

# **Chapter 6**

# Mass And Energy Analysis Of Control Volumes



## **Objectives**

- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.



## **Objectives**

- Identify the energy carried by a fluid stream crossing a control surface as the sum:
- Of internal energy, flow work, kinetic energy, and potential energy of the fluid
- And to relate the combination of the <u>internal</u> energy and the flow work to the property enthalpy.



# **Objectives**

- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteadyflow processes with particular emphasis on the uniform-flow process as the model.

## 6-1 CONSERVATION OF MASS

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

**Closed systems**: The mass of the system remain constant during a process.

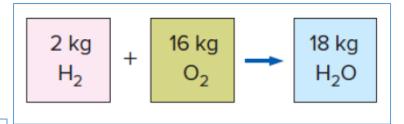
**Control volumes**: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.

# Mass *m* and energy *E* can be converted to each other according to

$$E = mc^2$$

where c is the speed of light in a vacuum, which is  $c = 2.9979 \times 10^8$  m/s.

The mass change due to energy change is negligible.



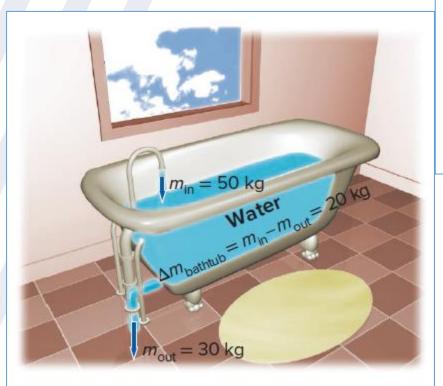
### FIGURE 6-1

Mass is conserved even during chemical reaction.



## **Conservation of Mass Principle**

$$\binom{\text{Total mass entering}}{\text{the CV during }\Delta t} - \binom{\text{Total mass leaving}}{\text{the CV during }\Delta t} = \binom{\text{Net change of mass}}{\text{within the CV during }\Delta t}$$



The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval  $\Delta t$  is equal to the net change (increase or decrease) in the total mass within the control volume during  $\Delta t$ .

$$m_{\rm in} - m_{\rm out} = \Delta m_{\rm CV}$$
 (kg)

$$\Delta m_{\rm cv} = m_{\rm final} - m_{\rm initial}$$

$$\dot{m}_{\rm in} - \dot{m}_{\rm out} = dm_{\rm CV/dt}$$

(kg/s)

These eequations are often referred to as the mass balance and are applicable to any control volume undergoing any kind of process.

### FIGURE 6-5

Conservation of mass principle for an ordinary bathtub.

## EXAMPLE 6-1 Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit (**Fig. 6–10**). If it takes 50 s to fill the bucket with water, determine (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.



#### FIGURE 6-10

Schematic for Example 6-1.

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#### SOLUTION

A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

### **Assumptions**

**1** Water is a nearly incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

### **Properties**

We take the density of water to be  $1000 \text{ kg/m}^3 = 1 \text{ kg/L}$ .

### **Analysis**

(a) Noting that 10 gal of water is discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}} \left( \frac{3.7854 \text{ L}}{1 \text{ gal}} \right) = \mathbf{0.757 \text{ L/s}}$$

$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = 0.757 \text{ kg/s}$$

(b) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi (0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2} \left(\frac{1 \text{m}^3}{1000 \text{ L}}\right) = 15.1 \text{ m/s}$$

### Discussion

It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.



# Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ( $m_{\rm CV}$  = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

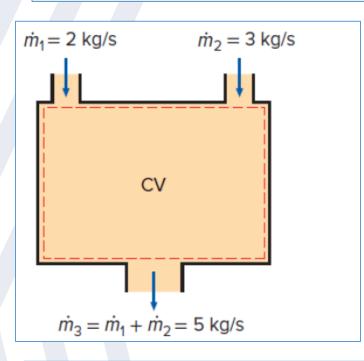


FIGURE 6-8

Conservation of mass principle for a two-inlet-one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate.* 

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$
 (kg/s) Multiple inlets and exits

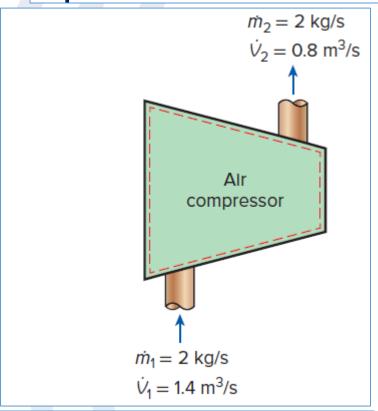
$$\dot{m}_1 = \dot{m}_2 \longrightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$
 Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

## **Special Case: Incompressible Flow**

### KENT STATE

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



### FIGURE 6-9

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

$$\sum_{\rm in} \dot{V} = \sum_{\rm out} \dot{V}$$

 $(m^3/s)$ 

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \longrightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow(single stream)

There is no such thing as a "conservation of volume" principle.

