Signals and Circuits

AERN 35500

Capacitors

Chapter 5: 5-4(Response of the RC circuit) pp. 230-233

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

Chapter 9: 9-5(Capacitor in DC circuits) and 9-6 (Capacitor in AC circuits) pp. 407-423

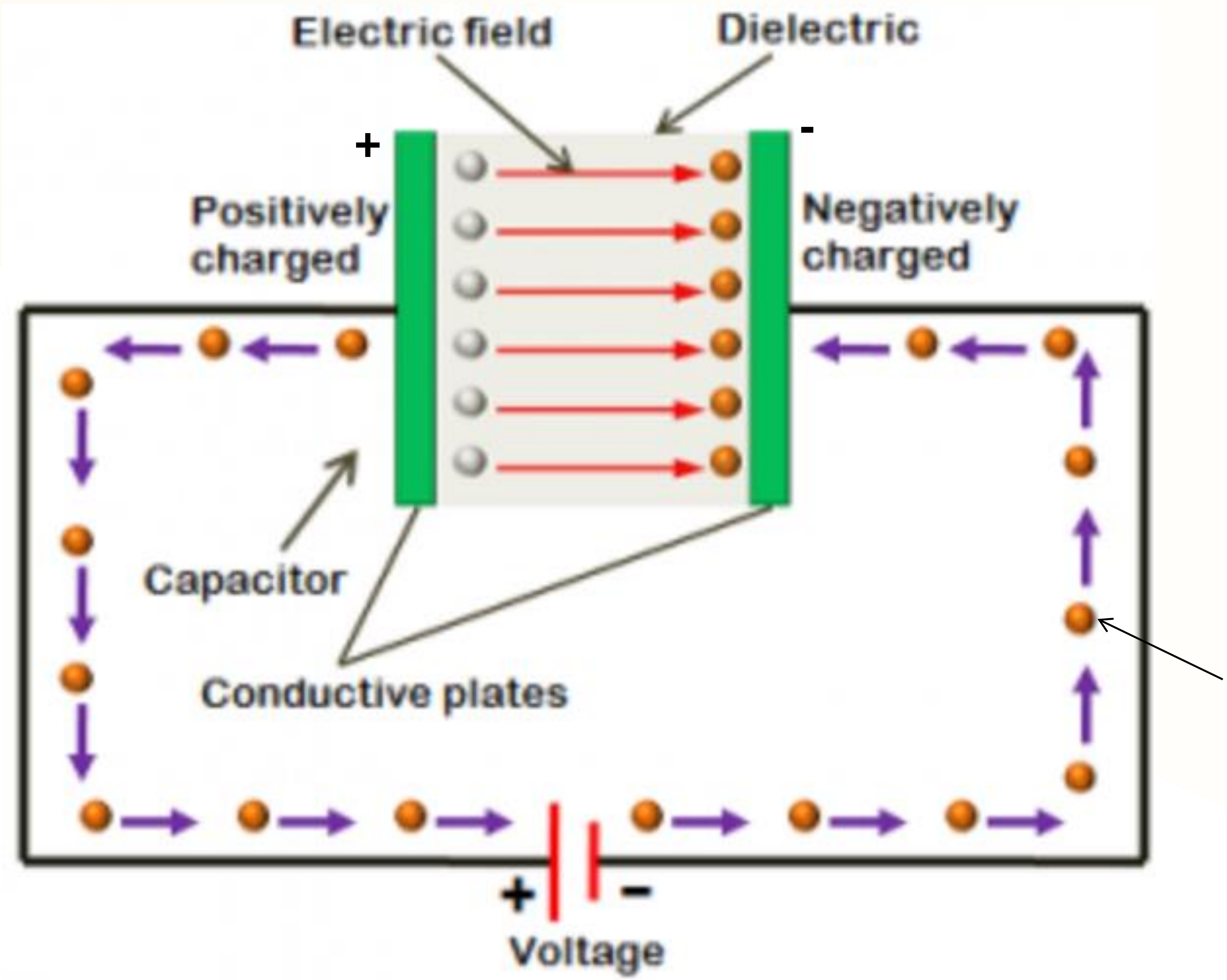
Floyd, T. L., and Buchla, D. M., *Electroics Fundamentals: Circuits, Devices & Applications*, 8th Edition, Pearson, 2009.

Web:
https://www.electronics-tutorials.ws/rc/rc_1.html
https://www.electronics-tutorials.ws/rc/rc_2.html
https://www.electronics-tutorials.ws/filter/filter_1.html

KENT STATE UNIVERSITY

Capacitor

How a capacitor stores the charge.



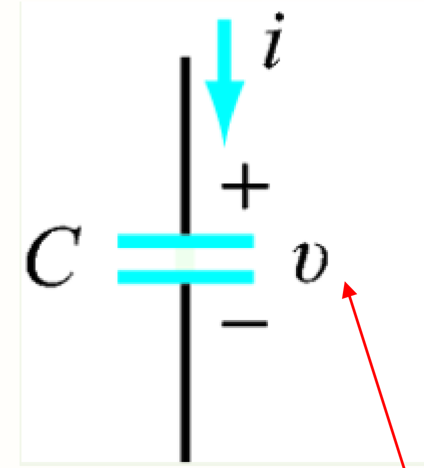
Electron

Capacitor

- The voltage across a capacitor cannot change instantaneously.

$$c = \frac{q}{v}$$

$$i = \frac{dq}{dt} = \frac{dcv}{dt} = c\frac{dv}{dt}$$



v is the voltage across the capacitor

The direction of *i* is entering the positive terminal of the capacitor.

