A Robust Entropy-Based Test of Asymmetry for Discrete and Continuous Processes

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Abstract

We consider a metric entropy capable of detecting deviations from symmetry that is suitable for both discrete and continuous processes. A test statistic is constructed from an integrated normed difference between nonparametric estimates of two density functions. The null distribution (symmetry) is obtained by resampling from an artificially lengthened series constructed from a rotation of the original series about its mean (median, mode). Simulations demonstrate that the test has correct size and good power in the direction of interesting alternatives, while applications to updated Nelson & Plosser (1982) data demonstrate its potential power gains relative to existing tests.

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1 Overview

Testing for asymmetric behavior present in a series or in conditional predictions has a rich history dating to the pioneering work by Crum (1923), Mitchell (1927), and Keynes (1936) who examined potential asymmetries present in a number of macroeconomic series. Interest has intensified and continues through the present day, and many macroeconomic series have been analysed and tested for asymmetric behavior in expansions and downturns. Recent work of Timmermann & Perez-Quiros (2001), Bai & Ng (2001), Premaratne & Bera (2005) and Belaire-Franch & Peiro (2003) are but a few such examples. There is also much recent interest in asymmetric behavior of prices as markets form and evolve toward greater competition. Some researchers subdivide asymmetry into categories such as sharpness, steepness, and deepness (see McQueen & Thorley (1993)). Though such categorizations of asymmetry may be of interest in their own right, our focus will rest upon consistent tests of asymmetries of any sort.

Existing tests are somewhat limited in application as they are designed for *continuous* processes. However, in applied settings one may encounter discrete processes, particularly in finance where price movements (i.e., differences) are often characterized as i) no change, ii) positive, or iii) negative. In this paper our goal is to provide a test that is robust to the underlying datatype, i.e., whether the data is categorical (namely, discrete) or continuous in nature.

We adapt the entropy measure in Granger, Maasoumi & Racine (2004) which is a normalization of the Bhattacharya-Matusita-Hellinger measure of "distance" between distributions, to test the direct and full hypothesis of symmetry. This is in the same spirit as the test proposed by Fan & Gencay (1995), but in contrast with many of the existing tests of symmetry that examine moments and other implications of symmetry, mostly in terms of odd order functions, as in Premaratne & Bera (2005) and Bai & Ng (2005). The latter report on well known difficulties with estimating higher order moments, such as kurtosis, as well as the greater power of tests which are based simultaneously on several odd order moments, not withstanding estimation and bias problems with estimating these moments. It seems that methods which are based on distributions, such as ours, take into account all the information that may be forthcoming from many moments without assuming the existence of such moments, and without much more difficulty in estimation, if any, in similar settings. We are also less likely to suffer from inadvertent introduction of special relations between moments that only hold for special distribution types, especially Gaussian and/or linear processes under composite null hypotheses. We implement our test with appropriate resampling techniques and based on recent kernel density estimation adapted for both continuous (see, e.g., Silverman (1986, chapter 3)) and discrete (see, e.g., Ouyang, Li & Racine (2006)) processes, respectively. We find that our test is correctly sized and has superb power, generally, and especially compared with many of the existing tests. These results are perhaps surprisingly robust to liberal degrees of asymmetry, kurtosis, and dependence in the processes, as well as for moderate to large sample sizes commonly examined in this area where large samples are needed for estimation of high order moments, assuming they exist. Existence of accessible software for our test puts in question the common invocation of "ease of use" for moment based tests which can mislead very badly in some realistic circumstances for financial and macroeconomic series.

The paper is organized as follows. First symmetry is discussed in general, and our test statistic is introduced for both continuous and discrete processes. Finite sample performance of a bootstrap implementation is examined next, again for both simulated continuous and discrete processes. Lastly, we re-examine well known empirical examples based on the Extended Nelson-Plosser US macroeconomic series, and we also consider an application involving discrete stock tick data.

2 Unconditional and Conditional Symmetry

Consider a (strictly) stationary series $\{Y_t\}_{t=1}^{T}$. Let μ_y denote a measure of central tendency, say $\mu_y = E[Y_t]$, let $f(y)$ denote the density function of the random variable Y_t, let $\tilde{Y}_t = -Y_t + 2\mu_y$ denote a rotation of Y_t about its mean, and let $f(\tilde{y})$ denote the density function of the random variable $\tilde{Y}_t$. Note that if $\mu_y = 0$ then $\tilde{Y}_t = -Y_t$, though in general this will not be so.

