Efficient pollution abatement in electricity markets with intermittent renewable energy

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Use of renewable energy has increased greatly ... We evaluate renewable energy technologies in a partial equilibrium setting that integrates their production profiles with variation in the market price of electricity. In order to discount the ..., we use a representative agent that prefers to smooth their electricity consumption over time. Our results find a highly non-linear elasticity of substitution between intermittent and fossil energy. Furthermore, we find that standard environmental policy such as carbon taxes and renewable subsidies can have important distributional consequences as a result of intermittency. Subsidizing research that improves battery technologies can complement present policy and greatly increase the substitutability of renewable and fossil energy.

I. Introduction

Renewable energy technologies have seen considerable adoption over the last few decades (EIA 2019). Unlike the alternatives, wind and solar power are particularly unique in that the amount of energy they supply is intermittent. Consequently, designing economically efficient policy to promote their adoption is not straightforward given that they cannot easily substitute for fossil fuel technologies such as coal power. Moreover, traditional economic measures such as the levelized cost of electricity (LCOE) fail to capture the true economic value of intermittent technologies, because they neglect to account for variation in output and prices over time (Joskow, 2011).

Some of the literature has overcome this critique by constructing numerical models that find the cheapest renewable technology set while accounting for intermittent supply. For instance, Musgens and Neuhoff (2006) model uncertain renewable output with intertemporal generation constraints, while Neuhoff et al. (2007) model temporal and spatial characteristics of wind output to optimize its deployment in the UK. On the other hand, other literature focuses on the effect of intermittent technologies on the market itself; Ambec and Crampes (2012) study the interaction between intermittent renewables and traditional reliable sources of energy in decentralized markets, and Chao (2011) models alternative pricing mechanisms for intermittent renewable energy sources. Additionally, Borenstein (2012) reviews the effects of present public policies used to promote renewables and the challenges posed by intermittency.

Our model comes closest to that of Helm and Mier (2019) who build a peak-load pricing model where the availability of renewable capacity varies stochastically. This stochastic variability models intermittency and negatively affects the adoption of renewables. Furthermore, they simplify their analysis by setting aside complications such as outage costs and rationing

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rules. In total, their model finds an S-shaped adoption curve for renewables as they get cheaper. In addition, they find that a Pigouvian tax can properly internalize the costs of fossil fuels in a setting with perfect competition. Likewise, we model the equilibrium of a market with dynamic prices and access to both renewable and fossil energy. However, our results differ in key ways, because we take a different approach to modeling intermittency.

To highlight this difference, we must first contrast the terms *intermittency* and *reliability*. By intermittency, we predictable changes in output related to physical constraints. For instance, while wind energy output varies over time, if we know the angle and speed of wind, we may precisely derive the quantity of energy that a wind farm generates.[1] At the same time, these forecasts may not be perfect and, as a result, we see unexpected variation in the output of a technology. This unpredictability may be better captured by the notion of reliability; specifically, we refer to definition provided by the US Department of Energy's ORNL (2004) – "Power reliability can be defined as the degree to which the performance of the elements in a bulk system results in electricity being delivered to customers within accepted standards and in the amount desired." This is more so related to the stochastic or unpredictable variation in the output of a technology. Unlike intermittency, this cannot be planned around with 100% certainty. A practical example may be a wind turbine's systems failing. While we may know the chances of this occurring, it's not possible to know when it will occur ahead of time; consequently, this may result in a temporary reduction in the quantity and quality of the electricity delivered – a loss of reliability. In short, when Helm and Mier define renewable output as equal to a baseline capacity multiplied by a uniform random variable, their model is closer to one of reliability rather than intermittency.

On the other hand, we model intermittency as defined above by allowing the output of renewable energy to differ between periods according to known function. For parsimony, we assume that the output of all technologies in each period does not vary stochastically; that is, we focus on intermittency rather than reliability. Next, we model the electricity sector using a representative firm that chooses and builds capacity from a set of electricity-generating technologies to maximize profit; some of these technologies are intermittent while others have constant output. Then, we consider a representative consumer who purchases varying quantities of electricity in each period in order to maximize utility. Generally, people prefer to spread their electricity consumption out over time. We model this preference by using a CES function of electricity consumption differentiated by period.[2] These two sides of the market reach an equilibrium through adjustments in prices of electricity in each period. All in all, we make many of the same assumptions that Helm and Mier (2019) do; we assume dynamic pricing,[3] no load rationing, and positive prices.

Then, we parametrize our model empirically. To start, we fit the parameters of the consumer's CES utility function using electricity consumption and price data for each US state. Specifically, we estimate the intertemporal elasticity of substitution for electricity consumption; this parameter plays a particularly important role in our model, since it captures the effects of intermittency on demand. Then, to model the supply side, we narrow our model to a two-period, two-technology setting with renewables and fossil energy. We proxy for renewable energy using solar and proxy for fossil energy using coal; then, we parametrize each accordingly. Finally, we implement our model numerically and solve for the equilibrium. We study the effects of

[1] Foley et al. (2012) provide a review of the literature on the forecasting of wind power. Over time, forecasts have gotten much more accurate allowing electricity grids to manage wind power intermittency ahead of time.

[2] The use of a CES function in this way has been explored earlier by Schwarz et al. (2002), Schwarz et al. (2002), Herriges et al. (1993), and King and Shatrawka (2011), Aubin et al. (1995), and Mohajeryami et al. (2016). Their papers empirically estimate the parameters for this function; we provide a more detailed discussion of the empirical literature in our Methodology.

[3] Helm and Mier provide a motivation for models incorporating the dynamic pricing of electricity. They argue such approaches to pricing will become the norm with further technological advances and coming regulatory changes.

technical change on the energy market and derive policy implications.

Overall, this treatment of intermittency produces some key results that contrast with those of Helm and Mier. First, while we also find that externalities can be handled through a Pigouvian tax, we further find that a combining both taxes on pollution and research subsidies for renewables is more optimal than either policy instrument alone. Second, we find that, as renewables get cheaper, their adoption is not necessarily S-shaped but can be concave or convex depending the elasticity of substitution in the consumer's utility function; with our estimated parameters, we find this relationship to be almost linear. Third, a significant amount of literature has assumed a CES structure between renewable and fossil energy (see Papageorgiou et al. (2017)); this assumption has been motivated, in part, by the need to capture imperfect substitutability between these two generation technologies as a result of intermittency. However, our direct model of intermittency disconfirms this assumption; the elasticity of substitution between these two energy technologies is far from constant as a *consequence* of intermittency. Moreover, it varies with the prices and quantities of energy generation technologies employed. Hence, we believe that optimal environmental policy, such as pollution taxes and research subsidies, should take into account the technologies powering local energy markets in order to effectively handle externalities.

II. Model

A. Consumers

Consumers purchase variable amounts of electricity Z_t over each period t. Additionally, assuming that the price of electricity in each period is held constant, they prefer to ration their electricity usage across each period in fixed proportions. And, lastly, we presume that consumers would be willing to shift their consumption from one period to another in response to a shift in prices. Overall, these assumptions can be captured using a a standard CES utility function

$$U = \left(\sum_t \alpha_t Z_t^{\phi} \right)^{1/\phi} \tag{1}$$

where $\sigma = 1/(1-\phi)$ is the intertemporal elasticity of substitution for electricity consumption. We define $\sum_t \alpha_t = 1$, so that α_t is the fraction of electricity consumption in period t when all prices are equal; naturally, $\alpha_t > 0$ for all t. We assume that any discounting over time is absorbed into α for parsimony.The budget constraint is given by

$$I = \sum_t p_t Z_t \tag{2}$$

where p_t is the price of electricity in period t and I is the income. Furthermore, since CES preferences are homothetic, we may aggregate the consumers into a single representative consumer. Consequently, the first order conditions four our representative consumer when maximizing utility given this budget constraint imply:

$$Z_t = \left(\frac{\alpha_t}{p_t} \right)^{\sigma} \frac{I}{P} \tag{3}$$

$$P = \sum_t \alpha_t^{\sigma} p_t^{1-\sigma} \tag{4}$$

where P is the price index. Note that this model naturally does not allow for blackouts in equilibrium, since the price of electricity in any period gets arbitrarily large as the quantity of energy consumed in that period approaches 0. Furthermore, note that prices must be positive; although this is sometimes violated in reality, we do not believe that this assumption significantly affects our analysis.

B. Firms

Secondly, we have firms maximizing profit by picking an optimal set of energy inputs. In reality, electricity markets are fairly competitive, so we can model the set of firms by using a single representative firm that sets marginal revenue equal to marginal cost. We define the quantity of deployed energy technology i as X_i, and we define its output per unit in period t as $\xi_{1,t}$. So, for example, if i is solar power, X_i would be the number of solar panels and $\xi_{i,t}$ may be kW generated per solar panel in period t. Consequently, the energy generated in period t, Z_t, is given by $\sum_i \xi_{i,t} X_i$. We define these variables in matrices to simplify notation:

$$X \equiv \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix},\ Z \equiv \begin{pmatrix} Z_1 \\ \vdots \\ Z_m \end{pmatrix},\ p \equiv \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix},\ \xi \equiv \begin{pmatrix} \xi_{1,1} & \dots & \xi_{1,m} \\ \vdots & \ddots & \\ \xi_{n,1} & & \xi_{n,m} \end{pmatrix}$$

where we have n technologies, m periods, and $Z \equiv \xi^T X$. A key element of this model is that X does not vary by time. This is important, because the time scale of our model must be fairly granular to study the effects of intermittency. So, for instance, we may have $t \in \{1, \ldots, 24\}$ representing each hour of the day. In such short time scales, we can safely assume that producers do not modify the quantity of the technology they employ. Moreover, this keeps the model parsimonious by preventing more complicated dynamics from entering the producers optimization problem.

Instead, our representative firm sets X once to maximize profit in all periods while facing the cost function $C(X_1, \ldots, X_n)$. Helm and Mier (2019) note that past literature has argued for concave cost functions in the energy market due to effects such as economics of scale and learning by doing; on the other hand, standard cost functions are generally convex. So, like Helm and Meir, we take an intermediate approach by using a linear cost function. Specifically, we have $C(X) \equiv \sum_i c_i X_i \equiv c^T X$ where c_i is the cost per unit of X_i. Total profit is given by

$$\Pi = p^T Z - c^T X \tag{5}$$

To simplify the algebra, we set the number of technologies equal to the number of periods ($n = m$). Additionally, we further require that the output per unit of each technology is unique and non-negative in each period; in other words, the output per unit of one technology is not a linear combination of those of the other technologies in our set. This then implies that ξ is of full rank and therefore invertible. Now, maximizing profit, we find the first order condition:

$$\frac{\partial \Pi}{\partial X} = 0 \implies p = \xi^{-1} c \tag{6}$$

Combining this FOC with the demand equation (Equation (3)) allows us to find the equilibrium. Generally, the equilibrium results for any number of technologies (n) are analytic, but they are difficult to interpret due to the number of parameters involved.