For steady flow of liquids, the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

# **Total Energy of a Flowing Fluid**

$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz$$
 (kJ/kg)

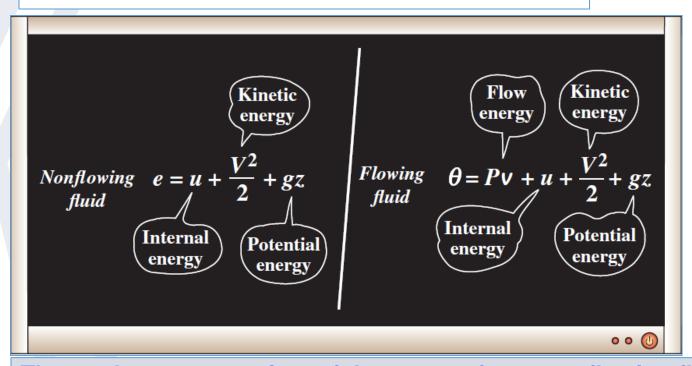
Energy of a **flowing fluid** 

(denoted by  $\theta$ )

$$\theta = Pv + e = Pv + (u + ke + pe)$$
  $h = u + Pv$ 

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz$$
 (kJ/kg)

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a non-flowing fluid and four parts for a flowing fluid.

A flowing liquid is that which has relative motion between its layers and pipe body.

It is of two types-

- **1.Streamlined==>** Type of flow in which the particle follow the path of its preceding particle's path. Its *Reynolds number* is from 1000 to 3000.
- 2. Turbulent ==> Type of flow in which particles of fluid don't have a specific path its *Reynolds number* is below 1000 or more than 3000.

A non flowing liquid is that fluid which doesn't have a relative motion with respect to inner body of vessel or pipe in which it is in touch with.

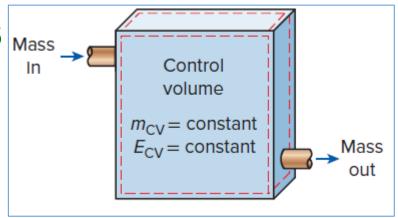
# 6-3 ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

Steady-flow process: A process during which a fluid flows through a control volume steadily.



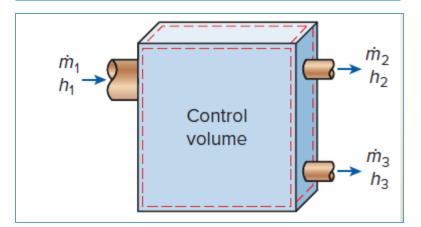
### **FIGURE 6-18**

Many engineering systems such as power plants operate under steady conditions.



### **FIGURE 6-19**

Under steady-flow conditions, the mass and energy contents of a control volume remains constant.



### **FIGURE 6-20**

Under steady-flow conditions, the fluid properties at an inlets or exits remains constant (do not change with time).



### Ex-6-3

### **EXAMPLE 6-3** Energy Transport by Mass

Steam is leaving a 4-L pressure cooker whose operating pressure is 150 kPa (**Fig. 6–17**). It is observed that the amount of liquid in the cooker has decreased by 0.6 L in 40 min after the steady operating conditions are established, and the cross-sectional area of the exit opening is 8 mm<sup>2</sup>. Determine (a) the mass flow rate of the steam and the exit velocity, (b) the total and flow energies of the steam per unit mass, and (c) the rate at which energy leaves the cooker by steam.



Pressure cooker

### FIGURE 6-17

Schematic for **Example 6–3**.

### SOLUTION

Steam leaves a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

### **Assumptions**

1 The flow is steady, and the initial start-up period is disregarded. 2 The kinetic and potential energies are negligible, and thus they are not considered. 3 Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at the cooker pressure.

### **Properties**

The properties of saturated liquid water and water vapor at 150 kPa are  $v_f = 0.001053 \text{ m}^3/\text{kg}$ ,  $v_g = 1.1594 \text{ m}^3/\text{kg}$ ,  $u_g = 2519.2 \text{ kJ/kg}$ , and  $h_g = 2693.1 \text{ kJ/kg}$  (**Table A–5**).

### Analysis

(a) Saturation conditions exist in a pressure cooker at all times after the steady operating conditions are established. Therefore, the liquid has the properties of saturated liquid and the exiting steam has the properties of saturated vapor at the operating pressure. The amount of liquid that has evaporated, the mass flow rate of the exiting steam, and the exit velocity are

$$m = \frac{\Delta V_{\text{liquid}}}{V_f} = \frac{0.6 \text{ L}}{0.001053 \text{ m}^3/\text{kg}} \left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = 0.570 \text{ kg}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{0.570 \text{ kg}}{40 \text{ min}} = 0.0142 \text{ kg/min} = 2.37 \times 10^{-4} \text{ kg/s}$$



(b) Noting that h = u + Pv and that the kinetic and potential energies are disregarded, the flow and total energies of the exiting steam are

$$e_{\text{flow}} = P v = h - u = 2693.1 - 2519.2 = 173.9 \text{ kJ/kg}$$
  
 $\theta = h + \text{ke} + \text{pe} \cong h = 2693.1 \text{ kJ/kg}$ 

Note that the kinetic energy in this case is ke =  $V^2/2$  = (34.3 m/s)<sup>2</sup>/2 = 588 m<sup>2</sup>/s<sup>2</sup> = 0.588 kJ/kg, which is small compared to enthalpy.

