Efficient pollution abatement in electricity markets with intermittent renewable energy

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I. Extended Abstract

Renewable energy technologies have seen considerable adoption over the last few decades (EIA 2019). Unlike the alternatives, wind and solar power are particularly unique in that the amount of energy they supply is intermittent. Consequently, designing economically efficient policy to promote their adoption is not straightforward given that they cannot easily substitute for traditional technologies such as coal power which handles base load demand. Some of the literature has approached the problem by constructing numerical models that find the cheapest renewable technology set while accounting for intermittent supply. Müsgens and Neuhoff (2006) model uncertain renewable output with intertemporal generation constraints, while Neuhoff, Cust, and Keats (2007) model temporal and spatial characteristics of wind output to optimize its deployment in the UK. On the other hand, other literature focuses on the effect of intermittent technologies on the market itself; Ambec and Crampes (2010) study the interaction between intermittent renewables and traditional reliable sources of energy in decentralized markets, and Chao (2011) models alternative pricing mechanisms for intermittent renewable energy sources. Additionally, Borenstein (2010) reviews the effects of present public policies used to promote renewables and the challenges posed by intermittency.

In this paper, we present a theoretical model of electricity markets with intermittent renewable energy and derive the optimal public policy to handle pollution externalities. In contrast with other theoretical models of intermittency which optimize a pubic electricity sector, our model assumes utility and profit maximization. Additionally, it represents the energy sector over multiple periods with electricity output for each technology varying over time for intermittent technologies. This approach is better suited for studying the dynamics of present US electricity markets which are primarily funded by private sector investment and have prices that vary over time. We first consider a simpler version of our model with two periods and two energy generation technologies, derive the comparative statics for this model, and detail the policy implications. Then, we produce numerical results for optimal policy prescriptions with a multi-period, multi-technology version of our model using electricity generation data on the PGM region.

Present models of the energy sector concerned with the adoption of renewable energy often discuss the elasticity of substitution between clean and dirty energy; this elasticity is meant to capture factors such as intermittency and reliability that impede perfect substitution between energy sources. Additionally, this elasticity is often estimated empirically or modelled in a CGE using a CES production function with capital inputs for each energy technology. However, this top down approach may be missing the actual source of the substitution effects while also being unrealistic. For instance, consider a simple case where energy output is a Cobb-Douglas function (a special case of CES) of two energy inputs: solar and coal. According to this function, increasing the amount of coal input causes the marginal product of solar input to rise; this makes

little sense in practical terms, since the output of additional solar panels should not be related to the amount of coal input. Moreover, this approach, while possibly relevant for other sectors' goods, weakens the accuracy of theoretical and CGE models focused on the energy sector.

However, it is still possible to produce a more accurate model of the electricity sector that can capture trade-offs between the energy output of different technologies from both cost and intermittency. To start, rather than energy production following a general CES function, we first assume that energy follows a linear production function where total energy is instead the sum of the energy output of each source. The purpose of using a linear production function is that increasing the input of one energy source does not change the marginal product of another source as it does in a CES function. Additionally, we split this production function up across periods; that is, in each period, the electricity generation is equal to the energy output of each source in that period. For intermittent technologies, energy output varies in each period, so total output would as well. Next, we assume that firms pick a profit-maximizing level of investment in each energy technology. This investment stays fixed in all periods, but the total energy output can vary over time due to intermittency; for instance, a solar plant provides a variable output by time of day, while nuclear plant cannot easily vary its output within 24 hours. Moreover, we assume that firms face linear costs and electricity prices are set equal to their marginal cost; the former assumption is temporarily made for mathematical tractability while the latter is realistic given the competitiveness of electricity markets.

While the production function described so far is linear, we may still find a non-zero elasticity of substitution between energy sources. That is, consider a representative agent with a CES utility function that captures intertemporal variation in the utility gained from energy consumption. Specifically, the utility function is composed of energy consumption differentiated by period; so, for example, in a two-period model, we may have off-peak consumption and on-peak consumption as our two goods. Because people prefer to consume energy at different proportions based on the time of day, this variation can be modeled through the share parameters in the CES function. Moreover, since people may substitute energy consumption across time by availability and prices, the CES elasticity parameter captures the intertemporal elasticity of substitution. Thus, in this model, the intermittency of an energy generation technology plays a key role in determining its substitutability with other technologies. For instance, solar power may be a good substitute for coal power during the day, but obviously not at night; consequently, this pair complements each other, since coal handles the base load while solar handles the peak load. Alternatively, wind and solar are a poor combination, since both produce intermittently; this pair is closer to being substitutes. All in all, the result of using CES preferences with temporally differentiated energy consumption is that we can capture substitutability/complementarity between energy sources in an accurate way.

