Signals and Circuits

ENGR 35500

Inductors

Chapter 5: 5-5(Response of the RL circuit) pp. 242-244

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

Chapter 11: 11-4(Inductor in DC circuits) and 11-5 (Inductor in AC circuits) pp. 508-524

Floyd, T. L., and Buchla, D. M., *Electroics Fundamentals: Circuits, Devices & Applications*, 8th Edition, Pearson, 2009.

Web:
https://www.electronics-tutorials.ws/inductor/lr-circuits.html

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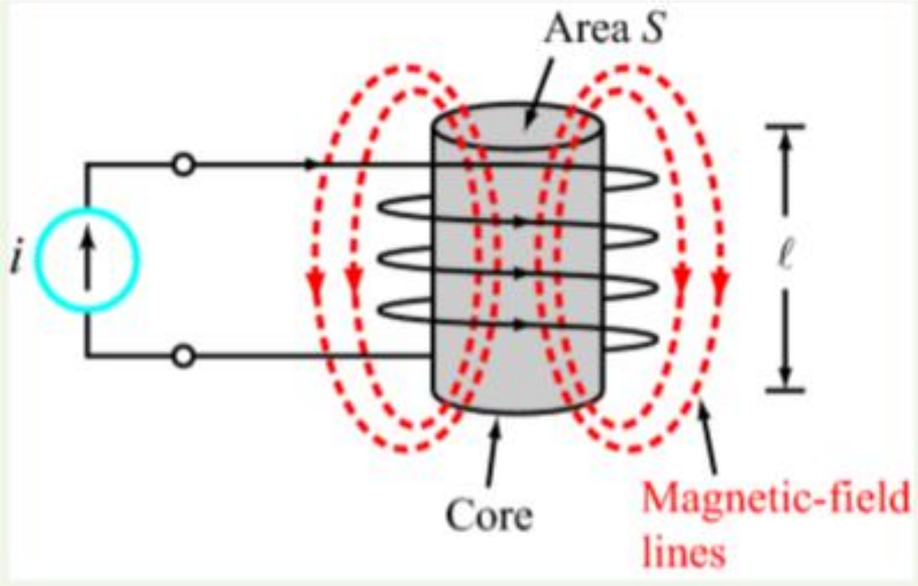
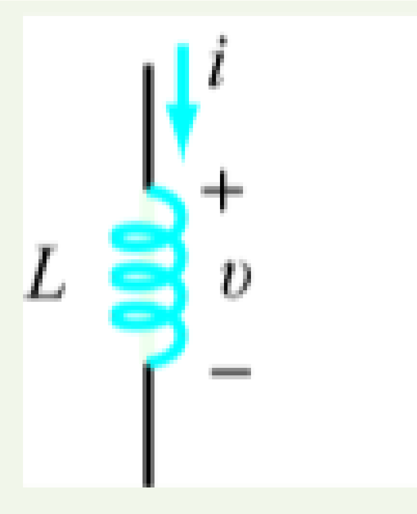
Inductor





$$v = L \frac{di}{dt}$$

An inductor is an electrical device composed of a coil of resistance-less wire wound around a supporting core whose material may be magnetic or non-magnetic. (e. g. solenoid)

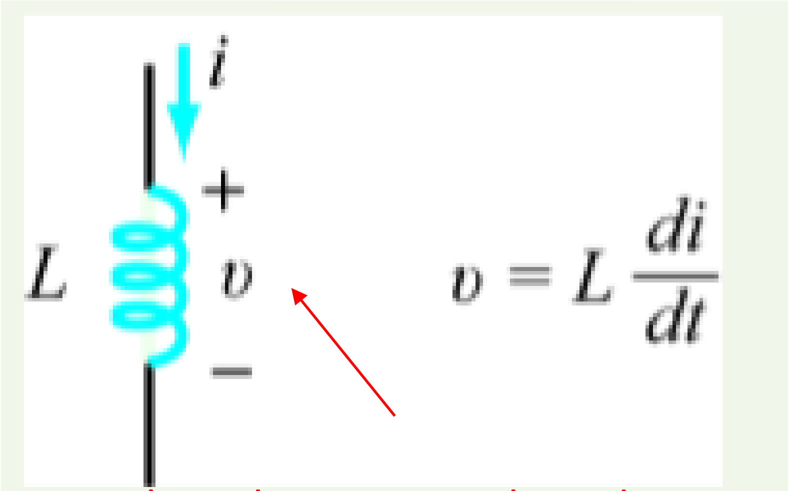
The current through the inductor cannot change instantaneously in zero time.

But the voltage can change instantaneously in zero time

Inductors

Electrical properties

$$L = \frac{\Lambda}{i}$$



v is the voltage across the inductor

The direction of *i* is entering the positive terminal of the inductor.

- According to Faradays law, if the magnetic-flux linkage in an inductor (or a circuit) changes with time, it induces a voltage v across the inductor's terminal.

$$v = \frac{d\Lambda}{dt} = \frac{diL}{dt} = L\frac{di}{dt}$$

- Under a stable DC condition, an inductor acts like a short circuit.

For stable DC
$$\frac{di}{dt} = 0$$
$$v = L\frac{di}{dt} = 0$$

- Get the current

$$\int_{t_0}^{t} \frac{di}{dt'}dt' = \frac{1}{L}\int_{t_0}^{t} v\,dt' \qquad i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\,dt'$$

- To get power $p(t) = vi = Li\frac{di}{dt}$

Sign of the power?

- To get energy change

$$w(t) = \int_{t_0}^{t} p\,dt' = L\int_{t_0}^{t}(i\frac{di}{dt'})\,dt' = L\int_{t_0}^{t}[\frac{d}{dt'}\left(\frac{1}{2}i^2\right)]\,dt' = \frac{1}{2}L\,(i_{(t)})^2 - \frac{1}{2}L(i_{(t_0)})^2$$

Sign of the energy?

