Chapter 5

Energy Analysis Of Closed Systems

Objectives-Continue

- **Examine the moving boundary work or PdV work commonly encountered in reciprocating devices such as automotive engines and compressors.**

- **Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.**

- **Develop the general energy balance applied to closed systems.**

- **Define the specific heat at constant volume and the specific heat at constant pressure.**

Objectives-Continue

- **Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.**

- **Describe incompressible substances and determine the changes in their internal energy and enthalpy.**

- **Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.**

Moving boundary work (*P dV* work):
The expansion and compression work in a piston-cylinder device.

Quasi-equilibrium process:
A process during which the system remains nearly in equilibrium at all times.

$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

W_b is positive → for expansion
W_b is negative → for compression
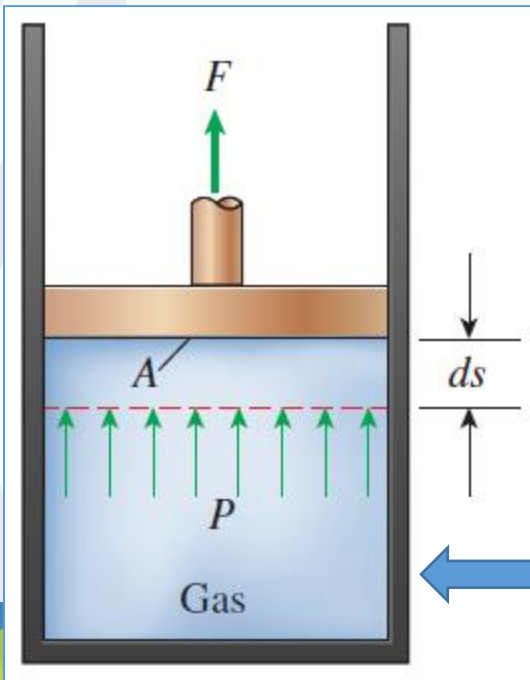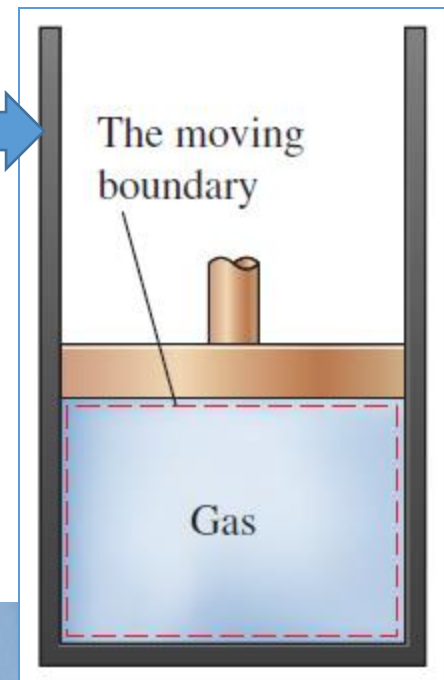
$$W_b = \int_1^2 P\,dV \qquad (\text{kJ})$$
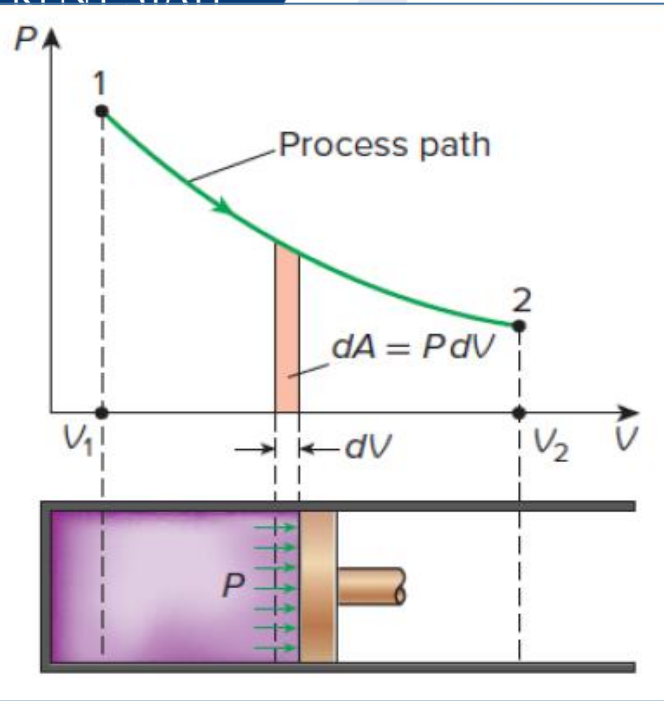
FIGURE 5-1

The work associated with a moving boundary is called *boundary work.*

FIGURE 5-2

A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount ds.

KENT STATE

The boundary work done during a process depends on the path followed as well as the end states.
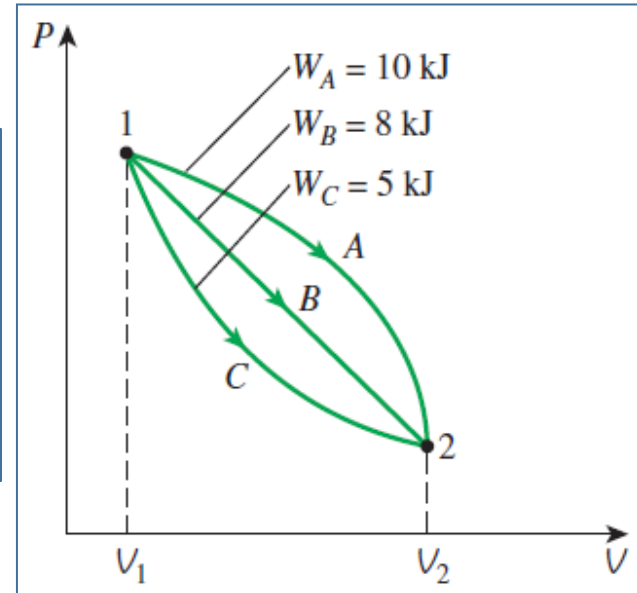
FIGURE 5-3

The area under the process curve on a P-V diagram represents the boundary work.

$$\text{Area} = A = \int_{1}^{2} dA = \int_{1}^{2} P \, dV$$

The area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.

FIGURE 5-5

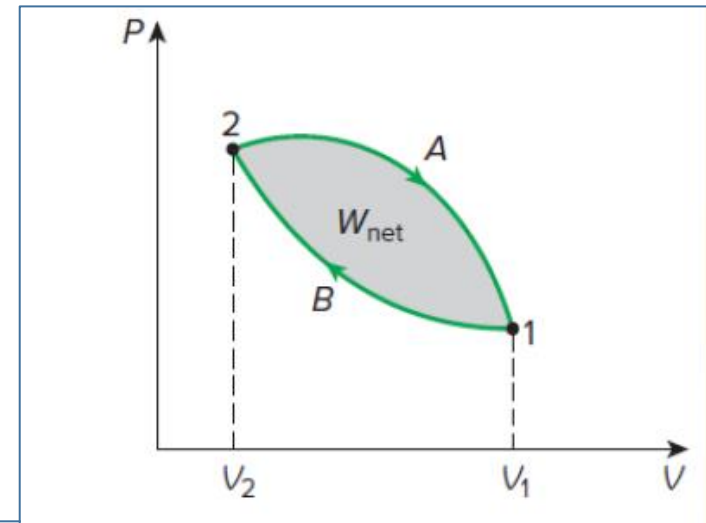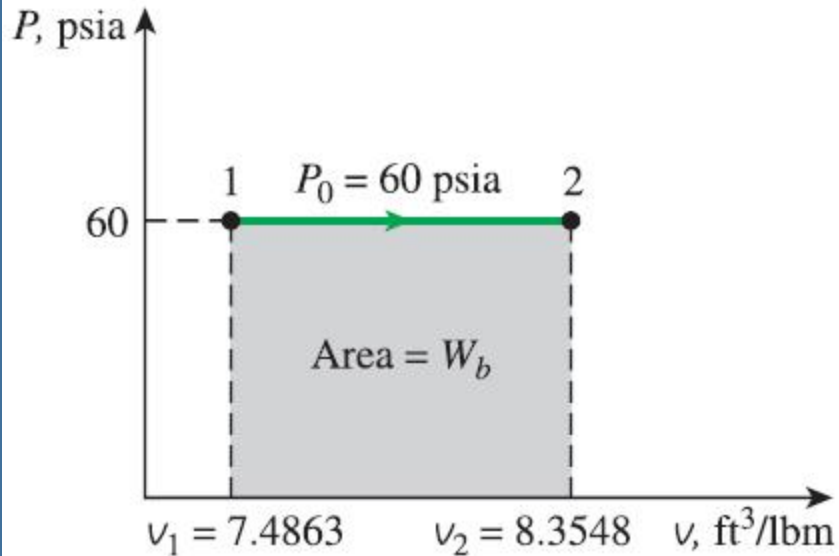**The net work done during a cycle is the difference between the work done by the system and the work done on the system.**

Boundary Work for a Constant-Pressure Process

H_2O

$m = 10$ lbm
$P = 60$ psia

Heat

P, psia

1    $P_0 = 60$ psia   2

60

Area $= W_b$

$v_1 = 7.4863$ $v_2 = 8.3548$ v, ft^3/lbm
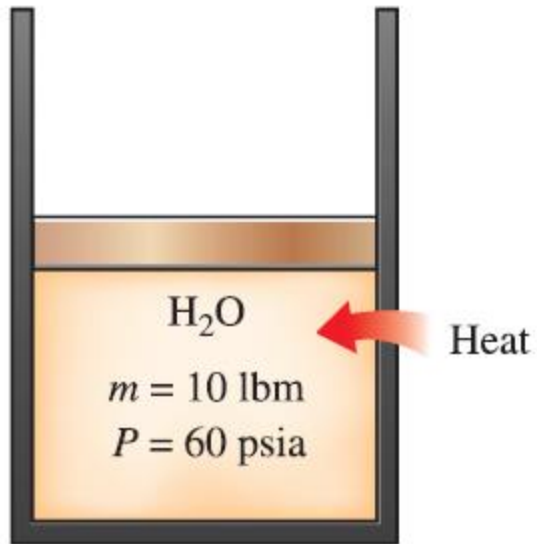
$$W_b = \int_1^2 P\, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

$$W_b = mP_0(v_2 - v_1)$$

Boundary Work for a Constant-Volume Process

Air
$P_1 = 500$ kPa
$T_1 = 150°C$

$P_2 = 400$ kPa
$T_2 = 65°C$

Heat

P, kPa

500 — 1

400 — 2

V
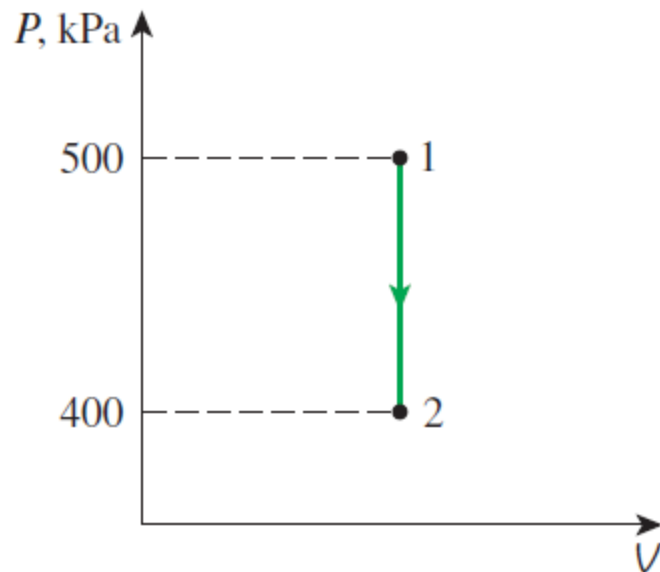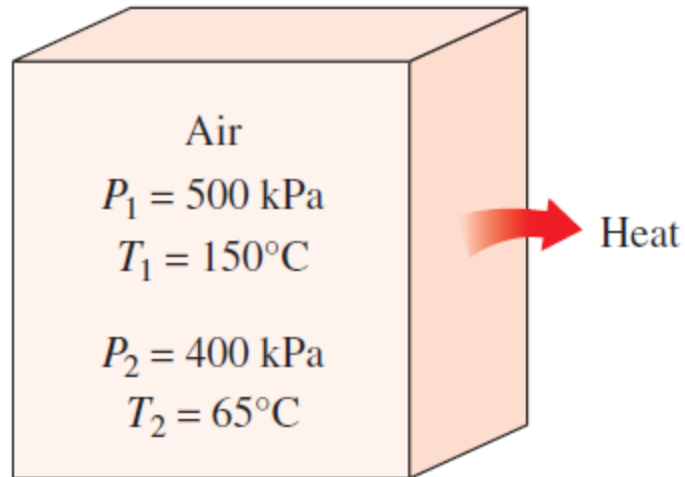
What is the boundary work for a constant-volume process?

