Estimation of CGE Parameters

Saketh Aleti

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Outline

- CES vs. CET Theory
- Fossil Fuel Results
- Renewables Results

Theory

CES/CET Problem

$$\textbf{Prod./Trans. Function}: Q = \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi}$$

$$\textbf{Budget Constraint}: B = \sum_i p_i \cdot X_i$$

- p_i are prices, θ is the scaling coefficient, α_i is the share coefficient, B is the budget
- The elasticity of substitution is $\sigma = 1/(1-\phi)$
- The elasticity of transformation is $\psi = 1/(\phi - 1)$

CES versus CET - CES Example

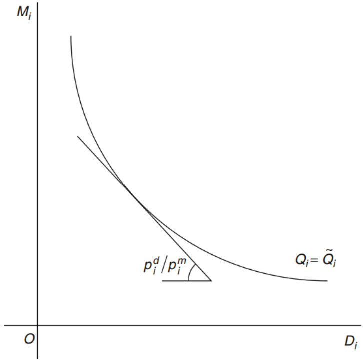


Figure 6.2 Isoquant of the CES function for the Armington composite good

- Isoquant of imported good M_i and domestic good D_i shows combination needed to produce a fixed quantity $\tilde{Q}_i$
- Consumer wants to satisfy a utility function including the composite Q_i
- Convexity shows diminishing returns to consuming only M_i or D_i

CES versus CET - CET Example

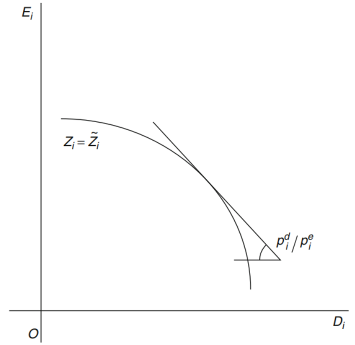


Figure 6.3 Isoquant of the CET function

- Isoquant of exports E_i and domestic supply D_i of some good i
- Shows combination needed to reach a GDP of $\tilde{Z}_i$
- Firms maximize profit by selling goods where they are most valued
- Concavity shows diminishing returns to selling everything domestically versus exporting everything

CES/CET Solution

CES/CET Problem

$$\textbf{Prod./Trans. Function}: \; Q = \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi}$$

$$\textbf{Budget Constraint}: \; B = \sum_i p_i \cdot X_i$$

$$\text{Profit Maximization} \implies \frac{X_i}{X_j} = \left(\frac{\alpha_i \cdot p_j}{\alpha_j \cdot p_i} \right)^{1/(1-\phi)}$$

$$\implies \ln\left(\frac{X_i}{X_j}\right) = \sigma \cdot \ln\left(\frac{\alpha_i \cdot p_j}{\alpha_j \cdot p_i}\right) = \psi \cdot \ln\left(\frac{\alpha_j \cdot p_i}{\alpha_i \cdot p_j}\right)$$

Methodology

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Inputs i and j in state s at time t
- X_i and X_j are the input quantities, p_i and p_j are the input prices
- Share parameters from previous equation are absorbed into the constant term
- This approach avoids endogeneity problems since the input ratios are not dependent on any variables except for price $\implies$ demand is fixed, supply movements trace out demand curve

Methodology (cont.)

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Elasticity of substitution/transformation is the same between all commodities in the production/transformation function
- Suppose coal (i), natural gas (j), and oil (k) are all part of the same production function for electricity

$$\begin{aligned}\implies \ln(X_i/X_j) =& \ \sigma \ln(p_j/p_i) + c_{1,s,t} + \varepsilon_{1,s,t} \\ = \ln(X_j/X_k) =& \ \sigma \ln(p_k/p_j) + c_{2,s,t} + \varepsilon_{2,s,t} \\ = \ln(X_i/X_k) =& \ \sigma \ln(p_k/p_i) + c_{3,s,t} + \varepsilon_{3,s,t}\end{aligned}$$

Methodology (cont.)

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Elasticity of substitution/transformation is the same between all commodities in the production/transformation function

$$\begin{aligned} \implies & corr(\ln(X_i/X_j),\ \ln(p_j/p_i)) \\ = & corr(\ln(X_j/X_k), \ln(p_k/p_j)) \\ = & corr(\ln(X_i/X_k), \ln(p_k/p_i)) \end{aligned}$$

- So, regressions are done by regressing all unique pairs with coefficients constrained to be equal

Methodology (cont.)

