Signals and Circuits

ENGR 35500

Equivalent Circuit

Chapter 2-3 (Equivalent Circuits) pp. 54-62

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

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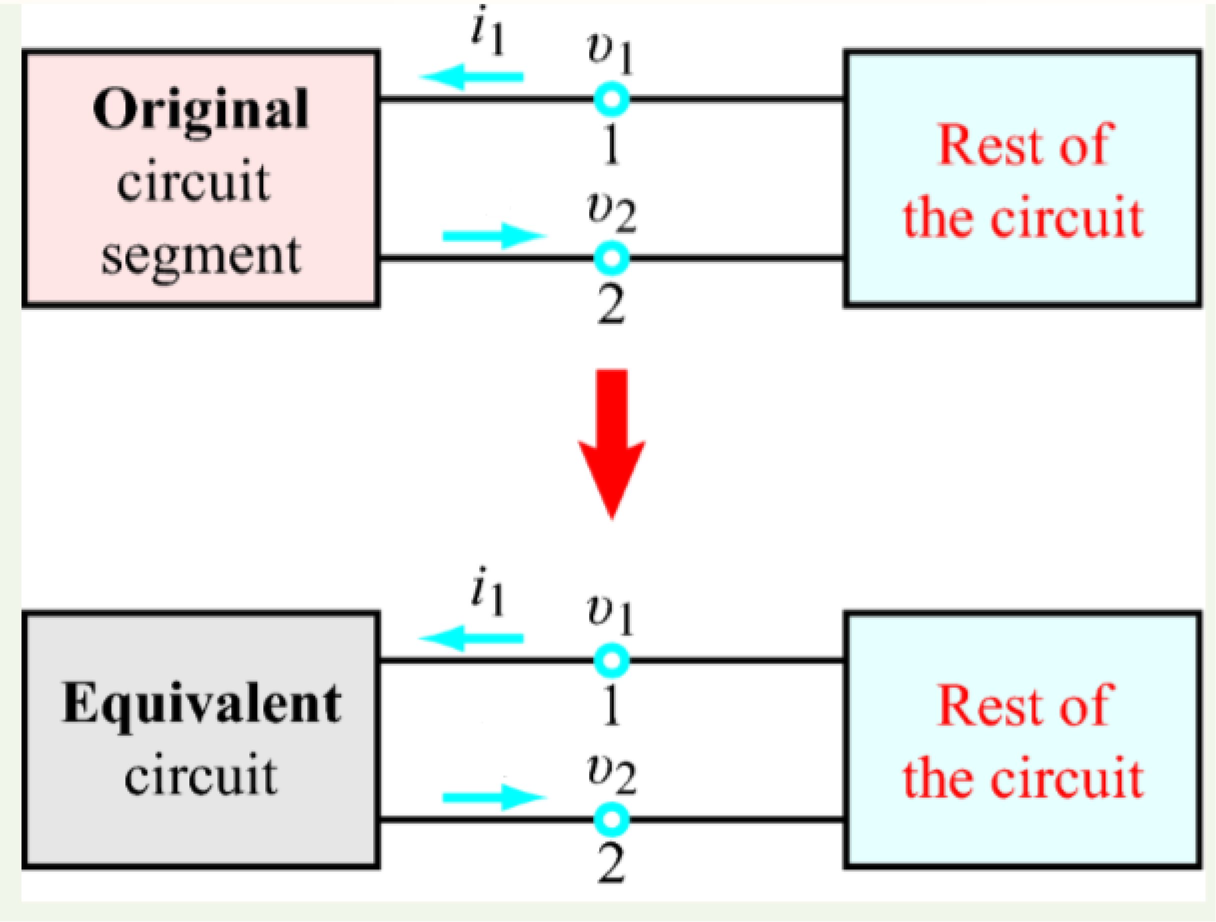
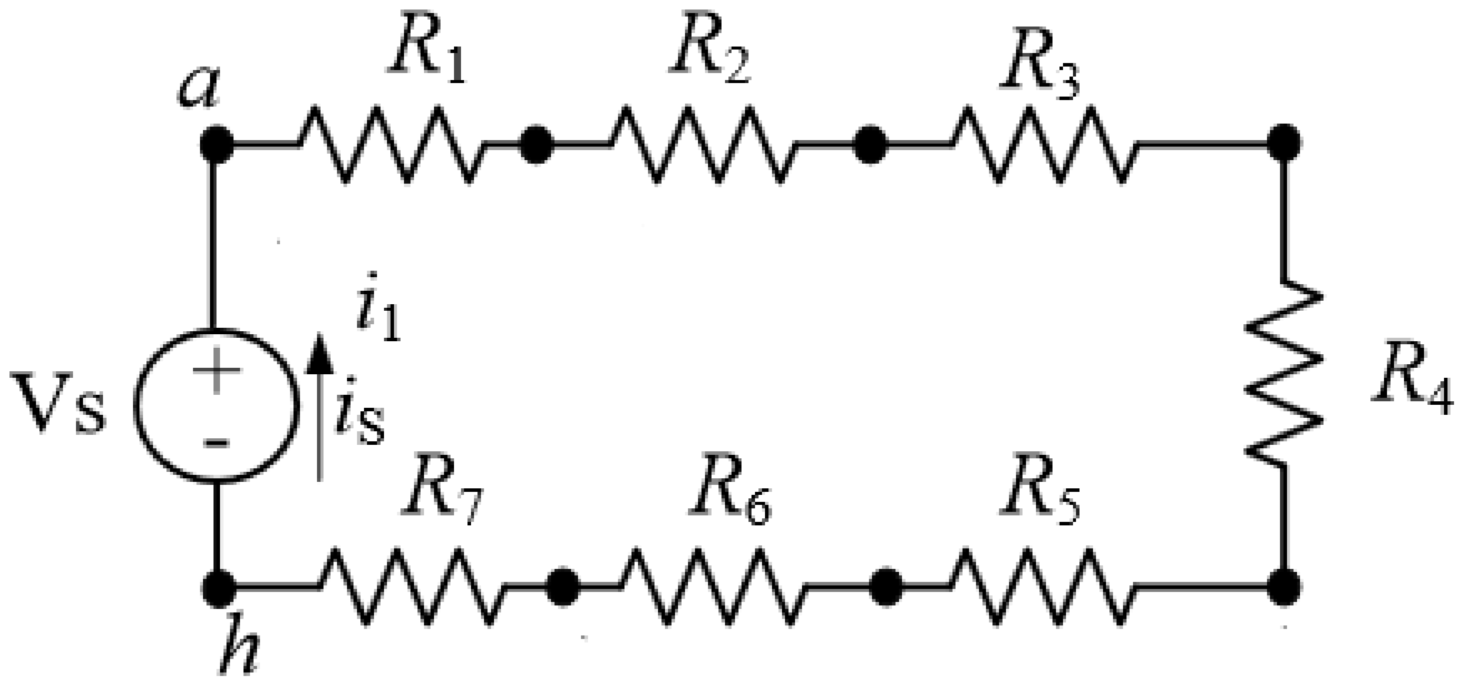
Circuit Equivalence