The first significant result in this model is that the optimal quantity of investment in intermittent renewable technology is concave with respect to both cost efficiency (cost per unit) and output efficiency (energy output per unit). So, for example, suppose some location has a coal-fired plant and is considering investing in a solar power plant. A 10% increase in the output per solar panel may increase the optimal quantity of solar panels by x%, while a 20% increase in solar efficiency will increase the optimal quantity of solar by y% where y ¡ 2x. Because of this concavity, exponential increases in efficiency of intermittent renewable sources over time, as Moore's Law may predict, is not enough to fully substitute out reliable energy technology. Hence, we argue that a full transition to renewable energy requires more emphasis on technologies such as nuclear, biomass, hydro, and geothermal energy sources; although these sources compete with intermittent energy technology, they are able fully substitute out for fossil fuel energy. Additionally, the reliable output of these technologies can handle base loads; this allows them to complement the intermittency of renewables such as wind and solar.

Secondly, suppose that we would like to promote clean energy adoption and replace dirty energy to reduce the pollution emissions. Because we have diminishing returns from both forms of efficiency, it is optimal to subsidize both research and the cost of intermittent renewable technology. This is because we expect research subsidies to increase both output efficiency and cost efficiency, while subsidies would increase the cost efficiency of renewables for a market participant. A mix of both instruments leads to improving both forms of efficiency at once, thus replacing dirty energy technology in the most cost-efficient way. Alternatively, since we would see symmetrical effects from taxing dirty energy sources rather than subsidizing clean ones, taxing dirty energy sources can substitute for renewable cost subsidies. Consequently, a carbon tax plus a research subsidy for renewables is another optimal choice for pollution abatement.

Finally, our model implies that the optimal level of subsidies/taxes on energy generation should vary by the state of the local energy market. That is, the change in optimal quantity of intermittent energy technology with respect to both types of efficiency is also a function of the efficiencies of the other technologies available. Consequently, the marginal benefit of a subsidy or tax varies spatially, since different geographies have access to different energy sources. For instance, one community may be relying on hydropower while another may be using coal power; the effect of a 1% cost subsidy on solar installments in these two communities would vary because of the differences in the communities' pre-existing energy generation technologies. Hence, local communities that aim to promote clean energy and reduce pollution should optimize their policy instruments to suit their local energy markets.

A. References

- Literature
- Description of model
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II. Outline

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Things I'd like to say:

- Because the marginal product of renewable energy inputs may be time-dependent, renewable energy may complement traditional, constant-output energy sources rather than fully substitute for it. Obvious arugment follows from base load output required at all times.
- How the conversion parameters for energy inputs affect their elasticity of substitution
- How variability in energy output decreases the value of particular energy sources. Obvious argument from concavity of utility implying risk aversion.

III. Introduction

Renewable energy technologies have seen considerable adoption over the last few decades (EIA 2019). Unlike the alternatives, wind and solar power are particularly unique in that the amount of energy they supply is intermittent. Consequently, designing economically efficient policy to promote their adoption is not straightforward given that they cannot easily substitute for traditional technologies such as coal power which handles base load demand. Moreover, simpler approaches, such as using the levelized cost of electricity (LCOE) to compare technologies is flawed in any setting with variable electricity prices. In particular, Joskow (2011) argues that

> "Instead [of using metrics like LCOE], the economics of all generating technologies, both intermittent and dispatchable, can be evaluated based on the expected market value of the electricity that they will supply, their total life-cycle costs, and their associated expected profitability. Such an analysis would reflect the actual expected production profiles of dispatchable and intermittent technologies, the value of electricity supplied at different times, and other costs of intermittency associated with reliable network integration. This is exactly the way investors in merchant generating plants evaluate whether or not to invest."

We structure our model with this critique in mind. Specifically, we develop a model where producers optimize the supply of each generation technology on the expected market value of the electricity produce. We do so by

Some of the literature has approached the problem by constructing numerical models that find the cheapest renewable technology set while accounting for intermittent supply. Musgens and Neuhoff (2006) model uncertain renewable output with intertemporal generation constraints, while Neuhoff et al. (2007) model temporal and spatial characteristics of wind output to optimize its deployment in the UK. On the other hand, other literature focuses on the effect of intermittent technologies on the market itself; Ambec and Crampes (2012) study the interaction between intermittent renewables and traditional reliable sources of energy in decentralized markets, and Chao (2011) models alternative pricing mechanisms for intermittent renewable energy sources. Additionally, Borenstein (2012) reviews the effects of present public policies used to promote renewables and the challenges posed by intermittency.

Our model comes closest to that of Helm and Mier (2019) who build a peak-load pricing model where the availability of renewable capacity varies stochastically. This stochastic variability models the intermittency and affects the adoption of renewables in a market with dynamic pricing. Furthermore, they simplify their analysis by setting aside complications such as outage costs and rationing rules. In total, their model finds an S-shaped adoption curve for renewables once their LCOE's reach those of fossil fuel energy. In addition, they find that a Pigouvian tax can properly internalize the costs of fossil fuels in a setting with perfect competition. Likewise, our model makes equivalent simplifications and finds similar results on an equilibrium in amarket with dynamic prices. But, our representation of the intermittency of renewables differs in a significant way.

