Signals and Circuits

ENGR 35500

Equivalent Circuit

Chapter 2-3 (Equivalent Circuits) pp. 54-62

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

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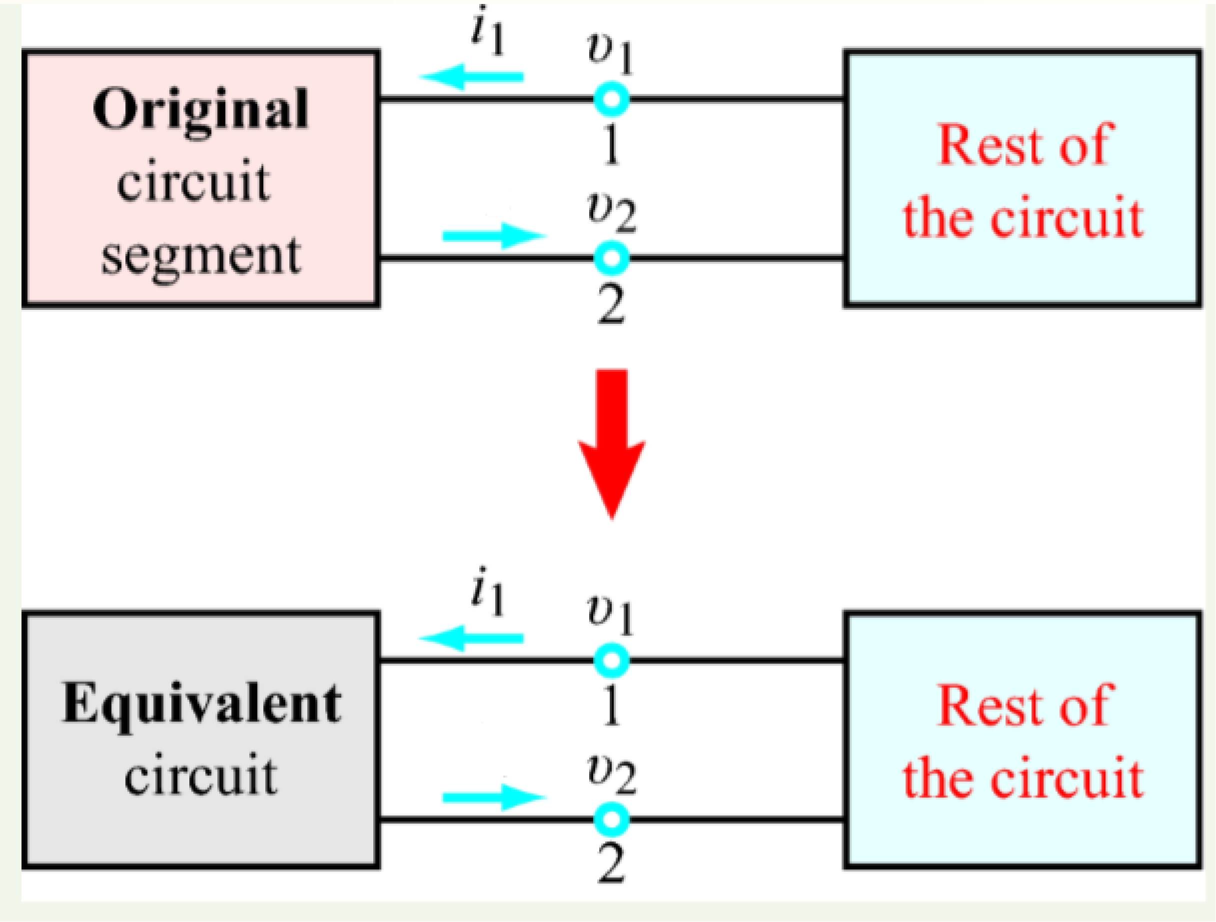
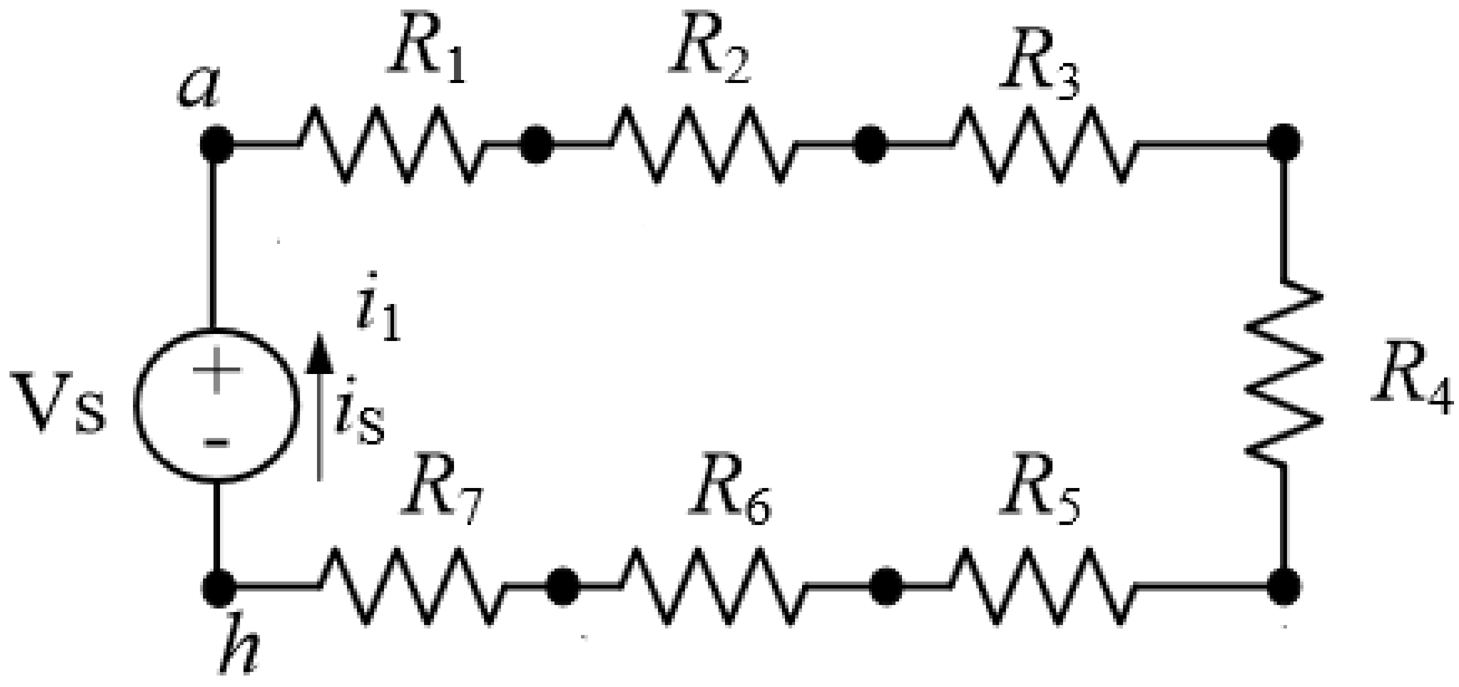
Circuit Equivalence