It is always Zero since

$$W_b = \int_1^2 P\, dV^{\;0} = 0$$

KENT STATE

Air
$V_1 = 0.4 \text{ m}^3$
$P_1 = 100 \text{ kPa}$
$T_0 = 80°C = \text{const.}$
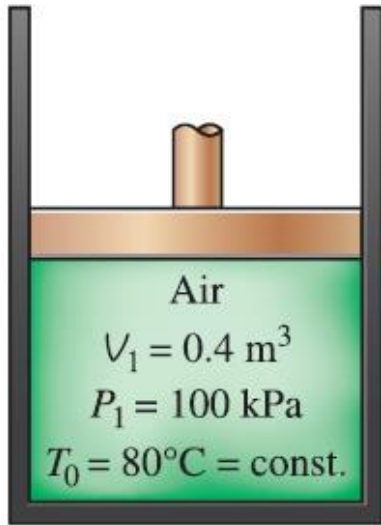
FIGURE 5–8

$$W_b = \int_1^2 P\, dV = \int_1^2 CV^{-1}\, \boldsymbol{dV} = \ln\left(\frac{V_2}{V_1}\right)$$

$$PV = mRT_0 = C \quad \textbf{or} \quad P = \frac{C}{V}$$

$$W_b = \int_1^2 P\, dV = \int_1^2 \frac{C}{V}\, \boldsymbol{dV} = C \int_1^2 \frac{dV}{V} = C \ln\frac{V_2}{V_1} = P_1 V_1 \ln\frac{V_2}{V_1}$$
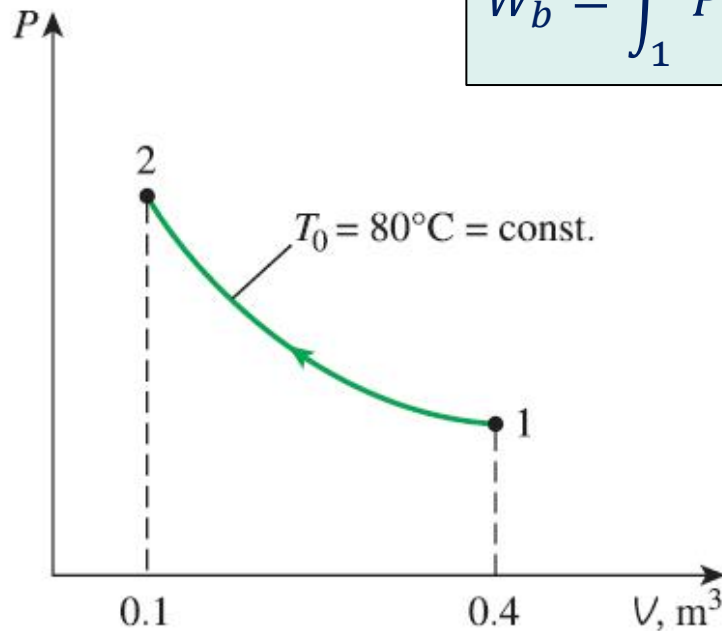
The c_v and c_p values of incompressible substances are identical and are denoted by c.

$$c_p = c_v = c$$

2

$T_0 = 80°C = \text{const.}$

1

0.1   0.4 V, m^3

$$W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \tag{5-6}$$

EXAMPLE 5-2   Boundary Work for a Constant-Pressure Process

A frictionless piston–cylinder device contains 10 lbm of steam at 60 psia and 320°F. Heat is now transferred to the steam until the temperature reaches 400°F. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process.

SOLUTION

Steam in a piston–cylinder device is heated and the temperature rises at constant pressure. The boundary work done is to be determined.

Assumption

The expansion process is quasi-equilibrium.

Analysis

A sketch of the system and the P-V diagram of the process are shown in **Fig. 5–7**. Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and, from **Eq. 5–2**

$$W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \tag{5-6}$$

H_2O
Heat
$m = 10$ lbm
$P = 60$ psia

P, psia

1 $P_0 = 60$ psia 2

60

Area $= W_b$

$v_1 = 7.4863$ $v_2 = 8.3548$ v, ft³/lbm
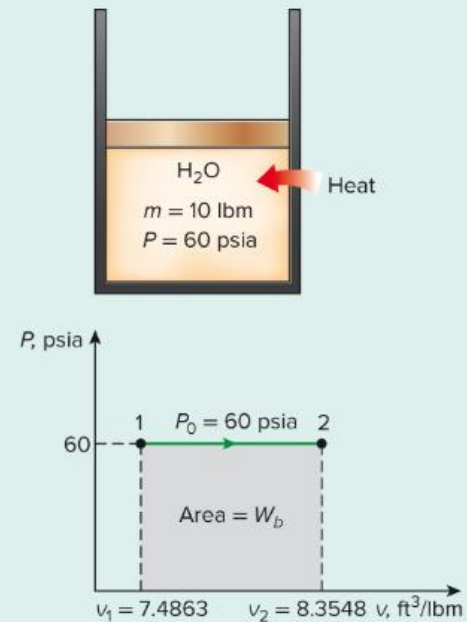
FIGURE 5–7
Schematic and P-V diagram for **Example 5–2**.

$$W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0(V_2 - V_1) \qquad \text{(5–6)}$$
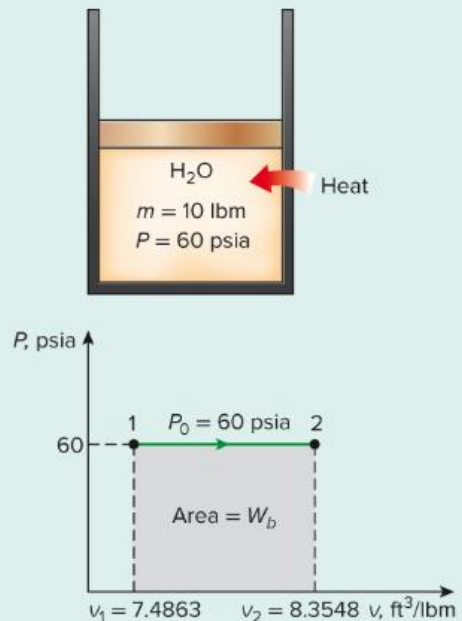
FIGURE 5–7
Schematic and P-V diagram for **Example 5–2**.

or

$$W_b = mP_0(v_2 - v_1)$$

since $V = mv$. From the superheated vapor table (Table A–6E), the specific volumes are determined to be $v_1 = 7.4863$ ft^3/lbm at state 1 (60 psia, 320°F) and $v_2 = 8.3548$ ft^3/lbm at state 2 (60 psia, 400°F). Substituting these values yields

$$W_b = (10 \text{ lbm})(60 \text{ psia})[(8.3548 - 7.4863) \text{ ft}^3/\text{lbm}]\left(\frac{1 \text{ Btu}}{5.404 \text{ psia·ft}^3}\right)$$

$$= 96.4 \text{ Btu}$$

Discussion
The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the P-V diagram, which is simply $P_0 \Delta V$ for this case.

Boundary Work for a Polytropic Process

During actual expansion and compression processes of gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. A process of this kind is called a **polytropic process** (**Fig. 5–9**). Below we develop a general expression for the work done during a polytropic process.

$$PV^n = C \qquad P = CV^{-n}$$

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\,\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \quad \textbf{For ideal gas}$$

(**Fig. 5–9**).

$$P_1V_1^n = P_2V_2^n$$

$$PV^n = \text{const.}$$

Gas

$$PV^n = C = \text{const.}$$
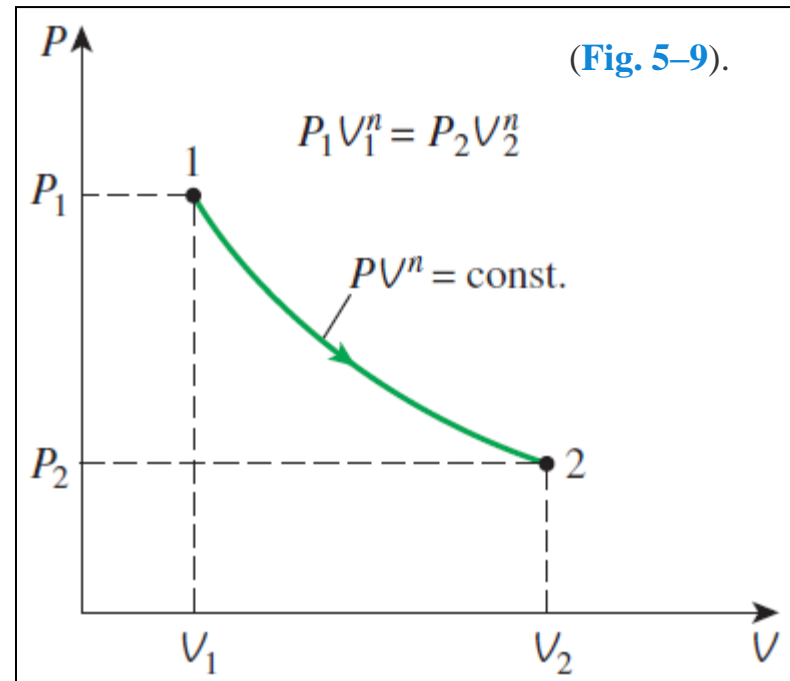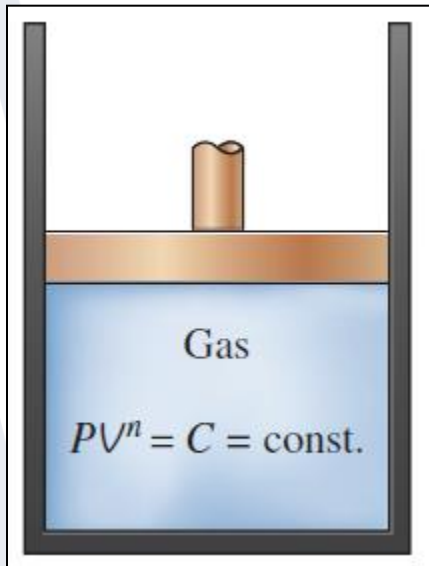
What is polytropic process?

For an ideal gas it means that the PV diagram of a gas can be described by the following equation : $PV^n = Constant$ where n is a constant.

n is known as the "*polytropic index*" for the process.

Examples of Polytropic process :

- Adiabatic Processes : $PV^\gamma = Constant$ Here $n = \gamma$ (no Heat Transfer)
- Isothermal process : $PV^1 = Constant$ (Boyle's law) here $n = 1$ (constant Temperature)
- Isobaric processes : $PV^0 = constant$ here $n = 0$ (constant Pressure)
- Isochoric process : $PV^\infty = constant$ here $n = \infty$ (constant Volume Process)

What does a Polytropic process really mean ?

Polytropic process can be also defined in this way — The process in which heat absorbed by the gas due to unit rise in temperature is constant — in other words a *constant specific heat process (C)*.

n is a function of C.
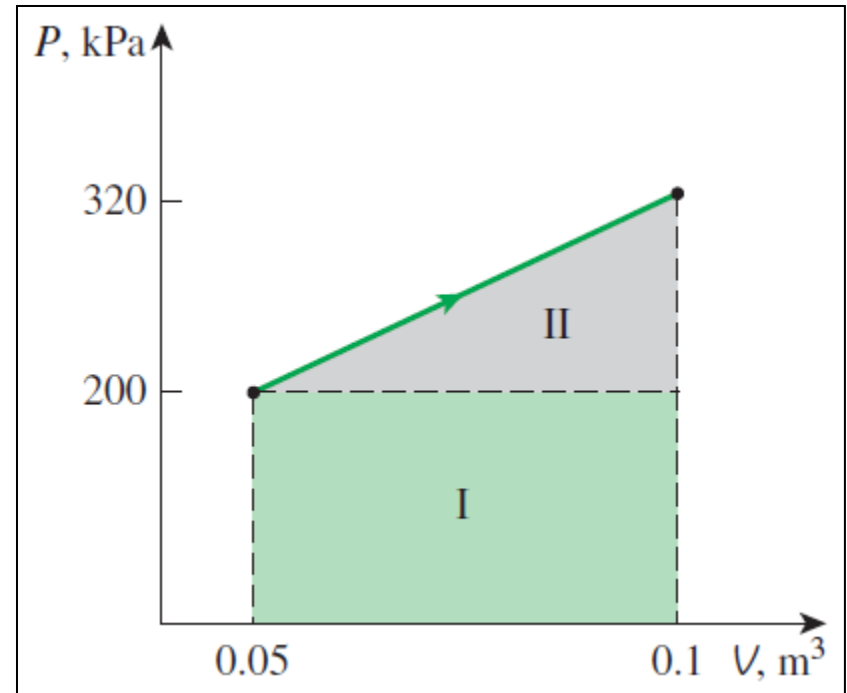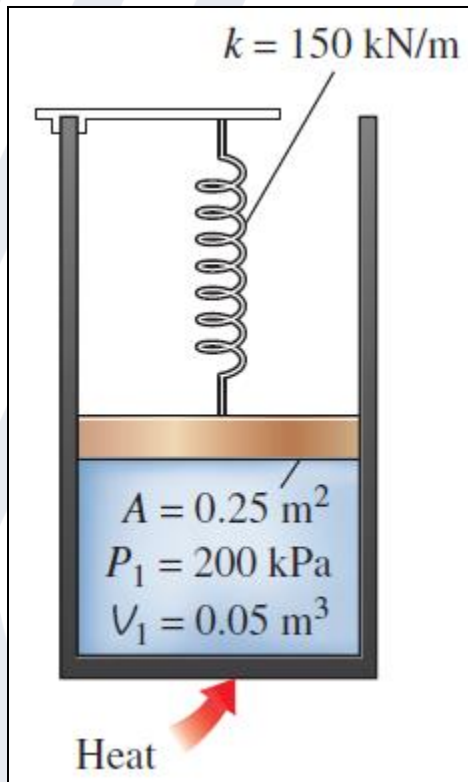
Expansion of a Gas against a Spring

EXAMPLE 5–4   Expansion of a Gas against a Spring

A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine (*a*) the final pressure inside the cylinder, (*b*) the total work done by the gas, and (*c*) the fraction of this work done against the spring to compress it.

SOLUTION

A gas in a piston–cylinder device equipped with a linear spring expands as a result of heating. The final gas pressure, the total work done, and the fraction of the work done to compress the spring are to be determined.

