Control Systems - ENGR 33041

Lecture 3: Laplace Transform-cont.

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Slides prepared based on

Control Systems Technology, C. Johnson and H. Malki

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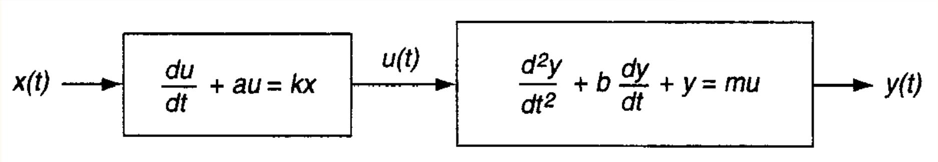
Recap



$$\frac{1}{m}\frac{d^3y}{dt^3} + \frac{a+b}{m}\frac{d^2y}{dt^2} + \frac{1+ab}{m}\frac{dy}{dt} + \frac{a}{m}y = kx$$

- A typical Control System might include several blocks in series, each represented by a differential equation.
- Combining all these blocks might result in a high-order differential equation, which cannot be solved by traditional differential equations methods.
- We learned Laplace Transform technique that allows us to transform the equations from time domain “t” to the Laplace transform domain “s” and solve them in “s” domain. Then, we apply Inverse Laplace Transform to transform the results back into the “t” domain.

3.5.1 Partial-Fraction Expansion

$$F(s) = \frac{9s + 39}{s^2 + 8s + 15}$$

Partial-Fraction Expansion

Take Common Denominator

$$F(s) = \frac{3}{s+5} + \frac{6}{s+3}$$

$$f(t) = 3e^{-5t} + 6e^{-3t}$$

TABLE 3.1
Laplace Transforms of Functions

Time Function	Laplace Transform
Impulse, $\delta(t)$	1
Step, $u(t) = 1$	$\frac{1}{s}$
k, k a constant number	$\frac{k}{s}$
Ramp, t	$\frac{1}{s^2}$
Polynomial, t^n	$\frac{n!}{s^{n+1}}$
Exponential, e^{-at}	$\frac{1}{s+a}$
Ramp exponential, te^{-at}	$\frac{1}{(s+a)^2}$
Polynomial exponential, $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine, $\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine, $\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
Damped sine, $e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine, $e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

Partial Fraction Expansion means expanding a complex expression or fraction into its partial fractions, such as the one shown here.

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Rational Function:

The ratio of two polynomials

$$P(s) = \frac{N(s)}{D(s)} = K\frac{s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$N(s) = s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0 = 0$

roots of $N(s)$ are $z_1, z_2, \ldots, z_m$

$N(s) = (s-z_1)(s-z_2)\ldots(s-z_m)$

roots of $D(s)$ are $p_1, p_2, \ldots, p_n$

$D(s) = (s-p_1)(s-p_2)\ldots(s-p_n)$

$$P(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}$$

Zeros: roots of numerator, m zeros

Poles: roots of denominator, n poles

If n>m, partial-fraction expansion gives

$$P(s) = \frac{K_1}{s-p_1} + \frac{K_2}{s-p_2} + \ldots + \frac{K_n}{s-p_n}$$

The amplitudes $K_1, K_2, \ldots, K_n$ are called the residues and could be real or complex numbers.

Residues K_i can be found as

$$K_i = (s - p_i)P(s)\Big|_{s=p_i} \tag{3.15}$$

$$P(s) = K\frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)}$$

To use partial-fraction expansion on a rational fraction $F(s) = K\frac{s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$

1. Find all the poles $P_1, P_2, \ldots P_n$ (roots of denominator)
2. Express $F(s)$ using its poles as $F(s) = \frac{K_1}{s - p_1} + \frac{K_2}{s - p_2} + \ldots + \frac{K_n}{s - p_n}$
3. Find residues K_i using $K_i = (s - p_i)F(s)\Big|_{s=p_i}$

