Chapter 11

FLUID STATICS

Objectives

- Calculate the forces exerted by a fluid at rest on plane or curved submerged surfaces
- Analyze the stability of floating and submerged bodies

11-1 INTRODUCTION TO FLUID STATICS

Fluid statics: Deals with problems associated with fluids at rest.

The fluid can be either gaseous or liquid.

Hydrostatics: When the fluid is a liquid.

Aerostatics: When the fluid is a gas.

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it.

11-1 INTRODUCTION TO FLUID STATICS

The only stress we deal with in fluid statics is the *normal stress*, which is the pressure, and the variation of pressure is due only to the weight of the fluid.

The topic of fluid statics has significance only in gravity fields.

The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics.

11-2 HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

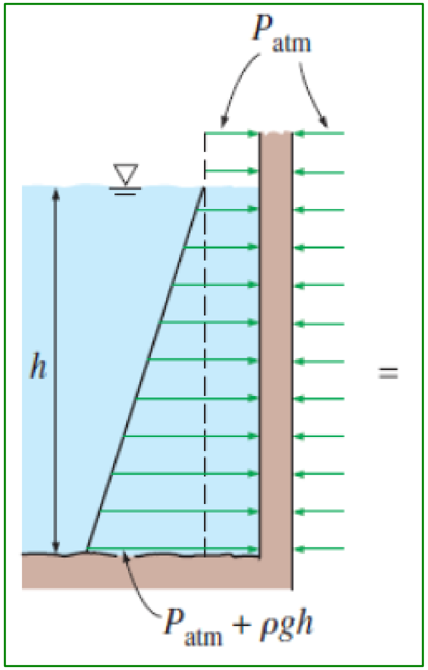
A plate, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface when exposed to a liquid.

On a *plane* surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application*, which is called the center of pressure**.**

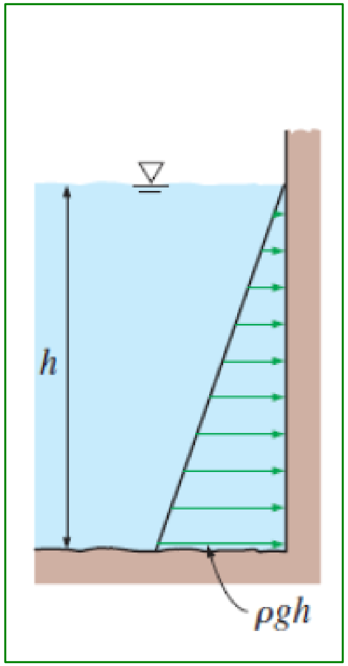
Hoover Dam.

11-2 Hydrostatic Forces On Submerged Plane Surfaces

When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

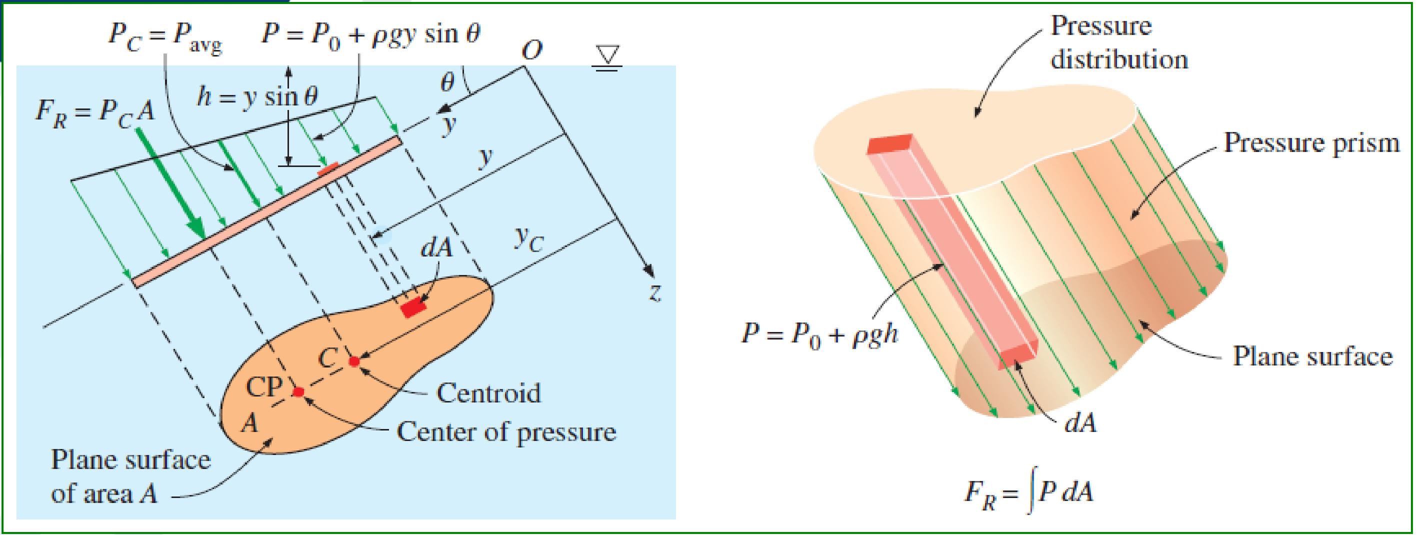


(a) P_{atm} **considered**



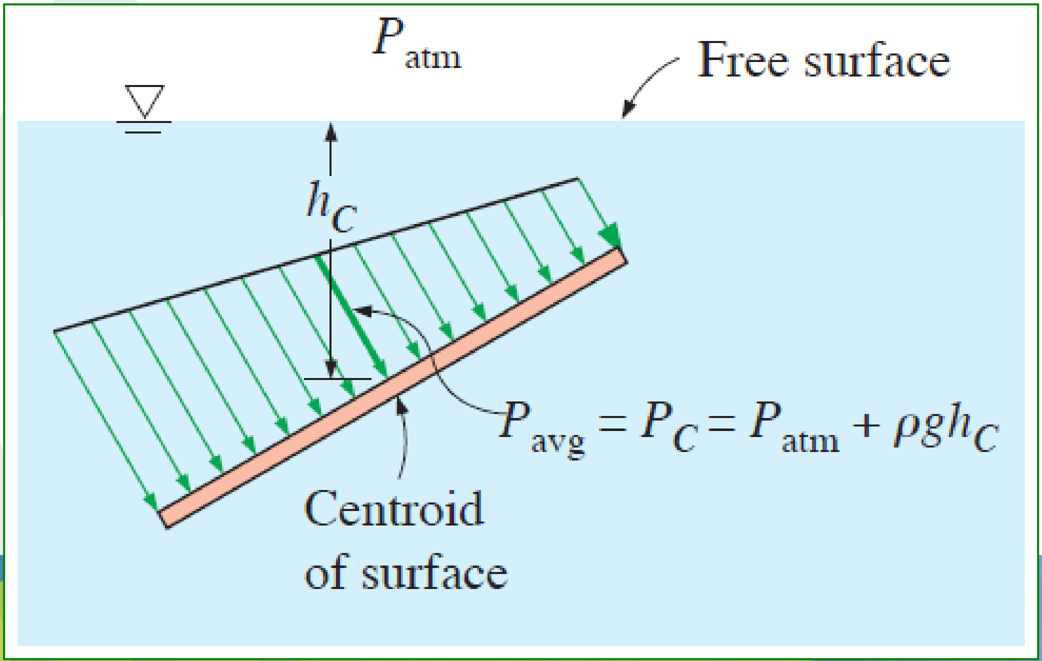
(b) P_{atm} **Substracted**

11-2 HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES-1



Hydrostatic force on an inclined plane surface completely submerged in a liquid.

$$F_R = (P_0 + \rho g y_c \sin\theta)A = (P_0 + \rho g h_c)A = P_C A = P_{avg} A$$

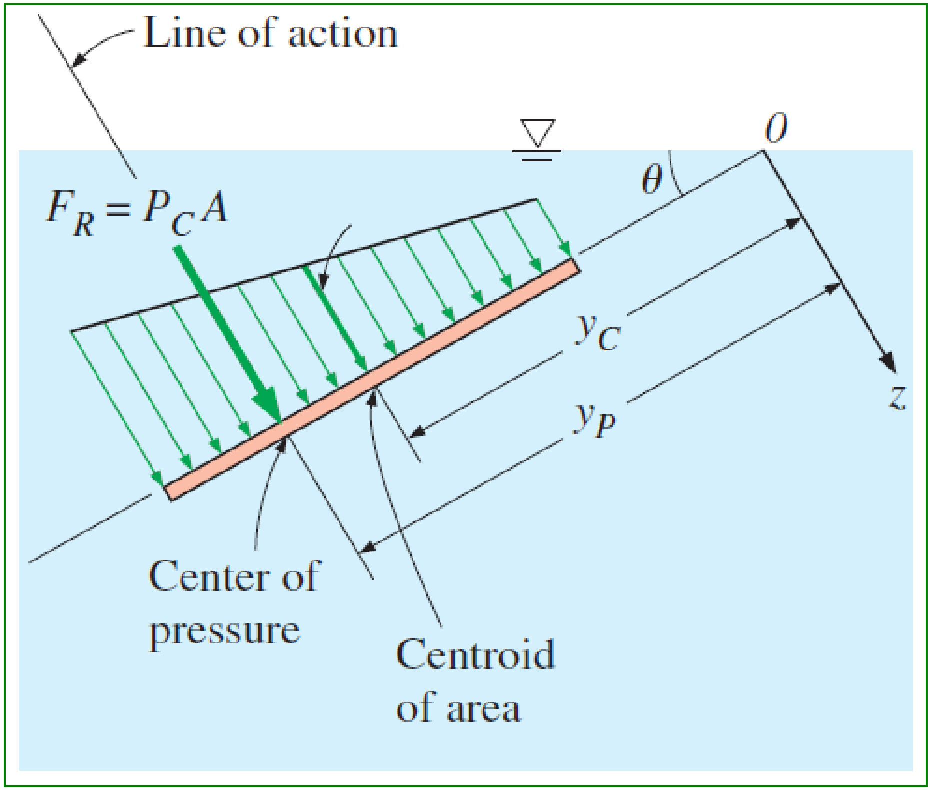


P_0 = atmospheric Pressure

The pressure at the centroid of a surface is equivalent to the average pressure on the surface.