We say a series is *symmetric about the mean* (median, mode) if $f(y) \equiv f(\tilde{y})$ almost surely. Tests for asymmetry about the mean therefore naturally involve testing the following null:

$$H_0 : f(y) = f(\tilde{y}) \text{ almost everywhere (a.e.)} \tag{1}$$

against the alternative:

$$H_1 : f(y) \neq f(\tilde{y}) \text{ on a set with positive measure.} \tag{2}$$

Though the mean has received particular attention when testing for deviations from symmetry, one could persuasively argue that deviations about the mode or median might seem a more natural characterization. One could of course clearly rotate a distribution around any of these measures of central tendency, and for what follows one simply would replace the mean with the appropriate statistic. Note that, for discrete processes, we must by necessity reflect about the median or mode to retain the original points of support (this feature would, in general, be lost were one instead to reflect a discrete process about its mean).

Tests for the presence of conditional asymmetry can be based upon standardized residuals obtained from a regression model (see Belaire-Franch & Peiro (2003)). Let

$$Y_t = h(\Omega_t, \beta) + \sigma(\Omega_t, \lambda)\epsilon_t, \tag{3}$$

denote a general location-scale model for this process, where Ω_t is a conditioning information set, $\sigma(\Omega_t, \lambda)$ the conditional standard deviation of Y_t, and ϵ_t is a zero mean unit variance error process independent of the elements of Ω_t. If $E[\epsilon|\Omega_t] = 0$ and $e = \epsilon/\sigma$ is suitably standardized, then tests for conditional asymmetry involve the following null:

$$H_0 : f(e) = f(-e) \text{ almost everywhere} \tag{4}$$

against the alternative:

$$H_1 : f(e) \neq f(-e) \text{ on a set with positive measure.} \tag{5}$$

Related work includes Bai & Ng (2001), who construct tests based on the empirical distribution of e_t and that of $-e_t$. Belaire-Franch & Peiro (2003) apply their test and other tests to the Nelson & Plosser (1982) data updated to include 1988. We shall make use of the updated data in Section 5.1 below.

3 An Entropy-Based Test of Asymmetry

Granger et al. (2004) considered a normalization of the Bhattacharya-Matusita-Hellinger measure of dependence given by

$$S_\rho = \frac{1}{2}\int_{-\infty}^{\infty} \left(f_1^{1/2} - f_2^{1/2}\right)^2 dy \tag{6}$$

where $f_1 = f(y)$ is the marginal density of the random variable Y and $f_2 = f(\tilde{y})$ that of $\tilde{Y}$, $\tilde{Y}$ being a rotation of Y about its mean.

Note that (6) presumes that the underlying datatype is continuous. However, without loss of generality, for a discrete process Y having support $\mathcal{Y} = \{0, 1, 2 \ldots, c-1\}$ for $c = 3, 5, \ldots$, this becomes

$$S_\rho = \frac{1}{2} \sum_{y \in \mathcal{Y}} \left(p_1^{1/2} - p_2^{1/2} \right)^2 \tag{7}$$

where $p_1 = p(y)$ is the marginal probability of the random variable Y and $p_2 = p(\tilde{y})$ that of $\tilde{Y}$, $\tilde{Y}$ being a rotation of Y about its median.

We consider a kernel-based implementation of equations (6) and (7), denoted $\hat{S}_\rho$, for the purposes of testing the null of symmetry. When Y is continuous we use standard Parzen kernel estimators, while when Y is discrete we use the estimator of Ouyang et al. (2006)

Rather than adopt asymptotic-based testing procedures, we elect to use a bootstrap resampling approach (see Efron (1982) and Hall (1992) for further details on bootstrap resampling procedures). We do so mainly because critical values obtained from the asymptotic null distribution do not depend on the bandwidth (due in part to the fact that the bandwidth is a quantity which vanishes asymptotically), while the value of the test statistic depends directly on the bandwidth. This is a serious drawback in practice, since the outcome of such asymptotic-based tests tends to be quite sensitive to the choice of bandwidth. This has been noted by a number of authors including Robinson (1991) who noted that "substantial variability in the [test statistic] across bandwidths was recorded," which would be most troubling in applied situations due, in part, to numerous competing approaches for data-driven bandwidth choice (see Jones, Marron & Sheather (1996) for an excellent survey article on bandwidth selection for kernel density estimates). Granger et al. (2004) contains further discussion and references on the relative merits of asymptotic inferences based on our statistic, and gives an outline of its consistency and asymptotic distribution which will carry over in the present setting under similar assumptions including, particularly, the stationarity of the underlying processes.