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Ex.3.15

Use partial-fraction expansion to find inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{9s + 39}{s^2 + 8s + 15}$ (b) $F(s) = \dfrac{2s+3}{s(s+1)(s+4)}$

Solution (a) $F(s) = \dfrac{9s + 39}{s^2 + 8s + 15}$

1. Find all the poles: $s^2 + 8s + 15 = 0 \longrightarrow p = \dfrac{-8 \pm \sqrt{8^2 - 4(15)(1)}}{2(1)} = -4 \pm 1$ → $p_1 = -3$, $p_2 = -5$

2. Express F(s) using its poles: $F(s) = \dfrac{9s + 39}{(s+3)(s+5)} = \dfrac{K_1}{s+3} + \dfrac{K_2}{s+5}$

3. Find residues K_1 *and* K_2:

$$K_1 = (\cancel{s+3})\frac{9s+39}{(\cancel{s+3})(s+5)}\bigg|_{s=-3} = \frac{9(-3)+39}{(-3+5)} = 6$$

$$K_2 = (s+5)\frac{9s+39}{(s+3)(s+5)}\bigg|_{s=-5} = \frac{9(-5)+39}{(-5+3)} = 3$$

$\Longrightarrow$ $F(s) = \dfrac{6}{s+3} + \dfrac{3}{s+5}$ $\xRightarrow{\text{Using Table 3.1}}$ $f(t) = 6e^{-3t} + 3e^{-5t}$

Solution (b) $F(s) = \dfrac{2s+3}{s(s+1)(s+4)}$

$$F(s) = \frac{2s+3}{s(s+1)(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+4}$$

$$K_1 = s\,F(s)\Big|_{s=0} = s\,\frac{2s+3}{s(s+1)(s+4)}\Big|_{s=0} = \frac{3}{(1)(4)} = 0.75$$

$$K_2 = (s+1)\,F(s)\Big|_{s=-1} = (s+1)\,\frac{2s+3}{s(s+1)(s+4)}\Big|_{s=-1} = \frac{2(-1)+3}{(-1)(-1+4)} = \frac{1}{-3} = -0.333$$

$$K_3 = (s+4)\,F(s)\Big|_{s=-4} = (s+4)\,\frac{2s+3}{s(s+1)(s+4)}\Big|_{s=-4} = \frac{2(-4)+3}{(-4)(-4+1)} = \frac{-5}{12} = -0.417$$

So, $F(s)$ can be written as : $F(s) = \dfrac{2s+3}{s(s+1)(s+4)} = \dfrac{0.75}{s} - \dfrac{0.333}{s+1} - \dfrac{0.417}{s+4}$

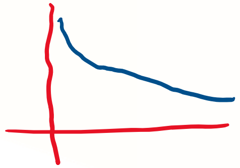
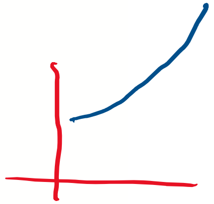
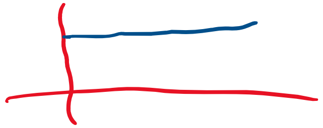
Then, the time function is found from the table as: $f(t) = 0.75 - 0.333e^{-t} - 0.417e^{-4t}$

Real Poles

- Partial-fraction expansion shows the nature of corresponding time function is dependent on the poles.
- The zeros convey information about the amplitudes.
- Real poles mean that the time dependence (function) has an exponential form:

Exponential, e^{-at}	$\dfrac{1}{s+a}$

Real Poles: $s - p = s + a;\quad p = -a$

- **If $p < 0$ ($a > 0$), e^{-at} will decay and system is stable.** 
- **If $p > 0$ ($a < 0$), e^{-at} is a growing exponential and instability will occur.** 
- **If p = 0 ($a = 0$), the system response stays constant.** 

Complex Poles:

When denominator has complex poles, the corresponding time function exhibits oscillations:

Damped sine, $e^{-at}\sin(\omega t)$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
Damped cosine, $e^{-at}\cos(\omega t)$	$\dfrac{s+a}{(s+a)^2+\omega^2}$

$$(s+a)^2+\omega^2 = s^2+2as+a^2+\omega^2 = 0 \Rightarrow p_{1,2} = \frac{-2a \pm \sqrt{(2a)^2-4(a^2+\omega^2)}}{2}$$

$$\Downarrow$$

$$p_1 = -a + j\omega \qquad p_2 = -a - j\omega \Leftarrow p_{1,2} = \frac{-2a \pm j2\omega}{2}$$

Real part: -*a*; describes exponential damping coefficient (stability).

Imaginary part: *ω;* describes frequency of oscillation.

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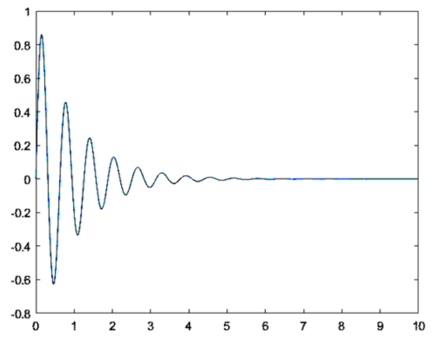
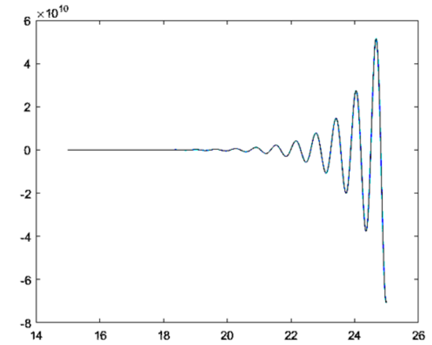
$$e^{-at}\cos(\omega t) \quad \text{or} \quad e^{-at}\sin(\omega t)$$

Real part: *-a*

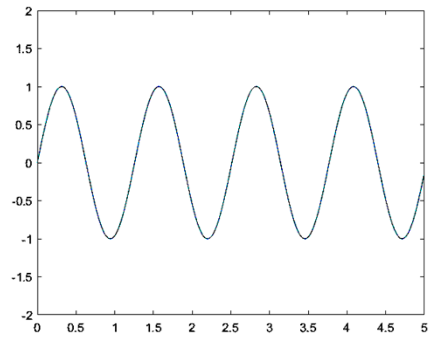
- If $a > 0$, e^{-at} will decay and system is stable. →



- If $a < 0$, e^{-at} is a growing exponential and instability will occur. →



- If $a = 0$, it is called the steady-state oscillation (constant oscillation without damping or growing). →



Imaginary part: *ω*

- ω is the frequency of oscillation.

Complex Pole Amplitudes

$a+bj \xrightarrow[\text{Conjugate}]{\text{complex}} a-bj$

$\Uparrow$

$$F(s) = \frac{N(s)}{(s-p_1)\ldots(s-a+j\omega)(s-a-j\omega)\ldots(s-p_n)} \quad \textbf{(3.18)}$$

$$F(s) = \frac{K_1}{s-p_1} + \cdots + \frac{K}{s-a+j\omega} + \frac{K*}{s-a-j\omega} + \cdots + \frac{K_n}{s-p_n} \quad \textbf{(3.19)}$$

K^* is complex Conjugate of K

Residue Formula: $K_i = (s-p_i)P(s)\Big|_{s=p_i}$ $\Rightarrow$ $K = (s-a+j\omega)F(s)\Big|_{s=a-j\omega}$ **(3.20)**

K is the residue of the complex pole $p = a - j\omega$

$\Downarrow$

$$\boxed{\begin{aligned} &\text{cos amplitude} = 2\text{Re}(K) && \textbf{(3.16)} \\ &\text{sin amplitude} = 2\text{Im}(K) && \textbf{(3.17)} \end{aligned}}$$

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Complex Pole Amplitudes

$$f(t) = K_1 e^{p_1 t} + \cdots + 2\,\mathrm{Re}(K) e^{at} \cos(\omega t) + 2\,\mathrm{Im}(K) e^{at} \sin(\omega t) + \cdots + K_n e^{p_n t} \quad (3.21)$$

So, *K* determines both the damped cosine and damped sine amplitudes.