Assumptions

1 The expansion process is quasi-equilibrium. **2** The spring is linear in the range of interest.

Analysis

A sketch of the system and the *P-V* diagram of the process are shown in **Fig. 5–10**.
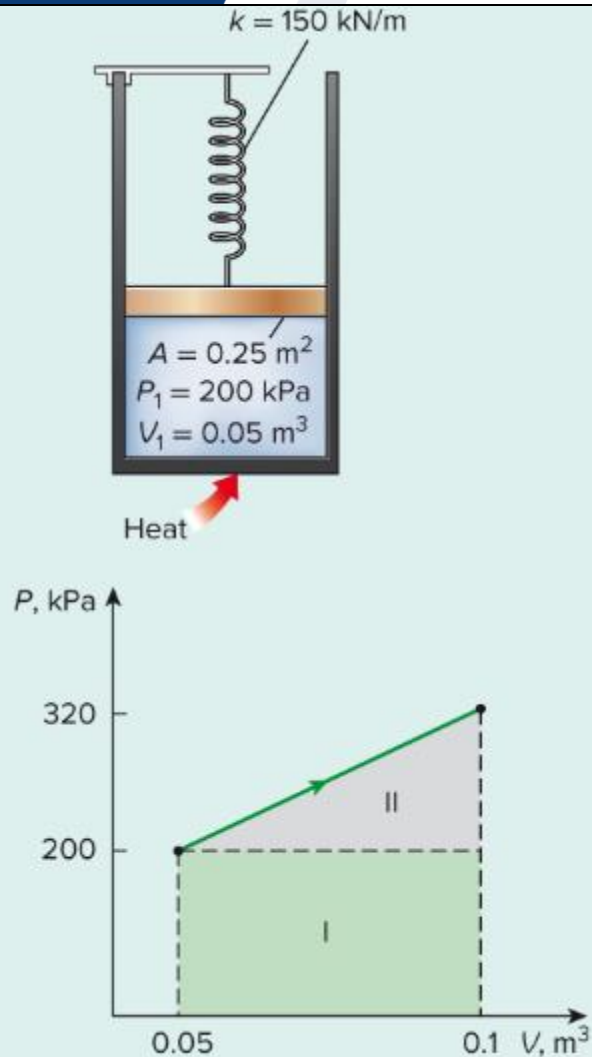
FIGURE 5–10
Schematic and *P-V* diagram for **Example 5–4**.

(*a*) The enclosed volume at the final state is

$$V_2 = 2V_1 = (2)(0.05 \text{ m}^3) = 0.1 \text{ m}^3$$

Then the displacement of the piston (and of the spring) becomes

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05) \text{ m}^3}{0.25 \text{ m}^2} = 0.2 \text{ m}$$

The force applied by the linear spring at the final state is

$$F = kx = (150 \text{ kN/m})(0.2 \text{ m}) = 30 \text{ kN}$$

The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30 \text{ kN}}{0.25 \text{ m}^2} = 120 \text{ kPa}$$

Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

$$200 + 120 = \mathbf{320 \text{ kPa}}$$

at the final state.

Expansion of a Gas against a Spring

$k = 150$ kN/m

$A = 0.25$ m^2
$P_1 = 200$ kPa
$V_1 = 0.05$ m^3

Heat

P, kPa

320

200

0.05   0.1 V, m^3
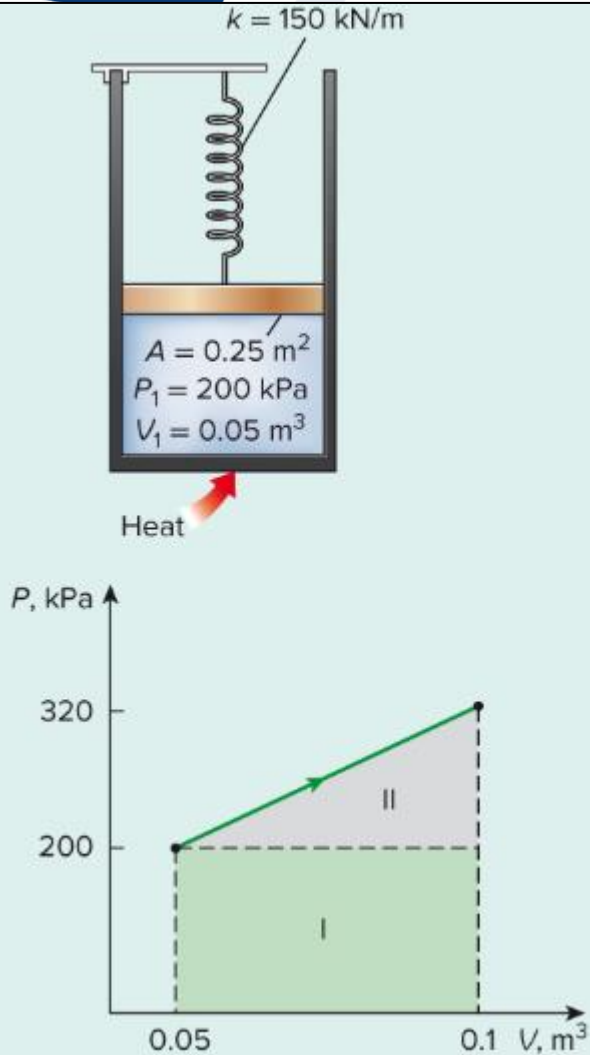
(b) An easy way of finding the work done is to plot the process on a P-V diagram and find the area under the process curve. From **Fig. 5–10** the area under the process curve (a trapezoid) is determined to be

$$W = \text{area} = \frac{(200 + 320) \text{ kPa}}{2} [(0.1 - 0.05) \text{ m}^3]\left(\frac{1 \text{ kJ}}{1 \text{ kPa·m}^3}\right) = \mathbf{13 \text{ kJ}}$$

Note that the work is done by the system.

(c) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus,

$$W_{\text{spring}} = \tfrac{1}{2}[(320 - 200) \text{ kPa}](0.05 \text{ m}^3)\left(\frac{1 \text{ kJ}}{1 \text{ kPa·m}^3}\right) = \mathbf{3 \text{ kJ}}$$

Discussion

This result could also be obtained from

$$W_{\text{spring}} = \tfrac{1}{2}k(x_2^2 - x_1^2) = \tfrac{1}{2}(150 \text{ kN/m})[(0.2 \text{ m})^2 - 0^2]\left(\frac{1 \text{ kJ}}{1 \text{ kN·m}}\right) = 3 \text{ kJ}$$

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}} \quad (\text{kJ})$$

$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat,work,and mass}}}$ $\underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal,kinetic,} \\ \text{potential,etc.,energies}}}$

Energy balance for any system undergoing any process

$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat,work,and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic,potential,etc.,energies}}} \quad (\text{kW})$

Energy balance in the rate form

The total quantities are related to the quantities per unit time is;

$$Q = \dot{Q}\Delta t, \quad W = \dot{W}\Delta t, \quad \textbf{and} \quad \Delta E = (dE/dt)\Delta t \quad (\text{kJ})$$

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}} \quad (\text{kJ/kg})$$

Energy balance per unit mass basis

$$\delta E_{\text{in}} - \delta E_{\text{out}} = dE_{\text{system}} \quad \textbf{or} \quad \delta e_{\text{in}} - \delta e_{\text{out}} = de_{\text{system}}$$

Energy balance in differential form

$$W_{\text{net,out}} = Q_{\text{net,in}} \quad \textbf{or} \quad \dot{W}_{\text{net,out}} = \dot{Q}_{\text{net,in}}$$

Energy balance for a cycle

$$Q = Q_{\text{net,in}} = Q_{\text{in}} - Q_{\text{out}}$$

$$Q_{\text{net,in}} - W_{\text{net,out}} = \Delta E_{\text{system}} \quad \textbf{or} \quad Q - W = \Delta E$$

$$W = W_{\text{net,out}} = W_{\text{out}} - W_{\text{in}}$$

Energy balance when sign convention is used: (i.e., heat input and work output are positive; heat output and work input are negative).

That is, the net work output during a cycle is equal to net heat input

General $\quad Q - W = \Delta E$

Stationary systems $\quad Q - W = \Delta U$

Per unit mass $\quad q - w = \Delta e$

Differential form $\quad \delta q - \delta w = de$

Various forms of the first-law relation for closed systems when sign convention is used.
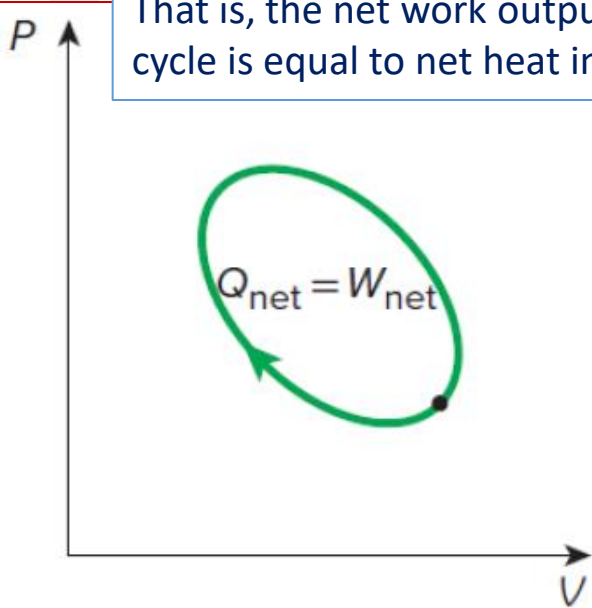
FIGURE 5-11

For a cycle $\Delta E = 0,$ thus $Q = W.$

The first law cannot be proven mathematically, but no process in nature is known to have violated the first law, and this should be taken as sufficient proof.

Energy balance for a constant-pressure expansion or compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

For a constant-pressure expansion or compression process:

$$\Delta U + W_b = \Delta H$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{net energy transfer} \\ \text{by heat,work,and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal,kinetic,} \\ \text{potential,etc.,energies}}}$$

$$Q - W_{\text{other}} - W_b = U_2 - U_1$$

$$Q - W = \Delta U + \Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0}$$

$$Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1$$

$$Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

$$H = U + PV$$

$$Q - W_{\text{other}} = H_2 - H_1$$

An example of constant-pressure process

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$
$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1)$$
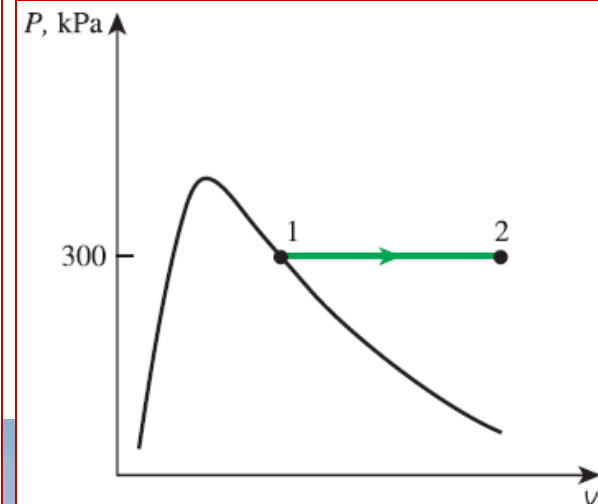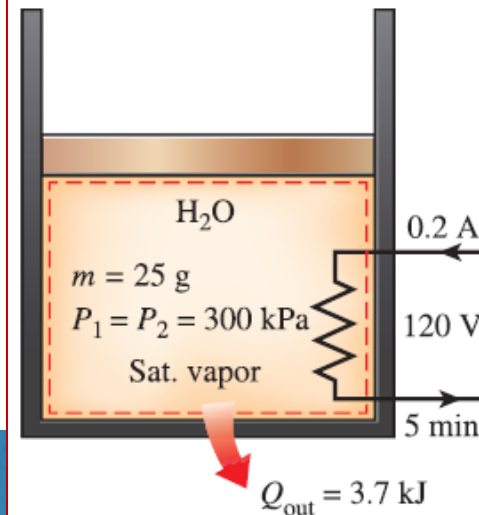
H_2O

0.2 A

$m = 25$ g

$P_1 = P_2 = 300$ kPa

120 V

Sat. vapor

5 min

$Q_{\text{out}} = 3.7$ kJ

P, kPa

300

1

2

V
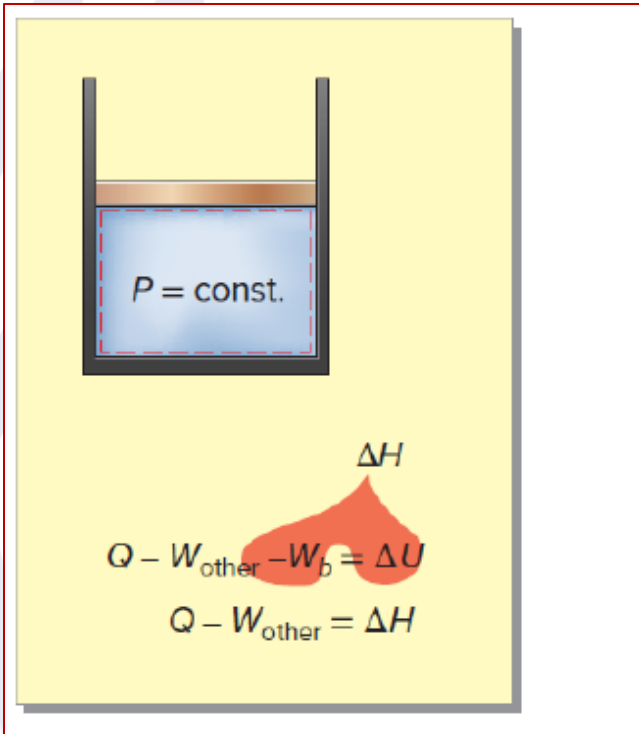
FIGURE 5-14

For a closed system undergoing a quasi-equilibrium, P = constant process, $\Delta U + W_b = \Delta H$. Note that this relation is NOT valid for closed systems processes during which pressure DOES NOT remain constant.
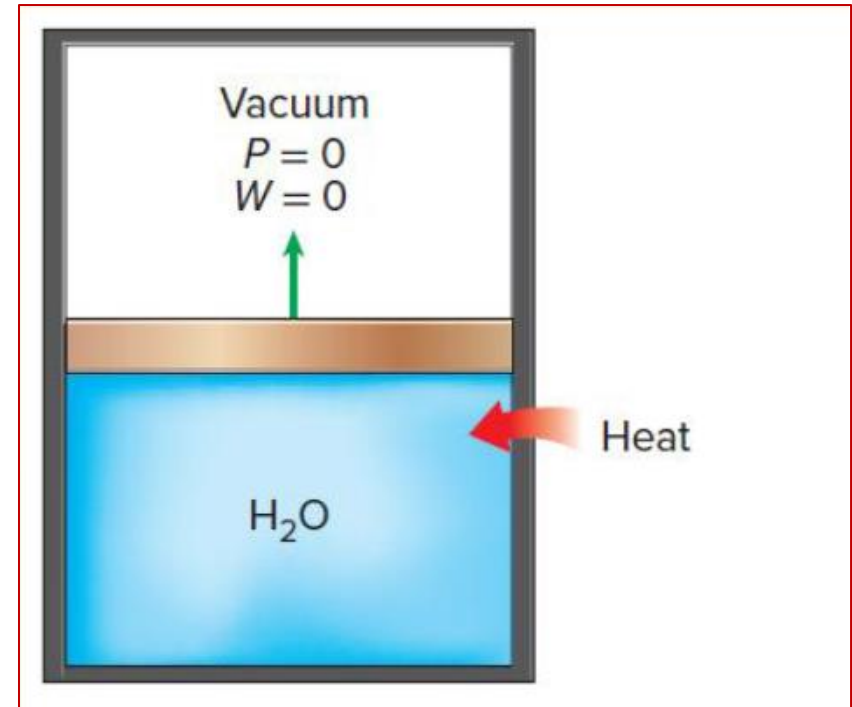
FIGURE 5-16

Expansion against a vacuum involves no work and thus no energy transfer.