C. Equilibrium

For more tractable results, we consider a simpler scenario where $n = m = 2$ and $\sigma = 1$ (Cobb-Douglas); this particular case is described in greater detail with proofs in Appendix A.A. Moreover, these parameters are of particular interest, because they simplify the model enough to allow us to derive comparative statics and determine which sets of parameters lead to edge cases.

To start, we discuss the conditions on the exogenous parameters required to avoid edge-cases. Alternatively, these are the conditions required for a technology to be economical. But, first, we define two recurring terms in our analysis: cost efficiency and output efficiency. For some arbitrary period t and technology i, we use cost efficiency to refer to $\xi_{i,t}/c_i$ and output efficiency to refer to $\xi_{i,t}$. So, for example, A technology is more cost efficient than another in a particular period when its output in that period per "dollar" spent is larger than that of the other technology in the same period.

Proposition 1 *Assume that, for all i (technologies) and t (periods), we have $\xi_{i,t} > 0\,, \alpha_t > 0\,,$ and $c_i > 0$. Furthermore, suppose that ξ is invertible. For technology i to be economical, it must meet three conditions with respect to some arbitrary period t. Firstly, it must the more cost effective than any of the other technologies in period t. Secondly, it must have have a comparative advantage in output efficiency in the same period t. And, thirdly, there must be sufficient amount of demand for energy in this period t.*

The condition on cost efficacy is fairly intuitive; one case of the contrapositive is that if a technology is not the most cost effective in any period, it will not be used. Alternatively, if a technology is the most cost effective in every single period, it will be the only technology used. The second condition regarding output efficiency actually stems from the invertibility of ξ. If we did not have ξ invertible, we either have at least one technology that does not produce in any period or we have at least one technology being a linear combination of the other technologies in terms of output. In the latter case, some technologies are not economical because their output can be replicated the other technologies in a more cost effective way. As an example, suppose ξ consisted of 3 technologies but was of rank 2. In this case, we may represent any of these technologies as a linear combination of the other two; call this combination the synthetic version of technology i. It is not possible for every synthetic technology i to be more expensive than its original; for at least one i, we must have a synthetic technology being cheaper or equal in price to the actual technology. In the first case, synthetic technology i is cheaper so we do not use the actual technology i. In the later case, we may still eliminate technology i or not use the other two technologies depending on the demand. Overall, in any case, ξ not invertible means that at least one technology is not used. Finally, the demand condition is straightforward; even if a technology is optimal in a certain period, if consumers do not sufficiently demand electricity during that period, then there is little reason to use that technology.

We may also derive the comparative statics for this simplified scenario.

Proposition 2 *Suppose we are not in an edge case, so that the conditions of Proposition 1 hold for each technology. The equilibrium quantity of a technology is increasing with its output efficiency and decreasing with its cost; at the same time, it is decreasing with the output efficiency of the other technologies and increasing with their cost. Also, suppose that some technology i is the most cost effective in period t. Then, its equilibrium quantity is increasing with respect to the demand share parameter α_t and decreasing with respect to the demand share parameters of the other periods. Furthermore, again assuming technology i is the most cost effective in period t, the comparative statics of Z_t and X_i are equivalent.*

The comparative statics with respect to X and its output efficiency and cost are not surprising. On the other hand, the statics for Z may not be immediately obvious. Instead, they follow from the fact that we have $Z \equiv \xi_T X$. That is, suppose we have an arbitary technology i that is the most cost effective source of electricity on period t. If consumers demanded that 100% of their energy arrive in period t, then relying solely on technology i for energy would be the most economical solution. Consequently, it seems intuitive that the comparative statics of X_i follow through to Z_t. This intuition just happens to apply even when other technologies are employed and there is demand in multiple periods. Similarly, the comparative statics for the share parameters of the utility function, α, travel in the opposite direction. A rise in α_t would directly raise the optimal quantity of Z_t; hence, whichever technology is most cost effective at producing in period t would be used more. We provide a more detailed and quantitative discussion of the comparative statics and edge cases in the appendix.

III. Empirical Methodology

In order to better understand the practical implications of our model, we empirically estimate its parameters and study its implications numerically. We are particularly interested in estimating σ, the intertemporal elasticity of substitution for electricity consumption. In our discussion, we explain in detail why σ is of interest; but, to summarize, σ implicitly determines how well renewables can substitute for fossil energy. Specifically, if $\sigma > 1$, then electricity consumption in different periods are substitutes; consequently, fossil energy and renewable energy are highly substitutable. On the other hand, if σ is around 0.5, electricity consumption in different periods are complements; so, fossil and renewable energy are far less substitutable due to the effects of intermittency.

The other parameters in our model, c, ξ, and α, are of secondary interest, since they are easier to obtain directly. To start, c is the cost per unit for each technology; estimates of this value for different technologies can be obtained directly from the literature. Similarly, ξ, the output per unit of technology, can be directly obtained from the literature. And, finally, α may be approximated directly from the data. That is, note that the demand equation from earlier (Equation 3) is:

$$Z_t = \left(\frac{\alpha_t}{p_t}\right)^{\sigma} \frac{I}{P}$$

where P is the price index and I is income. Retail customers pay fixed rates each month for electricity, hence p_t is constant within each month; and, we expect that income I does not vary significantly on a daily basis. Consequently, all variation in intramonthly demand is due to the share parameter α and the elasticity σ. Hence, after estimating σ, we can approximate α. However, this raises another problem, because, without prices varying each hour, we cannot estimate σ on an hourly basis.

A number of other papers have approached this problem using data from real-time pricing experiments. These include Schwarz et al. (2002), Herriges et al. (1993), and King and Shatrawka (2011).[4] The latter two papers estimate σ to be around 0.1 while the paper by Schwarz et al. obtain estimates around 0.04. All three papers study real-time electricity pricing programs for industrial consumers using similar methodologies. Additionally, Aubin et al. (1995) also provide estimates of the σ but using a different methodology; their results find elasticity of substitution below 0. Under a CES structure, this is problematic, because it would imply upward-sloping demand curves. Finally, Mohajeryami et al. (2016) also empirically estimates the share parameters for a CES function of this form, but do not estimate the elasticity of substitution.

Overall, the past literature has approached the estimation of σ by running regressions with variants of the CES demand equation. We are concerned about endogeneity, because producers may intertemporally substitute electricity generation. For instance, during the oil crisis of 1973, refineries increased gasoline stocks expecting future prices to be higher (Adelman, 1995). The existence of such behavior implies that estimates of σ would be biased downwards unless we properly control for endogeneity. This is particularly important because whether σ is closer to 0.1 or 1 significantly change the practical implications of our model.

So, we take a different approach by using a supply instrument to identify the CES demand parameters. Specifically, we use coal prices which affect the supply of electricity but not the demand. Furthermore, we estimate σ on a monthly basis. This decision is primarily due to data limitations, since we do not have access to the proprietary data on real-time pricing

[4] There has also been a large literature that directly estimates the price elasticity of electricity demand without imposing a CES functional form. These papers include Wolak and Patrick (2001), Zarnikau (1990), Woo et al. (1996), Zhou and Teng (2013), Reiss and White (2005), Fan and Hyndman (2011), and Deryugina and Mackay (2017). These papers estimate own-price elasticities, while some also estimate cross-price elasticities for electricity consumption at different times. Because they do not impose a CES structure, we cannot obtain estimates of σ from this literature.

experiments which the past literature has used. Although we are interested in understanding intertemporal substitution over a shorter time scale (since intermittency plays a larger role in shorter periods), estimates of σ on a monthly basis may still be applicable on a smaller time frame. For instance, Schwarz et al. (2002) estimate σ on a daily and hourly basis and find fairly close results; similarly, Herriges et al. (1993) also find no significant difference in their estimates of σ for these two intervals. That is, while a daily basis is 24 times larger than an hourly basis, the estimates for σ, suprisingly, do not appear to change. Hence, we expect our estimates of σ on a monthly basis to not be far from estimates on shorter time scales. At the same time, our estimates of σ will likely be larger than that of the literature, because we are controlling for endogeneity. We now define our econometric methodology in detail.

A. Theory

Firstly, recall the demand equation (Equation 3) from our general model:

$$Z_t = \left(\frac{\alpha_t}{p_t}\right)^{\sigma} \frac{I}{P}$$
$$P = \sum_t \alpha_t^{\sigma} p_t^{1-\sigma}$$

For any pair of electricity ouputs Z_t and Z_s, we have:

$$\frac{Z_t}{Z_s} = \left(\frac{\alpha_t\, p_s}{\alpha_s\, p_t}\right)^{\sigma}$$

Taking logs on both sides and letting i represent different observations, we may rewrite this in a form more suitable for estimation.

$$\ln(Z_{t,i}/Z_{s,i}) = -\sigma \ln(P_{t,i}/P_{s,i}) + \sigma \ln(\alpha_{t,i}/\alpha_{s,i})$$

Our data differentiates consumption for each state in the US, so we let i refer to a particular state. Additionally, most consumers pay monthly fixed rates for electricity, so we can, at most, estimate this equation on a monthly basis; hence, t and s refer to different months. Lastly, note that the state i is kept constant for each observation; this is because consumers within each state can substitute consumption across time, but consumers in different states do not substitute consumption with one another.

