Control Systems - ENGR 33041

Lecture 11B: Controller Design, Demo, and Final Project

Instructor:

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Slides prepared based on:

Modern Control Engineering, K. Ogata

The Fundamentals of Control Theory, B. Douglas

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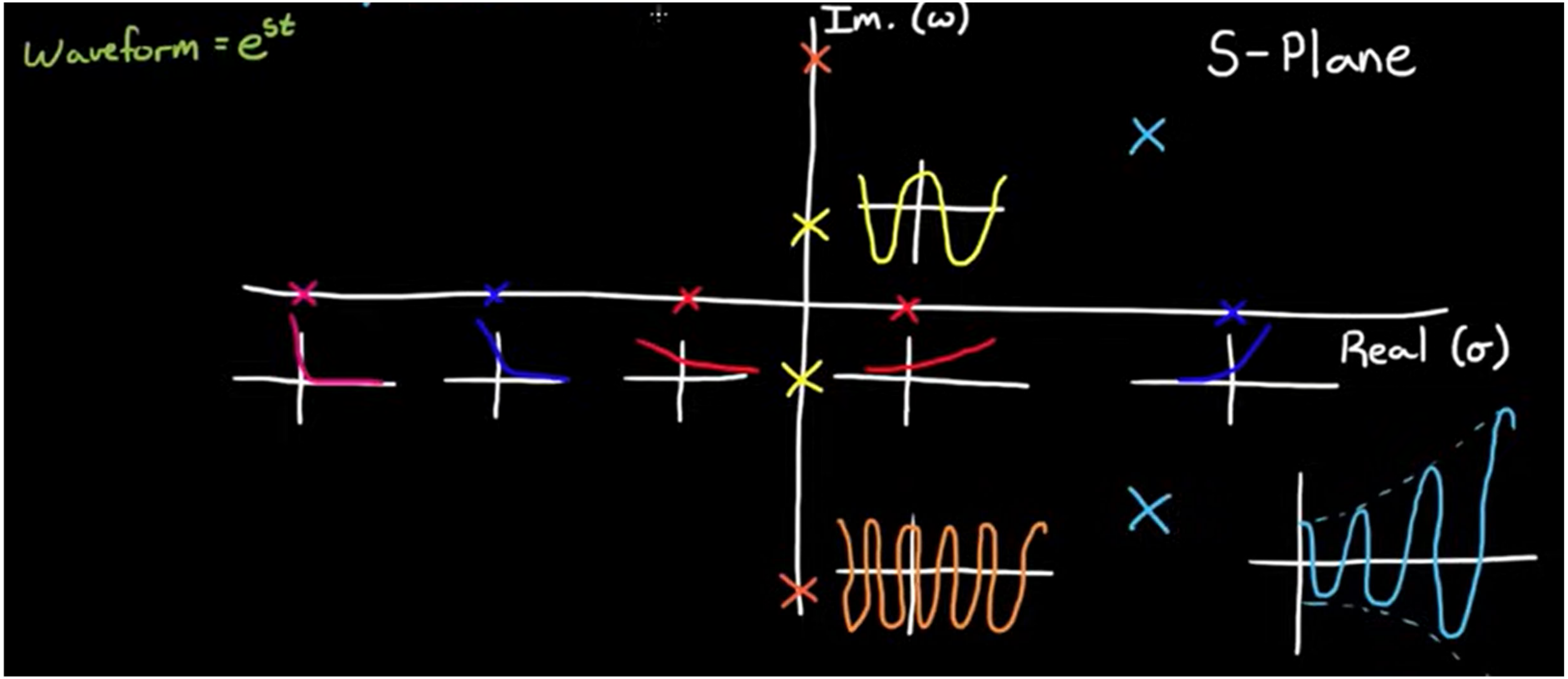
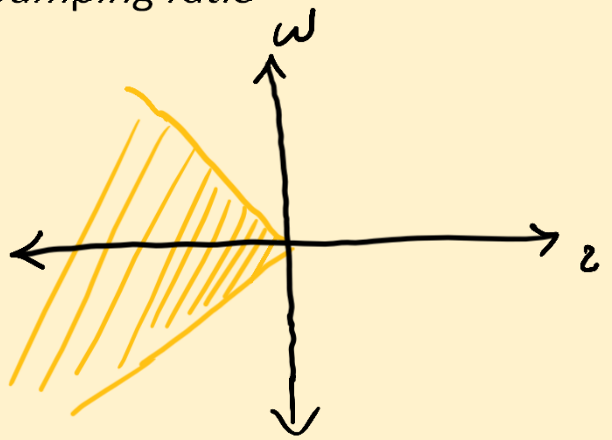
How to interpret the locations of the poles of a system in the s-plane?



