Pregunta 1 - Emanuel Esquivel Lopez

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$f(x) = \frac{1}{2}\cos(x) + 5\,\text{sen}(x)$, $\left[0, \frac{3\pi}{2}\right]$, $S = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

a) Calculamos los puntos,

$f(0) = \frac{1}{2}$, $f\left(\frac{\pi}{2}\right) = 5$, $f(\pi) = -\frac{1}{2}$, $f\left(\frac{3\pi}{2}\right) = -5$

$\Rightarrow$ tenemos $\underset{x_0\ y_0}{(0, 1/2)}$, $\underset{x_1\ y_1}{\left(\frac{\pi}{2}, 5\right)}$, $\underset{x_2\ y_2}{\left(\pi, \frac{-1}{2}\right)}$, $\underset{x_3\ y_3}{\left(\frac{3\pi}{2}, -5\right)}$

Calculamos.

$$P(x) = \sum_{k=0}^{n} y_k L_k(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-\frac{\pi}{2})(x-\pi)(x-\frac{3\pi}{2})}{(0-\frac{\pi}{2})(0-\pi)(0-\frac{3\pi}{2})} = \frac{-4x^3}{3\pi^3} + \frac{4x^2}{\pi^2} - \frac{11x}{3\pi} + 1$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-0)(x-\pi)(x-\frac{3\pi}{2})}{(\frac{\pi}{2}-0)(\frac{\pi}{2}-\pi)(\frac{\pi}{2}-\frac{3\pi}{2})} = \frac{4x^3}{\pi^3} - \frac{10x^2}{\pi^2} + \frac{6x}{\pi}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-0)(x-\frac{\pi}{2})(x-\frac{3\pi}{2})}{(\pi-0)(\pi-\frac{\pi}{2})(\pi-\frac{3\pi}{2})} = \frac{-4x^3}{\pi^3} + \frac{8x^2}{\pi^2} - \frac{3x}{\pi}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-0)(x-\frac{\pi}{2})(x-\pi)}{(\frac{3\pi}{2}-0)(\frac{3\pi}{2}-\frac{\pi}{2})(\frac{3\pi}{2}-\pi)} = \frac{4x^3}{3\pi^3} - \frac{2x^2}{\pi^2} + \frac{2x}{3\pi}$$

$$\Rightarrow P(x) = \frac{1}{2}\left(\frac{-4x^3}{3\pi^3} + \frac{4x^2}{\pi^2} - \frac{11x}{3\pi} + 1\right) + 5\left(\frac{4x^3}{\pi^3} - \frac{10x^2}{\pi^2} + \frac{6x}{\pi}\right) \ldots$$

$$-\frac{1}{2}\left(\frac{-4x^3}{\pi^3} + \frac{8x^2}{\pi^2} - \frac{3x}{\pi}\right) - 5\left(\frac{4x^3}{3\pi^3} - \frac{2x^2}{\pi^2} + \frac{2x}{3\pi}\right)$$

$$\boxed{P(x) = \frac{44x^3}{3\pi^3} - \frac{42x^2}{\pi^2} + \frac{79x}{3\pi} + \frac{1}{2}}$$

b) $f(x) = \frac{1}{2}\cos(x) + 5\,\text{sen}(x)$

$f^{(4)}(x) = 5\,\text{sen}(x) + \frac{1}{2}\cos(x)$

$\alpha = \max_{x\in[0,\frac{3\pi}{2}]} \left|\frac{f^{(4)}(x)}{4!}\right| = \frac{1}{4!}\left|5\,\text{sen}(x) + \frac{1}{2}\cos(x)\right|$

$\alpha = \frac{1}{4!} \max_{x\in[0,\frac{3\pi}{2}]}\left(5\,\text{sen}(x) + \frac{1}{2}\cos(x)\right)$

$\alpha = \frac{1}{4!}\cdot\frac{\sqrt{101}}{2} = \frac{\sqrt{101}}{48}$

$\beta = \max_{x\in[0,\frac{3\pi}{2}]} \cdot (x-0)(x-\pi/2)(x-\pi)(x-3\pi/2)$

$\beta = x^4 - 3\pi x^3 + \frac{11\pi^2}{4}x^2 - \frac{3\pi^3}{4}x \to g$ Puntos críticos.

$= 4x^3 - 9\pi x^2 + \frac{11\pi^2 x}{2} - \frac{3\pi^3}{4} \to g'$ $\quad x = \frac{3\pi}{4}, \left(\frac{3}{4} - \frac{\sqrt{5}}{4}\right)\pi, \left(\frac{3}{4} + \frac{\sqrt{5}}{4}\right)\pi$

$g\left(\frac{3\pi}{4}\right) = \frac{9\pi^4}{256}, \quad g\left\{\left(\frac{3}{4} - \frac{\sqrt{5}}{4}\right)\pi\right\} = \frac{\pi^4}{16}, \quad g\left\{\left(\frac{3}{4} + \frac{\sqrt{5}}{4}\right)\pi\right\} = \frac{\pi^4}{16}$

$\Rightarrow$ Por lo tanto $\beta = \max_{x\in[0,\frac{3\pi}{2}]} g(x) = \frac{\pi^4}{16}$

$\therefore \quad |f(x) - P(x)| \leq \alpha\cdot\beta \leq \frac{\sqrt{101}}{48}\cdot\frac{\pi^4}{16} = 1{,}27$

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