Estimation of CGE Parameters

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Theory

CES/CET Problem

$$\textbf{Prod./Trans. Function}: Q = \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi}$$

$$\textbf{Budget Constraint}: B = \sum_i p_i \cdot X_i$$

- p_i are prices, θ is the scaling coefficient, α_i is the share coefficient, B is the budget
- The elasticity of substitution is $\sigma = 1/(1-\phi)$
- The elasticity of transformation is $\psi = 1/(\phi - 1)$

CES versus CET - CES Example

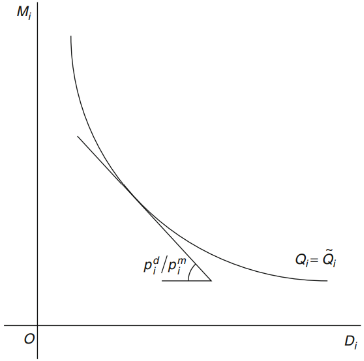


Figure 6.2 Isoquant of the CES function for the Armington composite good

- Isoquant of imported good M_i and domestic good D_i shows combination needed to produce a fixed quantity $\tilde{Q}_i$
- Consumer wants to satisfy a utility function including the composite Q_i
- Convexity shows diminishing returns to consuming only M_i or D_i

CES versus CET - CET Example

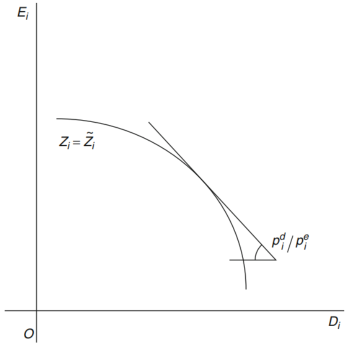


Figure 6.3 Isoquant of the CET function

- Isoquant of exports E_i and domestic supply D_i of some good i
- Shows combination needed to reach a GDP of $\tilde{Z}_i$
- Firms maximize profit by selling goods where they are most valued
- Concavity shows diminishing returns to selling everything domestically versus exporting everything

CES/CET Solution

CES/CET Problem

$$\textbf{Prod./Trans. Function}: Q = \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi}$$

$$\textbf{Budget Constraint}: B = \sum_i p_i \cdot X_i$$

$$\text{Profit Maximization} \implies \frac{X_i}{X_j} = \left(\frac{\alpha_i \cdot p_j}{\alpha_j \cdot p_i} \right)^{1/(1-\phi)}$$

$$\implies \ln\left(\frac{X_i}{X_j}\right) = \sigma \cdot \ln\left(\frac{\alpha_i \cdot p_j}{\alpha_j \cdot p_i}\right) = \psi \cdot \ln\left(\frac{\alpha_j \cdot p_i}{\alpha_i \cdot p_j}\right)$$

Methodology

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Inputs i and j in state s at time t
- X_i and X_j are the input quantities, p_i and p_j are the input prices
- Share parameters from previous equation are absorbed into the constant term
- This approach avoids endogeneity problems since the input ratios are not dependent on any variables except for price $\implies$ demand is fixed, supply movements trace out demand curve

Methodology (cont.)

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Elasticity of substitution/transformation is the same between all commodities in the production/transformation function
- Suppose coal (i), natural gas (j), and oil (k) are all part of the same production function for electricity

$$\begin{aligned}\implies & corr(\ln(X_i/X_j), \ln(p_j/p_i)) \\ =& corr(\ln(X_j/X_k), \ln(p_k/p_j)) \\ =& corr(\ln(X_i/X_k), \ln(p_k/p_i))\end{aligned}$$

Methodology (cont.)

CES $\implies$ Functional Form

$$\ln\left(\frac{X_{i,s,t}}{X_{j,s,t}}\right) = \sigma \cdot \ln\left(\frac{p_{j,s,t}}{p_{i,s,t}}\right) + c_{s,t} + \varepsilon_{s,t}$$

- Elasticity of substitution/transformation is the same between all commodities in the production/transformation function

$$\begin{aligned} \implies & corr(\ln(X_i/X_j), \ln(p_j/p_i)) \\ =& corr(\ln(X_j/X_k), \ln(p_k/p_j)) \\ =& corr(\ln(X_i/X_k), \ln(p_k/p_i)) \end{aligned}$$

- So, regressions are done by regressing all unique pairs with coefficients constrained to be equal

Methodology (cont.)

