

# Efficient pollution abatement in electricity markets with intermittent renewable energy

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## I. Introduction

Renewable energy technologies have seen considerable adoption over the last few decades (EIA 2019). Unlike the alternatives, wind and solar power are particularly unique in that the amount of energy they supply is intermittent. Consequently, designing economically efficient policy to promote their adoption is not straightforward given that they cannot easily substitute for fossil fuel technologies such as coal power. Moreover, traditional economic measures such as the levelized cost of electricity (LCOE) fail to capture the true economic value of intermittent technologies, because they neglect to account for variation in output and prices over time (Joskow, 2011).

Some of the literature has overcome this critique by constructing numerical models that find the cheapest renewable technology set while accounting for intermittent supply. For instance, Musgens and Neuhoff (2006) model uncertain renewable output with intertemporal generation constraints, while Neuhoff et al. (2007) model temporal and spatial characteristics of wind output to optimize its deployment in the UK. On the other hand, other literature focuses on the effect of intermittent technologies on the market itself; Ambec and Crampes (2012) study the interaction between intermittent renewables and traditional reliable sources of energy in decentralized markets, and Chao (2011) models alternative pricing mechanisms for intermittent renewable energy sources. Additionally, Borenstein (2012) reviews the effects of present public policies used to promote renewables and the challenges posed by intermittency.

Our model comes closest to that of Helm and Mier (2019) who build a peak-load pricing model where the availability of renewable capacity varies stochastically. This stochastic variability models intermittency and negatively affects the adoption of renewables. Furthermore, they simplify their analysis by setting aside complications such as outage costs and rationing rules. In total, their model finds an S-shaped adoption curve for renewables as they get cheaper. In addition, they find that a Pigouvian tax can properly internalize the costs of fossil fuels in a setting with perfect competition. Likewise, we model the equilibrium of a market with dynamic prices and access to both renewable and fossil energy. However, our results differ in key ways, because we take a different approach to modeling intermittency.

To highlight this difference, we must first contrast the terms *intermittency* and *reliability*. By intermittency, we predict changes in output related to physical constraints. For instance, while wind energy output varies over time, if we know the angle and speed of wind, we may

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precisely derive the quantity of energy that a wind farm generates.<sup>1</sup> At the same time, these forecasts may not be perfect and, as a result, we see unexpected variation in the output of a technology. This unpredictability may be better captured by the notion of reliability; specifically, we refer to definition provided by the US Department of Energy’s ORNL (2004) – “Power reliability can be defined as the degree to which the performance of the elements in a bulk system results in electricity being delivered to customers within accepted standards and in the amount desired.” This is more so related to the stochastic or unpredictable variation in the output of a technology. Unlike intermittency, this cannot be planned around with 100% certainty. A practical example may be a wind turbine’s systems failing. While we may know the chances of this occurring, it’s not possible to know when it will occur ahead of time; consequently, this may result in a temporary reduction in the quantity and quality of the electricity delivered – a loss of reliability. In short, when Helm and Mier define renewable output as equal to a baseline capacity multiplied by a uniform random variable, their model is closer to one of reliability rather than intermittency.

On the other hand, we model intermittency as defined above by allowing the output of renewable energy to differ between periods according to known function. But, for parsimony, we assume that the output of all technologies in each period does not vary stochastically; that is, we focus on intermittency rather than reliability. We then model the electricity sector using a representative firm that chooses and builds capacity from a set of electricity-generating technologies to maximize profit; some of these technologies are intermittent while others have constant output. Then, we consider a representative consumer who purchases varying quantities of electricity in each period in order to maximize utility. Generally, people prefer to spread their electricity consumption out over time; a standard CES function can capture this preference in a simple way. That is, we treat electricity as a heterogenous good which is differentiated by its delivery time; consumer utility is given by a CES function of electricity consumption in each period.<sup>2</sup> These two sides of the market reach an equilibrium through adjustments in prices of electricity in each period. All in all, we make many of the same assumptions that Helm and Mier (2019) do. We assume dynamic pricing,<sup>3</sup> no load rationing, and positive prices.

Then, we parametrize our model empirically. To start, we fit the parameters of the consumer’s CES utility function using electricity consumption and price data for each US state. Specifically, we estimate the intertemporal elasticity of substitution for electricity consumption; this parameter plays a particularly important role in our model, since it captures the effects of intermittency on demand. Then, to model the supply side, we narrow our model to a two-period, two-technology setting with renewables and fossil energy. We proxy for renewable energy using solar and proxy for fossil energy using coal; then, we parametrize each accordingly. Finally, we implement our model numerically and solve for the equilibrium. We study the effects of technical change on the energy market and derive policy implications.

Overall, this treatment of intermittency produces some key results that contrast with those of Helm and Mier. First, while we also find that externalities can be handled through a Pigouvian tax, we further find that a combining both taxes on pollution and research subsidies for renewables is more optimal than either policy instrument alone. Second, we find that, as renewables get cheaper, their adoption is not necessarily S-shaped but can be concave or convex depending the elasticity of substitution in the consumer’s utility function; with our estimated

<sup>1</sup> Foley et al. (2012) provide a review of the literature on the forecasting of wind power. Over time, forecasts have gotten much more accurate allowing electricity grids to manage wind power intermittency ahead of time.

<sup>2</sup> The use of a CES function as such has been explored earlier by Schwarz et al. (2002), Schwarz et al. (2002), Herriges et al. (1993), and King and Shatrawka (2011), Aubin et al. (1995), and Mohajeryami et al. (2016). Their papers empirically estimate the parameters for this function; we provide a more detailed discussion of the empirical literature in our Methodology.

<sup>3</sup> Helm and Mier provide a motivation for models incorporating the dynamic pricing of electricity. They argue such approaches to pricing will become the norm with further technological advances and coming regulatory changes.

parameters, we find this relationship to be almost linear. Third, a significant amount of literature has assumed a CES structure between renewable and fossil energy (see [Papageorgiou et al. \(2017\)](#)); this assumption has been motivated, in part, by the need to capture imperfect substitutability between these two generation technologies as a result of intermittency. However, our direct model of intermittency disconfirms this assumption; the elasticity of substitution between these two energy technologies is far from linear as a *consequence* of intermittency. In short, optimal taxes and subsidies along with the substitutability of renewable and fossil energy all vary with the state of the local energy market. Hence, we believe that optimal environmental policy must take into account local conditions far more than has previously been assumed.

## II. Model

### A. Consumers

Consumers purchase variable amounts of electricity  $Z_t$  over each period  $t$ . Additionally, assuming that the price of electricity in each period is held constant, they prefer to ration their electricity usage across each period in fixed proportions. And, lastly, we presume that consumers would be willing to shift their consumption from one period to another in response to a shift in prices. Overall, these assumptions can be captured using a standard CES utility function

$$U = \left( \sum_t \alpha_t Z_t^\phi \right)^{1/\phi} \quad (1)$$

where  $\sigma = 1/(1 - \phi)$  is the intertemporal elasticity of substitution for electricity consumption. We define  $\sum_t \alpha_t = 1$ , so that  $\alpha_t$  is the fraction of electricity consumption in period  $t$  when all prices are equal; naturally,  $\alpha_t > 0$  for all  $t$ . We assume that any discounting over time is absorbed into  $\alpha$  for parsimony. The budget constraint is given by

$$I = \sum_t p_t Z_t \quad (2)$$

where  $p_t$  is the price of electricity in period  $t$  and  $I$  is the income. Furthermore, since CES preferences are homothetic, we may aggregate the consumers into a single representative consumer. Consequently, the first order conditions for our representative consumer when maximizing utility given this budget constraint imply:

$$Z_t = \left( \frac{\alpha_t}{p_t} \right)^\sigma \frac{I}{P} \quad (3)$$

$$P = \sum_t \alpha_t^\sigma p_t^{1-\sigma} \quad (4)$$

where  $P$  is the price index. Note that this model naturally does not allow for blackouts in equilibrium, since the price of electricity in any period gets arbitrarily large as the quantity of energy consumed in that period approaches 0. Furthermore, note that prices must be positive; although this is sometimes violated in reality, we do not believe that this assumption significantly affects our analysis.

### B. Firms

Secondly, we have firms maximizing profit by picking an optimal set of energy inputs. In reality, electricity markets are fairly competitive, so we can model the set of firms by using a single representative firm that sets marginal revenue equal to marginal cost. We define the quantity of deployed energy technology  $i$  as  $X_i$ , and we define its output per unit in period  $t$  as  $\xi_{i,t}$ . So, for example, if  $i$  is solar power,  $X_i$  would be the number of solar panels and  $\xi_{i,t}$  may be kW

generated per solar panel in period  $t$ . Consequently, the energy generated in period  $t$ ,  $Z_t$ , is given by  $\sum_i \xi_{i,t} X_i$ . We define these variables in matrices to simplify notation:

$$X \equiv \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}, Z \equiv \begin{pmatrix} Z_1 \\ \vdots \\ Z_m \end{pmatrix}, p \equiv \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix}, \xi \equiv \begin{pmatrix} \xi_{1,1} & \dots & \xi_{1,m} \\ \vdots & \ddots & \vdots \\ \xi_{n,1} & & \xi_{n,m} \end{pmatrix}$$

where we have  $n$  technologies,  $m$  periods, and  $Z \equiv \xi^T X$ . A key element of this model is that  $X$  does not vary by time. This is important, because the time scale of our model must be fairly granular to study the effects of intermittency. So, for instance, we may have  $t \in \{1, \dots, 24\}$  representing each hour of the day. In such short time scales, we can safely assume that producers do not modify the quantity of the technology they employ. Moreover, this keeps the model parsimonious by preventing more complicated dynamics from entering the producers optimization problem.

Instead, our representative firm sets  $X$  once to maximize profit in all periods while facing the cost function  $C(X_1, \dots, X_n)$ . Helm and Mier (2019) note that past literature has argued for concave cost functions in the energy market due to effects such as economics of scale and learning by doing; on the other hand, standard cost functions are generally convex. So, like Helm and Meir, we take an intermediate approach by using a linear cost function. Specifically, we have  $C(X) \equiv \sum_i c_i X_i \equiv c^T X$  where  $c_i$  is the cost per unit of  $X_i$ . Total profit is given by

$$\Pi = p^T Z - c^T X \quad (5)$$

To simplify the algebra, we set the number of technologies equal to the number of periods ( $n = m$ ). Additionally, we further require that the output per unit of each technology is unique and non-negative in each period; in other words, the output per unit of one technology is not a linear combination of those of the other technologies in our set. This then implies that  $\xi$  is of full rank and therefore invertible. Now, maximizing profit, we find the first order condition:

$$\frac{\partial \Pi}{\partial X} = 0 \implies p = \xi^{-1} c \quad (6)$$

Combining this FOC with the demand equation (Equation (3)) allows us to find the equilibrium. Generally, the equilibrium results for any number of technologies ( $n$ ) are analytic, but they are difficult to interpret due to the number of parameters involved.

### C. Equilibrium

For more tractable results, we consider a simpler scenario where  $n = m = 2$  and  $\sigma = 1$  (Cobb-Douglas); this particular case is described in greater detail with proofs in [Appendix A.A](#). Moreover, these parameters are of particular interest, because they simplify the model enough to allow us to derive comparative statics and determine which sets of parameters lead to edge cases.