Control engineers have to design control systems to a set of requirements. These requirements often come in the form of damping ratio, exponential growth or decay, or natural frequency of the system.

Example 1:

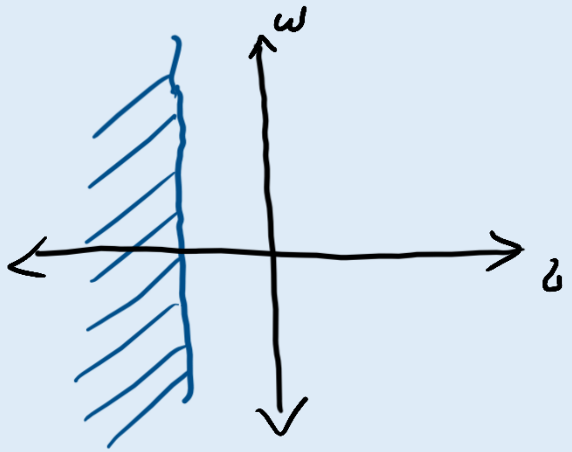
Damping ratio



Damping ratio must be in the cone to be greater than some specified value (e.g. $\zeta > 0.75$).

Example 2:

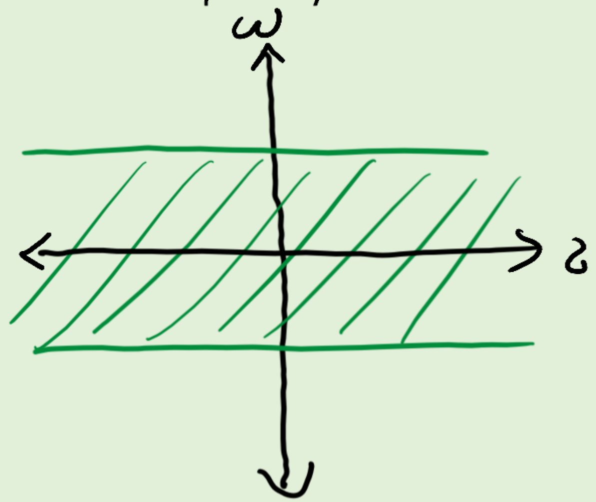
Time for exponential decay to half



All poles must be located to the left of that particular time to half line.

Example 3:

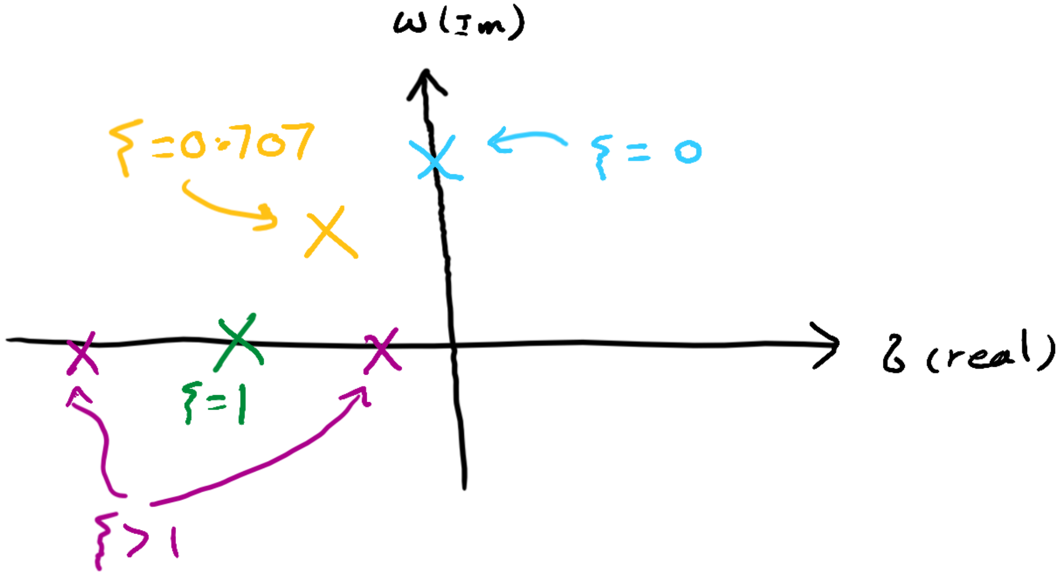
Natural frequency



The modes have to be slower than particular natural frequency (e.g., 1 rad/s).

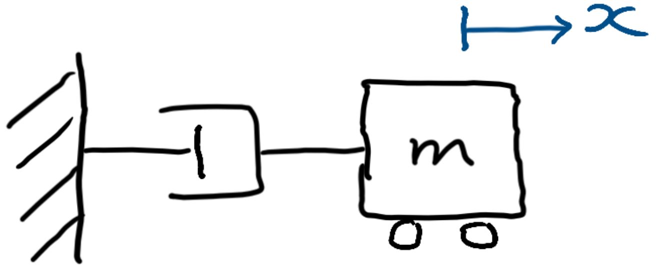
You may also be given any combination of the above requirements.

Side note: Do not be confused between the term damping ratio ζ and the term damping coefficient b. They are two different things.



ζ is unitless

$\underbrace{\hspace{6cm}}$ damping ratio



$b\dot{x} = \text{Force}$

$b\ [N \cdot s/m]$

$\underbrace{\hspace{6cm}}$ damping coefficient

Control System Design and Analysis using Root Locus Method

Now that we understand how pole locations affect the system, let's use the **root locus** method to "design" a control system and "analyze" it.

Reminder from Lecture 9:
Two questions to respond with the root locus method:

Design

What value of K should I choose to meet the system performance requirements? (That is, having poles in the correct location in the s-plane)

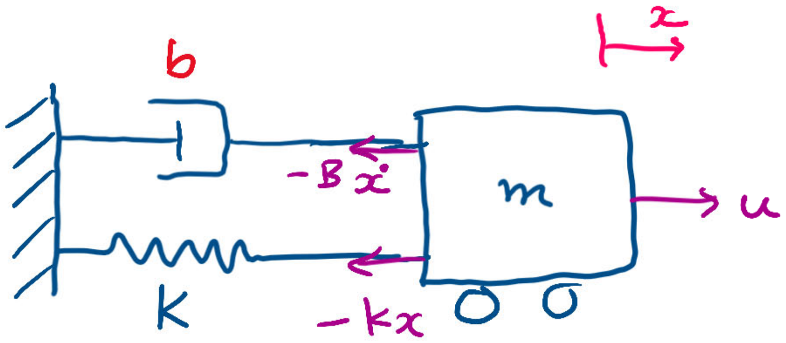
Analysis (Effect of Variations)

What is the effect of the variation of K on the poles of the system? (That is, how sensitive is the system to a value of K that is slightly off of what you have predicted?)

We as the control engineer want to design a mass-spring-damper system that meets certain requirements. The requirements are that the damping ratio must be greater than or equal to 0.75 ($\zeta \geq 0.75$).

- The Spring Department is in charge of choosing spring and they pick the spring constant as 1 $(k = 1)$.
- The mass is developed by the Marketing Department, and they say that the mass of 1 would sell the best $(m = 1)$.
- Now we get to pick our own damper, and with any damping coefficient we need, in order to meet the system requirements.

Solution:



From Newton's second law:

$$\sum F = ma \implies u - kx - b\dot{x} = m\ddot{x}$$

$$\implies m\ddot{x} - b\dot{x} + kx = u \implies m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = u$$

Taking Laplace Transform:

$$m s^2 X(s) + b s X(s) + k X(s) = U(s)$$

$$X(s)\left[ms^2+bs+k\right] = U(s) \Rightarrow \frac{U(s)}{X(s)} = \frac{1}{ms^2+bs+k} \Rightarrow \boxed{\frac{U(s)}{X(s)} = \frac{1}{s^2+bs+1}}$$

1

Now, the root locus method helps us see how the poles of the system move, when we sweep damping coefficient b from 0 to ∞.