- To get the stored energy

$$E(t) = \frac{1}{2}L(i_{(t)})^2$$

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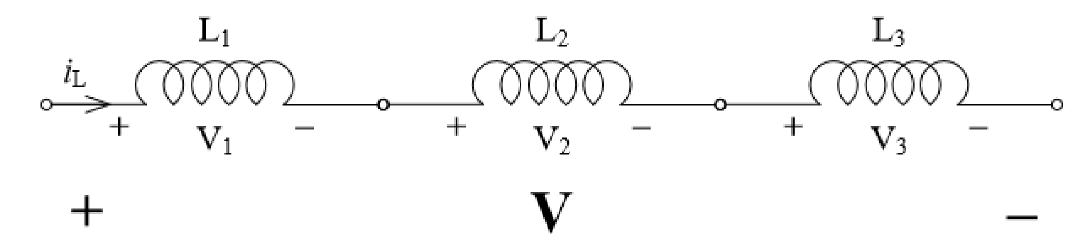
Inductors

➢ Inductors in series:



$$V = V_1 + V_2 + V_3$$

$$= (L_1 + L_2 + L_3)\frac{di_L(t)}{dt} \longleftarrow \text{KVL}$$

$$\Rightarrow L_{eq} = L_1 + L_2 + L_3$$

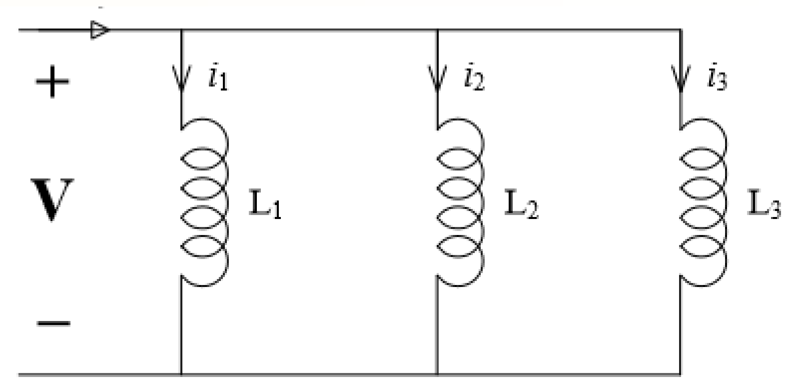
The equivalence relationship for inductors connected in series is similar in form to the relationship for resistors connected in series.

Voltage divider

$$V_x = \frac{L_x}{L_{eq}} V$$

The largest-value inductor in a series connection will have the largest voltage across it. The smallest-value inductor will have the smallest voltage across it.

Inductors



- <u>Inductors in parallel:</u>

$$i = i_1 + i_2 + i_3 \quad \longleftarrow \quad \text{KCL}$$

$$= \left[\frac{1}{L_1}\int_{t_0}^{t} V(t)dt + i_1(t_0)\right] + \left[\frac{1}{L_2}\int_{t_0}^{t} V(t)dt + i_2(t_0)\right] + \left[\frac{1}{L_3}\int_{t_0}^{t} V(t)dt + i_3(t_0)\right]$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} V(t)dt + \underbrace{[i_1(t_0) + i_2(t_0) + i_3(t_0)]}_{i(t_0)}$$

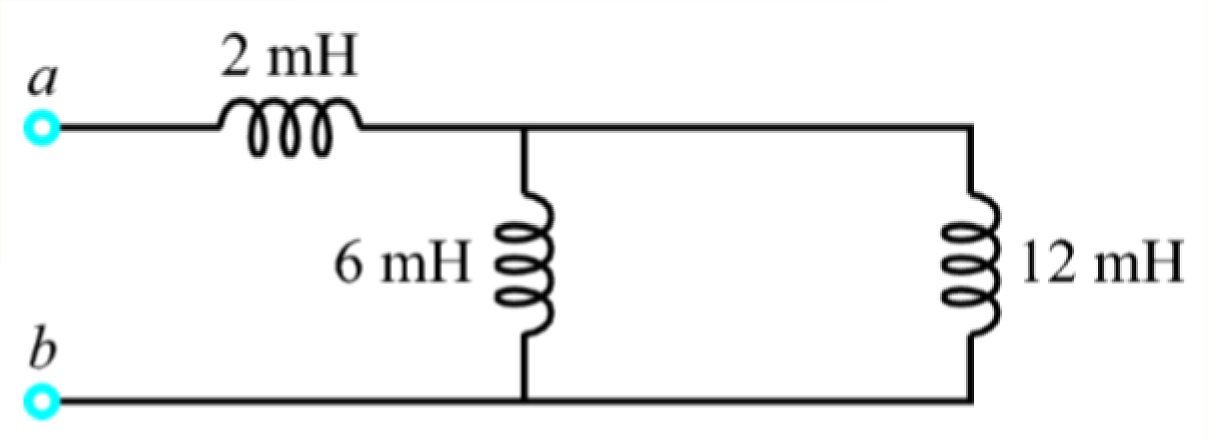
$$\Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

The equivalence relationship for inductors connected in parallel is similar in form to the relationship for resistors connected in parallel.

Inductors

E. g.

Find the equivalent circuit for the following circuit.

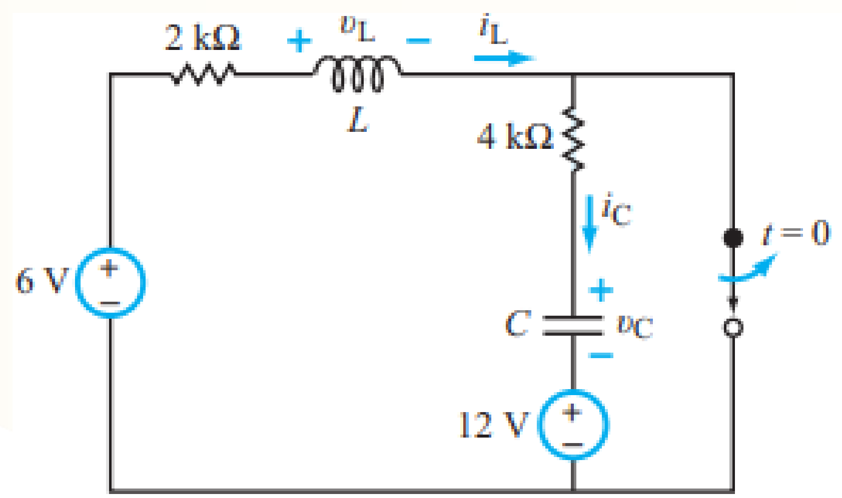


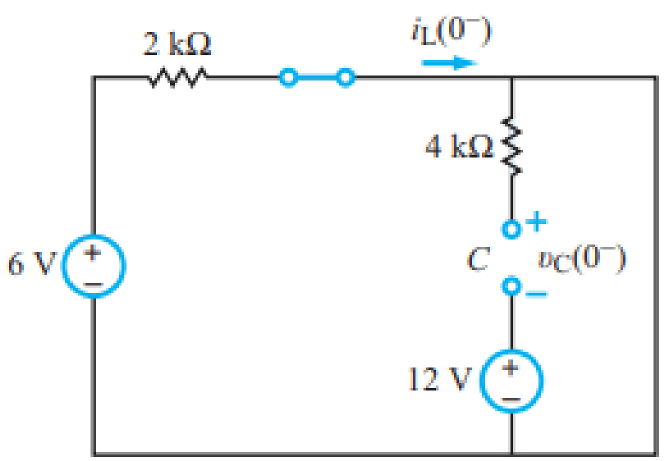
inductors

Note:
1. Under a stable DC condition, an inductor acts like a short circuit.
2. At the state changing moment, an inductor acts like a current source, and a capacitor acts like a voltage source