- If the imaginary part of K is zero, then only cosine is valid (sine is zero).
- If the real part of K is zero, then only sine is valid (cosine is zero).

Note:

If a=0, it will be the steady-state oscillation (constant oscillation) meaning that it will be pure sine and/or pure cosine function in time (no exponential term).

Recap of Partial-Fraction Expansion for Complex Poles:

- Complex poles always occur as a **pair** of complex conjugate poles. That means $p_1 = a - j\omega$ and $p_2 = a + j\omega$.

- When we have a pair of complex poles in the Laplace transform domain, the corresponding time function will have an oscillation and will be a combination of damped sine and damped cosine functions.

- We will apply the partial-fraction expansion technique and find the residues of complex poles as:

$$K = (s - a + j\omega)F(s)\Big|_{s = a - j\omega}$$

Assuming K is the residue of the complex pole $p = a - j\omega$:

$$\boxed{\begin{array}{l} \text{Amplitude of Damped Cosine} = 2 \times \text{real part } (K) \\ \\ \text{Amplitude of Damped Sine} = 2 \times \text{imaginary part } (K) \end{array}}$$

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Ex. 3.16 Use partial-fraction expansion to find the inverse Laplace transform of the following functions:

$$F(s) = \frac{4s+3}{s(s^2+2s+5)}$$

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Solution

$$F(s) = \frac{4s+3}{s(s^2+2s+5)}$$

$p_1 = 0$

$p_2 = -1 - 2j$

$p_3 = -1 + 2j$

First, we find the poles (roots of quadratic equation): $p = \dfrac{-2 \pm \sqrt{4-4(5)}}{2} = -1 \pm 2j$

So, the complex pole means there will be an oscillation term in the time response. $F(s)$ can now be written as the factors of its poles :

$$F(s) = \frac{4s+3}{s(s^2+2s+5)} = \frac{4s+3}{s(s+1+2j)(s+1-2j)} = \frac{K_1}{s} + \frac{K_2}{s+1+2j} + \frac{K_2^*}{s+1-2j}$$

$$K_1 = sF(s)\Big|_{s=0} = s\frac{4s+3}{s(s+1+2j)(s+1-2j)}\Bigg|_{s=0} = \frac{3}{(1+2j)(1-2j)} = \frac{3}{5} = 0.6$$

$$K_2 = \cancel{(s+1+2j)}\frac{4s+3}{s\cancel{(s+1+2j)}(s+1-2j)}\bigg|_{s=-1-2j} = \frac{4(-1-2j)+3}{(-1-2j)(-1-2j+1-2j)} = \frac{-1-8j}{4j(1+2j)}$$

Use online calculators or scientific calculator to simplify it: $\longrightarrow$ $K_2 = \dfrac{-1-8j}{4j(1+2j)} = -0.3 + 0.85j$

Then the cos term amplitude is $2\mathrm{Re}[K_2] = 2(-0.3) = -0.6$
and the sine term amplitude is $2\mathrm{Im}[K_2] = 2(0.85) = 1.7$

$$F(s) = \frac{0.6}{s} + \frac{-0.3+0.85j}{s+1+2j} + \frac{-0.3-0.85j}{s+1-2j}$$

Using inverse Laplace transform, the corresponding time function is:

$$\boxed{f(t) = 0.6 - 0.6\, e^{-t}\cos(2t) + 1.7\, e^{-t}\sin(2t)}$$

Useful online calculators:

https://www.mathsisfun.com/numbers/complex-number-calculator.html

https://web2.0calc.com/

Ex. 3.17

Figure 3.8 shows two blocks of a control system and the differential equations lating input and output. (a) Find the Laplace transform of the transfer function tween $x(t)$ and $y(t)$, and (b) assuming the input is an impulse, $x(t) = \delta(t)$, find time function of $y(t)$. Consider all the initial conditions to be zero.

