Machine Learning

Reinforcement Learning

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Reinforcement Learning For Prediction

Possible Options for Prediction

When we want to perform prediction and we do not know the environment dynamics or modeling the environment is too complex:

- Monte Carlo (first and every visit)

$$V(s_t) \leftarrow V(s_t) + \alpha(v_t - V(s_t))$$

- Temporal Difference

$$V(s_t) \leftarrow V(s_t) + \alpha(r_{t+1} + \gamma V(s_{t+1}) - V(s_t))$$

- $TD(\lambda)$ (eligibility traces)

$$V(s_t) \leftarrow V(s_t) + \alpha(v_t^\lambda - V(s_t))$$

with $v_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} v_t^{(n)}$

Exercise 9.5

Evaluate the value for the MDP with states $\mathcal{S} = \{A, B, C\}$ (C is terminal), actions $\mathcal{A} = \{h, r\}$ given the policy π and the following trajectories:

$$(A, h, 3) \to (B, r, 2) \to (B, h, 1) \to (C)$$
$$(A, h, 2) \to (A, h, 1) \to (C)$$
$$(B, r, 1) \to (A, h, 1) \to (C)$$

1. Can you tell without computing anything if by resorting to MC with every-visit and first-visit approach you will have different results?
2. Compute the values with the two aforementioned methods
3. Assume to consider a discount factor $\gamma = 1$ and compute the values by resorting to TD? Assume to start from zero values for each state and $\alpha = 0.1$

Exercise 9.6 (variant)

Evaluate the value for the MDP with states $\mathcal{S} = \{A, B, C\}$ (C is terminal), actions $\mathcal{A} = \{h, l\}$ given the policy π and the following trajectories:

$$(A, h, -1) \to (A, l, 4) \to (B, l, 1) \to (C)$$
$$(B, l, 4) \to (A, h, -3) \to (C)$$
$$(A, l, 1) \to (B, h, -2) \to (A, l, 1) \to (B, l, 1) \to (C)$$

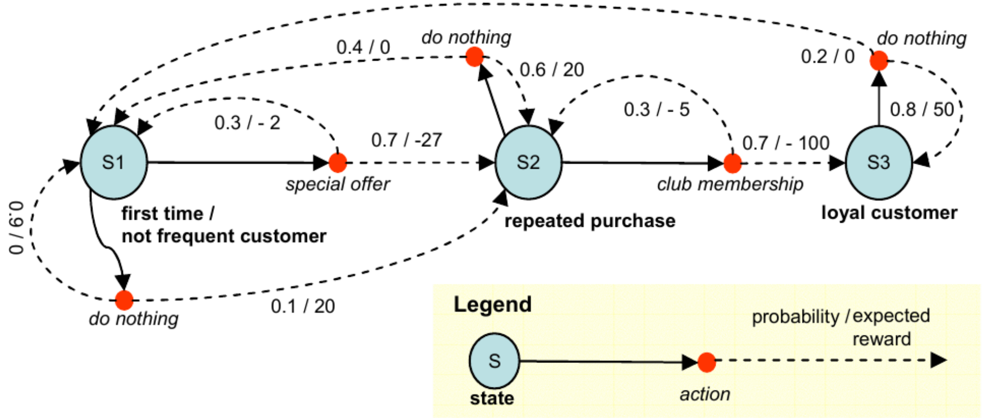
1. Compute the state-action value function $Q(A, r)$ by resorting to TD evaluation. Assume $\alpha = 0.5$, $\gamma = 1$, zero initial values.
2. Compute the state-action value function for every meaningful state-action pair by resorting to first-visit MC evaluation
3. What does the greedy policy prescribe according to the MC first-visit evaluation?
4. Assume to have performed the MC first-visit evaluation with an infinite number of trajectories from the same policy. What can we say about the optimal policy?

Reinforcement Learning For Control

Possible Options for Control

- Monte Carlo Control:
  - Policy evaluation: Monte Carlo Estimation
  - Policy improvement: ε-greedy
- SARSA:
  - Policy evaluation: Temporal Difference Estimation
  - Policy improvement: ε-greedy
- Q-learning: empirical version of Value Iteration

Example: Advertising Problem



RL Basic Elements

The elements needed to apply RL algorithms are:

- Dataset of transitions ($\{(s_n, a_n, r_n, s_{n+1})\}_{n=1}^{N}$) or model generating transitions
- Policy improvement step
- Evaluation (update) step