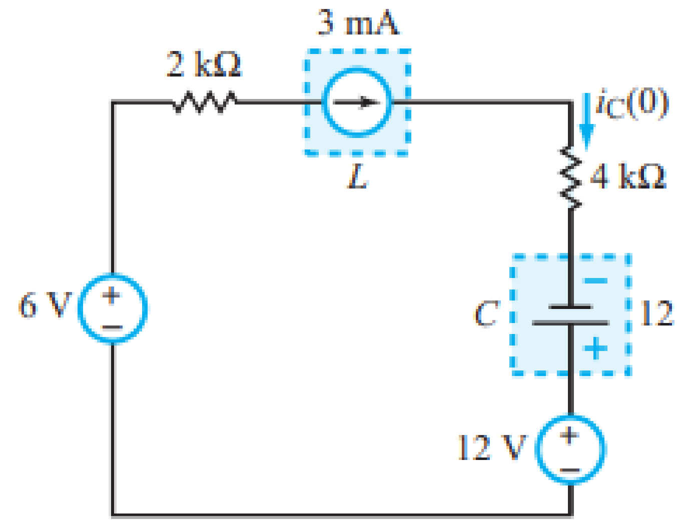
- E. G.

The switch in the circuit opens at t = 0 after it had been closed for a long time. Indicate the circuit states at t = 0^-, t = 0, and t = ∞.

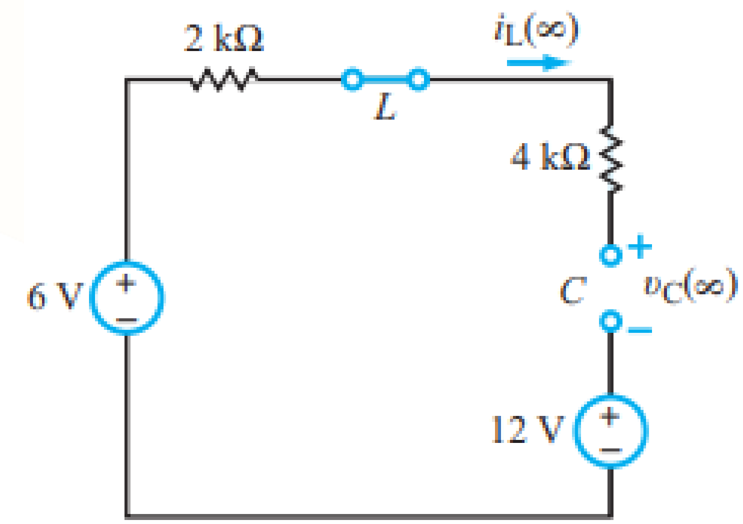




(a) At $t = 0^-$



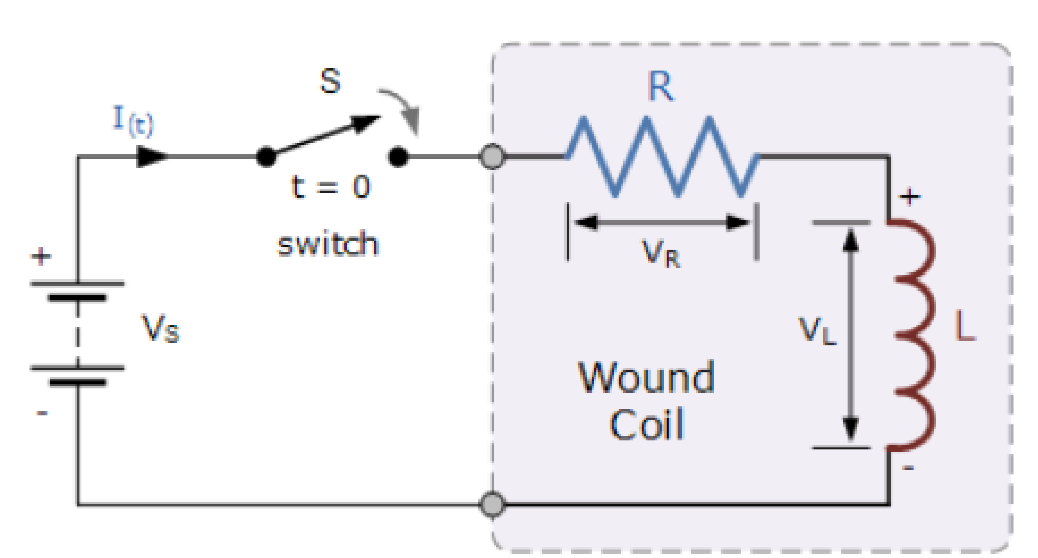
(b) At $t = 0$

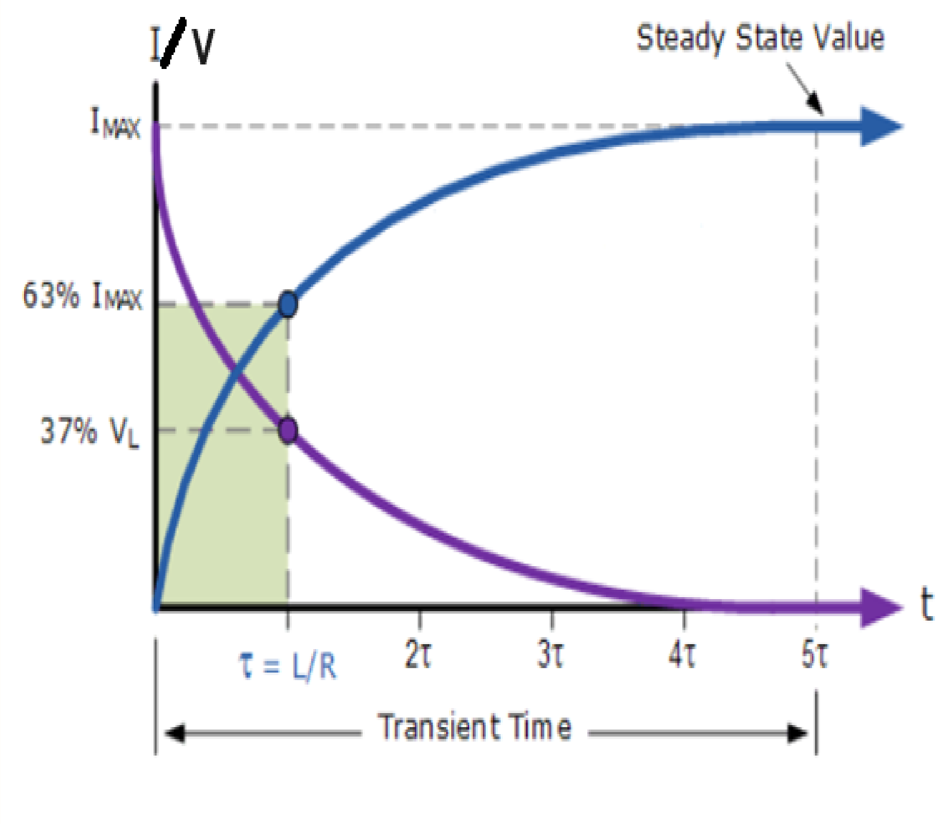


(c) At $t = \infty$

Inductors

RL series circuit charging





- Assume the switch is open for a long time

  t = 0, I = 0, and V_L = 0

- At the moment it closes,

  t = 0, I= 0, and then V_L= ?