Consider the sample of size $2T$ given by $Z = \{Y_1, \ldots, Y_T, \tilde{Y}_1, \ldots, \tilde{Y}_T\}$. We may construct the empirical distribution of $\hat{S}_\rho$ under the null of symmetry by noting that bootstrap samples drawn from Z, which we denote by Z^*, will be symmetric almost surely. We recompute $\hat{S}_\rho$ for each of the B resamples drawn from Z which we then order from smallest to largest and denote by $\hat{S}^*_{\rho,1}, \hat{S}^*_{\rho,2}, \ldots, \hat{S}^*_{\rho,B}$ where B is the number of bootstrap replications, say, $B = 399$ (see Davidson & MacKinnon (2000) for further details of the appropriate number of bootstrap replications). Given the set of B statistics computed under the null, we may then compute percentiles from the B (ordered) bootstrap statistics and use these as the basis for a test of asymmetry. For instance, to conduct the test at the 5% level, we reject H_0 if $\hat{S}_\rho > \hat{S}^*_{\rho,380}$ where $\hat{S}^*_{\rho,380}$ is the 95th percentile of the ordered bootstrap statistics that were generated under the null. Alternatively, we can compute empirical power via the proportion of the ordered bootstrap statistics that exceed the actual statistic.

A few words are in order regarding which of the many existing bootstrap resampling procedures are appropriate for the proposed test. Bootstrap resampling schemes fall into one of three classes, i) i.i.d. resampling (see, e.g., Efron (1982)), ii) i.n.i.d. resampling (see, e.g., Liu (1988)), and iii) stationary resampling (see, e.g., Künsch (1989), Politis & Romano (1994)). In practice, one needs to implement a resampling scheme that mimics the manner in which the sample in hand was drawn from its respective population. Both ii) and iii) require the user to set additional 'tuning' parameters (see, e.g., Künsch (1989) and Politis & White (2004)). One can of course test for symmetry in a variety of settings. For example, if interest lies in the distribution of bids at, say, a sealed auction, then i.i.d. resampling is naturally appropriate. However, for heterogeneous but otherwise independent bids one might use the so-called wild-bootstrap (Liu (1988)), while for sequential (stationary) bids that may be correlated, one might use the so-called stationary (Politis & Romano (1994)) or block (Künsch (1989)) bootstrap procedures. Of course, this places an additional burden on the practitioner, namely, that she must worry about appropriate values of incidental tuning parameters. Fortunately, a variety of such bootstrap resampling schemes are readily available in a number of popular software packages. Such flexibility allows practitioners to verify whether their results are robust to the underlying resampling scheme or not, and we advocate this approach in practice.

4 Finite-Sample Performance

We now consider the finite-sample performance of the kernel-based implementation of the test. For what follows, we conduct 1,000 Monte Carlo draws from each data generating process (DGP), and set the number of bootstrap replications underlying the test to $B = 399$. The bandwidth is selected via likelihood cross-validation (Silverman (1986, page 52)) which produces density estimators which are "optimal" according to the Kullback-Leibler criterion. Should one wish to use one of the many alternative methods of bandwidth selection (see, e.g., Jones et al. (1996)), one may do so at this stage without loss of generality.

4.1 Continuous Data Monte Carlo Simulation

We first consider the finite-sample behavior of the proposed test using the metric (6) for continuous Y. We shall consider both i.i.d. data ($Y_i = \epsilon_i$) and stationary dependent data ($Y_t = 0.5Y_{t-1} + \epsilon_t$). For i.i.d. data we use a standard bootstrap procedure that resamples with replacement from the empirical distribution of the data, while for the stationary dependent data we consider both the block bootstrap of Künsch (1989) and the stationary bootstrap of Politis & Romano (1994) using the recommended[1] blocksize that is equal to $3.15T^{1/3}$. Let

[1] For consistency, the (mean) block length should be proportional to $T^{1/3}$ (see Politis & Romano (1994) and Politis & White (2004)). The constant 3.15 was provided by Politis & Romano (1994) who considered a Gaussian $AR(1)$ process by way of example. The procedure suggested by Politis & White (2004) for selecting

the sample size assume values $n = 50, 100, 200$, and consider a range of DGP's: $N(120, 240)$ (symmetric), $t(2)$ (symmetric fat-tailed), $\chi^2(80)$ (asymmetric), $\chi^2(40)$ (asymmetric), $\chi^2(20)$ (asymmetric), $\chi^2(10)$ (asymmetric), $\chi^2(5)$ (asymmetric), and $\chi^2(1)$ (asymmetric). Figure 1 plots each asymmetric DGP to allow the reader to get a sense of the range of asymmetries considered, contrasting the symmetric $N(\mu, \sigma^2)$.

