I. Introduction

In CGE modeling, energy production often consists of a CES function of energy sources with an empirically estimated elasticity of substitution; this elasticity is meant to capture factors such as intermittency between sources. However, this top down approach may be missing the actual source of the elasticity of substitution. Rather than firms following a general CES function,[1] we may instead have firms simply following a linear production function where energy output is instead the sum of the input of each energy source multiplied by a conversion factor. Although more complex models than this could be conceived, the necessary element of such a model is that increasing the input of one energy source does not decrease the output of another source as it does in a CES function.[2] Without a CES function, we can still find a non-zero elasticity of substitution between energy sources from another source - household preferences. So, instead of a CES of energy inputs on the producer side, consumers might follow a utility function that captures their preferences for particular amounts of energy based on the time of day. Because agents prefer to have energy at different proportions based on the time of day, intermittency then plays a key role in determining the elasticity of substitution. The result of using such preferences in a CGE is that we can capture substitutability between energy sources without having to directly model intermittency on the production side.

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Things I'd like to say:

- Because the marginal product of renewable energy inputs may be time-dependent, renewable energy may complement traditional, constant-output energy sources rather than fully substitute for it. Obvious arugment follows from base load output required at all times.
- How the conversion parameters for energy inputs affect their elasticity of substitution
- How variability in energy output decreases the value of particular energy sources. Obvious argument from concavity of utility implying risk aversion.

[1] In practice, it is difficult to differentiate between firms following a CES production function versus a particular cost function that implies similar factor demand. For example, the cost function $C(X_1, X_2) = X_1^2 + X_2^2$ implies cost minimization when X_1/X_2 is proportional to the marginal rate of technical substitution. Similarly, X_1/X_2 is proportional to the ratio of their prices when using a CES function. Empirically, determining whether we have the above cost function, the latter production function, or both is not possible with only prices and quantities.

[2] Suppose we have the CES function $Y = (X_1^\phi + X_2^\phi)^{1/\phi}$. It's fairly simple to see that $\partial Y/\partial X_1$ is decreasing with X_2. But, realistically, it would not make sense if the energy output of the coal sector decreased with the size of the solar energy sector.

II. Model

First, suppose we define the energy output at time t as $Y(t)$. Consumers theoretically might prefer that some proportion of their energy arrive at time t_1 while some other fixed proportion arrive at t_2 if all prices at all times are equal. Additionally, suppose they would be willing to shift their consumption between different times of the day, thus substituting between $Y(t)$. Then, we can suppose that consumers follow CES utility function:

$$U = \left(\int_0^T \alpha(t) Y(t)^{\phi} dt \right)^{1/\phi} \tag{1}$$

where $\phi < 1$ to preserve concavity and $\phi > 0$ to allow for $Y(t) = 0$ at some t. The elasticity of substitution is $\sigma \equiv 1/(1-\phi)$ and $\alpha(t)$ captures the desire for particular proportions of energy use at various times of the day. Furthermore, T is a fixed value that represents the length of a day. Assuming a fixed budget of 1, we have the budget constraint:

$$\int_0^T Y(t) p(t) dt = 1 \tag{2}$$

Consequently, the Lagrangian may be set up as $U^{\phi} - \lambda(\int_0^T Y(t)p(t)dt - 1)$. In the following equations, we simplify the notation by letting $T = 1$. Solving to find the first order conditions at times t_1 and t_2, we see that:

$$\begin{aligned}
\frac{Y(t_1)}{Y(t_2)} &= \left(\frac{p(t_1)\,\alpha(t_2)}{p(t_2)\,\alpha(t_1)} \right)^{-\sigma} \\
1 &= Y(t_2)\, p(t_2)^{\sigma}\, \alpha(t_2)^{-\sigma} \int_0^1 p(t_1)^{1-\sigma}\, \alpha(t_1)^{\sigma}\, dt_1 \\
\implies Y(t_2) &= \frac{(\alpha(t_2)/p(t_2))^{\sigma}}{\int_0^1 p(t_1)^{1-\sigma}\, \alpha(t_1)^{\sigma}\, dt_1}
\end{aligned}$$

Now we define the price index $P \equiv \left(\int_0^1 p(t)^{1-\sigma}\, \alpha(t)^{\sigma}\, dt \right)^{1/(1-\sigma)}$ and simplify an earlier derivation to get the demand function:

$$Y(t) = (p(t)/\alpha(t))^{-\sigma}\, P^{1-\sigma} = \left(\frac{p(t)}{\alpha(t)P} \right)^{-\sigma} U \tag{3}$$

where U, our utility, is equivalent to the inverse of P.[3]

Next, we introduce energy sources into this model. For simplicity, suppose we only have two sources of energy, X_1 and X_2, which are converted into energy at rates $\xi_1(t)$ and $\xi_2(t)$; additionally, assume that the $\xi(t)$ functions are continuous with compact support.

[3] The derivation is:

$$\begin{aligned}
U^{\phi} &= \int_0^1 Y(t)^{\phi} dt = \int_0^1 (p(t)/\alpha(t))^{1-\sigma}\, P^{(\sigma-1)\phi} dt \\
&= P^{(\sigma-1)\phi} \int_0^1 (p(t)/\alpha(t))^{1-\sigma} dt = P^{(\sigma-1)\phi} P^{1-\sigma} = P^{-\phi}
\end{aligned}$$

Then, we have:

$$Y(t) \equiv \xi_1(t)X_1 + \xi_2(t)X_2 \tag{4}$$

$$Y = X_1 \int_0^1 \xi_1(t)dt + X_2 \int_0^1 \xi_1(t)dt \tag{5}$$

Firms pick X_1 and X_2 to maximize profit while facing the cost function $C(X_1, X_2)$; we set $\partial C/\partial X_i > 0$ to ensure convex costs. To simplify the algebra, suppose that we have the cost function $C = (c_1 X_1^2 + c_2 X_2^2)/2$. Then, we have:

$$\Pi = \int_0^1 Y(t)p(t)dt - (X_1^2 + X_2^2)/2 \tag{6}$$

The first order conditions for maximizing profit are:

$$\frac{\partial \Pi}{\partial X_i} = 0 \implies \int_0^1 \xi_i(t)\, p(t)dt = c_i X_i$$

Hence, we have the profit-maximizing condition:

$$\frac{X_1}{X_2} = \frac{c_2 \int_0^1 \xi_1(t)\, p(t)dt}{c_1 \int_0^1 \xi_2(t)\, p(t)dt} \tag{7}$$

In this case, since firms are setting the ratio X_1/X_2 to the ratio of their marginal profit, they are also fixing it to the ratio of their shadow prices faced by consumers. Therefore, with the cost function used here, the market equilibrium is equivalent to a centrally planned equilibrium.

Next, consider the marginal utility of X_1:

$$\frac{\partial U^\phi}{\partial X_1} = \int_0^1 \alpha(t)\, \phi\, Y(t)^{\phi-1} \xi_1(t)\, dt = \frac{\partial U^\phi}{\partial U} \frac{\partial U}{\partial X_1} \tag{8}$$

The marginal rate of substitution must then be:

$$MRS_{12} = \frac{\int_0^1 \alpha(t)\, Y(t)^{\phi-1}\, \xi_1(t)\, dt}{\int_0^1 \alpha(t)\, Y(t)^{\phi-1}\, \xi_2(t)\, dt} \tag{9}$$

References

Stern, D. (2012). "Interfuel Substitution: A Meta-Analysis." Journal of Economic Surveys 26 (2): 307–331. https://doi.org/10.1111/j.1467-6419.2010.00646.x.