

Parareal implementation

Consider the following ODE:

$$\begin{cases} \frac{du}{dt} = f(t, u), & t \in [t_0, t_N] \\ u(t_0) = u_0 \end{cases}$$

Introduce the following propagation operator:

- **Coarse propagator** \mathcal{G} (e.g. Forward Euler with $h \sim dT$)
- **Fine propagator** \mathcal{F} (e.g. Forward Euler with $dt = dT/500$)

Notation: $U_n^k \approx u(t_n)$ at the step k , with $t_n = t_0 + n * dT$

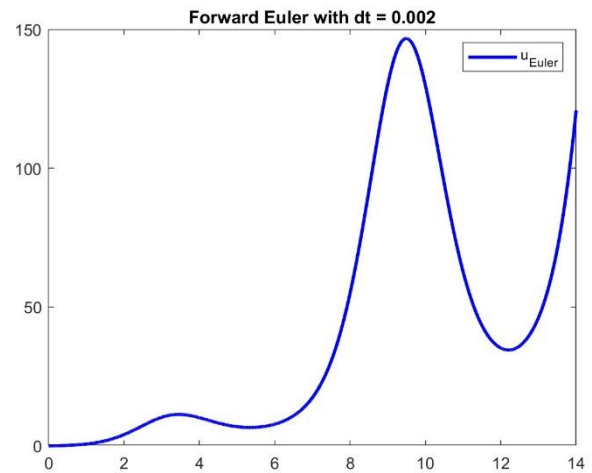
My parareal

$$\begin{cases} u' = u * \sin(t) + t, & t \in [t_0, t_N] \\ u(t_0) = 0 \end{cases}$$

Step ~ 0

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f = @(t,y) sin(t).*y + t;
t0 = 0; tN = 14;
y0 = 0;
dt = 0.002;

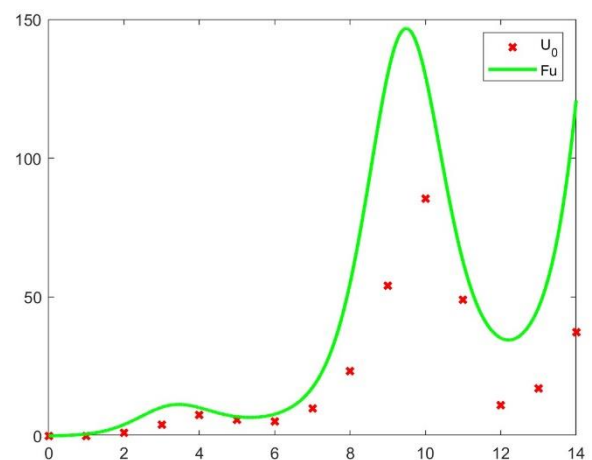
[t,y_fine] = fwd_Euler(t0,tN,y0,dt,f);
```



Step 1

Initial guess, first iteration of \mathcal{G}

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dT = 1;
[t_coarse,U_0] = fwd_Euler(t0,tN,u0,dT,f);
```



Definitions of u_{fine} and t_{fine}

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for m = 1 : (tN - t0) / dT % from 1 to number of coarse subintervals
    t_fine(m,:) = [t0 + (m-1) * dT : dt : t0 + m * dT]; % fine approx. of each subint.
end
% Notice: t_fine(m,end) = t_fine(m+1,1)

% L_coarse = n of coarse subintervals, L_fine = n of fine subinterval
u_fine = zeros(L_coarse, L_fine);

U = U_0;

```

Iterative Step

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for k = 1 : k_max

    % Evaluation of  $\mathcal{F}(t_{n+1}, t_n, U_n^k)$ 
    % Parallel step: computation of the fine solution on each subinterval
    parfor n = 1 : L_coarse - 1
        [t_fine(n,:), u_fine(n,:)] = fwd_Euler(t_coarse(n), t_coarse(n+1), U_0(n), dt, f);
    end

    % Evaluation of  $\mathcal{G}(t_{n+1}, t_n, U_n^k)$ 
    % Update of the coarse solution
    for h = 1 : L_coarse - 1
        du = sin(t_coarse(h)) * U(h) + t_coarse(h);
        U_k(h+1) = U(h) + dT * du;
    end

    % Evaluation of  $\mathcal{G}(t_{n+1}, t_n, U_n^{k+1})$ 
    for n = 1 : L_coarse - 1
        du = sin(t_coarse(n)) * U(n) + t_coarse(n);
        U(n+1) = U(n) + dT * du;

        % Correction step  $U_{n+1}^{k+1} = \mathcal{G}(t_{n+1}, t_n, U_n^{k+1}) + \mathcal{F}(t_{n+1}, t_n, U_n^k) - \mathcal{G}(t_{n+1}, t_n, U_n^k)$ 

        U(n+1) = U(n+1) + u_fine(n, end) - U_k(n+1);
    end

    U_0 = U

```

End

..Some plots now..

