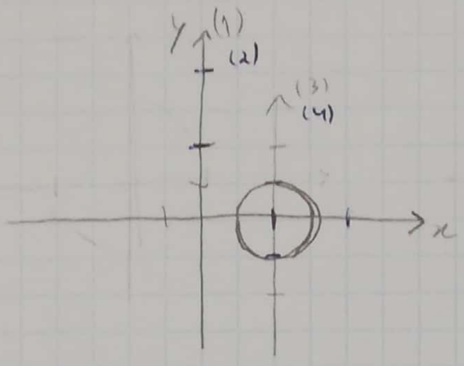
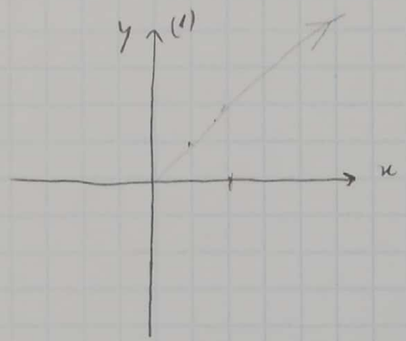
Ficha de Consolidação I - Transformações Geométricas

1.



a) glScale (2,2,2) aumenta a escala de cada eixo para o dobro.
glTranslate (1,0,0) move o centro do referencial para o ponto (1,0,0) - uma unidade para a direita
glScale (0.5, 0.5, 0.5) repõe a escala original de cada eixo desenhando-se assim a esfera de raio unitário.

2.



c) glRotate (45, 0.0, 0.0, 1.0) faz uma rotação de 45° em torno do eixo z
glTranslate (move o centro do referencial para o ponto (2,0,0)
glRotate (-45, 0.0, 0.0, 1.0) faz uma rotação de -45° em torno do eixo y.

3.

a) glTranslate (cos α, sen α, 0);
glRotate (α, 0, 0, 1.0);
desenha Casa ();

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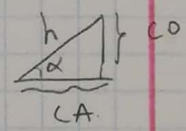
$$\text{sen}\ \alpha = \frac{CO}{H}$$

b) gl Rotate (α, 0, 0, 1.0);
gl Translate (1, 0, 0);
desenha Casa();



4.

i. $T_1 \times R_1 = R_1 \times T_1$ Afirmação falsa

$$T_1 \times R_1 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.7 & -0.7 & 0 & 0 \\ 0.7 & 0.7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.7 & 0 & 2 \\ 0.7 & 0.7 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \times T_1 = \begin{bmatrix} 0.7 & -0.7 & 0 & 0 \\ 0.7 & 0.7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.7 & 0 & 0.7 \\ 0.7 & 0.7 & 0 & 2.1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. $T_1 \times S_1 = S_1 \times T_1$ Afirmação falsa

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. $T_1 \times T_2 = T_2 \times T_1$ Afirmação verdadeira

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & 1 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_2+a_1 \\ 0 & 1 & 0 & b_2+b_1 \\ 0 & 0 & 1 & c_2+c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$//$$

$$\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & 1 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_1+a_2 \\ 0 & 1 & 0 & b_1+b_2 \\ 0 & 0 & 1 & c_1+c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv.

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & 1 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S_1 = \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1 \times S_1 = \begin{bmatrix} a_2 & 0 & 0 & a_1 \\ 0 & b_2 & 0 & b_1 \\ 0 & 0 & c_2 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & \frac{a_1}{a_2} \\ 0 & 1 & 0 & \frac{b_1}{b_2} \\ 0 & 0 & 1 & c_1/c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S_2 = \begin{bmatrix} a_2 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_2 \times T_2 = \begin{bmatrix} a_2 & 0 & 0 & a_1 \\ 0 & b_2 & 0 & b_1 \\ 0 & 0 & c_2 & c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Afirmação verdadeira

V. $R_1 \times R_2 = R_2 \times R_1$ Afirmação falsa

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) é a sequência incorreta.