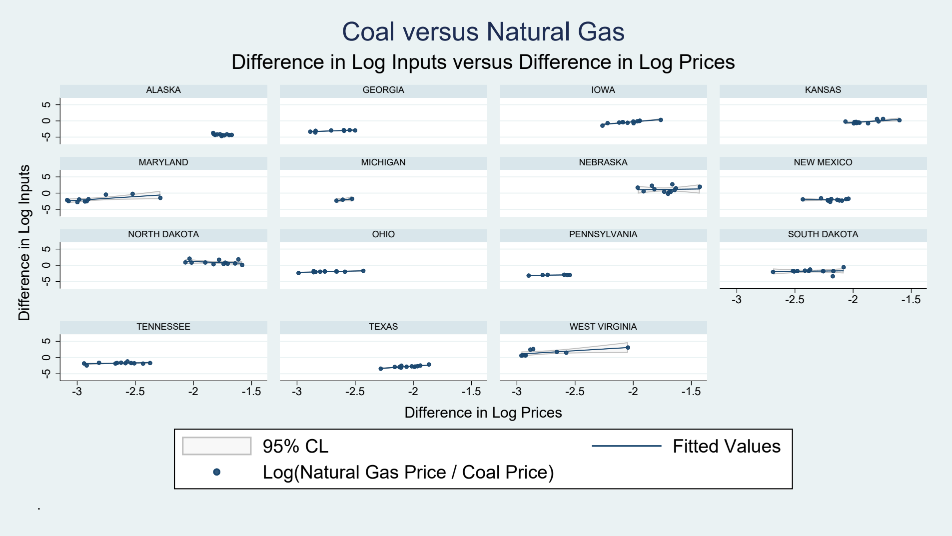
- Constrained regressions on $n * (n-1)/2 - 1$ sets of equations
- EG: (Coal/NatGas + Oil/NatGas), (Coal/NatGas + Oil/Coal), (Oil/Coal + Oil/NatGas)
- Theoretically, any set of equations in parenthesis should give the same results as another set of equations in parenthesis
- Data comes from the EIA for each state in 2016
  - Factor input quantities are set to the amount of each used to generate electricity
  - Factor prices are set to the be the average cost of the each input
  - Coal given thousand tons, Natural gas in thousand McF, and oil in thousand barrels

Fossil Fuel Results

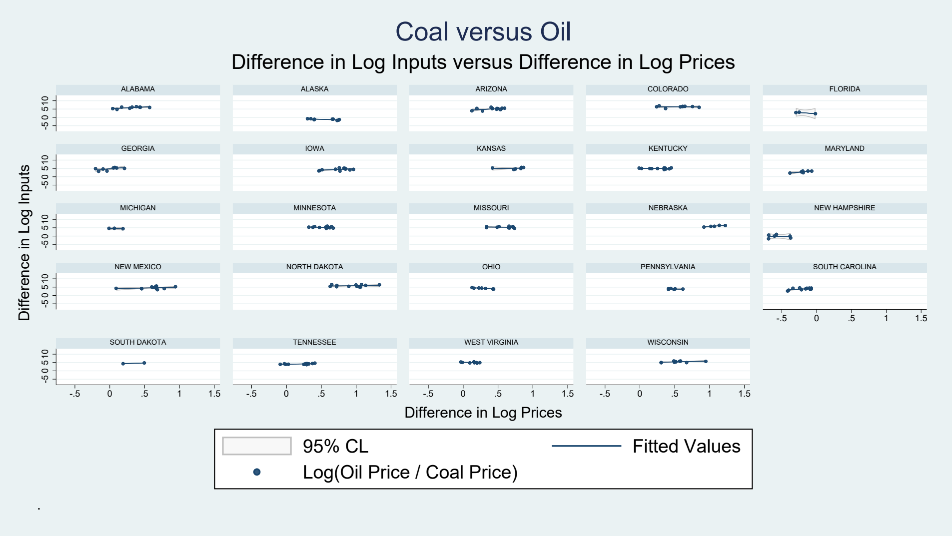
Table 1: Elasticity of Substitution for Fossil Fuels

	Dependent variable: Difference in Log Inputs					
	ln(Coal/NatGas)	ln(Coal/Oil)	ln(Coal/NatGas)	ln(Oil/NatGas)	ln(Coal/Oil)	ln(Oil/NatGas)
$\ln(p_{natgas}/p_{coal})$	1.002^{***} (0.209)		1.013^{***} (0.210)			
$\ln(p_{oil}/p_{coal})$		1.002^{***} (0.209)			1.004^{***} (0.209)	
$\ln(p_{natgas}/p_{oil})$				1.013^{***} (0.210)		1.004^{***} (0.209)
Constant	-2.429^{***} (0.398)	-1.782^{***} (0.190)	-2.411^{***} (0.399)	-0.623 (0.519)	-1.783^{***} (0.190)	-0.643 (0.517)
State Dummies	✓	✓	✓	✓	✓	✓
Observations	109	109	109	109	109	109
R^2	0.9284	0.9399	0.9284	0.8643	0.9399	0.8642
Chi^2	1414^{***}	1732^{***}	1427^{***}	706^{***}	1725^{***}	701^{***}