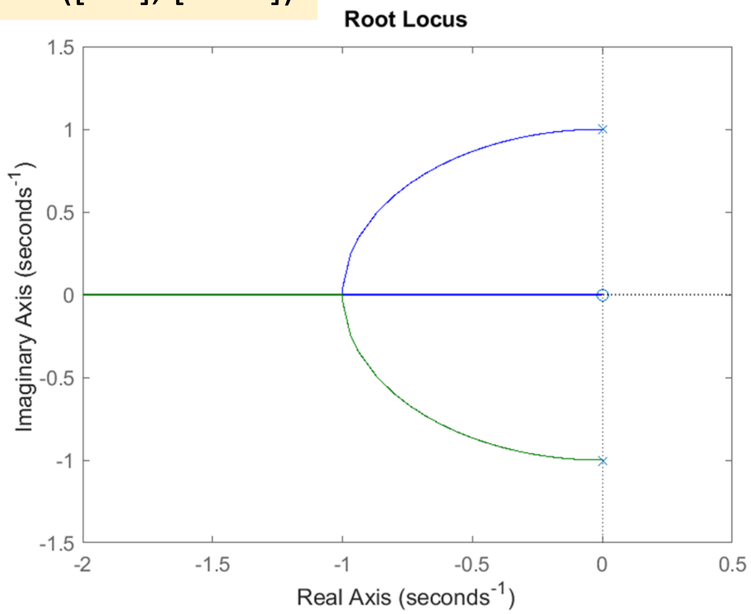
Characteritic Eq: $s^2+bs+1=0 \Rightarrow s^2+1+bs=0$