6.

a) $T_1 \times T_2 = T_2 \times T_1$

⊛

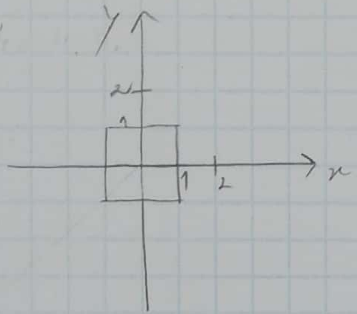
b) $S_1 \times S_2 = S_2 \times S_1$

$$\begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_1 b_1 & 0 & 0 & 0 \\ 0 & a_2 b_2 & 0 & 0 \\ 0 & 0 & a_3 b_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\parallel$$

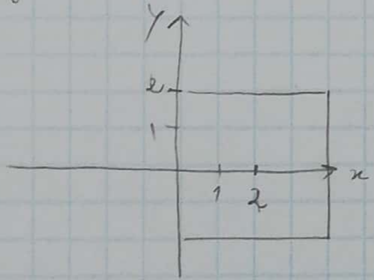
$$\begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} b_1 a_1 & 0 & 0 & 0 \\ 0 & b_2 a_2 & 0 & 0 \\ 0 & 0 & b_3 a_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

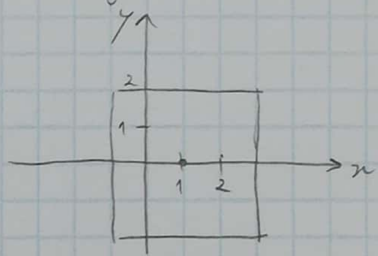


quadrado ()

glScale (2,2,2)
glTranslate (1,0,0)

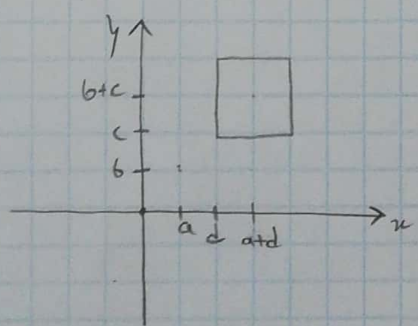


glTranslate (1,0,0)
glScale (2,2,2)

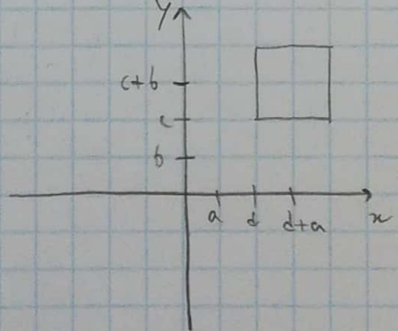


a) ⊛

glTranslate(a, b, 0)
glTranslate(d, c, 0)
quadrado ()

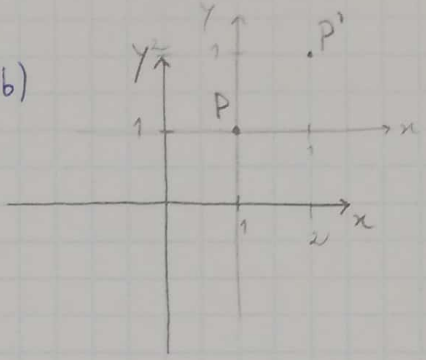


glTranslate (d, c, 0)
glTranslate (a, b, 0)
quadrado ()



7.

a) b)



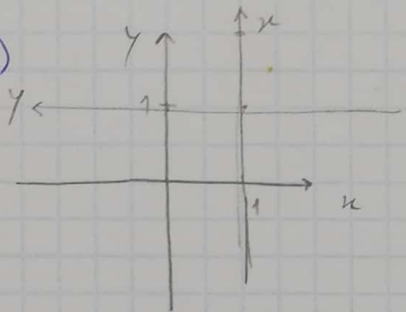
$$P' = \begin{bmatrix}1&0&1\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}2\\2\\1\end{bmatrix}$$

8.

a)

$$P' = \begin{bmatrix}0&-1&1\\1&0&1\\0&0&1\end{bmatrix}\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\3\\1\end{bmatrix}$$

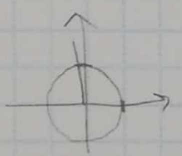
b)



$$\begin{bmatrix}0&-1&1\\1&0&1\\0&0&1\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

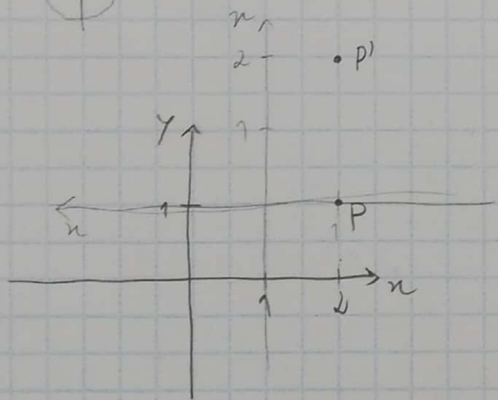
$$\begin{bmatrix}0&-1&1\\1&0&1\\0&0&1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$\begin{bmatrix}0&-1&1\\1&0&1\\0&0&1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix}$$