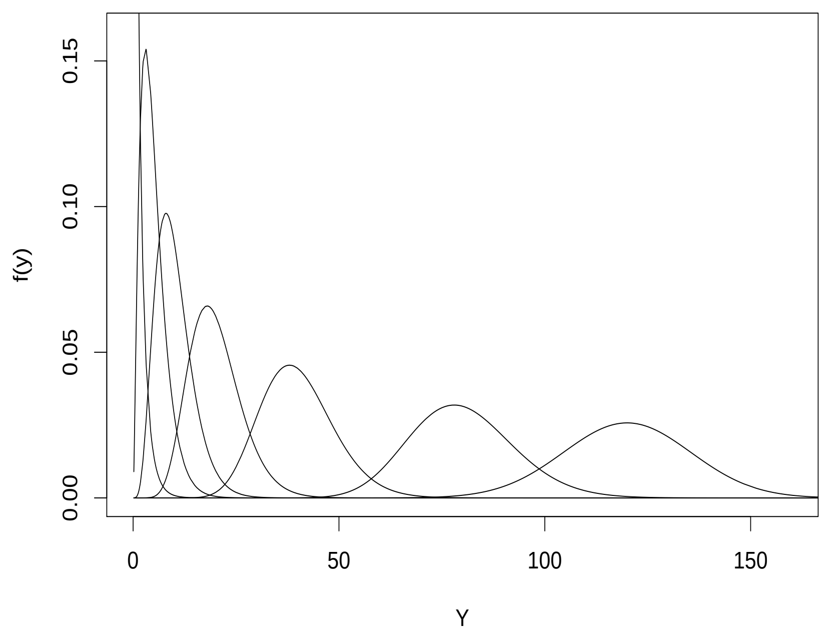

Figure 1: Simulated continuous distributions. The distributions are, from right to left, $N(120, 240)$, $\chi^2(80)$, $\chi^2(40)$, $\chi^2(20)$, $\chi^2(10)$, $\chi^2(5)$, and $\chi^2(1)$.

Tables 1 through 3 summarize the finite-sample performance of the proposed test conducted at nominal levels of $\alpha = 0.10, 0.05, 0.01$.

We observe from tables 1 through 3 that the test has correct size, though we note that

the block length is not applicable in this setting as it relies on visual inspection of the correlogram which is infeasible in a Monte Carlo setting due to the practical limitations of inspecting the correlogram for *each* Monte Carlo replication. Experimentation indicates that a slightly smaller constant of proportionality will produce a test that is correctly sized. Optimal selection of block length is an issue that is deserving of further study but is an issue that clearly lies beyond the scope of the current paper.

Table 1: Empirical rejection frequencies at levels $\alpha = 0.10, 0.05, 0.01$. The degree of asymmetry increases as we go from columns 2 and 3 (symmetric) to columns 4–9. Columns 2 and 3 reflect the empirical size of the test, columns 4–9 empirical power. $Y_i = \epsilon_i$, i.i.d. boostrap.

n	$N(\mu, \sigma^2)$	$t(2)$	$\chi^2(80)$	$\chi^2(40)$	$\chi^2(20)$	$\chi^2(10)$	$\chi^2(5)$	$\chi^2(1)$
				$\alpha = 0.10$				
50	0.105	0.100	0.157	0.259	0.411	0.598	0.835	0.958
100	0.090	0.098	0.284	0.445	0.700	0.912	0.990	0.995
200	0.098	0.123	0.478	0.741	0.945	0.997	1.000	1.000
400	0.101	0.108	0.726	0.946	0.998	1.000	1.000	1.000
				$\alpha = 0.05$				
50	0.048	0.041	0.096	0.136	0.270	0.426	0.690	0.886
100	0.055	0.041	0.169	0.316	0.554	0.817	0.967	0.984
200	0.039	0.050	0.337	0.611	0.897	0.988	1.000	1.000
400	0.047	0.046	0.614	0.916	0.993	1.000	1.000	1.000
				$\alpha = 0.01$				
50	0.008	0.007	0.028	0.029	0.087	0.175	0.381	0.605
100	0.014	0.006	0.053	0.120	0.276	0.542	0.833	0.933
200	0.008	0.007	0.137	0.350	0.696	0.946	0.997	0.999
400	0.012	0.004	0.373	0.739	0.979	1.000	1.000	1.000

the test is somewhat conservative (i.e., slightly undersized) for small sample sizes when using the block and stationary bootstrap for the autoregressive DGP. As might be expected, larger memory and persistence in the underlying processes requires larger sample sizes (200+) to reveal consistency of the tests. In general, however, there is considerable power, especially in cases of appreciable asymmetry. The Block bootstrap method does slightly better than the stationary bootstrap in our experiments. Columns 2 and 3 of tables 1 through 3 present results for the symmetric $N(\mu, \sigma^2)$ and $t(2)$ distributions for levels α = 0.10, 0.05, and 0.01. As the degree of asymmetry increases (moving from columns 4 through 9), we observe that power increases notably, and it increases uniformly with the sample size. In contrast, the state-of-art moment based tests, such as the Bai-Ng test (see Section 4.3 below) do not perform as well. The latter's power, for instance, improves for only some underlying distributions, with the largest sample sizes, and when tests are based on several high order