To start, we discuss the conditions on the exogenous parameters required to avoid edge-cases. Alternatively, these are the conditions required for a technology to be economical. But, first, we define two recurring terms in our analysis: cost efficiency and output efficiency. For some arbitrary period  $t$  and technology  $i$ , we use cost efficiency to refer to  $\xi_{i,t}/c_i$  and output efficiency to refer to  $\xi_{i,t}$ . So, for example, A technology is more cost efficient than another in a particular period when its output in that period per “dollar” spent is larger than that of the other technology in the same period.

**Proposition 1** *Assume that, for all  $i$  and  $t$ , we have  $\xi_{i,t}, \alpha_t, c_i > 0$ . Furthermore, suppose that  $\xi$  is invertible. For technology  $i$  to be economical, it must meet three conditions with respect to some arbitrary period  $t$ . Firstly, it must be more cost effective than any of the other technologies*

in period  $t$ . Secondly, it must have a comparative advantage in output efficiency in the same period  $t$ . And, thirdly, there must be sufficient amount of demand for energy in this period  $t$ .

The condition on cost efficacy is fairly intuitive; one case of the contrapositive is that if a technology is not the most cost effective in any period, it will not be used. Alternatively, if a technology is the most cost effective in every single period, it will be the only technology used. The second condition regarding output efficiency actually stems from the invertibility of  $\xi$ . If we did not have  $\xi$  invertible, we either have at least one technology that does not produce in any period or we have at least one technology being a linear combination of the other technologies in terms of output. In the latter case, some technologies are not economical because their output can be replicated the other technologies in a more cost effective way. As an example, suppose  $\xi$  consisted of 3 technologies but was of rank 2. In this case, we may represent any of these technologies as a linear combination of the other two; call this combination the synthetic version of technology  $i$ . It is not possible for every synthetic technology  $i$  to be more expensive than its original; for at least one  $i$ , we must have a synthetic technology being cheaper or equal in price to the actual technology. In the first case, synthetic technology  $i$  is cheaper so we do not use the actual technology  $i$ . In the later case, we may still eliminate technology  $i$  or not use the other two technologies depending on the demand. Overall, in any case,  $\xi$  not invertible means that at least one technology is not used. Finally, the demand condition is straightforward; even if a technology is optimal in a certain period, if consumers do not sufficiently demand electricity during that period, then there is little reason to use that technology.

We may also derive the comparative statics for this simplified scenario.

**Proposition 2** *Suppose we are not in an edge case, so that the conditions of Proposition 1 hold for each technology. The equilibrium quantity of a technology is increasing with its output efficiency and decreasing with its cost; at the same time, it is decreasing with the output efficiency of the other technologies and increasing with their cost. Also, suppose that some technology  $i$  is the most cost effective in period  $t$ . Then, its equilibrium quantity is increasing with respect to the demand share parameter  $\alpha_t$  and decreasing with respect to the demand share parameters of the other periods. Furthermore, again assuming technology  $i$  is the most cost effective in period  $t$ , the comparative statics of  $Z_t$  and  $X_i$  are equivalent.*

The comparative statics with respect to  $X$  and its output efficiency and cost are not surprising. On the other hand, the statics for  $Z$  may not be immediately obvious. Instead, they follow from the fact that we have  $Z \equiv \xi_T X$ . That is, suppose we have an arbitrary technology  $i$  that is the most cost effective source of electricity on period  $t$ . If consumers demanded that 100% of their energy arrive in period  $t$ , then relying solely on technology  $i$  for energy would be the most economical solution. Consequently, it seems intuitive that the comparative statics of  $X_i$  follow through to  $Z_t$ . This intuition just happens to follow through even when other technologies are employed and there is demand in multiple periods. Similarly, the comparative statics for the share parameters of the utility function,  $\alpha$ , travel in the opposite direction. A rise in  $\alpha_t$  would directly raise the optimal quantity of  $Z_t$ ; hence, whichever technology is most cost effective at producing in period  $t$  would be used more. We provide a more detailed and quantitative discussion of the comparative statics and edge cases in the appendix.

### III. Empirical Methodology

In order to better understand the practical implications of our model, we empirically estimate its parameters and study its implications numerically. We are particularly interested in estimating  $\sigma$ , the intertemporal elasticity of substitution for electricity consumption. In our [discussion](#), we explain in detail why  $\sigma$  is of interest; but, to summarize,  $\sigma$  implicitly determines how well renewables can substitute for fossil energy. Specifically, if  $\sigma \geq 1$ , then electricity consumption

in different periods are substitutes; consequently, fossil energy and renewable energy are highly substitutable. On the other hand, if  $\sigma$  is around 0.5, electricity consumption in different periods are complements; so, fossil and renewable energy are far less substitutable due to the effects of intermittency.

The other parameters in our model,  $c$ ,  $\xi$ , and  $\alpha$ , are of secondary interest, since they are easier to obtain directly. To start,  $c$  is the cost per unit for each technology; estimates of this value for different technologies can be obtained directly from the literature. Similarly,  $\xi$ , the output per unit of technology, can be directly obtained from the literature. And, finally,  $\alpha$  may be approximated directly from the data. That is, note that the demand equation from earlier (Equation 3) is:

$$Z_t = \left( \frac{\alpha_t}{p_t} \right)^\sigma \frac{I}{P}$$

where  $P$  is the price index and  $I$  is income. Retail customers pay fixed rates each month for electricity, hence  $p_t$  is constant within each month; and, we expect that income  $I$  does not vary significantly on a daily basis. Consequently, all variation in intramonthly demand is due to the share parameter  $\alpha$  and the elasticity  $\sigma$ . Hence, after estimating  $\sigma$ , we can approximate  $\alpha$ . However, this raises another problem, because, without prices varying each hour, we cannot estimate  $\sigma$  on an hourly basis.

A number of other papers have approached this problem using data from real-time pricing experiments. These include [Schwarz et al. \(2002\)](#), [Herriges et al. \(1993\)](#), and [King and Shatrawka \(2011\)](#).<sup>4</sup> The latter two papers estimate  $\sigma$  to be around 0.1 while the paper by [Schwarz et al.](#) obtain estimates around 0.04. All three papers study real-time electricity pricing programs for industrial consumers using similar methodologies. Additionally, [Aubin et al. \(1995\)](#) also provide estimates of the  $\sigma$  but using a different methodology; their results find  $\sigma < 0$  which is impossible since it would imply upward-sloping demand curves under a CES structure. Finally, [Mohajeryami et al. \(2016\)](#) also empirically estimates the share parameters for a CES function of this form, but do not estimate the elasticity of substitution.

A common flaw in these papers is that they do not account for endogeneity caused by a supply-side response to prices. For instance, coal plants are able to modify their generation between days; so, they might respond to an increase in future prices by storing coal until prices rise. That is, there may exist intertemporal substitution for electricity generation which would bias  $\sigma$  downward unless controlled for. This is particularly important, because whether  $\sigma$  is closer to 0.1 or 1 can make a significant difference with respect to the practical implications of our model.

Consequently, we provide our own estimates of  $\sigma$  estimated on a monthly basis. Regarding the time scale, this decision is primarily due to data limitations, since we do not have access to the proprietary data on real-time pricing experiments which the past literature has used. Although we are interested in understanding intertemporal substitution over a shorter time scale (since intermittency plays a larger role in shorter periods), estimates of  $\sigma$  on a monthly basis may still be applicable on a smaller time frame. For instance, [Schwarz et al. \(2002\)](#) estimate  $\sigma$  on a daily and hourly basis and find fairly close results; similarly, [Herriges et al. \(1993\)](#) also find no significant difference in their estimates of  $\sigma$  for these two intervals. That is, while a daily basis is 24 times larger than an hourly basis, the estimates for  $\sigma$ , suprisingly, do not appear to change. Hence, we expect our estimates of  $\sigma$  on a monthly basis to not be far from estimates on shorter time scales. We now define our econometric methodology in detail.

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<sup>4</sup> There has also been a large literature that directly estimates the price elasticity of electricity demand without imposing a CES functional form. These papers include [Wolak and Patrick \(2001\)](#), [Zarnikau \(1990\)](#), [Woo et al. \(1996\)](#), [Zhou and Teng \(2013\)](#), [Reiss and White \(2005\)](#), [Fan and Hyndman \(2011\)](#), and [Deryugina and Mackay \(2017\)](#). These papers estimate own-price elasticities, while some also estimate cross-price elasticities for electricity consumption at different times. Because they do not impose a CES structure, we cannot obtain estimates of  $\sigma$  from this literature.



### A. Theory

Firstly, recall the demand equation (Equation 3) from our general model:

$$Z_t = \left( \frac{\alpha_t}{p_t} \right)^\sigma \frac{I}{P}$$

$$P = \sum_t \alpha_t^\sigma p_t^{1-\sigma}$$

For any pair of electricity outputs  $Z_t$  and  $Z_s$ , we have:

$$\frac{Z_t}{Z_s} = \left( \frac{\alpha_t p_s}{\alpha_s p_t} \right)^\sigma$$

Taking logs on both sides and letting  $i$  represent different observations, we may rewrite this in a form more suitable for estimation.

$$\ln(Z_{t,i}/Z_{s,i}) = -\sigma \ln(P_{t,i}/P_{s,i}) + \sigma \ln(\alpha_{t,i}/\alpha_{s,i})$$

Our data differentiates consumption for each state in the US, so we let  $i$  refer to a particular state. Additionally, most consumers pay monthly fixed rates for electricity, so we can, at most, estimate this equation on a monthly basis; hence,  $t$  and  $s$  refer to different months. Lastly, note that the state  $i$  is kept constant for each observation; this is because consumers within each state can substitute consumption across time, but consumers in different states do not substitute consumption with one another.

In order to estimate this  $\sigma$ , we further modify the previous equation. Firstly, note that we cannot observe the demand shifter  $\alpha_{t,i}$  directly, so we must replace the  $\alpha$  terms with a set of controls that may be responsible for shifts in demand. So, still in general terms, our regression equation is now

$$\ln(Z_{t,i}/Z_{s,i}) = -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i} A_{t,i} + \gamma_{s,i} A_{s,i} + u_i$$

where  $A$  represents set of controls for changes in demand while  $u_i$  is a normal error term. Note that the control  $A_{t,i}$  replaces  $\sigma \ln(\alpha_{t,i})$  and likewise for the period  $s$  term; this substitution is valid because the  $\ln(\alpha_{t,i}) \in \mathbb{R}$  and the  $\sigma$  is simply absorbed into the estimated coefficient for  $\gamma_{t,i}$ . For the demand controls themselves, we choose to use heating (HDD) and cooling degree days (CDD) due to the aggregation of the data. That is, a more general control such as average temperature would not be able to directly capture intramonthly changes in demand, since variation in temperature would be lost when aggregated; on the other hand, CDDs and HDDs directly represent daily deviations in temperature even when totaled for each month. Additionally, demand for electricity may rise over time. Hence, we include, as a control, the difference in months between time  $t$  and  $s$ ; this is represented by  $\Delta_{t,s}$ . Finally, this panel requires us to consider fixed effects for each state, so we use a fixed effects panel regression. In total, the demand equation is:

$$\begin{aligned} \ln(Z_{t,i}/Z_{s,i}) &= -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i} A_{t,i} + \gamma_{s,i} A_{s,i} + \eta \Delta_{t,s} + u_i \\ &= -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i} (CDD_{t,i} + HDD_{t,i}) \\ &\quad + \gamma_{s,i} (CDD_{s,i} + HDD_{s,i}) + \eta \Delta_{t,s} + u_i \end{aligned}$$

Still, this equation may suffer from bias, since producers can also substitute production over time. For instance, it is possible to store fuel for electricity generation in the future when prices may rise. So, we define the following supply equation

$$\ln(Z_{t,i}/Z_{s,i}) = \beta \ln(P_{t,i}/P_{s,i}) + \xi \ln(C_{t,i}/C_{s,i}) + v_i$$

where  $C_{t,i}$  is the average cost of coal used for electricity generation in state  $i$  at time  $t$  and  $v_i$  is a normal error term. Coal prices are independent of the electricity demand error term  $u_i$ , since residential consumers do not generally use coal for electricity generation; on the other hand, shocks in the price of coal are linked with the supply of electricity. Hence, coal price is a theoretically valid instrument. In total, the reduced form equation is given by:

$$\ln(P_{t,i}/P_{s,i}) = (\beta + \sigma)^{-1} (\gamma_{t,i}A_{t,i} + \gamma_{s,i}A_{s,i} + \eta\Delta_{t,s} - \xi \ln(C_{t,i}/C_{s,i}) + u_i - v_i)$$

where  $A_{t,i}$  consists of CDDs and HDDs at time  $t$ .