$\operatorname{sen}\alpha = 1$, $\alpha = 90°$

c)



10.

a)
$$\begin{bmatrix}3&0&0&0\\0&2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1\\2\\3\\1\end{bmatrix} = \begin{bmatrix}3\\4\\3\\1\end{bmatrix}$$

b)
$$\begin{bmatrix}1&0&0&2\\0&1&0&2\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1\\2\\3\\1\end{bmatrix} = \begin{bmatrix}3\\4\\3\\1\end{bmatrix}$$

9.

a)
$$Rot = \begin{bmatrix}\cos(45°)&-\operatorname{sen}(45°)&0&0\\ \operatorname{sen}(45°)&\cos(45°)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}0.707&-0.707&0&0\\0.707&0.707&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}0.707\\0.707\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0.99\\0\\1\end{bmatrix}$$

b)
$$\begin{bmatrix}0.707&0.707&0&0\\-0.707&0.707&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

c)
$$\begin{bmatrix}0.7\\0.7\\0\\1\end{bmatrix}$$

Ficha de Consolidação II

1.

a) i) gluLookAt (0, sen α, cos α , 0,0,0, 0,1,0)

b)

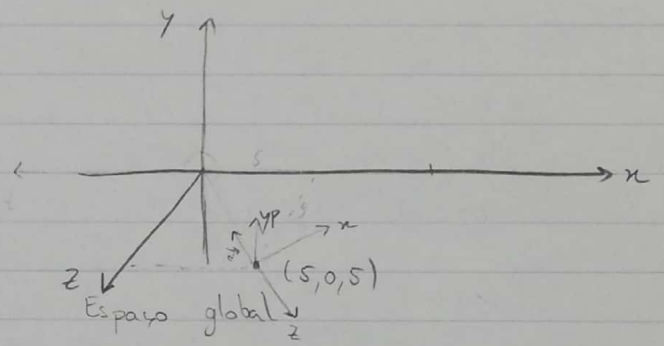
i) glRotate (α , 1 , 0,0)

ii) glTranslate (0 , - sen α, - cos α)

2.

a) V

b) F



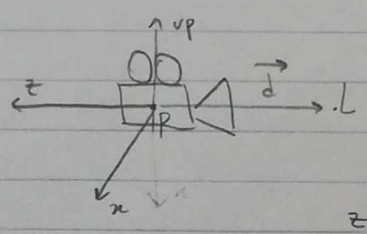
$z_c = -\sqrt{5^2+5^2} = -\sqrt{50}$

R : $(-\sqrt{50}, 0, 0)$

c) V

3.

a)



$d = l - p$

$z = -\vec{d}$

$up = (u_1, u_2, u_3)$ $x = up \times z \ / \ |up \times z|$

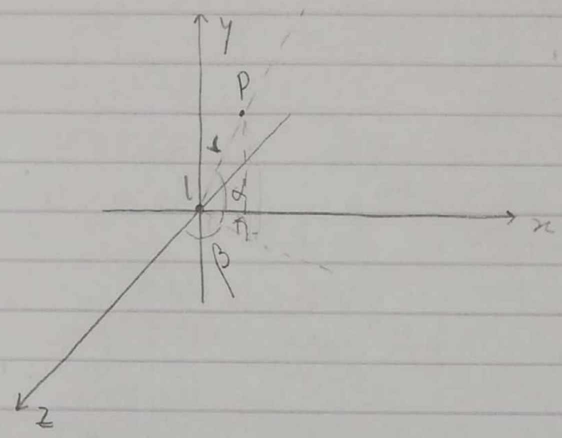
gluLookAt ($p_1 - x_1$, $p_2 - x_2$, $p_3 - x_3$, $l_1 - x_1$, $l_2 - x_2$, $l_3 - x_3$, u_1, u_2, u_3)

usar regra da mão direita para ver a ordem

b) $y = z \times x \,/\, |z \times x|$

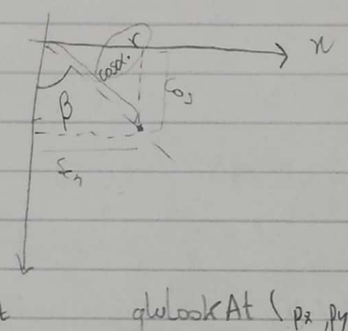
$\text{gluLookAt} = (p_1 + y_1,\ p_2 + y_2,\ p_3 + y_3,\ l_1 + y_1,\ l_2 + y_2,\ l_3 + y_3,\ u_1, u_2, u_3)$

4.