- Under dc stable conditions, a capacitor behaves like an open circuit.

For DC
$$\frac{dv}{dt} = 0$$

$$i = c\frac{dv}{dt} = 0$$

- To get voltage

$$\int_{t_0}^{t} \frac{dv}{dt'}dt' = \frac{1}{c}\int_{t_0}^{t} i\,dt'$$

$$v(t) = v(t_0) + \frac{1}{c}\int_{t_0}^{t} i\,dt'$$

- To get power

$$p(t) = vi = Cv\frac{dv}{dt}$$

Sign of the power?

- To get energy change

$$w(t) = \int_{t_0}^{t} p\,dt' = C\int_{t_0}^{t} (v\frac{dv}{dt'})\,dt' = C\int_{t_0}^{t} [\frac{d}{dt'}\left(\frac{1}{2}v^2\right)]\,dt' = \frac{1}{2}C(v(t))^2 - \frac{1}{2}C(v(t_0))^2$$

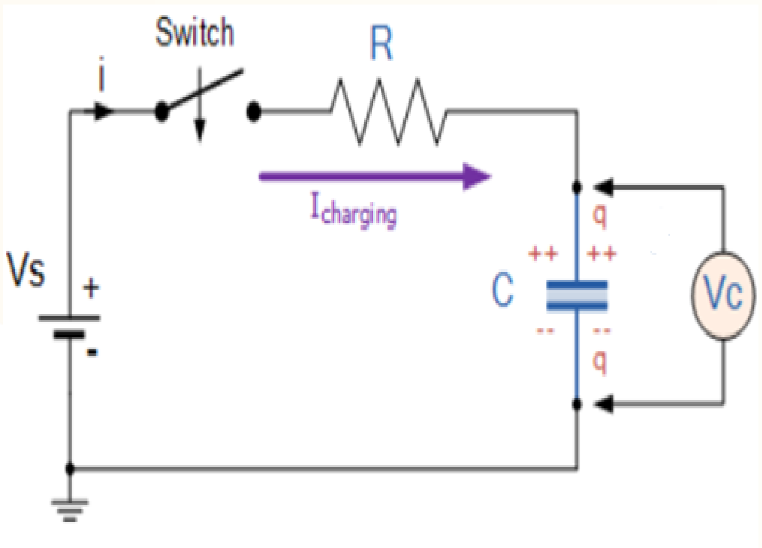
Sign of the ene

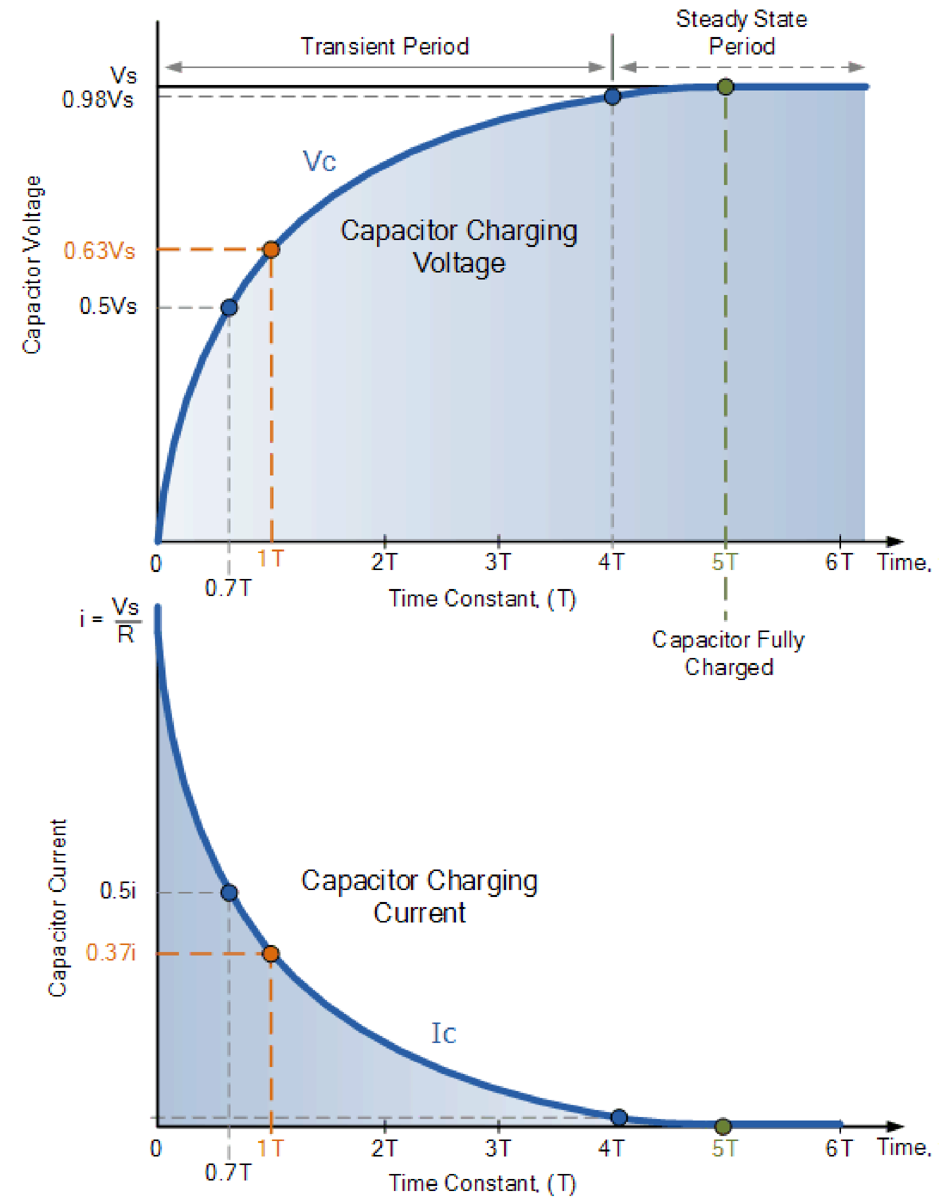
- To get energy in a capacitor

$$E(t) = \frac{1}{2}C(v(t))^2$$

Capacitor

RC charging circuit





Assume the switch is open for a long time

These are the initial conditions of the circuit, then t = 0, i = 0, q = 0, and V_C = 0

