# Modeling Circle Arcs with Cubic Bezier Curves

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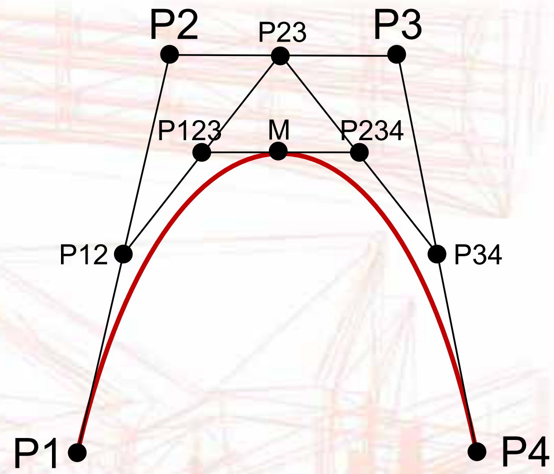
# Cubic Bezier Curves

- Formulation



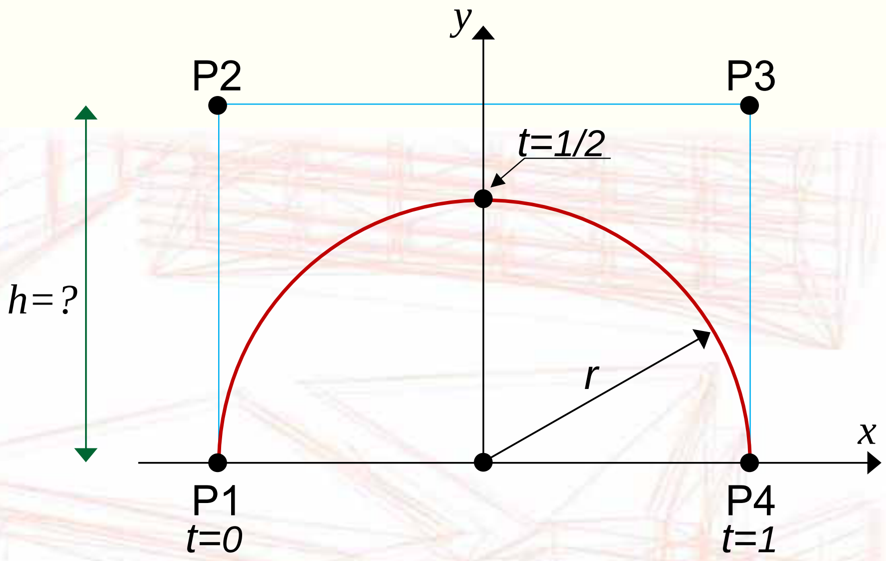

$$Q(t) = (x, y, z) = T.M_B.G_B = [t^3, t^2, t, 1].\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$\Downarrow$$

$$Q(t) = (1-t)^3.P_1 + 3.t.(1-t)^2.P_2 + 3.t^2.(1-t).P_3 + t^3.P_4$$

# Half Circle

- Known values
  - $P1=(-r, 0)$
  - $P2=(-r, h)$
  - $P3=(r, h)$
  - $P4=(r, 0)$




$$Q(t) = (1-t)^3.\boldsymbol{P_1} + 3.t.(1-t)^2.\boldsymbol{P_2} + 3.t^2.(1-t).\boldsymbol{P_3} + t^3.\boldsymbol{P_4}$$

$$Q(t) = (1-t)^3.(-r,0) + 3.t.(1-t)^2.(-r,\boldsymbol{h}) + 3.t^2.(1-t).(r,\boldsymbol{h}) + t^3.(r,0)$$

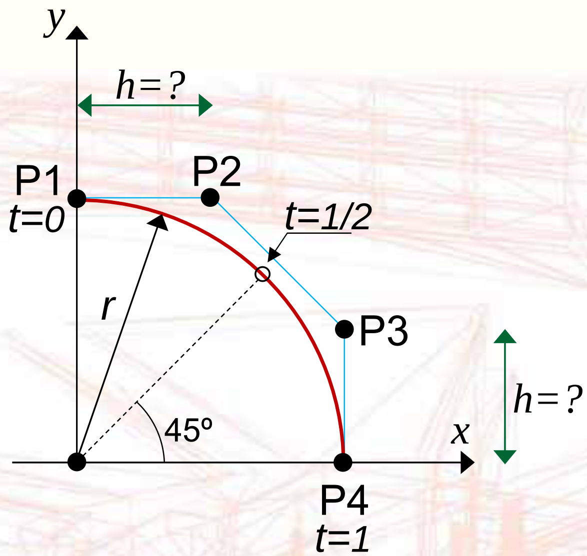
$$Q(1/2) = (0,r) = (1-1/2)^3.(-r,0) + 3.1/2.(1-1/2)^2.(-r,\boldsymbol{h}) + 3.1/2^2.(1-1/2).(r,\boldsymbol{h}) + 1/2^3.(r,0)$$

$$\begin{cases} 0 = (1-1/2)^3.(-r) + 3.1/2.(1-1/2)^2.(-r) + 3.1/2^2.(1-1/2).(r) + 1/2^3.(r) \\ \mathrm{r} = (1-1/2)^3.0 + 3.1/2.(1-1/2)^2.\boldsymbol{h} + 3.1/2^2.(1-1/2).\boldsymbol{h} + 1/2^3.0 \end{cases}$$

$$\boldsymbol{h} = \frac{\boldsymbol{4}}{\boldsymbol{3}}.\boldsymbol{r}$$

# Quarter of a Circle

- Known values
  - *P1=(0, r)*
  - *P2=(h, r)*
  - *P3=(r, h)*
  - *P4=(r, 0)*




$$Q(t) = (1-t)^3.\boldsymbol{P_1} + 3.t.(1-t)^2.\boldsymbol{P_2} + 3.t^2.(1-t).\boldsymbol{P_3} + t^3.\boldsymbol{P_4}$$

$$Q(t) = (1-t)^3.(0,r) + 3.t.(1-t)^2.(h,r) + 3.t^2.(1-t).(r,\boldsymbol{h}) + t^3.(r,0)$$

$$Q(1/2) = \left(\sqrt{2}/2.r, \sqrt{2}/2.r\right) = (1-1/2)^3.(0,r) + 3.1/2.(1-1/2)^2.(h,r) + 3.1/2^2.(1-1/2).(r,\boldsymbol{h}) + 1/2^3.(r,0)$$

$$\begin{cases} \sqrt{2}/2.r = (1-1/2)^3.(0) + 3.1/2.(1-1/2)^2.(\boldsymbol{h}) + 3.1/2^2.(1-1/2).(r) + 1/2^3.(r) \\ \sqrt{2}/2.r = (1-1/2)^3.(r) + 3.1/2.(1-1/2)^2.(r) + 3.1/2^2.(1-1/2).(\boldsymbol{h}) + 1/2^3.(0) \end{cases}$$

$$\boldsymbol{h = \frac{4}{3}(\sqrt{2}-1).r}$$