In order to estimate this σ, we further modify the previous equation. Firstly, note that we cannot observe the demand shifter $\alpha_{t,i}$ directly, so we must replace the α terms with a set of controls that may be responsible for shifts in demand. So, still in general terms, our regression equation is now

$$\ln(Z_{t,i}/Z_{s,i}) = -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i} A_{t,i} + \gamma_{s,i} A_{s,i} + u_i$$

where A represents set of controls for changes in demand while u_i is a normal error term. Note that the control $A_{t,i}$ replaces $\sigma \ln(\alpha_{t,i})$ and likewise for the period s term; this substitution is valid because the $\ln(\alpha_{t,i}) \in \mathbb{R}$ and the σ is simply absorbed into the estimated coefficient for $\gamma_{t,i}$. For the demand controls themselves, we choose to use heating (HDD) and cooling degree days (CDD) due to the aggregation of the data. That is, a more general control such as average temperature would not be able to directly capture intramonthly changes in demand, since variation in temperature would be lost when aggregated; on the other hand, CDDs and HDDs directly represent daily deviations in temperature even when totaled for each month. Additionally, demand for electricity may rise over time. Hence, we include, as a control, the difference in months between time t and s; this is represented by $\Delta_{t,s}$. Finally, this panel requires us to consider fixed effects for each state, so we use a fixed effects panel regression. In total, the demand equation is:

$$\begin{aligned}\ln(Z_{t,i}/Z_{s,i}) &= -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i}A_{t,i} + \gamma_{s,i}A_{s,i} + \eta\Delta_{t,s} + u_i \\ &= -\sigma \ln(P_{t,i}/P_{s,i}) + \gamma_{t,i}\left(CDD_{t,i} + HDD_{t,i}\right) \\ &\quad + \gamma_{s,i}\left(CDD_{s,i} + HDDs, i\right) + \eta\Delta_{t,s} + u_i\end{aligned}$$

Still, this equation may suffer from bias, since producers can also substitute production over time. For instance, it is possible to store fuel for electricity generation in the future when prices may rise. So, we define the following supply equation

$$\ln(Z_{t,i}/Z_{s,i}) = \beta \ln(P_{t,i}/P_{s,i}) + \xi \ln(C_{t,i}/C_{s,i}) + v_i$$

where $C_{t,i}$ is the average cost of coal used for electricity generation in state i at time t and v_i is a normal error term. Coal prices are independent of the electricity demand error term u_i, since residential consumers do not generally use coal for electricity generation; on the other hand, shocks in the price of coal are linked with the supply of electricity. Hence, coal price is a theoretically valid instrument. In total, the reduced form equation is given by:

$$\ln(P_{t,i}/P_{s,i}) = (\beta + \sigma)^{-1}\left(\gamma_{t,i}A_{t,i} + \gamma_{s,i}A_{s,i} + \eta\Delta_{t,s} - \xi \ln(C_{t,i}/C_{s,i}) + u_i - v_i\right)$$

where $A_{t,i}$ consists of CDDs and HDDs at time t.

Finally, we also consider a semiparametric specification. That is, we allow the error terms u_i and v_i to be non-normal and place the demand controls and instruments in unknown functions. So, overall, we have:

$$\begin{aligned}\ln(Z_{t,i}/Z_{s,i}) &= -\sigma \ln(P_{t,i}/P_{s,i}) + f\left(A_{t,i}, A_{s,i}, \Delta_{t,s}\right) + u_i \\ \ln(Z_{t,i}/Z_{s,i}) &= \beta \ln(P_{t,i}/P_{s,i}) + g\left(\ln(C_{t,i}/C_{s,i})\right) + v_i\end{aligned}$$

where f and g are unknown, bounded functions. We restrict $cov(u_i, v_i) = 0$ but allow for the controls and instruments to be correlated. The advantage of this specification is that we can account for the controls or instrument having any nonlinear effects on the regressands. **(More here on explaining the estimation)**

B. Data

We collect monthly data from the EIA (2019a) on retail electricity prices and consumption for each state in the US from 2011 to 2018. Additionally, also from the EIA, we obtain data on the average cost of coal for electricity generation for each state and month.[5] We deflated both electricity and coal prices over time using the PCECP Index provided by the US BEA (2019). Finally, we collect data on HDDs and CDDs from the NOAA's Climate Prediction Center (2019) for the same panel. Then, we merge these three data sets and trim 1% of outliers for a total of 818 observations for each month and state. We use this preliminary data set to construct the data required for our regressions. That is, each observation in our estimation equation belongs to a set (t, s, i) consisting of two time periods and a state. Hence, we construct each row in our regression data set using unique combinations of t, s where $t \neq s$ for each state i. This gives us a total of 6817 observations. All in all, each observation in our regression data set consists the following variables: state (i), date 1 (t), date 2 (s), the log difference in electricity consumption between month 1 and month 2, the log difference in the price of electricity, the log difference in the price of coal, the number of CDDs for each date, the number of HDDs for each date, and the difference in months between dates 1 and 2.

Table 1: OLS Regression Results

	Dependent variable: $\ln(Z_{t,i}/Z_{s,i})$					
	(1)	(2)	(3)	(4)	(5)	(6)
$-\ln(P_{t,i}/P_{s,i})$	0.937***	1.075***	1.098***	1.074***	1.263***	1.305***
	(0.030)	(0.026)	(0.026)	(0.224)	(0.172)	(0.169)
$\Delta_{t,s}$			0.0005***			0.0065*
			(0.0001)			(0.0003)
CDD_t		1.156***	1.163***		1.164***	1.174***
$(\times 1000^{-1})$		(0.017)	(0.017)		(0.072)	(0.075)
CDD_s		−1.143***	−1.158***		−1.200***	−1.224***
$(\times 1000^{-1})$		(0.025)	(0.026)		(0.096)	(0.098)
HDD_t		0.246***	0.245***		0.237***	0.236***
$(\times 1000^{-1})$		(0.007)	(0.007)		(0.031)	(0.031)
HDD_s		−0.267***	−0.265***		−0.268***	−0.263***
$(\times 1000^{-1})$		(0.008)	(0.008)		(0.038)	(0.038)
Intercept	0.028***	0.012*	0.026***			
	(0.004)	(0.006)	(0.007)			
State FEs				Yes	Yes	Yes
Observations	6,817	6,817	6,817	6,817	6,817	6,817
R^2	0.079	0.507	0.508	0.092	0.521	0.524
Adjusted R^2	0.079	0.506	0.508	0.085	0.518	0.520
F Statistic	582***	1399***	1172***	685***	1474***	1241***

Note: The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except $\Delta_{t,s}$. The unit of observation is a set (t, s, i) where $t \neq s$ are months and i is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The coefficient on $-\ln(P_{t,i}/P_{s,i})$ is the estimate of σ. The variable $\Delta_{t,s}$ is the difference in months between periods t and s. CDD_t and HDD_t refer to the total number of heating and cooling degree days in month t. We scale the coefficients on degree days for clarity. Robust standard errors are reported in parentheses. *p<0.05, **p<0.01, ***p<0.001

IV. Results

Based on the OLS results reported in Table 1, we estimate the intertemporal elasticity of substitution for electricity consumption $\hat{\sigma} = 1.305$ ($|t| > 24$) when accounting for all degree day covariates and state fixed effects. Additionally, we find that accounting for cooling and heating degree days captures a large amount of variation in demand for electricity; in the fixed effects regressions, adding these controls raises adjusted R^2 from 8.5% to 51.8%. Also, as expected, the coefficients on the degree day covariates are symmetrical; that is, the coefficient on CDD_t is approximately the same as the negative of that on CDD_s, and the same applies to HDD_t and HDD_s. [6] Furthermore, we found that electricity consumption seems to rise more in response

[5] The coal price data set contains a large number of missing values due to privacy concerns; however, we do not expect that these missing values are correlated with the data itself.

[6] It is important to note that the reason they are not perfectly equal is because our regression data consists of each unique *combination* of (t, s) for each i; that is, if we have an arbitrary observation (t, s, i) in our data, then (s, t, i) does not also appear. Including these observations would not affect σ and they also don't add any further information to our regression. However, they would make the magnitude of the estimated coefficients for

Table 2: IV (2SLS) Regression Results

	First-Stage *Dep. Variable:* $\ln(P_{t,i}/P_{s,i})$			Second-Stage *Dep. Variable:* $\ln(Z_{t,i}/Z_{s,i})$		
	(A.1)	(B.1)	(C.1)	(A.2)	(B.2)	(C.2)
$\ln(C_{t,i}/C_{s,i})$	-0.042^{***}	-0.018^{***}	-0.018^{***}			
	(0.002)	(0.002)	(0.002)			
$-\ln(P_{t,i}/P_{s,i})$				2.978^{***}	5.896^{***}	5.818^{***}
				(0.180)	(0.548)	(0.524)
$\Delta_{t,s}$			0.001^{***}			0.003^{***}
			(0.00004)			(0.0004)
CDD_t $(\times 1000^{-1})$		0.100^{***}	0.105^{***}		1.637^{***}	1.657^{***}
		(0.006)	(0.006)		(0.068)	(0.067)
CDD_s $(\times 1000^{-1})$		-0.096^{***}	-0.114^{***}		-1.688^{***}	-1.783^{***}
		(0.009)	(0.009)		(0.079)	(0.084)
HDD_t $(\times 1000^{-1})$		-0.048^{***}	-0.048^{***}		0.001	0.007
		(0.003)	(0.003)		(0.031)	(0.030)
HDD_s $(\times 1000^{-1})$		0.053^{***}	0.055^{***}		0.0001	0.007
		(0.003)	(0.003)		(0.035)	(0.034)
State FEs	Yes	Yes	Yes	Yes	Yes	Yes
Observations	6817	6817	6817	6817	6817	6817
R^2	0.061	0.264	0.294			
Adjusted R^2	0.061	0.264	0.293			
F Statistic	443^{***}	489^{***}	472^{***}			

Note: The log difference in coal price between period t and s, $\ln(C_{t,i}/C_{s,i})$, is used as an instrument in these regressions. The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except $\Delta_{t,s}$. The unit of observation is a set (t, s, i) where $t \neq s$ are months and i is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The coefficient on $\ln(P_{t,i}/P_{s,i})$ is an estimate of $-\sigma$. The variable $\Delta_{t,s}$ is the difference in months between periods t and s. CDD_t and HDD_t refer to the total number of heating and cooling degree days in month t. We scale the coefficients on degree days for clarity. Robust standard errors are reported in parentheses. *p<0.05, **p<0.01, ***p<0.001

to CDDs rather than HDDs. Next, we find that the sign on $\Delta_{t,s}$ is positive in all regressions, so electricity consumption rises over time independent of price. Specifically, fit (6) finds a coefficient of 0.000655 which implies that electricity consumption rises, on average, by $\approx 0.7\%$ per year. Finally, adding state fixed effects seems to raise the estimates of σ for all fits.

To account for endogeneity in the OLS results, we provide results for our IV specification in Table 2. Here, we find a much larger estimate of $\hat{\sigma} = 5.818$ ($|t| > 11.1$) when considering all covariates and fixed effects. F-Statistics on all three specifications are significantly larger than 10, which suggests that the instruments are not weak (Staiger and Stock, 1997). These results greatly differ from the literature's estimates probably as a consequence of controlling for endogeneity. That is, ignoring variation on the supply side would bias OLS estimates for the effect of price on demand downwards; by controlling for this, we find larger estimates of σ

CDD_t and CDD_s equivalent (and likewise for HDDs).

Table 3: Partially Linear IV Regression Results

	Instrument: $\ln(C_{t,i}/C_{s,i})$		
	(1)	(2)	(3)
$\hat{\sigma}$	2.9976***	1.2123***	0.8847***
	(0.169)	(0.052)	(0.044)
Time Control			Yes
Degree Day Controls		Yes	Yes
Observations	6817	6817	6817

Note: The log difference in coal price between period t and s, $\ln(C_{t,i}/C_{s,i})$, is used as an instrument in these regressions. The sample covers all 50 US states from 2011 to 2018; outliers are removed by trimming 1% of each variable except $\Delta_{t,s}$. The unit of observation is a set (t, s, i) where $t \neq s$ are months and i is a state; as discussed in the Data section, we take a random sample of 9000 observations from the data. The estimation procedure is described in the appendix. Robust standard errors are reported in parentheses. *p<0.05, **p<0.01, ***p<0.001

which is expected. Next, with respect to the demand controls, we find results similar to those of OLS. The magnitudes of the coefficients on DDs do not appear to be significantly different. Furthermore, we again find that CDDs affect electricity consumption more than HDDs. But, this time, the coefficients on heating degree days do not appear to be significant. Additionally, in contrast with the OLS results, we find a much larger estimate for the coefficient on $\Delta_{t,s}$ of 0.003; this implies that electricity consumption seems to rise by about 3.65% per year. So, overall, our IV results suggest much higher estimates of σ.