Then close the switch?

At the moment it closes, then t = 0, i = $\frac{V_s}{R}$, q = 0, and then V_C = 0

KVL

$$V_s - R \times i(t) - V_c(t) = 0$$

A long time after the closing

then t = ∞, i = 0, q = Q, and then Vc = Vs

τ (usually τ) is the time constant or time delay. (63% voltage rises)

τ = R x C   second = Ohm x Farads

4τ (usually τ) is the transient period; 98% charged

5τ (usually τ) is the steady state period; almost fully charged

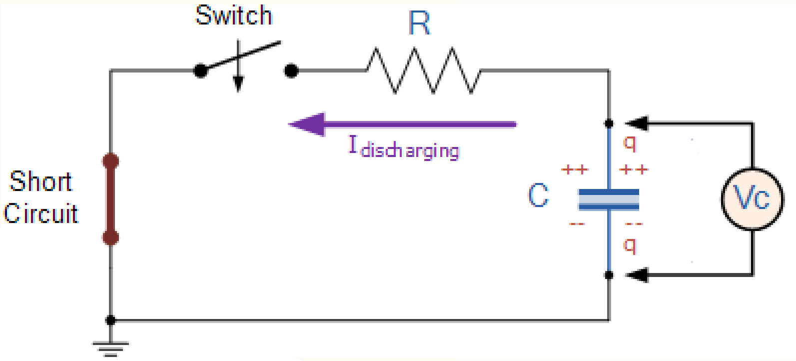
$$Vc = Vs\,(1 - e^{-t/RC})$$

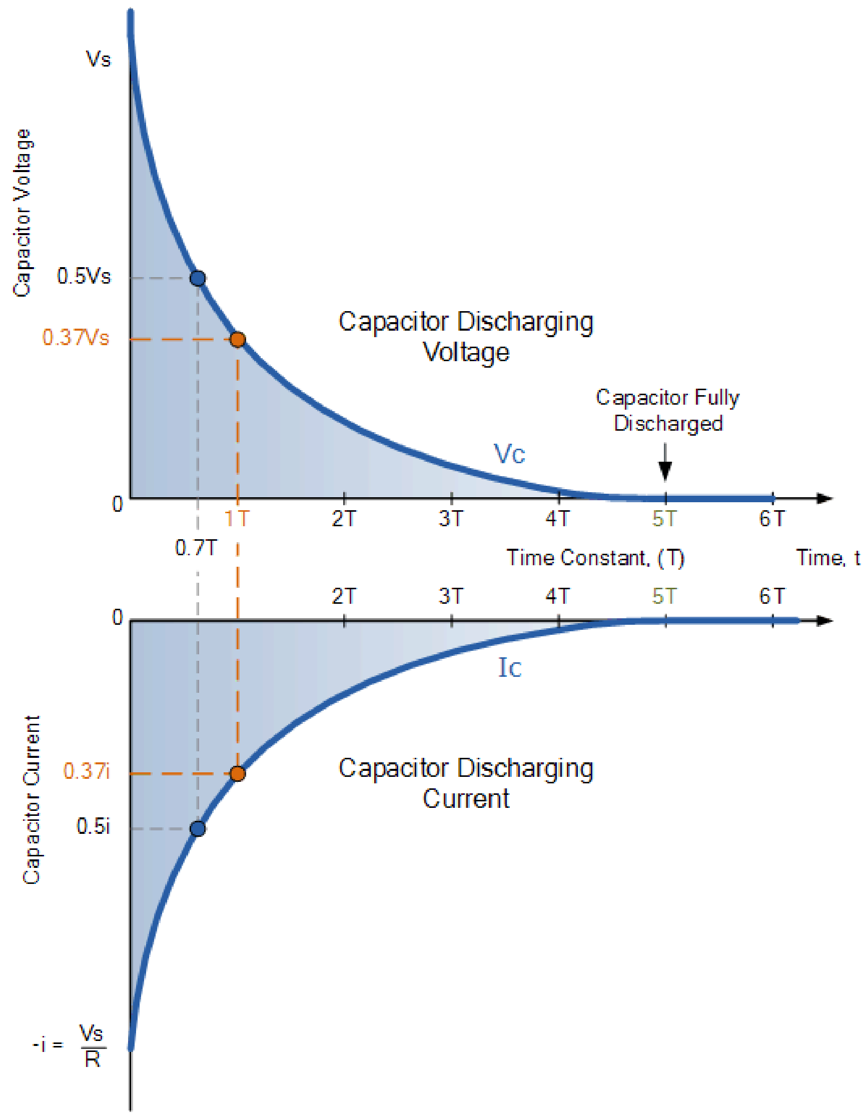
$$i_c = \frac{V_s}{R} e^{-t/RC}$$

KENT STATE UNIVERSITY

Capacitor

RC discharging circuit





Assume the capacitor is fully charged for a long time

These are the initial conditions of the circuit, then t = 0, i = 0, q = Q, and then Vc = Vs

Then replace the battery with a short wire, and close the switch?