(c) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

$$\dot{E}_{\text{mass}} = \dot{m}\theta = (2.37 \times 10^{-4} \text{ kg/s})(2693.1 \text{ kJ/kg}) = 0.638 \text{ kJ/s} = \mathbf{0.638 \text{ kW}}$$

### Discussion

The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is  $h_{fg}$ ) since it relates directly to the amount of energy supplied to the cooker.





## Mass and Energy balances for a steady-flow process

$$\sum_{\rm in} \dot{m} = \sum_{\rm out} \dot{m}$$

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$\underline{\dot{E}_{\rm in} - \dot{E}_{\rm out}}$$

Rate of net energy transfer in by heat, work, and mass

Rate of net energy transfer in

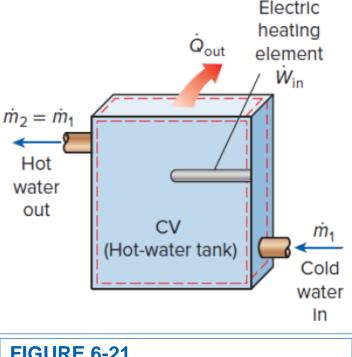
by heat, work, and mass

$$dE_{\text{system}}/dt^{\text{No(steady)}} = 0$$

Rate of change in internal, kinetic, potential,etc,.energies

(kW)

Rate of net energy transfer in by heat, work, and mass



### **FIGURE 6-21**

A water heater in steady operation.

### **Energy balance**

$$\dot{Q}_{\rm in} + \dot{W}_{\rm in} + \sum_{\rm in} \dot{m}\theta = \dot{Q}_{\rm out} + \dot{W}_{\rm out} + \sum_{\rm out} \dot{m}\theta$$

$$\dot{Q}_{\rm in} + \dot{W}_{\rm in} + \underbrace{\sum_{\rm in} \dot{m} \left( h + \frac{V^2}{2} + gz \right)}_{\rm for \ each \ inlet} = \dot{Q}_{\rm out} + \dot{W}_{\rm out} + \underbrace{\sum_{\rm out} \dot{m} \left( h + \frac{V^2}{2} + gz \right)}_{\rm for \ each \ exit}$$

### Energy balance relations with sign **conventions** (i.e., heat input and work output are positive)

$$\dot{Q} - \dot{W} = \sum_{\text{out}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}} - \sum_{\text{in}} \dot{m} \underbrace{\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}}$$

$$\dot{Q} - \dot{W} = \dot{m} \left[ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

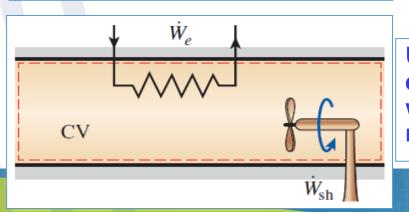
$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1 \qquad q = \dot{Q} / \dot{m}$$

$$q = \dot{Q}/\dot{m}$$

$$w = \dot{W}/\dot{m}$$

### when kinetic and potential energy changes are negligible



$$h = u + Pv$$

**Conversion Factor** 

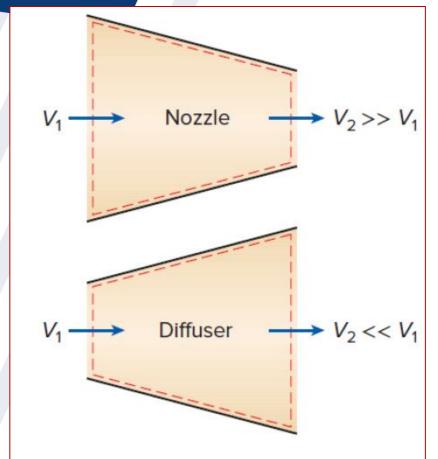
$$\frac{J}{kg} \equiv \frac{N \cdot m}{kg} \equiv \left(kg \frac{m}{s^2}\right) \frac{m}{kg} \equiv \frac{m^2}{s^2}$$

$$\left(\text{Also, } \frac{\text{Btu}}{\text{lbm}} = 25,037 \frac{\text{ft}^2}{\text{s}^2}\right)$$

The units m<sup>2</sup>/s<sup>2</sup> and J/kg are equivalent.

Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

### **Nozzles and Diffusers**



### **FIGURE 6-26**

Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies

- Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.
- A nozzle is a device that increases the velocity of a fluid at the expense of pressure.
- A diffuser is a device that increases the pressure of a fluid by slowing it down.
- The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Energy balance for a nozzle or diffuser:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

(since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ , and  $\Delta pe \cong 0$ )



# Deceleration of Air in a Diffuser



$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}} }_{\text{Rate of net energy transfer}}$$
 by heat ,work,and mass

$$dE_{\text{system}}/dt^{\text{0(steady)}} = 0$$
Rate of change in internal, kinetic, potential, etc., energies

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

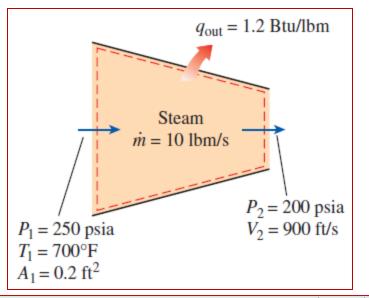
$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$
 (since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ , and  $\Delta pe \cong 0$ )