Two circuits connected between a pair of nodes are considered to be equivalent-as seen by the rest of the circuit-if they exhibit identical *i* - *v* characteristics at those nodes.

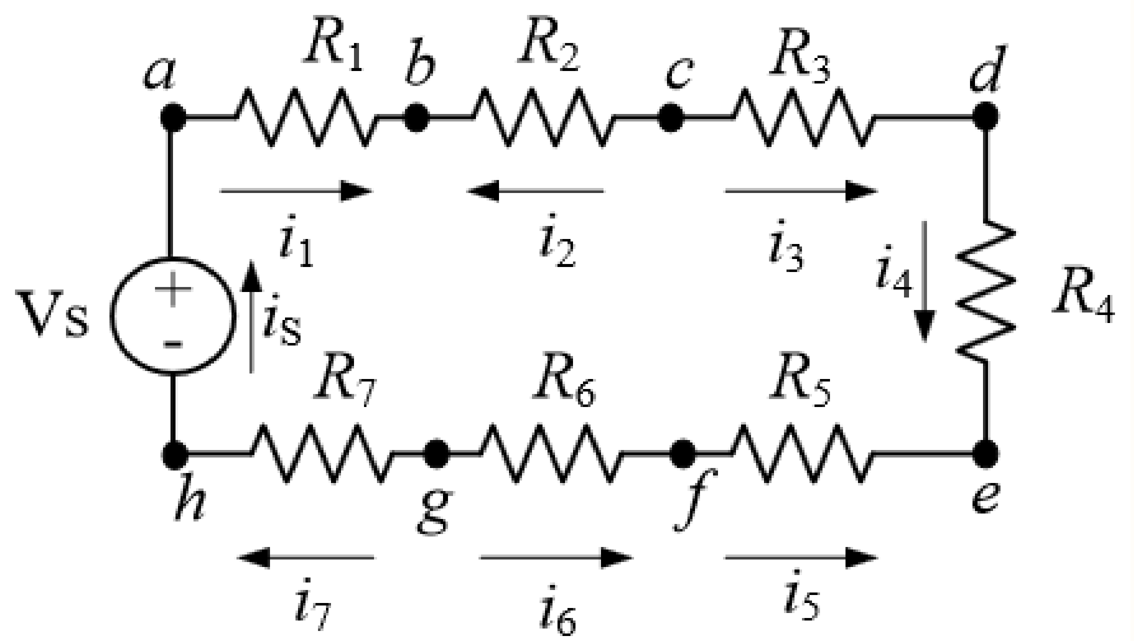
Resistors in Series

Find the relationship between the resistors, $\boldsymbol{i_s}$ and $\boldsymbol{V_s}$



Resistors in Series

Find the relationship between the resistors, ***i_s*** and ***V_s***



Share the same current: $i_S = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7$

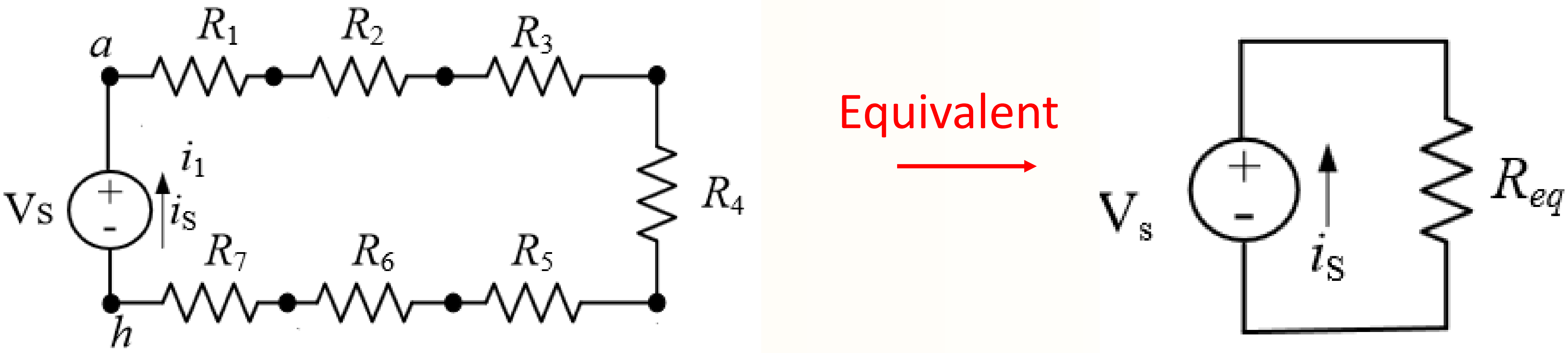
KVL: $-V_S + i_S R_1 + i_S R_2 + i_S R_3 + i_S R_4 + i_S R_5 + i_S R_6 + i_S R_7 = 0$

$\rightarrow \; V_S = i_S(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)$