Two circuits connected between a pair of nodes are considered to be equivalent-as seen by the rest of the circuit-if they exhibit identical *i* - *v* characteristics at those nodes.

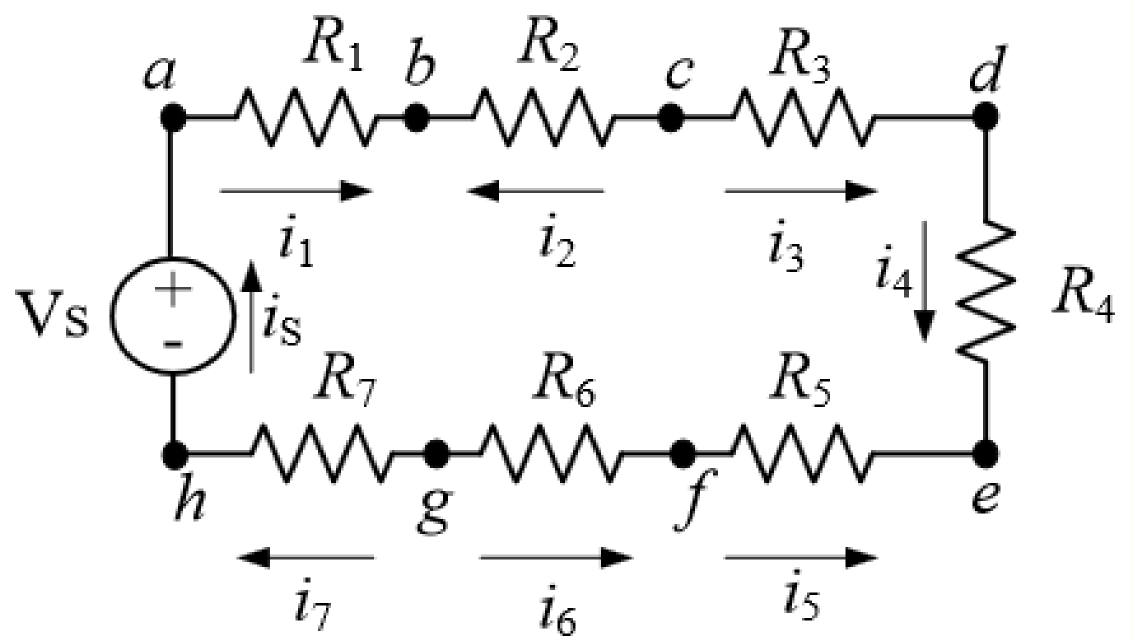
Resistors in Series

Find the relationship between the resistors, $\boldsymbol{i_s}$ and $\boldsymbol{V_s}$



Resistors in Series

Find the relationship between the resistors, ***i*** and ***V***



Share the same current: $i_S = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7$

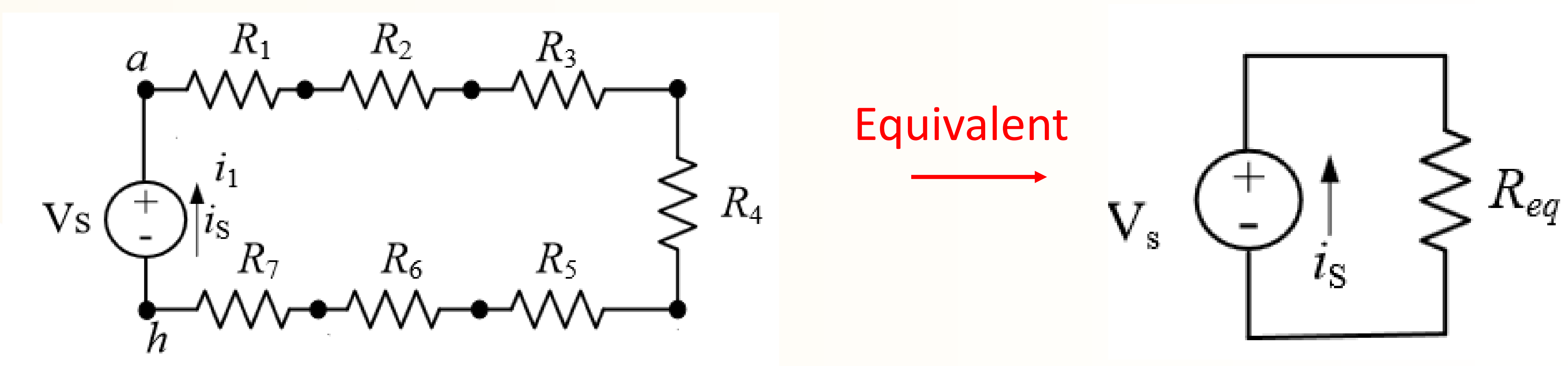
KVL: $-V_S + i_S R_1 + i_S R_2 + i_S R_3 + i_S R_4 + i_S R_5 + i_S R_6 + i_S R_7 = 0$

$\rightarrow \; V_S = i_S(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)$