To highlight this difference, we must first contrast the terms *intermittency* and *reliability*. By intermittency, we mean uncontrollable variation in output over time due to physical constraints; by reliability, we mean variation in output that is wholly unpredictable[1] and may be the result of failures that cannot be modeled stochastically. Consider wind energy as an example. The energy provided by a wind turbine necessarily varies over time due to differences in wind speed and direction. Moreover, the relationship between the force created by the wind and the amount of wind power generated must necessarily be a fixed, functional relationship; that is, *ceterius paribus*, two identical wind turbines under identical wind conditions should generate the same

[1]By unpredictible, we mean a notion similar to that of *Knightian uncertainity* (Warkins and Knight, 1922).

amount of energy. Consequently, the output of wind power can be forecasted[2] up to an error term. Because no forecast of wind (or solar) power can be perfect, some level of weather-related intermittency may be left in the error term. At the same time, the error term also captures unreliability; for instance, failures in the circuitry of wind turbines cannot be forecasted as a time series. So, overall, we may think of the energy output of a renewable technology as consisting of three components: a knowable/forecastable component related to intermittency, a stochastic component related to intermittency, and an unpredictible component related to reliability.

While Helm and Mier (2019) model the stochastic component of intermittency, we model the predictible component. More specifically, Helm and Mier model the capacity of renewable energy equal to a baseline capacity multiplied by a uniform random variable between 0 and 1. This is a valid approach given that their model consists of a single period of generation. On the other hand, we neglect stochastic intermittency for parsimony and develop a general, multi-period model where the output per unit of each technology follows a known function. We then narrow this to a two period, two technology model where our first technology represents fossil fuel energy with constant output in both periods while our second technology's output differs between each period. Additionally, while their model consists of two sets of consumers, one set which receives fixed price contracts and another paying real time prices, we consider one representative consumer. Specifically, this representative consumer's utility is given by CES function where each good is electricity consumption differentiated by time; this approach was first set forth by Mohajeryami et al. (2016). Interestingly, this treatment of intermittency produces some key results similar to that of Helm and Mier.

IV. Model

We model electricity as a heterogenous good which is differentiated by its delivery time. Then, we consider a representative consumer who purchases varying quantities of electricity in each period in order to maximize utility. Generally, people prefer to spread their electricity consumption out over time; the convexity of the indifference surfaces of a standard CES function can model this preference in a simple way. Next, we model the electricity sector using a representative firm that chooses a set of electricity-generating inputs to maximize profit; some of these inputs are intermittent - they generate a differing amounts of energy in each period. These two sides of the market reach an equilibrium through adjustments in prices of electricity in each period.[3] For simplicity, we do not allow for load rationing or negative prices. And, again for simplicity, we assume that the output of all technologies in each period does not vary stochastically.

A. General Model

Consumers purchase variable amounts of electricity Z_t over each period t. Assuming the price of electricity in each period is held constant, they prefer that a fixed proportion of their energy arrive in period t while some other fixed proportion arrive at $s \neq t$. Finally, we suppose that consumers would be willing to shift their consumption from one period to another in response to a shift in prices. In total, these assumptions can be captured using a representative consumer

[2] Foley et al. (2012) provide a review of the literature on the forecasting of wind power and advancements over time

[3] While, in reality, consumers generally pay fixed rates for electricity, Helm and Mier provide a motivation for models incorporating dynamic pricing; they argue such approaches to pricing will become the norm with further technological advances and coming regulatory changes.

with the standard CES utility function

$$U = \left(\sum_t \alpha_t Z_t^{\phi}\right)^{1/\phi} \tag{1}$$

where $\sigma = 1/(1-\phi)$ is the intertemporal elasticity of substitution for electricity consumption. We define $\sum_t \alpha_t = 1$, so that α_t is the fraction of electricity consumption in period t when all prices are equal; naturally, $\alpha_t > 0$ for all t. The budget constraint is given by

$$I = \sum_t p_t Z_t \tag{2}$$

where p_t is the price of electricity in period t and I is the income. Consequently, the first order conditions when maximizing utility given this budget constraint imply:

$$Z_t = \left(\frac{\alpha_t}{p_t}\right)^{\sigma} \frac{I}{P} \tag{3}$$

$$P = \sum_t \alpha_t^{\sigma} p_t^{1-\sigma} \tag{4}$$

where P is the price index. This model naturally does not allow for blackouts in equilibrium, since the price of electricity in any period gets arbitrarily large as the quantity of energy consumed in that period falls.