At the moment it closes, then t = 0, i =- $\frac{V_s}{R}$, q = Q, and Vc = Vs

KVL

$$R \times i(t) + V_c(t) = 0$$

A long time after the closing

then t = ∞, i = 0, q = 0, and Vc = 0

τ (usually τ) is the time constant or time delay. (63% voltage drop

$$\tau = R \times C$$

4τ (usually τ) is the transient period; 98% discharged

5τ (usually τ) is the steady state period; almost fully discharged

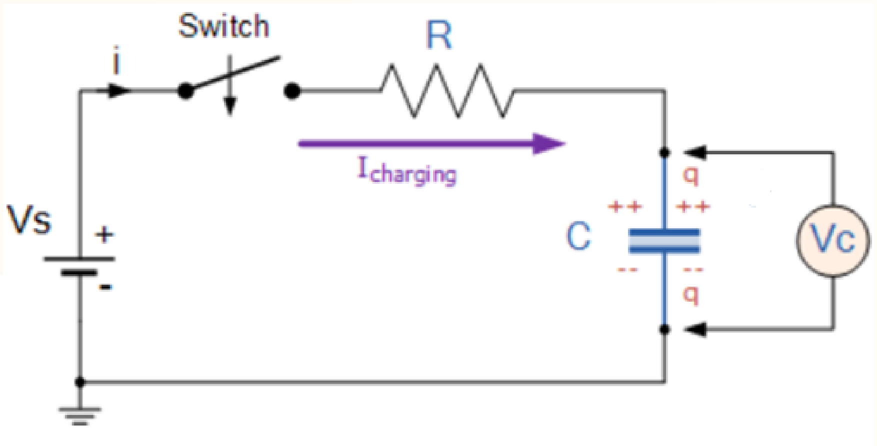
$$V_c = V_s e^{-t/\tau}$$

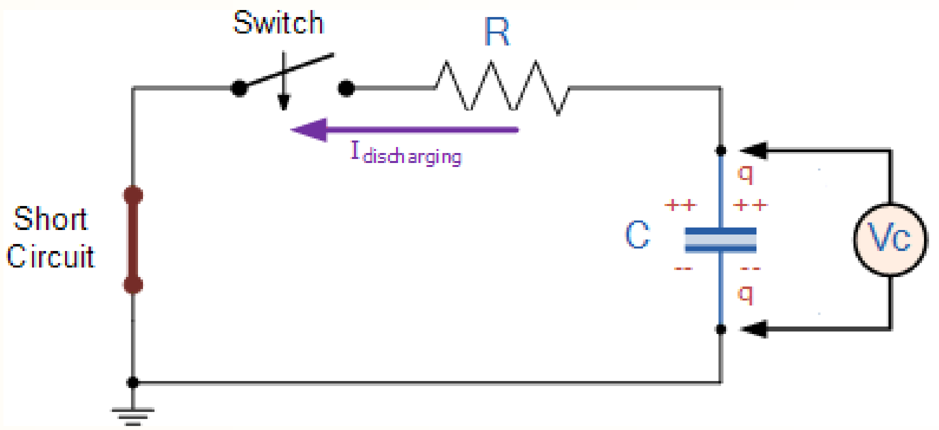
$$i_c = -\frac{V_s}{R} e^{-t/\tau}$$

KENT STATE UNIVERSITY

Capacitor

RC charging/discharging circuit





$$V(t) = V_F + (V_i - V_F)\, e^{-t/\tau}$$

$$i(t) = I_F + (I_i - I_F)\, e^{-t/\tau}$$

$$\tau = RC$$

where V(t) and i(t) are the instantaneous voltage and current;
V_i and I_i is the initial voltage and current;
V_F and I_F are the final voltage and current.

Capacitor

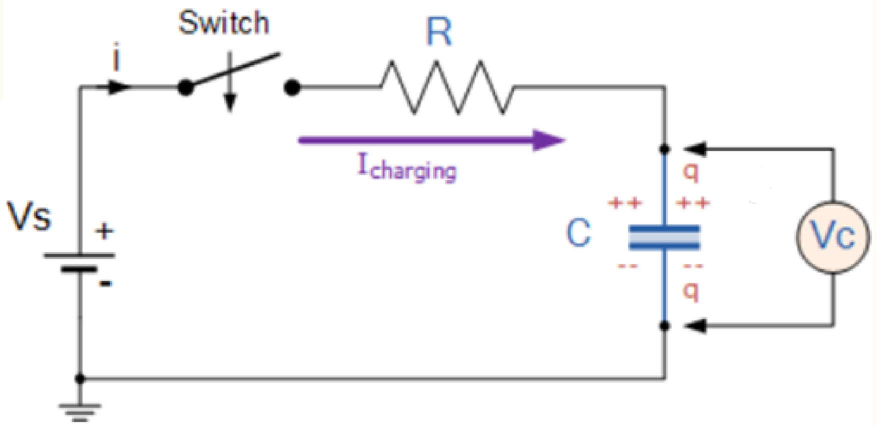
E.g.

$$V(t) = V_F + (V_i - V_F)\, e^{-t/\tau}$$

$$i(t) = I_F + (I_i - I_F)\, e^{-t/\tau}$$

$\tau = RC$

Determine the capacitor voltage after the switch is closed if the capacitor initially is uncharged. Draw the charging curving. C=50 uF; R=8.2 ohm; Vs= 50V.