For nodes a and h $\quad V_S = i_S(R_{eq})$

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

Resistors in Series



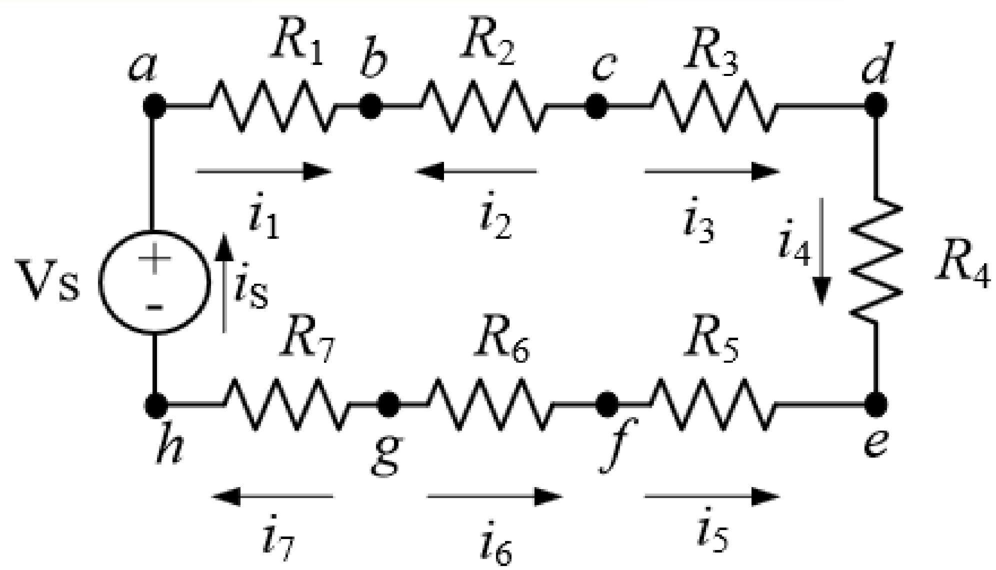
$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

$$\rightarrow \boxed{R_{eq} = \sum_{i=1}^{k} R_i = (R_1 + R_2 + \cdots + R_k)}$$

Multiple Resistors connected in series (experiencing the same current) can be combined into a single equivalent resistor R_{eq} whose resistance is equal to the sum of all of their individual resistances.

Resistors in Series

Find the voltage cross *R1* and *R5*



Ohm's law

$$V_1 = R_1 i_s \qquad (V_{ab})$$

$$V_5 = R_5 i_s \qquad (V_{ef})$$

And $R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$

$$V_s = i_s(R_{eq})$$

$$V_1 = \frac{R_1}{R_1 + R_2 + \cdots R_7} V_s$$

$$V_5 = \frac{R_5}{R_1 + R_2 + \cdots R_7} V_s$$

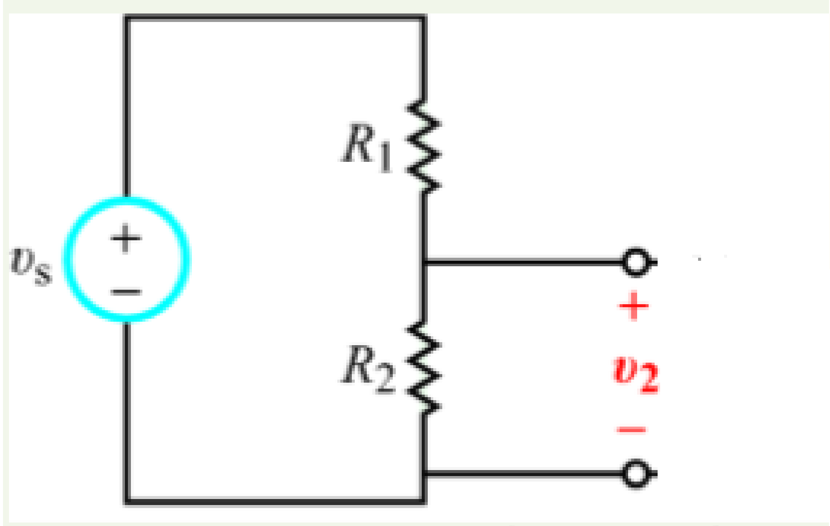
Resistors in Series

Voltage divider

The voltage across any individual resistor R_i in a series circuit is a proportionate fraction (R_i/R_{eq}) of the voltage of the entire group

$$V_i = \frac{R_i}{R_{eq}} V_s$$

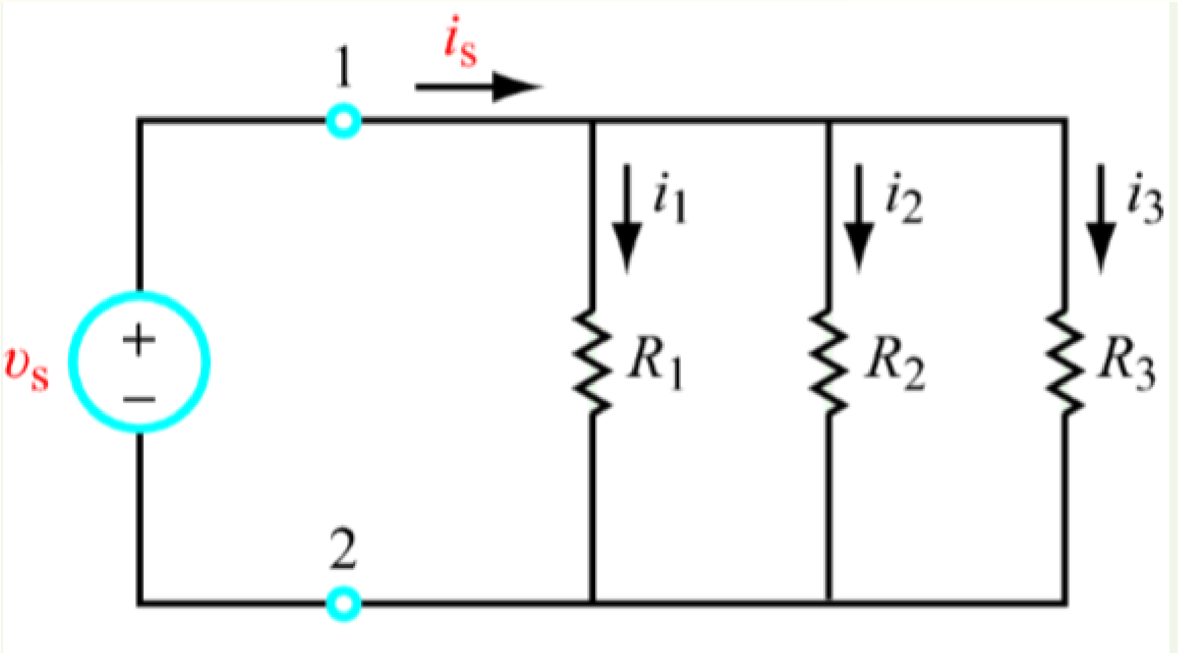
Example:



$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

Resistors in parallel

Find the relationship between i_s, v_s, R_1, R_2, R_3



Apply ohm's law

$$i_1 = \frac{v_s}{R_1} \qquad i_2 = \frac{v_s}{R_2} \qquad i_3 = \frac{v_s}{R_3}$$

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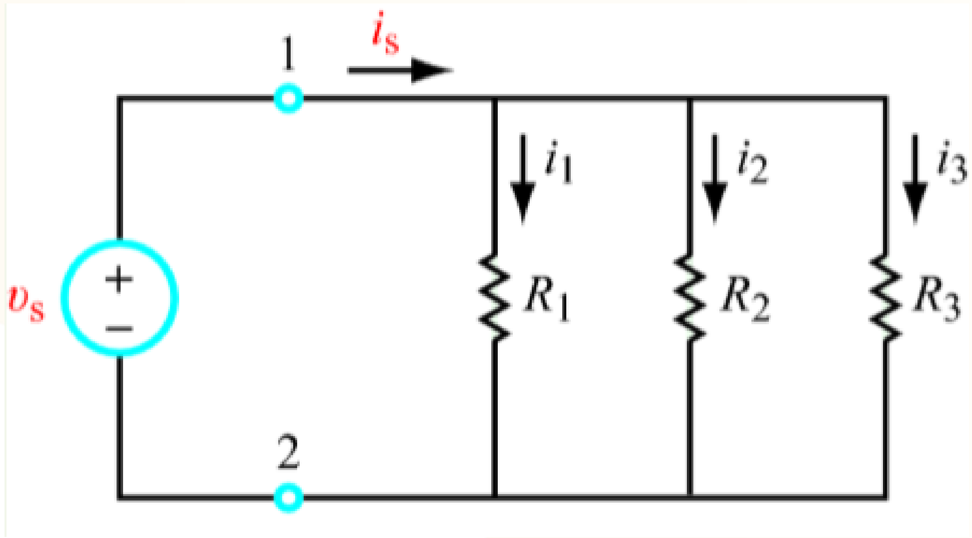
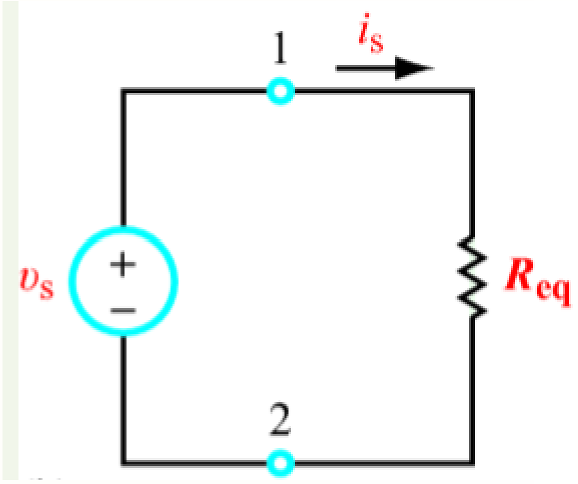
$$i_s = i_1 + i_2 + i_3 = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3}$$