Secondly, we have firms maximizing profit by picking an optimal set of energy inputs. In reality, electricity markets are fairly competitive, so we assume perfect competition. Hence, we can model the set of firms using a representative firm that sets marginal revenue equal to marginal cost. We define the quantity of deployed energy technology i as X_i, and we define its output per unit in period t as $\xi_{1,t}$. So, for example, if i is solar power, X_i would be the number of solar panels and $\xi_{i,t}$ may be kW generated per solar panel in period t. Consequently, the energy generated in period t, Z_t, is given by $\sum_i \xi_{i,t} X_i$. We define these variables in matrices to simplify notation:

$$X \equiv \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix},\ Z \equiv \begin{pmatrix} Z_1 \\ \vdots \\ Z_m \end{pmatrix},\ p \equiv \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix},\ \xi \equiv \begin{pmatrix} \xi_{1,1} & \dots & \xi_{1,m} \\ \vdots & \ddots & \\ \xi_{n,1} & & \xi_{n,m} \end{pmatrix}$$

where we have n technologies and m periods. Also, note that we have $Z \equiv \xi^T X$. Firms pick X to maximize profit while facing the cost function $C(X_1, \dots, X_n)$ where C is convex with respect to each input. Total profit is given by

$$\Pi = p^T Z - C(X_1, \dots, X_n) \tag{5}$$

To simplify the algebra, suppose we $n = m$. We further make the assumption that the output per unit of each technology is unique and non-negative in each period; in other words, the output per unit of one technology is not a linear combination of those of the other technology in our set. This then implies that ξ is of full rank and therefore invertible. Lastly, suppose that we have a linear cost function $C = \sum_i c_i X_i = c^T X$ where c_i is the cost per unit of X_i Now, maximizing profit, we find the first order condition:

$$\frac{\partial \Pi}{\partial X} = 0 \implies p = \xi^{-1} c \tag{6}$$

Combining both first order conditions allows us to find the equilibrium; however, in order to produce more comprehensible results, we consider a further simplification.

Equilibrium Results

Firstly, we restrict the utility function to its Cobb-Douglas form which is simply the case where the elasticity of substitution $\sigma = 1$. Secondly, we limit the number of periods and technologies to 2. And, thirdly, we normalize the prices such that the budget is 1. Consequently, our demand equations simplify to:

$$Z_t = \alpha_t / p_t \tag{7}$$
$$Z_s = \alpha_s / p_s \tag{8}$$

where t and s are our two periods. Next, solving for the FOC condition for profit maximizing (which remains the same), we have:

$$p = \xi^{-1} c$$
$$p = \begin{pmatrix} -\dfrac{c_1\,\xi_{2s} - c_2\,\xi_{1s}}{\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}} \\ \dfrac{c_1\,\xi_{2t} - c_2\,\xi_{1t}}{\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}} \end{pmatrix}$$

And, substituting back into our demand equations, we have the equilibrium quantities for Z and X.

$$Z = \begin{pmatrix} \dfrac{\alpha_t\,(\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s})}{c_2\,\xi_{1s} - c_1\,\xi_{2s}} \\ \dfrac{\alpha_s\,(\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s})}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \end{pmatrix} \implies X = \begin{pmatrix} \dfrac{\alpha_t\,\xi_{2s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} + \dfrac{\alpha_s\,\xi_{2t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \\ -\dfrac{\alpha_t\,\xi_{1s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} - \dfrac{\alpha_s\,\xi_{1t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \end{pmatrix}$$

Furthermore, we derive restrictions on the parameters ξ and c by assuming $Z, X > 0$. These restrictions are detailed in Table 1. There are two possible sets of symmetrical restrictions. The first set, Case 1, assumes that technology 2 is more cost effective in period t, while the second set, Case 2, assumes that technology 1 is more cost effective in period t. If a given set of parameters do not fall into either case, we are left with an edge case where one of the technologies is not used. Additionally, these inequalities compare two types of efficiency – output efficiency and cost efficiency; we define output efficiency as electricity output per unit of input and cost efficiency in terms of electricity output per dollar of input. We refer to the last set of restrictions as mixed, because they relate both cost and output efficiency.

Table 1: PARAMETER RESTRICTIONS FOR $Z, X > 0$

	Case 1	**Case 2**
Cost Efficiency Restrictions	$\xi_{2t}/c_2 > \xi_{1t}/c_1$ $\xi_{1s}/c_1 > \xi_{2s}/c_2$	$\xi_{2t}/c_2 < \xi_{1t}/c_1$ $\xi_{1s}/c_1 < \xi_{2s}/c_2$
Output Efficiency Restrictions	$\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t}$	$\xi_{2t}/\xi_{2s} < \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} < \xi_{2s}/\xi_{2t}$
Mixed Efficiency Restrictions	$\dfrac{\alpha_s\,(\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t\,(\xi_{2t}/c_2 - \xi_{1t}/c_1)} > \xi_{2s}/\xi_{2t}$ $\dfrac{\alpha_s\,(\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t\,(\xi_{2t}/c_2 - \xi_{1t}/c_1)} < \xi_{1s}/\xi_{1t}$	$\dfrac{\alpha_s\,(\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t\,(\xi_{2t}/c_2 - \xi_{1t}/c_1)} < \xi_{2s}/\xi_{2t}$ $\dfrac{\alpha_s\,(\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t\,(\xi_{2t}/c_2 - \xi_{1t}/c_1)} > \xi_{1s}/\xi_{1t}$

Note: The inequalities in this table assume that all elements of ξ are greater than 0. The full proof given below provides equivalent restrictions for the zero cases.