For a closed system undergoing a **cycle,** the initial and final states are identical, and thus $\Delta E_{\text{system}} = E_2 - E_1 = 0$. Then, the energy balance for a cycle simplifies to $E_{\text{in}} - E_{\text{out}} = 0$ or $E_{\text{in}} = E_{\text{out}}$. Noting that a closed system does not involve any mass flow across its boundaries,

The **enthalpy** of a system is equal to the system's internal energy plus the product of its pressure and volume. For processes at constant pressure, the heat absorbed or released equals the change in **enthalpy**.

Enthalpy
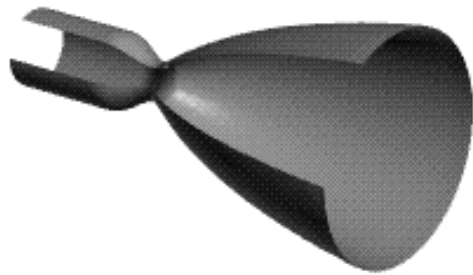
E = Internal Energy

T = Temperature

p = Pressure

V = Volume

Q = Heat Transfer

W = Work

C_p = Heat Capacity (constant pressure)

Define a new state variable that is a combination of other state variables.

$$\text{Enthalpy} = H = E + pV$$

$$\text{Specific Enthalpy} = \frac{\text{Enthalpy}}{\text{mass}} = h = e + pv$$

1st Law of Thermodynamics: $E_2 - E_1 = Q - W$

For a <u>constant pressure process</u>, the work is given: $W = p(V_2 - V_1)$

Substitute: $E_2 - E_1 = Q - p(V_2 - V_1)$

Re-group: $(E_2 + pV_2) - (E_1 + pV_1) = Q$

Heat Transfer at constant pressure: $Q = C_p(T_2 - T_1)$

Definition of Enthalpy: $(H_2 - H_1) = C_p(T_2 - T_1)$

(specific enthalpy) $(h_2 - h_1) = c_p(T_2 - T_1)$

EXAMPLE 5–5   Electric Heating of a Gas at Constant Pressure

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (a) Show that for a closed system the boundary work W_b and the change in internal energy ΔU in the first-law relation can be combined into one term, ΔH, for a constant-pressure process. (b) Determine the final temperature of the steam.

SOLUTION

Saturated water vapor in a piston–cylinder device expands at constant pressure as a result of heating. It is to be shown that $\Delta U + W_b = \Delta H$, and the final temperature is to be determined.

Assumptions

1 The tank is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$. Therefore, $\Delta E = \Delta U$ and internal energy is the only form of energy of the system that may change during this process. **2** Electrical wires constitute a very small part of the system, and thus the energy change of the wires can be neglected.

Analysis

We take the contents of the cylinder, including the resistance wires, as the *system* (**Fig. 5–13**). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work W_b. The pressure remains constant during the process and thus $P_2 = P_1$. Also, heat is lost from the system and electrical work W_e is done on the system.
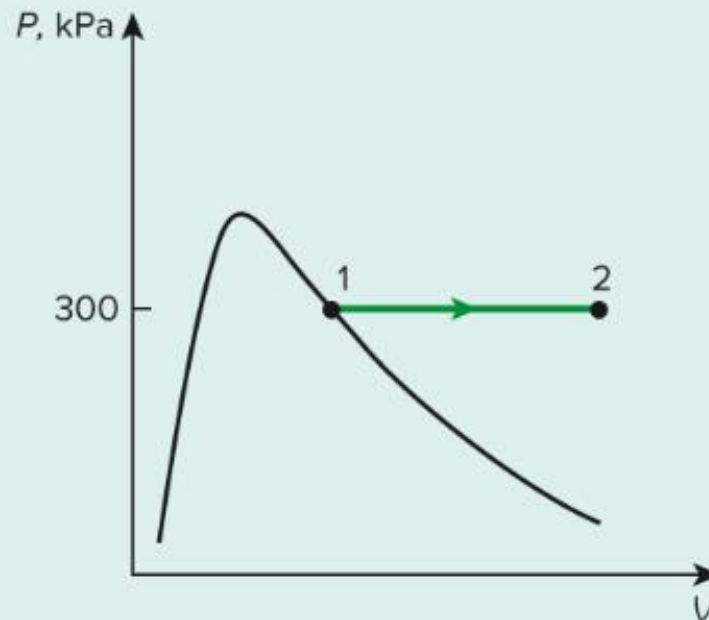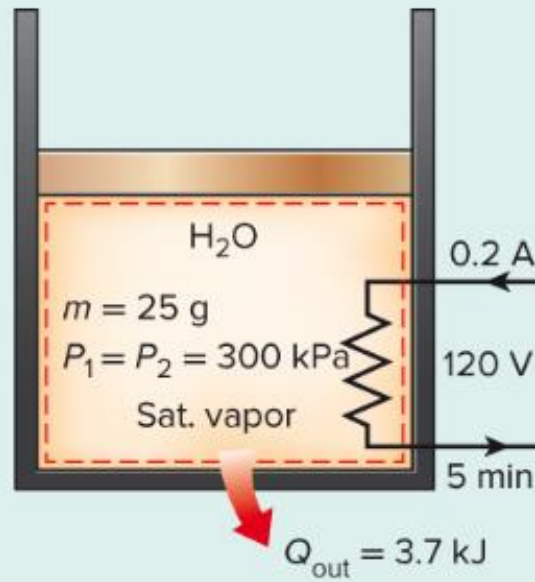
FIGURE 5–13
Schematic and P-V diagram for **Example 5–5**.

(a) This part of the solution involves a general analysis for a closed system undergoing a quasi-equilibrium constant-pressure process, and thus we consider a general closed system. We take the direction of heat transfer Q to be to the system and the work W to be done by the system.

FIGURE 5–14

For a closed system undergoing a quasi-equilibrium, P = constant process, $\Delta U + W_b = \Delta H$. Note that this relation is NOT valid for closed systems processes during which pressure DOES NOT remain constant.

(b) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = VI\,\Delta t = (120\text{ V})(0.2\text{ A})(300\text{ s})\left(\frac{1\text{ kJ/s}}{1000\text{ VA}}\right) = 7.2\text{ kJ}$$

$$\text{State 1:}\quad \left.\begin{array}{l} P_1 = 300\text{ kPa} \\ \text{sat. vapor} \end{array}\right\}\quad h_1 = h_{g\ @\ 300\text{ kPa}} = 2724.9\text{ kJ/kg} \quad\text{(Table A–5)}$$

constant-pressure quasi-equilibrium process since the boundary work is automatically taken care of by the enthalpy terms, and one no longer needs to determine it separately.

TABLE A–5

Saturated water—Pressure table

Press., P kPa	Sat. temp., T_{sat} °C	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
175	116.04	0.001057	1.0037	486.82	2037.7	2524.5	487.01	2213.1	2700.2	1.4850	5.6865	7.1716
200	120.21	0.001061	0.88578	504.50	2024.6	2529.1	504.71	2201.6	2706.3	1.5302	5.5968	7.1270
225	123.97	0.001064	0.79329	520.47	2012.7	2533.2	520.71	2191.0	2711.7	1.5706	5.5171	7.0877
250	127.41	0.001067	0.71873	535.08	2001.8	2536.8	535.35	2181.2	2716.5	1.6072	5.4453	7.0525
275	130.58	0.001070	0.65732	548.57	1991.6	2540.1	548.86	2172.0	2720.9	1.6408	5.3800	7.0207
300	133.52	0.001073	0.60582	561.11	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3200	6.9917
325	136.27	0.001076	0.56199	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2645	6.9650
350	138.86	0.001079	0.52422	583.89	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2128	6.9402

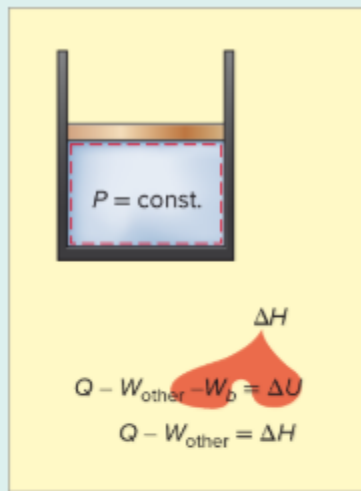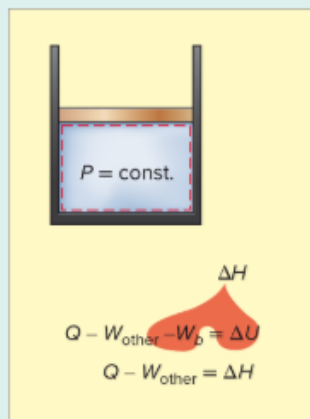**FIGURE 5–14**

For a closed system undergoing a quasi-equilibrium, P = constant process, $\Delta U + W_b = \Delta H$. Note that this relation is NOT valid for closed systems processes during which pressure DOES NOT remain constant.

(b) The only other form of work in this case is the electrical work, which can be determined from

$$W_e = VI\,\Delta t = (120\ \text{V})(0.2\ \text{A})(300\ \text{s})\left(\frac{1\ \text{kJ/s}}{1000\ \text{VA}}\right) = 7.2\ \text{kJ}$$

State 1: $\left.\begin{array}{l} P_1 = 300\ \text{kPa} \\ \text{sat. vapor} \end{array}\right\}$ $\quad h_1 = h_{g\ @\ 300\ \text{kPa}} = 2724.9\ \text{kJ/kg}$ (Table A–5)

The enthalpy at the final state can be determined directly from **Eq. 5–18** by expressing heat transfer from the system and work done on the system as negative quantities (since their directions are opposite to the assumed directions). Alternately, we can use the general energy balance relation with the simplification that the boundary work is considered automatically by replacing ΔU by ΔH for a constant-pressure expansion or compression process:

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - Q_{\text{out}} - W_b = \Delta U$$
$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) \qquad (\text{since } P = \text{constant})$$
$$7.2\ \text{kJ} - 3.7\ \text{kJ} = (0.025\ \text{kg})(h_2 - 2724.9)\ \text{kJ/kg}$$
$$h_2 = 2864.9\ \text{kJ/kg}$$

Now the final state is completely specified since we know both the pressure and the enthalpy. The temperature at this state is

State 2: $\left.\begin{array}{l} P_2 = 300\ \text{kPa} \\ h_2 = 2864.9\ \text{kJ/kg} \end{array}\right\}$ $\quad T_2 = \textbf{200°C}$ (Table A–6)

Therefore, the steam will be at 200°C at the end of this process.