$$\frac{i_s}{v_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in Parallel



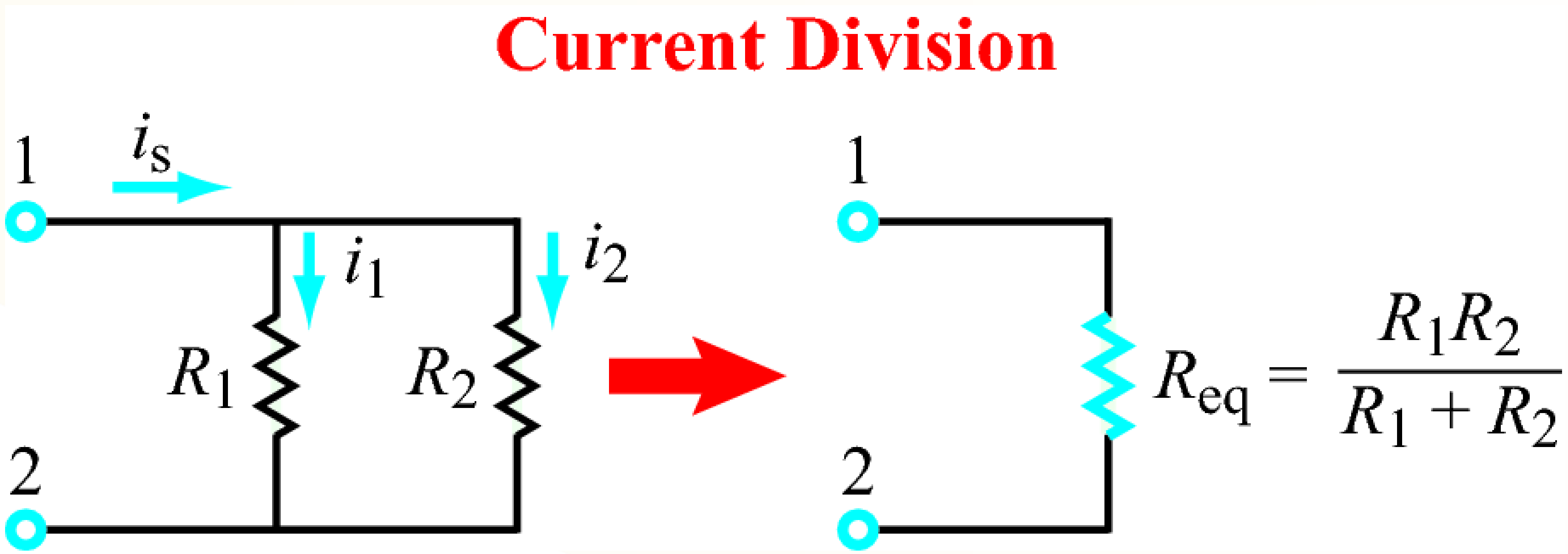
Equivalent



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

For short, we sometimes denote such a parallel R_1 and R_2 combination as $R_1 || R_2$.

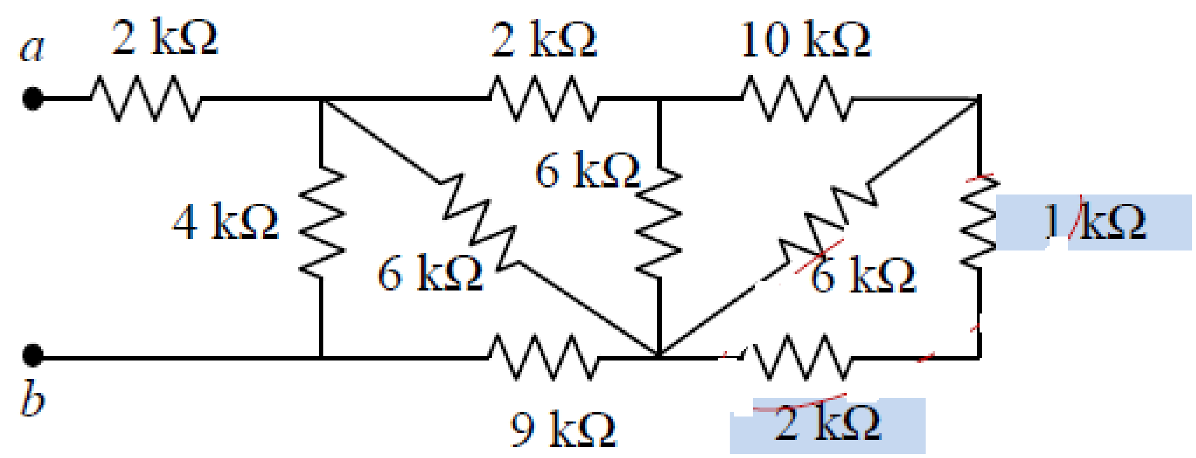
Current divider



$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

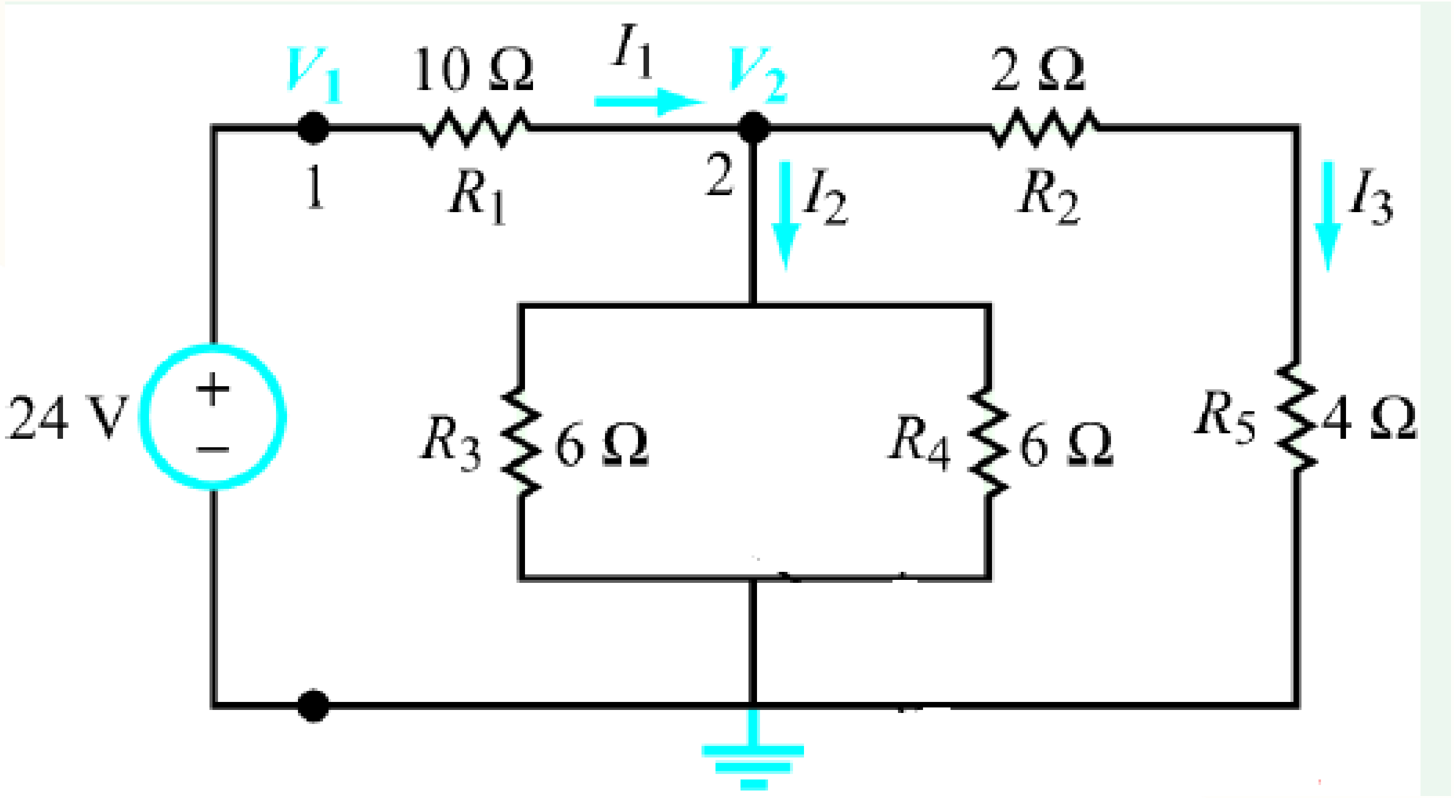
Exercise

Example: Find the equivalent resistance between “a” and “b”



Exercise

Use the equivalent-resistance approach to determine V_2, I_1, I_2, and I_3 in the circuit.