divided by s^2+1 $\Rightarrow$ $1+\dfrac{bs}{s^2+1}=0$

$[1 \quad 0]$

$[1 \quad 0 \quad 1]$

```
>> rlocus ([1 0], [1 0 1])
```



Let’s draw the radial lines for constant damping ratio $\zeta = 0.75$

```
>> sgrid (0.75, 0)
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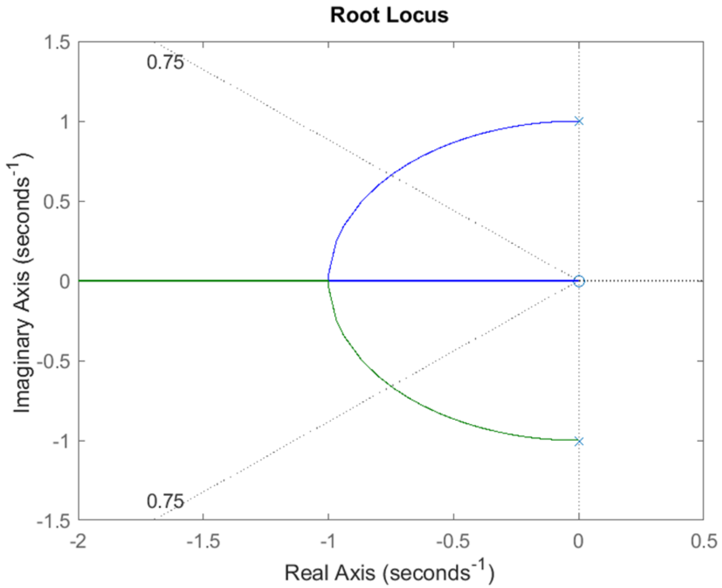
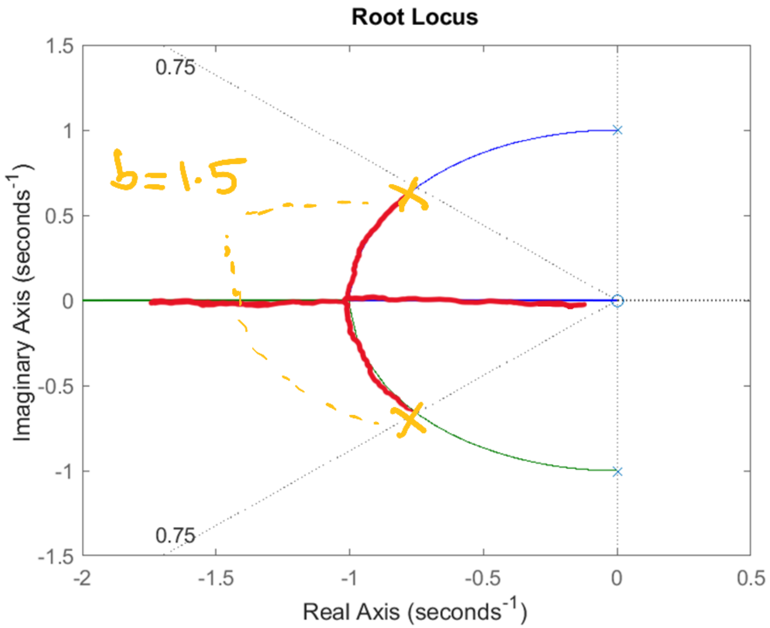


Now we need to choose a value of damping coefficient b that puts the poles of this transfer function somewhere along that red line.

We are going to choose b=1.5, because it allows us to buy the smallest lightest damper we can while still meeting the system requirement.

Therefore, we need the damping coefficient of b=1.5.



Now, we are about to ship our mass-spring-damper out to our customers all over the world when the Spring Department comes running to us in panic and says the spring constant k changes when it is subjected to temperature variation. When the spring gets really hot, the spring constant drops to 90% of its normal value and when it gets cold the coefficient raises to 110% of its normal value.

