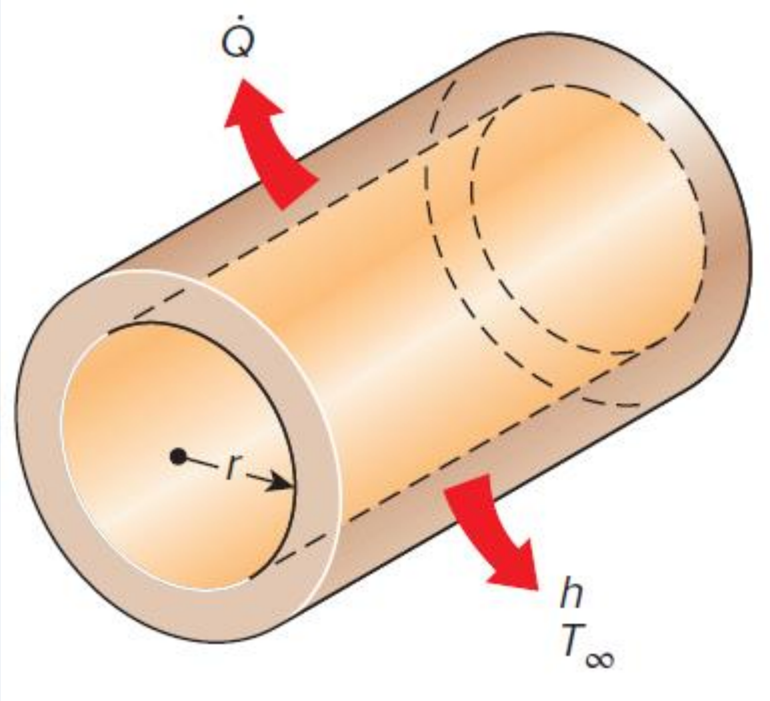
Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1) any plane wall normal to the *x*-axis is *isothermal* (i.e., to assume the temperature to vary in the *x*-direction only)

(2) any plane parallel to the *x*-axis is *adiabatic* (i.e., to assume heat transfer to occur in the *x*-direction only)

Do they give the same result?

17-4 Heat Conduction In Cylinders And Spheres



Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

Heat transfer through the pipe can be modeled as ***steady*** and ***one-dimensional.***

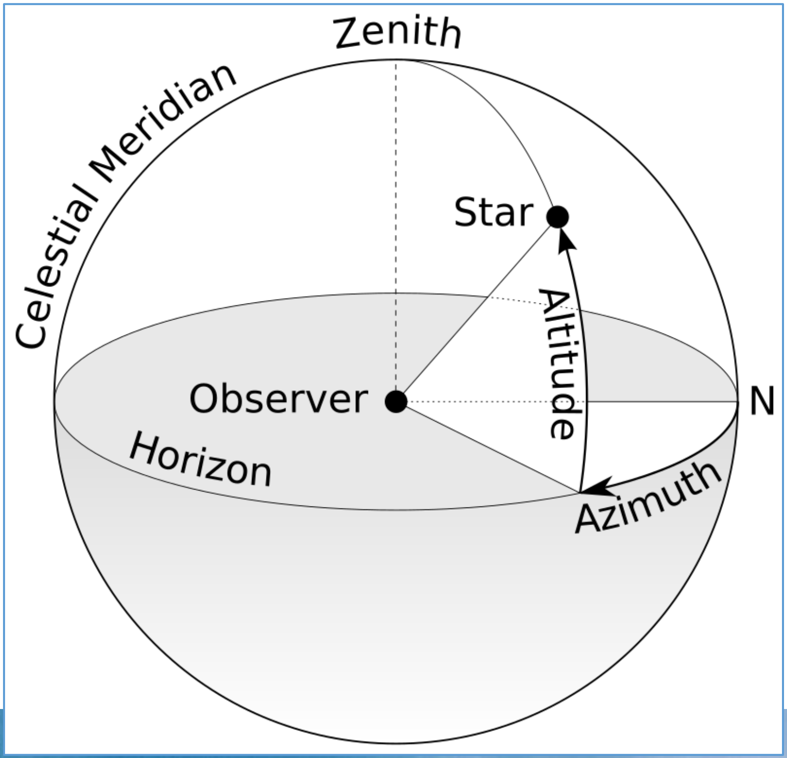
The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$.

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.