  $$-V_s + I \times R + V_L = 0 \qquad \text{KVL}$$

  $$V_L = V_s$$

- A long time after the closing

  then t = ∞, $V_L = L\frac{di}{dt} = 0$, $i = \frac{V_s}{R}$

τ (usually τ) is the time constant or time delay. (63% current rises)

$$\tau = \frac{L}{R} \qquad \text{Unit: s = H/}\Omega$$

4τ (usually τ) is the transient period; 98% charged

5τ (usually τ) is the steady state period; almost steady

$$I = \frac{V}{R}(1 - e^{-t/\tau})$$

$$V_L = Ve^{-t/\tau}$$

Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/lr-circuits.html

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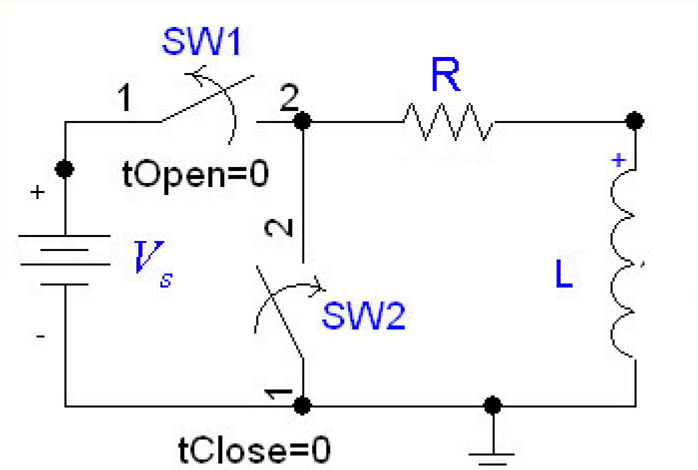
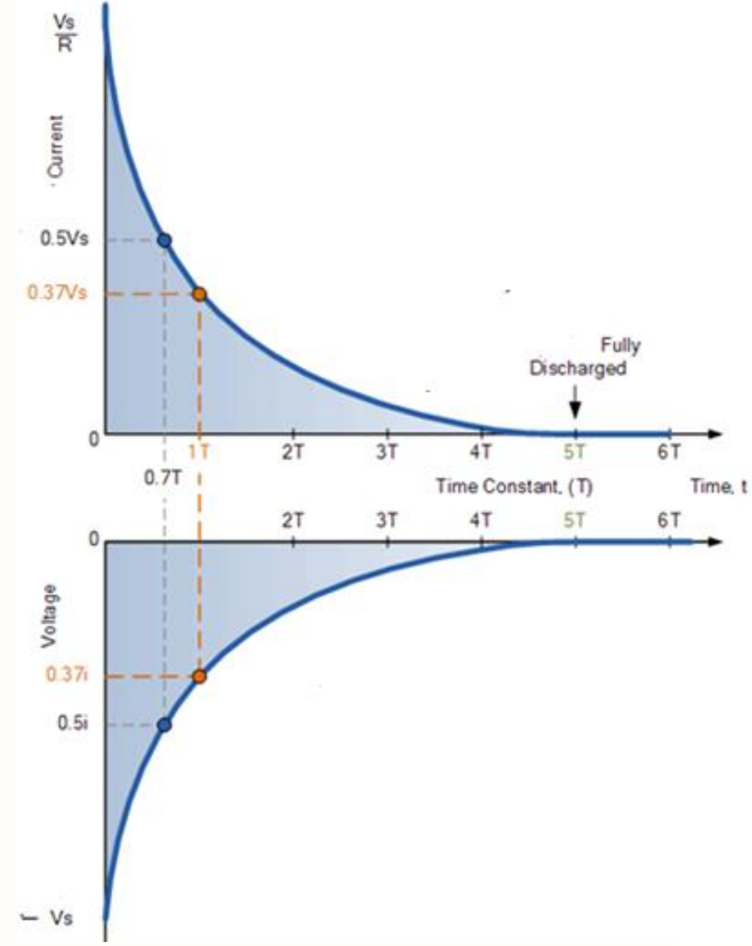
Inductors

RL series circuit discharging



Diagram?



- Assume the SW 1 is closed , SW2 is open for long time

  $t = 0^{-}$, i = $I = \frac{V_s}{R}$, and V_L = 0

- At the moment open SW1 and close SW 2

  t = 0, i =I, and then V_L= ?