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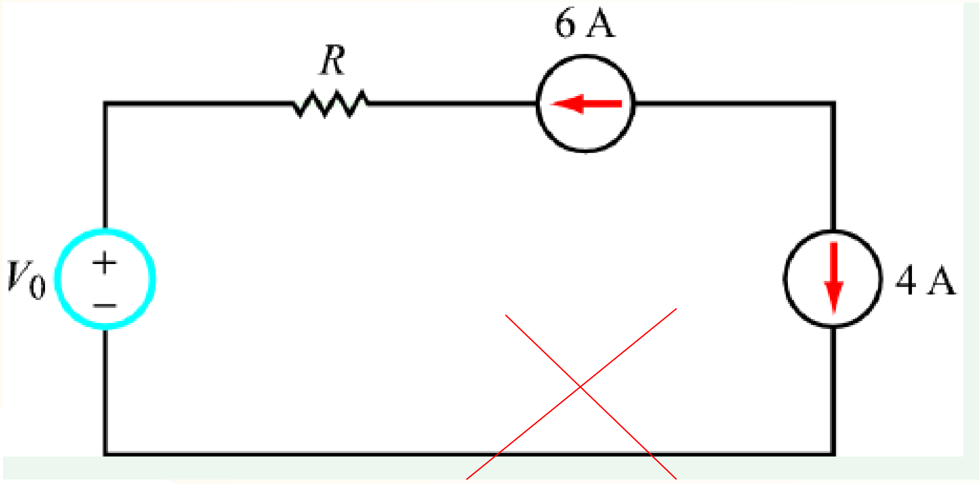
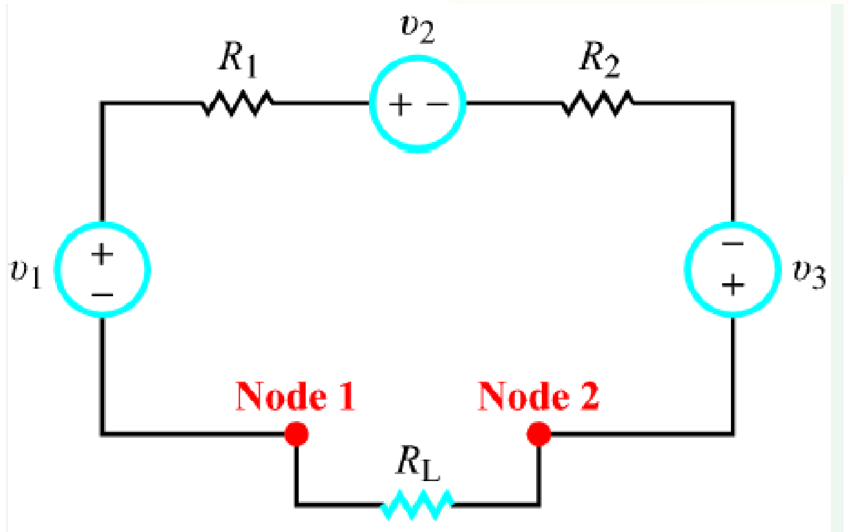
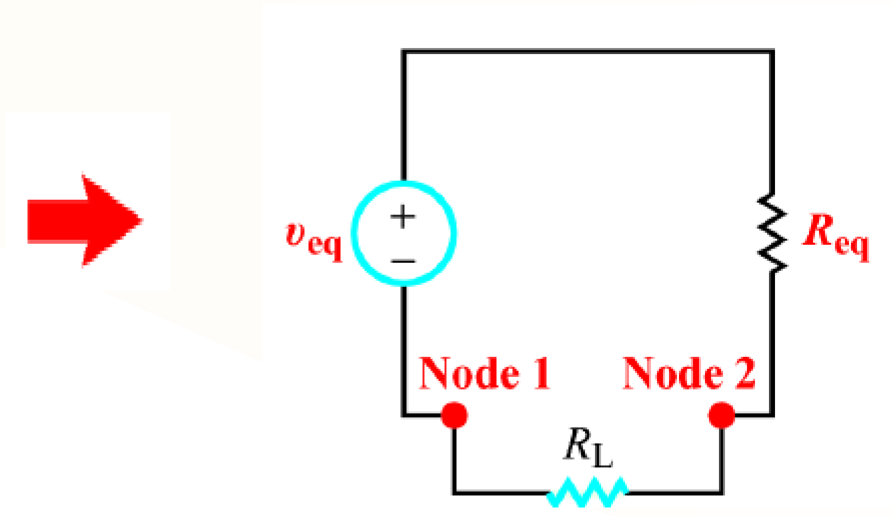
Source in series



?

If the two current sources provide the same value current and in the same direction, so ?





$$v_{eq} = v_1 - v_2 + v_3 \qquad R_{eq} = R_1 + R_2$$

Multiple voltage source connected in series can be combined into an equivalent voltage source whose voltage is equal to the algebraic sum of the voltages of the individual sources.

