Signals and Circuits

AERN 35500

AC

Chapter 7: 7-1(AC analysis) pp. 346-350

Ulaby, Fawwaz T., and Maharbiz, Michael M., *Circuits*, 2nd Edition, National Technology and Science Press, 2013.

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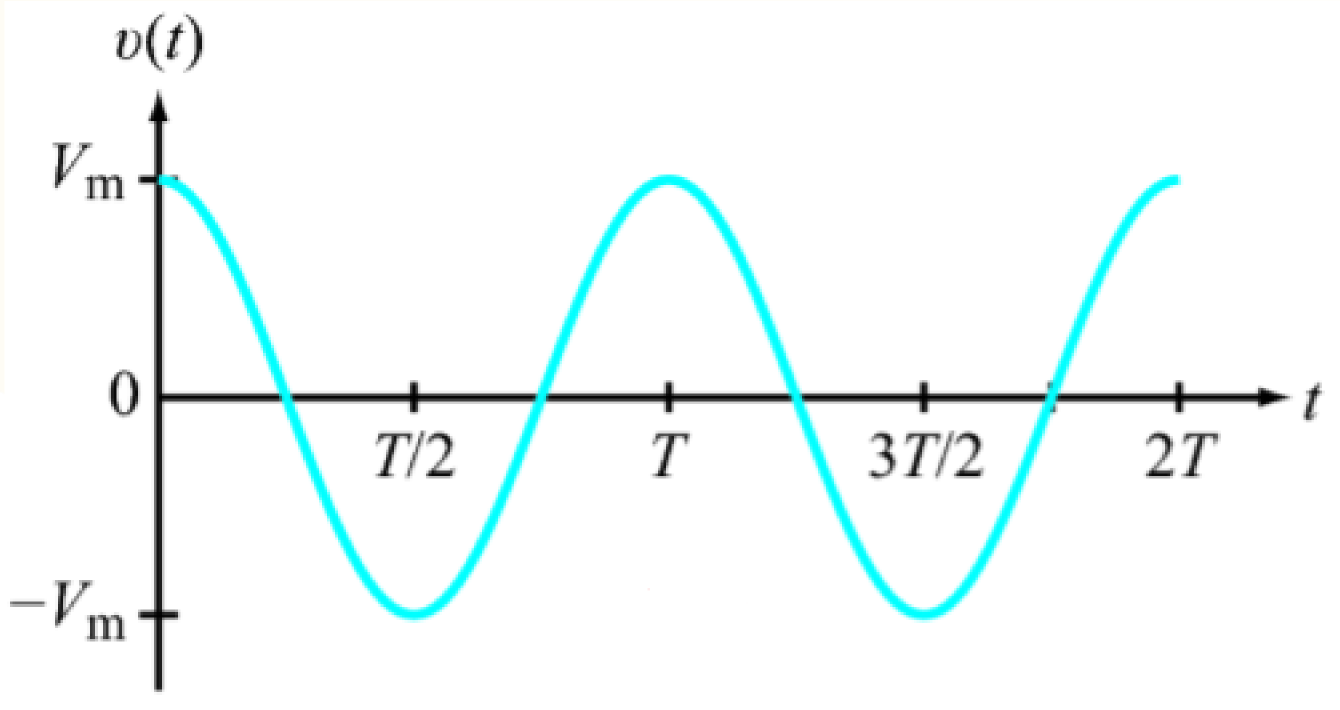
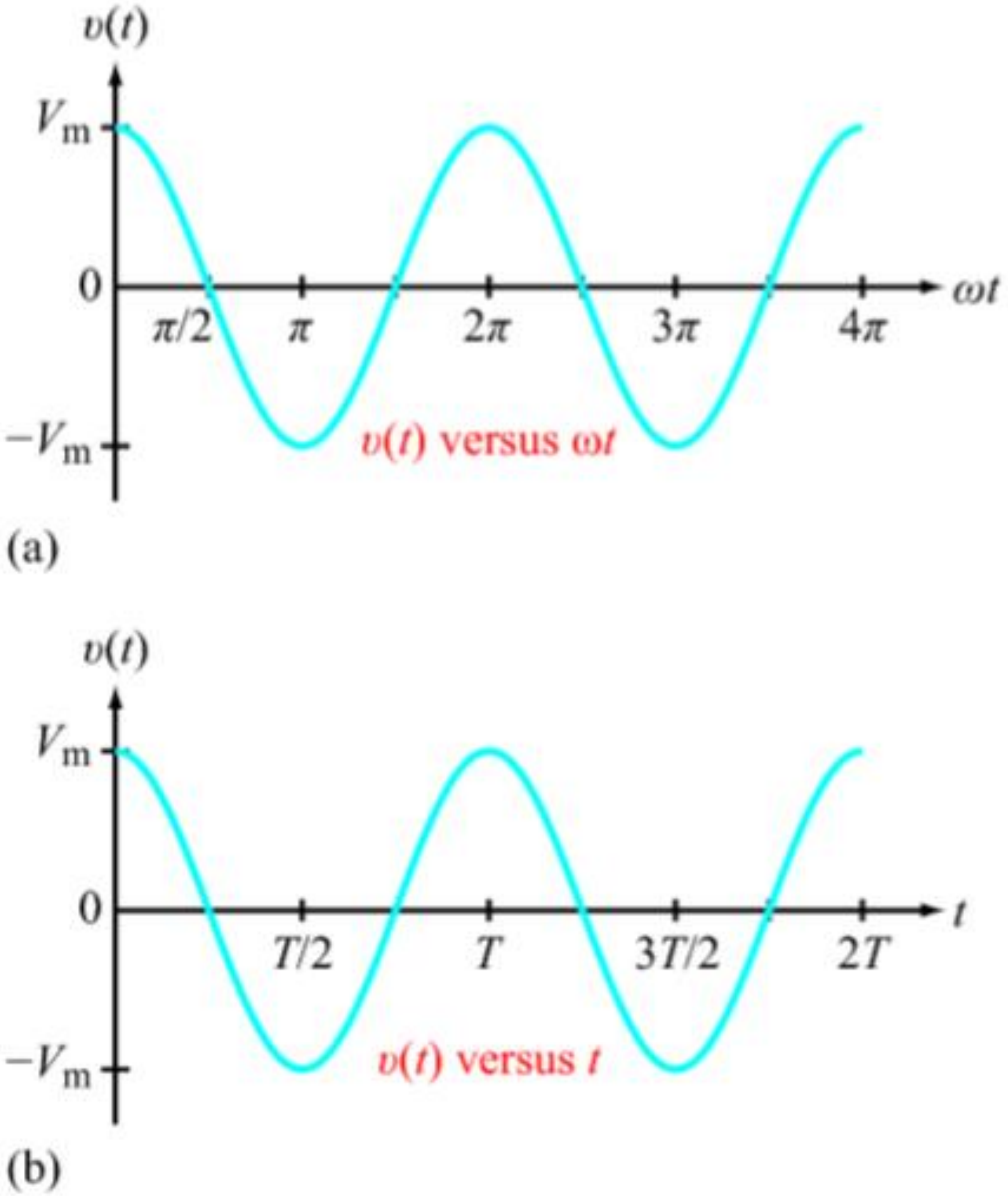
AC