11-2 Hydrostatic Forces On Submerged Plane Surfaces-2

The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.



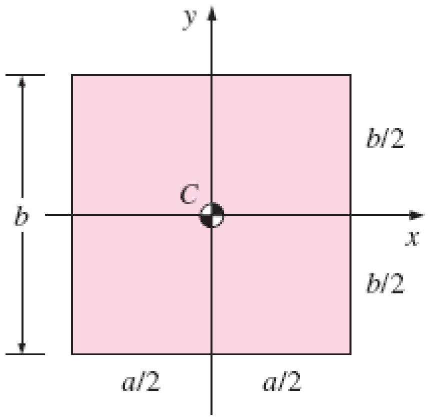
$$y_p = y_c + \frac{I_{xx,c}}{[y_c + P_0/(\rho g \sin\theta)]A}$$

$$y_P = y_C + \frac{I_{xx,C}}{y_C A}$$

$$I_{xx,o} = \int_A y^2 dA$$

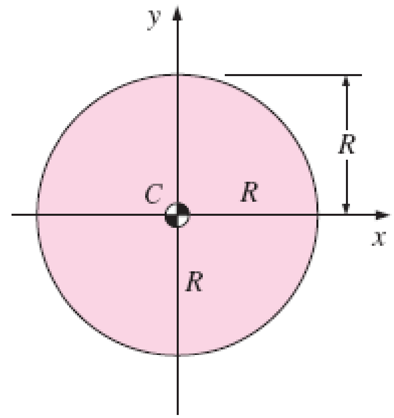
second moment of area (***area moment of inertia***) **about the *x*-axis.**

$$I_{xx,o} = I_{xx,c} + y_c^2 A$$



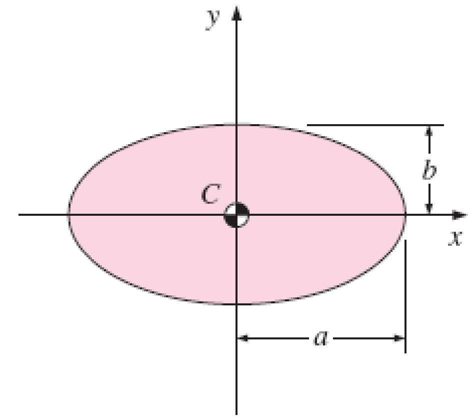
$A = ab,\ I_{xx,\,C} = ab^3/12$

(*a*) Rectangle



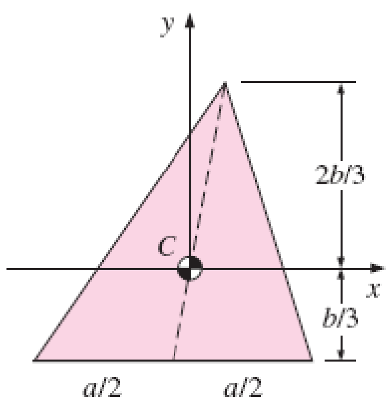
$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$

(*b*) Circle



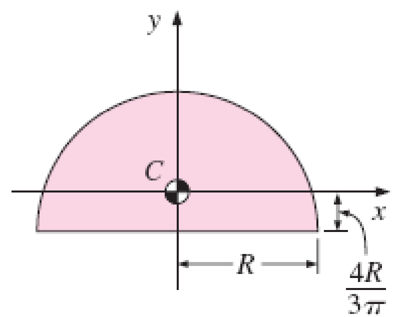
$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$

(*c*) Ellipse



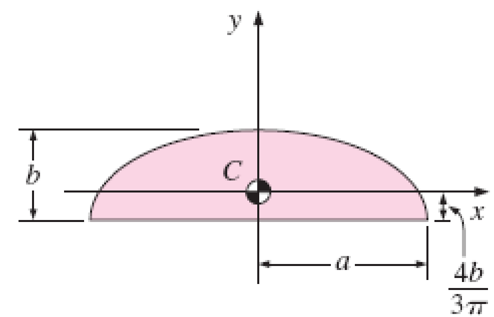
$A = ab/2,\ I_{xx,\,C} = ab^3/36$

(*d*) Triangle



$A = \pi R^2/2,\ I_{xx,\,C} = 0.109757R^4$

(*e*) Semicircle



$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$

(*f*) Semiellipse

The centroid and the centroidal moments of inertia for some common geometries.

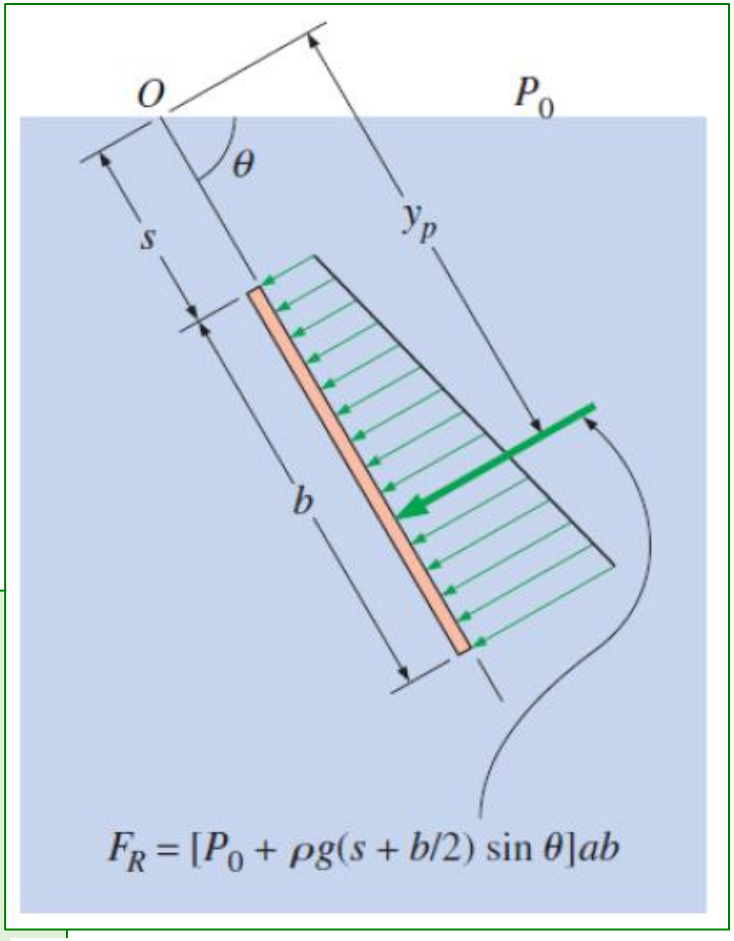
Special Case: Submerged Rectangular Plate

Hydrostatic force acting on the top surface of a submerged tilted rectangular plate.

$$y_P = s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin\theta)]ab}$$

$$= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]}$$

Eq. 11-9

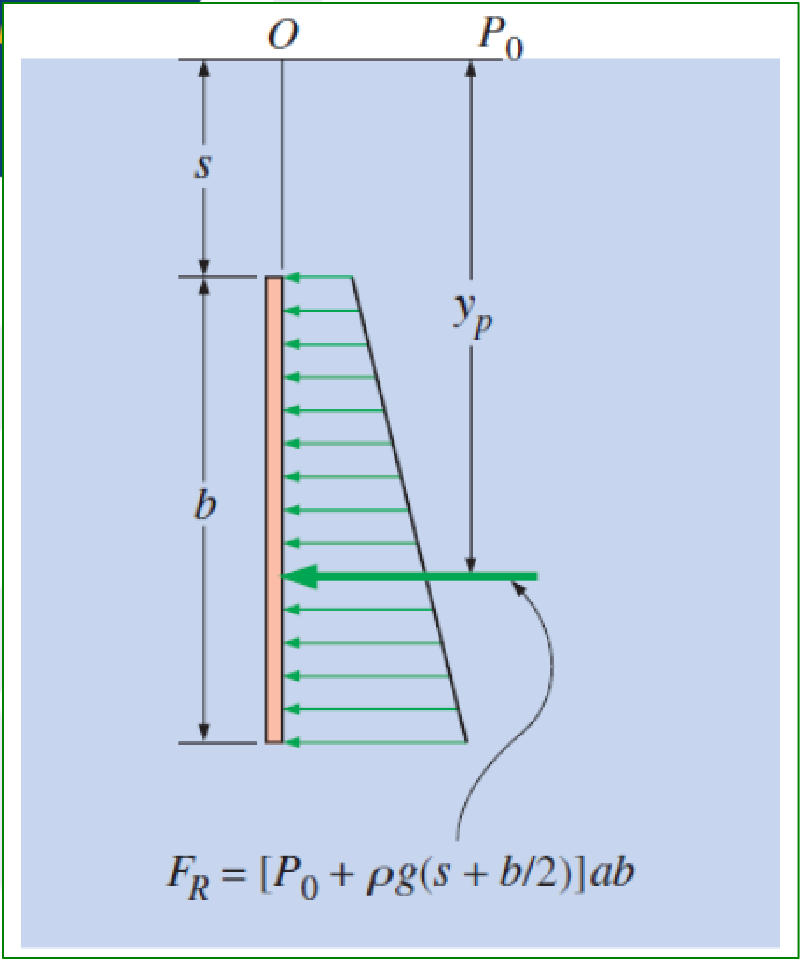


(a)Tilted plate

Tilted rectangular plate: $F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab$

Tilted rectangular plate $(s = 0)$***:*** $F_R = [P_0 + \rho g(b\sin\theta)/2]ab$

Special Case: Submerged Rectangular Plate-1



(b) **Vertical plate**

Hydrostatic force acting on the top surface of a submerged vertical rectangular plate.