For nodes a and h $\quad V_S = i_S(R_{eq})$

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

Resistors in Series



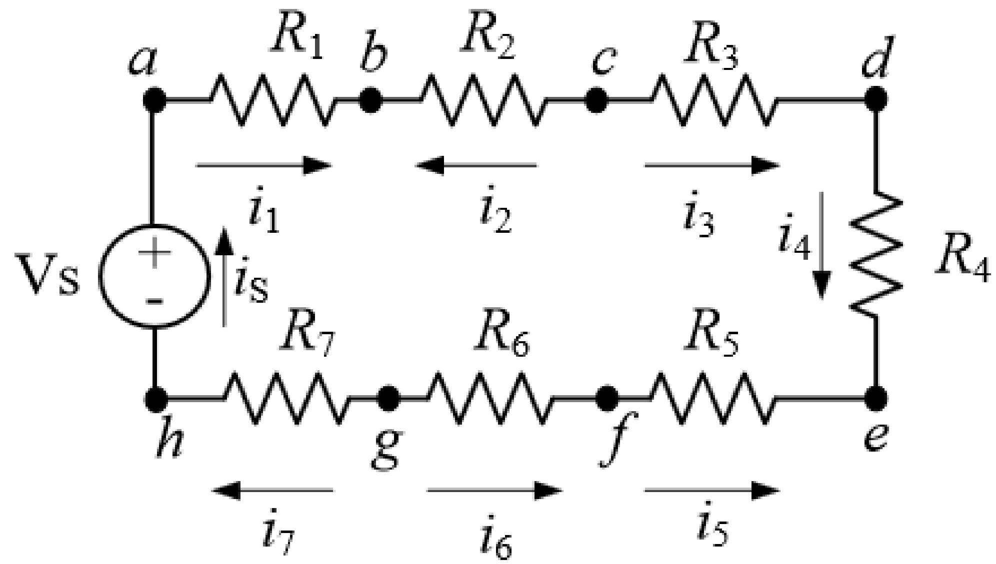
$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

$$\rightarrow \boxed{R_{eq} = \sum_{i=1}^{k} R_i = (R_1 + R_2 + \cdots + R_k)}$$

Multiple Resistors connected in series (experiencing the same current) can be combined into a single equivalent resistor R_{eq} whose resistance is equal to the sum of all of their individual resistances.

Resistors in Series

Find the voltage cross *R1* and *R5*



Ohm's law

$$V_{ab} = R_1 i_s$$

$$V_{ef} = R_5 i_s$$

And $R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$

$$V_s = i_s(R_{eq})$$

$$V_1 = \frac{R_1}{R_1 + R_2 + \cdots R_7} V_s$$

$$V_5 = \frac{R_5}{R_1 + R_2 + \cdots R_7} V_s$$

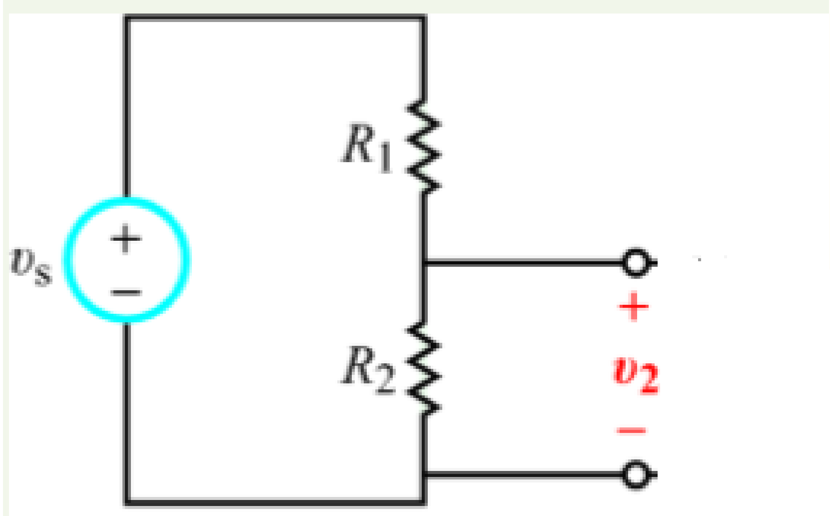
Resistors in Series

Voltage divider

The voltage across any individual resistor R_i in a series circuit is a proportionate fraction (R_i/R_{eq}) of the voltage of the entire group

$$V_i = \frac{R_i}{R_{eq}} V_s$$

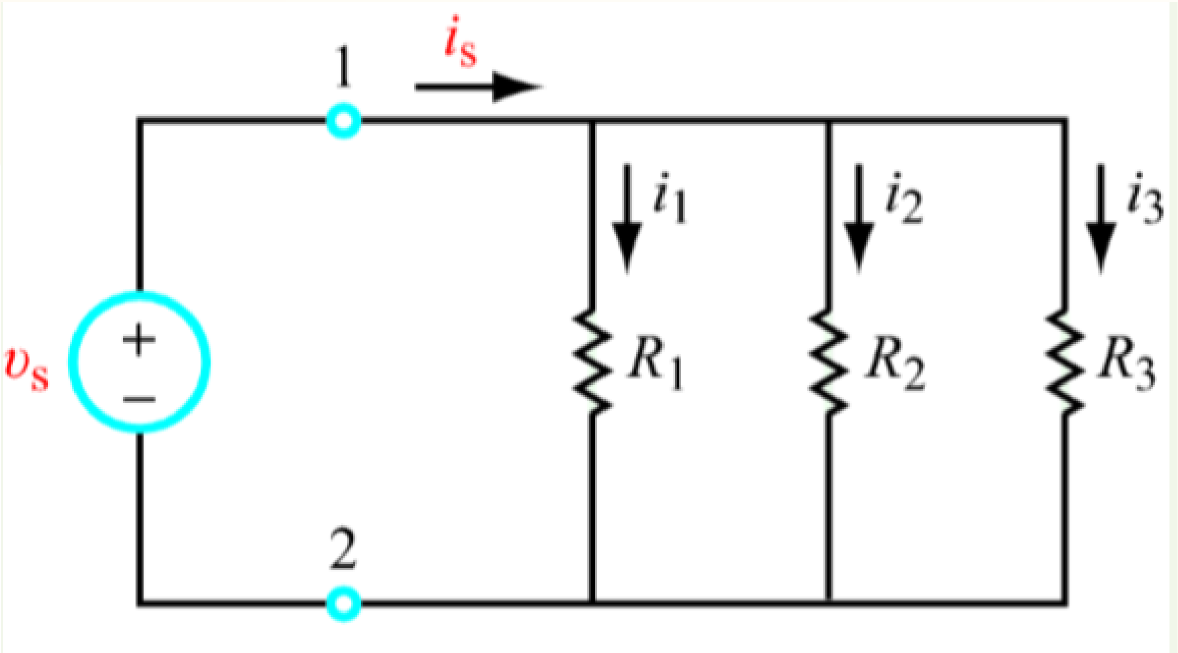
Example:



$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

Resistors in parallel

Find the relationship between i_s, v_s, R_1, R_2, R_3



Apply ohm's law

$$i_1 = \frac{v_s}{R_1} \qquad i_2 = \frac{v_s}{R_2} \qquad i_3 = \frac{v_s}{R_3}$$

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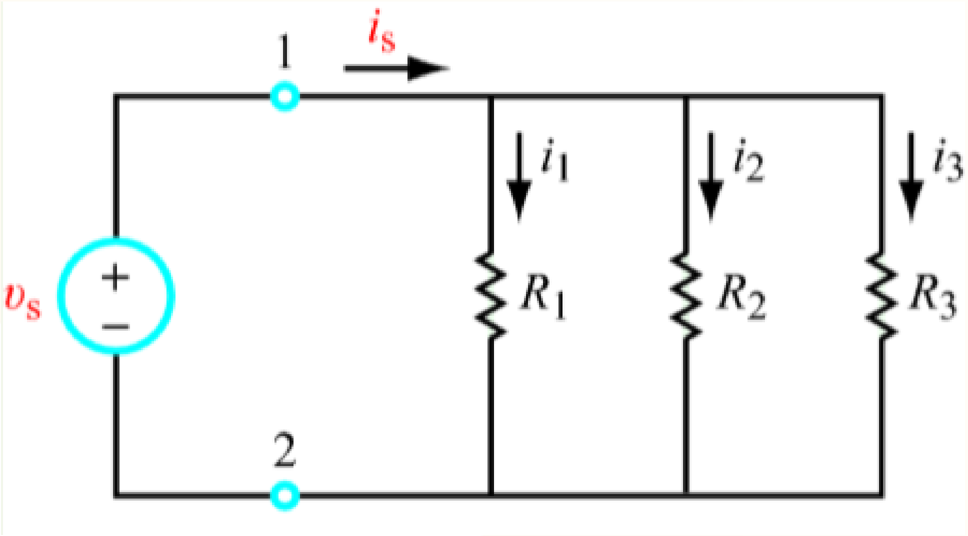
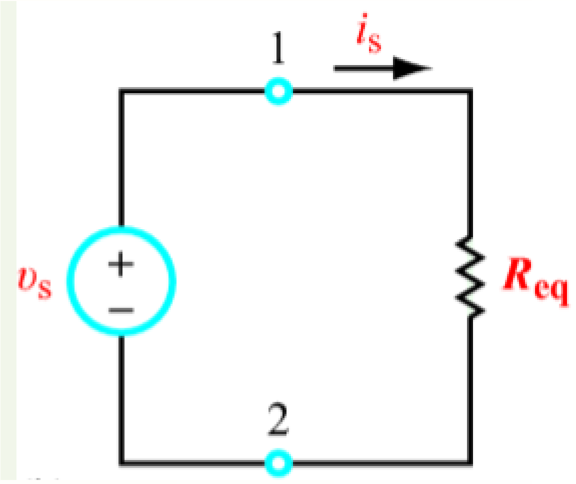
$$i_s = i_1 + i_2 + i_3 = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3}$$

$$\frac{i_s}{v_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in Parallel



Equivalent

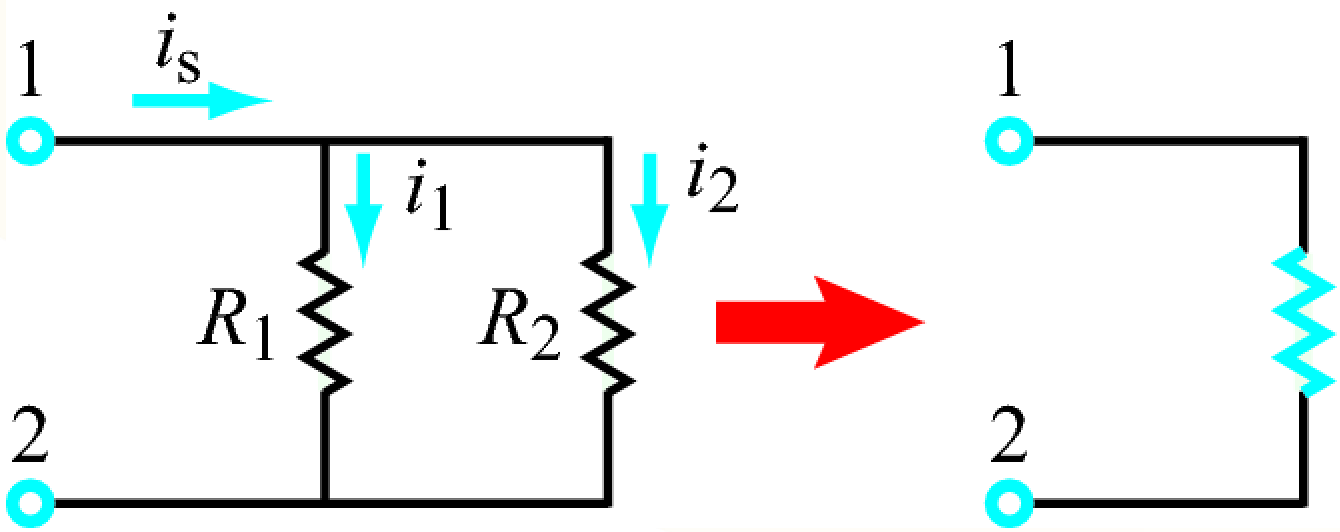


$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

For short, we sometimes denote such a parallel R_1 and R_2 combination as $R_1||R_2$.