Discussion

Strictly speaking, the potential energy change of the steam is not zero for this process since the center of gravity of the steam rose somewhat. Assuming an elevation change of 1 m (which is rather unlikely), the change in the potential energy of the steam would be 0.0002 kJ, which is very small compared to the other terms in the first-law relation. Therefore, in problems of this kind, the potential energy term is always neglected.
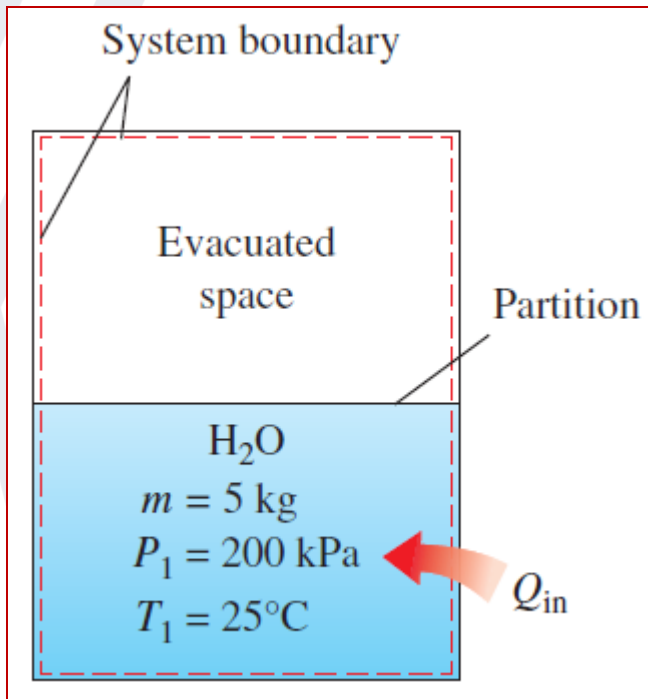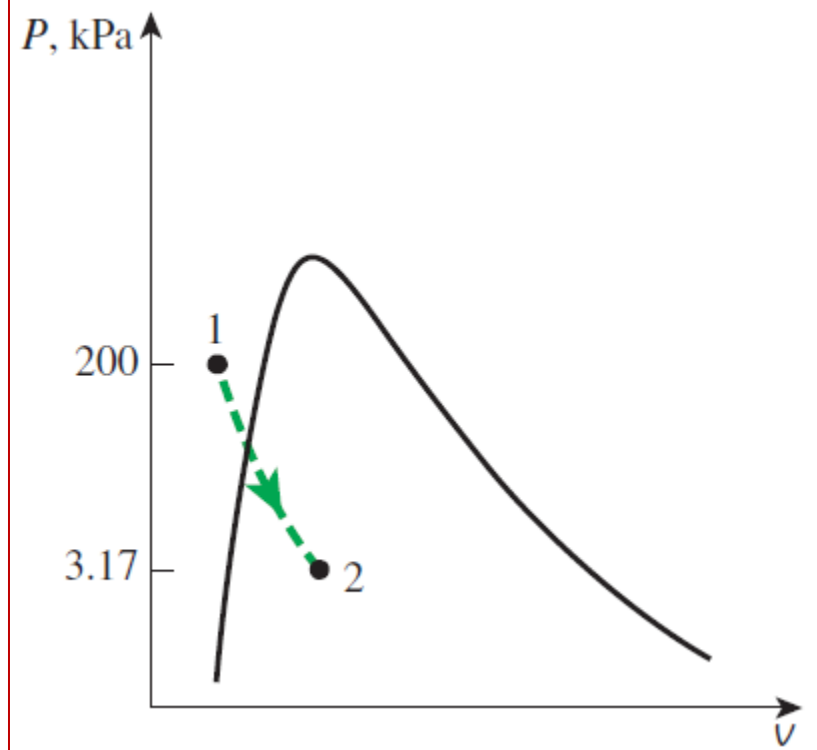
TABLE A-6

Superheated water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	P = 0.01 MPa (45.81°C)*				P = 0.05 MPa (81.32°C)				P = 0.10 MPa (99.61°C)			
Sat.†	14.670	2437.2	2583.9	8.1488	3.2403	2483.2	2645.2	7.5931	1.6941	2505.6	2675.0	7.3589
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.4	7.6953	1.6959	2506.2	2675.8	7.3611
150	19.513	2587.9	2783.0	8.6893	3.8897	2585.7	2780.2	7.9413	1.9367	2582.9	2776.6	7.6148
200	21.826	2661.4	2879.6	8.9049	4.3562	2660.0	2877.8	8.1592	2.1724	2658.2	2875.5	7.8356
250	24.136	2736.1	2977.5	9.1015	4.8206	2735.1	2976.2	8.3568	2.4062	2733.9	2974.5	8.0346
300	26.446	2812.3	3076.7	9.2827	5.2841	2811.6	3075.8	8.5387	2.6389	2810.7	3074.5	8.2172
400	31.063	2969.3	3280.0	9.6094	6.2094	2968.9	3279.3	8.8659	3.1027	2968.3	3278.6	8.5452
500	35.680	3132.9	3489.7	9.8998	7.1338	3132.6	3489.3	9.1566	3.5655	3132.2	3488.7	8.8362
600	40.296	3303.3	3706.3	10.1631	8.0577	3303.1	3706.0	9.4201	4.0279	3302.8	3705.6	9.0999
700	44.911	3480.8	3929.9	10.4056	8.9813	3480.6	3929.7	9.6626	4.4900	3480.4	3929.4	9.3424
800	49.527	3665.4	4160.6	10.6312	9.9047	3665.2	4160.4	9.8883	4.9519	3665.0	4160.2	9.5682
900	54.143	3856.9	4398.3	10.8429	10.8280	3856.8	4398.2	10.1000	5.4137	3856.7	4398.0	9.7800
1000	58.758	4055.3	4642.8	11.0429	11.7513	4055.2	4642.7	10.3000	5.8755	4055.0	4642.6	9.9800
1100	63.373	4260.0	4893.8	11.2326	12.6745	4259.9	4893.7	10.4897	6.3372	4259.8	4893.6	10.1698
1200	67.989	4470.9	5150.8	11.4132	13.5977	4470.8	5150.7	10.6704	6.7988	4470.7	5150.6	10.3504
1300	72.604	4687.4	5413.4	11.5857	14.5209	4687.3	5413.3	10.8429	7.2605	4687.2	5413.3	10.5229
	P = 0.20 MPa (120.21°C)				P = 0.30 MPa (133.52°C)				P = 0.40 MPa (143.61°C)			
Sat.	0.88578	2529.1	2706.3	7.1270	0.60582	2543.2	2724.9	6.9917	0.46242	2553.1	2738.1	6.8955
150	0.95986	2577.1	2769.1	7.2810	0.63402	2571.0	2761.2	7.0792	0.47088	2564.4	2752.8	6.9306
200	1.08049	2654.6	2870.7	7.5081	0.71643	2651.0	2865.9	7.3132	0.53434	2647.2	2860.9	7.1723
250	1.19890	2731.4	2971.2	7.7100	0.79645	2728.9	2967.9	7.5180	0.59520	2726.4	2964.5	7.3804
300	1.31623	2808.8	3072.1	7.8941	0.87535	2807.0	3069.6	7.7037	0.65489	2805.1	3067.1	7.5677
400	1.54934	2967.2	3277.0	8.2236	1.03155	2966.0	3275.5	8.0347	0.77265	2964.9	3273.9	7.9003
500	1.78142	3131.4	3487.7	8.5153	1.18672	3130.6	3486.6	8.3271	0.88936	3129.8	3485.5	8.1933
600	2.01302	3302.2	3704.8	8.7793	1.34139	3301.6	3704.0	8.5915	1.00558	3301.0	3703.3	8.4580
700	2.24434	3479.9	3928.8	9.0221	1.49580	3479.5	3928.2	8.8345	1.12152	3479.0	3927.6	8.7012
800	2.47550	3664.7	4159.8	9.2479	1.65004	3664.3	4159.3	9.0605	1.23730	3663.9	4158.9	8.9274
900	2.70656	3856.3	4397.7	9.4598	1.80417	3856.0	4397.3	9.2725	1.35298	3855.7	4396.9	9.1394
1000	2.93755	4054.8	4642.3	9.6599	1.95824	4054.5	4642.0	9.4726	1.46859	4054.3	4641.7	9.3396
1100	3.16848	4259.6	4893.3	9.8497	2.11226	4259.4	4893.1	9.6624	1.58414	4259.2	4892.9	9.5295
1200	3.39938	4470.5	5150.4	10.0304	2.26624	4470.3	5150.2	9.8431	1.69966	4470.2	5150.0	9.7102
1300	3.63026	4687.1	5413.1	10.2029	2.42019	4686.9	5413.0	10.0157	1.81516	4686.7	5412.8	9.8828

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat,work,and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal,kinetic,} \\ \text{potential,etc.,enrgies}}}$$

$$Q_{\text{in}} = \Delta U = m(u_2 - u_1)$$

Unrestrained Expansion of Water

System boundary

Evacuated space

Partition

H_2O
$m = 5$ kg
$P_1 = 200$ kPa
$T_1 = 25°C$

Q_{in}

P, kPa

200 — 1

3.17 — 2

v

EXAMPLE 5–6   Unrestrained Expansion of Water

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (*a*) the volume of the tank, (*b*) the final pressure, and (*c*) the heat transfer for this process.

SOLUTION

One half of a rigid tank is filled with liquid water while the other side is evacuated. The partition between the two parts is removed and water is allowed to expand and fill the entire tank while the temperature is maintained constant. The volume of the tank, the final pressure, and the heat transfer are to be to determined.

Assumptions

1 The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **2** The direction of heat transfer is to the system (heat gain, Q_{in}). A negative result for Q_{in} indicates the assumed direction is wrong and thus it is a heat loss. **3** The volume of the rigid tank is constant, and thus there is no energy transfer as boundary work. **4** There is no electrical, shaft, or any other kind of work involved.

Analysis

We take the contents of the tank, including the evacuated space, as the *system* (**Fig. 5–15**). This is a *closed system* since no mass crosses the system boundary during the process. We observe that the water fills the entire tank when the partition is removed (possibly as a liquid–vapor mixture).
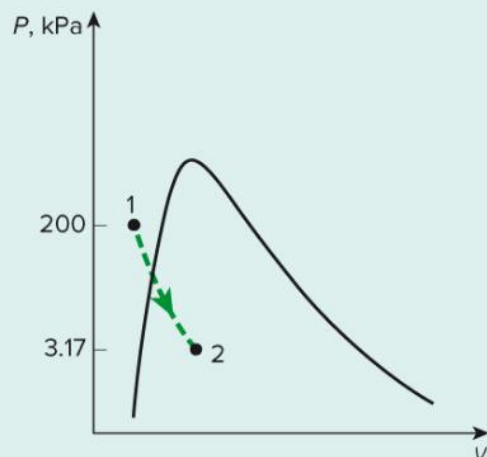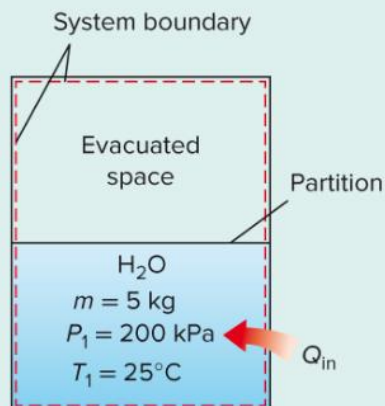
FIGURE 5–15
Schematic and P-v diagram for **Example 5–6**.

(a) Initially the water in the tank exists as a compressed liquid since its pressure (200 kPa) is greater than the saturation pressure at 25°C (3.1698 kPa). Approximating the compressed liquid as a saturated liquid at the given temperature, we find

$$v_1 \cong v_{f @ 25°C} = 0.001003 \text{ m}^3/\text{kg} \cong 0.001 \text{ m}^3/\text{kg} \qquad \text{(Table A–4)}$$

Then the initial volume of the water is

$$V_1 = mv_1 = (5 \text{ kg})(0.001 \text{ m}^3/\text{kg}) = 0.005 \text{ m}^3$$

The total volume of the tank is twice this amount:

$$V_{\text{tank}} = (2)(0.005 \text{ m}^3) = \mathbf{0.01 \ m^3}$$

(b) At the final state, the specific volume of the water is

$$v_2 = \frac{V_2}{m} = \frac{0.01 \text{ m}^3}{5 \text{ kg}} = 0.002 \text{ m}^3/\text{kg}$$

which is twice the initial value of the specific volume. This result is expected since the volume doubles while the amount of mass remains constant.

At 25°C: $v_f = 0.001003 \text{ m}^3/\text{kg}$ and $v_g = 43.340 \text{ m}^3/\text{kg}$ (Table A–4)

Since $v_f < v_2 < v_g$, the water is a saturated liquid–vapor mixture at the final state, and thus the pressure is the saturation pressure at 25°C:

$$P_2 = P_{\text{sat @ 25°C}} = \mathbf{3.1698 \ kPa} \qquad \text{(Table A–4)}$$

(c) Under stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

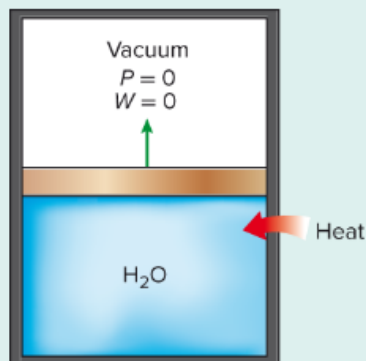$$Q_{\text{in}} = \Delta U = m(u_2 - u_1)$$

TABLE A–4

Saturated water—Temperature table

Temp., T °C	Sat. press., P_{sat} kPa	Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.416	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.868	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.183	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.609	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
105	120.90	0.001047	1.4186	440.15	2071.8	2511.9	440.28	2243.1	2683.4	1.3634	5.9319	7.2952
110	143.38	0.001052	1.2094	461.27	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193	7.2382
115	169.18	0.001056	1.0360	482.42	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7092	7.1829
120	198.67	0.001060	0.89133	503.60	2025.3	2528.9	503.81	2202.1	2706.0	1.5279	5.6013	7.1292
125	232.23	0.001065	0.77012	524.83	2009.5	2534.3	525.07	2188.1	2713.1	1.5816	5.4956	7.0771
130	270.28	0.001070	0.66808	546.10	1993.4	2539.5	546.38	2173.7	2720.1	1.6346	5.3919	7.0265
135	313.22	0.001075	0.58179	567.41	1977.3	2544.7	567.75	2159.1	2726.9	1.6872	5.2901	6.9773
140	361.53	0.001080	0.50850	588.77	1960.9	2549.6	589.16	2144.3	2733.5	1.7392	5.1901	6.9294
145	415.68	0.001085	0.44600	610.19	1944.2	2554.4	610.64	2129.2	2739.8	1.7908	5.0919	6.8827
150	476.16	0.001091	0.39248	631.66	1927.4	2559.1	632.18	2113.8	2745.9	1.8418	4.9953	6.8371
155	543.49	0.001096	0.34648	653.19	1910.3	2563.5	653.79	2098.0	2751.8	1.8924	4.9002	6.7927
160	618.23	0.001102	0.30680	674.79	1893.0	2567.8	675.47	2082.0	2757.5	1.9426	4.8066	6.7492
165	700.93	0.001108	0.27244	696.46	1875.4	2571.9	697.24	2065.6	2762.8	1.9923	4.7143	6.7067
170	792.18	0.001114	0.24260	718.20	1857.5	2575.7	719.08	2048.8	2767.9	2.0417	4.6233	6.6650
175	892.60	0.001121	0.21659	740.02	1839.4	2579.4	741.02	2031.7	2772.7	2.0906	4.5335	6.6242
180	1002.8	0.001127	0.19384	761.92	1820.9	2582.8	763.05	2014.2	2777.2	2.1392	4.4448	6.5841
185	1123.5	0.001134	0.17390	783.91	1802.1	2586.0	785.19	1996.2	2781.4	2.1875	4.3572	6.5447
190	1255.2	0.001141	0.15636	806.00	1783.0	2589.0	807.43	1977.9	2785.3	2.2355	4.2705	6.5059
195	1398.8	0.001149	0.14089	828.18	1763.6	2591.7	829.78	1959.0	2788.8	2.2831	4.1847	6.4678
200	1554.9	0.001157	0.12721	850.46	1743.7	2594.2	852.26	1939.8	2792.0	2.3305	4.0997	6.4302

Notice that even though the water is expanding during this process, the system chosen involves fixed boundaries only (the dashed lines) and therefore the moving boundary work is zero (**Fig. 5–16**). Then $W = 0$ since the system does not involve any other forms of work. (Can you reach the same conclusion by choosing the water as our system?) Initially,

$$u_1 \cong u_{f@\,25°C} = 104.83 \text{ kJ/kg}$$

Vacuum
$P = 0$
$W = 0$

Heat

H_2O

FIGURE 5–16
Expansion against a vacuum involves no work and thus no energy transfer.