Proof: We aim to derive conditions on ξ and c required to have positive Z and X, so we begin by assuming $X, Z > 0$. Second, since the equations so far are symmetrical, note that there be two symmetrical sets of potential restrictions we must impose on the parameters. Thus, we first assume the inequality $c_1\xi_{2t} - c_2\xi_{1t} > 0$ to restrict ourselves to one of the two cases. This assumption results in the denominator of Z_s being positive. Hence, we must also have $\xi_{1s}\xi_{2t} - \xi_{2s}\xi_{1t} > 0$ for $Z_s > 0$. This same term appears in the numerator for Z_t, hence its denominator must be positive: $c_2\xi_{1s} - c_1\xi_{2s} > 0$. Now, rewriting these inequalities, we have:

$$\begin{aligned}
c_1\xi_{2t} - c_2\xi_{1t} > 0 &\implies \xi_{2t}/c_2 > \xi_{1t}/c_1 \\
c_2\xi_{1s} - c_1\xi_{2s} > 0 &\implies \xi_{1s}/c_1 > \xi_{2s}/c_2 \\
\xi_{1s}\xi_{2t} - \xi_{2s}\xi_{1t} > 0 &\implies \xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t} \\
&\implies \xi_{1t}/\xi_{1s} < \xi_{2t}/\xi_{2s}
\end{aligned}$$

Note that the latter two restrictions can be derived from the former two. Additionally, we implicitly assume that we have $\xi > 0$. However, this is not necessary assumption, since ξ invertible only requires $\xi_{1t}\xi_{2s} > 0$ or $\xi_{1s}\xi_{2t} > 0$. Instead, we may leave the latter two inequalities in the form $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$ which remains valid when values of ξ are equal to 0. Lastly, the mixed efficiency restrictions come from $X > 0$. To start, for X_1, we have:

$$\begin{aligned}
X_1 > 0 &\implies (\alpha_t\xi_{2s})(c_1\xi_2 t - c_2\xi_1 t) + (\alpha_s\xi_{2t})(c_1\xi_{2s} - c_2\xi_{1s}) < 0 \\
&\implies (\alpha_t\xi_{2s})(c_1\xi_2 t - c_2\xi_1 t) < (\alpha_s\xi_{2t})(c_2\xi_{1s} - c_1\xi_{2s}) \\
&\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s(c_2\xi_{1s} - c_1\xi_{2s}))/(\alpha_t(c_1\xi_{2t} - c_2\xi_{1t})) \\
&\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s(\xi_{1s}/c_1 - \xi_{2s}/c_2))/(\alpha_t(\xi_{2t}/c_2 - \xi_{1t}/c_1))
\end{aligned}$$

Similarly, for X_2, note that only the numerators differ; ξ_{2s} is replaced with $-\xi_{1s}$ and ξ_{2t} is replaced with $-\xi_{1t}$. Hence, we have

$$\begin{aligned}
X_2 > 0 &\implies (\alpha_t\xi_{1s})(c_1\xi_2 t - c_2\xi_1 t) + (\alpha_s\xi_{1t})(c_1\xi_{2s} - c_2\xi_{1s}) > 0 \\
&\implies (\xi_{1s}/\xi_{1t}) > (\alpha_s(\xi_{1s}/c_1 - \xi_{2s}/c_2))/(\alpha_t(\xi_{2t}/c_2 - \xi_{1t}/c_1))
\end{aligned}$$

To double check, note that combining the inequalities from $X_1 > 0$ and $X_2 > 0$ leads to $\xi_{2s}/\xi_{2t} < \xi_{1s}/\xi_{1t}$. This is precisely the earlier result obtained from $Z > 0$. Again, it is important to note that we assume $\xi > 0$ for to simplify the inequalities of $X_1 > 0$ and $X_2 > 0$. Otherwise, we may leave the inequalities in their pre-simplified forms and they are still valid when $\xi_{1t}\xi_{2s} > 0$ or $\xi_{1s}\xi_{2t} > 0$.

■

Let us consider the set of restrictions belonging to Case 1. The first inequality, our initial assumption, states that technology 2 is relatively more cost effective in period t. The second inequality claims technology 1 is relatively more cost effective in period s. The implications are fairly straightforward; if a technology is to be used, it must have an absolute advantage in cost efficiency in at least one period. The third condition states that the relative output efficiency of technology 2 is greater than that of the first technology in period t. And, the fourth condition makes a symmetrical claim but for the technology 1 and period s. These latter two restrictions regarding output efficiency enter Z and X through p; they're simply a restatement of the invertibility of ξ and can also be derived through the cost efficiency restrictions.