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$





# Acceleration of Steam in a Nozzle



$$\underline{\dot{E}_{in} - \dot{E}_{out}} = \underline{dE_{system}/dt}^{\text{No(steady)}} = 0$$
Rate of net energy transfer by heat ,work, and mass
$$\underline{\dot{E}_{in} - \dot{E}_{out}} = \underline{dE_{system}/dt}^{\text{No(steady)}} = 0$$
Rate of change in internal, kinetic, potential, etc, energies

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{Q}_{\text{out}} + \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \quad \text{(since } \dot{W} = 0 \text{, and } \Delta \text{pe} \cong 0\text{)}$$

$$h_2 = h_1 - q_{\text{out}} - \frac{V_2^2 - V_1^2}{2}$$





### Ex-6-4

### **EXAMPLE 6-4** Deceleration of Air in a Diffuser

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m<sup>2</sup>. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (a) the mass flow rate of the air and (b) the temperature of the air leaving the diffuser.

### SOLUTION

Air enters the diffuser of a jet engine steadily at a specified velocity. The mass flow rate of air and the temperature at the diffuser exit are to be determined.

### **Assumptions**

**1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\rm CV} = 0$  and  $\Delta E_{\rm CV} = 0$ . **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The potential energy change is zero,  $\Delta$  pe = 0. **4** Heat transfer is negligible. **5** Kinetic energy at the diffuser exit is negligible. **6** There are no work interactions.

### Analysis

We take the *diffuser* as the system (**Fig. 6–27**). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .









### FIGURE 6-27

The diffuser of a jet engine discussed in **Example 6–4**.

- © Yunus Cengel
- (a) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(283 \text{ K})}{80 \text{ kPa}} = 1.015 \text{ m}^3/\text{kg}$$

Then,

$$\dot{m} = \frac{1}{V_1} V_1 A_1 = \frac{1}{1.015 \text{ m}^3/\text{kg}} (200 \text{ m/s})(0.4 \text{ m}^2) = 78.8 \text{ kg/s}$$

Since the flow is steady, the mass flow rate through the entire diffuser remains constant at this value.



(b) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\rm in} - \dot{E}_{\rm out}}_{\text{Rate of net energy transfer}} = \underbrace{dE_{\rm system}/dt}_{\text{Rate of change in internal, kinetic,}} 0 \text{ (steady)}_{\text{energy transfer}} = 0$$
Rate of change in internal, kinetic, potential, etc., energies

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$
 
$$\dot{m} \left( h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left( h_2 + \frac{V_2^2}{2} \right) \quad \text{(since } \dot{Q} \cong 0, \, \dot{W} = 0, \, \text{and } \Delta \text{pe} \cong 0 \text{)}$$
 
$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ( $V_2 << V_1$ ); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table (Table A–21) to be

$$h_1 = h_{@283 \text{ K}} = 283.14 \text{ kJ/kg}$$

Substituting, we get

$$h_2 = 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right)$$
  
= 303.14 kJ/kg

Ex-6-4

From Table A–21, the temperature corresponding to this enthalpy value is

$$T_2 = 303 \text{ K}$$

### **Discussion**

This result shows that the temperature of the air increases by about 20°C as it is slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.



## EXAMPLE 6-5

### Acceleration of Steam in a Nozzle

Steam at 250 psia and 700°F steadily enters a nozzle whose inlet area is 0.2 ft<sup>2</sup>. The mass flow rate of steam through the nozzle is 10 lbmys. Steam leaves the nozzle at 200 psia with a velocity of 900 ft/s. Heat losses from the nozzle per unit mass of the steam are estimated to be 1.2 Btu/lbm. Determine (a) the inlet velocity and (b) the exit temperature of the steam.

### SOLUTION

Steam enters a nozzle steadily at a specified flow rate and velocity. The inlet velocity of steam and the exit temperature are to be determined.

### **Assumptions**

**1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\rm CV}$  = 0 and  $\Delta E_{\rm CV}$  = 0. **2** There are no work interactions. **3** The potential energy change is zero,  $\Delta$  pe = 0.

### **Analysis**

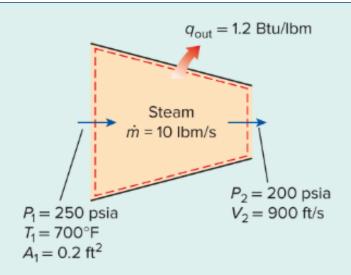
We take the *nozzle* as the system (**Fig. 6–28**). This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .







Ex-6-5



### FIGURE 6-28

Schematic for **Example 6–5**.

(a) The specific volume and enthalpy of steam at the nozzle inlet are

$$P_1 = 250 \text{ psia}$$
  $V_1 = 2.6883 \text{ ft}^3/\text{lbm}$  (Table A–6E)  $T_1 = 700^{\circ}\text{F}$   $h_1 = 1371.4 \text{ Btu/lbm}$ 

Then,

$$\dot{m} = \frac{1}{V_1} V_1 A_1$$
10 lbm/s =  $\frac{1}{2.6883 \text{ ft}^3/\text{lbm}} (V_1)(0.2 \text{ ft}^2)$ 

$$V_1 = 134.4 \text{ ft/s}$$





Ex-6-5

(b) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underline{\dot{E}_{in} - \dot{E}_{out}} = \underbrace{dE_{system}/dt}^{0 \text{ (steady)}} = 0$$
Rate of net energy transfer by heat, work, and mass potential, etc., energies

$$\dot{E}_{\mathrm{in}} = \dot{E}_{\mathrm{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{Q}_{\text{out}} + \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$
 (since  $\dot{W} = 0$ , and  $\Delta pe \approx 0$ )