Azimuthal Angle

An azimuth is an angular measurement in a spherical coordinate system. The vector from an observer to a point of interest is projected perpendicularly onto a reference plane; the angle between the projected vector and a reference vector on the reference plane is called the azimuth. [Wikipedia](https://en.wikipedia.org/wiki/Azimuth).



EXAMPLE 17–6 Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks (k = 0.72 W/m·K) separated by 3-cm-thick plaster layers (k = 0.22 W/m·K). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k = 0.026 W/m·K) on the inner side of the wall, as shown in Fig. 17–21. The indoor and the outdoor temperatures are 20°C and −10°C, respectively, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m^2·K and h_2 = 25 W/m^2·K, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

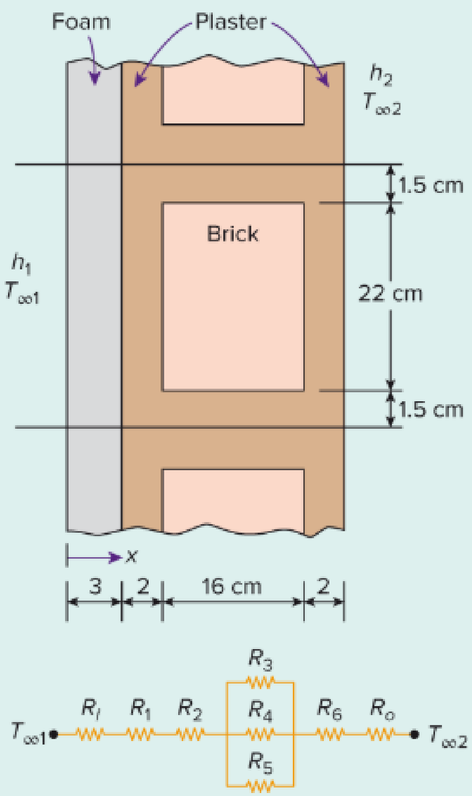


FIGURE 17–21
Schematic for Example 17–6.

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SOLUTION

The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions

1 Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties

The thermal conductivities are given to be k = 0.72 W/m·K for bricks, k = 0.22 W/m·K for plaster layers, and k = 0.026 W/m·K for the rigid foam.

Analysis

There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in **Fig. 17–21**. The individual resistances are evaluated as

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.40°\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 4.62°\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)}$$

$$= 0.36°\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m·K})(0.015 \times 1 \text{ m}^2)}$$

$$= 48.48°\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m·K})(0.22 \times 1 \text{ m}^2)} = 1.01°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2\text{·K})(0.25 \times 1 \text{ m}^2)} = 0.16°\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/°C}$$

which gives

$$R_{\text{mid}} = 0.97°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$R_{\text{total}} = R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o$$
$$= 0.40 + 4.62 + 0.36 + 0.97 + 0.36 + 0.16$$
$$= 6.87°\text{C/W}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{6.87°\text{C/W}} = 4.37 \text{ W} \quad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

or 4.37/0.25 = 17.5 W/m^2 area. The total area of the wall is A = 3 m × 5 m = 15 m^2. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \mathbf{263 \text{ W}}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion

In the above solution, we assumed the temperature at any cross section of the wall normal to the *x*-direction to be *isothermal*. We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the *x*-direction to be *adiabatic*. The thermal resistance network in this case will be as shown in **Fig. 17–22**. By following the approach outlined above, the total thermal resistance in this case is determined to be R_{total} = 6.97°C/W, which is very close to the value 6.85°C/W obtained before. Thus either approach gives roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