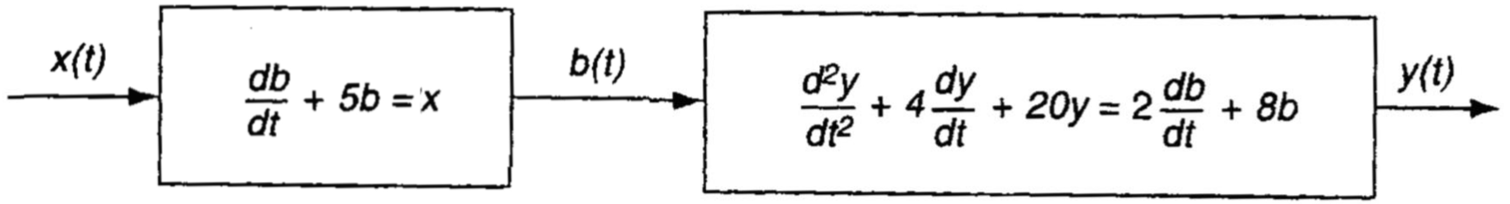


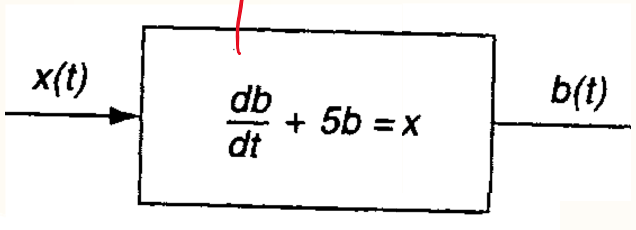
FIGURE 3.8
Differential equation blocks for example 3.14.

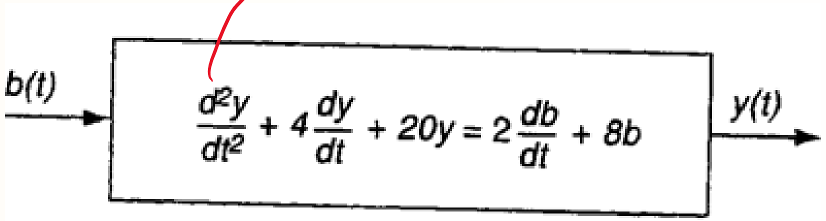
Solution:

a.

$$\mathcal{L}\left\{\frac{db}{dt}\right\} = S\,B(S)$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} = s^2\,Y(s)$$





$$\mathcal{L}\left\{\frac{db}{dt} + 5b\right\} = \mathcal{L}\{x\}$$

$$sB(s) + 5B(s) = X(s)$$

$$(s + 5)B(s) = X(s)$$

$$\frac{B(s)}{X(s)} = \frac{1}{s + 5} \qquad \text{i)}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 20y\right\} = \mathcal{L}\{2\frac{db}{dt} + 8b\}$$

$$s^2Y(s) + 4sY(s) + 20Y(s) = 2sB(s) + 8B(s)$$

$$(s^2 + 4s + 20)Y(s) = 2(s + 4)B(s)$$

$$\frac{Y(s)}{B(s)} = \frac{2(s + 4)}{s^2 + 4s + 20} \qquad \text{ii)}$$

Multiplying i) and ii) yields:

$$\boxed{\frac{Y(s)}{X(s)} = \frac{2(s + 4)}{(s + 5)(s^2 + 4s + 20)} \equiv F(s)}$$

b. If $x(t)$ is a delta function, then $X(s) = 1$ from table 3.1.

$$\frac{Y(s)}{X(s)} = \frac{2(s+4)}{(s+5)(s^2+4s+20)} \equiv F(s) \quad , \quad X(s) = 1$$

$$\Downarrow$$

$$Y(s) = \frac{2(s+4)}{(s+5)(s^2+4s+20)}$$

Inverse Laplace transform is needed to find the corresponding time-domain function. But before doing that, we need to do the partial-fraction expansion first to find its fractions (simpler terms).