Vertical rectangular plate: $F_R = [P_0 + \rho g(s + b/2)]ab$

Vertical rectangular plate $(s = 0)$***:*** $F_R = (P_0 + \rho g b/2)ab$

EXAMPLE 11–1 Hydrostatic Force Acting on the Door of a Submerged Car

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels (Fig. 11–9). The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

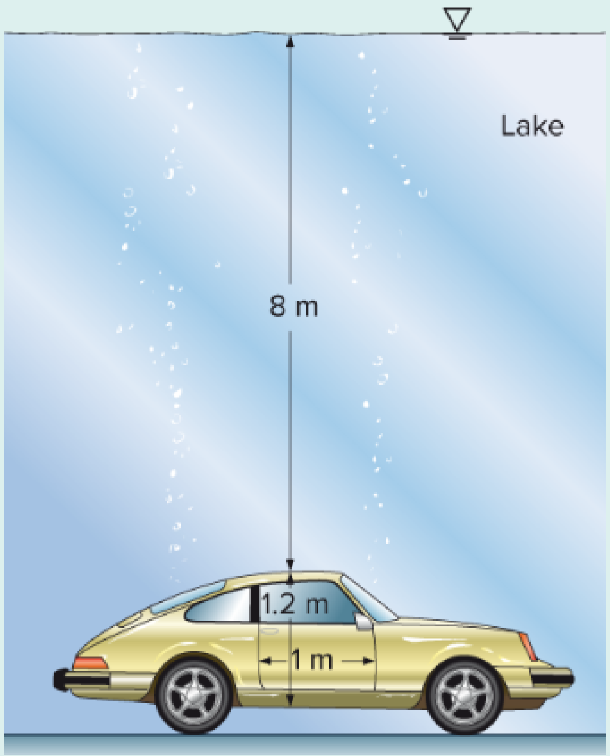


FIGURE 11–9
Schematic for Example 11–1.

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SOLUTION

A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions

1 The bottom surface of the lake is horizontal. **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. Therefore, atmospheric pressure cancels out in the calculations since it acts on both sides of the door. **5** The weight of the car is larger than the buoyant force acting on it.

Properties

We take the density of lake water to be 1000 kg/m^3 throughout.

Analysis

The average (gage) pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$P_{avg} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 + 1.2/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{84.4 \text{ kN/m}^2}$$

Then the resultant hydrostatic force on the door becomes

$$F_R = P_{avg}A = (84.4\ \text{kN/m}^2)\ (1\ \text{m} \times 1.2\ \text{m}) = \mathbf{101.3\ kN}$$

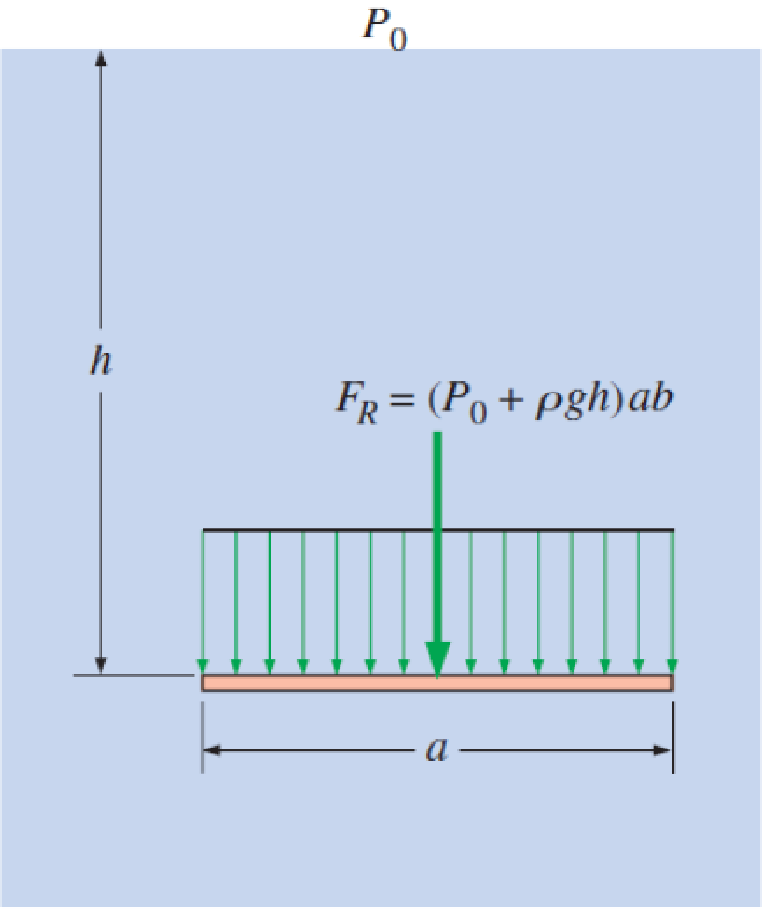
The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from **Eq. 11–9** by setting $P_0 = 0$, yielding

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61\ m}$$

Discussion

A strong person can lift 100 kg, which is a weight of 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN·m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN·m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

Special Case: Submerged Rectangular Plate-2

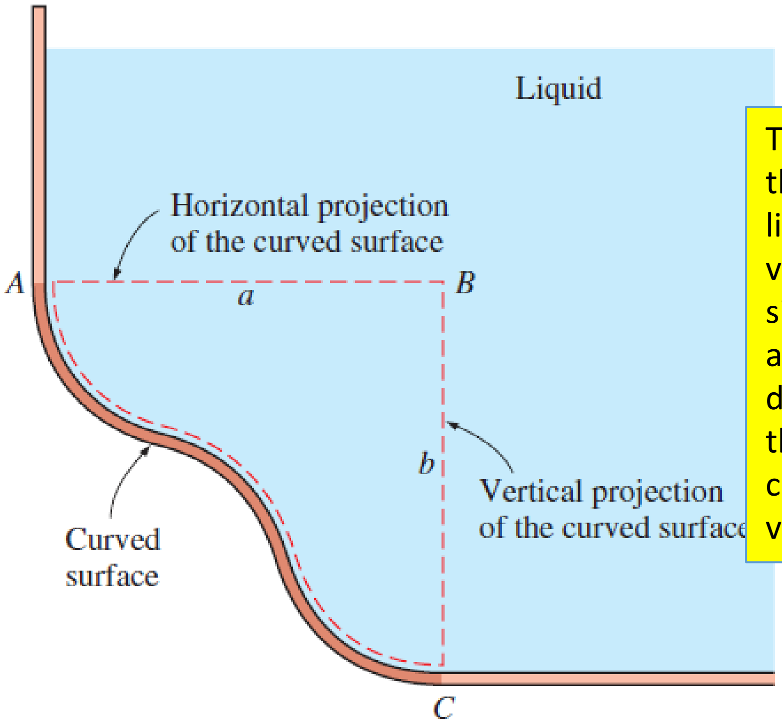


***(c)* Horizontal plate**

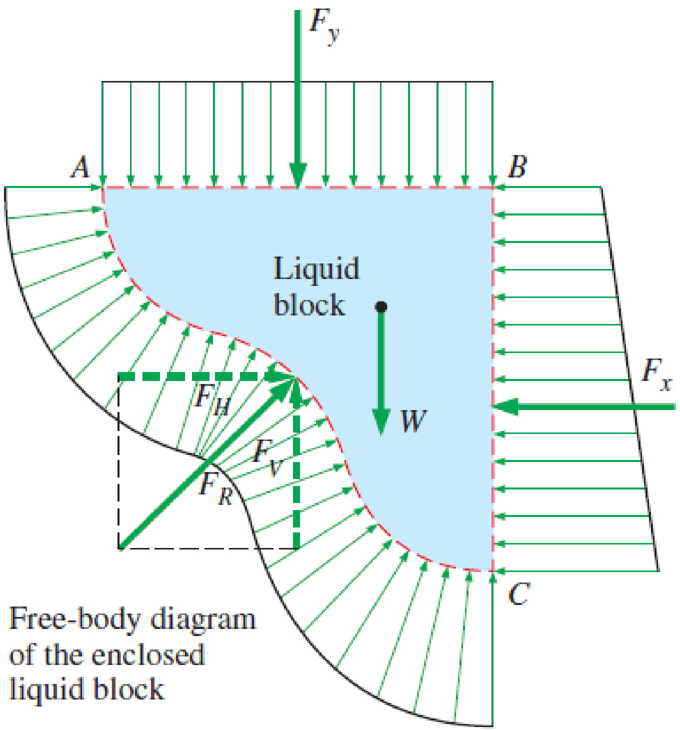
Hydrostatic force acting on the top surface of a submerged horizontal rectangular plate.

Horizontal rectangular plate: $F_R = (P_0 + \rho g h)ab$

11-3 HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES



The weight of the enclosed liquid block of volume V is simply W= ρgV, and it acts downward through the centroid of this volume



