1 *Signals, Systems and other supporting functions*

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2\, dt \qquad E_\infty = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \qquad P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} |x(t)|^2\, dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{+N} |x[n]|^2 \qquad x(t) = x(t+T),\ T\in\mathbb{R} \qquad x[n] = x[n+N],\ N\in\mathbb{Z}$$

$$x(t) = x_p(t) + x_i(t) \qquad x_p(t) = \frac{x(t)+x(-t)}{2} \qquad x_i(t) = \frac{x(t)-x(-t)}{2}$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[n] \qquad \delta[n] = u[n]-u[n-1] \qquad x(t-t_0)\delta(t) = x(t_0)\delta(t-t_0)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \qquad \delta(t) = \frac{d}{dt}u(t) \qquad \sum_{k=0}^{N-1}\alpha^k = \frac{1-\alpha^N}{1-\alpha}$$

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$$

$$A_{ik} = \frac{1}{(\sigma_i-k)!}\left[\frac{d^{\sigma_i-k}}{d\,u^{\sigma_i-k}}\left[(v-\rho_i)^{\sigma_i}G(v)\right]\right]_{v=\rho_i} \qquad B_{ik} = \frac{1}{(\sigma_i-k)!}(-\rho_i)^{\sigma_i-k}\left[\frac{d^{\sigma_i-k}}{d\,u^{\sigma_i-k}}\left[(1-{\rho_i}^{-1}v)^{\sigma_i}G(v)\right]\right]_{v=\rho_i}$$

2 LIT *Systems*

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \qquad y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \qquad y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau$$

3 CFS — *Continuous Fourier Series*

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{\jmath k\omega_0 t} \qquad a_k = \frac{1}{T}\int_T x(t)e^{-\jmath k\omega_0 t}\, dt$$

$$Ax(t)+By(t) \xrightarrow{CFS} Aa_k + Bb_k \qquad x(t-t_0) \xrightarrow{CFS} a_k e^{-\jmath k\omega_0 t_0}$$

$$e^{-\jmath l\omega_0 t}x(t) \xrightarrow{CFS} a_{k-l} \qquad x(-t) \xrightarrow{CFS} a_{-k}$$

$$x^*(t) \xrightarrow{CFS} a^*_{-k} \qquad x^*(-t) \xrightarrow{CFS} a^*_k$$

$$x(t)y(t) \xrightarrow{CFS} \sum_{l=-\infty}^{+\infty} a_l b_{k-l} \qquad \frac{d}{dt}x(t) \xrightarrow{CFS} \jmath k\omega_0 a_k$$

$$x_i(t) \xrightarrow{CFS} \jmath\,\mathrm{imag}\{a_k\} \qquad x_p(t) \xrightarrow{CFS} \mathrm{real}\{a_k\}$$

$$\mathrm{real}\{x(t)\} \xrightarrow{CFS} a_{kp} = \frac{1}{2}[a_k + a^*_{-k}] \qquad \jmath\,\mathrm{imag}\{x(t)\} \xrightarrow{CFS} a_{ki} = \frac{1}{2}[a_k - a^*_{-k}]$$

$$\int_{-\infty}^{t} x(t)\, dt \xrightarrow{CFS} \frac{1}{\jmath k\omega_0}a_k \qquad \frac{1}{T}\int_T |x(t)|^2\, dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Sinal periódico: $x(t) = x(t+T)$ onde, $x(t) = \begin{cases} 1 & , \quad |t| \leq T_1 \\ 0 & , \quad |t| > T_1 \end{cases} \quad \xrightarrow{CFS} \quad a_k = \dfrac{\sin(k\omega_0 T_1)}{k\pi}$