The quality at the final state is determined from the specific volume information:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.002 - 0.001}{43.34 - 0.001} = 2.3 \times 10^{-5}$$
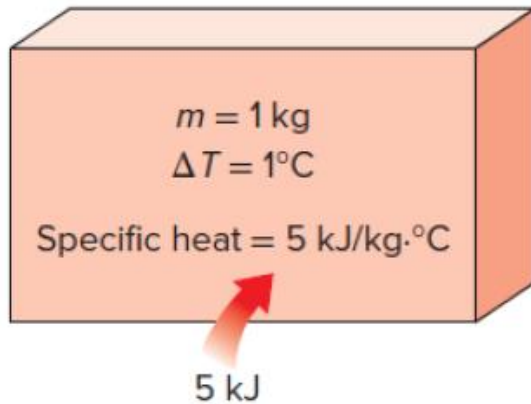
Then

$$u_2 = u_f + x_2 u_{fg}$$

$$= 104.83 \text{ kJ/kg} + (2.3 \times 10^{-5})(2304.3 \text{ kJ/kg})$$

$$= 104.88 \text{ kJ/kg}$$

Substituting yields

$$Q_{in} = (5 \text{ kg})[(104.88 - 104.83) \text{ kJ/ kg}] = \mathbf{0.25 \text{ kJ}}$$

Discussion

The positive sign indicates that the assumed direction is correct, and heat is transferred to the water.
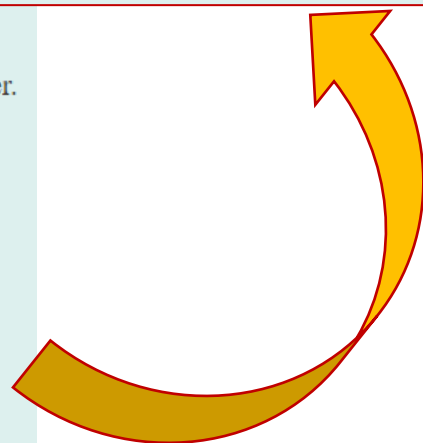
5-3 SPECIFIC HEATS

Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

1 kg Iron 20 → 30°C 4.5 kJ

1 kg Water 20 → 30°C 41.8 kJ

FIGURE 5-17

It takes different amounts of energy to raise the temperature of different substances by the same amount.

$m = 1\,kg$
$\Delta T = 1°C$

Specific heat = 5 kJ/kg·°C

5 kJ

FIGURE 5-18

Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).

$V = \text{constant}$
$m = 1\,kg$
$\Delta T = 1°C$
$c_v = 3.12\ \dfrac{kJ}{kg \cdot °C}$

3.12 kJ

$P = \text{constant}$
$m = 1\,kg$
$\Delta T = 1°C$
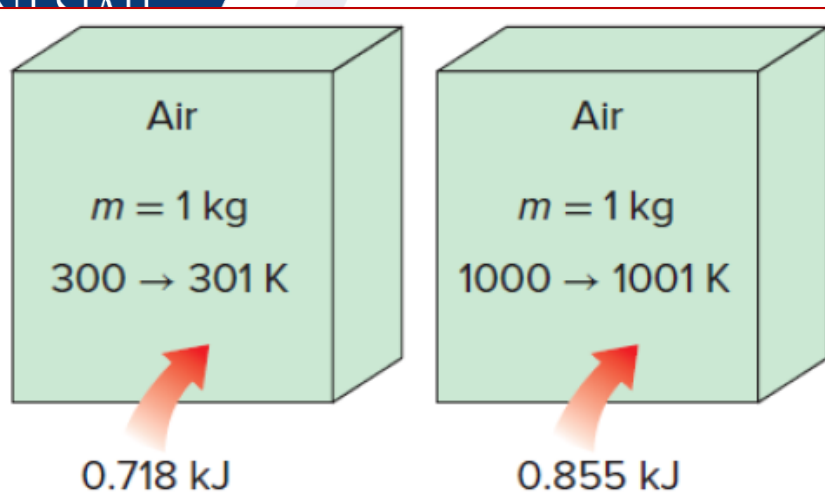$c_p = 5.19\ \dfrac{kJ}{kg \cdot °C}$

5.19 kJ

(2)
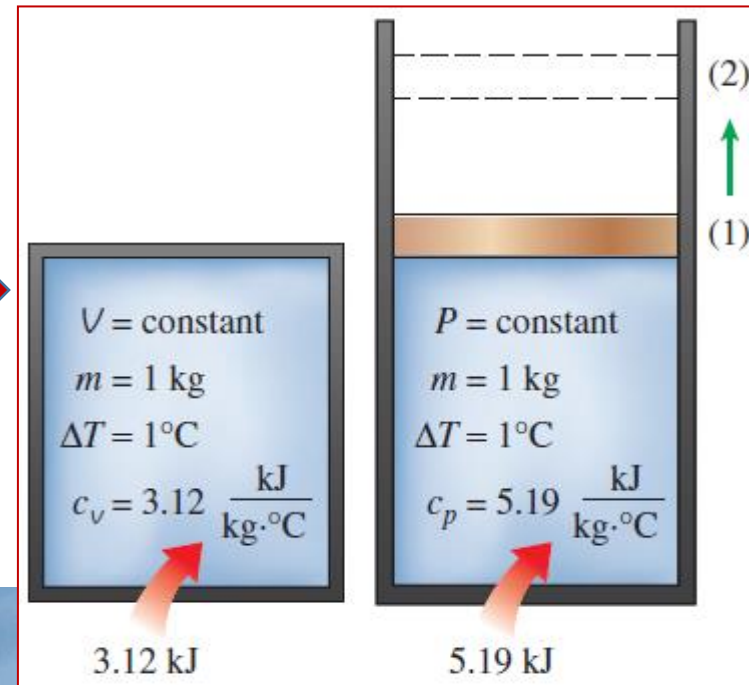
(1)

FIGURE 5-21

The specific heat of substance change with temperature.

- **The equations are valid for *any* substance undergoing *any* process.**
- **c_v and c_p are properties.**
- **c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.**
- **A common unit for specific heats is kJ/kg·°C or kJ/kg·K. Are these units identical?**

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

= the change in internal energy with temperature at constant volume

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

Formal definitions of c_v and c_p.

The heat capacity at constant pressure **CP** is **greater than** the heat capacity at constant volume **CV** , because when heat is added at constant pressure, the substance expands.

QV = **CV** $\triangle$T = $\triangle$U + W = $\triangle$U because no work is done.

Therefore, dU = **CV** dT and **CV** = dU/dT .

Joule Experiment: he reasoned that the internal energy is a function of temperature only and not a function of pressure or specific volume.
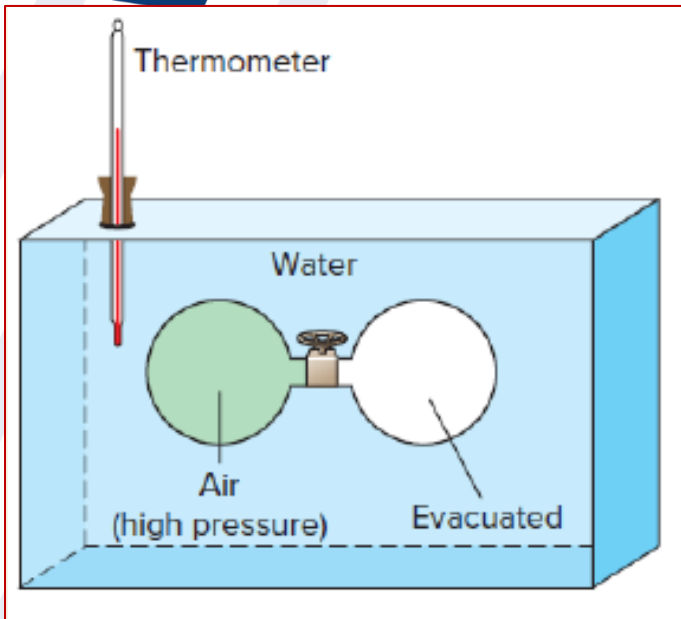
FIGURE 5-22
Schematic of the experimental apparatus used by Joule.

$$h = u + Pv \left.\vphantom{\begin{matrix}a\\b\end{matrix}}\right\} h = u + RT$$
$$Pv = RT$$

$$u = u(T) \qquad h = h(T)$$

$$du = C_v(T)\, dT \quad dh = C_p(T)\, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 C_v(T)\, dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_p(T)\, dT$$

Joule showed using this experimental apparatus that $u = u(T)$

$$u = u(T)$$
$$h = h(T)$$
$$c_v = c_v(T)$$
$$c_p = c_p(T)$$

For ideal gases, u, h, c_v, and c_p vary with temperature only.

Internal energy and enthalpy change of an ideal gas

# Joule's Experiment and First law of thermodynamics | noncyclic process | cyclic process

https://www.youtube.com/watch?v=Rcz_w4-hgVU

Watch this educational video

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.

- *u* and *h* data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.
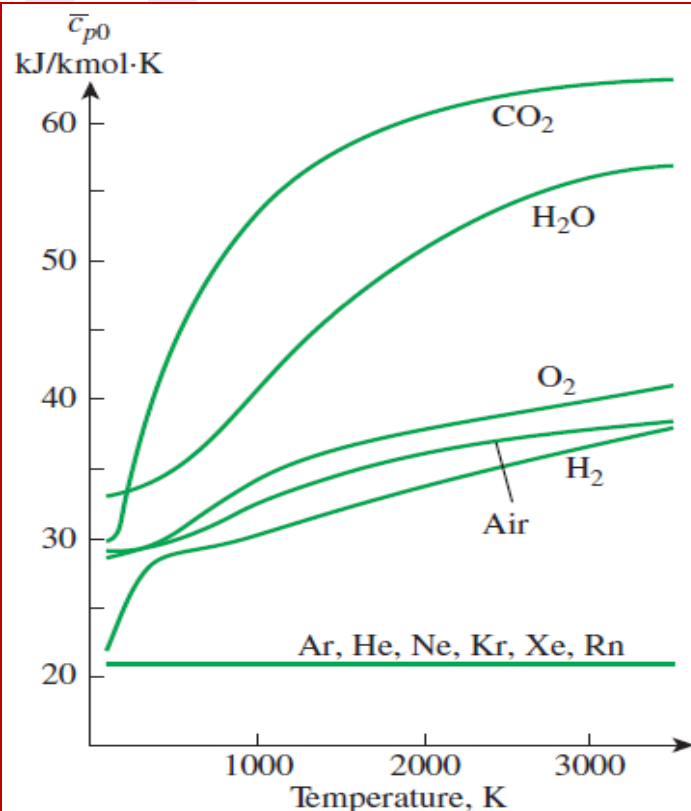
Real gas: Real gases are defined as the gases that do not obey gas laws at all standard pressure and temperature

Ideal-gas constant-pressure specific heats for some gases (see Table A–2c for c_p equations).