Current divider

Current Division

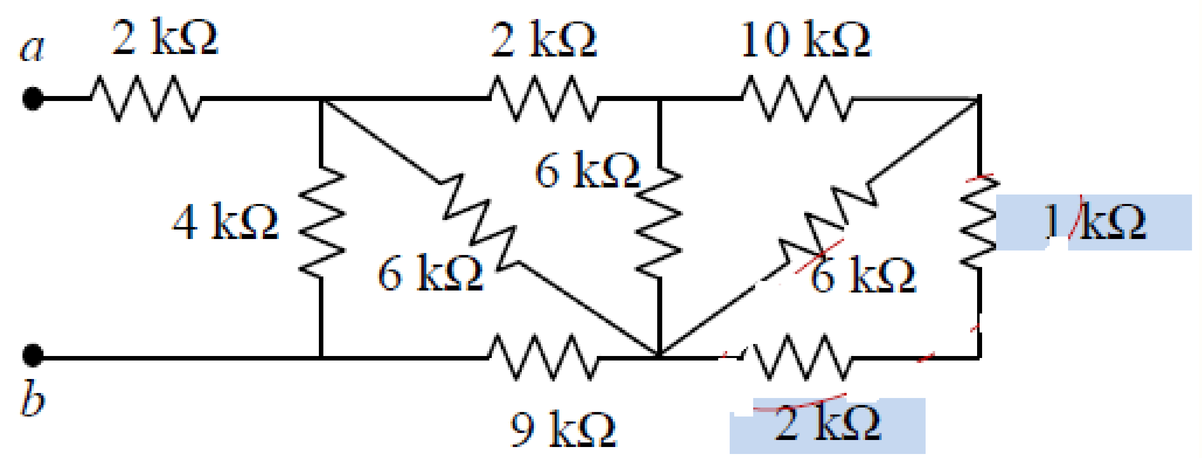


$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

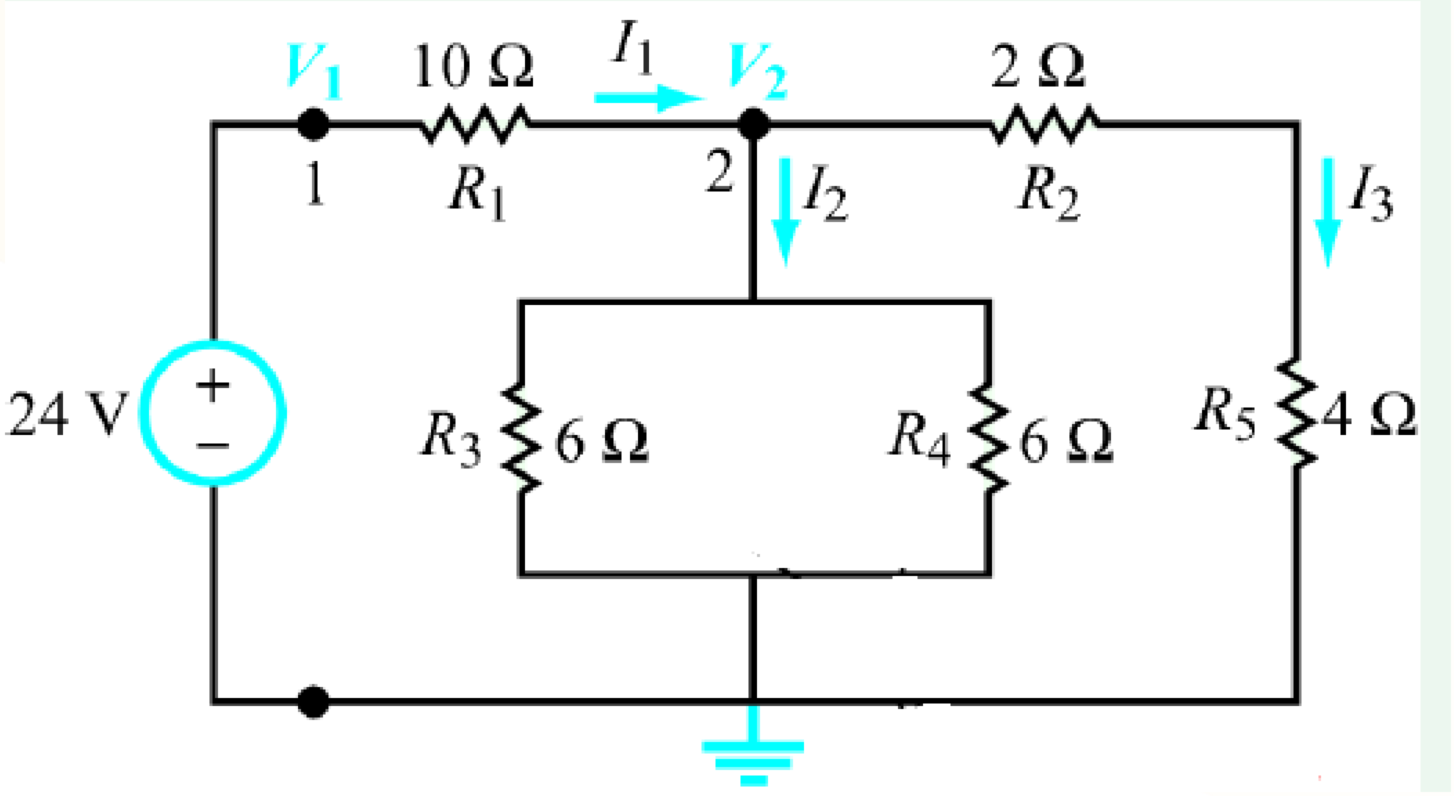
Exercise

Example: Find the equivalent resistance between “a” and “b”

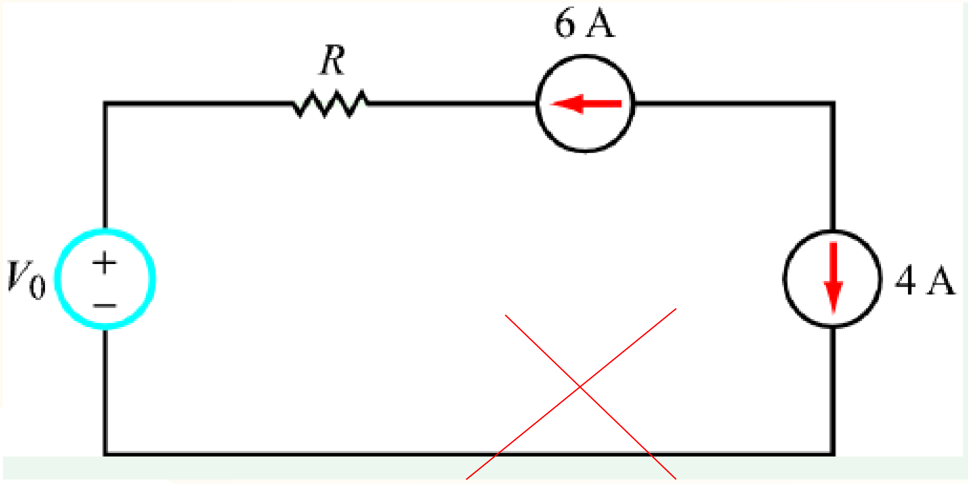


Exercise

Use the equivalent-resistance approach to determine V_2, I_1, I_2, and I_3 in the circuit.

