Extending Network training to regression tasks

```
clear all; close all; clc;
addpath('..\')
```

R->R function

We want the network to learn a real valued function

```
f = @(x) sin(10*x);
```

We start by sampling some training points in the interval [a_, b_]

```
n_train = 50;
a_ = 0; b_ = 1;
x = sort(a_ + (b_ - a_).*rand(n_train, 1))';
y = f(x);
```

Specify some hyperparameters of the network.

Note: we switched from `sigmoid` to `tanh` activation function

```
niter = 1e4;
shape = [1, 30, 30, 30, 30, 1];
sigma = @(t) tanh(t);
sigmaprime = @(t) 1 - tanh(t).^2;
eta = 0.005;
```

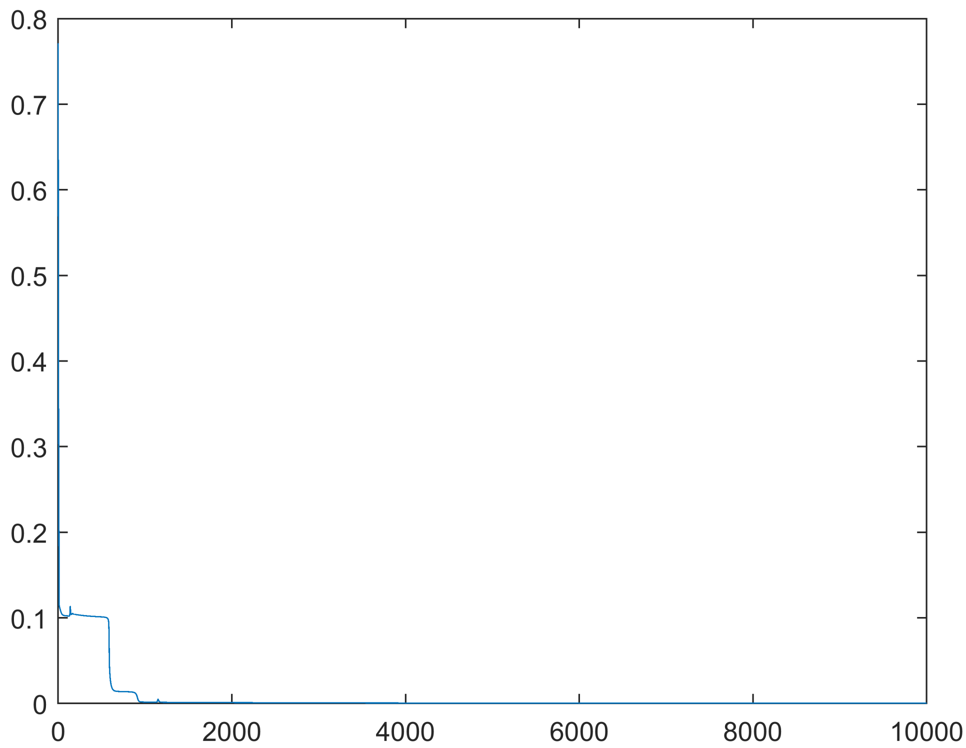
Training

Train the network (or load pre-trained weights)

```
% [costHistory, W, b] = GradientDescent(...
%     x, y, niter, sigma, sigmaprime, eta, shape);
% save sin10xNNparams.mat W b costHistory
load sin10xNNparams.mat
```

Plot the evolution of the cost function

```
figure
plot(linspace(0,niter,niter)', costHistory, '-')
```




```
fprintf('Cost Function: %f\n', costHistory(end));
```

```
Cost Function: 0.000257
```

Test

We sample some other data to perform the testing.

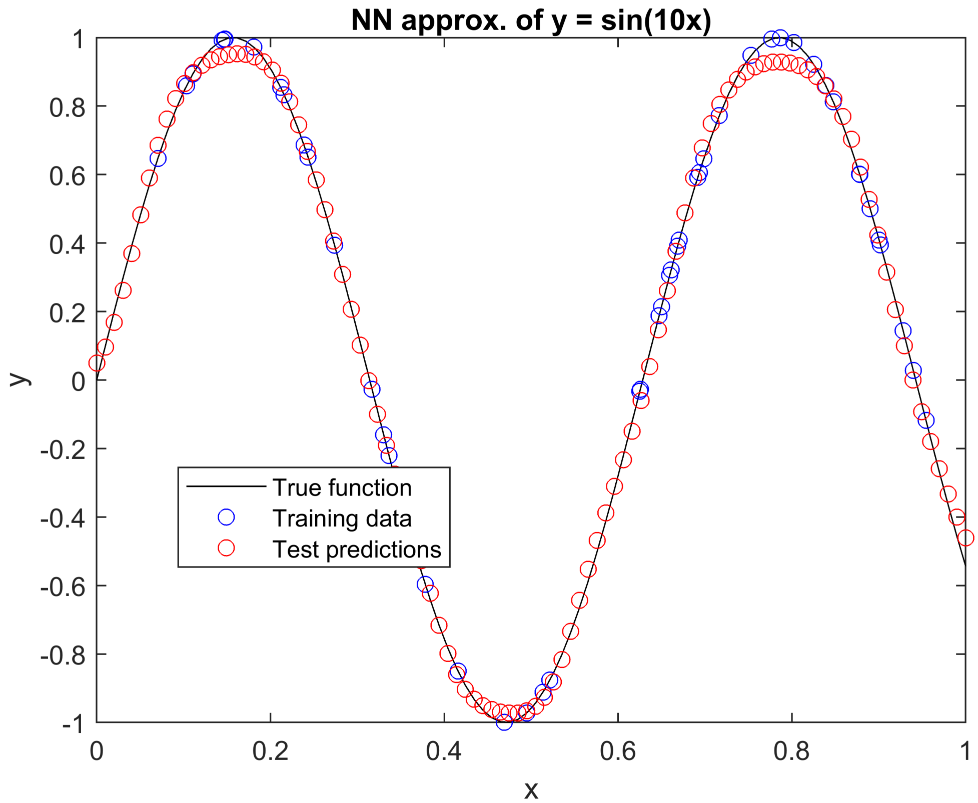
```
n_test = 100;
x_test = linspace(0, 1, n_test);
y_test = f(x_test);
```