The mixed efficiency restrictions are less intuitive. Firstly, note that $(\xi_{1s}/c_1 - \xi_{2s}/c_2)$ is the difference in cost efficiency for the two technologies in period s; this is equivalent to the increase in Z_s caused by shifting a marginal dollar towards technology 1. Similarly, the bottom term $(\xi_{2t}/c_2 - \xi_{1t}/c_1)$ represents the change in Z_t caused by shifting a marginal dollar towards technology 1. Both these terms are then multiplied by the share parameter of the utility function for their respective time periods. Furthermore, note that α_t (α_s) is the elasticity of utility with respect to Z_t (Z_t). Hence, in total, the mixed efficiency restrictions relate the relative cost efficiencies of each technology with their output efficiency and the demand for energy. So, for example, suppose that consumers prefer, *ceteris paribus*, that nearly all their electricity arrive in period t. This would imply α_t is arbitrarily large which results in the left-hand side of the fraction becoming arbitrarily small. This violates the first mixed efficiency restriction but not the second; consequently, use of the first technology, which is less cost effective in period t, approaches 0.

In more practical terms, suppose that our first technology was coal power and latter was solar power; additionally, assume period t and s are the peak and off-peak for a day. We know that the output efficiency of coal is constant through the day, while that of solar power is higher in period t – this ensures the output efficiency restriction: $\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$. Additionally, we can reasonably assume that coal is more cost effective than solar in the off-peak period when there is less sun; hence, the second cost efficiency restriction is satisfied. Thirdly, the first condition must be true for there to be an incentive to use solar power; that is, solar needs be cost effective during peak hours otherwise we hit an edge case where no solar is employed. Finally, we face the mixed efficiency condition, which essentially implies that there must be sufficient demand for electricity during period t, when solar is more effective, for it to be a feasible technology. So, overall, for a technology to be feasible, we have three conditions: it must the most cost effective in a particular period, its output efficiency must be maximized in the same period, and there must be sufficient amount of demand in that period.

Comparative Statics

The comparative statics are similarly intuitive. The equilibrium quantity of a technology is increasing with its output efficiency and decreasing with its cost. Additionally, the equilibrium quantities for a particular technology move in the opposite direction with respect to the output efficiency and cost of the other technologies. So, for example, an increase in the output efficiency of solar or a decrease in solar's cost will reduce the optimal quantity of coal power. To find the effects of α on X, we must assume one of the cases of restrictions shown in 1Table 1. So, assume Case 1 is true; this implies that X_2 is optimal in period t and X_1 is optimal in period s. Additionally, note that α determines demand for electricity in a period. Hence, when α_t rises, we see the optimal level of X_2 rise as well; likewise, X_1 rises with α_s. So, essentially, the optimal quantity of a technology rises with the demand shifter α of the period that the technology is optimal in. This concept carries through for the comparative statics of Z. When the output efficiency of technology 1 rises or its cost falls, we see output Z_s rise and output Z_t fall. This is because technology 1 is optimal in period s given the Case 1 restrictions. Moreover, we would see symmetrical results for the output with respect to the cost and output efficiency of technology 2. In total, the comparative statics are fairly straightforward.

Proof: We begin by deriving the comparative statics of the cost and efficiency parameters with respect to X. Firstly, we take derivatives with respect to the cost vectors:

$$\frac{\partial X_1}{\partial c} = \begin{pmatrix} -\dfrac{\alpha_t\,{\xi_{2\mathrm{s}}}^2}{(c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}})^2} - \dfrac{\alpha_s\,{\xi_{2\mathrm{t}}}^2}{(c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}})^2} \\ \dfrac{\alpha_t\,\xi_{1\mathrm{s}}\,\xi_{2\mathrm{s}}}{(c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}})^2} + \dfrac{\alpha_s\,\xi_{1\mathrm{t}}\,\xi_{2\mathrm{t}}}{(c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}})^2} \end{pmatrix} = \begin{pmatrix} <0 \\ >0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial c} = \begin{pmatrix} \dfrac{\alpha_t\,\xi_{1\mathrm{s}}\,\xi_{2\mathrm{s}}}{(c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}})^2} + \dfrac{\alpha_s\,\xi_{1\mathrm{t}}\,\xi_{2\mathrm{t}}}{(c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}})^2} \\ -\dfrac{\alpha_t\,{\xi_{1\mathrm{s}}}^2}{(c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}})^2} - \dfrac{\alpha_s\,{\xi_{1\mathrm{t}}}^2}{(c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}})^2} \end{pmatrix} = \begin{pmatrix} >0 \\ <0 \end{pmatrix}$$

The first and second terms of $\partial X_1/\partial c_1$ are clearly both negative independent of the restrictions on the parameters. Similarly, all terms of $\partial X_1/\partial c_2$ are positive independent of any restrictions.