Finally, we control for nonlinear effects using a partially linear IV regression reported in Table 3. Here, we find much smaller results than 2SLS when controlling for all possible covariates. Specifically, in fit (3), we have $\hat{\sigma} = 0.8847$ ($|t| > 20$). The estimates of σ with less controls are much larger. However, based on the OLS and IV results, it seems that both the time control and degree day controls are significant. Hence, fit (3) is likely the most appropriate specification. Additionally, all of these estimates are significantly different from the 2SLS results which suggests that the 2SLS results are not robust. To be precise, our controls and instrument likely have non-linear effects on price and quantity which cannot be captured by linear models; this may explain why the 2SLS results differ significantly from these semiparametric results. Alternatively, this difference may also be because the errors in our regressions are not Gaussian. In any case, we believe that our third semiparametric fit, $\hat{\sigma} = 0.8847$, is the most robust estimate of σ. We now discuss what this means for renewable energy policy in greater detail.

V. Discussion

A. The Elasticity of Substitution between Renewable and Fossil Energy

To understand the economic implications of these estimates for σ, we parametrize and numerically evaluate our model in a two-period, two-technology setting. In particular, we are concerned with how σ affects the substitutability of fossil and renewable energy. This is important, because sufficient substitutability between these two technologies is required to transition into greener economy in the future. For instance, Acemoglu et al. (2012) provide a model where they argue that, "When the two sectors [clean and dirty energy] are substitutable but not sufficiently so, preventing an environmental disaster requires a permanent policy intervention. Finally, when the two sectors are complementary, the only way to stave off a disaster is to stop long-run growth." However, note that while our model sets the intertemporal elasticity of substitution for electricity consumption σ to a fixed value, we make no direct assumptions about the elasticity of substitution between different energy technologies. Instead, this latter elasticity emerges as an indirect result of substitution between electricity consumption in periods. To illustrate this, we parametrize and estimate our model numerically.

Firstly, we let technology 1 be coal power and technology 2 be solar power; let period t represent peak hours and period s represent off-peak hours. We assume that, holding prices equal, consumers prefer that approximately 60% of their energy arrive in period t and the remaining 40% arrive in period s; that is, we have $\alpha_t = 0.6$ and $\alpha_s = 0.4$. Next, we normalize all quantity units to a MWh basis. Hence, ξ represents the percent of capacity utilized in each period; we assume coal uses 100% of its capacity in both periods, while solar can access 100% during peak hours and only 10% during the off-peak. Finally, we set cost parameters, given in \$ per MWh, equal to LCOE estimates for 2023 from the EIA (2019b); specifically, we use estimates for "Solar PV" and "Coal with 30% CCS" from Table 1b.[7] In total, we have the following parameters.

Example A: $\alpha_t = 0.6$, $\alpha_s = 0.4$, $\xi_1 = (1, 1)$, $\xi_2 = (1, 0.1)$, $c_1 = 104.3$, $c_2 = 60$.

We begin by exploring the implied elasticity of substitution between these two technologies. Recall, for any two commodities i and j, the elasticity of substitution $e_{i,j}$ is given by:

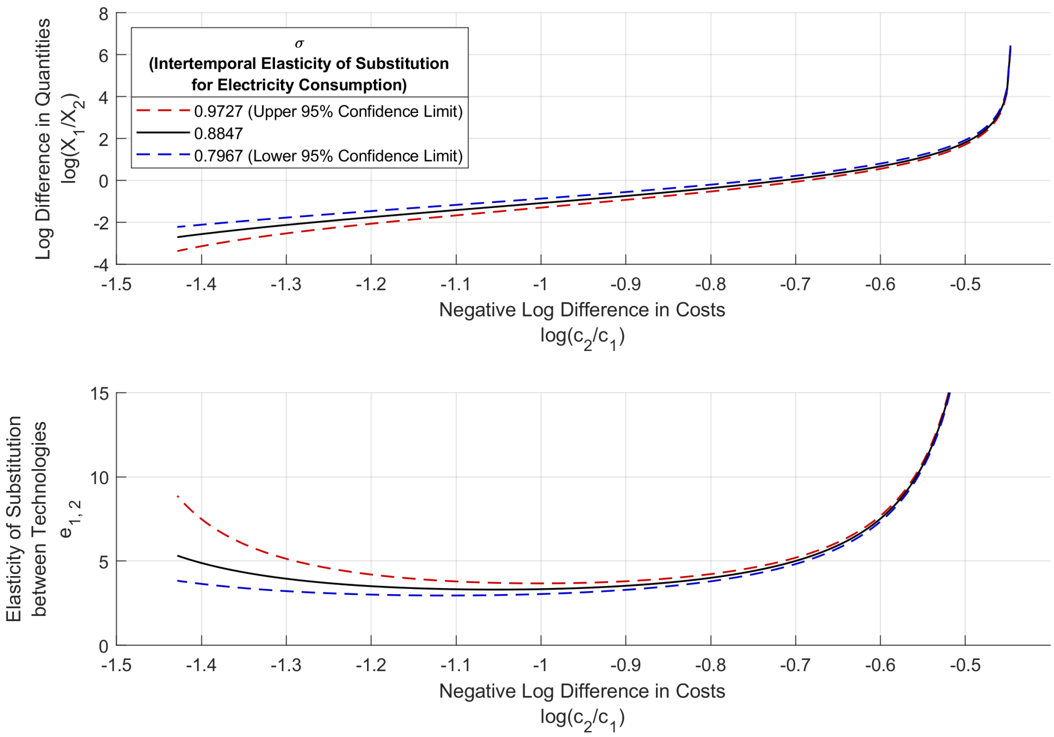
$$e_{i,j} = \frac{\partial \ln(X_i/X_j)}{\partial \ln(c_j/c_i)}$$

where c is their prices. This relationship is of particular interest, because many applied and theoretical models studying renewable and fossil energy impose a CES production structure between the two (see Papageorgiou et al. (2017)). That is, they assume that e is a fixed constant between renewable and fossil energy. In our model, this is not the case; while directly deriving the equation for $e_{1,2}$ in our model is analytically intractable, we numerically estimate how it varies with $\ln(c_2/c_1)$ and σ.

We plot our numerical estimates of $e_{1,2}$, the elasticity of substitution between solar and coal power, in Figure 1. These results were generated by computing the optimal quantities of each technology given a range of prices. Specifically, we held the price of solar capacity constant and varied the price of coal capacity from 50% to 200% of its original parametrized price. Then, we numerical differentiated $\log(X_1/X_2)$ with respect to $\log(c_2/c_1)$ to estimate

[7] The LCOE for a technology is equal to the sum of its lifetime costs divided by its lifetime energy output. While Joskow (2011) explains the flaws of comparing generation technologies solely on the basis of LCOE, our use of this measure is unrelated to his critique. That is, he argues that the economic value an intermittent technology should also account for when it produces electricity and the prices of electricity in those periods (see Table 2 of his paper). Our model does exactly this; but, we still need to use LCOE to parametrize the cost of developing capacity.

Figure 1: The Elasticity of Substition between Solar and Coal



Note: Technology 1 is coal and technology 2 is solar. The legend in the upper subplot also applies to the lower subplot. These results were obtained using the following parameters: $\alpha_t = 0.6$, $\alpha_s = 0.4$, $\xi_1 = (1,\ 1)$, $\xi_2 = (1,\ 0.1)$, $c_1 = 104.3$, $c_2 = 60$. Furthermore, we set the parameter for the intertemporal elasticity of substition for electricity consumption equal to our estimate $\hat{\sigma} = 0.8847$. In order to generate these numerical, we first found the optimal quantities of X over a range of prices $c_1^* \in (0.5\,c_1, 2\,c_1)$. Then, we obtained estimates of the elasticity of substition by numerically differentiating $\ln(X_1/X_2)$ with respect to $-\ln(c_1, c_2)$. That is, the elasticity of substitution between technology 1 and 2 is given by the slope of the upper subplot, and it is graphed in the lower subplot. Finally, we repeat this procedure with σ equal to two standard deviations above and below its estimated value; that is, the dashed lines represent $\sigma = 0.8847 \pm 2(0.044)$.

the elasticity of substitution between solar and coal.[8] This numerically computed elasticity of substitution between solar and coal power, $e_{solar,coal}$, is shown in the lower subplot. We repeat this process with different values of σ, the elasticity of substitution in the consumer's utility function. Specifically, we use our estimate of $\hat{\sigma} = 0.8847\,(0.044)$ and its 95% confidence interval $(0.7967, 0.9727)$. Finally, after finding the optimal values of our quantities X, we filter all observations of our numerical simulation that correspond to edge cases – solutions where $X, Z \leq 0$.

The first relationship we see in Figure 1 is that $e_{1,2}$ varies non-linearly with the relative costs of each technology; in particular, it appears to take on a hook shape. This shape is a result of both intermittency and costs. Intermittency reduces substitutability as overall energy generation becomes more intermittent. In other words, intermittency is bigger issue for consumers when their energy supply is not consistent. This causes $e_{1,2}$ to fall when the majority of energy generated comes from solar; naturally, the latter occurs when $\log(c_2/c_1)$ is small (solar is relatively cheap). At the same time, quantities become more sensitive to costs when costs greatly differ.[9] That is, $e_{1,2}$ rises when the relative prices of solar and coal are very different.

[8] Instead, we could have generated variation in $\log(c_2/c_1)$ data by holding solar at a constant price and varying the price of coal capacity. However, this does not change the results.

[9] We explore this point further in Appendix B in which we consider a numerical example where technologies

When these two effects interact, we see the elasticity of substitution between solar and coal take on a hook shape.