Transition Model

Let us model the transition model of the advertising problem from which we will get episodes used in the RL algorithms:

$$r : S \times A \to \mathbb{R}$$
$$P : S \to S$$

Especially, we need to define the generative process:

```
class Environment(object):
    ...
    def transition_model(self, a):
        ...
        return s_prime, inst_rew
```

Policy Improvement Step

The ε-greedy policy is:

```
def eps_greedy(s, Q, eps, allowed_actions):
  if np.random.rand() <= eps:
    a = % take a random action
  else:
    Q_s = Q[s, :].copy()
    Q_s[allowed_actions == 0] = - np.inf
    a = np.argmax(Q_s)
  return a
```

NB: we need to manage also the case in which the Q-values of more than one action have the same value in a state

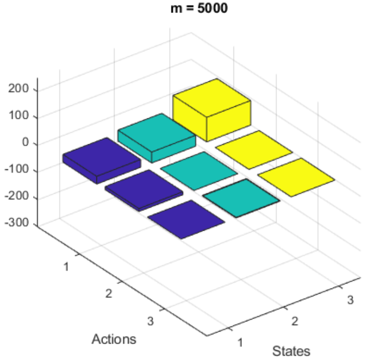
SARSA

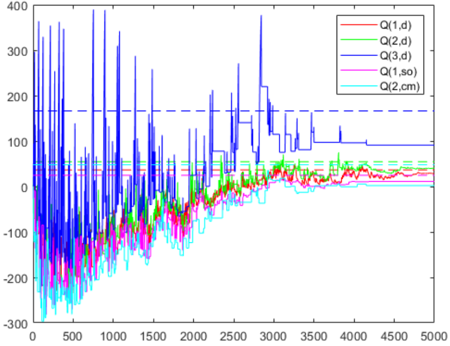
The SARSA algorithm iterates between:

- An environment step, using the transition model
- A policy improvement step, with the ϵ-greedy policy
- An evaluation step, with the TD update of the Q function:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\big(r + \gamma Q(s',a') - Q(s,a)\big)$$

SARSA - Results





Solutions Comparison

Is it a good solution?

SARSA			Exact		
40.2274	8.5816	0	36.3636	24.6818	0
67.3932	0	6.0867	54.5455	0	47.9545
79.7005	0	0	166.2338	0	0

Depending on the task we are interested in, we have a good or a poor one

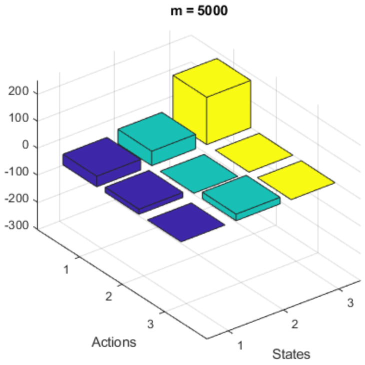
Q-learning

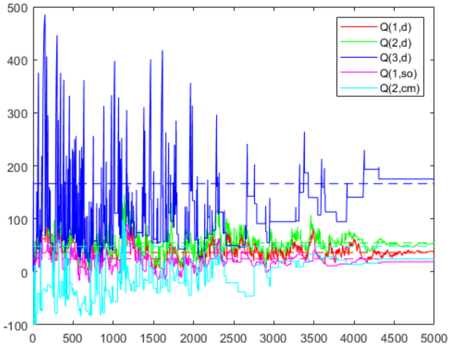
The Q-learning algorithm iterates over:

- An environment step, using the transition model
- A policy improvement step, with the ϵ-greedy policy
- An update with the Bellman optimality equation:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\big(r + \gamma \max_{\widetilde{a} \in \mathcal{A}} Q(s', \widetilde{a}) - Q(s,a)\big)$$

Q-learning - Results





Solutions Comparison

SARSA			Q-learning			Exact		
40.22	8.58	0	41.15	25.13	0	36.36	24.68	0
67.39	0	6.08	68.68	0	28.26	54.54	0	47.95
79.70	0	0	127.83	0	0	166.23	0	0

On-policy vs Off-policy

- With Q-learning I could have had a dataset to use for learning (off-policy learning)
- With SARSA I need to execute the ε-greedy policy at each time point (on-policy learning)

- How to use SARSA on a given transition dataset?
- Possible Solution: use importance sampling

Final Considerations

- What if I need to implement the previous two methods on a different environment?
- Replace `transition_model(s, a)` with the one corresponding to the new environment

- What other could I change in the learning process?
  - M time horizon / number of generated transitions
  - α learning rate
  - ε exploration incentive for ε-greedy