Since the structure of this problem is symmetrical with respect to X_1 and X_2, the same comparative statics apply but in reverse for X_1. Next, we derive comparative statics for each element of ξ.

$$\frac{\partial X_1}{\partial \xi} = \begin{pmatrix} \frac{\alpha_s\, c_2\, \xi_{2\mathrm{t}}}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{\alpha_t\, c_2\, \xi_{2\mathrm{s}}}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \\ \frac{-\alpha_s\, c_2\, \xi_{1\mathrm{t}}}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{-\alpha_t\, c_2\, \xi_{1\mathrm{s}}}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \end{pmatrix} = \begin{pmatrix} >0 & >0 \\ <0 & <0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial \xi} = \begin{pmatrix} \frac{-\alpha_s\, c_1\, \xi_{2\mathrm{t}}}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{-\alpha_t\, c_1\, \xi_{2\mathrm{s}}}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \\ \frac{\alpha_s\, c_1\, \xi_{1\mathrm{t}}}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{\alpha_t\, c_1\, \xi_{1\mathrm{s}}}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \end{pmatrix} = \begin{pmatrix} <0 & <0 \\ >0 & >0 \end{pmatrix}$$

Again, the signs are fairly straightforward. The optimal quantity of X_1 increases with its output efficiency in both periods; however, it decreases with the output efficiency of X_2 in both periods. Similarly, symmetrical results are shown for X_2. Next, we study the effects of α on X; this requires us to place some restrictions on the parameters, so we use those belonging to Case 1 in Table 1. With $\alpha \equiv (\alpha_t \;\; \alpha_s)^T$,

$$\frac{\partial X_1}{\partial \alpha} = \begin{pmatrix} \frac{\xi_{2\mathrm{s}}}{c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}} \\ \frac{\xi_{2\mathrm{t}}}{c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}} \end{pmatrix} = \begin{pmatrix} <0 \\ >0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial \alpha} = \begin{pmatrix} \frac{-\xi_{1\mathrm{s}}}{c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}} \\ \frac{-\xi_{1\mathrm{t}}}{c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}} \end{pmatrix} = \begin{pmatrix} >0 \\ <0 \end{pmatrix}$$

Note that our restrictions imply that $c_1\xi_{2t} - c_2\xi_{1t} > 0$ and $c_2\xi_{1s} - c_1\xi_{2s} > 0$. From here, the intuition is clear; we assume that X_2 is more cost efficient in period t, so increases in demand during period t (caused by increases in α_t) will increase the optimal quantity of X_2. And, the same applies to X_1 with respect to period s and α_s. Again, due to symmetry, the statics are reversed when the technologies are flipped. Similarly, the signs would also be flipped if we used the restrictions given by Case 2 instead.

Next, we derive the comparative statics for Z.

$$\frac{\partial Z_t}{\partial c} = \begin{pmatrix} \frac{\alpha_t\, \xi_{2\mathrm{s}}\, \left(\xi_{1\mathrm{s}}\, \xi_{2\mathrm{t}} - \xi_{1\mathrm{t}}\, \xi_{2\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \\ \frac{-\alpha_t\, \xi_{1\mathrm{s}}\, \left(\xi_{1\mathrm{s}}\, \xi_{2\mathrm{t}} - \xi_{1\mathrm{t}}\, \xi_{2\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \end{pmatrix} = \begin{pmatrix} >0 \\ <0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial c} = \begin{pmatrix} \frac{-\alpha_s\, \xi_{2\mathrm{t}}\, \left(\xi_{1\mathrm{s}}\, \xi_{2\mathrm{t}} - \xi_{1\mathrm{t}}\, \xi_{2\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} \\ \frac{\alpha_s\, \xi_{1\mathrm{t}}\, \left(\xi_{1\mathrm{s}}\, \xi_{2\mathrm{t}} - \xi_{1\mathrm{t}}\, \xi_{2\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} \end{pmatrix} = \begin{pmatrix} <0 \\ >0 \end{pmatrix}$$

From our restrictions, we have $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$, and all the results above follow from this.

$$\frac{\partial Z_t}{\partial \xi} = \begin{pmatrix} \frac{\alpha_t\, \xi_{2\mathrm{s}}}{c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}} & \frac{-\alpha_t\, \xi_{2\mathrm{s}}\, \left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \\ \frac{-\alpha_t\, \xi_{1\mathrm{s}}}{c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}} & \frac{\alpha_t\, \xi_{1\mathrm{s}}\, \left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)}{\left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)^2} \end{pmatrix} = \begin{pmatrix} <0 & <0 \\ >0 & >0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial \xi} = \begin{pmatrix} \frac{-\alpha_s\, \xi_{2\mathrm{t}}\, \left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{\alpha_s\, \xi_{2\mathrm{t}}}{c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}} \\ \frac{\alpha_s\, \xi_{1\mathrm{t}}\, \left(c_1\, \xi_{2\mathrm{s}} - c_2\, \xi_{1\mathrm{s}}\right)}{\left(c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}\right)^2} & \frac{-\alpha_s\, \xi_{1\mathrm{t}}}{c_1\, \xi_{2\mathrm{t}} - c_2\, \xi_{1\mathrm{t}}} \end{pmatrix} = \begin{pmatrix} >0 & >0 \\ <0 & <0 \end{pmatrix}$$

Again, recall that we have $c_1\xi_{2t} - c_2\xi_{1t} > 0$ and $c_2\xi_{1s} - c_1\xi_{2s} > 0$; the rest follows. And finally, we have:

$$\frac{\partial Z_t}{\partial \alpha} = \begin{pmatrix} \dfrac{-\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} \\ 0 \end{pmatrix} = \begin{pmatrix} > 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial \alpha} = \begin{pmatrix} 0 \\ \dfrac{\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \end{pmatrix} = \begin{pmatrix} 0 \\ > 0 \end{pmatrix}$$