Table 2: Empirical rejection frequencies at levels $\alpha = 0.10, 0.05, 0.01$. The degree of asymmetry increases as we go from columns 2 and 3 (symmetric) to columns 4–9. Columns 2 and 3 reflect the empirical size of the test, columns 4–9 empirical power. $Y_t = 0.5Y_{t-1} + \epsilon_t$, block bootstrap.

n	$N(\mu, \sigma^2)$	$t(2)$	$\chi^2(80)$	$\chi^2(40)$	$\chi^2(20)$	$\chi^2(10)$	$\chi^2(5)$	$\chi^2(1)$
				$\alpha = 0.10$				
50	0.074	0.102	0.013	0.028	0.050	0.159	0.337	0.767
100	0.072	0.092	0.011	0.040	0.105	0.377	0.718	0.984
200	0.086	0.110	0.047	0.118	0.362	0.756	0.969	1.000
400	0.106	0.097	0.211	0.350	0.748	0.975	1.000	1.000
				$\alpha = 0.05$				
50	0.022	0.030	0.002	0.005	0.019	0.060	0.133	0.472
100	0.022	0.037	0.000	0.011	0.042	0.193	0.480	0.937
200	0.033	0.038	0.017	0.051	0.201	0.575	0.911	1.000
400	0.043	0.032	0.096	0.225	0.611	0.951	1.000	1.000
				$\alpha = 0.01$				
50	0.001	0.002	0.000	0.001	0.004	0.003	0.007	0.054
100	0.002	0.005	0.000	0.001	0.002	0.016	0.088	0.499
200	0.002	0.002	0.001	0.005	0.029	0.172	0.566	0.985
400	0.003	0.003	0.022	0.063	0.275	0.754	0.988	1.000

moments simultaneously and/or when the test exploits additional information about the one sidedness of the asymmetry hypothesis that may be justified in some applications. Since the metric entropy test is never bettered, we do not see any reason to qualify its adoption. Our code is in R and freely available and freely adopted.

4.2 Discrete Data Monte Carlo Simulation

We now examine the finite-sample behavior of (7), and consider by way of example two trials from a Bernoulli process hence $Y \in \{0, 1, 2\}$. When $Pr(Y = 1) = 0.5$ the process has a symmetric marginal probability function, while when $Pr(Y = 1) \neq 0.5$ the marginal probability function is asymmetric. Figure 2 plots the probability functions for a range of values for $Pr(Y = 1)$.

Table 3: Empirical rejection frequencies at levels $\alpha = 0.10, 0.05, 0.01$. The degree of asymmetry increases as we go from columns 2 and 3 (symmetric) to columns 4–9. Columns 2 and 3 reflect the empirical size of the test, columns 4–9 empirical power. $Y_t = 0.5Y_{t-1} + \epsilon_t$, stationary bootstrap.

n	$N(\mu, \sigma^2)$	$t(2)$	$\chi^2(80)$	$\chi^2(40)$	$\chi^2(20)$	$\chi^2(10)$	$\chi^2(5)$	$\chi^2(1)$
				$\alpha = 0.10$				
50	0.076	0.083	0.027	0.035	0.071	0.114	0.273	0.760
100	0.067	0.068	0.013	0.023	0.091	0.290	0.623	0.975
200	0.074	0.083	0.033	0.070	0.281	0.661	0.959	1.000
400	0.083	0.091	0.161	0.329	0.697	0.975	1.000	1.000
				$\alpha = 0.05$				
50	0.023	0.024	0.008	0.009	0.025	0.040	0.101	0.419
100	0.014	0.014	0.004	0.004	0.030	0.118	0.313	0.859
200	0.020	0.024	0.009	0.024	0.135	0.432	0.846	1.000
400	0.024	0.035	0.072	0.170	0.495	0.911	1.000	1.000
				$\alpha = 0.01$				
50	0.000	0.002	0.000	0.000	0.000	0.001	0.003	0.048
100	0.000	0.000	0.001	0.000	0.000	0.010	0.019	0.214
200	0.001	0.001	0.001	0.003	0.014	0.066	0.276	0.886
400	0.001	0.004	0.007	0.028	0.124	0.528	0.921	1.000

For the simulations that follow, we vary $Pr(Y = 1)$ from 0.35 through 0.50 in increments of 0.01. We draw 1,000 Monte Carlo replications for each DGP and apply the proposed test. Empirical rejection frequencies are summarized in the form of power curves which we report in Figure 3.

It can be seen from Figure 3 that the test is correctly sized and has power increasing appreciably with the degree of asymmetry. This is to be expected since entropies are functions of many moments, when these moments exist. We would like to remind the reader that the proposed test is quite versatile, and affords practitioners with a simple test that can be used regardless of the underlying datatype.