AC is short for (Alternating current); it is associated with electric circuit whose currents and voltages vary sinusoidally with time.

Sinusoidal waveform



The voltage between two points in a circuit (or the current flowing through a branch) is said to have a sinusoidal waveform if its time variation is given by a sinusoidal function.

E. g.

$$v(t) = V_m cos\omega t$$

V_m — **Amplitude (absolute)**

ω — **Angular frequency (rad/s)**

ωt — **Argument can be in radians or in degrees**

$$\pi\ (rad) \approx\ 180^{\circ}$$

T — **Period (time for one cycle wave)**

$T = 2\pi/\omega$ **(Usually in second)**

$$v(t) = v(t + nT)$$

f — **Oscillation frequency (*Hz*)**

$$f = \frac{1}{T}$$

Sinusoidal waveform

Useful Trigonometric identities

$$\sin x = \pm\cos(x \mp 90^{\circ})$$

$$\cos x = \pm\cos(x \pm 90^{\circ})$$

$$\sin x = -\sin(x \pm 180^{\circ})$$

$$\cos x = -\cos(x \pm 180^{\circ})$$

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$2\sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2\sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2\cos x \cos y = \cos(x + y) + \cos(x - y)$$

Sinusoidal waveforms can be expressed in terms of either sine or cosine functions. We usually use cosine forms as a reference standard expression.

E. g.

$$i = 6sin(\omega t + 30^{\circ})$$
$$= 6cos(\omega t + 30^{\circ} - 90^{\circ})$$
$$= 6cos(\omega t - 60^{\circ})$$

Sinusoidal waveform

$v(t) = V_m \cos(\omega t + \emptyset)$

$\emptyset$ is called the phase angle

$\emptyset$ can be positive or negative. We usually add or subtract multiples of 2π radians (or -360°) to make is be between - π (-180°) and π (180°).