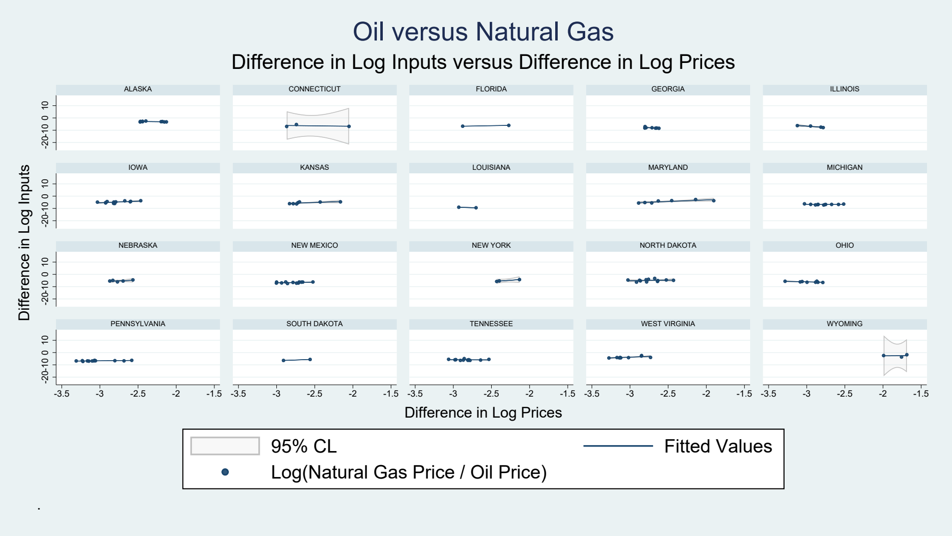
Fossil Fuel Results



Fossil Fuel Results



Fossil Fuel Results



Methodology - A Different Approach

CES Parameter Optimization

$$\min_{\theta,\phi,\alpha,\beta} ||-\ln(Q_{elec})+\ln(\theta)+\frac{1}{\phi}\cdot\ln\left(\alpha_{coal}X_{coal}^{\phi}+\alpha_{natgas}X_{natgas}^{\phi}+\alpha_{oil}X_{oil}^{\phi}\right)+\beta\cdot D_{state}||_2^2$$

- Can estimate all parameters at once using non-linear least squares
- D_{state} represents indicator variables for each state
- Can also check fit statistics to see if CES functional form matches the data

Table 2: CES Parameter Estimation for Fossil Fuels - Non-Linear

	Dependent Variable: Electricity Output from Fossil Fuels	
	$\ln(\theta) + \frac{1}{\phi} \cdot \ln\left(\alpha_{coal} X_{coal}^{\phi} + \alpha_{natgas} X_{natgas}^{\phi} + \alpha_{oil} X_{oil}^{\phi}\right) + \beta \cdot D_{state}$	
θ	1.144	0.676***
	(0.609)	(0.0941)
ϕ	0.917***	0.900***
	(0.0916)	(0.0658)
$\ln(\alpha_{coal})$	0.533	1.147
	(0.433)	(.)
$\ln(\alpha_{natgas})$	-1.323*	-0.925***
	(0.536)	(0.165)
$\ln(\alpha_{oil})$	1.308	-25.94
	(.)	(.)
State Dummies	✓	
Observations	518	518
R^2	0.934	0.796
Adj. R^2	0.928	0.795

Fossil Fuel Results

CES $\rightarrow$ Cobb-Douglas

$$\lim_{\phi \to 0} \theta \cdot \left(\sum_i \alpha_i \cdot X_i^{\phi} \right)^{1/\phi} = \theta \cdot \prod_i X_i^{\alpha_i}$$

- $\sigma = 1/(1-\phi)$
- Elasticity of substitution varied by approach
  - Ratios between commodities led to elasticities of substitution around 1 (Cobb-Douglas)
  - Non-linear least squares led to estimates around 10 (strong substitutes)
- Could try estimating the Translog Function
  - Second Order Taylor Polynomial of CES around $\phi = 0$
  - Works best when ϕ is expected to be around 0

Methodology

- Solar and wind elasticities are estimated using instrumental variables
- Instruments
    - Solar Supply: Average Solar Radiation, Land Area
    - Solar Demand: Population, Cooling/Heating Degree Days
    - Wind Supply: Average Wind Speed, Land Area
    - Wind Demand: Population, Cooling/Heating Degree Days

Table 3: Elasticities for Solar and Wind - 3SLS Estimation

	Dependent Variable			
	ln(Solar Net Gen)	ln(Solar Net Gen)	ln(Wind Net Gen)	ln(Wind Net Gen)
log(Electricity Price)	14.46***	-4.619*	4.997***	-16.98***
(*cents/kWh*)	(1.597)	(1.911)	(1.487)	(4.086)
log(Average Solar Radiation)	101.5*			
(kWh/m^2)	(51.33)			
Log(Average Wind Speed)			0.678***	
(m/s)			(0.0811)	
Land Area	1.14e-11***		1.41e-11***	
(m^2)	(1.16e-12)		(9.79e-13)	
Log(Population)		1.085***		1.166***
		(0.136)		(0.286)
Log(Cooling Degree Days)		-0.164		-0.322
		(0.102)		(0.191)
Log(Heating Degree Days)		-0.169		-0.0660
		(0.101)		(0.182)
Constant	-66.97***	7.541	-24.11***	65.21***
	(7.367)	(7.963)	(6.961)	(16.81)
State Dummies	✓	✓	✓	✓
Obs	504	504	504	504
Chi^2	109.3***	81.32***	76.64***	41.26***

Questions?