4 DFS — *Discrete Fourier Series*

$$x[n] = \sum_{k=0}^{N-1} a_k e^{\jmath k \Omega_0 n} \qquad a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-\jmath k \Omega_0 n}$$

$$Ax[n] + By[n] \xrightarrow{DFS} Aa_k + Bb_k \qquad x[n-n_0] \xrightarrow{DFS} a_k e^{-\jmath k \Omega_0 n_0}$$

$$x[n]e^{\jmath n \Omega_0 l} \xrightarrow{DFS} a_{k-l} \qquad x^*[n] \xrightarrow{DFS} a^*_{-k}$$

$$x[-n] \xrightarrow{DFS} a_{-k} \qquad x[n]y[n] \xrightarrow{DFS} \sum_{l=<N>} a_l b_{k-l}$$

$$\sum_{r=<N>} x[r]y[n-r] \xrightarrow{DFS} N a_k b_k \qquad x[n] - x[n-1] \xrightarrow{DFS} (1 - e^{-\jmath k \Omega_0}) a_k$$

$$\sum_{k=-\infty}^{n} x[n] \xrightarrow{DFS} \frac{1}{(1 - e^{-\jmath k \Omega_0})} a_k \qquad \text{real}\{x[n]\} \xrightarrow{DFS} a_{kp} = \frac{1}{2}[a_k + a^*_{-k}]$$

$$\jmath\,\text{imag}\{x[n]\} \xrightarrow{DFS} a_{ki} = \frac{1}{2}[a_k - a^*_{-k}] \qquad x_p[n] \xrightarrow{DFS} \text{real}\{a_k\}$$

$$x_i[n] \xrightarrow{DFS} \jmath\,\text{imag}\{a_k\} \qquad \frac{1}{N}\sum_{n=<N>} |x[n]|^2 = \sum_{k=<N>} |a_k|^2$$

5 CTFT — *Continuous-time Fourier Transform*

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\jmath\omega) e^{\jmath \omega t}\, d\omega \qquad X(\jmath\omega) = \int_{-\infty}^{+\infty} x(t) e^{-\jmath \omega t}\, dt$$

$$Ax(t) + By(t) \xrightarrow{CTFT} AX(\jmath\omega) + BY(\jmath\omega) \qquad x(t-t_0) \xrightarrow{CTFT} e^{-\jmath\omega t_0} X(\jmath\omega)$$

$$x^*(t) \xrightarrow{CTFT} X^*(-\jmath\omega) \qquad x(t) \in \mathbb{R} \xrightarrow{CTFT} X(\jmath\omega) = X^*(-\jmath\omega)$$

$$\text{par}\{x(t)\} \xrightarrow{CTFT} \text{real}\{X(\jmath\omega)\} \qquad \text{impar}\{x(t)\} \xrightarrow{CTFT} \jmath\,\text{imag}\{X(\jmath\omega)\}$$

$$\frac{d}{dt}x(t) \xrightarrow{CTFT} \jmath\omega X(\jmath\omega) \qquad \int_{-\infty}^{t} x(t)\, dt \xrightarrow{CTFT} \frac{1}{\jmath\omega} X(\jmath\omega) + \pi X(0)\delta(\omega)$$

$$x(\alpha t) \xrightarrow{CTFT} \frac{1}{|\alpha|} X(\frac{\jmath\omega}{\alpha}) \qquad x(-t) \xrightarrow{CTFT} X(-\jmath\omega)$$

$$X(t) \xrightarrow{CTFT} 2\pi x(-\jmath\omega) \qquad tx(t) \xrightarrow{CTFT} \jmath \frac{d}{d\omega} X(\jmath\omega)$$

$$x(t)y(t) \xrightarrow{CTFT} \frac{1}{2\pi} X(\jmath\omega) * Y(\jmath\omega) \qquad x(t) * y(t) \xrightarrow{CTFT} X(\jmath\omega)Y(\jmath\omega)$$