Finally, we also consider a semiparametric specification. That is, we allow the error terms  $u_i$  and  $v_i$  to be non-normal and place the demand controls and instruments in unknown functions. So, overall, we have:

$$\begin{aligned} \ln(Z_{t,i}/Z_{s,i}) &= -\sigma \ln(P_{t,i}/P_{s,i}) + f(A_{t,i}, A_{s,i}, \Delta_{t,s}) + u_i \\ \ln(Z_{t,i}/Z_{s,i}) &= \beta \ln(P_{t,i}/P_{s,i}) + g(\ln(C_{t,i}/C_{s,i})) + v_i \end{aligned}$$

where  $f$  and  $g$  are unknown, bounded functions. We restrict  $cov(u_i, v_i) = 0$  but allow for the controls and instruments to be correlated. The advantage of this specification is that we can account for the controls or instrument having any nonlinear effects on the regressands. **(More here on explaining the estimation)**

## B. Data

We collect monthly data from the [EIA \(2019a\)](#) on retail electricity prices and consumption for each state in the US from 2011 to 2018. Additionally, also from the EIA, we obtain data on the average cost of coal for electricity generation for each state and month.<sup>5</sup> We deflated both electricity and coal prices over time using the PCECP Index provided by the [US BEA \(2019\)](#). Finally, we collect data on HDDs and CDDs from the NOAA's [Climate Prediction Center \(2019\)](#) for the same panel. Then, we merge these three data sets and trim 1% of outliers for a total of 826 observations for each month and state. We use this preliminary data set to construct the data required for our regressions. That is, each observation in our estimation equation belongs to a set  $(t, s, i)$  consisting of two time periods and a state. Hence, we construct each row in our regression data set using unique combinations of  $t, s$  where  $t \neq s$  for each state  $i$ . Due to the number of potential combinations and thus observations, we restrict our data set to a random sample of 9000 observations.<sup>6</sup> All in all, each observation in our regression data set consists the following variables: state ( $i$ ), date 1 ( $t$ ), date 2 ( $s$ ), the log difference in electricity consumption between month 1 and month 2, the log difference in the price of electricity, the log difference in the price of coal, the number of CDDs for each date, the number of HDDs for each date, and the difference (in months) between dates 1 and 2.

## IV. Results

Based on the OLS results reported in [Table 1](#), we estimate the intertemporal elasticity of substitution for electricity consumption  $\hat{\sigma} = 0.609$  ( $|t| > 24$ ) when accounting for all degree day covariates and state fixed effects. Interestingly, we find that the estimated coefficients for all three specifications do not differ when considering fixed effects. Additionally, as expected, the coefficients on the degree day covariates are symmetrical; that is, the coefficient on  $CDD_t$  is approximately the same as the negative of that on  $CDD_s$ , and the same applies to  $HDD_t$  and  $HDD_s$ . On the other hand, we found that electricity consumption seems to rise more in

<sup>5</sup> The coal price data set contains a large number of missing values due to privacy concerns; however, we do not expect that these missing values are correlated with the data itself.

<sup>6</sup> We reran our regressions with several different samples of 9000 and found that our estimated coefficients did not significantly change; hence we believe this sample size is sufficient.



**Table 1: OLS REGRESSION RESULTS**

	<i>Dependent variable: <math>\ln(Z_{t,i}/Z_{s,i})</math></i>					
	(1)	(2)	(3)	(4)	(5)	(6)
$-\ln(P_{t,i}/P_{s,i})$	0.751*** (0.030)	0.481*** (0.026)	0.607*** (0.026)	0.750*** (0.224)	0.481** (0.172)	0.607*** (0.169)
$\Delta_{t,s}$			0.001*** (0.0001)			0.001*** (0.0002)
$CDD_t$ ( $\times 1000^{-1}$ )		0.990*** (0.016)	1.003*** (0.016)		0.990*** (0.087)	1.002*** (0.086)
$CDD_s$ ( $\times 1000^{-1}$ )		-0.999*** (0.016)	-1.011*** (0.016)		-0.999*** (0.087)	-1.012*** (0.085)
$HDD_t$ ( $\times 1000^{-1}$ )		0.306*** (0.005)	0.294*** (0.005)		0.307*** (0.030)	0.295*** (0.030)
$HDD_s$ ( $\times 1000^{-1}$ )		-0.309*** (0.005)	-0.296*** (0.005)		-0.308*** (0.029)	-0.295*** (0.029)
Intercept	0.001 (0.003)	0.002 (0.006)	0.002 (0.006)			
State FEs				Yes	Yes	Yes
Observations	9,000	9,000	9,000	9,000	9,000	9,000
R <sup>2</sup>	0.070	0.580	0.596	0.070	0.580	0.596
Adjusted R <sup>2</sup>	0.070	0.579	0.596	0.065	0.577	0.594

*Note:* The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except  $\Delta_{t,s}$ . The unit of observation is a set  $(t, s, i)$  where  $t \neq s$  are months and  $i$  is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The coefficient on  $\ln(P_{t,i}/P_{s,i})$  is an estimate of  $-\sigma$ . The variable  $\Delta_{t,s}$  is the difference in months between periods  $t$  and  $s$ .  $CDD_t$  and  $HDD_t$  refer to the total number of heating and cooling degree days in month  $t$ . We scale the coefficients on degree days for clarity. Robust standard errors are reported in parentheses. \*p<0.05, \*\*p<0.01, \*\*\*p<0.001

response to CDDs rather than HDDs. Lastly, we find that the sign on  $\Delta_{t,s}$  is positive in all regressions; this implies that electricity consumption rises over time independent of price.

To account for endogeneity in the OLS results, we provide results for our IV specification in Table 2. Here, we find a much larger estimate  $\hat{\sigma} = 11.430$  ( $|t| > 2.1$ ) when considering all covariates and fixed effects. F-Statistics on all three specifications are significantly larger than 10, which suggests that the instruments are not weak (Staiger and Stock, 1997). With respect to the demand controls, we find results similar to those of OLS. State fixed effects do not appear to affect the estimates, demand increases over time, and CDDs raise electricity consumption more than HDDs.

Finally, we control for nonlinear effects using a partially linear IV regression reported in Table 3. Here, we find results similar to that of OLS;  $\hat{\sigma} = 0.413$  ( $|t| > 48$ ) when considering all controls. The estimate of  $\sigma$  without any controls is not significantly different from that in specifications with controls. However, all of these estimates are significantly different from the IV results; hence, the IV regressions do not appear to be robust. Consequently, we opt to use these estimates of  $\sigma$  in our model. Since these estimates show  $\hat{\sigma} \in (0, 1)$ , it appears that

**Table 2:** IV (2SLS) REGRESSION RESULTS

	First-Stage			Second-Stage		
	<i>Dep. Variable: <math>\ln(P_{t,i}/P_{s,i})</math></i>			<i>Dep. Variable: <math>\ln(Z_{t,i}/Z_{s,i})</math></i>		
	(A.1)	(B.1)	(C.1)	(A.2)	(B.2)	(C.2)
$\ln(C_{t,i}/C_{s,i})$	-0.060*** (0.002)	0.004* (0.002)	0.004* (0.002)			
$-\ln(P_{t,i}/P_{s,i})$				0.711*** (0.106)	13.987 (8.384)	11.430* (5.330)
$\Delta_{t,s}$		0.001*** (0.00002)	0.001*** (0.00002)			0.007* (0.003)
$CDD_t$ ( $\times 1000^{-1}$ )		0.151*** (0.007)	0.151*** (0.007)		3.064* (1.288)	2.620** (0.798)
$CDD_s$ ( $\times 1000^{-1}$ )		-0.149*** (0.007)	-0.149*** (0.007)		-3.034* (1.264)	-2.609*** (0.788)
$HDD_t$ ( $\times 1000^{-1}$ )		-0.057*** (0.003)	-0.057*** (0.003)		-0.407 (0.447)	-0.311 (0.301)
$HDD_s$ ( $\times 1000^{-1}$ )		0.058*** (0.003)	0.058*** (0.003)		0.423 (0.458)	0.324 (0.308)
State FEs	Yes	Yes	Yes	Yes	Yes	Yes
Observations	9,000	9,000	9,000	9,000	9,000	9,000
R <sup>2</sup>	0.066	0.441	0.441			
Adjusted R <sup>2</sup>	0.066	0.441	0.441			
F Statistic	678***	1,213***	1,197***			

*Note:* The log difference in coal price between period  $t$  and  $s$ ,  $\ln(C_{t,i}/C_{s,i})$ , is used as an instrument in these regressions. The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except  $\Delta_{t,s}$ . The unit of observation is a set  $(t, s, i)$  where  $t \neq s$  are months and  $i$  is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The coefficient on  $\ln(P_{t,i}/P_{s,i})$  is an estimate of  $-\sigma$ . The variable  $\Delta_{t,s}$  is the difference in months between periods  $t$  and  $s$ .  $CDD_t$  and  $HDD_t$  refer to the total number of heating and cooling degree days in month  $t$ . We scale the coefficients on degree days for clarity. Robust standard errors are reported in parentheses. \*p<0.05, \*\*p<0.01, \*\*\*p<0.001

electricity consumption in different months complement one another.