$$F_R = \sqrt{F_H^2 + F_V^2}$$

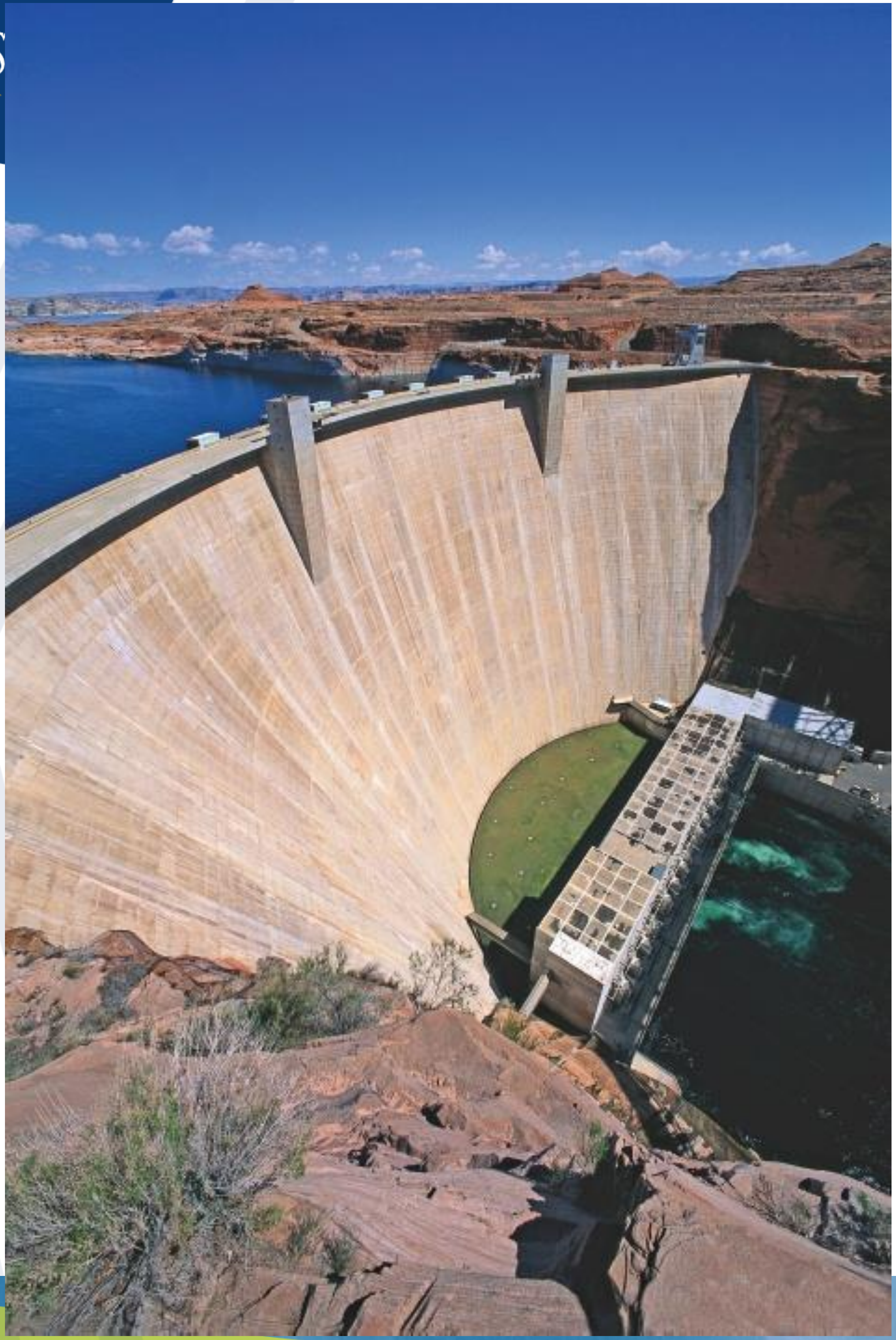
$$\propto = F_V / F_H$$

Tangent angle

Determination of the hydrostatic force acting on a submerged curved surface.

Horizontal force component on curved surface: $F_H = F_x$

Vertical force component on curved surface: $F_V = F_y + W$



11-3 HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES-1

In many structures of practical application, the submerged surfaces are not flat, but curved as here at Glen Canyon Dam in Utah and Arizona.

EXAMPLE 11–2 A Gravity-Controlled Cylindrical Gate A long solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in Fig. 11–15. When the water level reaches 5 m, the gate opens by turning about the hinge at point A. Determine (a) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (b) the weight of the cylinder per meter length of the cylinder

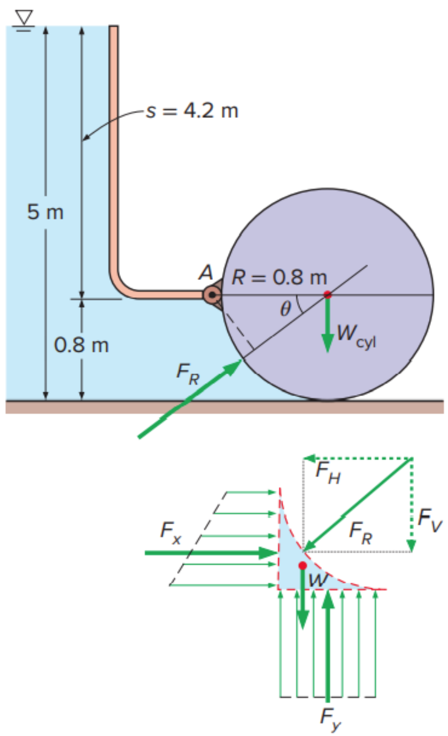


FIGURE 11–15
Schematic for Example 11–2 and the free-body diagram of the liquid underneath the cylinder.

SOLUTION The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per meter length are to be determined.

Assumptions **1** Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis (*a*) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{avg}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)$$

$$= 36.1\ \text{kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{bottom} A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(5\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right)$$

$$= 39.2\ \text{kN}$$

Weight (downward) of fluid block for 1 m width into the page:

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1 \text{ m})$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})^2(1 - \pi/4)(1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)$$
$$= 1.3 \text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9 \text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3 \text{ kN}}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

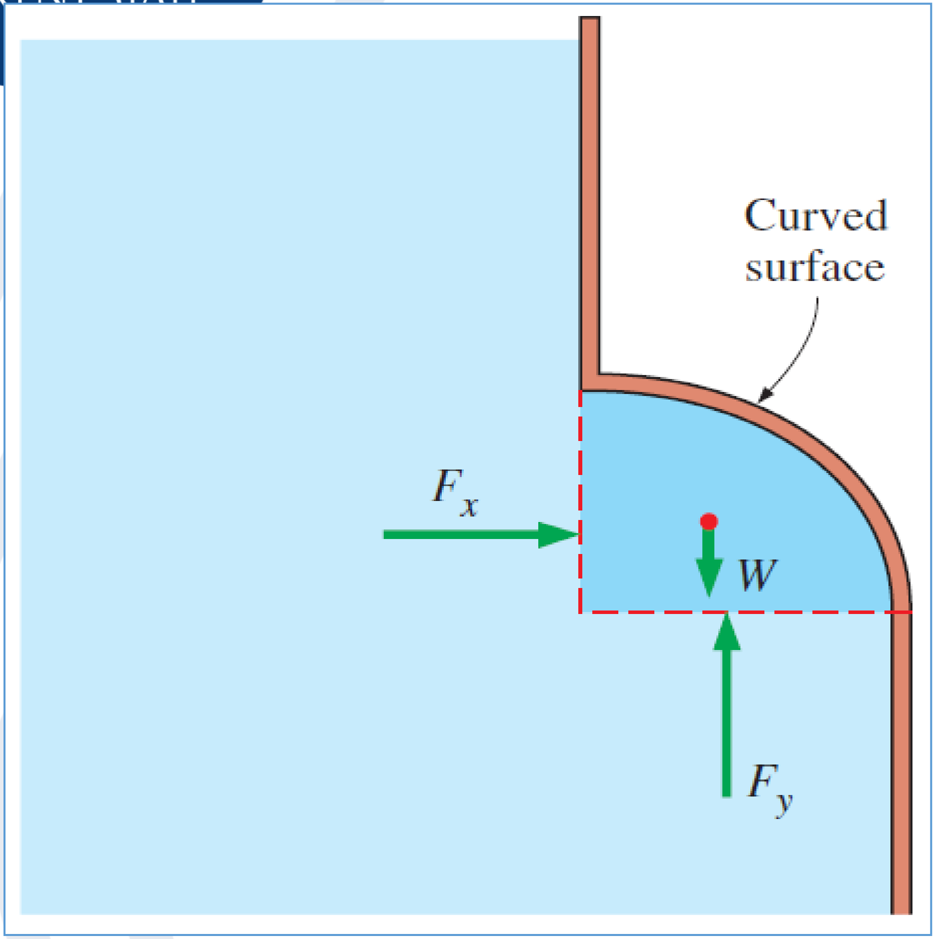
Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN/m length of the cylinder, and its line of action passes through the center of the cylinder making an angle 46.4° with the horizontal.

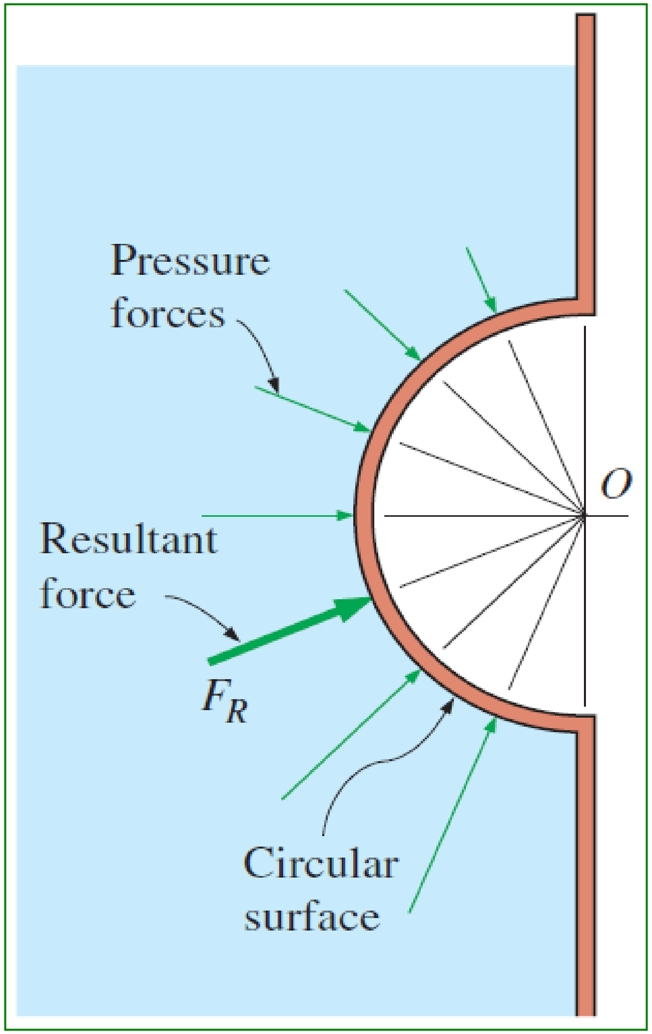
(*b*) When the water level is 5 meter high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3 \text{ kN}) \sin 46.4° = \mathbf{37.9 \text{ kN}}$$

Discussion The weight of the cylinder per meter length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg/m length and to a density of 1921 kg/m^3 for the material of the cylinder.