Hot: $k = 0.9\, k_{norm}$

Cold: $k = 1.1\, k_{norm}$

We've messed up!

Now, we need to know how sensitive our design is to this variation. We can use the root locus method to determine how sensitive the system is to changes in the spring constant. We can replace the spring constant with the variable "k" in the transfer function and sketch the root locus in MATLAB by changing the value of k from 90% to 110% of its normal value.

$TF = \dfrac{1}{ms^2 + bs + k}$ $m = 1$, $b = 1.5$

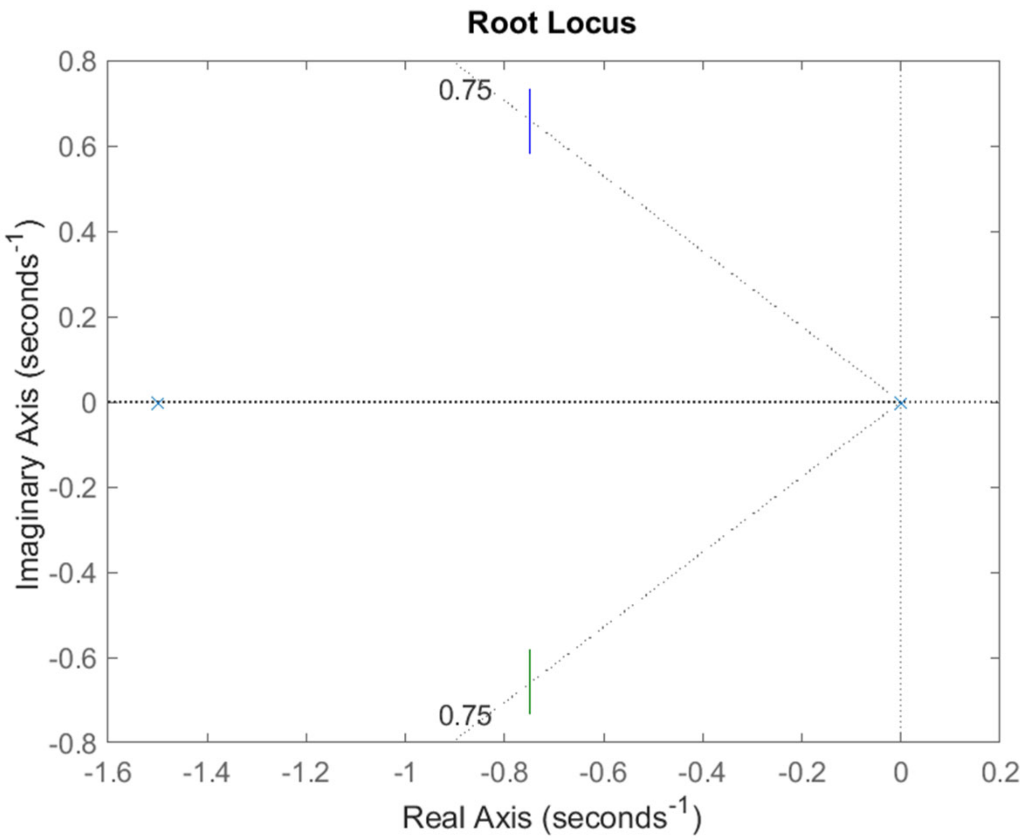
$\Rightarrow TF = \dfrac{1}{s^2 + 1.5s + k}$ for $0.9k$ to $1.1k$ of ideal constant

i.e., $0.9 \leq k \leq 1.1$

Characteristic Eq: $s^2 + 1.5s + K = 0$ divided by $s^2 + 1.5s$

$$\Rightarrow \quad 1 + K \frac{1 \rightarrow [1]}{s^2 + 1.5s \rightarrow [1 \;\; 1.5 \;\; 0]} = 0 \quad \text{for } 0.9 \leq K \leq 1.1$$