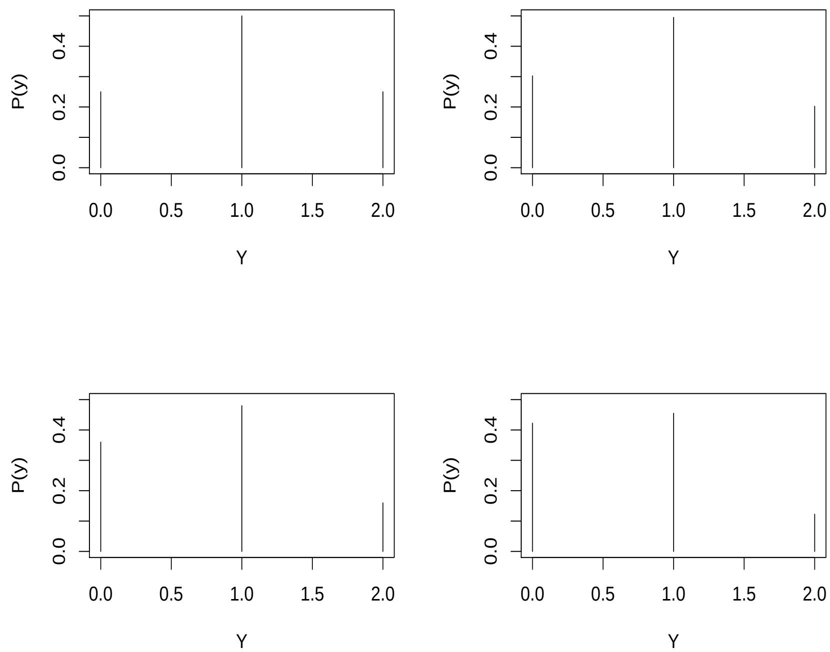

Figure 2: Simulated discrete distributions. The distributions are, from top left, top right, bottom left, and bottom right, those for two draws from a Bernoulli trial with $Pr(Y = 1) = 0.50, 0.45, 0.40$, and 0.35 respectively.

4.3 Comparison With Existing Tests.

Bai & Ng (2005) consider three moment-based test statistics for testing symmetry for *continuous* processes. We briefly compare our proposed test with their tests. As we have already demonstrated that our test is correctly sized, we restrict attention to power comparisons. We consider samples of sizes $n = 100$ and 200. Briefly, $\hat{\pi}_3^*$ and $\hat{\pi}_3^{**}$ are Bai & Ng's (2005) one-and two-sided tests for symmetry (skewness) while $\hat{\mu}_{35}$ is a joint test of the third and fifth central moments. Bai & Ng (2005) consider a range of distributions. We replicate their simulations which correspond to DGPs A1 (lognormal), A2 (exponential), A3 (χ^2_2), and A4 (generalized lambda with $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 1.4$, and $\lambda_4 = 0.25$) in Bai & Ng (2005, Table 1). All tests are conducted at the 5% level. Results are reported in Table 4 below. Bai &

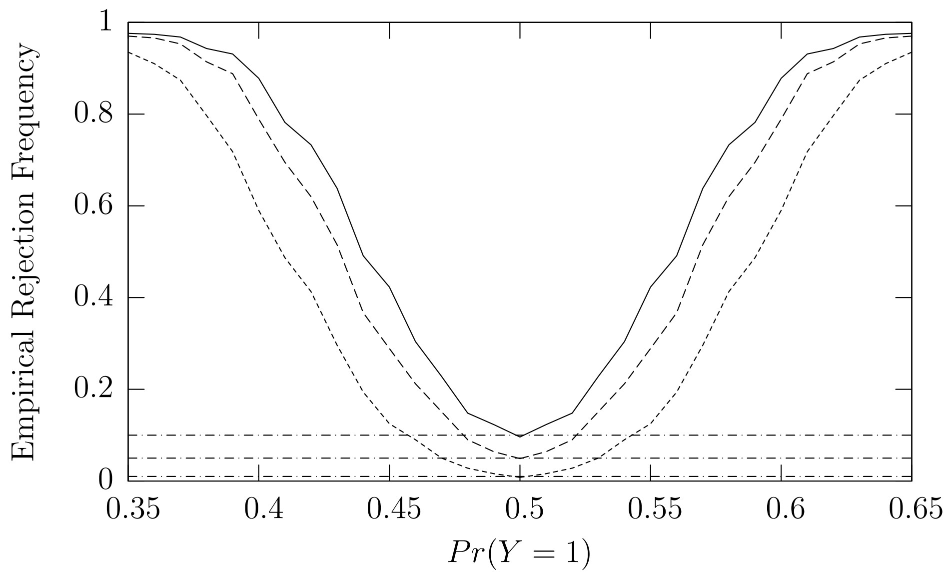

Figure 3: Power curves for (7) for $n = 100$ when $Pr(Y = 1) \in [0.35, 0.65]$. The dotted lines are the nominal levels of the test ($\alpha = 0.01, 0.05, 0.10$).

