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1- $V = \frac{\pi h^2 [3R - h]}{3}$; $R = 3\,m$

$V = 30\,m^3$

$$V = \frac{\pi h^2 3R - \pi h^3}{3}$$

$$3V = \pi h^2 3R - \pi h^3$$

$$0 = -\pi h^3 + 3R\pi h^2 - 3V$$

$$0 = \pi h^3 - 9\pi h^2 + 90$$

$\rightarrow f(h) = \pi h^3 - 9\pi h^2 + 90$

$\rightarrow f'(h) = 3\pi h^2 - 18\pi h$

$V = A \cdot h$ $\quad A = \pi r^2$

$A = 9\pi$

$$h = \frac{V}{A} = \frac{30}{9\pi} = 1{,}06\,m \leftarrow h_0$$

$\Rightarrow h_1 = 1{,}06 - \frac{f(1{,}06)}{f'(1{,}06)} = 1{,}06 - \frac{61{,}97}{-49{,}35}$

$h_1 = 2{,}31\,m$ $\quad e = \left|\frac{1{,}06 - 2{,}31}{2{,}31}\right| = 0{,}54$

$h_2 = 2{,}31 - \frac{f(2{,}31)}{f'(2{,}31)} = 2{,}31 - \frac{-22{,}15}{-80{,}35}$

$h_2 \approx 2{,}03$ $\quad e = \left|\frac{2{,}31 - 2{,}03}{2{,}03}\right| = 0{,}13$

$h_3 = 2{,}03 - \frac{f(2{,}03)}{f'(2{,}03)} = 2{,}03 - \frac{-0{,}23}{-75{,}96}$

$h_3 = 2{,}02$ $\quad e = \left|\frac{2{,}03 - 2{,}02}{2{,}02}\right| = 1{,}5 \times 10^{-3}$

$\therefore$ $h = 2{,}02\,m$ con error $1{,}5 \times 10^{-3}$

2- $\begin{cases} 4x + y - z = 13 \\ x - 5y - z = -8 \\ 2x - y - 6z = -2 \end{cases}$ $\quad \bar{x}^0 = (0,0,0)$

$$\left(\begin{array}{ccc|c} 4 & 1 & -1 & 13 \\ 1 & -5 & -1 & -8 \\ 2 & -1 & -6 & -2 \end{array}\right) \qquad b = [13, -8, -2]'$$

$$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -6 \end{pmatrix}, \; D^{-1} = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & -1 & 0 \end{pmatrix}, \; U = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x^{k+1} = -D^{-1}(L+U)x^k + D^{-1}b \\ x^0 \end{cases}$$

$$\Rightarrow \begin{cases} p^1 = -\begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \begin{pmatrix} 13 \\ -8 \\ -2 \end{pmatrix} \\ [0,0,0]' \end{cases}$$

$$p^1 \left[+\frac{13}{4}, +\frac{8}{5}, +\frac{1}{3}\right]' \quad e = \|b - Ap^1\|_2 = 5{,}18$$

$$p^2 = -\begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{13}{4} \\ -8/5 \\ -1/3 \end{pmatrix} + \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \begin{pmatrix} 13 \\ -8 \\ -2 \end{pmatrix}$$

$$p^2 = (2{,}93,\ 2{,}18,\ 0{,}61)' \quad e = \|b - Ap^2\|_2 = 2{,}12$$

$$p^3 = -\begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2{,}93 \\ 2{,}18 \\ 0{,}61 \end{pmatrix} + \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & -1/5 & 0 \\ 0 & 0 & -1/6 \end{pmatrix} \begin{pmatrix} 13 \\ -8 \\ -2 \end{pmatrix}$$

$$p^3 = (2{,}86,\ 2{,}06,\ 0{,}46)' \quad e = \|b - Ap^3\|_2 = 2{,}09$$

Por lo que la solucion del sistema es

$$\begin{cases} x = 2{,}86 \\ y = 2{,}06 \\ z = 0{,}46 \end{cases}$$