Additionally, note that the elasticity of substitution $e_{1,2}$ between solar and coal becomes larger and more non-linear as σ rises. Firstly, the reason why it becomes larger is fairly intuitive. When a consumer's intertemporal elasticity of substitution for electricity consumption σ is large, they will be more sensitive to changes in the price of electricity and more willing to substitute their electricity consumption across periods. As a result, the intermittency of solar becomes less of an issue for consumers, but its relative price with respect to coal will play a bigger role. Consequently, this implies that the elasticity of substitution $e_{1,2}$ will be larger. Hence, in the lower subplot of Figure 1, we see that $e_{1,2}$ rises with σ when holding relative prices of each technology constant. Secondly, we find that $e_{1,2}$ becomes more u-shaped as σ rises. This is because larger values of σ weaken the disincentive created by intermittency. Consequently, the shape of the elasticity of substitution $e_{1,2}$ becomes more dependent on the relative costs. As argued earlier, when costs differ greatly between technologies, the elasticity of substitution $e_{1,2}$ is higher; this effect causes $e_{1,2}$ to take on a u-shape. So, in total, as σ rises, we expect $e_{1,2}$ to become increasingly u-shaped.

B. Implications for Pollution Abatement

Overall, our results have shown that the traditional assumption of a CES relationship between renewable and fossil energy is inappropriate. However, since e takes on a fairly simple shape, we can still adjust the findings of past models to account for variation in the elasticity of substitution. For example, consider again the results of Acemoglu et al. (2012).

Firstly, they find that the short-run cost of policy intervention is increasing with the elasticity of substitution between clean and dirty technologies.[10] We find that the elasticity of substitution between renewable and fossil energy is decreasing with intermittency. Consequently, this implies that regions with access to clean, reliable energy (such as hydro and geothermal energy) will require the most expensive interventions. Additionally, their paper states that the cost of delaying intervention is increasing with the elasticity of substitution e. Similarly, this implies delaying policy intervention is the most costly in regions with access to non-intermittent, clean energy.

Secondly, Acemoglu et al. argue that, when the discount rate and elasticity of substitution between clean and dirty energy (e) is sufficiently low, a disaster cannot be avoided under laissez-faire.[11] In our model, it is possible for e to start at a large value and transition to much smaller values as clean energy becomes cheaper; this occurs when the source of clean energy is highly intermittent. On the other hand, if clean energy is not intermittent, it will be highly substitutable with dirty energy no matter their relative prices. This latter case would likely not lead to disaster but the former case (with high intermittency) would. Hence, we offer an alternative interpretation of this proposition: when the discount rate is sufficiently low and the intermittency of clean energy is sufficiently high, a disaster cannot be avoided under laissez-faire.

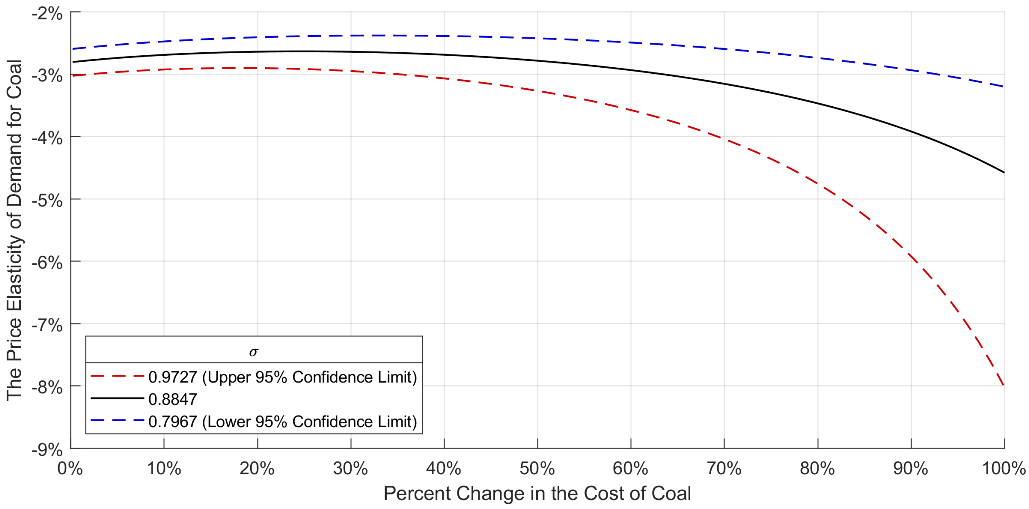
Finally, Acemoglu et al. find that, "when the elasticity of substitution is high, since in this case a relatively small carbon tax is sufficient to redirect R&D towards clean technologies." Recall that, in our model, we see that e changes with relative prices; when fossil fuels are relatively cheap, e is large. Hence, this implies we need a relatively small carbon tax early on to spur research. As technological change makes renewables cheaper, e will fall and thus the carbon tax needs to rise. On the other hand, Acemoglu et al. show numerically that an optimal carbon

are minimally intermittent. That is, we again consider a two-period, two-technology example, but assume that ξ_1 and ξ_2 are nearly constant over time. We plot our results in Figure B.1 and find that the elasticity of substitution takes on a u-shape.

[10] See Corollary 1 of their paper for a proof and intuitive explanation.

[11] See Proposition 9 of their paper.

Figure 2: The Price Elasticity of Demand for Coal Power



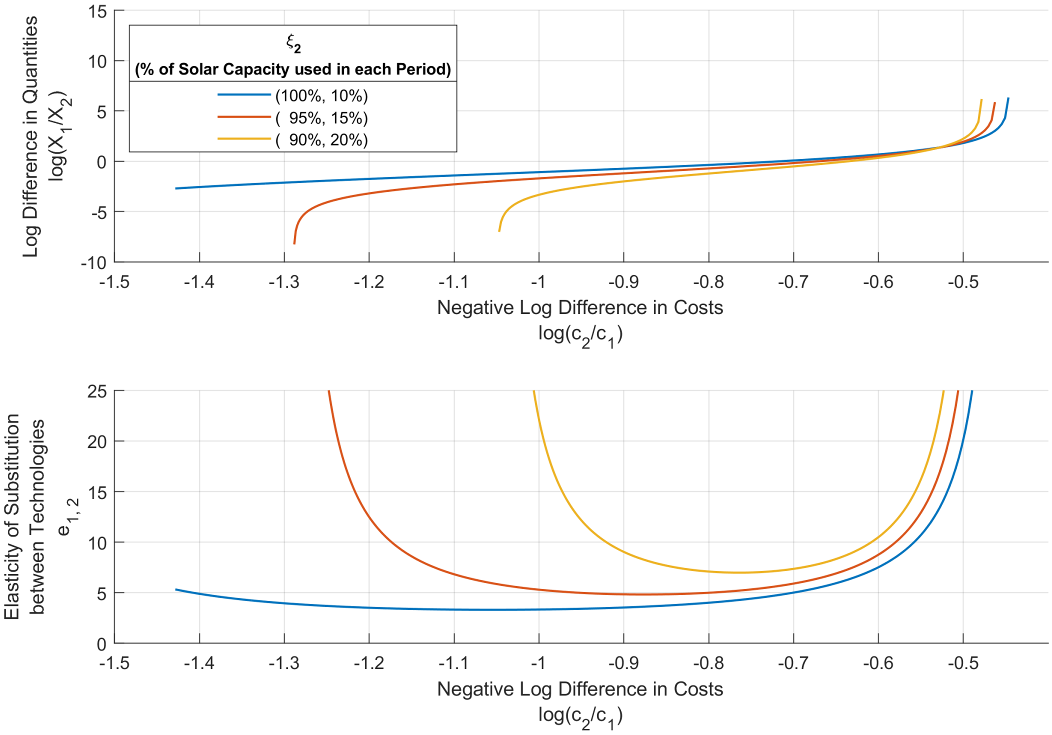
Note: These results were obtained using the following parameters: $\alpha_t = 0.6$, $\alpha_s = 0.4$, $\xi_1 = (1, 1)$, $\xi_2 = (1, 0.1)$, $c_1 = 104.3$, $c_2 = 60$, $\sigma = 0.5$. We generate these results by finding the optimal quantity of coal, X_1, over a range of percent changes in its price c_1. Then, on the y-axis, we plot the log difference in X_1 divided by the log difference in its price. This is equivalent to the price elasticity of demand for X_1.

tax should decreases over time when e is sufficiently large. Hence, an intermediate solution may be to maintain a constant carbon tax over time.

Carbon taxes can have important distributional consequences. Earlier, we showed that the elasticity of substitution between renewable and fossil fuels (e) is non-constant and falls with intermittency of present generation. This further implies that the elasticity of demand for generation technologies is non-constant as well. Specifically, we find that the elasticity of demand for a reliable source of energy increases in magnitude with respect to its own price. Using the earlier numerical example of solar and coal power, we show this explicitly in Figure 2. As the price of coal power rises significantly beyond its current price, demand gets more elastic as its price becomes the primary factor disincentivizing its use. But, initially, demand for coal power is inelastic since it is needed to fill the gaps in solar power's intermittent output. This should be of concern for policy makers considering a pollution tax. Consumers in regions without access to clean and reliable energy will have an inelastic demand for fossil fuels; hence, they will bear significant welfare losses from a tax on fossil fuels. On the other hand, consumers in regions that have access to dispatchable, renewable energy such as hydropower will have a relatively elastic demand for fossil fuels, because they can easily manage added intermittency; hence, they will lose relatively less welfare from a carbon tax. All in all, if a carbon tax were implemented federally and its revenue were distributed equally, it may nevertheless function as an inequitable, between-state welfare transfer due to differences in the availability of renewable energy technology. This same argument applies to renewable subsidies and carbon quotas.

However, there alternative policies that can mitigate this distributional side effect. One such policy is a research subsidy for improving battery technologies. Reducing battery costs, improving their storage capacity, and reducing their energy loss can allow intermittent renewables to far more easily substitute for traditional, fossil energy. To understand the magnitude of this effect, we again provide a numerical example using solar and coal. We model solar energy with batteries by shifting a portion of the original solar energy output across periods. That is, we initially parametrized solar with $\xi_2 = (1, 0.1)$ implying that it functions at 100% of its potential capacity during the peak and at 10% of its potential capacity during the off-peak. We shift a fraction of its peak capacity to the off-peak, thereby modeling battery storage and deployment, by considering $\xi_2 = (0.95, 0.15)$ and $\xi_2 = (0.90, 0.20)$; this is equivalent to transferring 5% and

Figure 3: The Effect of Battery Storage on the Elasticity of Substition between Solar and Coal



Note: Technology 1 is coal and technology 2 is solar. The legend in the upper subplot also applies to the lower subplot. These results were obtained using the following parameters: $\alpha_t = 0.6$, $\alpha_s = 0.4$, $\xi_1 = (1,\ 1)$, $\xi_2 = (1,\ 0.1)$, $c_1 = 104.3$, $c_2 = 60$. Furthermore, we set the parameter for the intertemporal elasticity of substition for electricity consumption equal to our estimate $\hat{\sigma} = 0.8847$. In order to generate these numerical, we first found the optimal quantities of X over a range of prices $c_1^* \in (0.5\,c_1, 2\,c_1)$. Then, we obtained estimates of the elasticity of substition by numerically differentiating $\ln(X_1/X_2)$ with respect to $-\ln(c_1, c_2)$. That is, the elasticity of substitution between technology 1 and 2 is given by the slope of the upper subplot, and it is graphed in the lower subplot. Finally, we repeat this procedure with $\xi_2 = (0.95, 0.15)$ and $\xi_2 = (0.90, 0.20)$ to simulate the effects of shifting solar power output using batteries.