Dividing by the mass flow rate  $\dot{m}$  and substituting,  $h_2$  is determined to be

$$h_2 = h_1 - q_{\text{out}} - \frac{V_2^2 - V_1^2}{2}$$

$$= (1371.4 - 1.2) \text{ Btu/lbm} - \frac{(900 \text{ ft/s})^2 - (134.4 \text{ ft/s})^2}{2} \left(\frac{1 \text{ Btu/lbm}}{25,037 \text{ ft}^2/\text{s}^2}\right)$$

$$= 1354.4 \text{ Btu/lbm}$$

Then,

$$P_2 = 200 \text{ psia}$$
  
 $h_2 = 1354.4 \text{ Btu/lbm}$   $T_2 = 662.0 \text{°F}$  (Table A–6E)

### **Discussion**

Note that the temperature of steam drops by 38.0°F as it flows through the nozzle. This drop in temperature is mainly due to the conversion of internal energy to kinetic energy. (The heat loss is too small to cause any significant effect in this case.)





## **Turbines and Compressors**



**FIGURE 6-29** 

Turbine blades attached to the turbine shaft.

Turbine drives the electric generator In steam, gas, or hydroelectric power plants.

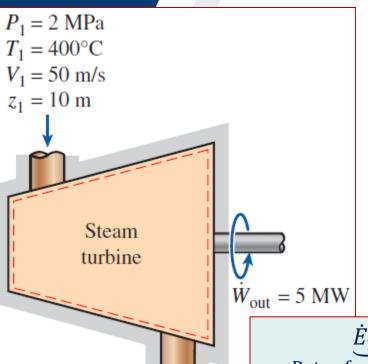
As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as pumps and fans, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A *compressor* is capable of compressing the gas to very high pressures.

A *fan* increases the pressure of a gas slightly and is mainly used to mobilize a gas.

**Pumps** work very much like compressors except that they handle liquids instead of gases.



 $P_2 = 15 \text{ kPa}$ 

 $V_2 = 180 \text{ m/s}$ 

 $x_2 = 0.90$ 

 $z_2 = 6 \text{ m}$ 

# Power Generation by a Steam Turbine

$$\underline{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}} = \underbrace{dE_{\text{system}}/dt^{\text{lo(steady)}}}_{\text{Rate of net energy transfer by heat ,work,and mass}} = \underbrace{dE_{\text{system}}/dt^{\text{lo(steady)}}}_{\text{Rate of change in internal,kinetic, potential,etc,.energies}} = 0$$

 $\dot{E}_{\rm in} = \dot{E}_{\rm out}$ 

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{W}_{\text{out}} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) \left(\text{since } \dot{Q} = 0\right)$$

$$w_{\text{out}} = -\left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = -(\Delta h + \Delta \text{ke} + \Delta \text{pe})$$



### **EXAMPLE** 6–6 Compressing Air by a Compressor

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

### SOLUTION

Air is compressed steadily by a compressor to a specified temperature and pressure. The power input to the compressor is to be determined.

### **Assumptions**

**1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\rm CV} = 0$  and  $\Delta E_{\rm CV} = 0$ . **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The kinetic and potential energy changes are zero,  $\Delta ke = \Delta pe = 0$ .

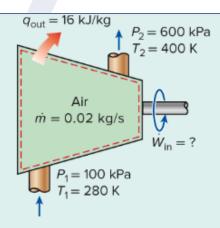
### **Analysis**

We take the *compressor* as the system (**Fig. 6–30**). This is a *control* volume since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ . Also, heat is lost from the system and work is supplied to the system.









### FIGURE 6-30

Schematic for **Example 6–6**.

Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\underline{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}} = \underbrace{dE_{\text{system}}/dt}^{0 \text{ (steady)}} = 0$$
Rate of net energy transfer by heat, work, and mass potential, etc., energies 
$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$E_{\text{in}} = E_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad \text{(since } \Delta \text{ke} = \Delta \text{pe} \cong 0\text{)}$$

$$\dot{W}_{\text{in}} = \dot{m}q_{\text{out}} + \dot{m}(h_2 - h_1)$$

The enthalpy of an ideal gas depends on temperature only, and the enthalpies of the air at the specified temperatures are determined from the air table (**Table A–21**) to be

$$h_1 = h_{@280 \text{ K}} = 280.13 \text{ kJ/kg}$$
  
 $h_2 = h_{@400 \text{ K}} = 400.98 \text{ kJ/kg}$ 

Substituting, the power input to the compressor is determined to be

$$\dot{W}_{in} = (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg}$$
  
= 2.74 kW

#### Discussion

Note that the mechanical energy input to the compressor manifests itself as a rise in enthalpy of air and heat loss from the compressor.