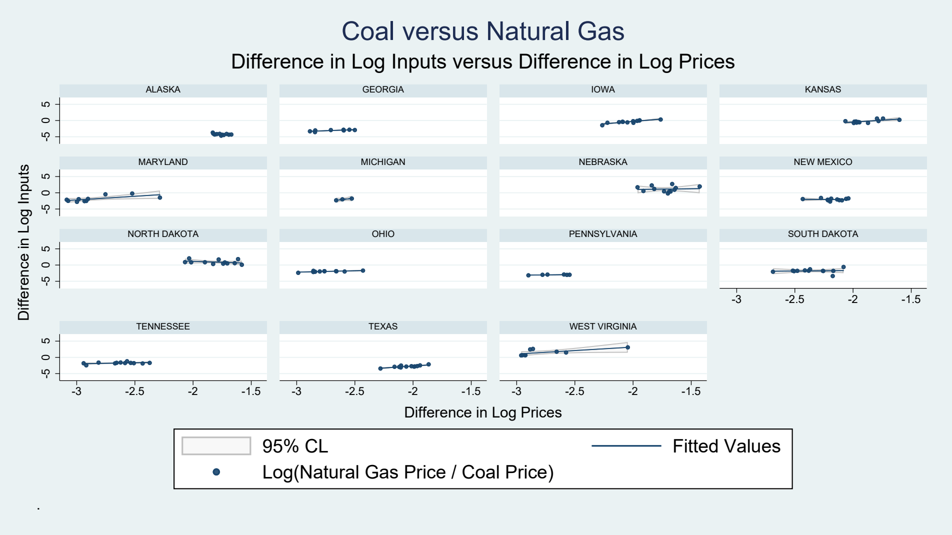
- Constrained regressions on $n * (n-1)/2 - 1$ sets of equations
- EG: (Coal/NatGas + Oil/NatGas), (Coal/NatGas + Oil/Coal), (Oil/Coal + Oil/NatGas)
- Theoretically, any set of equations in parenthesis should give the same results as another set of equations in parenthesis
- Data comes from the EIA for each state in 2016
  - Factor input quantities are set to the amount of each used to generate electricity
  - Factor prices are set to the be the average cost of the each input
  - Coal given thousand tons, Natural gas in thousand McF, and oil in thousand barrels

Fossil Fuel Results

Table 1: Elasticity of Substitution for Fossil Fuels

	Dependent variable: Difference in Log Inputs					
	ln(Coal/NatGas)	ln(Coal/Oil)	ln(Coal/NatGas)	ln(Oil/NatGas)	ln(Coal/Oil)	ln(Oil/NatGas)
$\ln(p_{natgas}/p_{coal})$	1.002***		1.013***			
	(0.209)		(0.210)			
$\ln(p_{oil}/p_{coal})$		1.002***			1.004***	
		(0.209)			(0.209)	
$\ln(p_{natgas}/p_{oil})$				1.013***		1.004***
				(0.210)		(0.209)
Constant	−2.429***	−1.782***	−2.411***	−0.623	−1.783***	−0.643
	(0.398)	(0.190)	(0.399)	(0.519)	(0.190)	(0.517)
State Dummies	✓	✓	✓	✓	✓	✓
Observations	109	109	109	109	109	109
R^2	0.9284	0.9399	0.9284	0.8643	0.9399	0.8642
Chi^2	1414***	1732***	1427***	706***	1725***	701***