```
>> rlocus ([1], [1 1.5 0], [0.9 1.1])
>> sgrid (0.75, 0)
```



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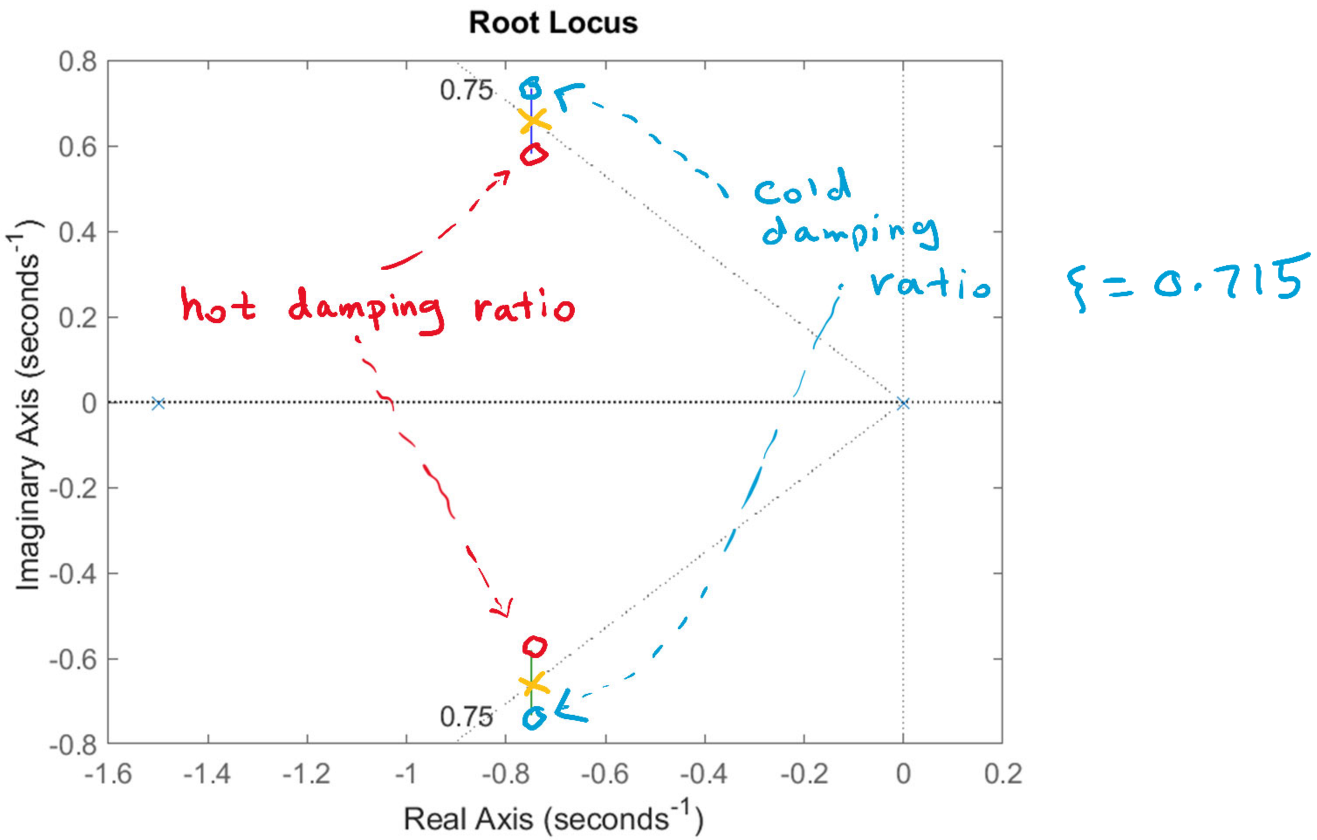
Clearly, our designed mass-spring-damper system does not meet the requirement when it is subjected to the cold temperature.



Now, our team has a decision to make:

- Do we restrict the operating temperature on the low end, so that the device still performs?
- Do we add a warning that the performance will be degraded in low temperatures.
- Do we call back all of our parts and redesign the system, so it works for all reasonable temperature.

Unfortunately, the root locus method cannot help with that!

Final Project: Automated Steering Control Design

- **Problem**

One of the basic problems in automated steering control is a lane change maneuver. The steering input controls the lateral motion of the vehicle (Figure 1). The automated steering control system uses information about the vehicle position relative to the center of the current lane to determine the steering wheel angle. A lateral force on the vehicle (and, hence, a lateral acceleration) is created as the wheels turn. The automated steering controller is designed to steer the vehicle from the center of the current lane to the center of an adjacent lane.

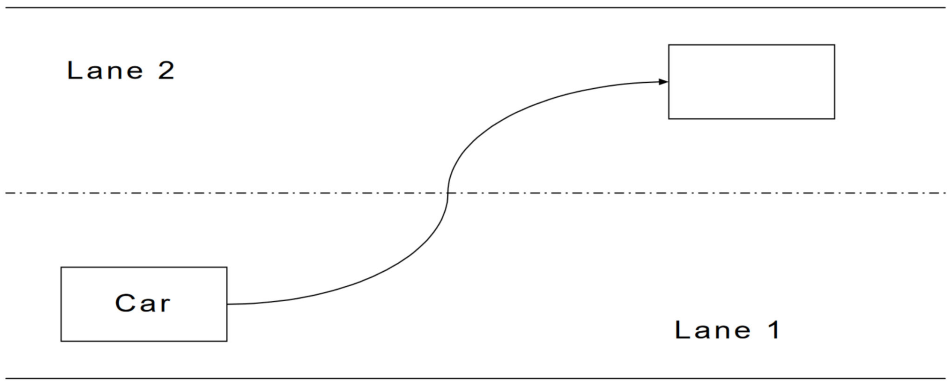


Figure 1: Lane change maneuver

Assuming a relatively perfect measurement sensor, the block diagram of the closed-loop steering control problem is shown in Figure 2. The vehicle has a transfer function $\mathrm{G}_{\mathrm{p}}(\mathrm{s}) = \frac{0.1}{s(s+1)}$, the steering actuator has a transfer function $\mathrm{G}_{\mathrm{a}}(\mathrm{s}) = \frac{10}{s+10}$, and the proportional controller has a transfer function $\mathrm{G}_{\mathrm{c}}(\mathrm{s}) = K$. The signals in Figure 2 are:

x(t): lateral position (units: lanes)
e(t): lateral position error (units: lanes)
r(t): desired lateral position (units: lanes)
u(t): steering angle (units: degrees)

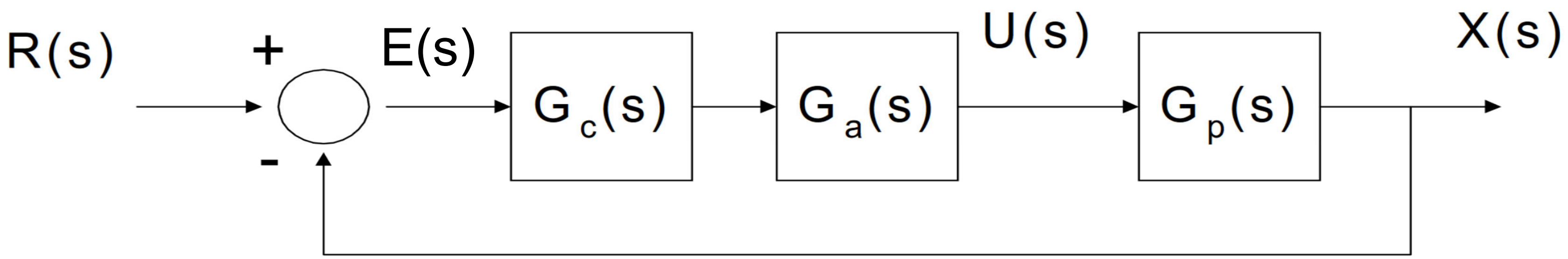


Figure 2: Closed loop block diagram for steering control problem

The objective of the design project is to design an automated steering control system; that is, to determine a value for the controller gain K. The selection of K is based on the vehicle's motion during a lane change maneuver.

The specifications for the steering control design are that the vehicle completes the lane change maneuver quickly and safely without causing the passenger's discomfort. From the systems engineering point of view, these specifications require that the step response of the vehicle's lateral position has a small rise time and minimal overshoot. Furthermore, the comfort of passengers is closely related to the lateral acceleration during the lane change maneuver. Specifically, passenger comfort requires that the lateral acceleration is small. Equivalently, it can be shown that the lateral acceleration is proportional to the steering input $u(t)$ and, therefore, passenger comfort requires that the steering input is small.

The **specifications** on the control system design can be divided into two categories:

1. Safety: The closed loop system must have less than 10 % overshoot in the unit step response.