TABLE 3.1
Laplace Transforms of Functions

Time Function	Laplace Transform
Impulse, $\delta(t)$	1
Step, $u(t) = 1$	$\frac{1}{s}$
k, k a constant number	$\frac{k}{s}$
Ramp, t	$\frac{1}{s^2}$
Polynomial, t^n	$\frac{n!}{s^{n+1}}$
Exponential, e^{-at}	$\frac{1}{s+a}$
Ramp exponential, te^{-at}	$\frac{1}{(s+a)^2}$
Polynomial exponential, $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine, $\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine, $\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
Damped sine, $e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine, $e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

b. To do partial-fraction expansion, we need to find the poles (roots of denominator) first: $F(s) = \dfrac{2(s+4)}{(s+5)(s^2+4s+20)}$

$$(s+5)(s^2+4s+20) = 0$$

$$s+5 = 0 \rightarrow s = -5$$

$$s^2+4s+20 = 0 \rightarrow s = \frac{-4 \pm \sqrt{16-4(20)}}{2} = -2 \pm 4j$$

$$\rightarrow \quad F(s) = \frac{K_1}{s+5} + \frac{K_2}{s+2+4j} + \frac{K_2^*}{s+2-4j}$$

$$K_1 = (s+5)F(s)|_{s=-5} = \frac{2(-5+4)}{-5^2+4(-5)+20} = -0.08$$

$$K_2 = (s+2+4j)F(s)|_{s=-2-4j} = \frac{2(-2-4j+4)}{(-2-4j+5)(-2-4j+2-4j)} \rightarrow \quad K_2 = 0.04 + 0.22j$$

$$\rightarrow \quad F(s) = \frac{-0.08}{s+5} + \frac{0.04+0.22j}{s+2+4j} + \frac{0.04-0.22j}{s+2-4j}$$

amplitude of the cosine part is 2Re(0.04 + 0.22j) = 0.08

amplitude of the sine part is 2Im(0.04+0.22j) = 0.44

$$\rightarrow \quad \boxed{f(t) = -0.08e^{-5t} + 0.08e^{-2t}\cos(4t) + 0.44e^{-2t}\sin(4t)}$$

Ex. 3.18 Find the impulse response of the Laplace transform found in Ex. 3.17 using MATLAB

impulse response ≡ response to the impulse input

≡ finding output when input = $\delta(t)$

Solution

```
>> syms s t
>> F= 2*(s+4)/((s+5)*(s^2+4*s+20));
>> f=ilaplace (F)  % MATLAB command to find inverse Laplace transform
>> simplify (f)
>> pretty (ans)
```

```
ans=
           /              sin(4 t) 11 \
exp(-2 t) | cos(4 t) + ----------- | 2
           \                   2      /    exp(-5 t) 2
-------------------------------------- - -----------
                  25                         25
```

Partial-Fraction Expansion in MATLAB

https://www.youtube.com/watch?v=jdjDeYr_CdU

Partial Fraction Expansion

- Evaluate the Partial Fraction Expansion using ***residue*** command

  Syntax: [r, p, g] = residue(num,den);

  - r is a vector for the partial values = residues
  - P is a vector for the poles
  - g is a vector for the polynomial coefficients

$$H(s) = \frac{s^3 + 2s^2 - s}{s^2 + 3s + 2} = s - 1 - \frac{2}{s+2} + \frac{2}{s+1}$$

Ex. 3.19 Use MATLAB to find the residues and poles for this Laplace transform:

Solution

$$F(s) = \frac{4(s+9)}{s^4 + 9s^3 + 45s^2 + 87s + 50}$$

```
>> num = conv([4],[1 9])
>> den = [ 1 9 45 87 50]
>> [r p k]= residue (num,den)
```