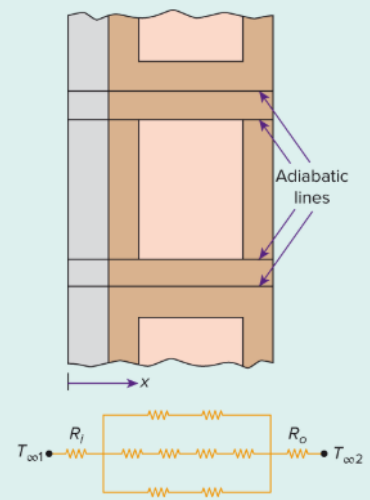
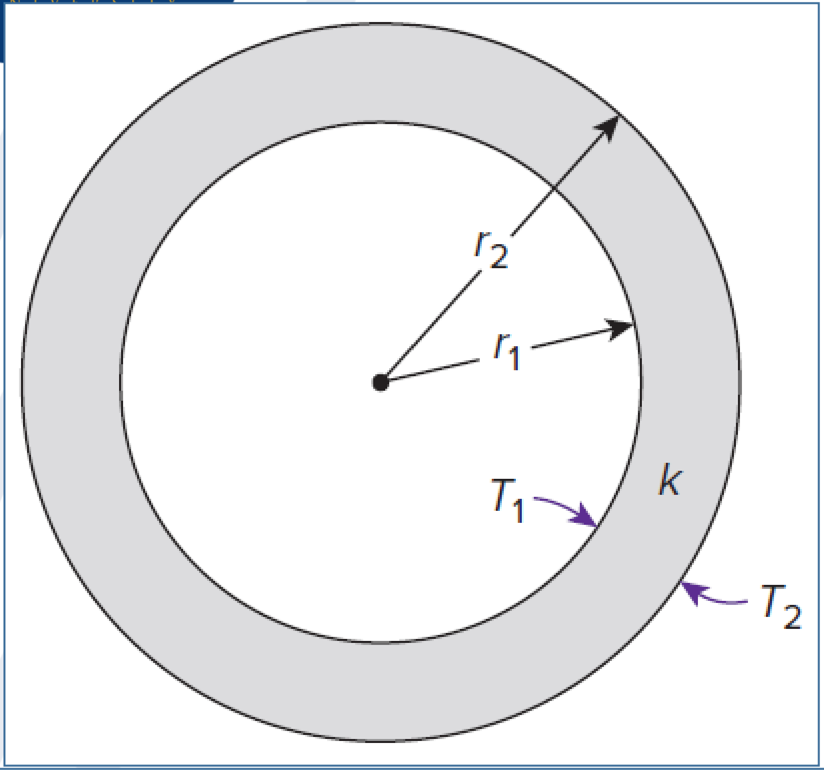


FIGURE 17–22
Alternative thermal resistance network for **Example 17–6** for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.

17-3 Generalized Thermal Resistance Networks-2



$$\dot{Q}_{\text{cond,cyl}} = -kA\frac{dT}{dr} \quad (\text{W})$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond,cyl}}}{A}\, dr = -\int_{T=T_1}^{T_2} k\, dT$$

$A = 2\pi r L$

$$\dot{Q}_{cond,cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad (\text{W})$$

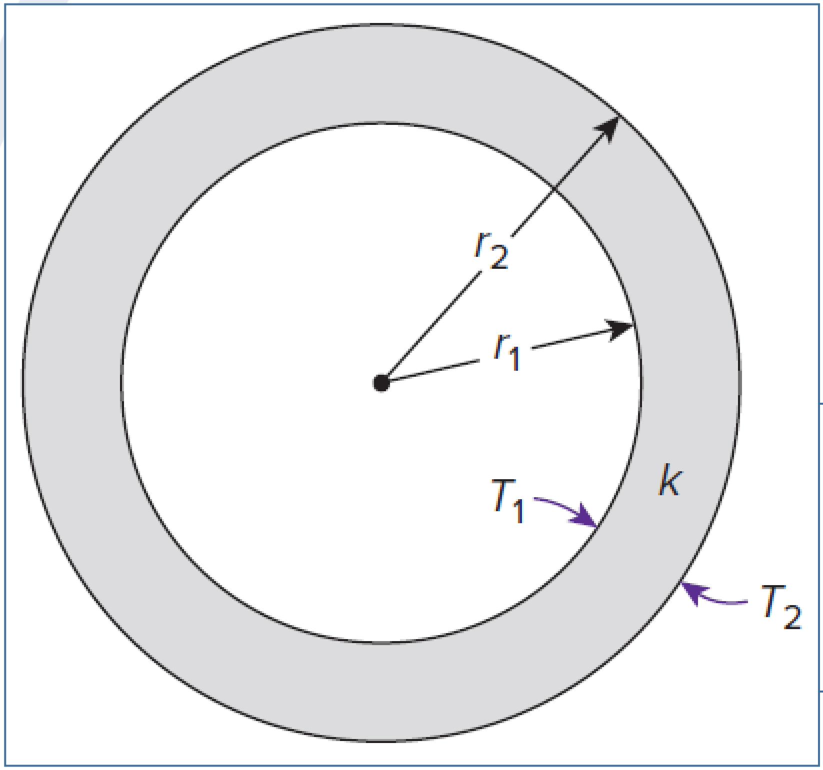
A long cylindrical pipe with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond,cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad (\text{W})$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius}/\text{Inner radius})}{2\pi \times \text{Lenght} \times \text{Thermal conductivity}}$$