10% of peak solar output to off-peak hours. We ignore the energy losses and costs of battery storage for simplicity. We plot our results in Figure 3. It is immediately clear that making solar output more consistent through the day results in it being far more substitutable with coal. Moreover, we can see that intermittency no longer causes $e_{1,2}$ to taper off around 5 as solar becomes the dominant source of electricity. Rather, with batteries, cost plays a much larger role in determining the optimal quantity of each technology even when solar is relatively cheap. Overall, shifting even a small fraction of solar output using batteries can significantly improve solar power's substitutability with coal.

Consequently, batteries can complement reductions in the cost of intermittent renewables and mitigate the distributional effects of a carbon tax. As shown in Figure 3, the elasticity of substitution between solar and coal rises significantly as solar becomes less intermittent; this implies that a change in price of solar would have a far greater effect when combined with batteries for solar output. Hence, if policy makers aim to promote the use of renewables, they should subsidize research that reduces the cost of renewables as well as research that improves battery technology. Using both policy instruments can be more effective than either alone. Additionally, the second benefit of batteries is that they mitigate the distributional problems of a carbon tax. This is because, by reducing the intermittency of renewables, they make demand for fossil fuels less inelastic. In other words, regions without clean, consistent-output renewables can instead employ intermittent technologies with batteries to transition away from fossil fuel energy; this reduces welfare losses that they would have otherwise experienced from intermittency.

VI. Conclusion

. . . Throughout the paper, we considered coal and solar power as examples of fossil and renewable energy. Moreover, For instance, we discussed how to handle pollution when energy sources are intermittent; we found that, rather than a standard Pigouvian tax, we would be better off using a combination of research subsidies and carbon taxes. One implicit but important point here is that optimal policy must be tuned for differences in local electricity markets. That is, even if externalities are global and research spills over, optimal policy differs on a local scale since generation technologies differ on a local scale. . . .

. . . An aim for future research may be to develop a model of clean and dirty energy that incorporates both predictable and stochastic intermittency in a multi-period setting.

VII. Appendix A: Supplementary Proofs

A. Cobb-Douglas Case with Two Periods & Two Technologies

In this section, we consider a simpler case of our general model to better understand its implications. Firstly, we restrict the utility function to its Cobb-Douglas form which is simply the case where the elasticity of substitution $\sigma = 1$. Secondly, we limit the number of periods and technologies to 2. And, thirdly, we normalize the prices such that our repesentative consumer's income I is 1.

Equilibrium Results

Firstly, our demand equations simplify to:

$$Z_t = \alpha_t / p_t \tag{7}$$

$$Z_s = \alpha_s / p_s \tag{8}$$

where t and s are our two periods. Next, solving for the FOC condition for profit maximization, we have:

$$p = \xi^{-1} c$$

$$p = \begin{pmatrix} -\dfrac{c_1\,\xi_{2s} - c_2\,\xi_{1s}}{\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}} \\ \dfrac{c_1\,\xi_{2t} - c_2\,\xi_{1t}}{\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s}} \end{pmatrix}$$

And, substituting back into our demand equations, we find the equilibrium quantities for Z and X.

$$Z = \begin{pmatrix} \dfrac{\alpha_t\ (\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s})}{c_2\,\xi_{1s} - c_1\,\xi_{2s}} \\ \dfrac{\alpha_s\ (\xi_{1s}\,\xi_{2t} - \xi_{1t}\,\xi_{2s})}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \end{pmatrix} \implies X = \begin{pmatrix} \dfrac{\alpha_t\,\xi_{2s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} + \dfrac{\alpha_s\,\xi_{2t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \\ -\dfrac{\alpha_t\,\xi_{1s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} - \dfrac{\alpha_s\,\xi_{1t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} \end{pmatrix}$$

Furthermore, we derive restrictions on the parameters ξ and c by assuming $Z, X > 0$. These restrictions are detailed in Table 1. There are two possible sets of symmetrical restrictions. The first set, Case 1, assumes that technology 2 is more cost effective in period t, while the second set, Case 2, assumes that technology 1 is more cost effective in period t. If a given set of parameters do not fall into either case, we are left with an edge case where one of the technologies is not used. Additionally, these inequalities compare two types of efficiency – output efficiency and cost efficiency; we define output efficiency as electricity output per unit of input and cost efficiency in terms of electricity output per dollar of input. We refer to the last set of restrictions as mixed, because they relate both cost and output efficiency.

Proof: We aim to derive conditions on ξ and c required to have positive Z and X, so we begin by assuming $X, Z > 0$. Second, since the equations so far are symmetrical, note that there be two symmetrical sets of potential restrictions we must impose on the parameters. Thus, we first assume the inequality $c_1\xi_{2t} - c_2\xi_{1t} > 0$ to restrict ourselves to one of the two cases. This assumption results in the denominator of Z_s being positive. Hence, we must also have $\xi_{1s}\xi_{2t} - \xi_{2s}\xi_{1t} > 0$ for $Z_s > 0$. This same term appears in the numerator for Z_t, hence its denominator must be positive: $c_2\xi_{1s} - c_1\xi_{2s} > 0$. Now, rewriting these inequalities, we have:

$$\begin{aligned} c_1\xi_{2t} - c_2\xi_{1t} > 0 &\implies \xi_{2t}/c_2 > \xi_{1t}/c_1 \\ c_2\xi_{1s} - c_1\xi_{2s} > 0 &\implies \xi_{1s}/c_1 > \xi_{2s}/c_2 \\ \xi_{1s}\xi_{2t} - \xi_{2s}\xi_{1t} > 0 &\implies \xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t} \\ &\implies \xi_{1t}/\xi_{1s} < \xi_{2t}/\xi_{2s} \end{aligned}$$

Table 4: PARAMETER RESTRICTIONS FOR $Z, X > 0$

	Case 1	Case 2
Cost Efficiency Restrictions	$\xi_{2t}/c_2 > \xi_{1t}/c_1$ $\xi_{1s}/c_1 > \xi_{2s}/c_2$	$\xi_{2t}/c_2 < \xi_{1t}/c_1$ $\xi_{1s}/c_1 < \xi_{2s}/c_2$
Output Efficiency Restrictions	$\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} > \xi_{2s}/\xi_{2t}$	$\xi_{2t}/\xi_{2s} < \xi_{1t}/\xi_{1s}$ $\xi_{1s}/\xi_{1t} < \xi_{2s}/\xi_{2t}$
Mixed Efficiency Restrictions	$\frac{\alpha_s\left(\xi_{1s}/c_1 - \xi_{2s}/c_2\right)}{\alpha_t\left(\xi_{2t}/c_2 - \xi_{1t}/c_1\right)} > \xi_{2s}/\xi_{2t}$ $\frac{\alpha_s\left(\xi_{1s}/c_1 - \xi_{2s}/c_2\right)}{\alpha_t\left(\xi_{2t}/c_2 - \xi_{1t}/c_1\right)} < \xi_{1s}/\xi_{1t}$	$\frac{\alpha_s\left(\xi_{1s}/c_1 - \xi_{2s}/c_2\right)}{\alpha_t\left(\xi_{2t}/c_2 - \xi_{1t}/c_1\right)} < \xi_{2s}/\xi_{2t}$ $\frac{\alpha_s\left(\xi_{1s}/c_1 - \xi_{2s}/c_2\right)}{\alpha_t\left(\xi_{2t}/c_2 - \xi_{1t}/c_1\right)} > \xi_{1s}/\xi_{1t}$

Note: The inequalities in this table assume that all elements of ξ are greater than 0. The full proof given below provides equivalent restrictions for the zero cases.

Note that the latter two restrictions can be derived from the former two. Additionally, we implicitly assume that we have $\xi > 0$. However, this is not necessary assumption, since ξ invertible only requires $\xi_{1t}\xi_{2s} > 0$ or $\xi_{1s}\xi_{2t} > 0$. Instead, we may leave the latter two inequalities in the form $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$ which remains valid when values of ξ are equal to 0. Lastly, the mixed efficiency restrictions come from $X > 0$. To start, for X_1, we have:

$$\begin{aligned} X_1 > 0 &\implies (\alpha_t\xi_{2s})(c_1\xi_2t - c_2\xi_1t) + (\alpha_s\xi_{2t})(c_1\xi_{2s} - c_2\xi_{1s}) < 0 \\ &\implies (\alpha_t\xi_{2s})(c_1\xi_2t - c_2\xi_1t) < (\alpha_s\xi_{2t})(c_2\xi_{1s} - c_1\xi_{2s}) \\ &\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s(c_2\xi_{1s} - c_1\xi_{2s}))/(\alpha_t(c_1\xi_{2t} - c_2\xi_{1t})) \\ &\implies (\xi_{2s}/\xi_{2t}) < (\alpha_s(\xi_{1s}/c_1 - \xi_{2s}/c_2))/(\alpha_t(\xi_{2t}/c_2 - \xi_{1t}/c_1)) \end{aligned}$$

Similarly, for X_2, note that only the numerators differ; ξ_{2s} is replaced with $-\xi_{1s}$ and ξ_{2t} is replaced with $-\xi_{1t}$. Hence, we have

$$\begin{aligned} X_2 > 0 &\implies (\alpha_t\xi_{1s})(c_1\xi_2t - c_2\xi_1t) + (\alpha_s\xi_{1t})(c_1\xi_{2s} - c_2\xi_{1s}) > 0 \\ &\implies (\xi_{1s}/\xi_{1t}) > (\alpha_s(\xi_{1s}/c_1 - \xi_{2s}/c_2))/(\alpha_t(\xi_{2t}/c_2 - \xi_{1t}/c_1)) \end{aligned}$$

To double check, note that combining the inequalities from $X_1 > 0$ and $X_2 > 0$ leads to $\xi_{2s}/\xi_{2t} < \xi_{1s}/\xi_{1t}$. This is precisely the earlier result obtained from $Z > 0$. Again, it is important to note that we assume $\xi > 0$ for to simplify the inequalities of $X_1 > 0$ and $X_2 > 0$. Otherwise, we may leave the inequalities in their pre-simplified forms and they are still valid when $\xi_{1t}\xi_{2s} > 0$ or $\xi_{1s}\xi_{2t} > 0$.

■

Let us consider the set of restrictions belonging to Case 1. The first inequality, our initial assumption, states that technology 2 is relatively more cost effective in period t. The second inequality claims technology 1 is relatively more cost effective in period s. The implications are fairly straightforward; if a technology is to be used, it must have an absolute advantage in cost efficiency in at least one period. The third condition states that the relative output efficiency of technology 2 is greater than that of the first technology in period t. And, the fourth condition makes a symmetrical claim but for the technology 1 and period s. These latter two restrictions regarding output efficiency enter Z and X through p; they're simply a restatement of the invertibility of ξ and can also be derived through the cost efficiency restrictions.