These are fairly trivial, since $Z_t = \alpha_t/p_t$ (and $Z_s = \alpha_s/p_s$) and prices are positive.
■

Technical Change and Market Shares

Like Helm and Mier (2019) we find an S-shaped adoption curve for renewable energy driven by intermittency.[4]

Externalities

Often times, a social planner may also be interested in using other policy instruments such as research subsidies. In fact, Acemoglu et al. (2012) find that both carbon taxes and research subsidies are necessary to promote long-run structural change in the energy production sector and avert a climate disaster. While our model focuses on production on a shorter time scale, we find a similar result – a combination of both taxes and subsidies may be more effective in promoting a shift towards clean energy.

Firstly, consider a case where a carbon tax is our only policy instrument. Additionally, suppose that our first technology produced negative externalities. Specifically, the social cost of pollution is given by $S(X) = \gamma X_1$ where γ is the monetary damage per unit of X_1. A social planner is interested in balancing the damages caused by pollution with the surplus of the private sector and the revenue from the tax. That is, they aim to solve

$$\max_{\tau}\ CS + PS - S(X) + \tau X_1$$

where τ is the tax on X_1, so that its final price is $c_1 + \tau$ and the tax revenue is given by τX_1. The optimal tax is a Pigouvian tax; τ should be set equal to the marginal social cost of pollution γ. This is an expected, standard solution in most models.

Proof: We aim to maximize welfare with respect to the tax τ, hence our first order condition is:

$$\begin{aligned} 0 &= \frac{\partial CS}{\partial \tau} + \frac{\partial PS}{\partial \tau} - \frac{\partial S(X)}{\partial \tau} + \frac{\partial \tau X_1}{\partial \tau} \\ &= \left(\underbrace{\frac{\partial CS}{\partial p} + \frac{\partial PS}{\partial p}}_{=0} - \frac{\partial S(X)}{\partial p} + \frac{\partial \tau X_1}{\partial p} \right) \frac{\partial p}{\partial \tau} \\ &= -\gamma \frac{\partial X_1}{\partial p} + \tau \frac{\partial X_1}{\partial p} \end{aligned}$$

[4] Helm and Mier (2019) reference Geroski (1957) who provides a survey of the literature on adoption of new technologies. He finds that S-shaped adoption curves are often generated by information cascades. On the other hand, our model and that of Helm and Mier find S-shaped curves as a result of intermittency slowing down adoption.

where the derivatives of producer and consumer surplus are eliminated by the envelope theorem. Therefore, we must have $\tau = \gamma$.

■

Now, suppose that we may also subsidize research which improves the output efficiency of a technology. In particular, we let the output efficiency of technology 2 be $(\xi_{2t}, \xi_{2s})(1+\beta)$ where β is the percent improvement created by research. And, we let the cost of research be given by a convex, differentiable function $G(\beta)$.

Proof: First, note that we have:

$$X_1 = \frac{\alpha_t\,(1+\beta)\,\xi_{2\mathrm{s}}}{(c_1+\tau)\,(1+\beta)\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}}} + \frac{\alpha_s\,(1+\beta)\,\xi_{2\mathrm{t}}}{(c_1+\tau)\,(1+\beta)\;\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}}}$$

We aim to maximize welfare with respect to the tax τ and the research efficiency modifier β, hence our first order condition is:

$$\begin{aligned}
0 &= \frac{\partial CS}{\partial \tau} + \frac{\partial PS}{\partial \tau} - \frac{\partial S(X)}{\partial \tau} + \frac{\partial \tau X_1}{\partial \tau} - \frac{\partial G(\beta)}{\partial \beta} \\
&= \left(\underbrace{\frac{\partial CS}{\partial p} + \frac{\partial PS}{\partial p}}_{=0} - \frac{\partial S(X)}{\partial p} + \frac{\partial \tau X_1}{\partial p} - \frac{\partial G(\beta)}{\partial p} \right) \frac{\partial p}{\partial \tau} \\
&= -\gamma \frac{\partial X_1}{\partial p} + \tau \frac{\partial X_1}{\partial p} - \frac{\partial G(\beta)}{\partial p} \\
\tau &= \gamma + \frac{\partial G(\beta)}{\partial X_1}
\end{aligned}$$

where the derivatives of producer and consumer surplus are eliminated by the envelope theorem. Equivalently, if we take the derivative of the welfare function with respect to β, we find that:

$$G(\beta) = -\gamma + \frac{\partial \tau}{\partial X_1}$$

■

V. Conclusion

An aim for future research may be to develop a model of clean and dirty energy that incorporates both predictable and stochastic intermittency in a multi-period setting.

VI. Notes

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VII. Scratch

VIII. Literature

- Delarue et al. (2010) develop a model that distinguishes between power and energy in order to split up costs and risks into fixed and variable factors.
- Shahriari and Blumsack (2018) look at interactions between different solar and wind installations (covar matrix), while many other energy models do not.

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