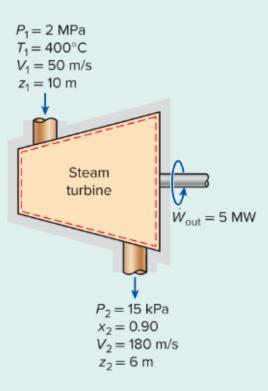




### Ex-6-7

### **EXAMPLE 6–7** Power Generation by a Steam Turbine

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are as indicated in Fig. 6–31.



### **FIGURE 6-31**

Schematic for **Example 6–7**.

- (a) Compare the magnitudes of  $\Delta h$ ,  $\Delta ke$ , and  $\Delta pe$ .
- (b) Determine the work done per unit mass of the steam flowing through the turbine.
- (c) Calculate the mass flow rate of the steam.



### Ex-6-7

### SOLUTION

The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

### **Assumptions**

**1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\rm CV}$  = 0 and  $\Delta E_{\rm CV}$  = 0. **2** The system is adiabatic and thus there is no heat transfer.

### Analysis

We take the *turbine* as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that there is only one inlet and one exit and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ . Also, work is done by the system. The inlet and exit velocities and elevations are given, and thus the kinetic and potential energies are to be considered.

(a) At the inlet, steam is in a superheated vapor state, and its enthalpy is

$$P_1 = 2 \text{ MPa}$$
  
 $T_1 = 400^{\circ}\text{C}$   $h_1 = 3248.4 \text{ kJ/kg}$  (Table A-6)

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)] \text{ kJ/kg} = 2361.01 \text{ kJ/kg}$$

Then

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4) \text{ kJ/kg} = -887.39 \text{ kJ/kg}$$



$$\Delta ke = \frac{V_2^2 - V_1^2}{2} = \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 14.95 \text{ kJ/kg}$$

$$\Delta pe = g(z_2 - z_1) = (9.81 \text{ m/s}^2)[(6 - 10) \text{ m}] \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = -0.04 \text{ kJ/kg}$$

(b) The energy balance for this steady-flow system can be expressed in the rate form as

$$\frac{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}{E_{\text{in}} - \dot{E}_{\text{out}}} = \underbrace{dE_{\text{system}}/dt}^{0 \text{ (steady)}} = 0$$
Rate of net energy transfer by heat, work, and mass potential, etc., energies

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{W}_{\text{out}} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$
 (since  $\dot{Q} = 0$ )

Dividing by the mass flow rate  $\dot{m}$  and substituting, the work done by the turbine per unit mass of the steam is determined to be

$$w_{\text{out}} = -\left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = -(\Delta h + \Delta \text{ke} + \Delta \text{pe})$$
$$= -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = 872.48 \text{ kJ/kg}$$

(c) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{out}}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = 5.73 \text{ kg/s}$$

### **Discussion**

Two observations can be made from these results. First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. This is typical for most engineering devices. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high. Yet the change in kinetic energy is a small fraction of the change in enthalpy (less than 2 percent in our case) and is therefore often neglected.



# EXAMPLE 6-9 Mixing of Hot and Cold Waters in a Shower

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.

### SOLUTION

In a shower, cold water is mixed with hot water at a specified temperature. For a specified mixture temperature, the ratio of the mass flow rates of the hot to cold water is to be determined.

### **Assumptions**

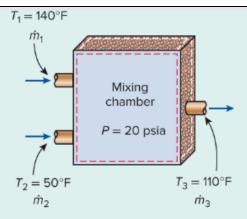
**1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\rm CV}$  = 0 and  $\Delta E_{\rm CV}$  = 0. **2** The kinetic and potential energies are negligible, ke  $\cong$  pe  $\cong$  0. **3** Heat losses from the system are negligible and thus  $\dot{Q}\cong$  0. **4** There is no work interaction involved.

### **Analysis**

We take the *mixing chamber* as the system (**Fig. 6–36**). This is a *control volume* since mass crosses the system boundary during the process. We observe that there are two inlets and one exit.







### **FIGURE 6-36**

Schematic for Example 6-9.

Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows:

Mass balance: 
$$\dot{m}_{in} - \dot{m}_{out} = dm_{system}/dt$$
 = 0  
 $\dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$ 

Energy balance: 
$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic,}} = 0$$

Rate of net energy transfer by heat, work, and mass potential, etc., energies

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$
 
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \qquad \text{(since } \dot{Q} \cong 0, \, \dot{W} = 0, \, \text{ke} \cong \text{pe} \cong 0\text{)}$$

Combining the mass and energy balances,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$



Dividing this equation by  $\dot{m}_2$  yields

$$yh_1 + h_2 = (y + 1)h_3$$

where  $y = \dot{m}_1/\dot{m}_2$  is the desired mass flow rate ratio.

The saturation temperature of water at 20 psia is 227.92°F. Since the temperatures of all three streams are below this value ( $T < T_{sat}$ ), the water in all three streams exists as a compressed liquid (**Fig. 6–37**). A compressed liquid can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f@ 140^{\circ}F} = 107.99 \text{ Btu/lbm}$$
  
 $h_2 \cong h_{f@ 50^{\circ}F} = 18.07 \text{ Btu/lbm}$   
 $h_3 \cong h_{f@ 110^{\circ}F} = 78.02 \text{ Btu/lbm}$ 

### **FIGURE 6-37**

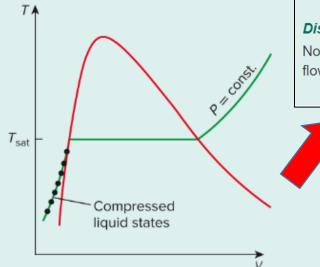
A substance exists as a compressed liquid at temperatures below the saturation temperatures at the given pressure.