$$\mathrm{I} \times \mathrm{R} + \mathrm{V_L} = 0 \qquad \text{KVL}$$

$$\mathrm{V_L} = -IR = -\mathrm{V_S}$$

- A long time after opening SW1 and closing SW2

  then t = ∞, V_L=L$\frac{di}{dt}$=0, i=0

τ (usually τ) is the time constant or time delay. (63% current decrease

$$\tau = \frac{L}{R} \qquad \text{Unit: s = H/Ω}$$

4τ (usually τ) is the transient period; 98% charged

5τ (usually τ) is the steady state period; almost steady

$$i_L = \frac{\mathrm{V_s}}{\mathrm{R}} e^{-t/\tau}$$

$$V_L = -\mathrm{V_s} e^{-t/\tau}$$

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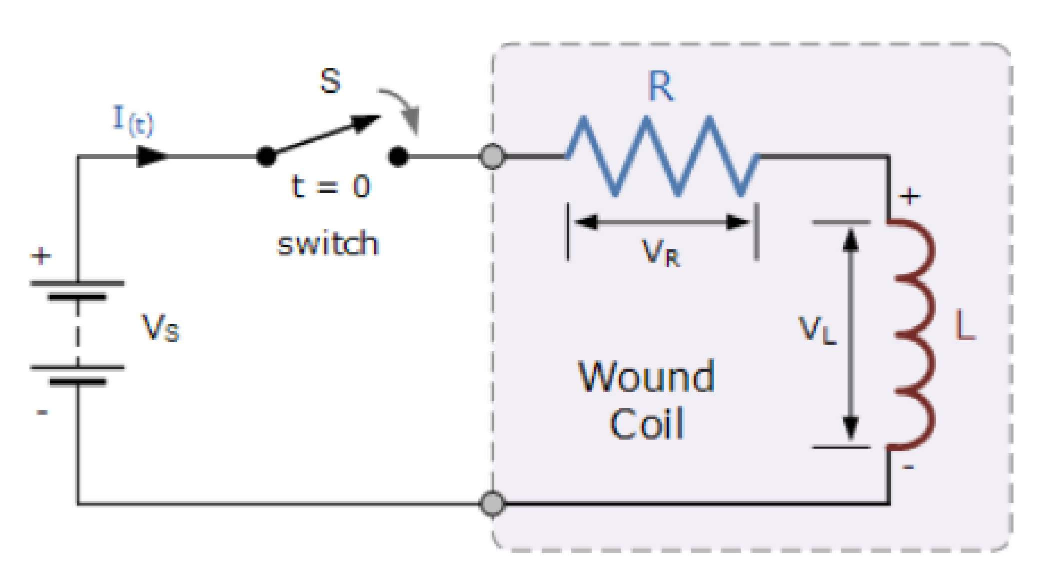
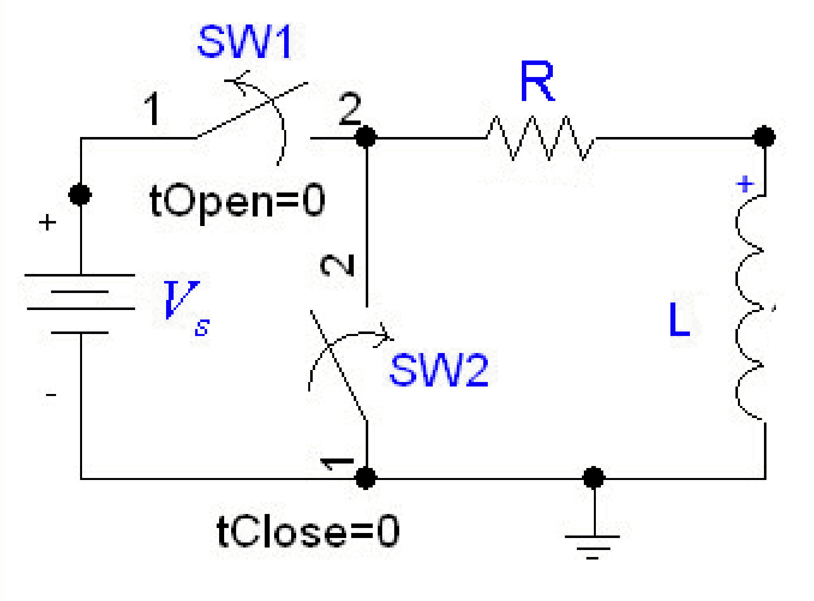
Inductors

RL charging/discharging circuit





$$V = V_F + (V_i - V_F)\, e^{-t/\tau}$$

$$i = I_F + (I_i - I_F)\, e^{-t/\tau}$$

where V and i are the instantaneous voltage and current;
V_i and I_i is the initial voltage and current;
V_F and I_F are the final voltage and current.

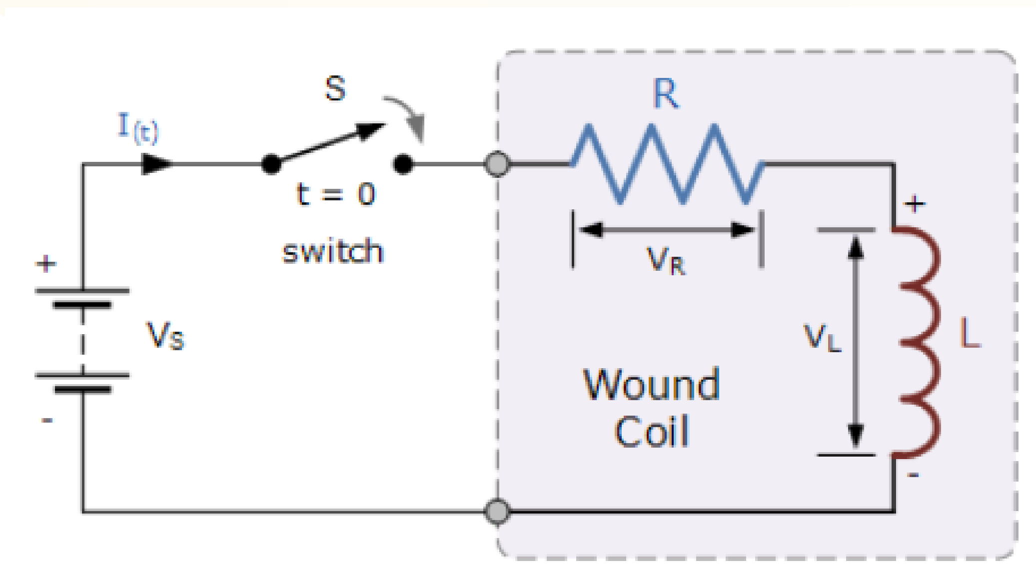
Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/lr-circuits.html

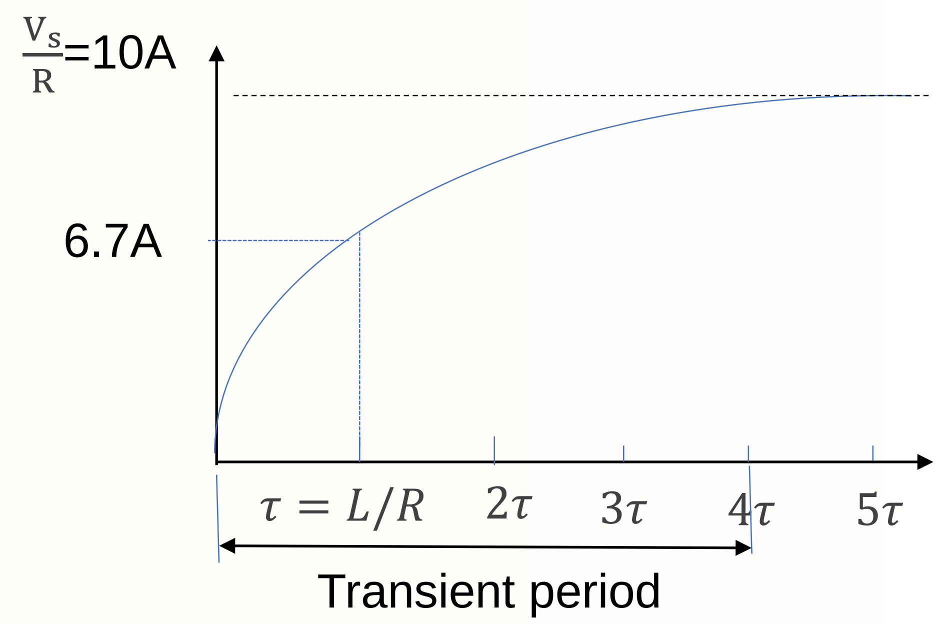
Inductor

$$V = V_F + (V_i - V_F)\, e^{-t/\tau}$$

$$i = I_F + (I_i - I_F)\, e^{-t/\tau}$$

E.g. A coil which has an inductance of 40mH and a resistance of 2Ω is connected together to form a LR series circuit. If they are connected to a 20V DC supply, what is the transient time? Draw the charging curving.