6 CTFT — Pares de Transformadas

$$\sum_{k=-\infty}^{+\infty} a_k e^{\jmath\omega_0 t} \xrightarrow{CTFT} 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0) \qquad e^{\jmath\omega_0 t} \xrightarrow{CTFT} 2\pi\delta(\omega-\omega_0)$$

$$\cos(\omega_0 t) \xrightarrow{CTFT} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] \qquad \sin(\omega_0 t) \xrightarrow{CTFT} \frac{\pi}{\jmath}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$$

$$x(t)=1 \xrightarrow{CTFT} 2\pi\delta(\omega) \qquad \delta(t) \xrightarrow{CTFT} 1$$

$$u(t) \xrightarrow{CTFT} \frac{1}{\jmath\omega}+\pi\delta(\omega) \qquad \sum_{n=-\infty}^{+\infty}\delta(t-nT) \xrightarrow{CTFT} \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$$

$$\frac{\sin(Wt)}{\pi t} \xrightarrow{CTFT} X(\jmath\omega)=\begin{cases}1 & , \quad |\omega|\le W\\ 0 & , \quad |\omega|>W\end{cases} \qquad \delta(t-t_0) \xrightarrow{CTFT} e^{-\jmath\omega t_0}$$

$$e^{-\alpha t}u(t), \quad \Re\{\alpha\}>0 \xrightarrow{CTFT} \frac{1}{\alpha+\jmath\omega}$$

$$te^{-\alpha t}u(t), \quad \Re\{\alpha\}>0 \xrightarrow{CTFT} \frac{1}{(\alpha+\jmath\omega)^2}$$

$$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), \quad \Re\{\alpha\}>0 \xrightarrow{CTFT} \frac{1}{(\alpha+\jmath\omega)^n}$$

Sinal periódico: $x(t)=x(t+T)$ onde, $x(t)=\begin{cases}1 & , \quad |t|\le T_1\\ 0 & , \quad |t|>T_1\end{cases} \xrightarrow{CTFT} \sum_{k=-\infty}^{+\infty}\frac{2\sin(k\omega_0 T_1)}{k}\delta(\omega-k\omega_0)$

$$x(t)=\begin{cases}1 & , \quad |t|\le T_1\\ 0 & , \quad |t|>T_1\end{cases} \xrightarrow{CTFT} \frac{2\sin(\omega T_1)}{\omega}$$

$$\int_{-\infty}^{+\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(\jmath\omega)|^2\,d\omega$$

7 DTFT — *Discrete-time Fourier Transform*

$$x[n]=\frac{1}{2\pi}\int_{2\pi}X(\Omega)e^{\jmath\Omega n}\,d\Omega \qquad X(\Omega)=\sum_{n=-\infty}^{+\infty}x[n]e^{-\jmath\Omega n}$$

$$Ax[n]+By[n] \xrightarrow{DTFT} AX(\Omega)+BY(\Omega) \qquad x[n-n_0] \xrightarrow{DTFT} e^{-\jmath\Omega n_0}X(\Omega)$$

$$x^*[n] \xrightarrow{DTFT} X^*(-\Omega) \qquad x[-n] \xrightarrow{DTFT} X(-\Omega)$$

$$x[n]-x[n-1] \xrightarrow{DTFT} (1-e^{\jmath\Omega})X(\Omega) \qquad \sum_{k=-\infty}^{n}x[k] \xrightarrow{DTFT} \frac{1}{1-e^{\jmath\Omega}}X(\jmath\omega)+\pi X(0)\sum_{k=-\infty}^{+\infty}\delta(\Omega-2\pi k)$$

$$nx[n] \xrightarrow{DTFT} \jmath\frac{d}{d\Omega}X(\Omega)$$

$$x[n]y[n] \xrightarrow{DTFT} \frac{1}{2\pi}X(\Omega)*Y(\Omega) \qquad x[n]*y[n] \xrightarrow{DTFT} X(\Omega)Y(\Omega)$$