**Table 3:** PARTIALLY LINEAR IV REGRESSION RESULTS

	<i>Instrument: <math>\ln(C_{t,i}/C_{s,i})</math></i>		
	(1)	(2)	(3)
$\hat{\sigma}$	0.5676*** (0.1036)	0.4511*** (0.0042)	0.4129*** (0.0085)
Time Control			Yes
Degree Day Controls		Yes	Yes
Observations	9,000	9,000	9,000

*Note:* The log difference in coal price between period  $t$  and  $s$ ,  $\ln(C_{t,i}/C_{s,i})$ , is used as an instrument in these regressions. The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except  $\Delta_{t,s}$ . The unit of observation is a set  $(t, s, i)$  where  $t \neq s$  are months and  $i$  is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The estimation procedure is described in the appendix. Robust standard errors are reported in parentheses. \*p<0.05, \*\*p<0.01, \*\*\*p<0.001

## V. Discussion

### The Elasticity of Substitution between Renewable and Fossil Energy

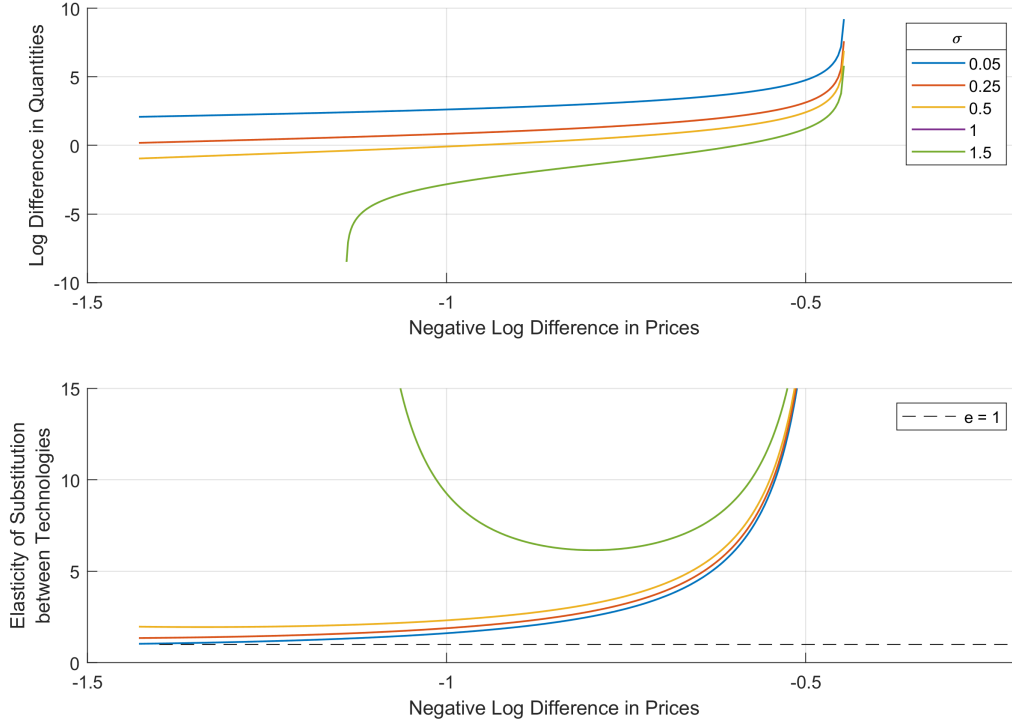
To understand the economic implications of these estimates for  $\sigma$ , we parametrize and numerically evaluate our model in a two-period, two-technology setting. In particular, we are concerned with how  $\sigma$  affects the substitutability of fossil and renewable energy. This is particularly important, because sufficient substitutability between these two technologies is required to transition to greener economy in the future. For instance, [Acemoglu et al. \(2012\)](#) argues that, “When the two sectors [clean and dirty energy] are substitutable but not sufficiently so, preventing an environmental disaster requires a permanent policy intervention. Finally, when the two sectors are complementary, the only way to stave off a disaster is to stop long-run growth.” However, note that, while our model sets the intertemporal elasticity of substitution for electricity consumption  $\sigma$  to a fixed value, we make no direct assumptions about the elasticity of substitution between different energy technologies. Rather, an interesting relationship regarding the substitutability/complementarity between technologies emerges indirectly as a result of intermittency. To illustrate this, we now proceed with the numerical model.

Firstly, we let technology 1 be coal power and technology 2 be solar power; again, period  $t$  represents peak hours and period  $s$  represents off-peak hours. We assume that, holding prices equal, consumers prefer that 60% of their energy arrive in period  $t$  and the remaining 40% arrive in period  $s$ ; that is, we have  $\alpha_t = 0.6$  and  $\alpha_s = 0.4$ . Additionally, we normalize the units of the quantities to a MWh basis. Hence,  $\xi$  represents the percent of capacity utilized in each period; we assume coal uses 100% of its capacity in both periods, while solar can access 100% during peak hours and only 10% during the off-peak. Finally, we set cost parameters (given in \$ per MWh) equal to LCOE estimates for 2023 from the [EIA \(2019b\)](#); specifically, we use estimates for “Solar PV” and “Coal with 30% CCS” from Table 1b.<sup>7</sup> In total, we have the following parameters.

**Example 1:**  $\alpha_t = 0.6$ ,  $\alpha_s = 0.4$ ,  $\xi_1 = (1, 1)$ ,  $\xi_2 = (1, 0.1)$ ,  $c_1 = 104.3$ ,  $c_2 = 60$ .

<sup>7</sup> The LCOE for a technology is equal to the sum of its lifetime costs divided by its lifetime energy output. While [Joskow \(2011\)](#) explains the flaws of comparing generation technologies solely on the basis of LCOE, our use of this measure is unrelated to his critique. That is, he argues that the economic value an intermittent technology should account for when it produces electricity and the prices of electricity in those periods (see Table 2 of his paper). Our model does exactly this.

**Figure 1: THE ELASTICITY OF SUBSTITUTION BETWEEN SOLAR AND COAL**



*Note:* The y-axis of the first plot is equivalent to  $\log(X_1/X_2)$  and the x-axis of both plots is equivalent to  $\log(c_2/c_1)$ . The legend in the upper subplot also applies to the lower subplot. These results were obtained using the following parameters:  $\alpha_t = 0.6$ ,  $\alpha_s = 0.4$ ,  $\xi_1 = (1, 1)$ ,  $\xi_2 = (1, 0.1)$ ,  $c_1 = 104.3$ ,  $c_2 = 60$ . We plot results for various values of the intertemporal elasticity of substitution for electricity consumption  $\sigma$ , which is used in the consumer's CES utility function. In order to numerically estimate these relationships, we first found the optimal quantities of  $X$  over a range of prices  $c_1^* \in (0.5 c_1, 2 c_1)$ . Then, we obtained estimates of the elasticity of substitution by numerically differentiating  $\log(X_1/X_2)$  with respect to  $-\log(c_1, c_2)$ . That is, the elasticity of substitution between technology 1 and 2 is given by the slope of the upper subplot, and it is graphed in the lower subplot.

We begin by exploring the implied elasticity of substitution between these two technologies. Recall, for any two commodities  $i$  and  $j$ , the elasticity of substitution  $e_{i,j}$  is given by:

$$e_{i,j} = \frac{\partial \ln(X_1/X_2)}{\partial \ln(c_2/c_1)}$$

where  $c$  is their prices. This relationship is of particular interest, because many applied and theoretical models studying renewable and fossil energy impose a CES production structure between the two. That is, they assume that  $e$  is a fixed constant different pairs of generation technologies. In our model, this is not the case; while directly deriving the equation for  $e$  in our model is analytically intractable, we numerically estimate it for varying costs and  $\sigma$ .

We plot our numerical estimates of  $e_{solar,coal}$ , the elasticity of substitution between solar and coal power, in Figure 1. These results were generated by computing the optimal quantities of each technology given a range of prices. Specifically, we held the price of solar capacity constant and varied the price of coal capacity from 50% to 200% of its original parametrized price. Then, we numerical differentiated  $\log(X_1/X_2)$  with respect to  $\log(c_2/c_1)$  to estimate the elasticity of substitution between solar and coal.<sup>8</sup> This numerically computed elasticity of substitution between solar and coal power,  $e_{solar,coal}$ , is shown in the lower subplot. We repeat

<sup>8</sup> Instead, we could have generated variation in  $\log(c_2/c_1)$  data by holding solar at a constant price and varying the price of coal capacity. However, this does not change the results.

this process with different values of  $\sigma$ , the elasticity of substitution in the consumer's utility function. Finally, we drop all observations of our numerical simulation that correspond to edge cases.

Our results show that  $e$  is clearly non-linear, hence assuming a CES production structure between clean and dirty energy is not an accurate assumption. Furthermore, the relationship between log quantities and log prices becomes increasingly non-linear as  $\sigma$  rises. This is somewhat intuitive; when energy consumption can be substituted across periods, the market is more sensitive to the cost efficiency of technologies and less sensitive to when they produce. For instance, suppose we had two technologies that each produced in a different period; if consumers are less willing to substitute consumption across periods, both technologies are likely to be employed. On the other hand, when consumers are open to substitution, they will simply consume the majority of their electricity in whichever period it is most cost effective to do so. Consequently, price plays bigger role than intermittency as  $\sigma$  rises. This is further evident in the lower subplots; note that the numerical estimates of  $e$  rise and become more u-shaped as  $\sigma$  rises.<sup>9</sup> When consumers are less concerned about when they receive their energy, they are far more open to substituting between coal and solar power. On the other hand, for lower values of  $\sigma$ , we find that the substitutability of coal and solar power diminishes as solar becomes progressively cheaper. To clarify, note that the x-axis of the lower subplot is  $\log(c_2/c_1)$ ; since  $c_2$  is the price of solar, cheaper solar energy implies lower values of  $\log(c_2/c_1)$  which corresponds to lower values of  $e$ . This can be explained by  $\sigma < 1$  implying a complementary relationship between electricity consumption over time. Consumers with such preferences will be relatively less sensitive to prices and more interested in spreading their electricity consumption across each period. Thus, solar is less substitutable for coal in this case, because its intermittency conflicts with consumers wanting to smooth their electricity consumption over time.

Additionally, note that we assumed earlier that the estimates of  $\sigma$  on a monthly basis would be fairly close to those on a daily or hourly basis. But, our results showed estimates of  $\hat{\sigma} \approx 0.5$  which differs from the literatures' estimates which fall between 0.01 and 0.25. Does this impact our numerical results? Alternatively, if our estimates were closer to past estimates, would there be significantly different economic implications? Based on our numerical model, the answer seems to be no. That is, the relationship between  $e$  and  $\log(c_2/c_1)$  in [Figure 1](#) does not significantly differ when varying  $\sigma$  in  $\{0.05, 0.25, 0.5\}$ . Hence, even if our assumption is incorrect or past estimates are biased, our model's results do not significantly differ within this range of estimates for  $\sigma$ .

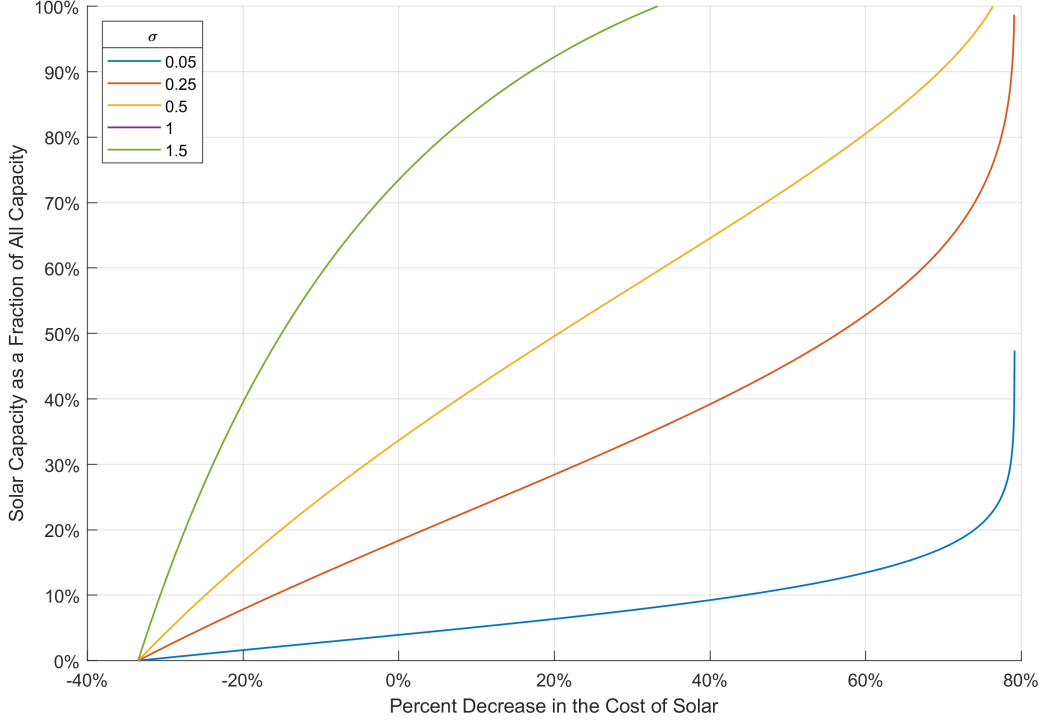
## Technical Change and the Adoption of Renewable Energy