11-3 HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES-2





When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

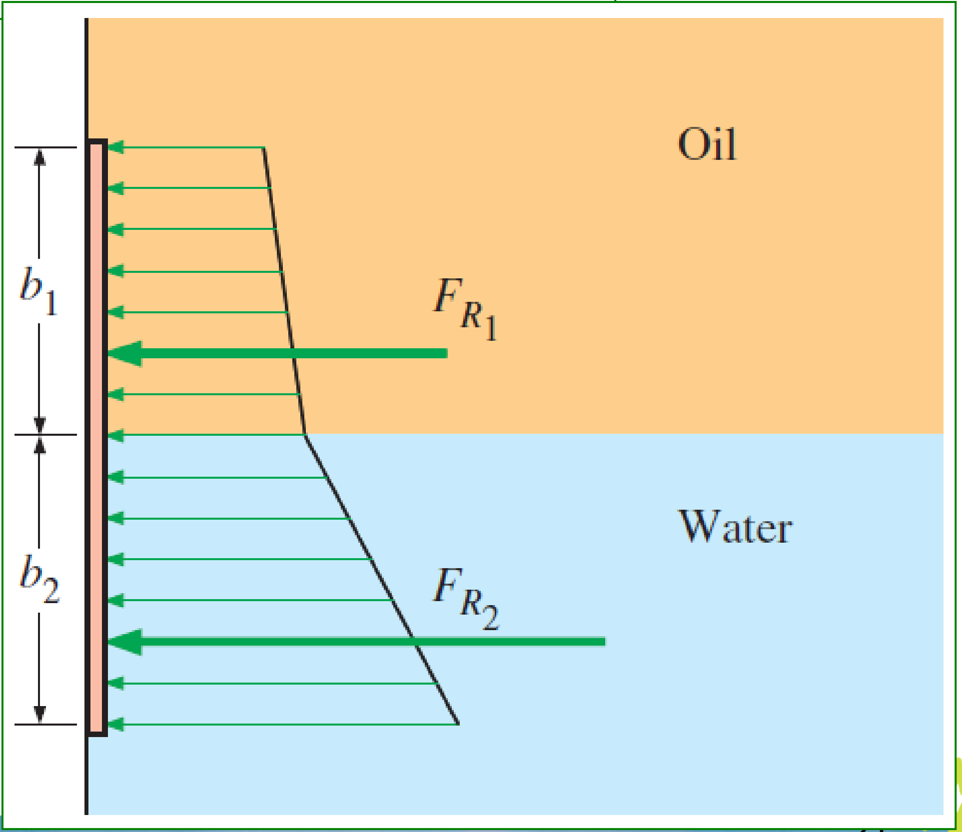
11-3 HYDROSTATIC FORCES ON SUBMERGED CURVED SURFACES-3

in a multilayered fluid of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as

Plane surface in a multilayered fluid: $F_R = \sum F_{R,i} = \sum P_{C,i} A_i$

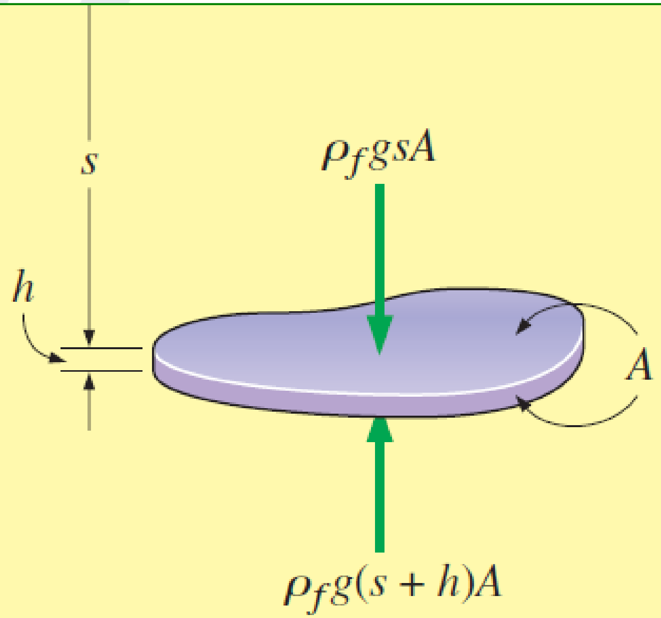
$$P_{C,i} = P_0 + \rho_i g h_{C,i}$$

The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.



11-4 BUOYANCY AND STABILITY

Buoyant force: The upward force a fluid exerts on a body immersed in it. The buoyant force is caused by the increase of pressure with depth in a fluid.



The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface.

It is also independent of the density of the solid body.

A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f gsA = \rho_f ghA = \rho_f gV$$

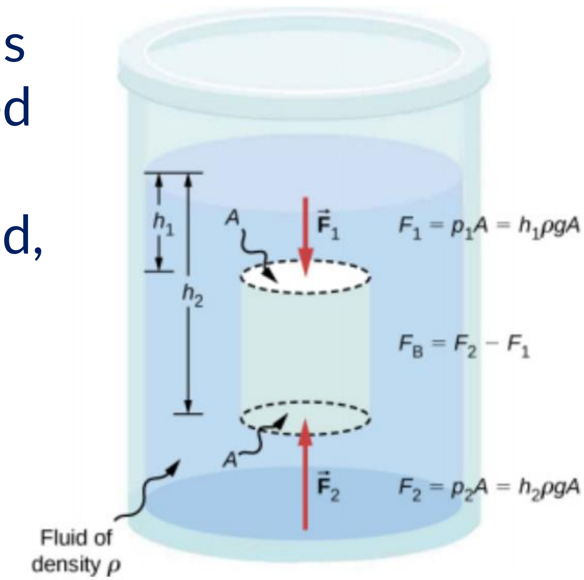
Buoyant force

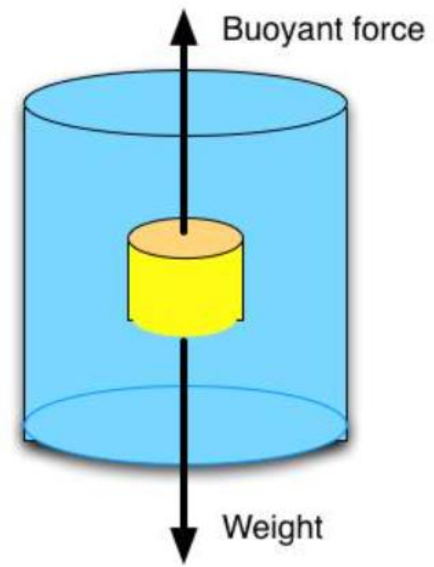
Buoyant force is the upward force a fluid exerts on an object. Archimedes' Principle is the fact that buoyant force is equal to the weight of the displaced fluid.

The buoyant force is equal to the weight of the displaced water by **Archimedes' Principle:**

$$F^{BF}_{fluid \to solid} = \rho_{fluid}\, g V_{solid}$$

Since the buoyant force is the weight of the displaced water, it is the mass of the water, determined from the density and the volume of the water displaced by the solid, times the gravitational field (in N/kg).