8 DTFT — Pares de Transformadas

$$x[n] = \sum_{n=<N>} a_k e^{\jmath\Omega_0 n} \xrightarrow{DTFT} X(\Omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\Omega - k\Omega_0)$$

$$e^{\jmath\Omega_0 n} \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} 2\pi\delta(\Omega - \Omega_0 - 2\pi l)$$

$$\cos(\Omega_0 n) \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} \pi[\delta(\Omega - \Omega_0 - 2\pi l) + \delta(\Omega + \Omega_0 - 2\pi l)]$$

$$\sin(\Omega_0 n) \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} \frac{\pi}{\jmath}[\delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)]$$

$$x[n] = 1 \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} 2\pi\delta(\Omega - 2\pi l)$$

$$\delta[n] \xrightarrow{DTFT} 1$$

$$u[n] \xrightarrow{DTFT} \frac{1}{1 - e^{\jmath\Omega}} + \sum_{l=-\infty}^{+\infty} \pi\delta(\Omega - 2\pi l)$$

$$\sum_{n=-\infty}^{+\infty} \delta[n - nN] \xrightarrow{DTFT} \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\Omega - \frac{2\pi k}{T})$$

$$\frac{\sin(Wn)}{\pi n} \xrightarrow{DTFT} X(\jmath\Omega) = \begin{cases} 1 & , \quad 0 \leq |\Omega| \leq W \\ 0 & , \quad W < |\Omega| \leq \pi \end{cases}$$

$$\delta[n - n_0] \xrightarrow{DTFT} e^{-\jmath\Omega n_0}$$

$$\alpha^n u[n],\ |\alpha| < 1 \xrightarrow{DTFT} \frac{1}{1 - \alpha e^{-\jmath\Omega}}$$

$$(n+1)\alpha^n u[n],\ |\alpha| < 1 \xrightarrow{DTFT} \frac{1}{(1 - \alpha e^{-\jmath\Omega})^2}$$

$$\frac{(n+r-1)!}{n!(r-1)!}\alpha^n u[n],\ |\alpha| < 1 \xrightarrow{DTFT} \frac{1}{(1 - \alpha e^{-\jmath\Omega})^r}$$

9 Z — *Z-Transform*

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} \quad , \qquad ROC : R$$

$$\alpha x_1[n] + \beta x_2[n] \xrightarrow{Z} \alpha X_1(z) + \beta X_2(z) \quad , \qquad ROC : (R_1 \cap R_2)$$

$$x[n - n_0] \xrightarrow{Z} z^{-n_0} X(z) \quad , \qquad ROC : R \pm \{z = 0\}$$

$$z_0^n x[n] \xrightarrow{Z} X\left(\frac{z}{z_0}\right) \quad , \qquad ROC : |z_0| \cdot R$$

$$e^{\jmath\omega_0 n} x[n] \xrightarrow{Z} X(e^{-\jmath\omega_0} z) \quad , \qquad ROC : R$$

$$x[-n] \xrightarrow{Z} X(z^{-1}) \quad , \qquad ROC : R^{-1}$$

$$x_{(k)}[n] = \begin{cases} x[r] & , \quad n = rk \\ 0 & , \quad n \neq rk \end{cases} \xrightarrow{Z} X(z^k) \quad , \qquad ROC : R^{1/k}$$

$$x^*[n] \xrightarrow{Z} X^*(z^*) \quad , \quad ROC: R$$

$$x_1[n] * x_2[n] \xrightarrow{Z} X_1(z) \cdot X_2(z) \quad , \quad ROC: (R_1 \cap R_2)$$

$$x[n] - x[n-1] \xrightarrow{Z} \left(1 - z^{-1}\right) X(z) \quad , \quad ROC: (R \cap |z| > 0)$$

$$\sum_{k=-\infty}^{n} x[k] \xrightarrow{Z} \frac{1}{1 - z^{-1}} X(z) \quad , \quad ROC: (R \cap |z| > 1)$$

$$n x[n] \xrightarrow{Z} -z \frac{dX(z)}{dz} \quad , \quad ROC: R$$

10 Z — Pares de Transformadas

$$\delta[n] \xrightarrow{Z} 1 \quad , \quad \forall z$$

$$u[n] \xrightarrow{Z} \frac{1}{1 - z^{-1}} \quad , \quad |z| > 1$$

$$-u[-n-1] \xrightarrow{Z} \frac{1}{1 - z^{-1}} \quad , \quad |z| < 1$$

$$\alpha^n u[n] \xrightarrow{Z} \frac{1}{1 - \alpha z^{-1}} \quad , \quad |z| > |\alpha|$$

$$-\alpha^n u[-n-1] \xrightarrow{Z} \frac{1}{1 - \alpha z^{-1}} \quad , \quad |z| < |\alpha|$$

$$n\alpha^n u[n] \xrightarrow{Z} \frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2} \quad , \quad |z| > |\alpha|$$

$$-n\alpha^n u[-n-1] \xrightarrow{Z} \frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2} \quad , \quad |z| < |\alpha|$$