Fossil Fuel Results



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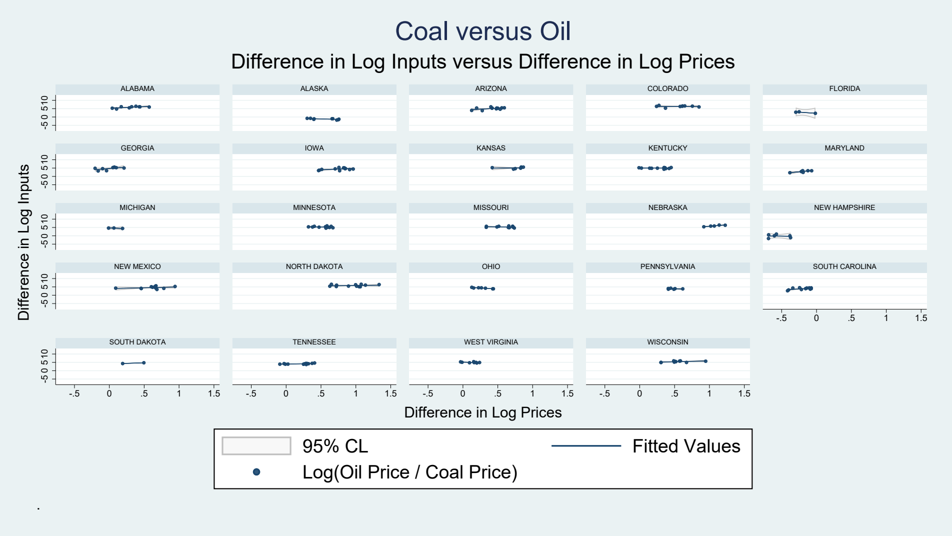
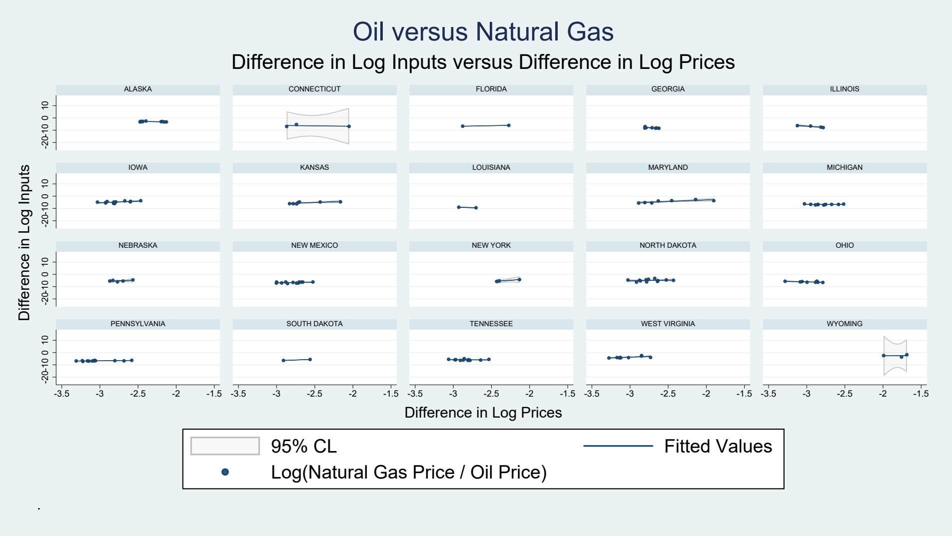
Fossil Fuel Results



Fossil Fuel Results



Methodology - A Different Approach

CES Parameter Optimization

$$\min_{\theta,\phi,\alpha,\beta} \left|\left| -\ln\left(Q_{elec}\right) + \ln(\theta) + \frac{1}{\phi} \cdot \ln\left(\alpha_{coal} X_{coal}^{\phi} + \alpha_{natgas} X_{natgas}^{\phi} + \alpha_{oil} X_{oil}^{\phi}\right) + \beta \cdot D_{state} \right|\right|_2^2$$

- Can estimate all parameters at once using non-linear least squares
- D_{state} represents indicator variables for each state
- Can also check fit statistics to see if CES functional form matches the data

Results

Table 2: CES Parameter Estimation for Fossil Fuels - NLS

	Dependent Variable: Electricity Output from Fossil Fuels	
	$\ln(\theta) + \frac{1}{\phi} \cdot \ln\left(\alpha_{coal} X_{coal}^{\phi} + \alpha_{natgas} X_{natgas}^{\phi} + \alpha_{oil} X_{oil}^{\phi}\right) + \beta \cdot D_{state}$	
θ	1.144 (0.609)	0.676*** (0.0941)
ϕ	0.917*** (0.0916)	0.900*** (0.0658)
$\ln(\alpha_{coal})$	0.533 (0.433)	1.147 (.)
$\ln(\alpha_{natgas})$	-1.323* (0.536)	-0.925*** (0.165)
$\ln(\alpha_{oil})$	1.308 (.)	-25.94 (.)
State Dummies	✓	
Observations	518	518
R^2	0.934	0.796
Adj. R^2	0.928	0.795