***Conduction resistance* of the cylinder layer**

17-3 Generalized Thermal Resistance Networks-3



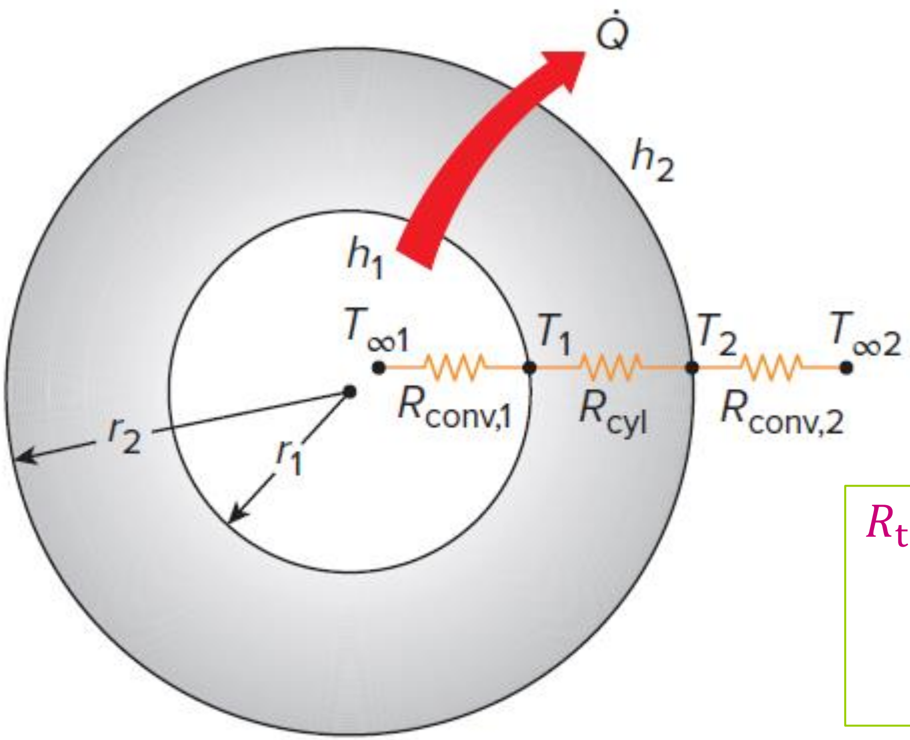
A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond,sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius}) - (\text{Thermal conductivity})}$$

Conduction resistance **of the spherical layer**

17-3 Generalized Thermal Resistance Networks-4



$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{cyl}} + R_{\text{conv},2}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

for a *cylindrical* layer

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{cyl}} + R_{\text{conv},2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *spherical* layer

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{sph}} + R_{\text{conv},2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