KENT STATE UNIVERSITY

Capacitor

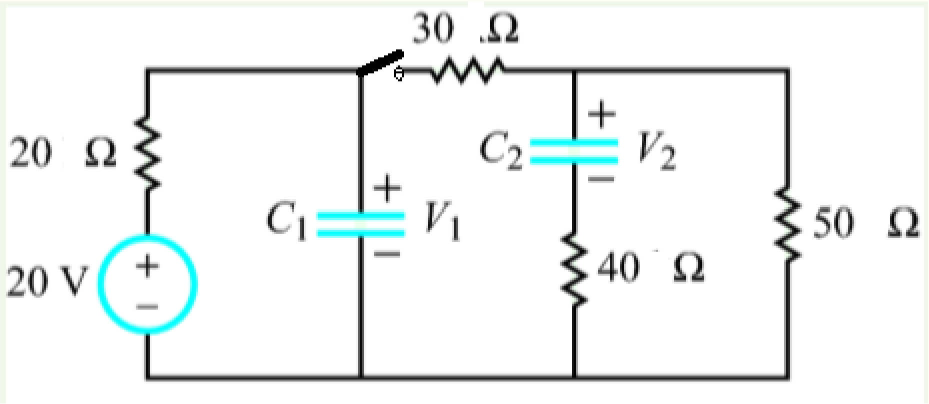
E.g.

The switch has been closed for a long time(It is a stable circuit initially). Determine the capacitor voltage V_2 after the switch is suddenly opened. Draw the discharging curving. C_1 =40 uF; C_2 =50 uF.

$$V(t) = V_F + (V_i - V_F)\, e^{-t/\tau}$$

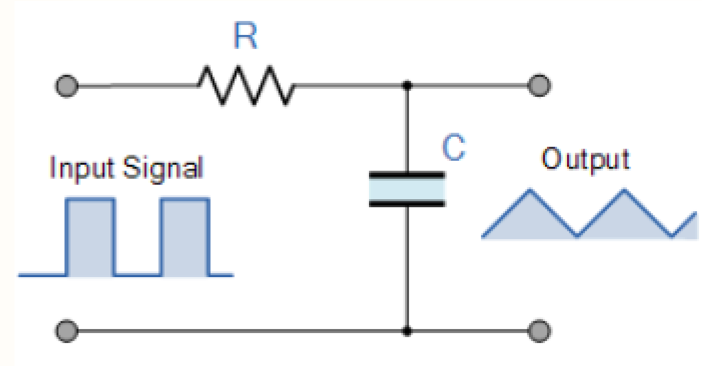
$$i(t) = I_F + (I_i - I_F)\, e^{-t/\tau}$$