Find the time function that this transform represents and plot it

```
>> syms s t
>> F=(4*(s+9)/(s^4+9*s^3+45*s^2+87*s+50))
>> f=ilaplace(F)
 >> simplify (f)
>> pretty (ans)
>> fplot(ans, [0,10]); % plot it from 0 to 10 s
```

Repeated Poles *multiple poles*

$$F(s) = \frac{N(s)}{(s-p_1)\ \dots\ (s-p_i)^m\ \dots (s-p_n)}$$

- When there are repeated poles, for example a pole p_i that occurs “m” times, then every power of the pole (from “1” to “m”) must be included when doing partial-fraction expansion:

$$F(s) = \frac{K_1}{s-p_1} + \cdots + \frac{K_{i1}}{s-p_i} + \frac{Ki_2}{(s-p_i)^2} + \cdots + \frac{K_{im}}{(s-p_i)^m} + \cdots + \frac{K_n}{s-p_n}$$

- **Residue formula for repeated poles:**

K_{ij} can be found as

$$\boxed{K_{ij} = \frac{1}{(m-j)!}\frac{d^{m-j}}{ds^{m-j}}\left[(s-p_i)^m F(s)\right]\Big|_{s=p_i}}$$

Ex. 3.18 Find the time function of the following Laplace transform,

$$F(s) = \frac{4s(s+5)}{(s+2)(s+8)^2}$$

Solution:

$$F(s) = \frac{K_1}{s+2} + \frac{K_{21}}{s+8} + \frac{K_{22}}{(s+8)^2}$$

$$K_1 = (s+2)F(s)\big|_{s=-2} = \left.\frac{4s(s+5)}{(s+8)^2}\right|_{s=-2} = \frac{4(-2)(-2+5)}{(-2+8)^2}$$

$$K_1 = -0.67$$

Reminder:

The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$\nearrow \frac{d}{ds}$

$$K_{21} = \frac{1}{(2-1)!}\frac{d^{2-1}}{ds^{2-1}}\left[(s+8)^2F(s)\right]\Bigg|_{s=-8} = \frac{1}{1!}\frac{d}{ds}\left[\frac{4s(s+5)}{(s+2)}\right]\Bigg|_{s=-8}$$

$$K_{21} = \frac{4(s+2)(2s+5) - 4s(s+5)}{(s+2)^2} = \frac{4(-8+2)(-16+5)-4(-8)(-8+5)}{(-8+2)^2}$$

$$K_{21} = 4.67$$

$\nearrow 1$

$$K_{22} = \frac{1}{(2-2)!}\frac{d^{2-2}}{ds^{2-2}}\left[(s+8)^2F(s)\right]\Bigg|_{s=-8} = \frac{1}{0!}\left[\frac{4s(s+5)}{(s+2)}\right]\Bigg|_{s=-8}$$

$$K_{22} = -16$$

$$F(s) = \frac{-0.67}{s+2} + \frac{4.67}{s+8} - \frac{16}{(s+8)^2}$$

$$\boxed{f(t) = -0.67e^{-2t} + 4.67e^{-8t} - 16te^{-8t}}$$

TABLE 3.1
Laplace Transforms of Functions

Time Function	Laplace Transform
Impulse, $\delta(t)$	1
Step, $u(t) = 1$	$\frac{1}{s}$
k, k a constant number	$\frac{k}{s}$
Ramp, t	$\frac{1}{s^2}$
Polynomial, t^n	$\frac{n!}{s^{n+1}}$
Exponential, e^{-at}	$\frac{1}{s+a}$
Ramp exponential, te^{-at}	$\frac{1}{(s+a)^2}$
Polynomial exponential, t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
Sine, $\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine, $\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
Damped sine, $e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped cosine, $e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

- Homework 3 is due Sep. 12, 11 am and needs to be submitted on Canvas.

- Some of the HW problems need access to MATLAB software.