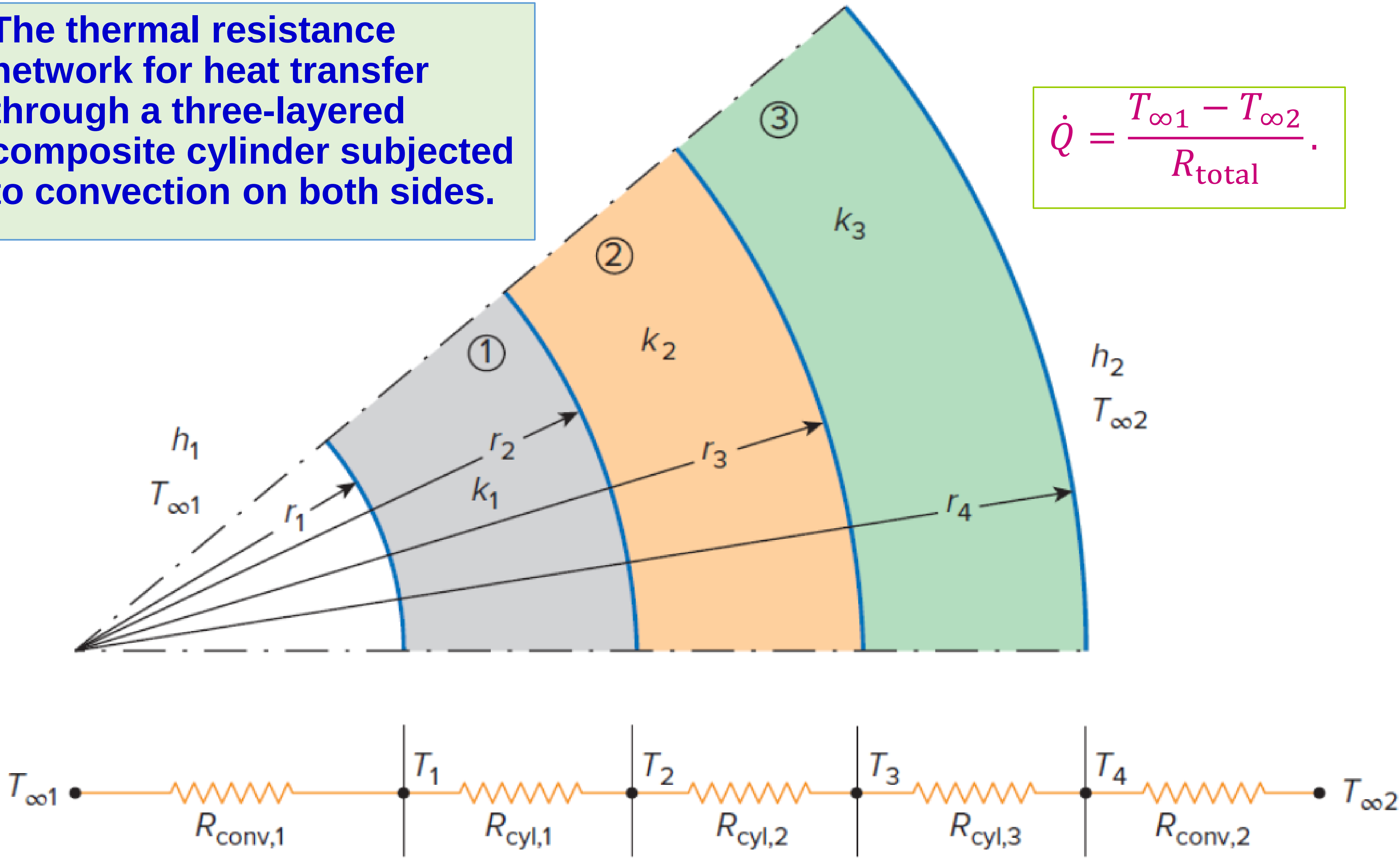
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

Multilayered Cylinders and Spheres

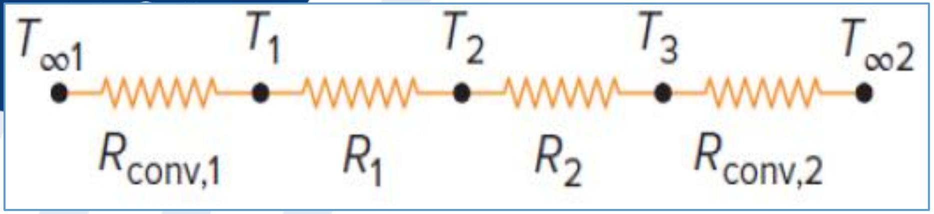
$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1A_1} + \frac{\ln(r_2/r_1)}{2\pi Lk_1} + \frac{\ln(r_3/r_2)}{2\pi Lk_2} + \frac{\ln(r_4/r_3)}{2\pi Lk_3} + \frac{1}{h_2A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}.$$



Multilayered Cylinders and Spheres-1



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1} = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}} = \cdots$$

The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

Once heat transfer rate *Q* has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{cyl,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

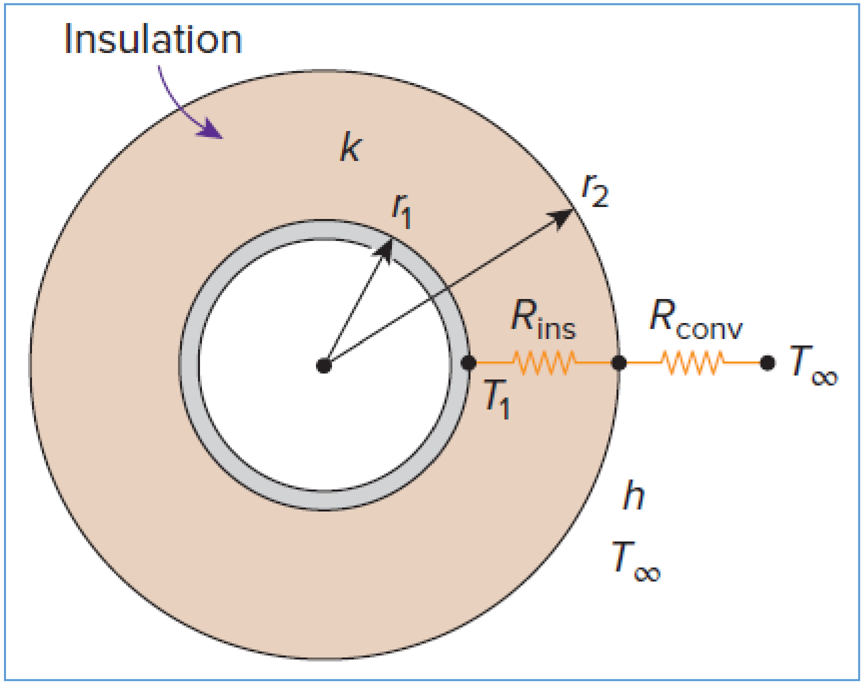
$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv,2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$