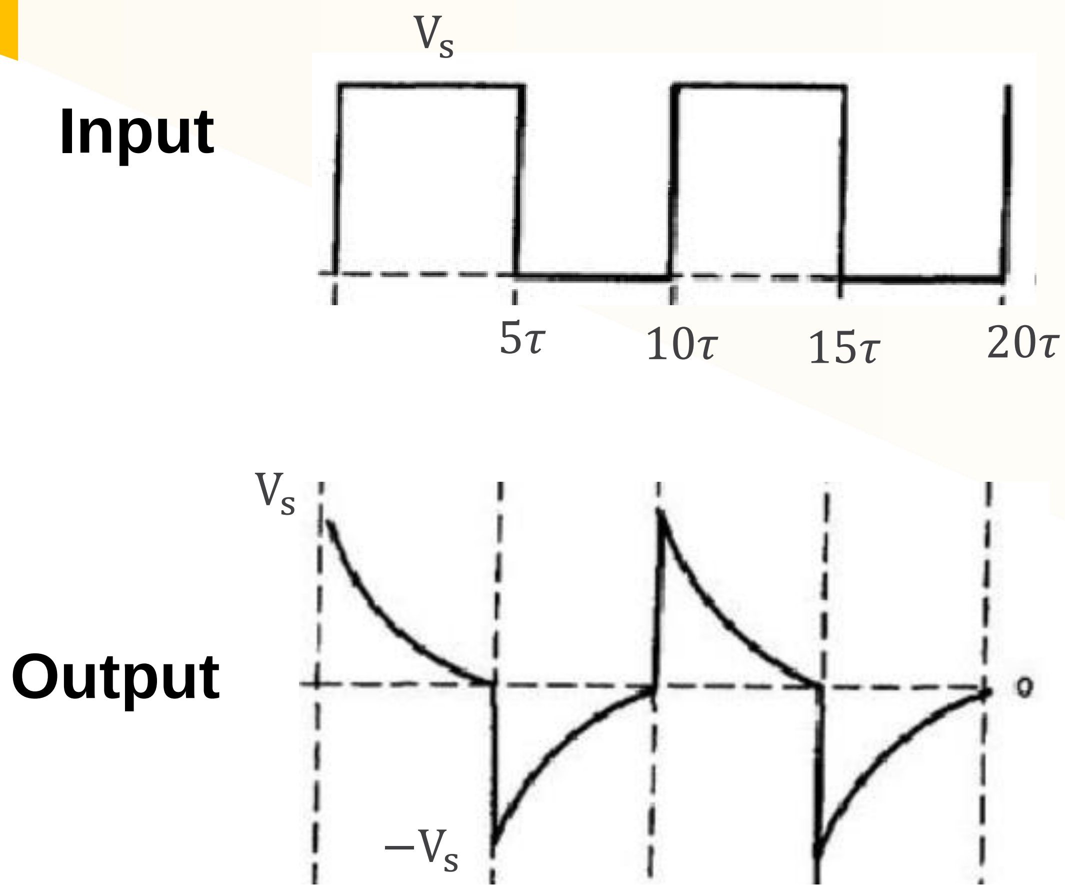


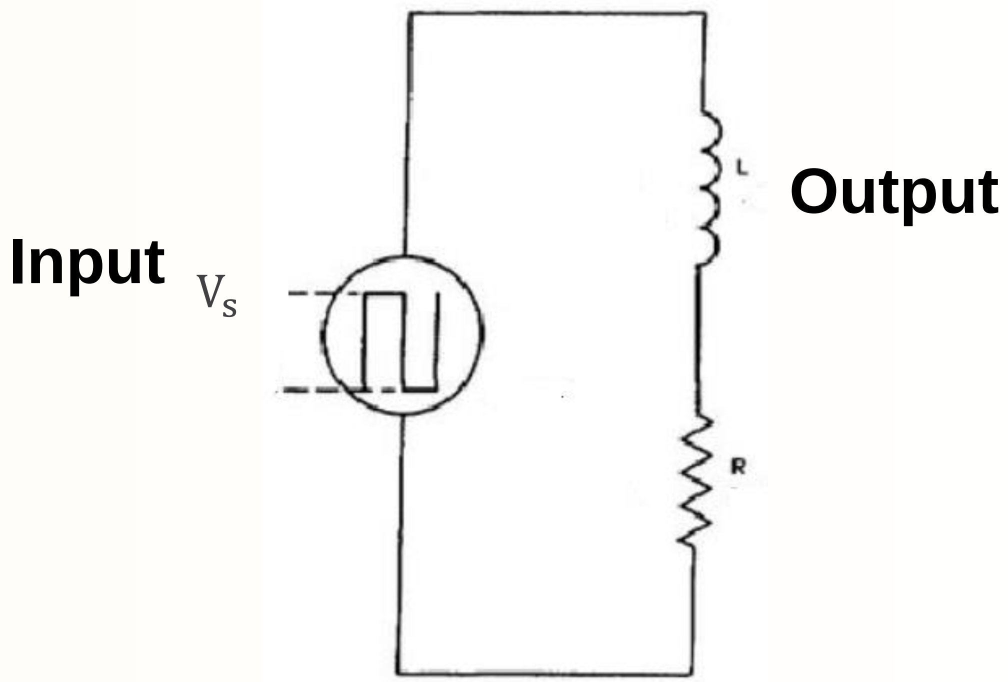
$t_T = 4\tau$=4*L/R=4*40mH/2=80ms

Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/lr-circuits.html

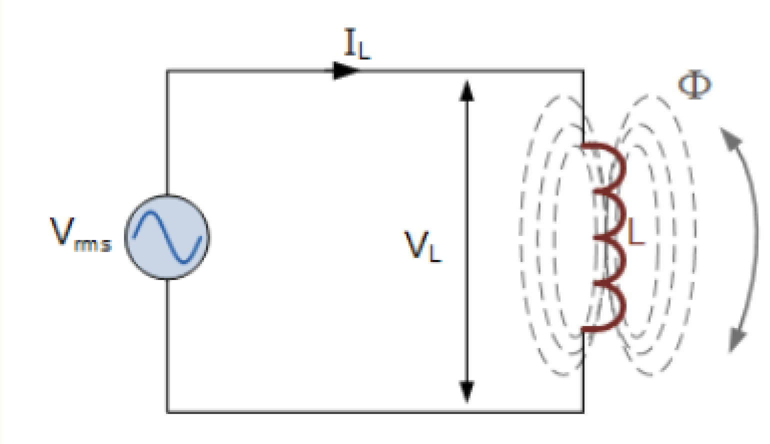
inductor

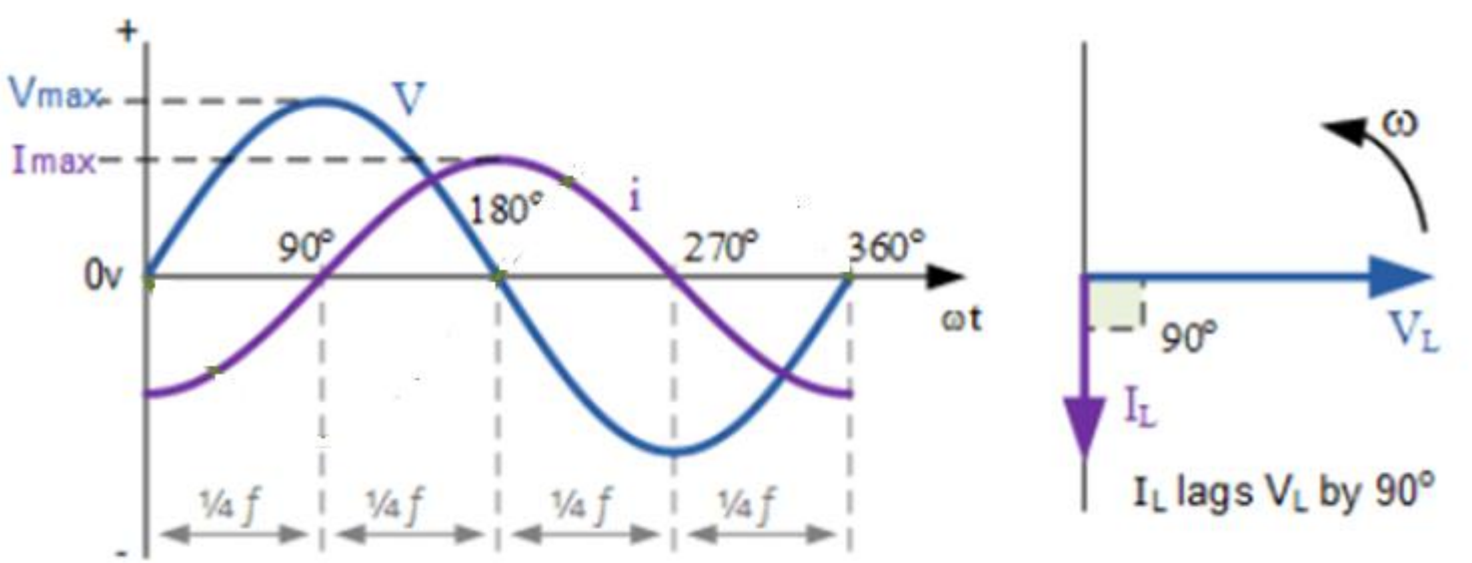
RL response to square wave





Pure inductor in AC





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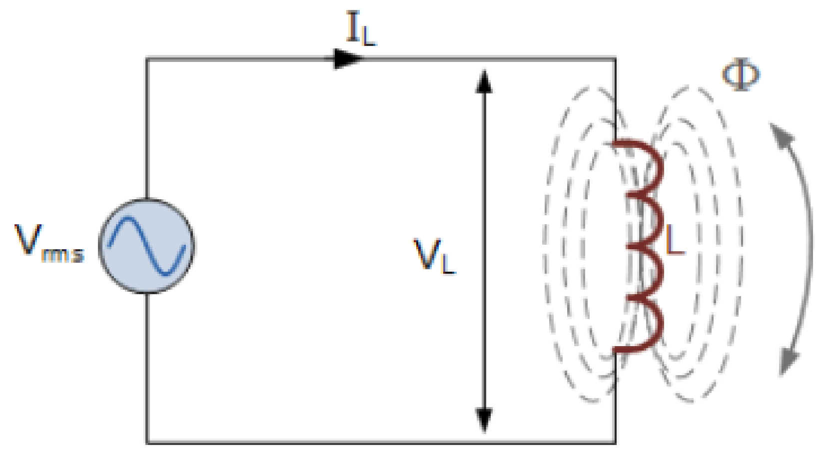
Pure inductor in AC

Inductive Reactance



With the same AC source, we used DMM and found the current decreases as the inductance increases;