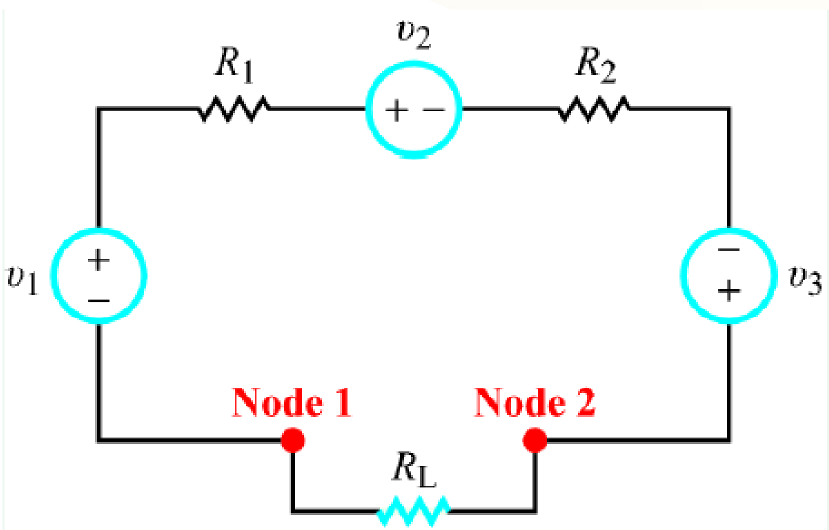


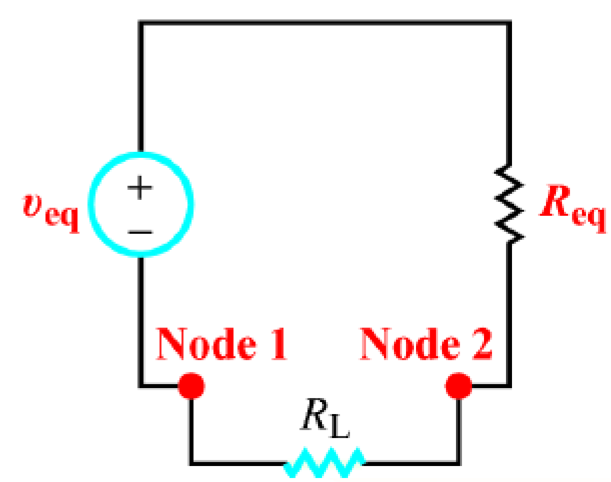
Source in series



?

If the two current sources provide the same value current and in the same direction, so ?

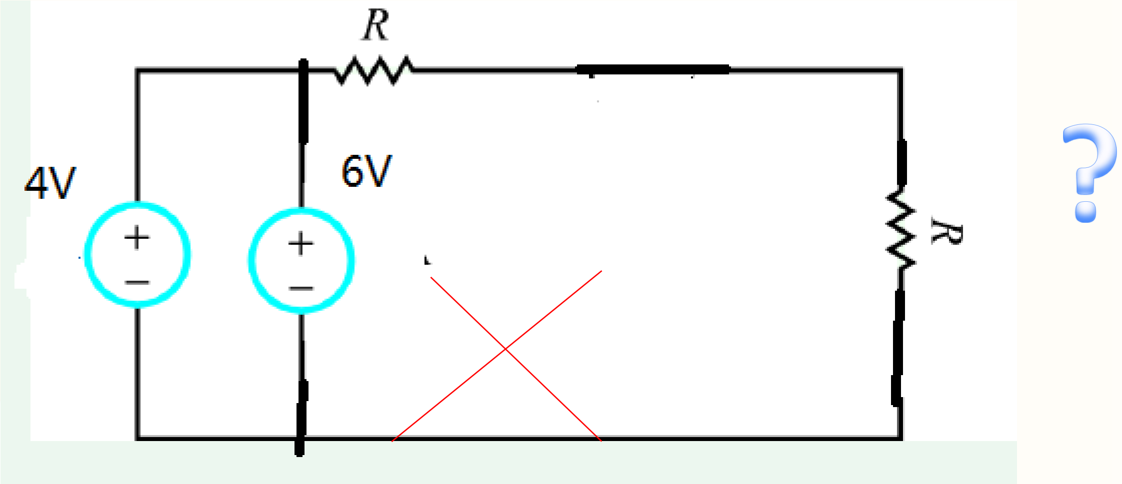




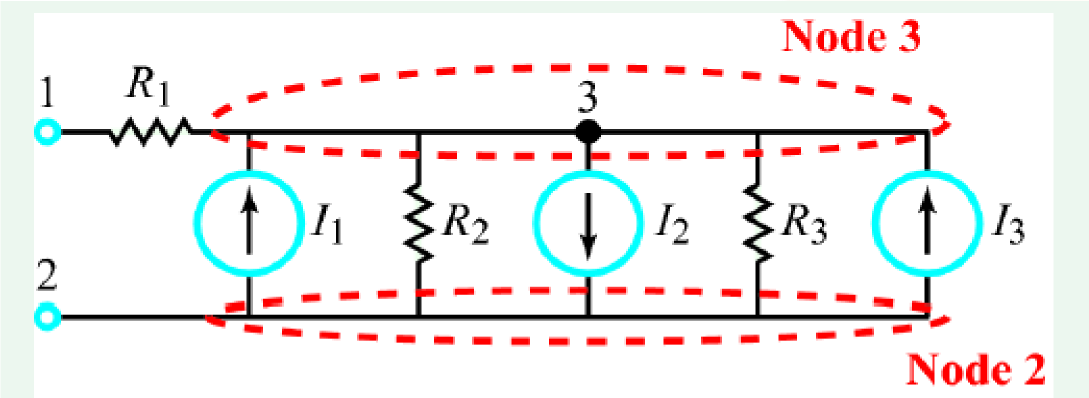
$$v_{eq} = v_1 - v_2 + v_3 \qquad R_{eq} = R_1 + R_2$$