Solving for *y* and substituting yields

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = 2.0$$

#### Discussion

Note that the mass flow rate of the hot water must be twice the mass flow rate of the cold water for the mixture to leave at 110°F.







TABI	E A-13											
Supe	rheated ref	rigerant-1	134a									
Τ	v	и	h	s	v	и	h	S	V	и	h	s
°C	m³/kg	kJ/kg	kJ/kg	kJ/kg·K	m³/kg	kJ/kg	kJ/kg	kJ/kg-K	m³/kg	kJ/kg	kJ/kg	kJ/kg-
$P = 0.06 \text{ MPa} (T_{\text{sat}} = -36.95^{\circ}\text{C})$					$P = 0.10 \text{ MPa} (T_{\text{sat}} = -26.37^{\circ}\text{C})$				$P = 0.14 \text{ MPa} (T_{\text{sat}} = -18.77^{\circ}\text{C})$			
Sat.	0.31108	209.13		0.9645	0.19255			0.9519	0.14020		239.19	0.944
-20	0.33608	220.62	240.78	1.0175	0.19841	219.68		0.9721	0.1.020			0.0
-10		227.57		1.0478	0.20743	226.77	247.51		0.14605	225.93	246.37	0.972
0	0.36476	234.67		1.0775	0.21630			1.0333	0.15263	233.25	254.61	1.003
10	0.37893	241.94		1.1067	0.22506	241.32	263.82		0.15908	240.68		1.033
20	0.39302		272.95		0.23373		272.18		0.16544		271.40	1.062
30	0.40705		281.39		0.24233	256.46	280.69		0.17172		279.99	1.091
40	0.42102		289.99		0.25088		289.36		0.17794		288.72	1.119
50	0.43495		298.75		0.25937		298.17		0.18412		297.59	1.147
60	0.44883		307.68		0.26783	280.36	307.15		0.19025	279.97	306.61	1.175
70			316.77		0.27626	288.65		1.2306	0.19635		315.78	1.202
80	0.47651		326.02	1.2998	0.28465				0.20242		325.11	1.228
	0.49032	306.02		1.3261	0.29303	305.71	335.01	1.2836	0.20847		334.59	1.255
100		314.76		1.3521	0.30138	314.48		1.3097	0.21449		344.22	1.281
$P = 0.18 \text{ MPa} (T_{\text{sat}} = -12.73^{\circ}\text{C})$					P = 0.				$P = 0.24 \text{ MPa} (T_{\text{sat}} = -5.38^{\circ}\text{C})$			
Cat								0.9379				
Sat. -10	0.11049 0.11189	225.01	242.90 245.18	0.9398 0.9485	0.09995	224.51 224.57		0.9379	0.08398	227.17	247.32	0.934
-10	0.11169		253.59	0.9485	0.10481		253.07		0.08617	221.20	251.98	0.952
_	0.11722		262.05	4	0.10481		261.60		0.08017	239.00	260.66	
20				1.0400	0.11418	16-36	270.20	1.0304	0.09423	246.76	269.38	1.013
	0.12748		279.27		0.11418	255.16		1.0596	0.09423	254.63		
40			288.07	1.0976	0.11374		287.74	1.0882	0.10193	262.61	287.07	
	0.14230		297.00	1.1257	0.12766	271.16	296.70	1.1164	0.10193	270.73	296.09	
60	0.14230		306.07	1.1533	0.12706		305.79	1.11441	0.10970	278.98	305.24	
70			315.28	1.1806	0.13200	287.75	315.03	1.1714	0.10342	287.38	314.53	1.155
80	0.15136		324.65	1.2075	0.13041	296.27	324.41	1.1714	0.11675	295.93	323.95	1.182
	0.16149		334.16	1.2340	0.14504	304.93	333.94	1.2250	0.12038	304.62	333.51	1.209
	0.16622		343.82	1.2603	0.14933	313.75	343.62	1.2513	0.12038	313.46	343.22	1.235
100	0.10022	313.50	343.02									
$P = 0.28 \text{ MPa} (T_{\text{sat}} = -1.25^{\circ}\text{C})$					$P = 0.32 \text{ MPa} (T_{\text{sat}} = 2.46^{\circ}\text{C})$				$P = 0.40 \text{ MPa } (T_{\text{sat}} = 8.91^{\circ}\text{C})$			
Sat.		229.49			0.06368	231.55	251.93	0.9303	0.051266	235.10	255.61	0.927
0	0.07282		250.85	0.9362								
10	0.07646			0.9681	0.06609	237.56		0.9545	0.051506			
20	0.07997	246.15	268.54	0.9987	0.06925	245.51	267.67		0.054213	244.19		0.962
30	0.08338	254.08	277.42	1.0285	0.07231	253.52		1.0158	0.056796	252.37	275.09	0.993
40		262.12		1.0577	0.07530			1.0452	0.059292			
50	0.09000			1.0862	0.07823		294.87	1.0739	0.061724			
60	0.09324		304.69	1.1143	0.08111	278.17		1.1022	0.064104		302.98	
70	0.09644			1.1419	0.08395			1.1299	0.066443			
80	0.09961		323.48	1.1690	0.08675		323.00	1.1572	0.068747		322.04	1.137
90	0.10275		333.07	1.1958	0.08953	303.99	332.64	1.1841	0.071023		331.75	1.164
100	0.10587	313.17	342.81	1.2223	0.09229	312.87	342.41		0.073274	312.28	341.59	1.190
110	0.10897	322.18	352.69	1.2484	0.09503	321.91	352.31	1.2368	0.075504	321.35	351.55	1.217
120	0.11205	331.34	362.72	1.2742	0.09775	331.08	362.36	1.2627	0.077717	330.56	361.65	1.243
130	0.11512	340.65	372.88	1.2998	0.10045	340.41	372.55	1.2883	0.079913	339.92	371.89	1.268
140	0.11818	350.11	383.20	1.3251	0.10314	349.88		1.3136	0.082096	349.42	382.26	1.294