2. Passenger comfort: The maximum steering input must be less than 4 degrees.

Design Project

The problem of designing an automated steering controller is investigated in this project:

Step 1: Examine the closed-loop step response (lane change response) for various values of the proportional controller gain K through MATLAB simulations. That is, investigate the step response of the system for various values of $K > 0$. How does the change of K affect the response?

Step 2: Sketch the root locus of the system and find the range of K for stability.

Step 3: Design a proportional controller, i.e., determine the value of K, so that the closed loop system has the fastest response (smallest rise time in the step response) while satisfying the two specifications of safety and passenger comfort as described in the previous slide.

Step 4: For the system designed in Step 3, report the values of gain K, maximum overshoot percentage, maximum steering input, and rise time.

Step 5: Sketch the Bode plot of the closed-loop system designed in Step 3 and find the bandwidth of the system.

Final Project Report: Due is **Tuesday, December 10, 2024, 3:00 pm** (hard deadline, no late submission is accepted).

The final project report needs to include the materials in the following order:

- Background information about the problem
- Methods and calculations for design and analysis
- Results including figures and/or tables.
- Explanation of results and findings (Discussion)
- Conclusions
- References, if any

Report Format:

Length: 8-10 pages (everything included)
Font Type: Times New Roman or Arial
Font size: 11 or 12
Line spacing: double spaced
Page Margin: 1 inch at each side (top, bottom, right, and left)

The final project report must be submitted on Canvas, as a single word or pdf file.

Teammate Selection for Final Project

- **Please click on the following link (or copy and paste the link in your web browser), login with your KSU account, and fill out the teammate selection form:**

- **https://forms.gle/GN49mMaSw55kxyfN7**

While the information collected in the form is used for building teams,
it is not guaranteed that you will be assigned to your preferred teammates.

- **Exam 3** will be held in class on **December 5, 2024.** It will cover Lectures 8, 9, and 10. 11A and 11B.

- The exam is closed-book and closed-note. However, you are allowed to have 1 regular page (U.S. letter size) of notes/formula for any formula you might need during the exam.

- No MATLAB Questions in Exam 3. Scientific calculator is allowed in the exam.

Course Evaluation

1. Login to Canvas and click on the **Course Evaluation** for Control Systems class.

2. Fill out the Course Evaluation and take a screenshot of your submission (last page).

3. Upload your screenshot on Canvas (under assignment) by **Wednesday, November 27, 11:59 pm.**

If > 90% of the class fill out the Course Evaluation by Nov. 27, I'll add 5 bonus points to your final grade!

This is the end of Lectures. In the rest of the semester, you will work on the final project and prepare yourself for Exam 3.

I hope you have enjoyed this class and your overall experience has been positive. If you have comments or suggestions to improve the class in the future, I'd be happy to hear your feedback via email or in person.