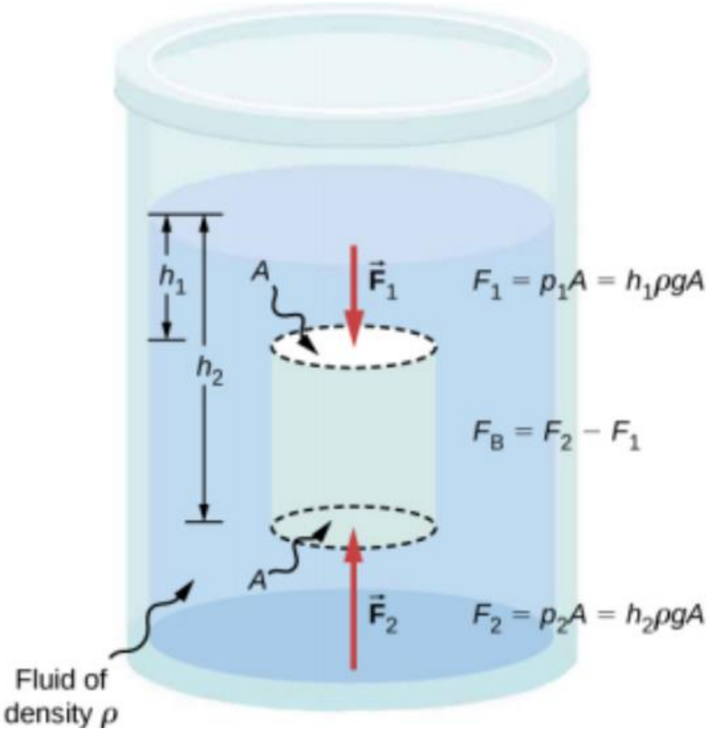
Archimedes' Principle

The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes' principle** is

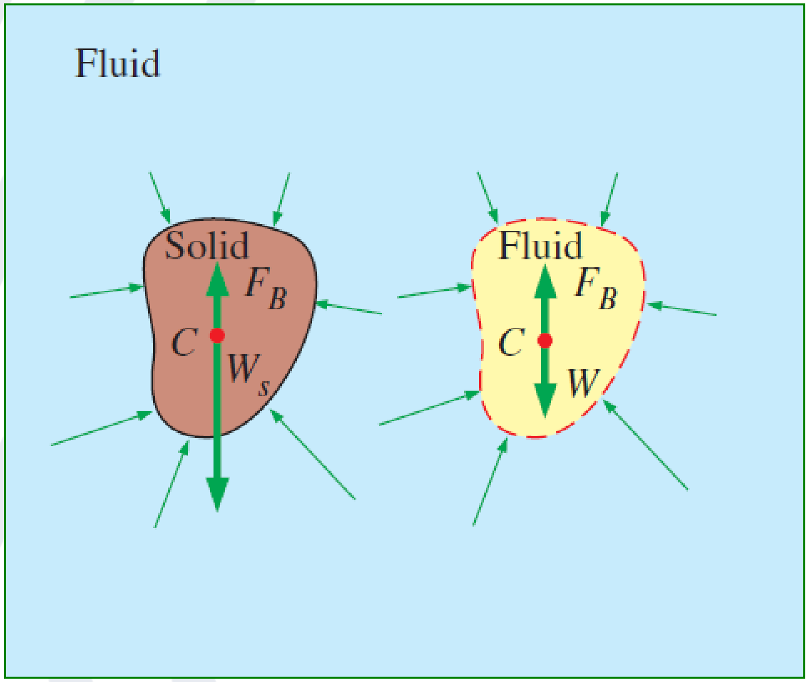
$$F_B = w_{fl},$$

Where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

- Archimedes' principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

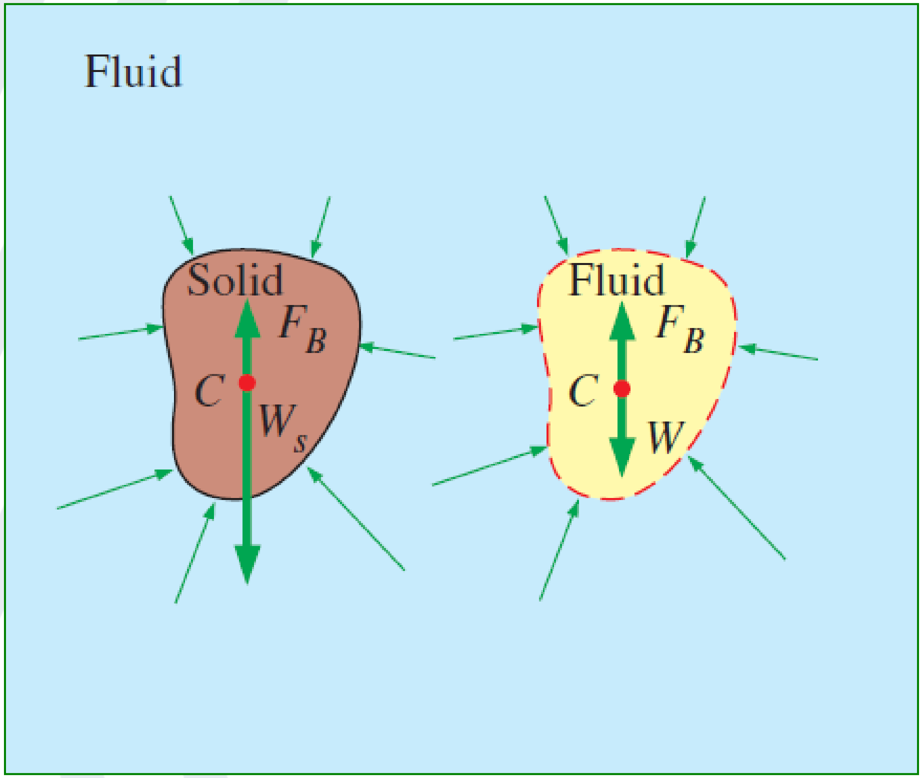


11-4 BUOYANCY AND STABILITY-1



- The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical.
- The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W of the displaced fluid but is opposite in direction.

11-4 BUOYANCY AND STABILITY-1



- For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces.
- (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.)

EXAMPLE 11–4 Weight Loss of an Object in Seawater

A crane is used to lower weights into the sea (density = 1025 kg/m^3) for an underwater construction project (**Fig. 11–21**). Determine the tension in the rope of the crane due to a rectangular 0.4-m × 0.4-m × 3-m concrete block (density = 2300 kg/m^3) when it is (*a*) suspended in the air and (*b*) completely immersed in water.

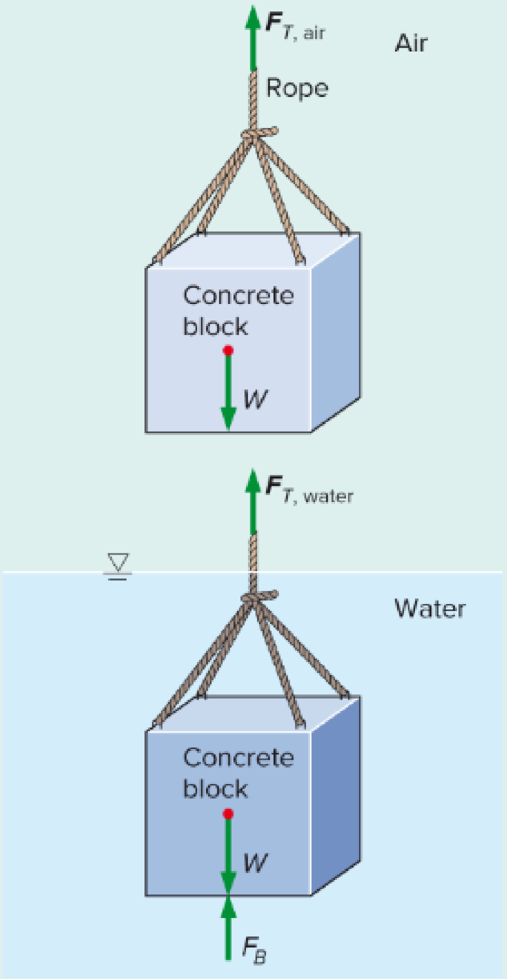


FIGURE 11–21

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SOLUTION

A concrete block is lowered into the sea. The tension in the rope is to be determined before and after the block is in water.

Assumptions

1 The buoyant force in air is negligible. **2** The weight of the ropes is negligible.

Properties

The densities are given to be 1025 kg/m^3 for seawater and 2300 kg/m^3 for concrete.

Analysis

(*a*) Consider a free-body diagram of the concrete block. The forces acting on the concrete block in air are its weight and the upward pull action (tension) by the rope. These two forces must balance each other, and thus the tension in the rope must be equal to the weight of the block:

$$V = (0.4\text{ m})(0.4\text{ m})(3\text{ m}) = 0.48\text{ m}^3$$

$$F_{T,\text{air}} = W = \rho_{\text{concrete}}\, gV$$

$$V = (0.4\ \text{m})(0.4\ \text{m})(3\ \text{m}) = 0.48\ \text{m}^3$$

$$F_{T,\,\text{air}} = W = \rho_{\text{concrete}}\, gV$$

$$= (2300\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.48\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{10.8\ kN}$$

(*b*) When the block is immersed in water, there is the additional force of buoyancy acting upward. The force balance in this case gives

$$F_B = \rho_f\, gV = (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.48\ \text{m}^3)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 4.8\ \text{kN}$$

$$F_{T,\,\text{water}} = W - F_B = 10.8 - 4.8 = \mathbf{6.0\ kN}$$

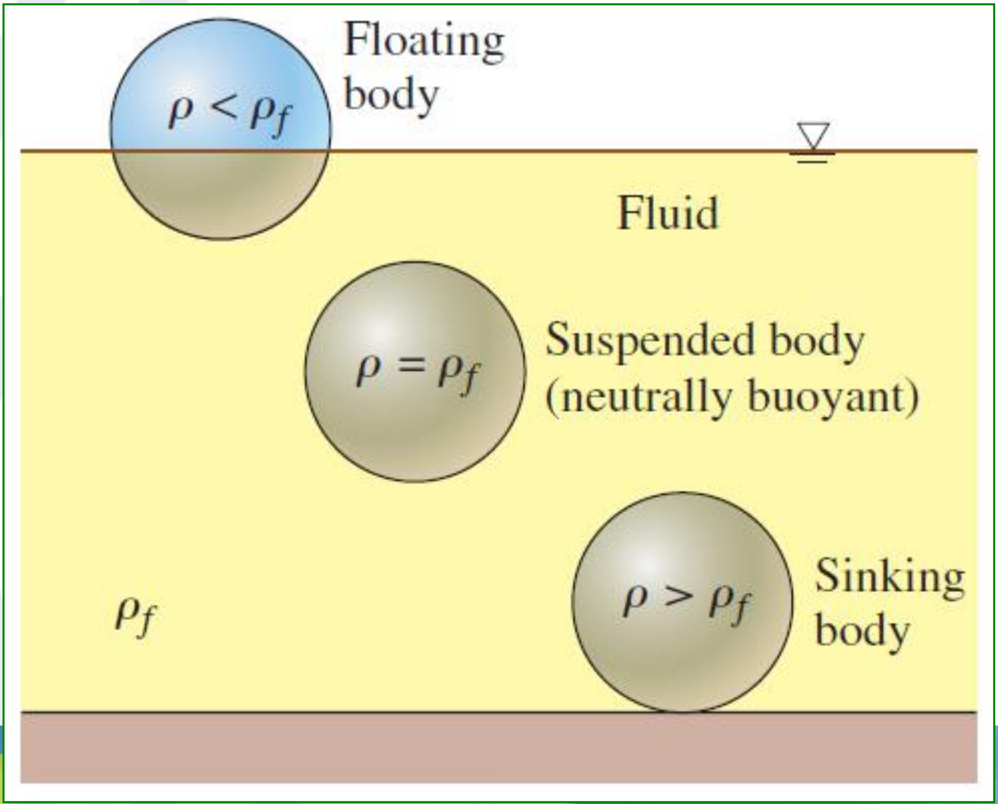
Discussion

Note that the weight of the concrete block, and thus the tension of the rope, decreases by (10.8 – 6.0)/10.8 = 55 percent in water.