Specific Volume at 200 KPa and 20 °C





TABLE A-13E												
Superheated refrigerant-134a (Concluded)												
Т	ν	и	h	s	v	и	h	s	v	и	h	s
°F	ft <sup>3</sup> /lbm	Btu/lbm	Btu/Ibm	Btu/Ibm-R	ft³/lbm	Btu/lbm	Btu/lbm	Btu/Ibm-R	ft³/lbm	Btu/lbm	Btu/lbm	Btu/lbm-R
	P = 90 psia (T <sub>sat</sub> = 72.78°F)					100 psia (	$T_{\rm sat} = 79.$	12°F)	$P = 120 \text{ psia } (T_{\text{sat}} = 90.49^{\circ}\text{F})$			
Sat.				0.22011				0.21981	0.39681	106.37	115.18	0.21928
80				0.22332				0.22018	0.41010	100.40	117.50	0.00064
100				0.23191				0.22902				0.22364
120				0.24009				0.23735	0.43692			
140 160				0.24799 0.25565				0.24535 0.25310				0.24059 0.24853
180				0.25565				0.25310	0.48563			
200				0.20313				0.26802				0.26370
220				0.27763				0.27525	0.55206			
240				0.28469				0.28234	0.57312			
260				0.29164				0.28932	0.59379			
280				0.29849				0.29620	0.61413			
300				0.30524				0.30297				0.29898
320				0.31191				0.30966	0.65402			0.30571
	$P = 140 \text{ psia } (T_{\text{sat}} = 100.51^{\circ}\text{F})$					60 psia (	$T_{\rm sat} = 109$	9.50°F)	P = 180 psia (T <sub>sat</sub> = 117.69°F)			
Sat.	0.33800	107.52	116.28	0.21883	0.29339	108.51	117.20	0.21840	0.25833	109.38	117.98	0.21799
120				0.22775				0.22339				0.21912
140				0.23630	0.32774							0.22852
160				0.24444				0.24070				0.23720
180				0.25229				0.24872				0.24542
200	0.44743	129.66	141.25	0.25990	0.38494	129.13	140.52	0.25647	0.33619	128.58	139.77	0.25332
220	0.46657	134.13	146.22	0.26731	0.40234	133.65	145.56	0.26399	0.35228	133.16	144.89	0.26095
240	0.48522	138.65	151.22	0.27457	0.41921	138.21	150.62	0.27133	0.36779	137.76	150.01	0.26838
260	0.50345	143.22	156.26	0.28168				0.27851				0.27564
280				0.28866				0.28555				0.28275
300				0.29553				0.29248				0.28972
320				0.30230				0.29929				0.29658
340				0.30898	0.49828			0.30600	0.43980			0.30333
360	0.59041	167.03	182.33	0.31557	0.51338	166.75	181.95	0.31262	0.45347	166.47	181.57	0.30998
$P = 200 \text{ psia } (T_{\text{sat}} = 125.22^{\circ}\text{F})$					P = 3				$P = 400 \text{ psia } (T_{\text{sat}} = 179.86^{\circ}\text{F})$			
Sat.	0.23001	110.13	118.64	0.21757	0.14279	112.61	120.54	0.21517	0.09643	113.36	120.50	0.21164
140	0.24541	113.86	122.94	0.22483								
160				0.23386				0.21747				
180				0.24231				0.22803				0.21174
200				0.25037				0.23734				0.22473
220				0.25813				0.24596				0.23502
240				0.26566				0.25412				0.24420
260				0.27300				0.26193				0.25272
280				0.28017				0.26949				0.26079
300				0.28720				0.27683				0.26853
320				0.29410				0.28399	0.17423			
340				0.30089				0.29100		157.97		0.28328
360	0.40552	100.18	181.19	0.30758	0.26159	164./1	179.23	0.29788	0.18951	163.16	177.19	0.29037



## **Summary**

- Conservation of mass
  - Mass and volume flow rates
  - Mass balance for a steady-flow process
  - Mass balance for incompressible flow
- Flow work and the energy of a flowing fluid
  - Energy transport by mass
- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
  - Nozzles and Diffusers
  - Turbines and Compressors
  - Throttling valves
  - Mixing chambers and Heat exchangers
  - Pipe and Duct flow
- Energy analysis of unsteady-flow processes