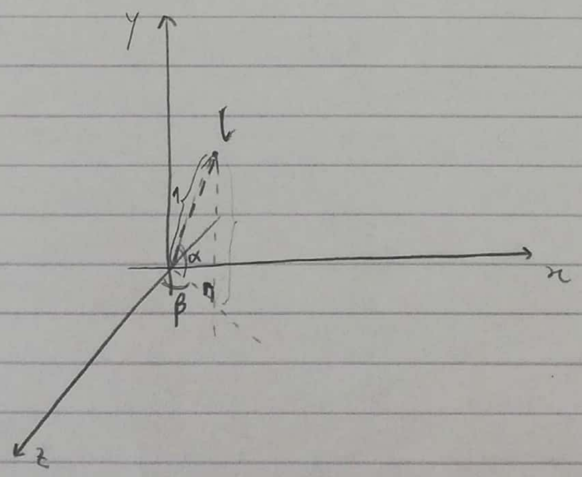
$P_x = \text{sen}\beta \cdot \cos\alpha\, r$
$P_y = \text{sen}\alpha \cdot r$
$P_z = \cos\beta \cdot \cos\alpha \cdot r$

cos = A/H sen = O/H SOCATOA



$\text{gluLookAt}(p_x, p_y, p_z, 0, 0, 0, up_x, up_y, up_z)$

5.

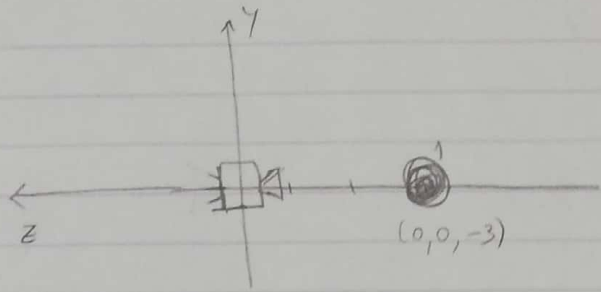


$L_x = P_x + \cos\alpha \cdot \text{sen}\beta$
$L_y = P_y + \text{sen}\,\alpha$
$L_z = P_z + \cos\alpha \cdot \cos\beta$

$\text{gluLookAt}(p_x, p_y, p_z, L_x, L_y, L_z, up_x, up_y, up_z$

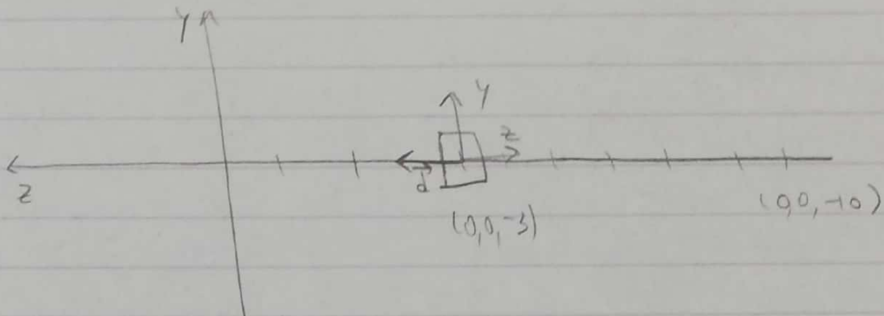
6.

a)