Ng (2005, pg. 54) write "the $\hat{\mu}_{35}$ test is to be recommended when symmetry is the main concern." It can be seen from Table 4 that the metric entropy test is not dominated in any case, but it has substantially greater power in a number of situations.

5 Applications

5.1 Testing for Asymmetry in U.S. Macroeconomic Time Series

In order to examine the behavior of the test on actual time series, we employed the extended Nelson & Plosser (1982) data on a number of US macroeconomic series. First, we applied our tests to the original series.[2] Tables 5 and 6 present results for the proposed S_ρ test for unconditional and conditional asymmetry for the updated Nelson & Plosser (1982) data.

[2] In the dataset, all series except the interest rate are logged, and we refer to these variables as the "original series" for what follows.

Table 4: Comparison with Bai-Ng (2005). Empirical rejection frequencies, $\alpha = 0.05$ (power).

Distribution	$\hat{\pi}_3^{**}$	$\hat{\pi}_3^{*}$	$\hat{\mu}_{35}$	$\hat{S}_\rho$
	$n = 100$			
lognormal	0.43	0.63	0.62	0.97
χ^2_2	0.74	0.88	0.96	0.99
exponential	0.75	0.89	0.96	1.00
Generalized λ	0.85	0.93	0.64	0.90
	$n = 200$			
lognormal	0.52	0.69	0.81	1.00
χ^2_2	0.90	0.96	1.00	1.00
exponential	0.91	0.97	1.00	1.00
Generalized λ	1.00	1.00	0.99	1.00

For the unconditional series we simply rotated each series about its mean, while for the conditional series we employed an $AR(P)$ process with lag order selected via SIC as in Belaire-Franch & Peiro (2003), where $e_t = Y_t - \hat{\delta}_0 - \sum_{j=1}^{p} \hat{\delta}_i Y_{t-j}$. Should the lag order $P = 0$, the test statistics and percentiles for conditional symmetry will be equivalent to those for the unconditional series (ignoring bootstrap resampling error).[3] We deploy Politis & Romano's (1994) stationary bootstrap procedure to handle potential dependence in the series.

For the original series, we observe from Table 5 that three series (Velocity, Bond Yields, and S&P 500) are deemed asymmetric by S_ρ. Belaire-Franch & Peiro (2003) detect asymmetry for only two series (Employment Rate and Unemployment Rate). There is a vast literature on whether there is a unit root in these series, and also on modelling asymmetric behavior of US GNP and other macroeconomic series, as exemplified by and discussed in Timmermann & Perez-Quiros (2001).

Below we report the unconditional tests for both the "original series", as well as their

[3]SIC optimal lag orders ranged from 0 through 5. Additionally, we systematically investigated lag orders from 1 through 5, and results were qualitatively unchanged. SIC optimal lag orders were 1, 1, 1, 0, 3, 3, 1, 5, 1, 0, 1, 0, 0, and 0, for each series listed in Table 6, respectively.

Table 5: Unconditional symmetry tests and percentiles under the null of symmetry.

Series	$\hat{S}_\rho$	p_{90}	p_{95}	p_{99}	P
Real GNP	0.030	0.429	0.568	0.763	0.886
Nominal GNP	0.108	0.352	0.441	0.601	0.457
Real p/c GNP	0.062	0.370	0.510	0.718	0.762
Industrial Production	0.035	0.340	0.430	0.652	0.923
Employment Rate	0.024	0.302	0.385	0.532	0.929
Unemployment Rate	0.018	0.046	0.056	0.081	0.400
GNP Price Deflator	0.165	0.262	0.319	0.514	0.327
CPI	0.234	0.300	0.353	0.474	0.220
Nominal Wages	0.107	0.394	0.497	0.705	0.592
Real Wages	0.084	0.568	0.725	0.934	0.820
Money Stock	0.043	0.335	0.435	0.620	0.873
Velocity	0.162	0.134	0.172	0.248	0.057
Bond Yields	0.359	0.257	0.302	0.387	0.014
S&P 500	0.280	0.267	0.334	0.539	0.088

first differences, for two reasons. First, asymmetry in a few of the original series, the real point of interest in macroeconomics and finance, appears to be present. Secondly, the issue of "nonstationarity" in some of these series is widely regarded as controversial and unsettled. This complicates the choice of prewhitening and filtering of the series, an issue that is most recently discussed by Dagum & Giannerini (2006). Nevertheless, many scholars believe that one should first difference these series (in logs, except where noted above) to make them stationary, as we have assumed them to be in our theoretical assertions. To examine this issue, Tables 7 and 8 present results for the proposed S_ρ test for unconditional and conditional asymmetry using the first difference (of the logs) of the updated Nelson & Plosser (1982) data. Again, potential dependence in each transformed series is accommodated by applying the stationary bootstrap of Politis & Romano (1994) when constructing the null distribution of the test. For the "unconditional" differenced series we again simply rotate the series about its mean. Indeed, we now find little evidence of asymmetry in the differenced log of

Table 6: Conditional symmetry tests and percentiles under the null of symmetry.