Let the test data flow throught the network to get the predictions

```
y_pred = PredictRegression(W, b, sigma, x_test);
```

Plot results

```
figure;
plot(x_test, y_test, 'k-');
hold on;
scatter(x, y, 'bo'); % Training data (blue dots)
scatter(x_test, y_pred, 'ro'); % Test predictions (red dots)
legend('True function', 'Training data', 'Test predictions', Location='best');
xlabel('x');
ylabel('y');
title('NN approx. of y = sin(10x)');
```



Compute errors

```
errorL2 = norm(y_pred-y_test);
errorLp = norm(y_pred-y_test, 10);
errorInf = norm(y_pred-y_test, "inf");
disp(['error L^2: ', num2str(errorL2), ' error L^\infty: ', num2str(errorInf)]);
```

```
error L^2: 0.29794 error L^\infty: 0.082748
```

R^2->R function

```
clear all; clc;
```

Now, the same task can be poerformed in a more involved case, considering a function from $\mathbb{R}^2 \mapsto \mathbb{R}$ (that can still be visualized).

```
f = @(x1, x2) sin(10 * x1) + cos(10 * x2);
```

As done before, we sample some training data.

```
n_train = 1000;
x1 = rand(1, n_train);
x2 = rand(1, n_train);
x = [x1; x2];
y = f(x1, x2);
```

and specify some hyperparameters of the network

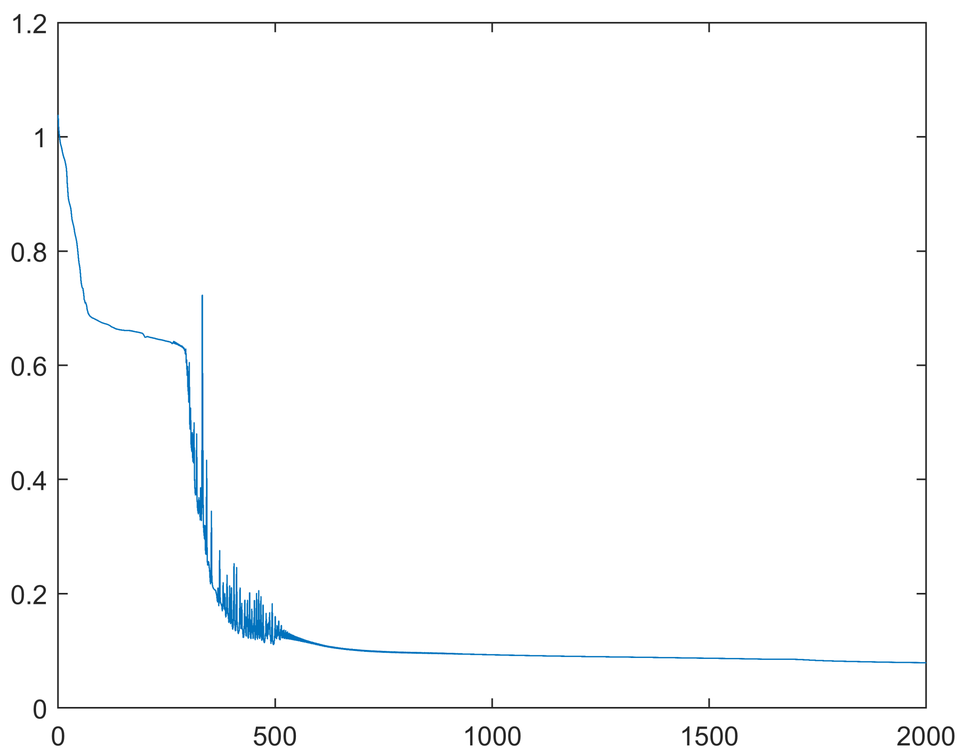
```
niter = 2e3;
shape = [2, 32, 64, 64, 32, 1];
sigma = @(t) tanh(t);
sigmaprime = @(t) 1 - tanh(t).^2;
eta = 0.01;
```