Air		
T, K	u, kJ/kg	h, kJ/kg
0	0	0
.	.	.
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24

In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Internal energy and enthalpy change when specific heat is taken constant at an average value

$$u_2 - u_1 = c_{v,\text{avg}}(T_2 - T_1)$$ **(kJ/kg)**
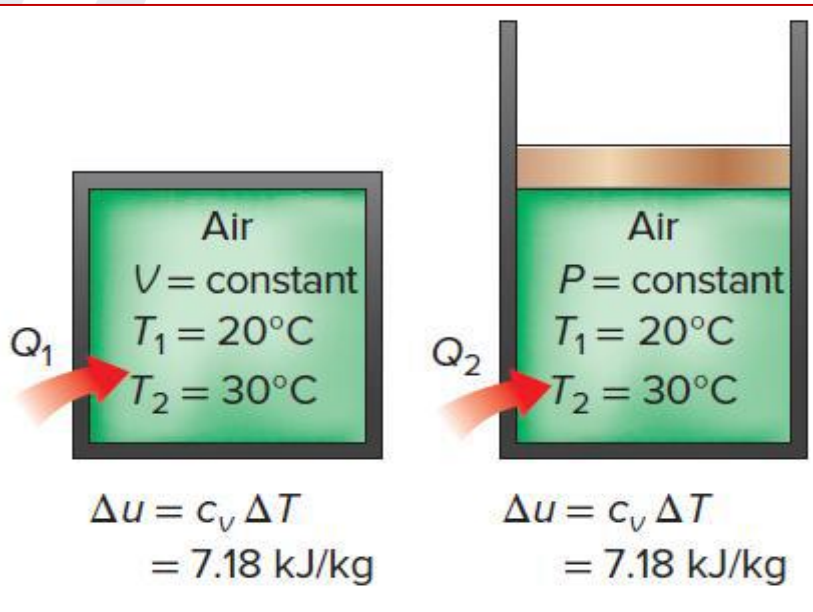
$$h_2 - h_1 = c_{p,\text{avg}}(T_2 - T_1)$$

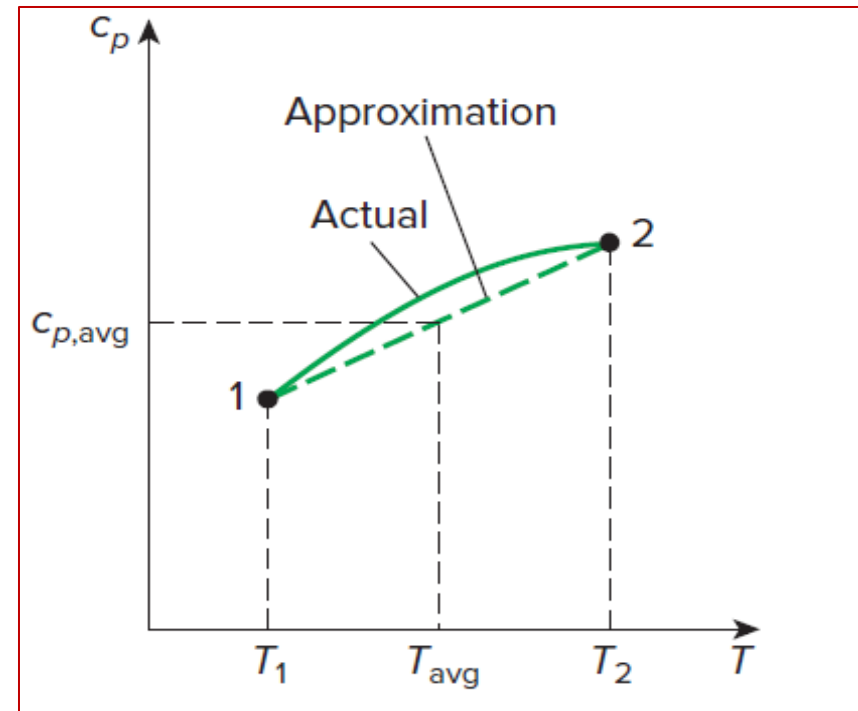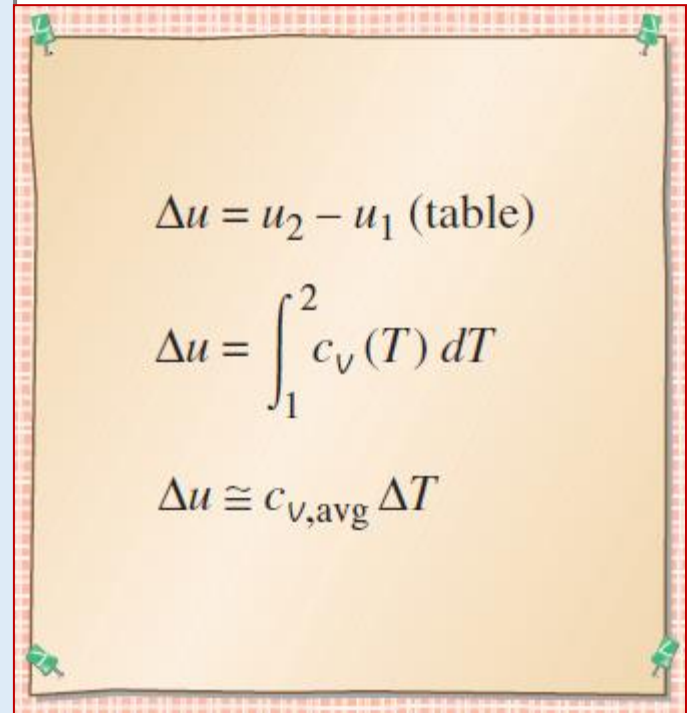**FIGURE 5-26**

For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

Air
$V = \text{constant}$
$T_1 = 20°C$
$T_2 = 30°C$

Q_1

$\Delta u = c_v \, \Delta T$
$= 7.18 \text{ kJ/kg}$

Air
$P = \text{constant}$
$T_1 = 20°C$
$T_2 = 30°C$

Q_2

$\Delta u = c_v \, \Delta T$
$= 7.18 \text{ kJ/kg}$

FIGURE 5-27

The relation $\Delta u = c_v \Delta T$ is valid for _any_ kind of process, constant-volume or not.

1. By using the tabulated u and h data. This is the easiest and most accurate way when tables are readily available.

2. By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are very accurate.

3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are reasonably accurate if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T)\, dT$$

$$\Delta u \cong c_{v,\text{avg}} \Delta T$$

Three ways of calculating Δu.

Specific Heat Relations of Ideal Gases

$$h = u + RT$$

$$\boldsymbol{d}h = du + R\,dT$$

$$dh = c_p\,dT \text{ and } du = c_v\,dT$$

The relationship between c_p, c_v and R

$$c_p = c_v + R \qquad (\text{kJ/kg} \cdot \text{K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\text{kJ/kmol} \cdot \text{K})$$

$$k = \frac{c_p}{c_v} \quad \textbf{Specific heat ratio}$$

Air at 300 K

$$c_v = 0.718 \text{ kJ/kg·K}$$
$$R = 0.287 \text{ kJ/kg·K} \Big\} \; c_p = 1.005 \text{ kJ/kg·K}$$

or

$$\bar{c}_v = 20.80 \text{ kJ/kmol·K}$$
$$R_u = 8.314 \text{ kJ/kmol·K} \Big\} \; \bar{c}_p = 29.114 \text{ kJ/kmol·K}$$

- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

The c_p of an ideal gas can be determined from a knowledge of c_v and R.

Incompressible substance: A substance whose specific volume (or density) is constant. **Solids and liquids** are incompressible substances.
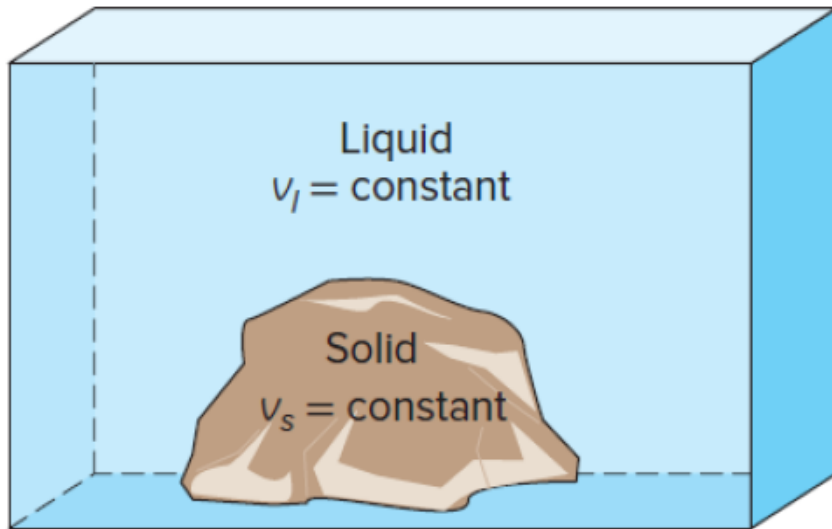

FIGURE 5-33
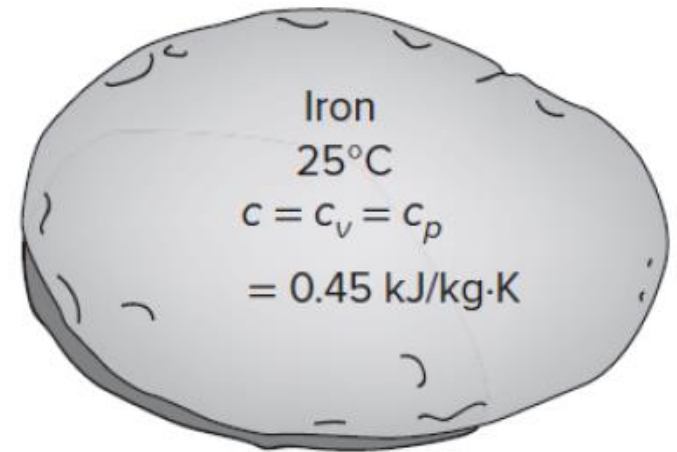The specific volumes of incompressible substances remain constant during the process.

FIGURE 5-34
The c_v and c_p **values of incompressible substances are identical and are denoted by c.**

Internal Energy Changes

$$du = c_v \, dT = c(T) \, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) \, dT \qquad \text{(kJ/kg)}$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad \text{(kJ/kg)}$$

Enthalpy Changes

$$h = u + Pv$$

$$dh = du + v\,dP + P\,dv^{\nearrow 0} = du + v\,dP$$

$$\Delta h = \Delta u + v\Delta P \cong c_{\text{avg}}\Delta T + v\Delta P \qquad \text{(kJ/kg)}$$

A substance whose specific volume (or density) is constant is called an **incompressible substance.**

For $solids$, the term $v\Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$. For $liquids$, two special cases are commonly encounterd:

1. $Constant-pressure\ processes$, as in heaters $(\Delta P = 0)$: $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$
2. $Constant-temperature\ processes$, as in pumps $(\Delta T = 0)$: $\Delta h = v\Delta P$

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{\text{sat}@T})$$

The enthalpy of a compressed liquid

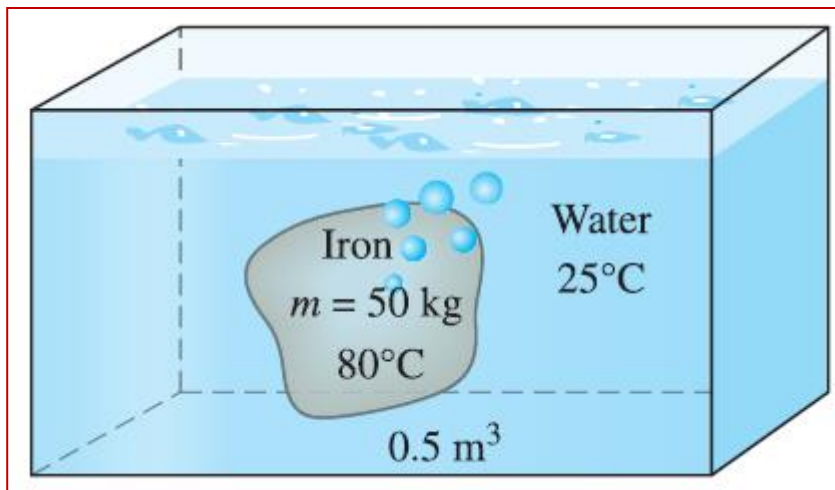**Usually, a more accurate relation than** $h_{@P,T} \cong h_{f@T}$

Cooling of an Iron Block by Water

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat,work,and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal,kinetic,} \\ \text{potential,etc.,energies}}}$$

$$0 = \Delta U$$

$$\Delta U_{\text{sys}} = \Delta U_{\text{iron}} + \Delta U_{\text{water}} = 0$$

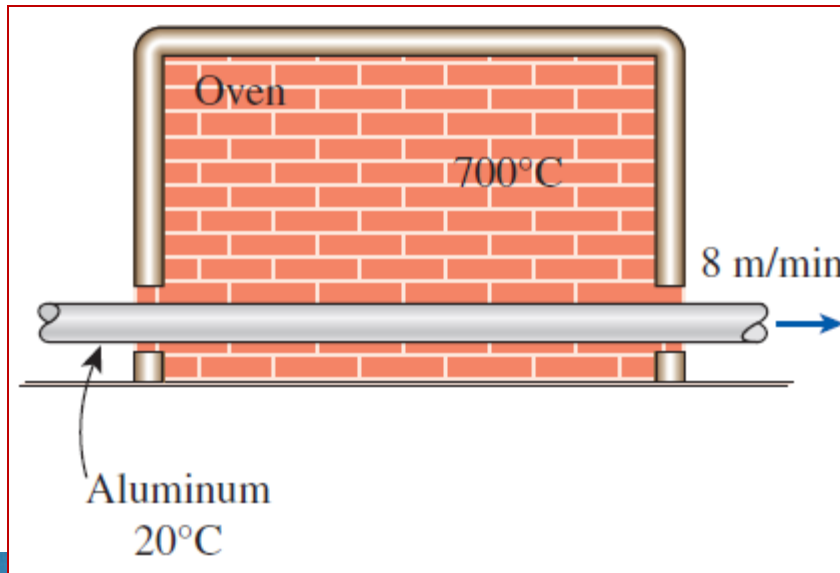$$[mc(T_2 - T_1)]_{\text{iron}} + [mc(T_2 - T_1)]_{\text{water}} = 0$$

Iron
$m = 50$ kg
$80°C$

Water
$25°C$

0.5 m^3

Heating of Aluminum Rods in a Furnace

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat,work,and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal,kinetic,} \\ \text{potential,etc.,energies}}}$$

$$Q_{in} = \Delta U_{rod} = m(u_2 - u_1)$$
$$Q_{in} = mc(T_2 - T_1)$$

$$\dot{Q}_{in} = Q_{in}/\Delta t$$

Oven

700°C

8 m/min

Aluminum
20°C

Air

$V_1 = 400$ L
$P_1 = 150$ kPa
$T_1 = 27°C$

Q

P, kPa

2   3
350 ┄┄┄●━━━━━━━▶●

A

1
150 ┄┄┄●

0.4   0.8   V, m³

FIGURE 5–32
Schematic and P-V diagram for
Example 5–10.