Fossil Fuel Results

CES $\to$ Cobb-Douglas

$$\lim_{\phi \to 0} \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi} = \theta \cdot \prod_i X_i^{\alpha_i}$$

- Elasticity of substitution ($\sigma = \frac{1}{1-\phi}$) varied by approach
  - Ratios between commodities led to σ around 1 (Cobb-Douglas)
  - Non-linear least squares led to estimates around 10 (strong substitutes)
- Could try estimating the Translog Function
  - Second Order Taylor Polynomial of CES around $\phi = 0$
  - Works best when $\phi \approx 0 \implies$ true function is almost Cobb-Douglas

Methodology - Renewables

- Still working on getting production costs for solar and wind
- Currently have solar and wind elasticities are estimated using 3SLS
- Structural Equation Model:
    - Solar Supply
    - Solar Demand
    - Wind Supply
    - Wind Demand

Methodology - Renewables

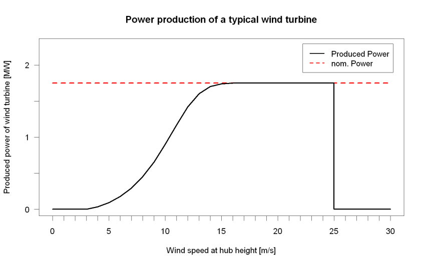
- Instrumental Variables Approach
  - Are price and quantity changes caused by shifts in demand or shifts in supply?
  - $Q_s = \beta_1 \cdot P + \gamma \cdot Z + \varepsilon_1$
  - $Q_d = \beta_2 \cdot P + \varepsilon_2$
  - Z is an instrument in the supply equation $\implies$ identifies the demand curve
  - Must have Z has independent of demand and good at explaining supply
- Instruments for each equation
  - Solar Supply: Average Solar Radiation, Land Area
  - Solar Demand: Population, Cooling/Heating Degree Days
  - Wind Supply: Average Wind Speed, Land Area
  - Wind Demand: Population, Cooling/Heating Degree Days

Methodology - Renewables

- Solar Supply Instruments
  - Solar energy generation $\approx$ Efficiency $\cdot$ Area $\cdot$ **Radiation**
  - Taking logs $\implies$ ln(*solar generation*) is linear with ln(*solar radiation*)
  - **Land area** does not seem to have a log-log relationship with ln(*solar generation*)
- Solar Demand Instruments
  - **Population** likely multiplicative with solar generation $\implies$ linear function with logs
  - **Cooling/Heating Degree Days** - number of degrees that a day's average temperature is below/above room temperature

Methodology - Renewables

- Wind Supply Instruments
  - **Land area** used again here
  - Wind generation seems to increase exponentially with **wind speed** in theory
  - Again, taking logs $\implies$ ln(*wind generation*) is linear with (*average wind speed*)



- Wind Demand Instruments are the same as those for solar

Table 3: Elasticities for Solar and Wind - 3SLS Estimation

	Dependent Variable			
	ln(Solar Net Gen)	ln(Solar Net Gen)	ln(Wind Net Gen)	ln(Wind Net Gen)
log(Electricity Price)	14.46***	-4.619*	4.997***	-16.98***
(*cents/kWh*)	(1.597)	(1.911)	(1.487)	(4.086)
log(Average Solar Radiation)	101.5*			
(kWh/m^2)	(51.33)			
Log(Average Wind Speed)			0.678***	
(m/s)			(0.0811)	
Land Area	1.14e-11***		1.41e-11***	
(m^2)	(1.16e-12)		(9.79e-13)	
Log(Population)		1.085***		1.166***
		(0.136)		(0.286)
Log(Cooling Degree Days)		-0.164		-0.322
		(0.102)		(0.191)
Log(Heating Degree Days)		-0.169		-0.0660
		(0.101)		(0.182)
Constant	-66.97***	7.541	-24.11***	65.21***
	(7.367)	(7.963)	(6.961)	(16.81)
State Dummies	✓	✓	✓	✓
Obs	504	504	504	504
Chi^2	109.3***	81.32***	76.64***	41.26***

Future Work

- Working on non-linear least squares approach for renewables
- Adding biomass, hydroelectric, nuclear, and geothermal elasticities
- Elasticity of substitution between clean and dirty energy
- Can verify final results with price elasticity estimates

Questions?