V

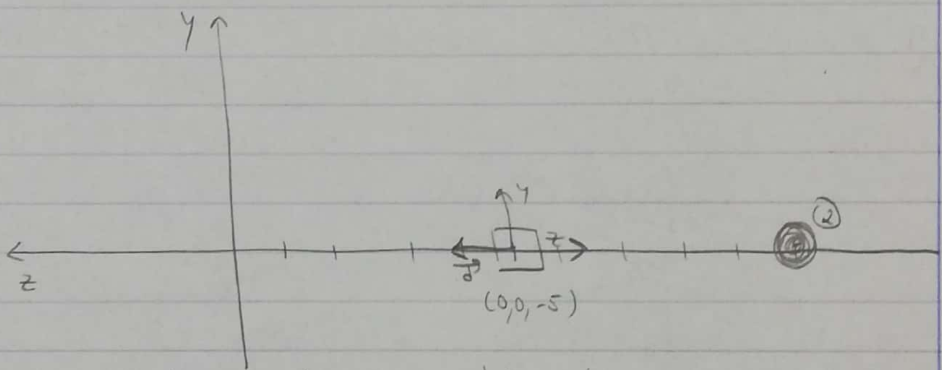
b)



R = (0,0,7)

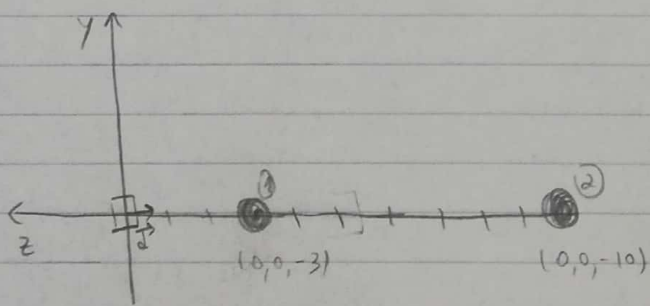
F

c)



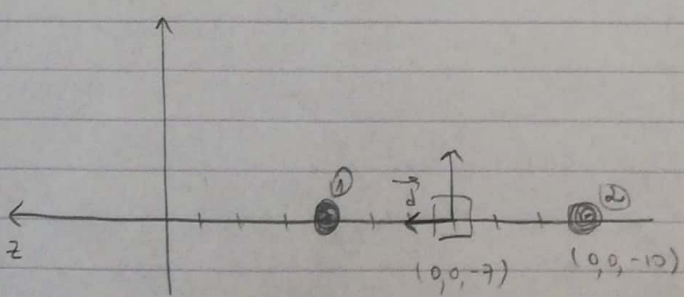
V - A câmara está a olhar no sentido contrário.

d)

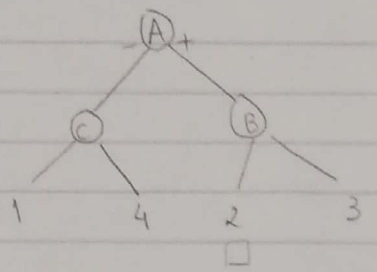


V - A esfera 1 está a tapar a 2

e) F

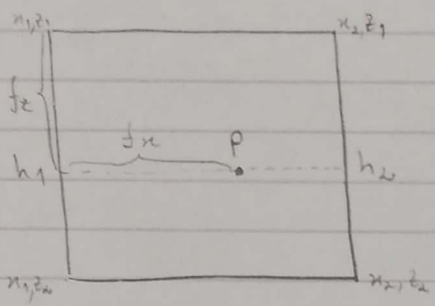


7.



4, 1, 3, 2

8.



$$h_1 = (1 - f_z)\, h(x_1, z_1) + f_z \cdot h(x_1, z_2)$$

$$h_2 = (1 - f_z)\, h(x_2, z_1) + f_z \cdot h(x_2, z_2)$$

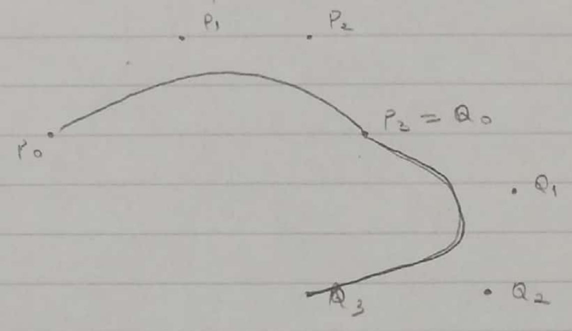
$$h_p = (1 - f_x)\, h_1 + f_x h_2$$

9.

Ficha de Consolidação - Curvas e Superfícies

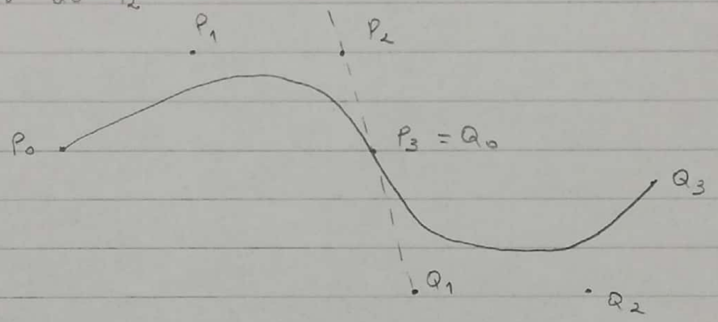
1.