τ=RC

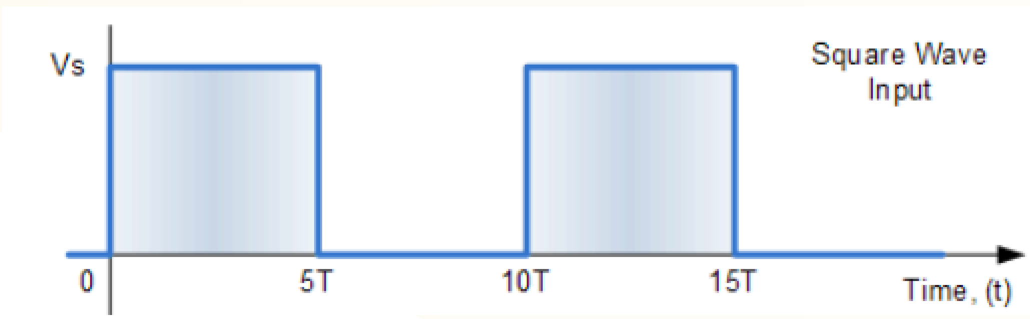


Capacitor

Response to square wave

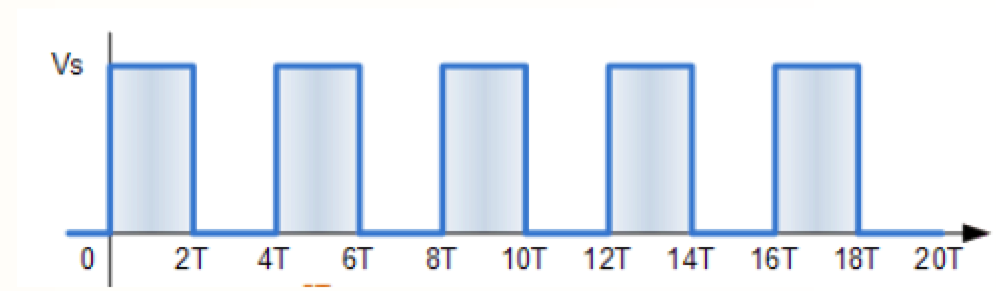


10 RC Input Waveform



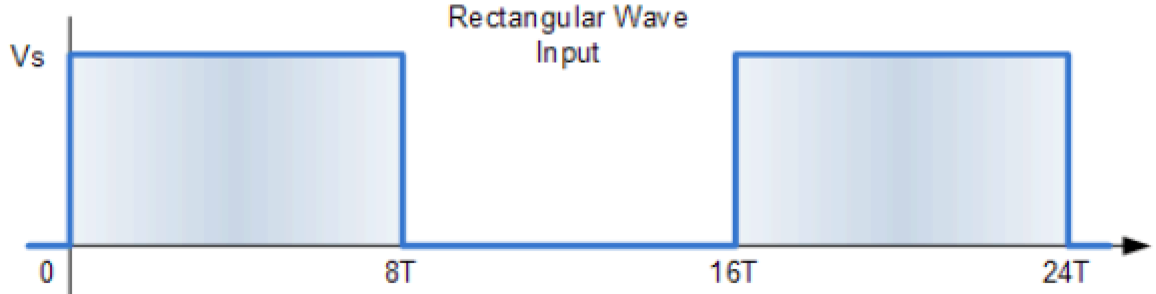
Vs

4RC Input Waveform



Vs

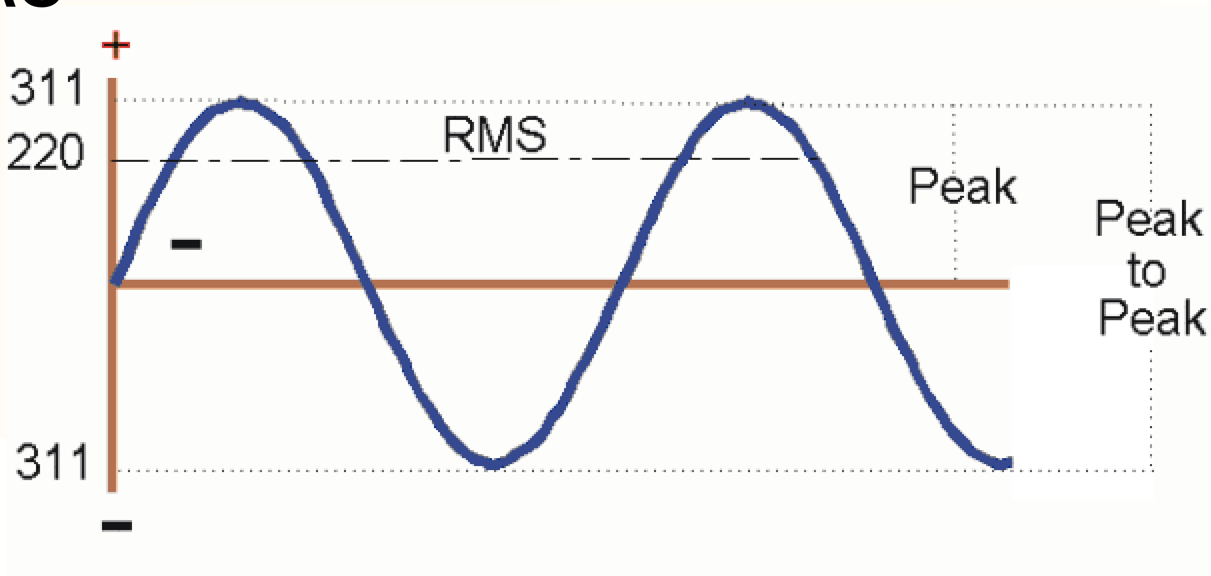
16 RC Input Waveform



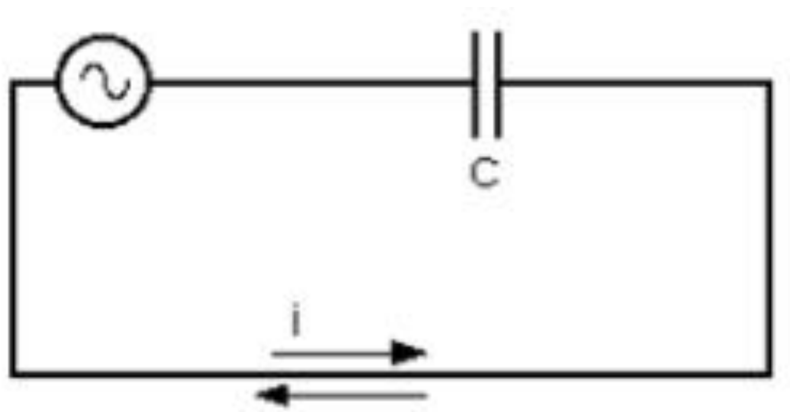
Vs

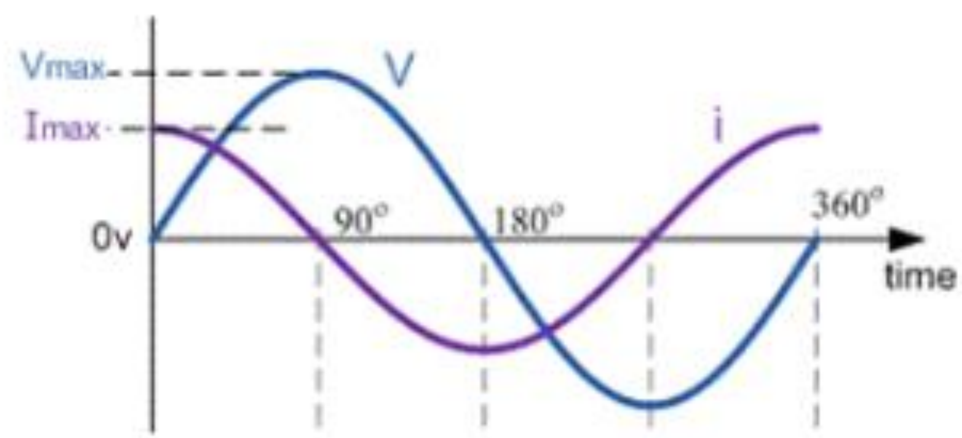
Capacitor in AC

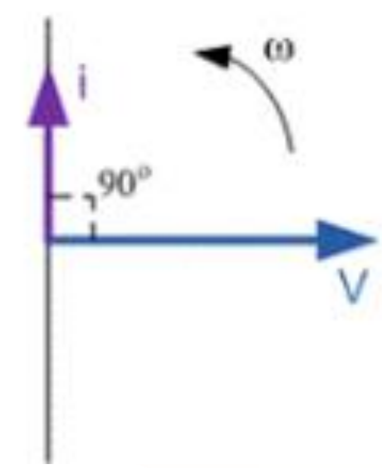
220 V AC



AC with a capacitor

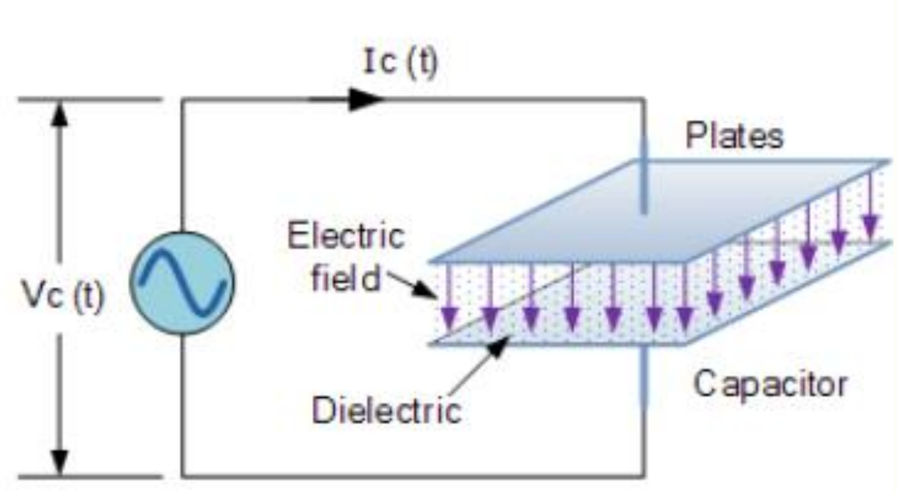






Capacitor in AC

Capacitive Reactance



With the same AC source, we used DMM and found the current increases as the capacitance increases;