11-4 BUOYANCY AND STABILITY-2

For *floating* bodies, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body:

$$F_B = W \longrightarrow \rho_f g V_{\text{sub}} = \rho_{\text{avg,body}} g V_{\text{total}} \longrightarrow \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{avg,body}}}{\rho_f}$$

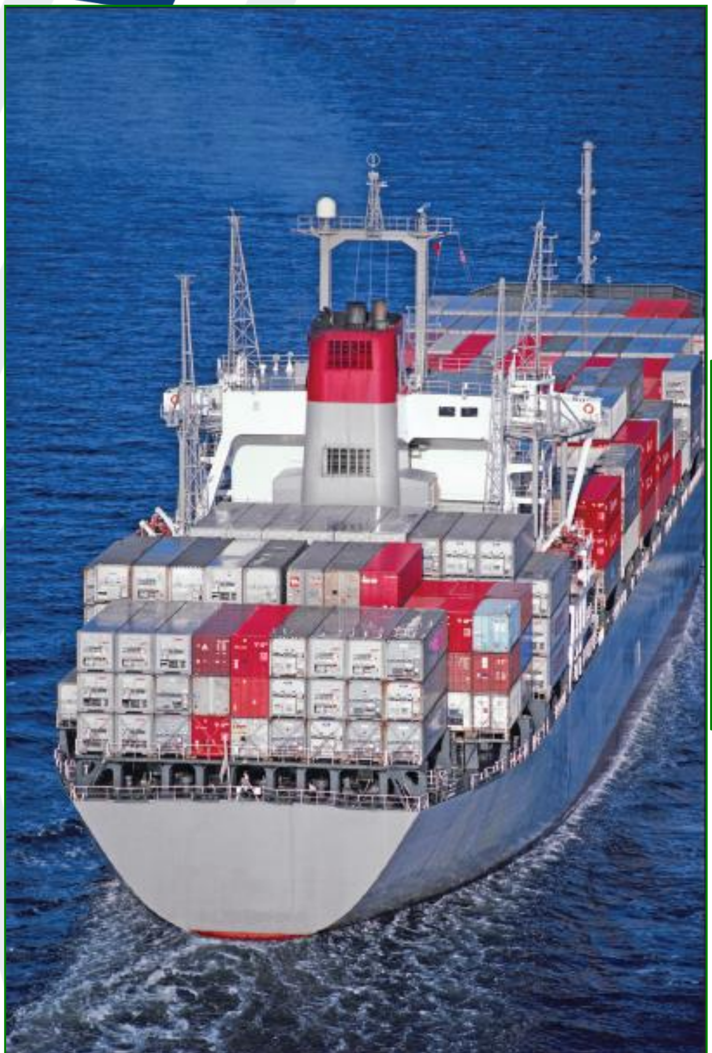


A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

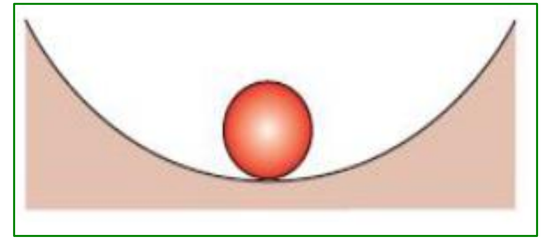
11-4 BUOYANCY AND STABILITY-3

- The altitude of a hot air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air.
- When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

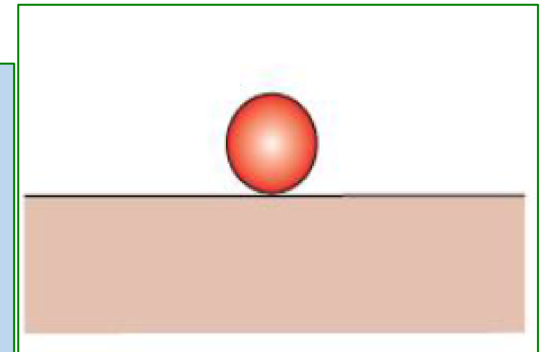
Stability of Immersed and Floating Bodies



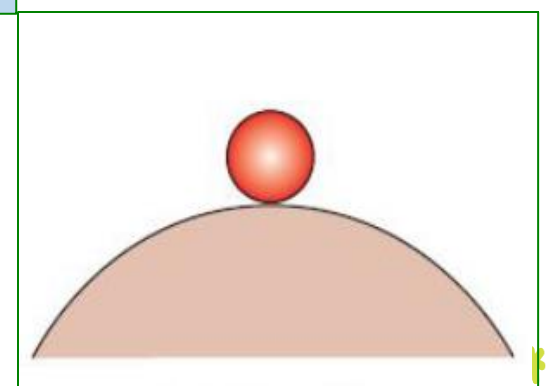
Stability is easily understood by analyzing a ball on the floor.



(a) **Stable**



(b) **Neutrally stable**



(c) **Unstable**

For floating bodies such as ships, stability is an important consideration for safety.

Stability of Immersed and Floating Bodies-1

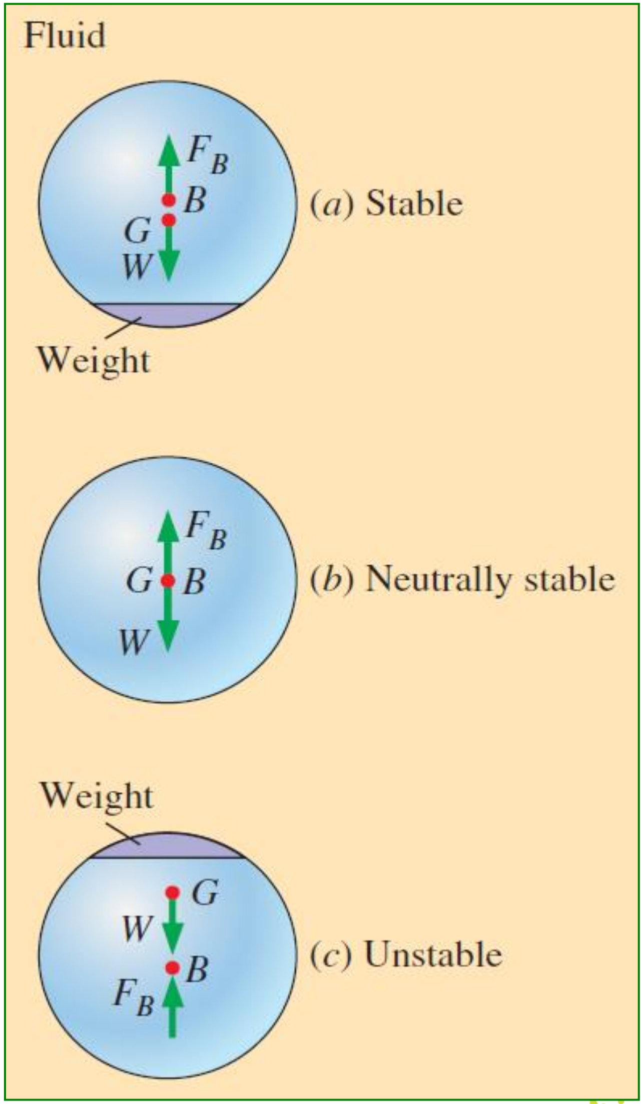
A floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.

An immersed neutrally buoyant body is

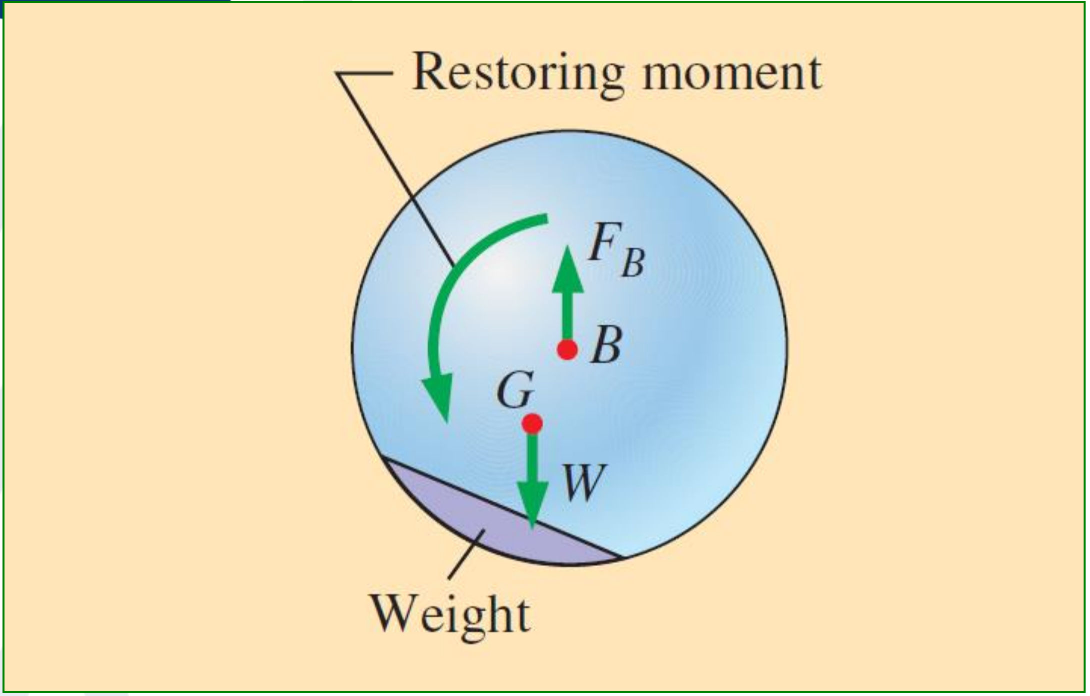
a) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body,

b) neutrally stable if *G* and *B* are coincident, and

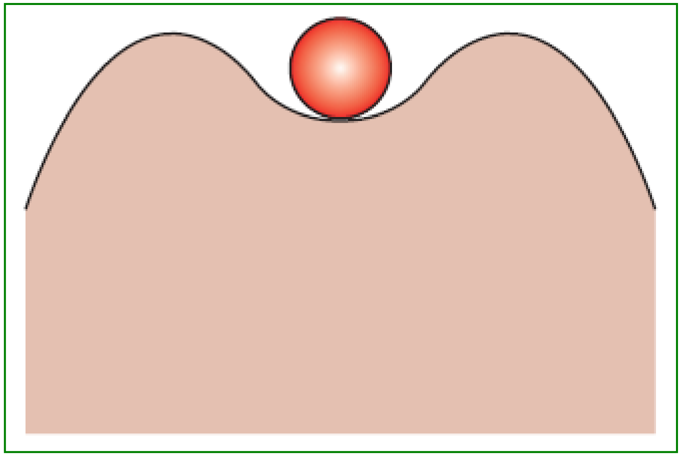
c) unstable if *G* is directly above *B*.