1.1) O primeiro ponto de controlo da segunda curva coincide com o último ponto da primeira curva.



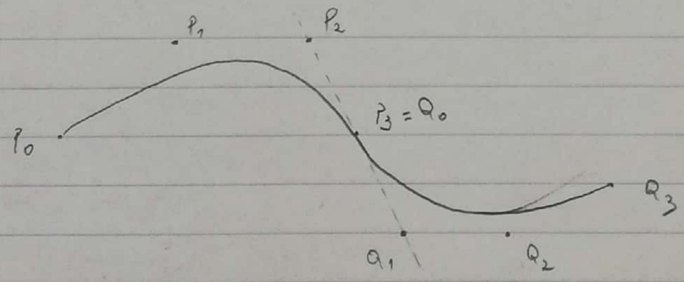
restrições de 1.1 mantêm-se em 1.2

1.2) O Q_1 tem de estar alinhado com a linha que liga o Q_0 ao P_2



restrições de 1.2 mantêm-se em 1.3

1.3) A distância entre P_2 e P_3 é igual à distância entre Q_0 e Q_1

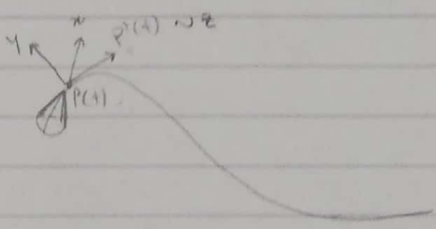


2. O facto de a soma ser 1 garante que a curva se encontra dentro da caixa convexa formada pelos pontos. Isto é útil para o culling pois apenas precisamos de verificar se a caixa se encontra no view frustum.

Estudar as fichas anteriores!

3.

3.1)



$y_0 = (0,1,0)$

$$z_i = \frac{P'(t)}{\|P'(t)\|} \qquad x_i = \frac{y_{i-1} \times z_i}{\|y_{i-1} \times z_i\|} \qquad y_i = z_i \times x_i$$

$$R = \begin{bmatrix} \begin{bmatrix} x_i \end{bmatrix} & \begin{bmatrix} y_i \end{bmatrix} & \begin{bmatrix} z_i \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.2) Se o objeto estiver inicialmente virado para o eixo do x, este viajaria sempre perpendicular à curva.
Para lidar com esta situação poderíamos começar por realizar uma rotação de $-90°$ sobre o eixo do y <u>ou</u>
Calculando a matriz da seguinte forma: $y_0 = (0,1,0)$

$$x_i = \frac{P'(t)}{\|P'(t)\|} \qquad z_i = \frac{x_i \times y_{i-1}}{\|x_i \times y_{i-1}\|} \qquad y_i = z_i \times x_i$$

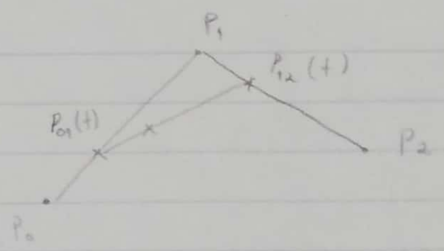
4.

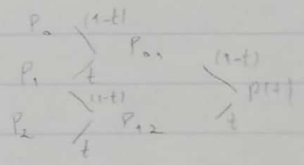
$$P = [u^3\ u^2\ u\ 1]\ M P M^T \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix}$$

$$u' = \frac{\delta P}{\delta u} = [3u^2\ 2u\ 1\ 0]\ M P M\ V$$

$$v' = \frac{\delta P}{\delta v} = U\ M P M^T \begin{bmatrix} 3v^2 \\ 2v \\ 1 \\ 0 \end{bmatrix} \qquad \vec{n} = \frac{u' \times v'}{\|u' \times v'\|}$$

5.





$P_{01}(t) = (1-t)P_0 + t P_1$

$P_{12}(t) = (1-t)P_1 + t P_2$

$P(t) = P_{01}(t)(1-t) + t P_{12}(t)$

6.