With the same inductance and AC voltage amplitude, we used DMM and found the current decreases as the AC source frequency increases.

The **opposition** to sinusoidal current in an inductor is called **inductive reactance.**

$$X_L = 2\pi f L$$

X_L Inductive Reactance, (Ω)

π (pi) = 3.142 (decimal) or as 22÷7 (fraction)

f = Frequency in Hertz, (Hz)

L is the Inductance (H)

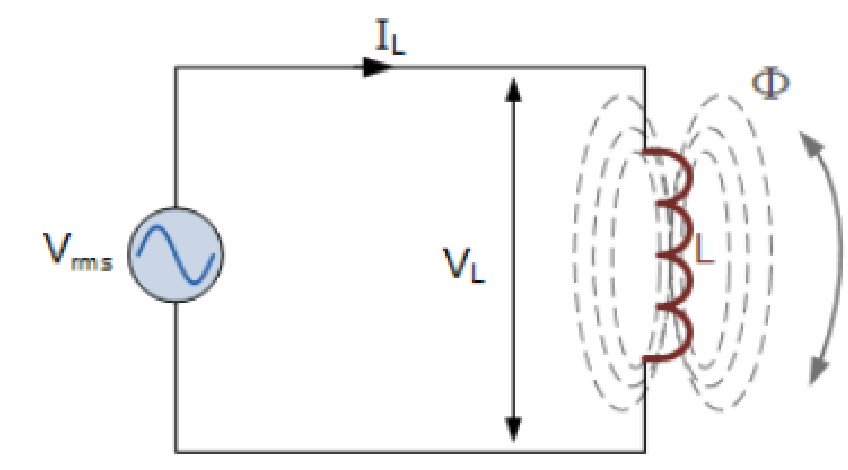
Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/ac-inductors.html

Pure inductor in AC

Inductive Reactance

E. g.

Calculate the inductive reactance value of a 5mH inductor at a frequency of 1kHz and again at a frequency of 10kHz.