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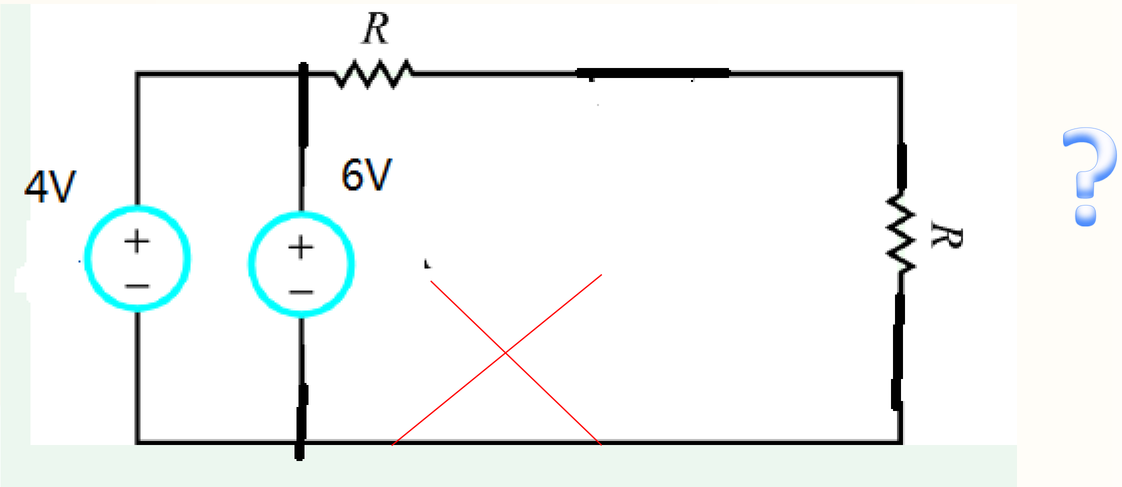
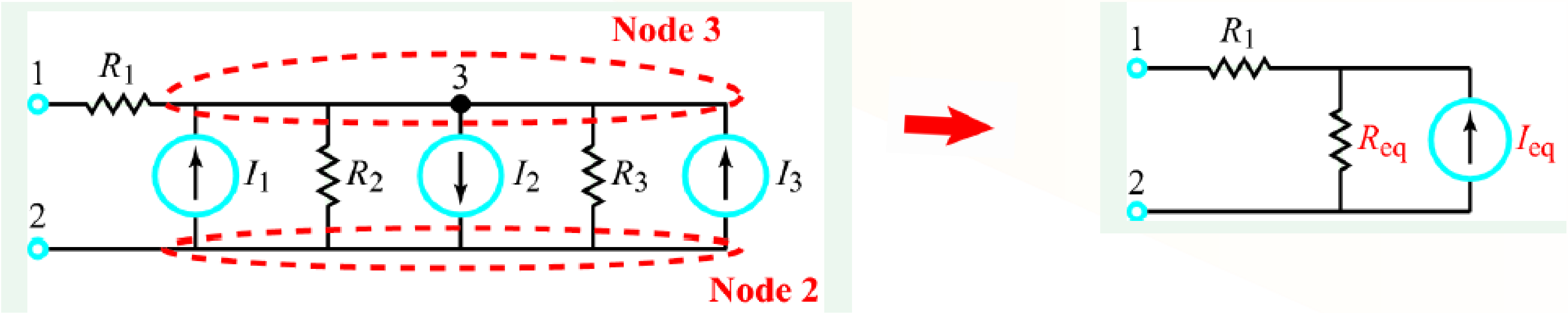
Source in parallel



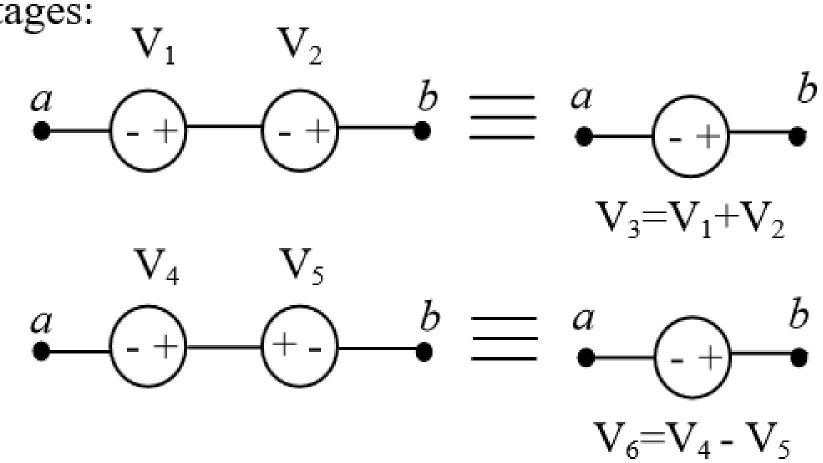
If the two voltage sources provide the some voltage and in the same direction, so ?



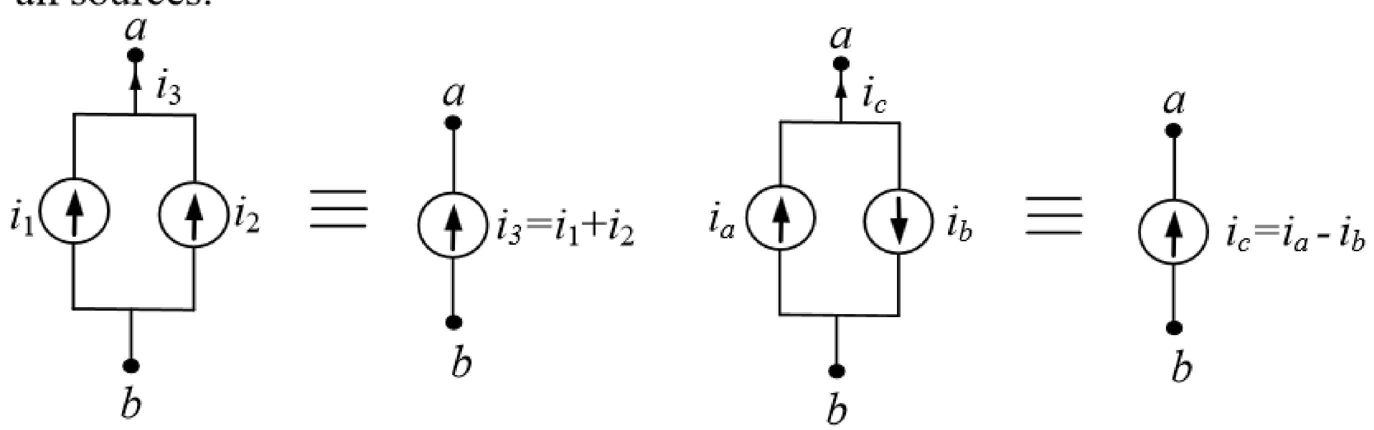
$$R_{eq} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} \qquad I_{eq} = I_1 - I_2 + I_3$$