With the same capacitance and AC voltage amplitude, we used DMM and found the current increases as the AC source frequency increases.

The **opposition** to sinusoidal current in a capacitor is called **capacitive reactance.**

$$Xc = \frac{1}{2\pi f C}$$

Where:

Xc = Capacitive Reactance in Ohms, (Ω)

π (pi) = 3.142 (decimal) or as 22÷7 (fraction)

f = Frequency in Hertz, (Hz)

C = Capacitance in Farads, (F)

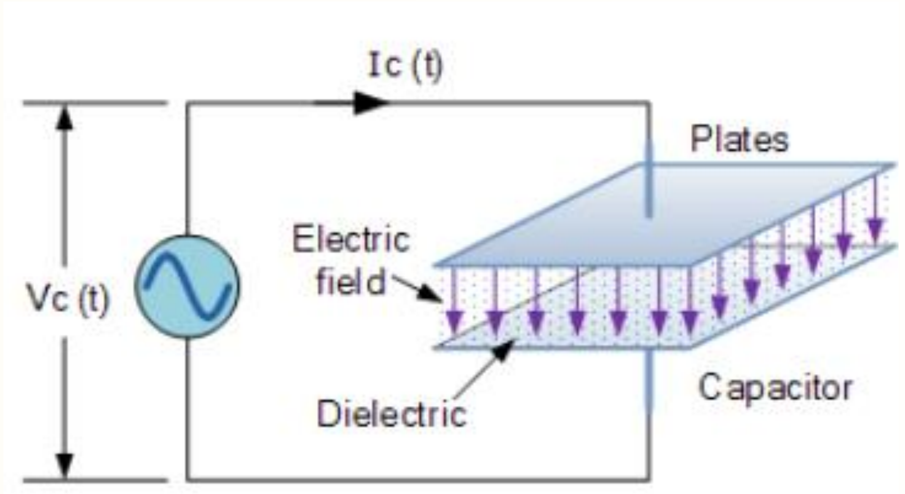
KENT STATE UNIVERSITY

Capacitor in AC

Capacitive Reactance

E. g.

Calculate the capacitive reactance value of a 220nF capacitor at a frequency of 1kHz and again at a frequency of 20kHz.