$$X_L = 2\pi f L$$

Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/ac-inductors.html

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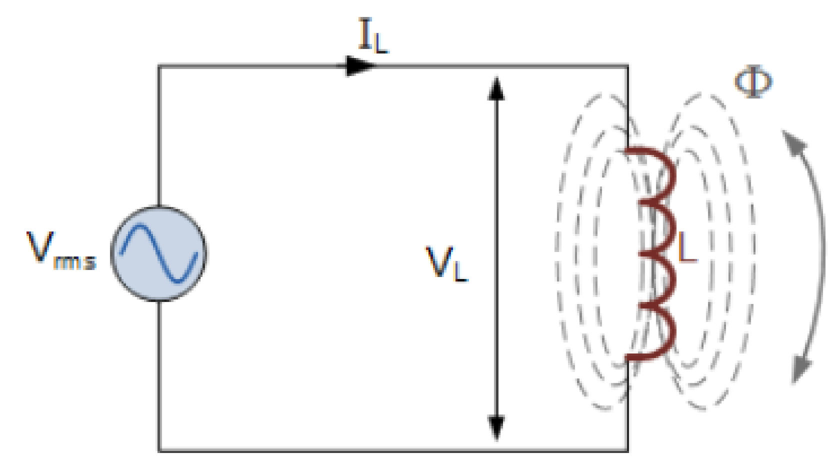
Pure inductor in AC

Ohm’s law in AC

$$I = \frac{V}{X_L}$$



$$I = \frac{V}{X_L}$$

When applying Ohm’s law in ac circuits, you much express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on (This is not in time domain).

E. g.

$V_{1rms} = 5\ V$ $\quad f = 10\ kHz$ $\quad L = 100mH$ $\quad I_{rms} = ?$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{5}{2\pi \times 10 \times 100} = 0.796\text{mA}$$

Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/inductor/ac-inductors.html

Inductor in AC

Inductive reactance for inductors in series in AC

$$X_{LT} = X_{L_1} + X_{L_2} + \cdots + X_{L_{1n}}$$

Inductive reactance for inductors in parallel in AC

$$\frac{1}{X_{LT}} = \frac{1}{X_{L_1}} + \frac{1}{X_{L_2}} \cdots + \frac{1}{X_{L_n}}$$

Voltage divider in AC

$$V_x = \frac{X_{L_1}}{X_{L_1} + X_{L_2} \cdots X_{L_n}} V$$

Current divider in AC?

Inductor in AC

Instantaneous power

$$p(t) = v(t)i(t)$$

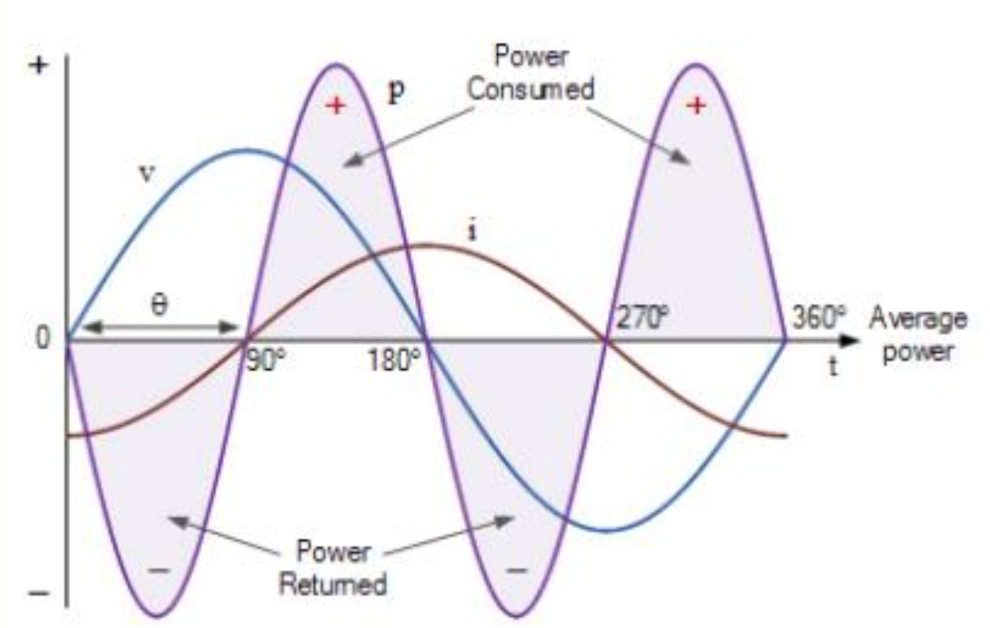
True power (one cycle power)

$$p_{ture} = 0$$

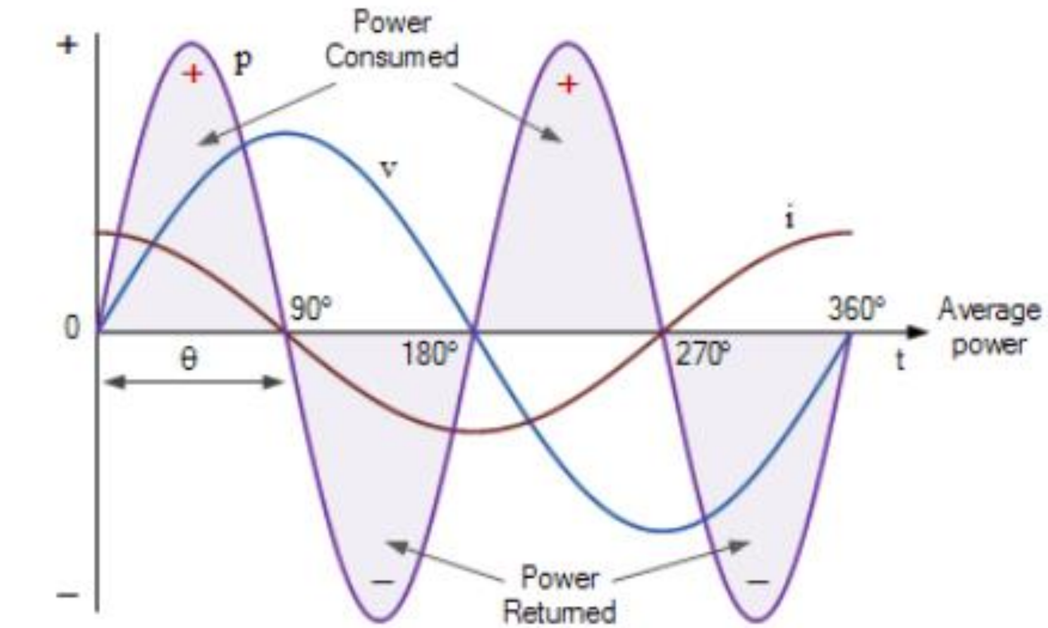
Reactive power (the rate at which an inductor stores or returns energy)

$$p_r = V_{rms}I_{rms} = \frac{V_{rms}^2}{X_L} = I_{rms}^2 X_L$$

AC power waveforms for a pure inductor



AC power waveforms for a pure capacitor



Figures: **Electronics Tutorials**: https://www.electronics-tutorials.ws/accircuits/power-in-ac-circuits.html