Source in Series/parallel

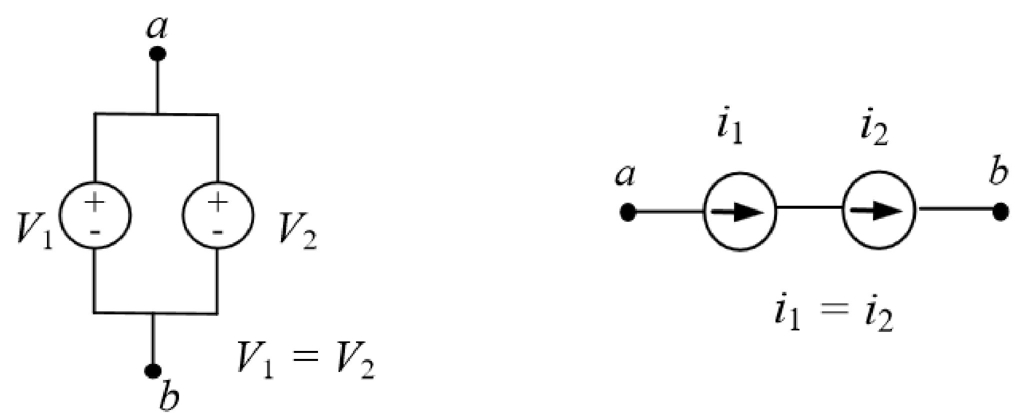
- Series voltage sources have a total voltage equal to the algebraic sum of sources voltages:



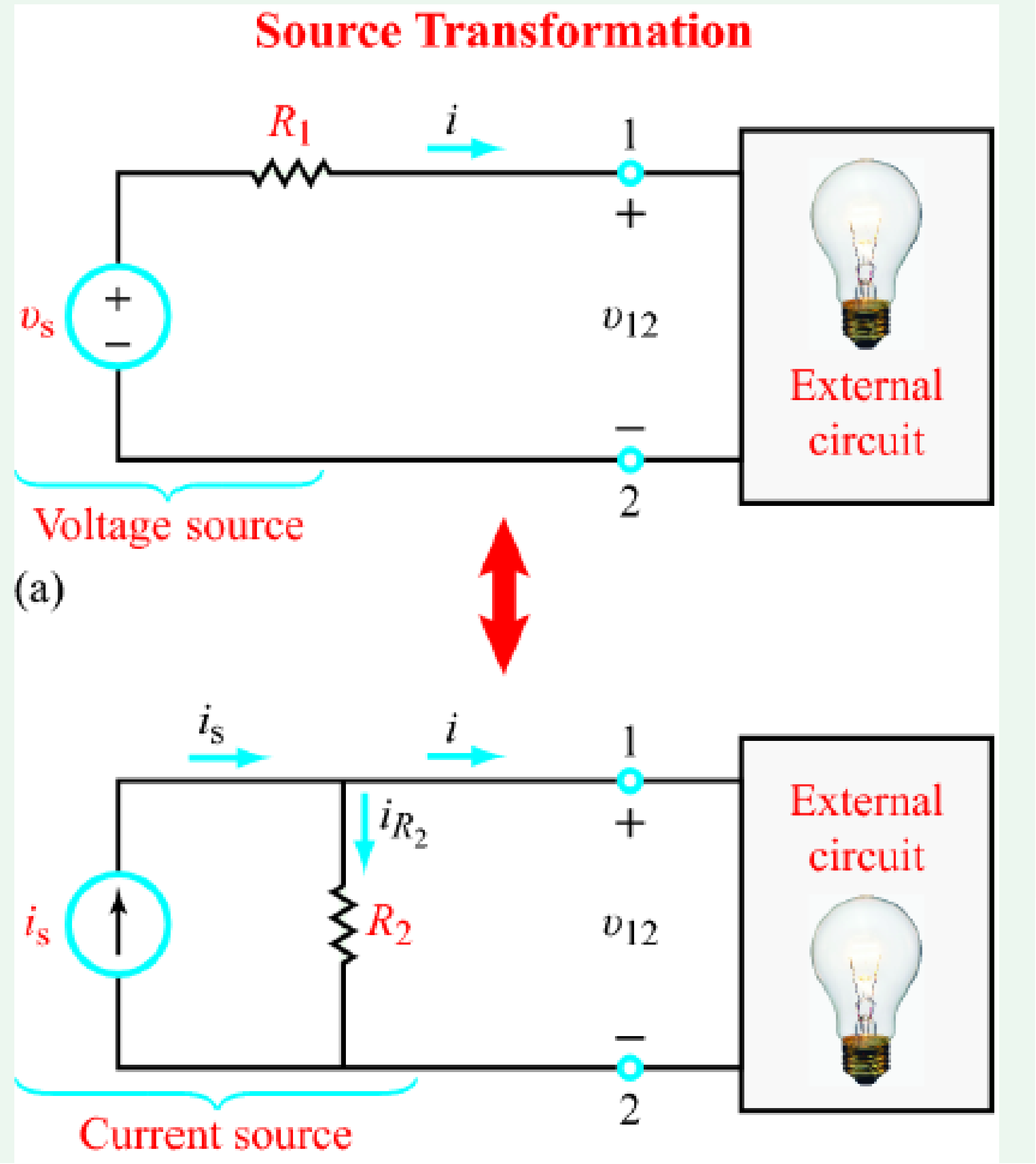
- Parallel current sources have a total current equal to the algebraic sum of all sources:



- Parallel voltage sources should have the same voltage and series current sources should have the same currents:



Source Transformation



$$i_s = v_s / R_1$$
$$R_2 = R_1$$

Exercise