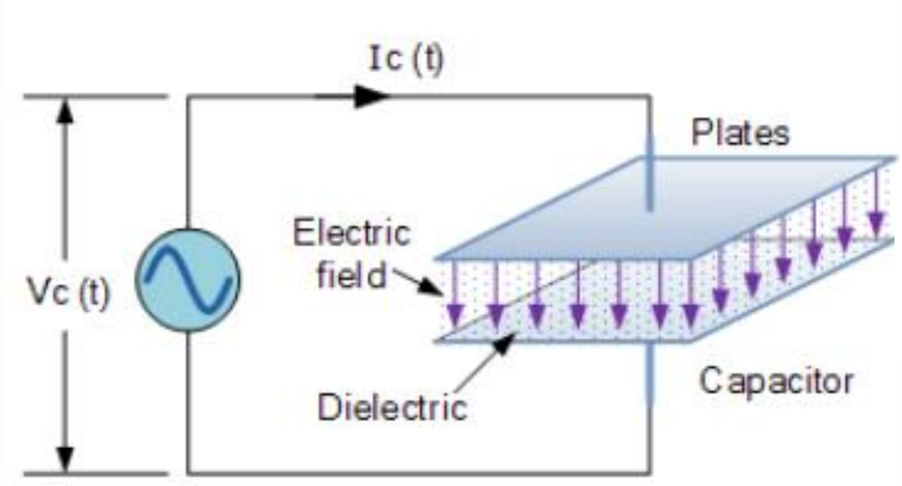


$$Xc = \frac{1}{2\pi f C}$$

Capacitor in AC

Ohm's law in AC

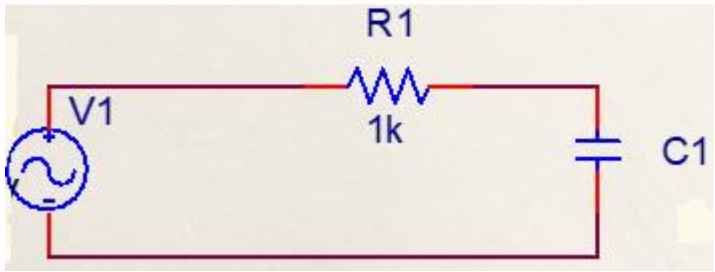
$$I = \frac{V}{X_c}$$



$$Xc = \frac{1}{2\pi fC}$$

When applying Ohm's law in ac circuits, you much express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on (This is not in time domain).

E. g.



$$V_{1rms} = 10\ V$$

$$f = 10\ kHz$$

$$C1 = 0.0056uF$$

$$R_1 = 1k\Omega$$

$$I_{rms} = ?$$

$$V_{1rms} = 10\ V$$

$$f = 10\ kHz$$

$$I_{rms} = ?$$

Capacitor in AC

Capacitive reactance for capacitors in series in AC

$$X_{CT} = X_{C_1} + X_{C_2} + \cdots + X_{C_n}$$

Capacitive reactance for capacitors in parallel in AC

$$\frac{1}{X_{CT}} = \frac{1}{X_{C_1}} + \frac{1}{X_{C_2}} + \frac{1}{X_{C_3}} + \cdots + \frac{1}{X_{C_n}}$$

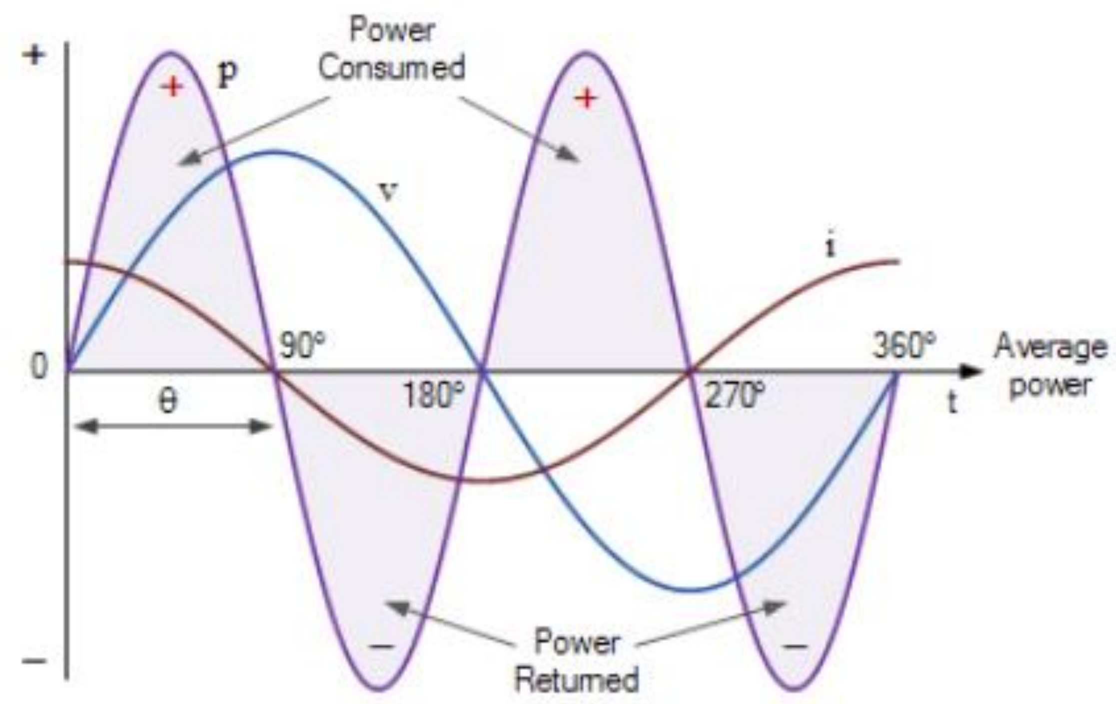
Voltage divider in AC

$$V_x = \frac{X_{cx}}{X_{c1} + X_{c2} \cdots X_{cn}} V$$

Current divider in AC?

Capacitor in AC

AC power waveforms for a pure capacitor



Instantaneous power

$$p(t) = v(t)i(t)$$

True power (one cycle power)

$$p_{ture} = 0$$

Reactive power (the rate at which a capacitor stores or returns energy)

$$p_r = V_{rms}I_{rms} = \frac{V_{rms}^2}{X_c} = I_{rms}^2 X_c$$