We now do a similar exercise but instead study the adoption (or market share/diffusion) of renewables. Specifically, using the same technologies, solar and coal, along with the same parameters, we study how the cost of each technology affects the market share of solar. First, we study the market share of solar with respect to its own price. Our results are shown in [Figure 2](#). We find that, for values of  $\sigma \geq 1$ , the market share of solar energy appears to be convex with respect to increases in cost efficiency. This relationship somewhat persists through to  $\sigma = 0.5$ ; here, the market share appears to follow an inverted S-shape that starts concave and becomes convex. But, overall, the curve for  $\sigma = 0.5$  seems almost linear. On the other hand, when  $\sigma$  is sufficiently low, this relationship is more consistently convex. Interestingly, these dynamics with respect to  $\sigma$  seem to be a product of intermittency. That is, as we discussed earlier, when consumers have high  $\sigma$ , they are more willing to shift their electricity consumption and are thus relatively more sensitive to changes in price. Consequently, the rate of adoption (the slope of the market share curve) is larger for higher  $\sigma$  around moderate changes in price;

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<sup>9</sup> Recall that  $\sigma < 1$  implies that electricity consumption in different periods are complements,  $\sigma = 1$  implies to a Cobb-Douglas relationship, and  $\sigma > 1$  implies that electricity consumption in different periods are substitutes.

**Figure 2: THE MARKET SHARE OF SOLAR ENERGY VERSUS OWN PRICE**



*Note:* These results were obtained using the following parameters:  $\alpha_t = 0.6$ ,  $\alpha_s = 0.4$ ,  $\xi_1 = (1, 1)$ ,  $\xi_2 = (1, 0.1)$ ,  $c_1 = 104.3$ ,  $c_2 = 60$ . To generate these values, we first vary the price of solar relative to its original parametrized cost  $c_2$ . Then, we find the optimal capacity of solar as a fraction of the total optimal capacity of all technologies (solar and coal) for each set of prices. And, finally, we repeat this process and plot the results for different values of  $\sigma$ , the consumer's intertemporal elasticity of electricity consumption.

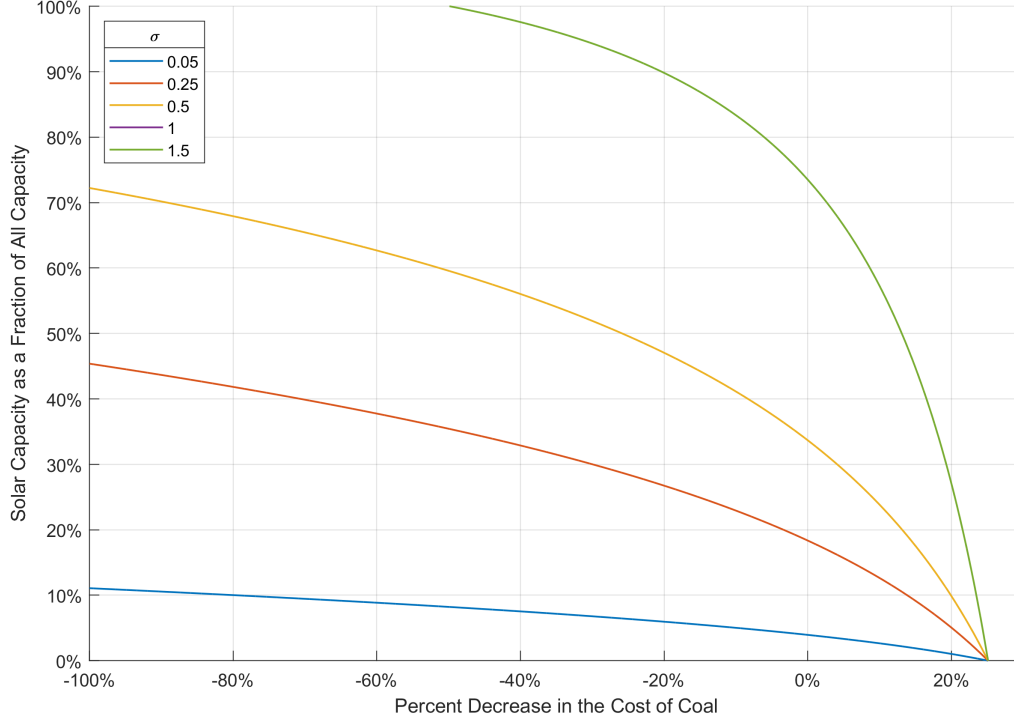
however, this relationship between adoption and  $\sigma$  changes as price falls more drastically. To understand why, see [Figure 3](#), where we compare the market share of solar with respect to the price of coal. We find that solar market share is concave for all values of  $\sigma$ ; consequently, the market share of coal must be convex with respect to its own price. Hence, it seems that market share with respect to own price is convex for non-intermittent technologies, but may be concave for intermittent ones. Furthermore, the importance of intermittency in this context is driven by  $\sigma$ ; that is, intermittency plays a bigger role in determining market share when consumers prefer, on the margin, smoother electricity consumption than electricity cost savings. This result directly contrasts with [Helm and Mier \(2019\)](#) who find S-shaped market diffusion for renewable energy; this shape is a result of their representation of intermittency as unpredictable, stochastic variability in the output of renewable energy. On the other hand, because we represent intermittency in a different way, our market share curves differ significantly. In any case, these market share curves have important implications in the context of promoting renewable energy adoption.

## Externalities

Suppose we are interested in handling externalities. We may assume that our first technology produces pollution according to the function  $S(X) = \gamma X_1$  where  $\gamma$  is the monetary damage per unit of  $X_1$ . A social planner then aims to balance the damages caused by pollution with the surplus of the private sector and the revenue from the tax. Hence, their objective function is



**Figure 3: THE MARKET SHARE OF SOLAR ENERGY VERSUS THE PRICE OF COAL**



*Note:* These results were obtained using the following parameters:  $\alpha_t = 0.6$ ,  $\alpha_s = 0.4$ ,  $\xi_1 = (1, 1)$ ,  $\xi_2 = (1, 0.1)$ ,  $c_1 = 104.3$ ,  $c_2 = 60$ . To generate these values, we first vary the price of coal relative to its original parametrized cost  $c_1$ . Then, we find the optimal capacity of solar as a fraction of the total optimal capacity of all technologies (solar and coal) for each set of prices. And, finally, we repeat this process and plot the results for different values of  $\sigma$ , the consumer's intertemporal elasticity of electricity consumption.

given by

$$\max_{\tau} CS + PS - S(X) + \tau X_1$$

where  $CS$  is consumer surplus,  $PS$  is producer surplus,  $S(X)$  is the cost of pollution,  $\tau$  is the tax on  $X_1$ , and  $\tau X_1$  is the tax revenue. As usual, the optimal tax is a Pigouvian tax;  $\tau$  should be set equal to the marginal social cost of pollution  $\gamma$ .

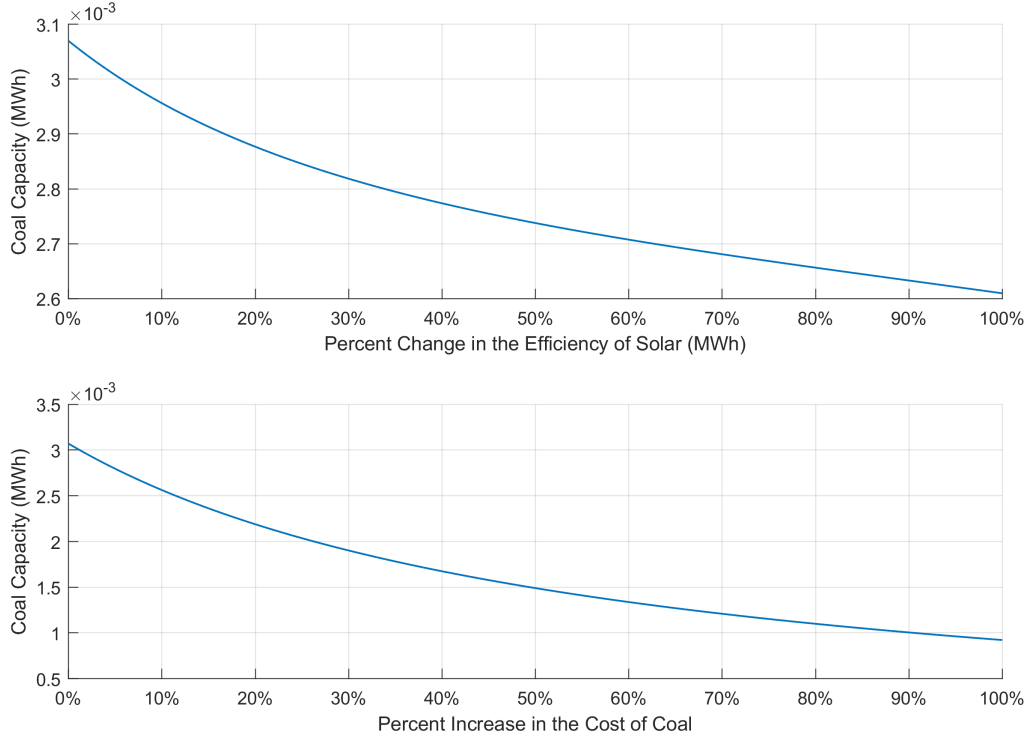
However, given the presence of intermittency, is there a better way to handle pollution? For example, [Acemoglu et al. \(2012\)](#) model intermittency using a CES production function between clean and dirty energy. They find that both carbon taxes and research subsidies are necessary to promote long-run structural change in the energy production sector and avert a climate disaster. The necessity of both instruments is a consequence of their model spanning multiple periods; that is, assuming we have CES production structure between clean and dirty energy in a one-period model, a single instrument is enough to optimally handle externalities.

<sup>10</sup> However, while our model focuses on production on a shorter time scale, we find a similar result – a combination of both taxes and subsidies may be more effective in promoting a shift towards clean energy. Moreover, the optimality of combining both instruments is a result of intermittency.

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<sup>10</sup> Recall that, with a CES production structure, the optimal quantity of technology  $i$  is given by  $X_i = \left(\frac{\xi_i}{c_i}\right)^\sigma \frac{B}{P}$  where  $\xi_i$  is its output per unit,  $B$  is the budget, and  $P$  is the price index. Neglecting effects on the price index, the own-price elasticity of any good is  $-\sigma$ . Furthermore, for the same structure, we have the FOC:  $X_i/X_j = \left(\frac{\xi_i c_j}{\xi_j c_i}\right)^\sigma$ . This implies that the cross elasticity of  $X_i$  with respect to the output efficiency of technology  $j$ ,  $\xi_j$ , is also  $-\sigma$ . Hence, a tax on the cost of  $X_i$  or a subsidy that increases the efficiency  $X_j$  can be used to deal with externalities; but, both are not necessary to do so.