11 Filtro IIR

$$y[n] = \sum_{k=1}^{N-1} b_k y[n-k] + \sum_{k=0}^{N-1} a_k x[n-k] \qquad s_k = \omega_c e^{\jmath \frac{\pi}{2N}(2k+N+1)}, \quad k = 0, \ldots, 2N-1$$

$$H_s(s) = \sum_{k=0}^{N-1} \frac{A_k}{s - s_k} \qquad H_z(z) = T \sum_{k=0}^{N-1} \frac{A_k}{1 - e^{s_k T} z^{-1}}$$

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} \qquad z = \frac{2 + sT}{2 - sT}$$

$$\omega = \frac{2}{T} \operatorname{tg} \frac{\Omega}{2} \qquad \Omega = 2 \operatorname{arctg} \frac{\omega T}{2}$$

$$H_b(s) H_b(-s) = \frac{1}{1 + \left(\frac{s}{\jmath \omega_c}\right)^{2N}} \qquad |H_b(\jmath\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

12 Filtro FIR

Janela retangular $$w[n] = \begin{cases} 1 & , \quad 0 \le n \le M \\ 0 & , \quad \text{outro n} \end{cases}$$

Janela Barlett $$w[n] = \begin{cases} \frac{2n}{M} & , \quad 0 \le n \le M/2 \\ 2 - \frac{2n}{M} & , \quad M/2 < n \le M \\ 0 & , \quad \text{outro n} \end{cases}$$

Janela Hanning $$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M) & , \quad 0 \le n \le M \\ 0 & , \quad \text{outro n} \end{cases}$$

Janela Hamming $$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M) & , \quad 0 \le n \le M \\ 0 & , \quad \text{outro n} \end{cases}$$

Janela de Kaiser $$w[n] = \begin{cases} \frac{I_0\left(\beta\left(1-[(n-\alpha)/\alpha]^2\right)^{1/2}\right)}{I_0(\beta)} & , \quad 0 \le n \le M \\ 0 & , \quad \text{outro n} \end{cases}$$

onde $\alpha = M/2$

$$A = -20\log(\delta), \qquad \text{onde } \delta \text{ é a atenuação do filtro}$$

$$\beta = \begin{cases} 0.1102(A-8.7) & , \quad A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21) & , \quad 21 \le A \le 50 \\ 0.0 & , \quad A < 21 \end{cases}$$

$$\Delta\Omega = \Omega_s - \Omega_p$$

$$M = \frac{A-8}{2.285\Delta\Omega}$$

Resposta impulsional do filtro ideal $$h[n] = \frac{\sin(\Omega_c n)}{\pi n}$$

13 DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] W_N^{-kn}$$

onde $W_N = e^{-\jmath 2\pi/N}$

COMPARISON OF COMMONLY USED WINDOWS

Window Type	Peak Sidelobe Amplitude (Relative)	Approximate Width of Mainlobe	Peak Approximation Error $20 \log_{10} \delta$ (dB)	Equivalent Kaiser Window β	Transition Width of Equivalent Kaiser Window
Rectangular	-13	$4\pi/(M+1)$	-21	0	$1.81\pi/M$
Bartlett	-25	$8\pi/M$	-25	1.33	$2.37\pi/M$
Hanning	-31	$8\pi/M$	-44	3.86	$5.01\pi/M$
Hamming	-41	$8\pi/M$	-53	4.86	$6.27\pi/M$
Blackman	-57	$12\pi/M$	-74	7.04	$9.19\pi/M$