Stability of Immersed and Floating Bodies-2

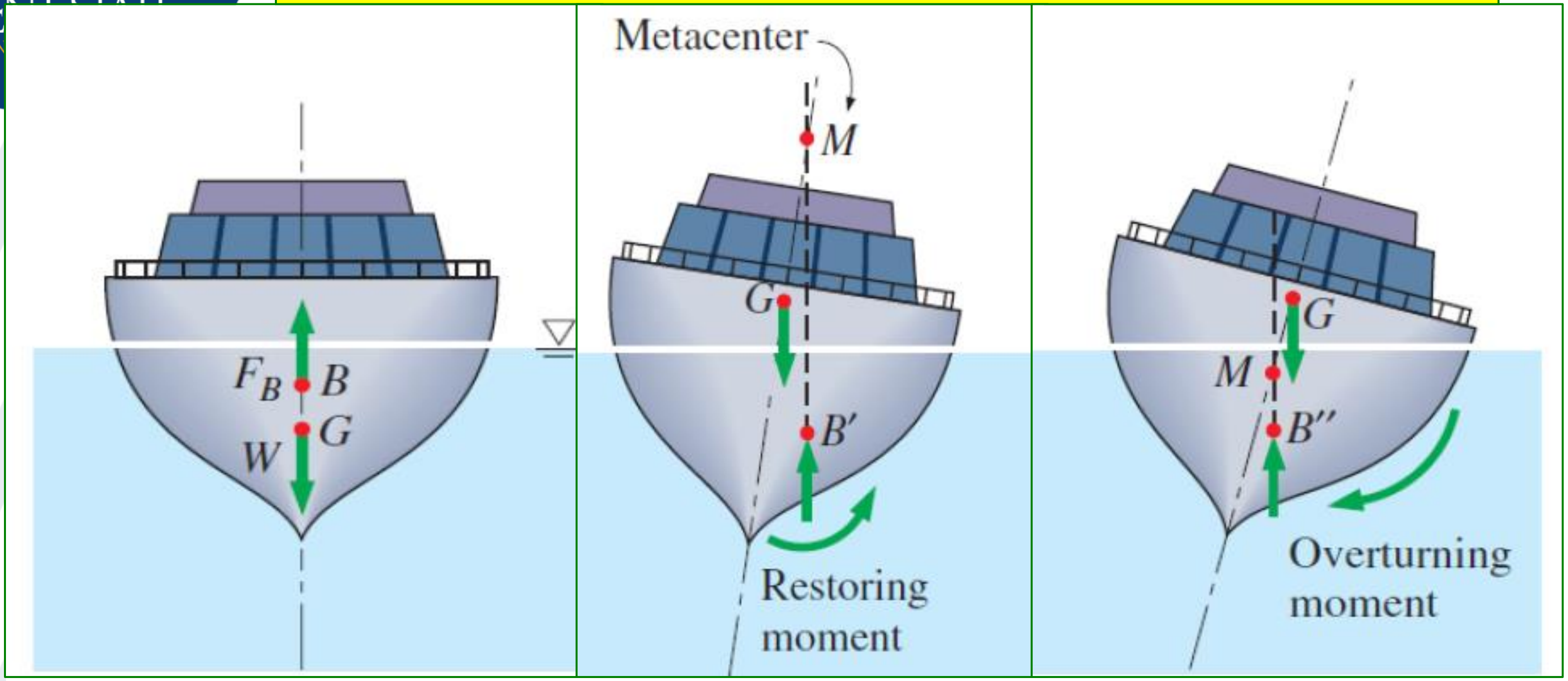


When the center of gravity *G* of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.



A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.

Stability of Immersed and Floating Bodies-3



(a) **Stable** *(b)* **stable** *(c)* **Unstable**

A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

Metacentric height *GM:* The distance between the center of gravity *G* and the metacenter *M*—the intersection point of the lines of action of the buoyant force through the body before and after rotation.

The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

Summary

- Introduction to Fluid Statics
- Hydrostatic Forces on Submerged Plane Surfaces
- Hydrostatic Forces on Submerged Curved Surfaces
- Buoyancy and Stability

SUMMARY

Fluid statics deals with problems associated with fluids at rest, and it is called *hydrostatics* when the fluid is a liquid. The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous fluid is equal to the product of the pressure P_C at the centroid of the surface and the area A of the surface and is expressed as

$$F_R = (P_0 + \rho g h_C)A = P_C A = P_{avg} A$$

where $h_C = y_C \sin\theta$ is the *vertical distance* of the centroid from the free surface of the liquid. The pressure P_0 at the surface of the liquid is usually atmospheric pressure, which cancels out in most cases since it acts on both sides of the plate. The point of intersection of the line of action of the resultant force and the surface is the *center of pressure*. The vertical location of the line of action of the resultant force is given by

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area.

A fluid of density P_f exerts an upward force on a body immersed in it. This force is called the *buoyant force* and is expressed as

$$F_B = \rho_f g V$$

where V is the volume of the body. This is known as *Archimedes' principle* and is expressed as—the buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body; it acts upward through the centroid of the displaced volume. In a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface. For *floating* bodies, the submerged volume fraction of the body is equal to the ratio of the average density of the body to the density of the fluid.

11–16 A 6-m-high, 5-m-wide rectangular plate blocks the end of a 5-m-deep freshwater channel, as shown in **Fig. P11–16**. The plate is hinged about a horizontal axis along its upper edge through a point *A* and is restrained from opening by a fixed ridge at point *B*. Determine the force exerted on the plate by the ridge.

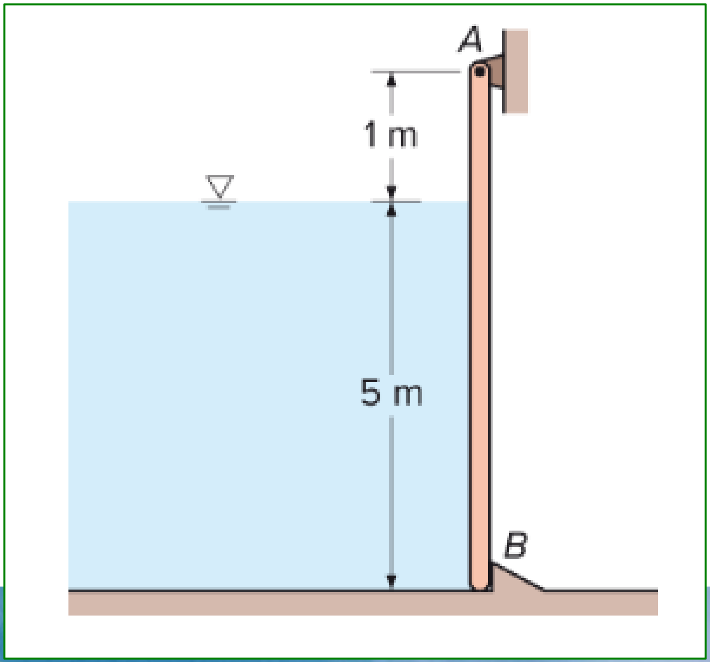


Fig. P11–16

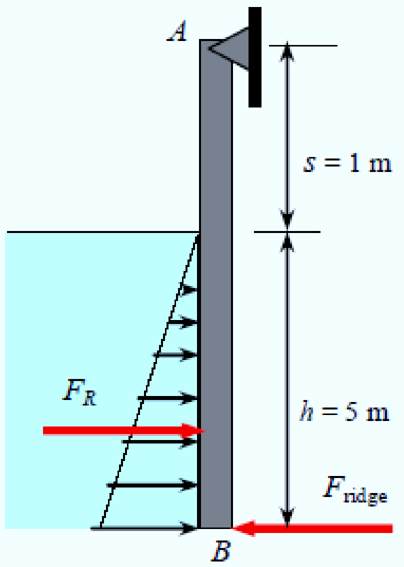
11-16

Solution A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point B. The force exerted to the plate by the ridge is to be determined.

Assumptions Atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = \rho g (h/2)$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(5/2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 24.53\,\text{kN/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = (24.53\,\text{kN/m}^2)(5\,\text{m}\times 5\,\text{m}) = 613.1\,\text{m}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(5\,\text{m})}{3} = 3.333\,\text{m}$$

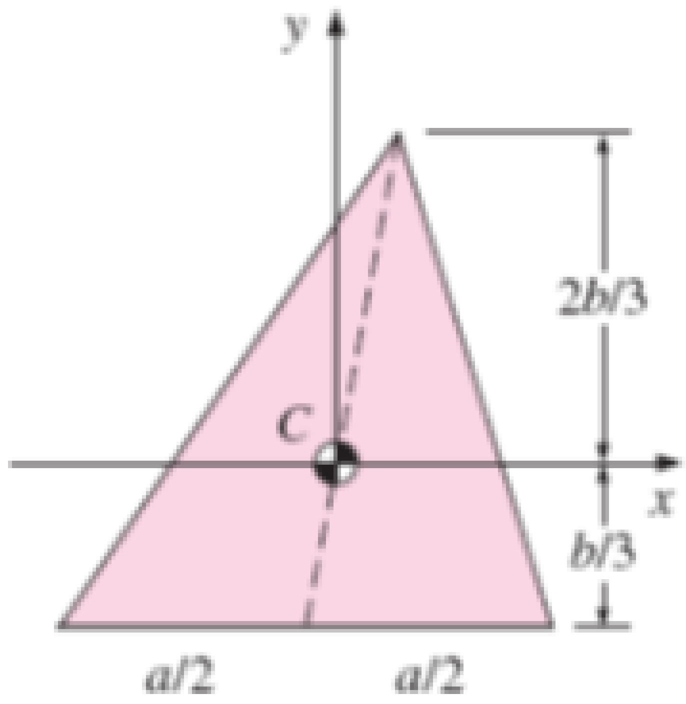
Taking the moment about point A and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = F_{\text{ridge}}\,\overline{AB}$$

Solving for F_{ridge} and substituting, the reaction force is determined to be

$$F_{\text{ridge}} = \frac{s + y_P}{\overline{AB}} F_R = \frac{(1 + 3.333)\,\text{m}}{6\,\text{m}}(613.1\,\text{kN}) = \mathbf{443\ kN}$$

Discussion The difference between F_R and F_{ridge} is the force acting on the hinge at point A.



$A = ab/2,\ I_{xx,C} = ab^3/36$

(*d*) Triangle