**Figure 4:** TECHNICAL CHANGE AND THE OPTIMAL QUANTITY OF COAL



*Note:* These results were obtained using the following parameters:  $\alpha_t = 0.6$ ,  $\alpha_s = 0.4$ ,  $\xi_1 = (1, 1)$ ,  $\xi_2 = (1, 0.1)$ ,  $c_1 = 104.3$ ,  $c_2 = 60$ ,  $\sigma = 0.5$ . For the first subplot, we graph the optimal quantity of coal while varying  $\xi_2$ , the output efficiency of solar, by different factors; so, for instance, at 100% on the x-axis, we have  $\xi_2 = (2, 0.2)$ . Similarly, in the second subplot, we do the same but with the cost of coal,  $c_1$ .

For clarity, let us first consider the cases in our framework where only one instrument is necessary. The first case is when no two technologies produce in the same period. That is, we can say that the output efficiency matrix,  $\xi$ , is diagonal without loss of generality; this further implies that we have a CES structure between technologies where the elasticity of substitution is same as that for consumers,  $\sigma$ .<sup>11</sup> Consequently, if one of these technologies creates negative externalities, the optimal solution is simply use a single instrument such as a Pigouvian tax. On the other hand, the second case occurs when all technologies produce in every period but with time-invariant output. This results in an edge case where we only employ whichever technology has the highest cost efficiency (output per dollar). Consequently, we only need one or no instruments depending on whether the technology employed creates negative externalities. Essentially this latter case describes a situation where we have no intermittency at all. On the other hand, the former case describes a situation where we have, in a sense, maximized relative intermittency. But, with respect to both cases, we only need one instrument to handle externalities.

In all other situations, where we have a mix of intermittent and constant-output technologies, using multiple instruments can be more effective than just one. That is, first suppose that research subsidies increase the output efficiency  $\xi_i$  of some technology  $i$ . So, output efficiency is then given by  $\xi_i (1 + \beta_i)$  where  $\beta_i$  is the percent increase in the output efficiency of technology  $i$  as a result of research. Now, consider again a two-period setting with coal and solar with the same parameters as those given in [Example 1](#). As discussed earlier, one solution to externalities is to set a Pigouvian tax on coal equal to its marginal social damage. However, we may be able to reduce our reliance on coal in a more effective (welfare-maximizing) way by subsidizing

<sup>11</sup> A proof is provided in [Appendix A.C](#)

research in solar as well. We can intuit this to be a potential solution based on [Figure 4](#); in the first subplot, we show the effects of research into solar on the use of coal, while, in the second subplot, we show the effect of taxes on the use of coal. Essentially, with respect to reducing the use of coal power, there are diminishing returns to both pollution taxes and solar research subsidies. Hence, the pollution-reduction benefits of both instruments are convex.<sup>12</sup> On the other hand, the deadweight loss created by taxation is generally convex; since research subsidies must be funded by tax revenue, its costs must necessarily be convex as well. In short, both instruments have convex benefits and costs, which implies that optimal policy is likely a combination of both.

In total, besides for edge cases, this suggests that optimal policy may be to incorporate both

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<sup>12</sup> To be more precise, we numerically compute the Hessian for  $X_1$  with respect to  $\beta_2$ , the percent change in the output efficiency of solar from research, and the tax rate,  $\tau$ . We find that the Hessian is negative definite. Consequently, since reducing  $X_1$  creates welfare by reducing the social cost of pollution  $\gamma X_1$ , the Hessian of the benefits,  $-\gamma X_1$  is positive definite and thus convex.

## VI. Conclusion

Throughout the paper, we considered coal and solar power as examples of fossil and renewable energy. Moreover, For instance, we discussed how to handle pollution when energy sources are intermittent; we found that, rather than a standard Pigouvian tax, we would be better off using a combination of research subsidies and carbon taxes. One implicit but important point here is that optimal policy must be tuned for differences in local electricity markets. That is, even if externalities are global and research spills over, optimal policy differs on a local scale since generation technologies differ on a local scale.

An aim for future research may be to develop a model of clean and dirty energy that incorporates both predictable and stochastic intermittency in a multi-period setting.

## VII. Appendix A: Supplementary Proofs

### A. Cobb-Douglas Case with Two Periods & Two Technologies

In this section, we consider a simpler case of our general model to better understand its implications. Firstly, we restrict the utility function to its Cobb-Douglas form which is simply the case where the elasticity of substitution  $\sigma = 1$ . Secondly, we limit the number of periods and technologies to 2. And, thirdly, we normalize the prices such that our representative consumer's income  $I$  is 1.

#### Equilibrium Results

Firstly, our demand equations simplify to:

$$Z_t = \alpha_t / p_t \quad (7)$$

$$Z_s = \alpha_s / p_s \quad (8)$$

where  $t$  and  $s$  are our two periods. Next, solving for the FOC condition for profit maximization, we have:

$$p = \xi^{-1} c$$

$$p = \left( \begin{array}{c} -\frac{c_1 \xi_{2s} - c_2 \xi_{1s}}{\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s}} \\ \frac{c_1 \xi_{2t} - c_2 \xi_{1t}}{\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s}} \end{array} \right)$$

And, substituting back into our demand equations, we find the equilibrium quantities for  $Z$  and  $X$ .

$$Z = \left( \begin{array}{c} \frac{\alpha_t (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{c_2 \xi_{1s} - c_1 \xi_{2s}} \\ \frac{\alpha_s (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{c_1 \xi_{2t} - c_2 \xi_{1t}} \end{array} \right) \Rightarrow X = \left( \begin{array}{c} \frac{\alpha_t \xi_{2s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} + \frac{\alpha_s \xi_{2t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} \\ -\frac{\alpha_t \xi_{1s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} - \frac{\alpha_s \xi_{1t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} \end{array} \right)$$

Furthermore, we derive restrictions on the parameters  $\xi$  and  $c$  by assuming  $Z, X > 0$ . These restrictions are detailed in [Table 1](#). There are two possible sets of symmetrical restrictions. The first set, Case 1, assumes that technology 2 is more cost effective in period  $t$ , while the second set, Case 2, assumes that technology 1 is more cost effective in period  $t$ . If a given set of parameters do not fall into either case, we are left with an edge case where one of the technologies is not used. Additionally, these inequalities compare two types of efficiency – output efficiency and cost efficiency; we define output efficiency as electricity output per unit of input and cost efficiency in terms of electricity output per dollar of input. We refer to the last set of restrictions as mixed, because they relate both cost and output efficiency.

**Proof:** We aim to derive conditions on  $\xi$  and  $c$  required to have positive  $Z$  and  $X$ , so we begin by assuming  $X, Z > 0$ . Second, since the equations so far are symmetrical, note that there be two symmetrical sets of potential restrictions we must impose on the parameters. Thus, we first assume the inequality  $c_1 \xi_{2t} - c_2 \xi_{1t} > 0$  to restrict ourselves to one of the two cases. This assumption results in the denominator of  $Z_s$  being positive. Hence, we must also have  $\xi_{1s} \xi_{2t} - \xi_{2s} \xi_{1t} > 0$  for  $Z_s > 0$ . This same term appears in the numerator for  $Z_t$ , hence its denominator must be positive:  $c_2 \xi_{1s} - c_1 \xi_{2s} > 0$ . Now, rewriting these inequalities, we have:

$$\begin{aligned} c_1 \xi_{2t} - c_2 \xi_{1t} > 0 &\implies \xi_{2t}/c_2 > \xi_{1t}/c_1 \\ c_2 \xi_{1s} - c_1 \xi_{2s} > 0 &\implies \xi_{1s}/c_1 > \xi_{2s}/c_2 \\ \xi_{1s} \xi_{2t} - \xi_{2s} \xi_{1t} > 0 &\implies \xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t} \\ &\implies \xi_{1t}/\xi_{1s} < \xi_{2t}/\xi_{2s} \end{aligned}$$

**Table 4:** PARAMETER RESTRICTIONS FOR  $Z, X > 0$

	Case 1	Case 2
<b>Cost Efficiency Restrictions</b>	$\xi_{2t}/c_2 > \xi_{1t}/c_1$ $\xi_{1s}/c_1 > \xi_{2s}/c_2$	$\xi_{2t}/c_2 < \xi_{1t}/c_1$ $\xi_{1s}/c_1 < \xi_{2s}/c_2$
<b>Output Efficiency Restrictions</b>	$\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t}$	$\xi_{2t}/\xi_{2s} < \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} < \xi_{2s}/\xi_{2t}$
<b>Mixed Efficiency Restrictions</b>	$\frac{\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1)} > \xi_{2s}/\xi_{2t}$ $\frac{\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1)} < \xi_{1s}/\xi_{1t}$	$\frac{\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1)} < \xi_{2s}/\xi_{2t}$ $\frac{\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)}{\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1)} > \xi_{1s}/\xi_{1t}$

*Note:* The inequalities in this table assume that all elements of  $\xi$  are greater than 0. The full proof given below provides equivalent restrictions for the zero cases.

Note that the latter two restrictions can be derived from the former two. Additionally, we implicitly assume that we have  $\xi > 0$ . However, this is not necessary assumption, since  $\xi$  invertible only requires  $\xi_{1t}\xi_{2s} > 0$  or  $\xi_{1s}\xi_{2t} > 0$ . Instead, we may leave the latter two inequalities in the form  $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$  which remains valid when values of  $\xi$  are equal to 0. Lastly, the mixed efficiency restrictions come from  $X > 0$ . To start, for  $X_1$ , we have:

$$\begin{aligned}
X_1 > 0 &\implies (\alpha_t \xi_{2s})(c_1 \xi_{2t} - c_2 \xi_{1t}) + (\alpha_s \xi_{2t})(c_1 \xi_{2s} - c_2 \xi_{1s}) < 0 \\
&\implies (\alpha_t \xi_{2s})(c_1 \xi_{2t} - c_2 \xi_{1t}) < (\alpha_s \xi_{2t})(c_2 \xi_{1s} - c_1 \xi_{2s}) \\
&\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s (c_2 \xi_{1s} - c_1 \xi_{2s})) / (\alpha_t (c_1 \xi_{2t} - c_2 \xi_{1t})) \\
&\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)) / (\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1))
\end{aligned}$$

Similarly, for  $X_2$ , note that only the numerators differ;  $\xi_{2s}$  is replaced with  $-\xi_{1s}$  and  $\xi_{2t}$  is replaced with  $-\xi_{1t}$ . Hence, we have

$$\begin{aligned}
X_2 > 0 &\implies (\alpha_t \xi_{1s})(c_1 \xi_{2t} - c_2 \xi_{1t}) + (\alpha_s \xi_{1t})(c_1 \xi_{2s} - c_2 \xi_{1s}) > 0 \\
&\implies (\xi_{1s}/\xi_{1t}) > (\alpha_s (\xi_{1s}/c_1 - \xi_{2s}/c_2)) / (\alpha_t (\xi_{2t}/c_2 - \xi_{1t}/c_1))
\end{aligned}$$

To double check, note that combining the inequalities from  $X_1 > 0$  and  $X_2 > 0$  leads to  $\xi_{2s}/\xi_{2t} < \xi_{1s}/\xi_{1t}$ . This is precisely the earlier result obtained from  $Z > 0$ . Again, it is important to note that we assume  $\xi > 0$  for to simplify the inequalities of  $X_1 > 0$  and  $X_2 > 0$ . Otherwise, we may leave the inequalities in their pre-simplified forms and they are still valid when  $\xi_{1t}\xi_{2s} > 0$  or  $\xi_{1s}\xi_{2t} > 0$ . ■

Let us consider the set of restrictions belonging to Case 1. The first inequality, our initial assumption, states that technology 2 is relatively more cost effective in period  $t$ . The second inequality claims technology 1 is relatively more cost effective in period  $s$ . The implications are fairly straightforward; if a technology is to be used, it must have an absolute advantage in cost efficiency in at least one period. The third condition states that the relative output efficiency of technology 2 is greater than that of the first technology in period  $t$ . And, the fourth condition makes a symmetrical claim but for the technology 1 and period  $s$ . These latter two restrictions regarding output efficiency enter  $Z$  and  $X$  through  $p$ ; they're simply a restatement of the invertibility of  $\xi$  and can also be derived through the cost efficiency restrictions.