EXAMPLE 5–10   Heating of a Gas at Constant Pressure

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 5–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (a) the final temperature, (b) the work done by the air, and (c) the total heat transferred to the air.

SOLUTION   Air in a piston–cylinder device with a set of stops is heated until its volume is doubled. The final temperature, work done, and the total heat transfer are to be determined.

Assumptions   1 Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. 2 The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. 3 The volume remains constant until the piston starts moving, and the pressure remains constant afterwards. 4 There are no electrical, shaft, or other forms of work involved.

Analysis   We take the contents of the cylinder as the *system* (Fig. 5–32). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work, W_b. Also, the boundary work is done by the system, and heat is transferred to the system.

(a) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \longrightarrow \frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_3}$$
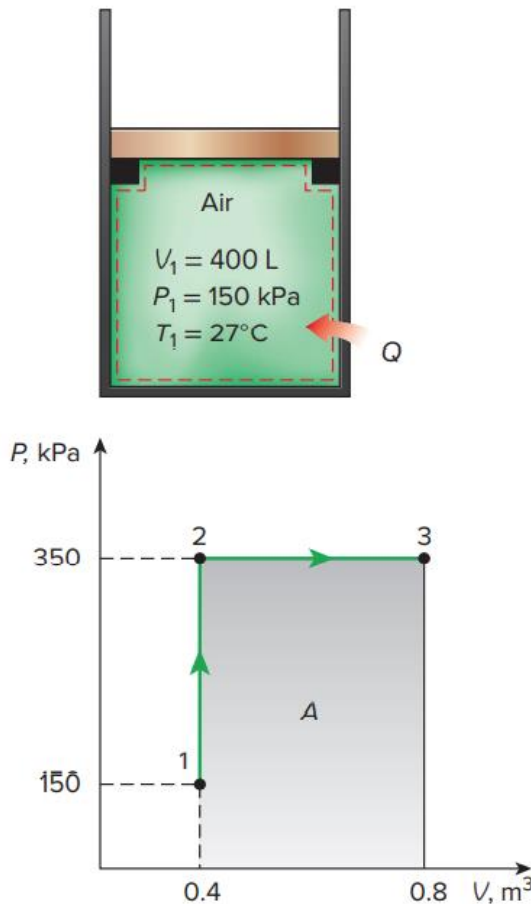
$$T_3 = \textbf{1400 K}$$

EXAMPLE 5–10

(b) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a P-V diagram, shown in Fig. 5–32:

$$A = (V_2 - V_1)P_2 = (0.4 \text{ m}^3)(350 \text{ kPa}) = 140 \text{ m}^3 \cdot \text{kPa}$$

Therefore,

$$W_{13} = \textbf{140 kJ}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

(c) Under the stated assumptions and observations, the energy balance on the system between the initial and final states (process 1–3) can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_3 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.4 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 0.697 \text{ kg}$$

The internal energies are determined from the air table (Table A–21) to be

$$u_1 = u_{@ 300 \text{ K}} = 214.07 \text{ kJ/kg}$$

$$u_3 = u_{@ 1400 \text{ K}} = 1113.52 \text{ kJ/kg}$$

Thus,

$$Q_{\text{in}} - 140 \text{ kJ} = (0.697 \text{ kg})[(1113.52 - 214.07) \text{ kJ/kg}]$$

$$Q_{\text{in}} = \textbf{767 kJ}$$

EXAMPLE 5–11   Enthalpy of Compressed Liquid

Determine the enthalpy of liquid water at 100°C and 15 MPa (*a*) by using compressed liquid tables, (*b*) by approximating it as a saturated liquid, and (*c*) by using the correction given by **Eq. 5–38**.

SOLUTION

The enthalpy of liquid water is to be determined exactly and approximately. the enthalpy of the compressed liquid can be expressed as

Analysis

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{sat@T}) \tag{5–38}$$

At 100°C, the saturation pressure of water is 101.42 kPa, and since $P > P_{sat}$, the water exists as a compressed liquid at the specified state.

(*a*) From compressed liquid tables, we read

$$\left. \begin{array}{l} P = 15 \text{ MPa} \\ T = 100°C \end{array} \right\} \quad h = \textbf{430.39 kJ/kg} \qquad \text{(Table A–7)}$$

This is the exact value.

(*b*) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain

$$h \cong h_{f@100°C} = \textbf{419.17 kJ/kg}$$

This value is in error by about 2.6 percent.

(*c*) From **Eq. 5–38**,

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{sat@T})$$
$$= (419.17 \text{ kJ/kg}) + (0.001 \text{ m}^3/\text{kg})[(15,000 - 101.42) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa·m}^3}\right)$$
$$= \textbf{434.07 kJ/kg}$$

Discussion

Note that the correction term reduced the error from 2.6 to about 1 percent in this case. However, this improvement in accuracy is often not worth the extra effort involved.

EXAMPLE 5–12 Cooling of an Iron Block by Water

A 50-kg iron block at 80°C is dropped into an insulated tank that contains 0.5 m^3 of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.

SOLUTION

An iron block is dropped into water in an insulated tank. The final temperature when thermal equilibrium is reached is to be determined.

Assumptions

1 Both water and the iron block are incompressible substances. **2** Constant specific heats at room temperature can be used for water and the iron. **3** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **4** There are no electrical, shaft, or other forms of work involved. **5** The system is well-insulated and thus there is no heat transfer.

Analysis

We take the entire contents of the tank as the *system* (**Fig. 5–35**). This is a *closed system* since no mass crosses the system boundary during the process. We observe that the volume of a rigid tank is constant, and thus there is no boundary work. The energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$0 = \Delta U$$

Iron
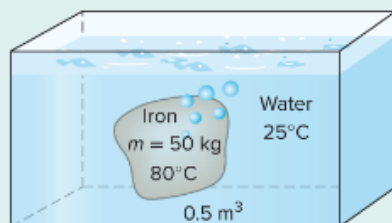m = 50 kg
80°C

Water
25°C

0.5 m^3

FIGURE 5–35
Schematic for **Example 5–12**.

The total internal energy U is an extensive property, and therefore it can be expressed as the sum of the internal energies of the parts of the system. Then the total internal energy change of the system becomes

$$\Delta U_{\text{sys}} = \Delta U_{\text{iron}} + \Delta U_{\text{water}} = 0$$

$$[mc(T_2 - T_1)]_{\text{iron}} + [mc(T_2 - T_1)]_{\text{water}} = 0$$

The specific volume of liquid water at or about room temperature can be taken to be 0.001 m^3/kg. Then the mass of the water is

$$m_{\text{water}} = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 500 \text{ kg}$$

The specific heats of iron and liquid water are determined from **Table A–3** to be $c_{\text{iron}} = 0.45$ kJ/kg·°C and $c_{\text{water}} = 4.18$ kJ/kg·°C. Substituting these values into the energy equation, we obtain

$$(50 \text{ kg})(0.45 \text{ kJ/kg·°C})(T_2 - 80°\text{C}) + (500 \text{ kg})(4.18 \text{ kJ/kg·°C})$$
$$(T_2 - 25°\text{C}) = 0$$

$$T_2 = \mathbf{25.6°C}$$

Therefore, when thermal equilibrium is established, both the water and iron will be at 25.6°C.

Discussion

The small rise in water temperature is due to its large mass and large specific heat.

EXAMPLE 5–13 Heating of Aluminum Rods in a Furnace

Long cylindrical aluminum rods (ρ = 2700 kg/m^3 and c_p = 0.973 kJ/kg·K) of 5-cm diameter are heat treated from 20°C to an average temperature of 400°C by drawing them at a velocity of 8 m/min through a long oven (**Fig. 5–36**). Determine the rate of heat transfer to the rods in the oven.
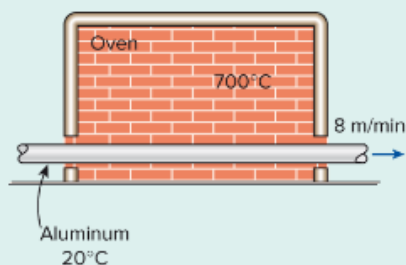

FIGURE 5–36
Schematic for **Example 5–13**

SOLUTION

Aluminum rods are to be heated in an oven to a specified average temperature. The rate of heat transfer to the rods is to be determined.

Assumptions

1 The thermal properties of the rods are constant. **2** There are no changes in kinetic and potential energies. **3** The balls are at a uniform temperature when they leave the oven.

Analysis

Aluminum rods pass through the oven at a constant speed of 8 m/min. That is, an external observer will see that an 8-m long section of cold rods enters and an 8-m long section of hot rods leaves the oven every minute. We take the 8-m long section of the rod as the system. The energy balance for this closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{in} = \Delta U_{rod} = m(u_2 - u_1)$$

$$Q_{in} = mc(T_2 - T_1)$$

The density and specific heat of the rods are given to be ρ = 2700 kg/m^3 and c = 0.973 kJ/kg·K = 0.973 kJ/kg·°C. The amount of heat transfer to an 8-m long section of the rod as it is heated to the specified temperature is determined to be

$$m = \rho V = \rho \frac{\pi D^2}{4} L = (2700 \text{ kg/m}^3) \frac{\pi (0.05 \text{ m})^2}{4} (8 \text{ m}) = 42.41 \text{ kg}$$

$$Q_{in} = mc(T_2 - T_1) = (42.41 \text{ kg})(0.973 \text{ kJ/kg·°C})(400 - 20)°C$$
$$= 15{,}680 \text{ kJ (per 8-m section)}$$

Considering that an 8-m long section of the rods is heated every minute, the rate of heat transfer to the rods in the oven becomes

$$\dot{Q}_{in} = Q_{in}/\Delta t = 15{,}680 \text{ kJ/min} = \textbf{261 kJ/s}$$

Discussion

This problem can also be solved by working with the *rate* form of the equations as

$$\dot{m} = \rho \dot{V} = \rho \frac{\pi D^2}{4} L/\Delta t = \rho \frac{\pi D^2}{4} V = (2700 \text{ kg/m}^3) \frac{\pi (0.05 \text{ m})^2}{4} (8 \text{ m/min})$$
$$= 42.41 \text{ kg/min}$$

$$\dot{Q}_{in} = \dot{m} c(T_2 - T_1) = (42.41 \text{ kg/min})(0.973 \text{ kJ/kg·°C})(400 - 20)°C$$
$$= 15{,}680 \text{ kJ/min}$$

which is identical to the result obtained before.

Summary

- **Moving boundary work**
  - W_b for an isothermal process
  - W_b for a constant-pressure process
  - W_b for a polytropic process
- **Energy balance for closed systems**
  - Energy balance for a constant-pressure expansion or compression process
- **Specific heats**
  - Constant-pressure specific heat, c_p
  - Constant-volume specific heat, c_v
- **Internal energy, enthalpy, and specific heats of ideal gases**
  - Specific heat relations of ideal gases
- **Internal energy, enthalpy, and specific heats of solids and liquids**

SUMMARY

Work is the energy transferred as a force acts on a system through a distance. The most common form of mechanical work is the *boundary work,* which is the work associated with the expansion and compression of substances. On a *P-V* diagram, the area under the process curve represents the boundary work for a quasi-equilibrium process. Various forms of boundary work are expressed as follows:

1. General $\quad W_b = \int_1^2 P\, dV$

2. Isobaric process

$$W_b = P_0(V_2 - V_1) \qquad (P_1 = P_2 = P_0 = \text{constant})$$

3. Polytropic process

$$W_b = \frac{P_2 V_2 - P_1 V_1}{1 - n} \ (n \neq 1) \qquad (PV^n = \text{constant})$$

4. Isothermal process of an ideal gas

$$W_b = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$= mRT_0 \ln \frac{V_2}{V_1} \qquad (PV = mRT_0 = \text{constant})$$

The first law of thermodynamics is essentially an expression of the conservation of energy principle, also called the energy balance. The general energy balances for *any system* undergoing *any process* can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

It can also be expressed in the *rate form* as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}}$$

Taking heat transfer *to* the system and work done *by* the system to be positive quantities, the energy balance for a closed system can also be expressed as

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

where

$$W = W_{other} + W_b$$
$$\Delta U = m(u_2 - u_1)$$
$$\Delta KE = \tfrac{1}{2} m(V_2^2 - V_1^2)$$
$$\Delta PE = mg(z_2 - z_1)$$

For a *constant-pressure process,* $W_b + \Delta U = \Delta H$. Thus,

$$Q - W_{other} = \Delta H + \Delta KE + \Delta PE$$

Note that the relation above is limited to constant pressure processes of a closed system, and is NOT valid for processes during which pressure varies.

The amount of energy needed to raise the temperature of a unit mass of a substance by one degree is called the *specific heat at constant volume* c_v for a constant-volume process and the *specific heat at constant pressure* c_p for a constant-pressure process. They are defined as

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \quad \text{and} \quad c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

For ideal gases u, h, c_v, and c_p are functions of temperature alone. The Δu and Δh of ideal gases are expressed as

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\, dT \cong c_{v,\text{avg}}(T_2 - T_1)$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\, dT \cong c_{p,\text{avg}}(T_2 - T_1)$$

For ideal gases, c_v and c_p are related by

$$c_p = c_v + R$$

where R is the gas constant. The *specific heat ratio k* is defined as

$$k = \frac{c_p}{c_v}$$

For *incompressible substances* (liquids and solids), both the constant-pressure and constant-volume specific heats are identical and denoted by c:

$$c_p = c_v = c$$

The Δu and Δh of imcompressible substances are given by

$$\Delta u = \int_1^2 c(T)\, dT \cong c_{\text{avg}}(T_2 - T_1)$$

$$\Delta h = \Delta u + v\Delta P$$