The mixed efficiency restrictions are less intuitive. Firstly, note that $(\xi_{1s}/c_1 - \xi_{2s}/c_2)$ is the difference in cost efficiency for the two technologies in period s; this is equivalent to the increase in Z_s caused by shifting a marginal dollar towards technology 1. Similarly, the bottom term $(\xi_{2t}/c_2 - \xi_{1t}/c_1)$ represents the change in Z_t caused by shifting a marginal dollar towards technology 1. Both these terms are then multiplied by the share parameter of the utility function for their respective time periods. Furthermore, note that α_t (α_s) is the elasticity of utility with respect to Z_t (Z_t). Hence, in total, the mixed efficiency restrictions relate the relative cost efficiencies of each technology with their output efficiency and the demand for energy. So, for example, suppose that consumers prefer, *ceteris paribus*, that nearly all their electricity arrive in period t. This would imply α_t is arbitrarily large which results in the left-hand side of the fraction becoming arbitrarily small. This violates the first mixed efficiency restriction but not the second; consequently, use of the first technology, which is less cost effective in period t, approaches 0.

In more practical terms, suppose that our first technology is coal power and the latter is solar power. Although coal power is dispatchable, it does not easily ramp up or down within a day; hence, it is reasonable to apply our model where capacities are fixed over time so long as our time frame is sufficiently short. Hence, we now assume periods t and s represent the peak and off-peak for a day. And, we expect that there is more available solar radiation during peak hours than off-peak hours, since peak hours are usually during the middle of the day. This implies that the the output efficiency of solar power is higher in period t due to more available solar radiation. Additionally, since the energy output of a unit of coal is independent of time, we know that the output efficiency of coal is constant. In total, this implies that we have met the output efficiency restrictions, since we have $\xi_{2t}/\xi_{2s} > \xi_{1t}/\xi_{1s}$. Next, we can reasonably assume that coal is more cost effective than solar in the off-peak period when there is less sun; hence, the second cost efficiency restriction is satisfied. Then, for there to be an incentive to use solar power, we must satisfy the first cost-efficiency condition; that is, solar needs be cost effective during peak hours otherwise we hit an edge case where no solar is employed. And, finally, solar must also satisfy the mixed efficiency condition, which essentially implies that there must be sufficient demand for electricity during period t, when solar is more effective, for it to be a feasible technology. So, overall, for a technology to be economical, it must meet three conditions: it must the most cost effective technology for a particular period, it must have a comparative advantage in output efficiency in the same period, and there must be sufficient amount of demand in that period.

Comparative Statics

The comparative statics are similarly intuitive. The equilibrium quantity of a technology is increasing with its output efficiency and decreasing with its cost per unit. Additionally, the equilibrium quantities for a particular technology move in the opposite direction with respect to the output efficiency and cost of the other technologies. For a practical example, consider again coal and solar power from before. An increase in the output efficiency of solar or a decrease in solar power's cost will reduce the optimal quantity of coal power. Likewise, as coal power's efficiency improves, it's adoption rises. To find the effects of α on X, we must assume one of the cases of restrictions shown in Table 1. So, again, let us assume Case 1 is true; this implies that X_2 is the most cost effective technology in period t and likewise for X_1 in period s. Firstly, note that α determines demand for electricity in a period. Hence, when α_t rises, we see the optimal level of X_2 rise as well; likewise, X_1 rises with α_s. In short, the optimal quantity of a technology rises linearly with the demand for electricity in the period it specializes in. Moreover, these relationships are reversed with respect to demand in each technology's suboptimal period. So, for example, we would expect the use of solar energy to rise when demand for electricity during peak hours rises, and it would fall when demand for energy in the off-peak rises. On the other hand, use coal power would rise with off-peak demand and fall with peak demand. This concept carries through for the comparative statics of Z. When the output efficiency of technology 1 rises or its cost falls, we see output Z_s rise and output Z_t fall. This is because technology 1 is optimal in period s given the Case 1 restrictions. Likewise, we see symmetrical results for the output with respect to the cost and output efficiency of technology 2; improvements in the efficiency of X_2 result in greater output in Z_t and smaller output in Z_s.

Proof: We begin by deriving the comparative statics of the cost and efficiency parameters with respect to X. Firstly, we take derivatives with respect to the cost vectors:

$$\frac{\partial X_1}{\partial c} = \begin{pmatrix} \frac{-\alpha_t\,{\xi_{2s}}^2}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} - \frac{\alpha_s\,{\xi_{2t}}^2}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} < 0 \\ \frac{\alpha_t\,\xi_{1s}\,\xi_{2s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} + \frac{\alpha_s\,\xi_{1t}\,\xi_{2t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} > 0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial c} = \begin{pmatrix} \frac{\alpha_t\,\xi_{1s}\,\xi_{2s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} + \frac{\alpha_s\,\xi_{1t}\,\xi_{2t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} > 0 \\ \frac{-\alpha_t\,{\xi_{1s}}^2}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} - \frac{\alpha_s\,{\xi_{1t}}^2}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} < 0 \end{pmatrix}$$

The first and second terms of $\partial X_1/\partial c_1$ are clearly both negative independent of the restrictions on the parameters. Similarly, all terms of $\partial X_1/\partial c_2$ are positive independent of any restrictions. Since the structure of this problem is symmetrical with respect to X_1 and X_2, the same comparative statics apply but in reverse for X_1. Next, we derive comparative statics for each element of ξ.

$$\frac{\partial X_1}{\partial \xi} = \begin{pmatrix} \frac{\alpha_s\,c_2\,\xi_{2t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} > 0 & \frac{\alpha_t\,c_2\,\xi_{2s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} > 0 \\ \frac{-\alpha_s\,c_2\,\xi_{1t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} < 0 & \frac{-\alpha_t\,c_2\,\xi_{1s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} < 0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial \xi} = \begin{pmatrix} \frac{-\alpha_s\,c_1\,\xi_{2t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} < 0 & \frac{-\alpha_t\,c_1\,\xi_{2s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} < 0 \\ \frac{\alpha_s\,c_1\,\xi_{1t}}{(c_1\,\xi_{2t} - c_2\,\xi_{1t})^2} > 0 & \frac{\alpha_t\,c_1\,\xi_{1s}}{(c_1\,\xi_{2s} - c_2\,\xi_{1s})^2} > 0 \end{pmatrix}$$

Again, the signs are fairly straightforward. The optimal quantity of X_1 increases with its output efficiency in both periods; however, it decreases with the output efficiency of X_2 in both periods. Similarly, symmetrical results are shown for X_2. Next, we study the effects of α on X; this requires us to place some restrictions on the parameters, so we use those belonging to Case 1 in Table 1. With $\alpha \equiv (\alpha_t \;\; \alpha_s)^T$,

$$\frac{\partial X_1}{\partial \alpha} = \begin{pmatrix} \frac{\xi_{2s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} < 0 \\ \frac{\xi_{2t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} > 0 \end{pmatrix}$$

$$\frac{\partial X_2}{\partial \alpha} = \begin{pmatrix} \frac{-\xi_{1s}}{c_1\,\xi_{2s} - c_2\,\xi_{1s}} > 0 \\ \frac{-\xi_{1t}}{c_1\,\xi_{2t} - c_2\,\xi_{1t}} < 0 \end{pmatrix}$$

Note that our restrictions imply that $c_1\xi_{2t} - c_2\xi_{1t} > 0$ and $c_2\xi_{1s} - c_1\xi_{2s} > 0$. From here, the intuition is clear; we assume that X_2 is more cost efficient in period t, so increases in demand during period t (caused by increases in α_t) will increase the optimal quantity of X_2. And, the same applies to X_1 with respect to period s and α_s. Again, due to symmetry, the statics are reversed when the technologies are flipped. Similarly, the signs would also be flipped if we used the restrictions given by Case 2 instead.

Next, we derive the comparative statics for Z. From our restrictions, we have $\xi_{1s}\xi_{2t} > \xi_{2s}\xi_{1t}$.

All the results above follow from this inequality and the cost efficiency restrictions.

$$\frac{\partial Z_t}{\partial c} = \begin{pmatrix} \dfrac{\alpha_t\,\xi_{2\mathrm{s}}\,\left(\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)^2} > 0 \\ \dfrac{-\alpha_t\,\xi_{1\mathrm{s}}\,\left(\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)^2} < 0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial c} = \begin{pmatrix} \dfrac{-\alpha_s\,\xi_{2\mathrm{t}}\,\left(\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)^2} < 0 \\ \dfrac{\alpha_s\,\xi_{1\mathrm{t}}\,\left(\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)^2} > 0 \end{pmatrix}$$

$$\frac{\partial Z_t}{\partial \xi} = \begin{pmatrix} \dfrac{\alpha_t\,\xi_{2\mathrm{s}}}{c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}} < 0 & \dfrac{-\alpha_t\,\xi_{2\mathrm{s}}\,\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)}{\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)^2} < 0 \\ \dfrac{-\alpha_t\,\xi_{1\mathrm{s}}}{c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}} > 0 & \dfrac{\alpha_t\,\xi_{1\mathrm{s}}\,\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)}{\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)^2} > 0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial \xi} = \begin{pmatrix} \dfrac{-\alpha_s\,\xi_{2\mathrm{t}}\,\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)^2} > 0 & \dfrac{\alpha_s\,\xi_{2\mathrm{t}}}{c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}} > 0 \\ \dfrac{\alpha_s\,\xi_{1\mathrm{t}}\,\left(c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}\right)}{\left(c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}\right)^2} < 0 & \dfrac{-\alpha_s\,\xi_{1\mathrm{t}}}{c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}} < 0 \end{pmatrix}$$

Again, recall that we have $c_1\xi_{2t} - c_2\xi_{1t} > 0$ and $c_2\xi_{1s} - c_1\xi_{2s} > 0$; the rest follows. And finally, we have:

$$\frac{\partial Z_t}{\partial \alpha} = \begin{pmatrix} \dfrac{-\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}}{c_1\,\xi_{2\mathrm{s}}-c_2\,\xi_{1\mathrm{s}}} > 0 \\ 0 \end{pmatrix}$$

$$\frac{\partial Z_s}{\partial \alpha} = \begin{pmatrix} 0 \\ \dfrac{\xi_{1\mathrm{s}}\,\xi_{2\mathrm{t}}-\xi_{1\mathrm{t}}\,\xi_{2\mathrm{s}}}{c_1\,\xi_{2\mathrm{t}}-c_2\,\xi_{1\mathrm{t}}} > 0 \end{pmatrix}$$

These are fairly trivial, since $Z_t = \alpha_t/p_t$ (and $Z_s = \alpha_s/p_s$) and prices are positive.
■

B. CES Production as a Special Case

Our framework nests the case where there exists a CES production structure between each technology. This occurs when each technology can only produce in a single, unique period; note that this is not a realistic scenario. For instance, this would occur if we had one technology that can only output electricity during the day and another that only outputs electricity at night. Anyways, in this case, the CES production function's elasticity parameter will be equivalent to the that of the consumer's CES utility function – the intertemporal elasticity of substitution for electricity consumption.