17-5 Radius Of Insulation Critical

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

The heat transfer from the pipe may increase or decrease, depending on which effect dominates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - \infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$

EXAMPLE 17–8 Heat Loss through an Insulated Steam Pipe

Steam at $T_{\infty 1}$ = 320°C flows in a cast iron pipe (k = 80 W/m·K) whose inner and outer diameters are D_1 = 5 cm and D_2 = 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with k = 0.05 W/m·K. Heat is lost to the surroundings at $T_{\infty 2}$ = 5°C by natural convection and radiation, with a combined heat transfer coefficient of h_2 = 18 W/m^2·K. Taking the heat transfer coefficient inside the pipe to be h_1 = 60 W/m^2·K, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION

A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions

1 Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

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Properties

The thermal conductivities are given to be k = 80 W/m·K for cast iron and k = 0.05 W/m·K for glass wool insulation.

Analysis

The thermal resistance network for this problem involves four resistances in series and is given in Fig. 17–29. Taking L = 1 m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

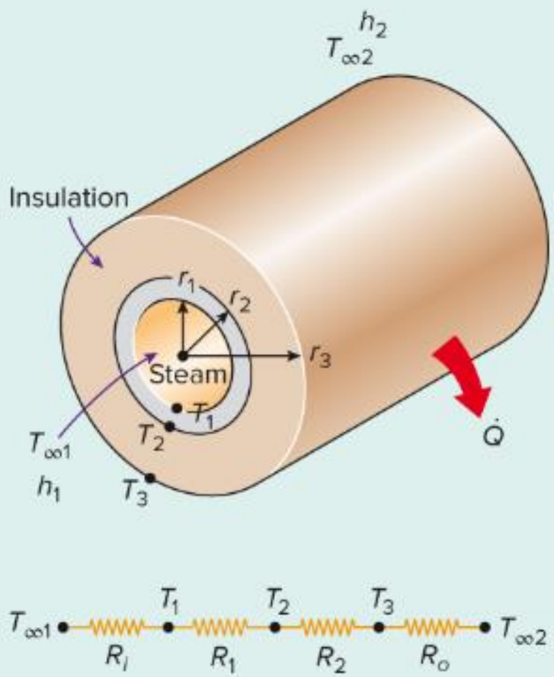

FIGURE 17–29
Schematic for Example 17–8.

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(60\ \text{W/m}^2\cdot\text{K})(0.157\ \text{m}^2)} = 0.106°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\ \text{W/m}\cdot\text{K})(1\ \text{m})} = 0.0002°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\ \text{W/m}\cdot\text{K})(1\ \text{m})} = 2.35°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18\ \text{W/m}^2\cdot\text{K})(0.361\ \text{m}^2)} = 0.154°\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)°\text{C}}{2.61°\text{C/W}} = \mathbf{121\ W} \quad \text{(per meter pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length *L*.

The temperature drops across the pipe and the insulation are determined from **Eq. 17–17** to be

$$\Delta T_{\text{pipe}} = \dot{Q}R_{\text{pipe}} = (121\ \text{W})(0.0002°\text{C/W}) = \mathbf{0.02°C}$$

$$\Delta T_{\text{insulation}} = \dot{Q}R_{\text{insulation}} = (121\ \text{W})(2.35°\text{C/W}) = \mathbf{284°C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

Discussion

Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

Summary

- **Steady Heat Conduction in Plane Walls**
  - Thermal Resistance Concept
  - Thermal Resistance Network
  - Multilayer Plane Walls
- **Thermal Contact Resistance**
- **Generalized Thermal Resistance Networks**
- **Heat Conduction in Cylinders and Spheres**
  - Multilayered Cylinders and Spheres