Training

```
% [costHistory, W, b] = GradientDescent(...
%         x, y, niter, sigma, sigmaprime, eta, shape);
% save R2function W b costHistory
load R2function.mat
```

Plot of the cost function:

```
figure
plot(linspace(0,niter,niter)', costHistory, '-')
```




```
fprintf('Cost Function: %f\n', costHistory(end));
```

```
Cost Function: 0.079276
```

Test

As done before, we sample some test data

```
n_test = 30;
x1_test = linspace(0, 1, n_test);
x2_test = linspace(0, 1, n_test);
```

```
[X1_test, X2_test] = meshgrid(x1_test, x2_test);
X_test = [X1_test(:), X2_test(:)]';
Y_test = f(X1_test,X2_test);
```

And, using the network, we compute the predictions:

```
y_pred = PredictRegression(W, b, sigma, X_test);
```

Plot results

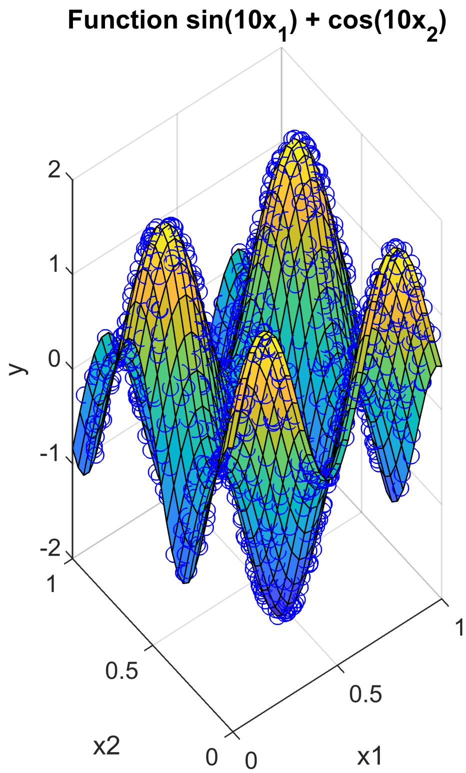
Since it's more difficult to visualize overlaffing 3D data, we plot the training points and the approximated functions in two different subplots.

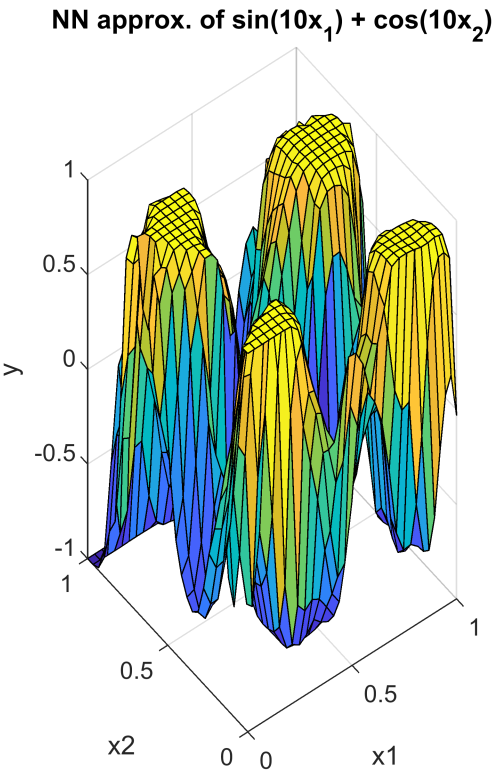
Plot training data and exact function:

```
%%
figure;
subplot(1, 2, 1)
scatter3(x1, x2, y, 'bo'); % training data
hold on;
surf(X1_test, X2_test, Y_test); % exact function
title("Function sin(10x_1) + cos(10x_2)")
xlabel('x1'); ylabel('x2'); zlabel('y');
% legend('Training data')
```

Plot the approximated function in the test points:

```
y_pred_matrix = reshape(y_pred, [30, 30]);
subplot(1, 2, 2)
surf(X1_test, X2_test, y_pred_matrix); % test predictions
title("NN approx. of sin(10x_1) + cos(10x_2)")
% legend('Test predictions');
xlabel('x1'); ylabel('x2'); zlabel('y');
```





Compute errors

```
errorL2 = norm(Y_test(:) - y_pred)
```

```
errorL2 = 891.7554
```

```
errorInf = norm(Y_test(:) - y_pred, "inf")
```

```
errorInf = 1.8355e+03
```

```
disp(['error L^2: ', num2str(errorL2), ' error L^\infty: ', num2str(errorInf)]);
```

```
error L^2: 891.7554 error L^\infty: 1835.5088
```