Proof: Firstly, note that we can reindex our technologies such that ξ is diagonal, since each technology only produces in one period. Hence, without loss of generality, we have diagonal ξ. Next, we may say that the electricity output in period i is given by $Z_i = \xi_{i,i}X_i$. Now, recall that the FOC for profit-maximization is given by $p = \xi^{-1}c$, hence we have $p_i = c_i/\xi_{i,i}$. Combining this equations with the FOC for utility maximization, we have:

$$\begin{aligned}\frac{Z_i}{Z_j} &= \left(\frac{\alpha_i p_j}{\alpha_j p_i}\right)^{\sigma} \\ \implies \frac{X_i}{X_j} &= \left(\frac{\alpha_i p_j \xi_{j,j}^{1/\sigma}}{\alpha_j p_i \xi_{i,i}^{1/\sigma}}\right)^{\sigma} \\ \implies \frac{X_i}{X_j} &= \left(\frac{\alpha_i c_j \xi_{j,j}^{1/\sigma-1}}{\alpha_j c_i \xi_{i,i}^{1/\sigma-1}}\right)^{\sigma}\end{aligned}$$

By definition, the elasticity of substitution between any two, arbitrary technologies i and j is constant. Moreover, it can be shown that this FOC can be rearranged to give the following demand equation for each technology i

$$X_i = \left(\frac{\beta_i}{c_i}\right)^{\sigma} \frac{I}{P} \tag{9}$$

$$P = \sum_t \beta_i^{\sigma} p_i^{1-\sigma} \tag{10}$$

where $\beta_i = \alpha_i \xi_{i,i}^{-\phi}$, $\sigma = 1/(1-\phi)$, and I is the consumers income. So, in total, accounting for both the producer and consumer's objectives, we are essentially solving for:

$$\begin{aligned} V &= \left(\sum_i \beta_i X_i^{\phi}\right)^{(1/\phi)} \\ \text{such that} \quad I &= \sum_i c_i X_i \end{aligned}$$

This is a standard CES function.

■

C. Asymptotic Elasticity of Substitution

Suppose we are in a two-period, two-technology setting with $\sigma = 1$. Furthermore, suppose that the output of our first technology is constant in both periods, $\xi_{1t} = \xi_{2t}$, but the output of our second technology is zero in the second period $\xi_{2s} = 0$. And, assume we have the parameter restrictions mentioned in earlier in Section A that ensure $X, Z > 0$. This is a simple case where we have (1) a constant output reliable technology and (2) a completely intermittent technology. We now show that, in this case, the elasticity of substitution approaches 1 as the relative cost of our second technology c_2/c_1 approaches 0.

Proof: Firstly, note that from earlier we have:

$$X = \left(\begin{array}{c} \dfrac{\alpha_t\,\xi_{2\mathrm{s}}}{c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}}} + \dfrac{\alpha_s\,\xi_{2\mathrm{t}}}{c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}}} \\ -\dfrac{\alpha_t\,\xi_{1\mathrm{s}}}{c_1\,\xi_{2\mathrm{s}} - c_2\,\xi_{1\mathrm{s}}} - \dfrac{\alpha_s\,\xi_{1\mathrm{t}}}{c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_{1\mathrm{t}}} \end{array}\right)$$

Let $\xi_1 = \xi_{1t} = \xi_{2t}$ and note that $\xi_{2s} = 0$. Hence, we have:

$$X = \left(\begin{array}{c} \dfrac{\alpha_s\,\xi_{2\mathrm{t}}}{c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_1} \\ \dfrac{\alpha_t}{c_2} - \dfrac{\alpha_s\,\xi_1}{c_1\,\xi_{2\mathrm{t}} - c_2\,\xi_1} \end{array}\right)$$

Now, note that X_1/X_2 is given by:

$$\begin{aligned}
\frac{X_1}{X_2} &= \frac{\alpha_s \xi_{2t} \left(c_2(c_1\xi_{2t} - c_2\xi_1)\right)}{\left(\alpha_t(c_1\xi_{2t} - c_2\xi_1) - c_2\alpha_s\xi_1\right)\left(c_1\xi_{2t} - c_2\xi_{1t}\right)} \\
&= \frac{\alpha_s c_2 \xi_{2t}}{\alpha_t(c_1\xi_{2t} - c_2\xi_1) - c_2\alpha_s\xi_1} \\
&= \frac{-\alpha_s\, c_2\, \xi_{2\mathrm{t}}}{\xi_1\ (\alpha_s\, c_2 + \alpha_t\, c_2) - \alpha_t\, c_1\, \xi_{2\mathrm{t}}}
\end{aligned}$$

Next, we can see that:

$$\frac{\partial \log(X_1/X_2)}{c_1} = \frac{\alpha_t\, \xi_{2\mathrm{t}}}{\xi_1\ (\alpha_s\, c_2 + \alpha_t\, c_2) - \alpha_t\, c_1\, \xi_{2\mathrm{t}}}$$

The elasticity of substitution is given by:

$$\begin{aligned}
\frac{\partial \log(X_1/X_2)}{\partial \log(c_2/c_1)} &= \frac{\partial \log(X_1/X_2)}{\partial c_1} \frac{\partial c_1}{\partial \log(c_2/c_1)} \\
&= \frac{\alpha_t\, \xi_{2\mathrm{t}}}{\xi_1\ (\alpha_s\, c_2 + \alpha_t\, c_2) - \alpha_t\, c_1\, \xi_{2\mathrm{t}}} \frac{\partial c_1}{\partial \log(c_2/c_1)} \\
&= \frac{-c_1 \alpha_t\, \xi_{2\mathrm{t}}}{\xi_1\ (\alpha_s\, c_2 + \alpha_t\, c_2) - \alpha_t\, c_1\, \xi_{2\mathrm{t}}} \\
&= \left(\frac{c_2\xi_1}{-c_1\alpha_t\xi_{2t}} + 1\right)^{-1}
\end{aligned}$$

where $\alpha_t + \alpha_s = 1$ by definition (Equation (1)). Finally, it is simple to see that:

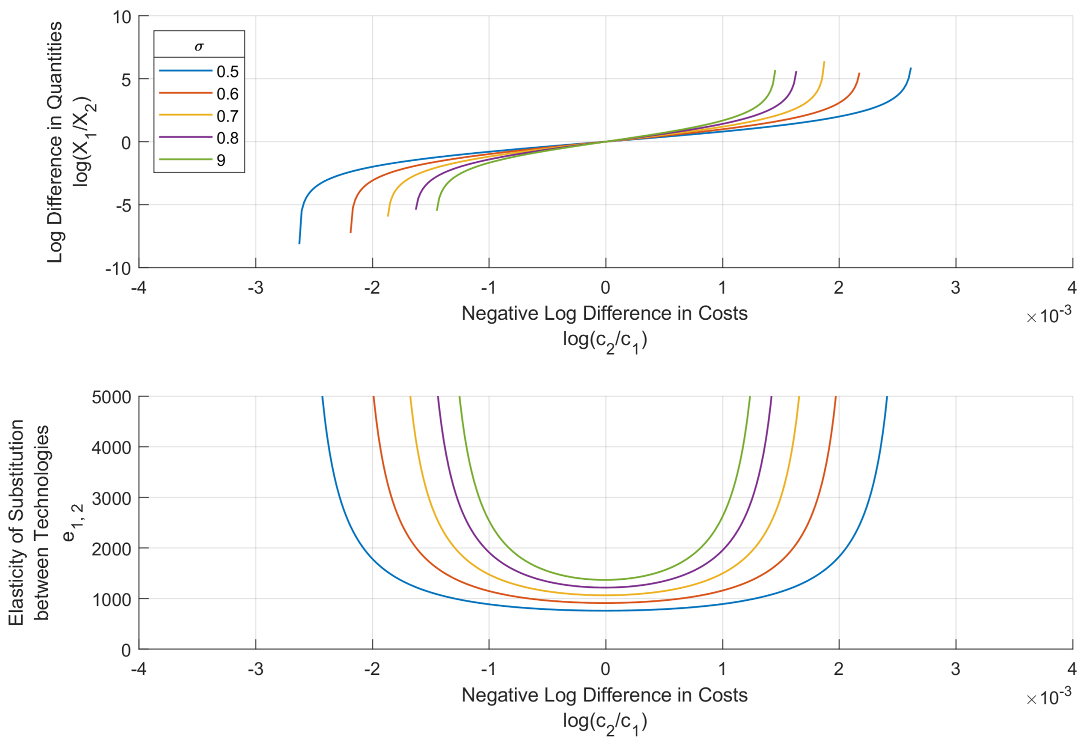
$$\lim_{c_2/c_1 \to 0} \frac{\partial \log(X_1/X_2)}{\partial \log(c_2/c_1)} = 1$$

■

VIII. Appendix B: Supplementary Figures

Figure B.1 below models the elasticity of substitution for two technologies that are close to being non-intermittent. That is, for technology 1, we have $\xi_1 = (0.95,\, 1)$, and, for technology 2, we have $\xi_2 = (1,\, 0.95)$. We further set their costs, c_1 and c_2, to equal values and allow $\alpha_t = \alpha_s$. This example illustrates how the elasticity of substitution between technologies $e_{1,2}$ would appear with minimal intermittency. We can see that the $e_{1,2}$ takes on a u-shape.

Figure B.1: The Elasticity of Substition Between Two Minimally Intermittent Technologies



Note: The y-axis of the first plot is equivalent to $\log(X_1/X_2)$ and the x-axis of both plots is equivalent to $\log(c_2/c_1)$. Technology 1 and 2 represent two arbitrary technologies that are practically non-intermittent. The legend in the upper subplot also applies to the lower subplot. These results were obtained using the following parameters: $\alpha_t = 0.5$, $\alpha_s = 0.5$, $\xi_1 = (0.95,\, 1)$, $\xi_2 = (1,\, 0.95)$, $c_1 = 100$, $c_2 = 100$. In order to generate these numerical, we first found the optimal quantities of X over a range of prices $c_1^* \in (0.5\, c_1, 1.5\, c_1)$. Then, we obtained estimates of the elasticity of substition by numerically differentiating $\ln(X_1/X_2)$ with respect to $-\ln(c_1, c_2)$. That is, the elasticity of substitution between technology 1 and 2 is given by the slope of the upper subplot, and it is graphed in the lower subplot. Finally, we repeat this procedure for various values of σ.

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