The mixed efficiency restrictions are less intuitive. Firstly, note that  $(\xi_{1s}/c_1 - \xi_{2s}/c_2)$  is the difference in cost efficiency for the two technologies in period  $s$ ; this is equivalent to the increase in  $Z_s$  caused by shifting a marginal dollar towards technology 1. Similarly, the bottom term  $(\xi_{2t}/c_2 - \xi_{1t}/c_1)$  represents the change in  $Z_t$  caused by shifting a marginal dollar towards



technology 1. Both these terms are then multiplied by the share parameter of the utility function for their respective time periods. Furthermore, note that  $\alpha_t$  ( $\alpha_s$ ) is the elasticity of utility with respect to  $Z_t$  ( $Z_s$ ). Hence, in total, the mixed efficiency restrictions relate the relative cost efficiencies of each technology with their output efficiency and the demand for energy. So, for example, suppose that consumers prefer, *ceteris paribus*, that nearly all their electricity arrive in period  $t$ . This would imply  $\alpha_t$  is arbitrarily large which results in the left-hand side of the fraction becoming arbitrarily small. This violates the first mixed efficiency restriction but not the second; consequently, use of the first technology, which is less cost effective in period  $t$ , approaches 0.

In more practical terms, suppose that our first technology is coal power and the latter is solar power. Although coal power is dispatchable, it does not easily ramp up or down within a day; hence, it is reasonable to apply our model where capacities are fixed over time so long as our time frame is sufficiently short. Hence, we now assume periods  $t$  and  $s$  represent the peak and off-peak for a day. And, we expect that there is more available solar radiation during peak hours than off-peak hours, since peak hours are usually during the middle of the day. This implies that the output efficiency of solar power is higher in period  $t$  due to more available solar radiation. Additionally, since the energy output of a unit of coal is independent of time, we know that the output efficiency of coal is constant. In total, this implies that we have met the output efficiency restrictions, since we have  $\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$ . Next, we can reasonably assume that coal is more cost effective than solar in the off-peak period when there is less sun; hence, the second cost efficiency restriction is satisfied. Then, for there to be an incentive to use solar power, we must satisfy the first cost-efficiency condition; that is, solar needs be cost effective during peak hours otherwise we hit an edge case where no solar is employed. And, finally, solar must also satisfy the mixed efficiency condition, which essentially implies that there must be sufficient demand for electricity during period  $t$ , when solar is more effective, for it to be a feasible technology. So, overall, for a technology to be economical, it must meet three conditions: it must be the most cost effective technology for a particular period, it must have a comparative advantage in output efficiency in the same period, and there must be sufficient amount of demand in that period.

## Comparative Statics

The comparative statics are similarly intuitive. The equilibrium quantity of a technology is increasing with its output efficiency and decreasing with its cost per unit. Additionally, the equilibrium quantities for a particular technology move in the opposite direction with respect to the output efficiency and cost of the other technologies. For a practical example, consider again coal and solar power from before. An increase in the output efficiency of solar or a decrease in solar power's cost will reduce the optimal quantity of coal power. Likewise, as coal power's efficiency improves, its adoption rises. To find the effects of  $\alpha$  on  $X$ , we must assume one of the cases of restrictions shown in Table 1. So, again, let us assume Case 1 is true; this implies that  $X_2$  is the most cost effective technology in period  $t$  and likewise for  $X_1$  in period  $s$ . Firstly, note that  $\alpha$  determines demand for electricity in a period. Hence, when  $\alpha_t$  rises, we see the optimal level of  $X_2$  rise as well; likewise,  $X_1$  rises with  $\alpha_s$ . In short, the optimal quantity of a technology rises linearly with the demand for electricity in the period it specializes in. Moreover, these relationships are reversed with respect to demand in each technology's suboptimal period. So, for example, we would expect the use of solar energy to rise when demand for electricity during peak hours rises, and it would fall when demand for energy in the off-peak rises. On the other hand, use coal power would rise with off-peak demand and fall with peak demand. This concept carries through for the comparative statics of  $Z$ . When the output efficiency of technology 1 rises or its cost falls, we see output  $Z_s$  rise and output  $Z_t$  fall. This is because technology 1 is optimal in period  $s$  given the Case 1 restrictions. Likewise, we see symmetrical results for the output with respect to the cost and output efficiency of technology 2; improvements in the

efficiency of  $X_2$  result in greater output in  $Z_t$  and smaller output in  $Z_s$ .

**Proof:** We begin by deriving the comparative statics of the cost and efficiency parameters with respect to  $X$ . Firstly, we take derivatives with respect to the cost vectors:

$$\begin{aligned}\frac{\partial X_1}{\partial c} &= \begin{pmatrix} \frac{-\alpha_t \xi_{2s}^2}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} - \frac{\alpha_s \xi_{2t}^2}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 \\ \frac{\alpha_t \xi_{1s} \xi_{2s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} + \frac{\alpha_s \xi_{1t} \xi_{2t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 \end{pmatrix} = \begin{pmatrix} < 0 \\ > 0 \end{pmatrix} \\ \frac{\partial X_2}{\partial c} &= \begin{pmatrix} \frac{\alpha_t \xi_{1s} \xi_{2s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} + \frac{\alpha_s \xi_{1t} \xi_{2t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 \\ \frac{-\alpha_t \xi_{1s}^2}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} - \frac{\alpha_s \xi_{1t}^2}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 \end{pmatrix} = \begin{pmatrix} > 0 \\ < 0 \end{pmatrix}\end{aligned}$$

The first and second terms of  $\partial X_1/\partial c_1$  are clearly both negative independent of the restrictions on the parameters. Similarly, all terms of  $\partial X_1/\partial c_2$  are positive independent of any restrictions. Since the structure of this problem is symmetrical with respect to  $X_1$  and  $X_2$ , the same comparative statics apply but in reverse for  $X_1$ . Next, we derive comparative statics for each element of  $\xi$ .

$$\begin{aligned}\frac{\partial X_1}{\partial \xi} &= \begin{pmatrix} \frac{\alpha_s c_2 \xi_{2t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 & \frac{\alpha_t c_2 \xi_{2s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} > 0 \\ \frac{-\alpha_s c_2 \xi_{1t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 & \frac{-\alpha_t c_2 \xi_{1s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} < 0 \end{pmatrix} = \begin{pmatrix} > 0 & > 0 \\ < 0 & < 0 \end{pmatrix} \\ \frac{\partial X_2}{\partial \xi} &= \begin{pmatrix} \frac{-\alpha_s c_1 \xi_{2t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 & \frac{-\alpha_t c_1 \xi_{2s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} < 0 \\ \frac{\alpha_s c_1 \xi_{1t}}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 & \frac{\alpha_t c_1 \xi_{1s}}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} > 0 \end{pmatrix} = \begin{pmatrix} < 0 & < 0 \\ > 0 & > 0 \end{pmatrix}\end{aligned}$$

Again, the signs are fairly straightforward. The optimal quantity of  $X_1$  increases with its output efficiency in both periods; however, it decreases with the output efficiency of  $X_2$  in both periods. Similarly, symmetrical results are shown for  $X_2$ . Next, we study the effects of  $\alpha$  on  $X$ ; this requires us to place some restrictions on the parameters, so we use those belonging to Case 1 in Table 1. With  $\alpha \equiv (\alpha_t \ \alpha_s)^T$ ,

$$\begin{aligned}\frac{\partial X_1}{\partial \alpha} &= \begin{pmatrix} \frac{\xi_{2s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} < 0 \\ \frac{\xi_{2t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} > 0 \end{pmatrix} = \begin{pmatrix} < 0 \\ > 0 \end{pmatrix} \\ \frac{\partial X_2}{\partial \alpha} &= \begin{pmatrix} \frac{-\xi_{1s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} > 0 \\ \frac{-\xi_{1t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} < 0 \end{pmatrix} = \begin{pmatrix} > 0 \\ < 0 \end{pmatrix}\end{aligned}$$

Note that our restrictions imply that  $c_1 \xi_{2t} - c_2 \xi_{1t} > 0$  and  $c_2 \xi_{1s} - c_1 \xi_{2s} > 0$ . From here, the intuition is clear; we assume that  $X_2$  is more cost efficient in period  $t$ , so increases in demand during period  $t$  (caused by increases in  $\alpha_t$ ) will increase the optimal quantity of  $X_2$ . And, the same applies to  $X_1$  with respect to period  $s$  and  $\alpha_s$ . Again, due to symmetry, the statics are reversed when the technologies are flipped. Similarly, the signs would also be flipped if we used the restrictions given by Case 2 instead.

Next, we derive the comparative statics for  $Z$ .

$$\begin{aligned}\frac{\partial Z_t}{\partial c} &= \begin{pmatrix} \frac{\alpha_t \xi_{2s} (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} > 0 \\ \frac{-\alpha_t \xi_{1s} (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} < 0 \end{pmatrix} = \begin{pmatrix} > 0 \\ < 0 \end{pmatrix} \\ \frac{\partial Z_s}{\partial c} &= \begin{pmatrix} \frac{-\alpha_s \xi_{2t} (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 \\ \frac{\alpha_s \xi_{1t} (\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s})}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 \end{pmatrix} = \begin{pmatrix} < 0 \\ > 0 \end{pmatrix}\end{aligned}$$

From our restrictions, we have  $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$ , and all the results above follow from this.

$$\begin{aligned}\frac{\partial Z_t}{\partial \xi} &= \begin{pmatrix} \frac{\alpha_t \xi_{2s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} < 0 & \frac{-\alpha_t \xi_{2s} (c_1 \xi_{2t} - c_2 \xi_{1t})}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} < 0 \\ \frac{-\alpha_t \xi_{1s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} > 0 & \frac{\alpha_t \xi_{1s} (c_1 \xi_{2t} - c_2 \xi_{1t})}{(c_1 \xi_{2s} - c_2 \xi_{1s})^2} > 0 \end{pmatrix} = \begin{pmatrix} < 0 & < 0 \\ > 0 & > 0 \end{pmatrix} \\ \frac{\partial Z_s}{\partial \xi} &= \begin{pmatrix} \frac{-\alpha_s \xi_{2t} (c_1 \xi_{2s} - c_2 \xi_{1s})}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} > 0 & \frac{\alpha_s \xi_{2t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} > 0 \\ \frac{\alpha_s \xi_{1t} (c_1 \xi_{2s} - c_2 \xi_{1s})}{(c_1 \xi_{2t} - c_2 \xi_{1t})^2} < 0 & \frac{-\alpha_s \xi_{1t}}{c_1 \xi_{2t} - c_2 \xi_{1t}} < 0 \end{pmatrix} = \begin{pmatrix} > 0 & > 0 \\ < 0 & < 0 \end{pmatrix}\end{aligned}$$

Again, recall that we have  $c_1\xi_{2t} - c_2\xi_{1t} > 0$  and  $c_2\xi_{1s} - c_1\xi_{2s} > 0$ ; the rest follows. And finally, we have:

$$\begin{aligned}\frac{\partial Z_t}{\partial \alpha} &= \begin{pmatrix} \frac{-\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s}}{c_1 \xi_{2s} - c_2 \xi_{1s}} \\ 0 \end{pmatrix} = \begin{pmatrix} > 0 \\ 0 \end{pmatrix} \\ \frac{\partial Z_s}{\partial \alpha} &> 0 = \begin{pmatrix} 0 \\ \frac{\xi_{1s} \xi_{2t} - \xi_{1t} \xi_{2s}}{c_1 \xi_{2t} - c_2 \xi_{1t}} > 0 \end{pmatrix} = \begin{pmatrix} 0 \\ > 0 \end{pmatrix}\end{aligned}$$

These are fairly trivial, since  $Z_t = \alpha_t/p_t$  (and  $Z_s = \alpha_s/p_s$ ) and prices are positive.