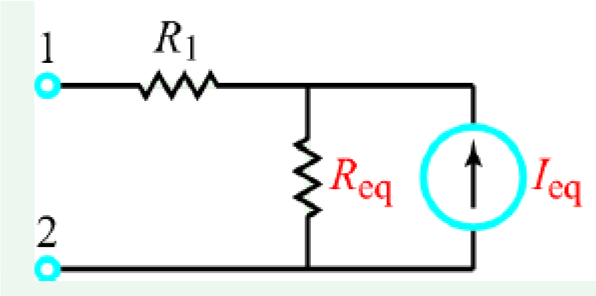
Multiple voltage source connected in series can be combined into an equivalent voltage source whose voltage is equal to the algebraic sum of the voltages of the individual sources.

Source in parallel



If the two voltage sources provide the some voltage and in the same direction, so ?





$$R_{eq} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} \qquad I_{eq} = I_1 - I_2 + I_3$$

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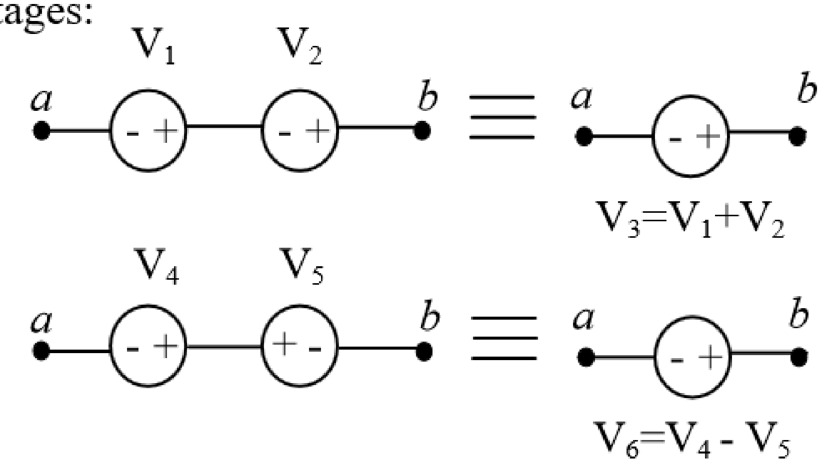
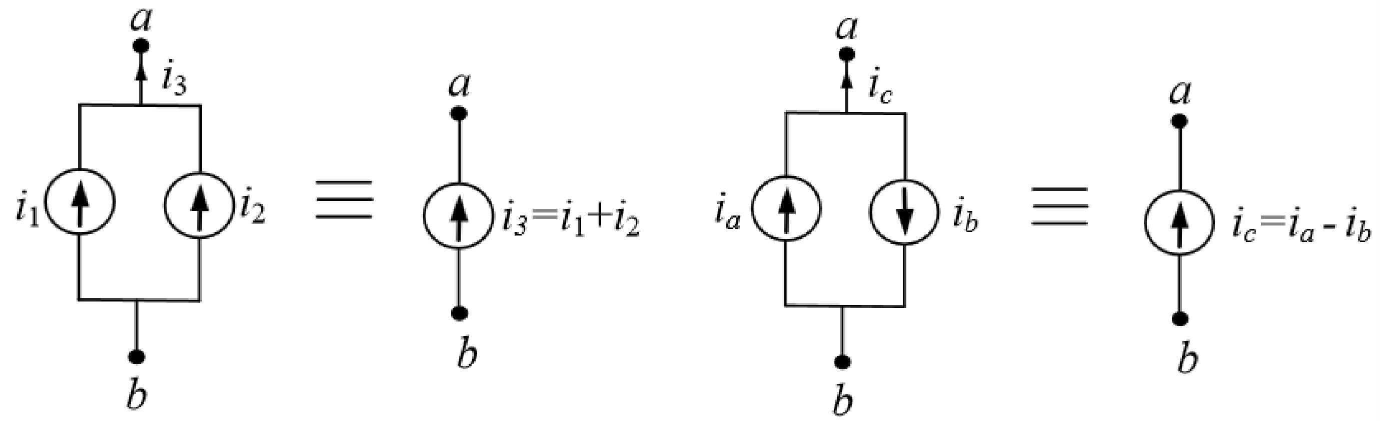
Source in Series/parallel

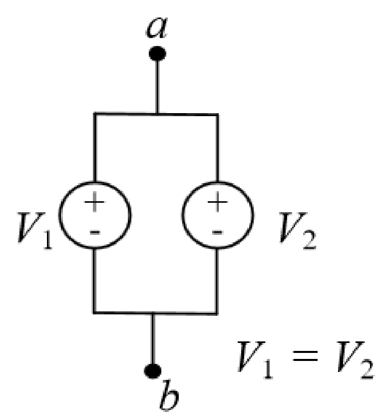
- Series voltage sources have a total voltage equal to the algebraic sum of sources voltages:

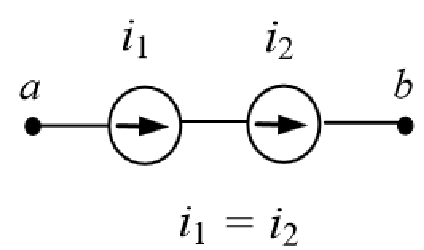


- Parallel current sources have a total current equal to the algebraic sum of all sources:

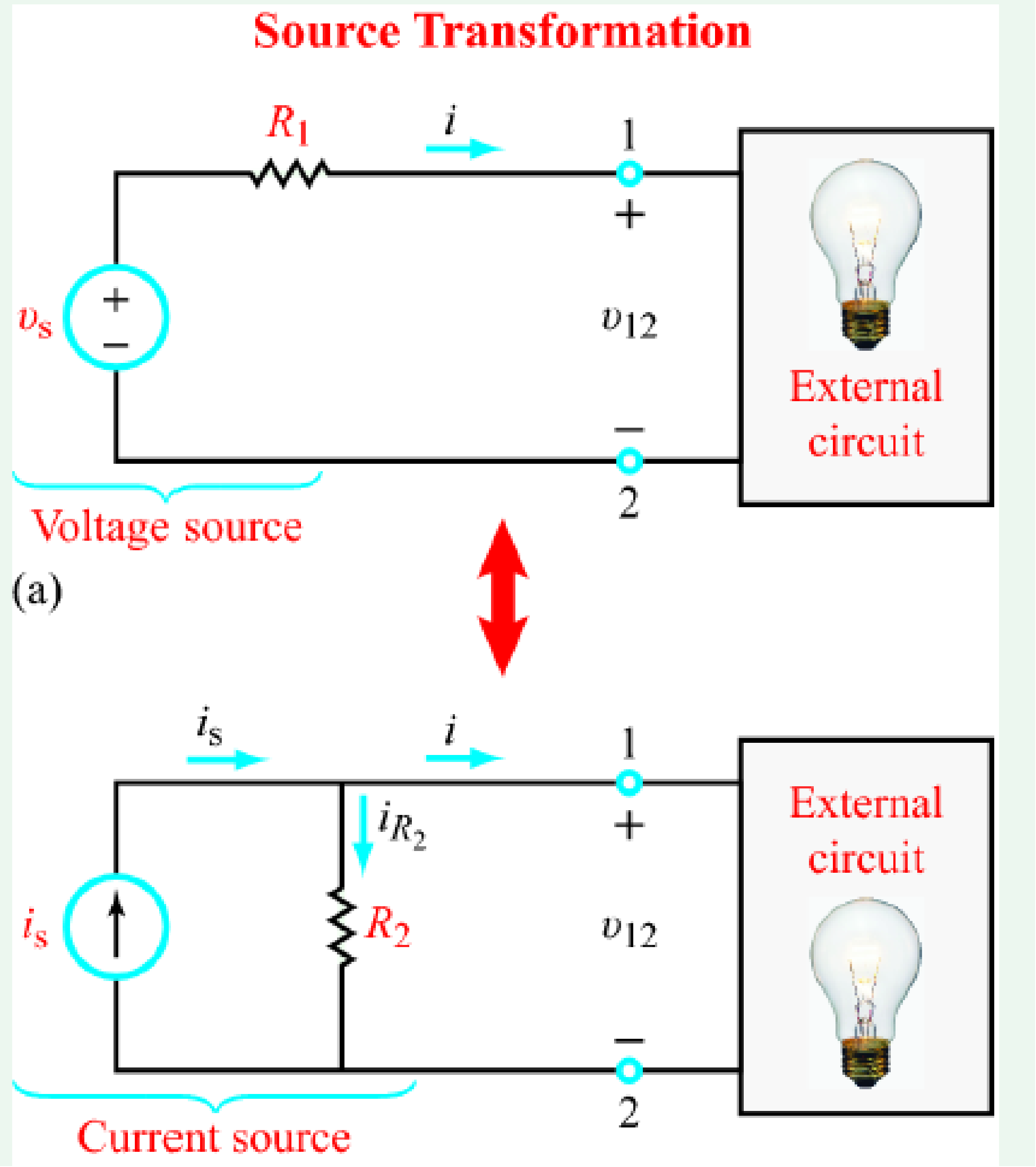


- Parallel voltage sources should have the same voltage and series current sources should have the same currents:



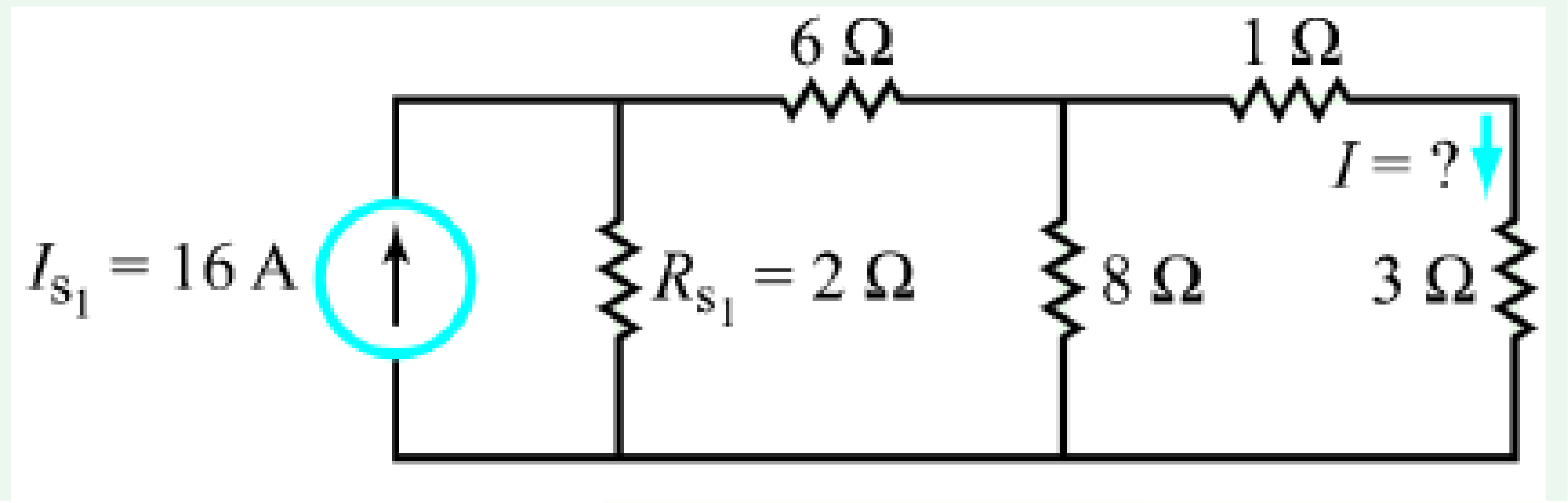


Source Transformation



$$i_s = v_s / R_1$$
$$R_2 = R_1$$

Exercise



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