Phase lead/lag

$$v_1(t) = V_1 \cos(\omega t + \emptyset_1)$$

$$v_2(t) = V_2 \cos(\omega t + \emptyset_2)$$

v_2 leads v_1 by ($\emptyset_2$-$\emptyset_1$)

$v_2$lags v_1 by ($\emptyset_1$-$\emptyset_2$)

$(\emptyset_2 - \emptyset_1) \in [-180^\circ, +180^\circ]$

v_2 and v_1 are in phase if $\emptyset_2 = \emptyset_1$

v_2 and v_1 are in phase − opposition if $\emptyset_2 = \emptyset_1 \pm 180^\circ$

Sinusoidal waveform

Phase lead/lag

$$v_1(t) = V_1\cos(\omega t + \emptyset_1)$$
$$v_2(t) = V_2\cos(\omega t + \emptyset_2)$$

v_2 leads v_1 by ($\emptyset_2$-$\emptyset_1$)

$v_2$lags v_1 by ($\emptyset_1$-$\emptyset_2$)

$$(\emptyset_2 - \emptyset_1) \in [-180^{\circ}, +180^{\circ}]$$

v_2 and v_1 are in phase. if $\emptyset_2$=$\emptyset_1$

v_2 and v_1 are in phase $-$ opposition. if $\emptyset_2$=$\emptyset_1 \pm 180^{\circ}$

E. g.

$$v_1(t) = 8\cos(\omega t + 30^{\circ})$$

$$v_2(t) = 15\sin(\omega t - 40^{\circ})$$

Does $v_1(t)\ lead v_2(t)?\ How\ much?$

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Power in AC circuits

Instantaneous power

The definition of power is:

$P = I \times V$

In this case $i = I_p \sin(2\pi f t + \theta)$

And $v = V_p \sin(2\pi f t + \theta)$

And then the instantaneous power is going to be:

$p = I_p V_p \sin^2(2\pi f t + \theta)$

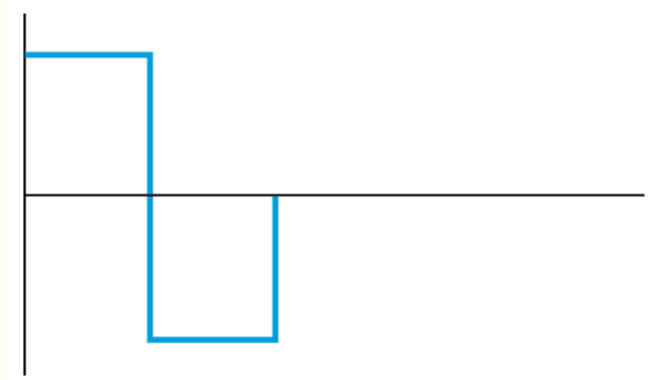
Average power

$$P_{avg} = \frac{\int_0^T p\,dt}{T} = \frac{\int_0^T I_p V_p \sin^2(2\pi f t + \theta)\,dt}{T} = I_p V_p (0.707)^2$$

Nonsinusoidal waveforms

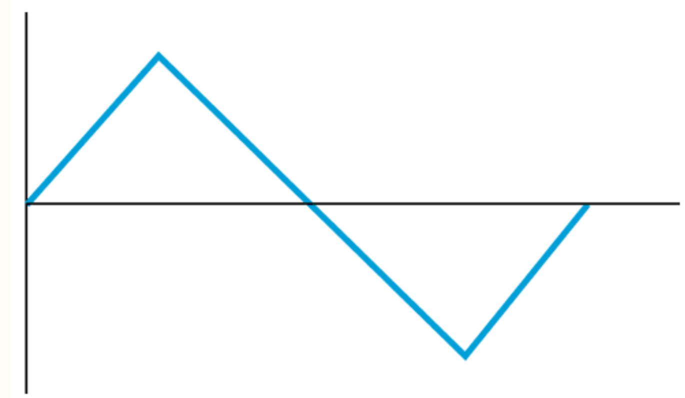
Square waveform

Useful as an electronic signal because its characteristics are easily changed.



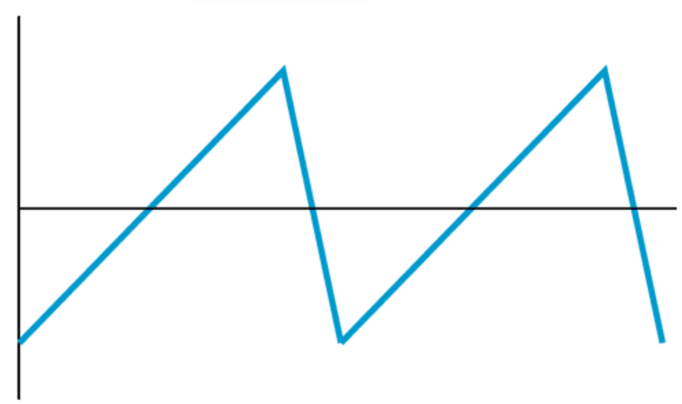
Triangular waveform

Used primarily as electronic signals.



Sawtooth waveform

Used to sweep the electron beam across the screen, creating an image, as in television sets.



$$f(t) = f(t + nT)$$