■

### B. Optimal Taxation

The following proof is a derivation of the optimal tax  $\tau$  on  $X_1$  in the case where it produces externalities.

**Proof:** We aim to maximize welfare with respect to the tax  $\tau$ , hence our first order condition is:

$$\begin{aligned}0 &= \frac{\partial CS}{\partial \tau} + \frac{\partial PS}{\partial \tau} - \frac{\partial S(X)}{\partial \tau} + \frac{\partial \tau X_1}{\partial \tau} \\ &= \left( \underbrace{\frac{\partial CS}{\partial p} + \frac{\partial PS}{\partial p}}_{=0} - \frac{\partial S(X)}{\partial p} + \frac{\partial \tau X_1}{\partial p} \right) \frac{\partial p}{\partial \tau} \\ &= -\gamma \frac{\partial X_1}{\partial p} + \tau \frac{\partial X_1}{\partial p}\end{aligned}$$

where the derivatives of producer and consumer surplus are eliminated by the envelope theorem. Therefore, we must have  $\tau = \gamma$ .

■

### C. CES Production as a Special Case

Our framework nests the case where there exists a CES production structure between each technology. This happens when no two technologies produce in the same period and we have the same number of periods as technologies. Additionally, the CES production function's elasticity parameter in this case will be equivalent to the consumer's CES utility elasticity parameter (the intertemporal elasticity of substitution for electricity consumption).

**Proof:** Firstly, note that we can reindex our technologies such that  $\xi$  is diagonal, since each technology only produces in one period. Hence, without loss of generality, we have diagonal  $\xi$ . Next, we may say that the electricity output in period  $i$  is given by  $Z_i = \xi_{i,i} X_i$ . Now, recall that the FOC for profit-maximization is given by  $p = \xi^{-1} c$ , hence we have  $p_i = c_i / \xi_{i,i}$ . And, finally, with the FOC for utility maximization, we have:

$$\begin{aligned}\frac{Z_i}{Z_j} &= \left( \frac{\alpha_i p_j}{\alpha_j p_i} \right)^\sigma \\ \frac{X_i}{X_j} &= \left( \frac{\alpha_i p_j \xi_{j,j}^{1/\sigma}}{\alpha_j p_i \xi_{i,i}^{1/\sigma}} \right)^\sigma \\ \frac{X_i}{X_j} &= \left( \frac{\alpha_i c_j \xi_{j,j}^{1/\sigma-1}}{\alpha_j c_i \xi_{i,i}^{1/\sigma-1}} \right)^\sigma\end{aligned}$$

By definition, the elasticity of substitution between any two, arbitrary technologies  $i$  and  $j$  is constant. Moreover, it can be shown that this FOC can be rearranged to give the following demand equation for each technology  $i$

$$X_i = \left( \frac{\beta_i}{c_i} \right)^\sigma \frac{I}{P} \quad (9)$$

$$P = \sum_t \beta_i^\sigma p_i^{1-\sigma} \quad (10)$$

where  $\beta_i = \alpha_i \xi_{i,i}^{-\phi}$ ,  $\sigma = 1/(1 - \phi)$ , and  $I$  is the consumers income. So, in total, accounting for both the producer and consumer's objectives, we are essentially solving for:

$$\begin{aligned}V &= \left( \sum_i \beta_i X_i^\phi \right)^{(1/\phi)} \\ \text{such that } I &= \sum_i c_i X_i\end{aligned}$$

This is a standard CES function.

■

## VIII. Literature

- Delarue et al. (2010) develop a model that distinguishes between power and energy in order to split up costs and risks into fixed and variable factors.
- Shahriari and Blumsack (2018) look at interactions between different solar and wind installations (covar matrix), while many other energy models do not.

## References

Acemoglu, D., Aghion, P., Bursztyn, L., & Hemous, D. (2012). The Environment and Directed Technical Change. *American Economic Review*, 102(1), 131–166. <https://doi.org/10.1257/aer.102.1.131>

- Ambec, S., & Crampes, C. (2012). Electricity provision with intermittent sources of energy. *Resource and Energy Economics*, 34(3), 319–336. <https://doi.org/10.1016/j.reseneeco.2012.01.001>
- Aubin, C., Fougere, D., Husson, E., & Ivaldi, M. (1995). Real-time pricing of electricity for residential customers: Econometric analysis of an experiment. *Journal Of Applied Econometrics*, 10, S171–S191.
- Borenstein, S. (2012). The Private and Public Economics of Renewable Electricity Generation. *Journal of Economic Perspectives*, 26(1), 67–92. <https://doi.org/10.1257/jep.26.1.67>
- Chao, H. (2011). Efficient pricing and investment in electricity markets with intermittent resources. *Energy Policy*, 39(7), 3945–3953. <https://doi.org/10.1016/j.enpol.2011.01.010>
- Climate Prediction Center (2019). Degree Days Statistics. National Oceanic and Atmospheric Administration. [https://www.cpc.ncep.noaa.gov/products/analysis\\_monitoring/cdus/degree-days/](https://www.cpc.ncep.noaa.gov/products/analysis_monitoring/cdus/degree-days/)
- Delarue, E., De Jonghe, C., Belmans, R., & D’Haeseleer, W. (2010). Applying portfolio theory to the electricity sector: Energy versus power. *Energy Economics*, 33(1), 12–23. <https://doi.org/10.1016/j.eneco.2010.05.003>
- Deryugina, T., Mackay, A., & Reif, J. (2017). The Long-Run Dynamics of Electricity Demand: Evidence from Municipal Aggregation. NBER Working Paper Series. <https://doi.org/10.3386/w23483>
- Electricity Information Administration (2019). Net generation, United States, all sectors, annual. Electricity Data Browser. <https://www.eia.gov/electricity/data/browser>
- Electricity Information Administration (2019). Levelized Cost and Levelized Avoided Cost of New Generation. Annual Energy Outlook 2019. [https://www.eia.gov/outlooks/aeo/pdf/electricity\\_generation.pdf](https://www.eia.gov/outlooks/aeo/pdf/electricity_generation.pdf)
- Fan, S., & Hyndman, R. (2011). The price elasticity of electricity demand in South Australia. *Energy Policy*, 39(6), 3709–3719. <https://doi.org/10.1016/j.enpol.2011.03.080>
- Foley, A., Leahy, P., Marvuglia, A., & Mckeogh, E. (2012). Current methods and advances in forecasting of wind power generation. *Renewable Energy*, 37(1), 1–8. <https://doi.org/10.1016/j.renene.2011.05.033>
- Geroski, P. (2000). Models of technology diffusion. *Research Policy*. Elsevier B.V. [https://doi.org/10.1016/S0048-7333\(99\)00092-X](https://doi.org/10.1016/S0048-7333(99)00092-X)
- Helm, C., & Mier, M. (2019). On the efficient market diffusion of intermittent renewable energies. *Energy Economics*, 80, 812–830. <https://doi.org/10.1016/j.eneco.2019.01.017>
- Herriges, J., Baladi, S., Caves, D., & Neenan, B. (1993). The Response of Industrial Customers to Electric Rates Based Upon Dynamic Marginal Costs. *The Review of Economics and Statistics*, 75(3), 446. <https://doi.org/10.2307/2109458>
- Joskow, P. (2011). Comparing the Costs of Intermittent and Dispatchable Electricity Generating Technologies. *American Economic Review*, 101(3), 238–241. <https://doi.org/10.1257/aer.101.3.238>
- King, Kathleen, and Peter Shatrawka (1994). "Customer Response to a Permanent Time-Varying Pricing Program in the United Kingdom." Laurits R. Christensen Associates, Madison, Wisconsin.

- Mohajeryami, S., Moghaddam, I., Doostan, M., Vatani, B., & Schwarz, P. (2016). A novel economic model for price-based demand response. *Electric Power Systems Research*, 135, 1–9. <https://doi.org/10.1016/j.epsr.2016.03.026>
- Musgens, F., & Neuhoﬀ, K. (2006). *Modelling Dynamic Constraints in Electricity Markets and the Costs of Uncertain Wind Output*. Faculty of Economics.
- Neuhoﬀ, K., Cust, J. & Keats, K. (2007). Implications of intermittency and transmission constraints for renewables deployment. *Cambridge Working Papers in Economics* 0711, Faculty of Economics, University of Cambridge.
- ORNL (2004). Measurement practices for reliability and power quality. A toolkit of reliability measurement practices. U.S. Department of Energy. <https://info.ornl.gov/sites/publications/Files/Pub57467.pdf>
- Papageorgiou, C., Saarn, M., & Schulte, P. (2017). Substitution between clean and dirty energy inputs: a macroeconomic perspective.(Author abstract). *Review of Economics and Statistics*, 99(2), 201–212. [https://doi.org/10.1162/REST\\_a\\_00623](https://doi.org/10.1162/REST_a_00623)
- Reiss, P., & White, M. (2005). Household Electricity Demand, Revisited. *The Review of Economic Studies*, 72(252), 853. Retrieved from <http://search.proquest.com/docview/204335314/>
- Schwarz, P., Taylor, T., Birmingham, M., & Dardan, S. (2002). Industrial response to electricity real-time prices: short run and long run. *Economic Inquiry*, 40(4), 597–610. <https://doi.org/10.1093/ei/40.4.597>
- Shahriari, M., & Blumsack, S. (2018). The capacity value of optimal wind and solar portfolios. *Energy*, 148, 992–1005. <https://doi.org/10.1016/j.energy.2017.12.121>
- Zhou, S. & Teng, F., (2013). Estimation of urban residential electricity demand in China using household survey data.(Statistical data). *Energy Policy*, 61, 394–402.
- Staiger, D., & Stock, J. (1997). Instrumental Variables Regression with Weak Instruments. *Econometrica*, 65(3), 557–586. <https://doi.org/10.2307/2171753>
- U.S. Bureau of Economic Analysis (2019). Personal consumption expenditures: chain-type price index. FRED, Federal Reserve Bank of St. Louis. <https://fred.stlouisfed.org/series/PCEPI>, April 27, 2019.
- Wolak, F., & Patrick, R. (2001). The Impact of Market Rules and Market Structure on the Price Determination Process in the England and Wales Electricity Market. NBER Working Paper Series. <https://doi.org/10.3386/w8248>
- Woo, C., Chow, P., & Horowitz, I. (1996). Optional real-time pricing of electricity for industrial firms. *Pacific Economic Review*, 1(1), 79–92. <https://doi.org/10.1111/j.1468-0106.1996.tb00175.x>
- Zarnikau, J. (1990). Customer Responsiveness to Real-Time Pricing of Electricity. *The Energy Journal*, 11(4), 99. <https://doi.org/10.5547/ISSN0195-6574-EJ-Vol11-No4-6>