Series	$\hat{S}_\rho$	p_{90}	p_{95}	p_{99}	P
Real GNP	0.015	0.023	0.028	0.037	0.261
Nominal GNP	0.043	0.069	0.082	0.107	0.332
Real p/c GNP	0.014	0.021	0.026	0.036	0.242
Industrial Production	0.035	0.388	0.468	0.628	0.914
Employment Rate	0.018	0.021	0.025	0.037	0.140
Unemployment Rate	0.001	0.011	0.014	0.021	0.849
GNP Price Deflator	0.005	0.022	0.027	0.042	0.638
CPI	0.014	0.030	0.037	0.052	0.509
Nominal Wages	0.002	0.021	0.030	0.046	0.859
Real Wages	0.084	0.547	0.697	0.890	0.785
Money Stock	0.006	0.019	0.025	0.039	0.496
Velocity	0.162	0.133	0.166	0.249	0.054
Bond Yields	0.359	0.261	0.298	0.408	0.023
S&P 500	0.280	0.255	0.349	0.500	0.082

these macroeconomic series. As might be expected from this finding, further differencing by the $AR(p)$ model of these already first differenced series produces symmetric residuals. Comparing these results with those of "pseudo differencing" of an $AR(p)$ process in Table 6, simple differencing is somewhat more effective in symmetrizing. Obviously, a wide range of models other than $AR(p)$ have been fit to these series over the last two decades. We did not pursue whether (standardized) residuals from such models would be symmetric. This would be a challenging and large project that would more reflect on the adequacy of other model specifications, than the "symmetry" of the macroeconomic series, the main interest of this section.

In interpreting and comparing our findings with the existing folklore it is worthwhile to remember that our hypothesis encompasses all the implications of "symmetry" for the whole distribution, and thus our test has the power to reject "symmetry" even if certain implications of symmetry, such as skewness, may not be rejected by other tests.

Table 7: Unconditional symmetry tests and percentiles under the null of symmetry for the differenced series.

Series	$\hat{S}_\rho$	p_{90}	p_{95}	p_{99}	P
Real GNP	0.016	0.021	0.028	0.039	0.231
Nominal GNP	0.054	0.074	0.086	0.123	0.229
Real p/c GNP	0.014	0.019	0.025	0.036	0.228
Industrial Production	0.039	0.046	0.060	0.090	0.158
Employment Rate	0.019	0.021	0.028	0.041	0.118
Unemployment Rate	0.020	0.024	0.028	0.037	0.146
GNP Price Deflator	0.003	0.018	0.023	0.036	0.746
CPI	0.025	0.038	0.047	0.071	0.229
Nominal Wages	0.004	0.019	0.027	0.044	0.662
Real Wages	0.001	0.008	0.010	0.016	0.841
Money Stock	0.007	0.020	0.026	0.046	0.491
Velocity	0.018	0.020	0.025	0.039	0.119
Bond Yields	0.089	0.097	0.114	0.143	0.136
S&P 500	0.012	0.022	0.026	0.034	0.260

Table 8: Conditional symmetry tests and percentiles under the null of symmetry for the differenced series.

Series	$\hat{S}_\rho$	p_{90}	p_{95}	p_{99}	P
Real GNP	0.005	0.018	0.024	0.034	0.437
Nominal GNP	0.023	0.032	0.036	0.050	0.194
Real p/c GNP	0.004	0.015	0.020	0.030	0.473
Industrial Production	0.039	0.050	0.063	0.079	0.181
Employment Rate	0.006	0.018	0.023	0.037	0.536
Unemployment Rate	0.008	0.013	0.016	0.024	0.298
GNP Price Deflator	0.009	0.019	0.023	0.032	0.480
CPI	0.009	0.018	0.020	0.030	0.428
Nominal Wages	0.014	0.024	0.029	0.039	0.355
Real Wages	0.001	0.007	0.009	0.013	0.849
Money Stock	0.008	0.018	0.021	0.029	0.454
Velocity	0.018	0.020	0.026	0.036	0.125
Bond Yields	0.089	0.095	0.110	0.136	0.127
S&P 500	0.012	0.020	0.025	0.032	0.248

5.2 Testing for Asymmetry in Daily Stock Movements

Often interest lies in a particular aspect of price movements of financial assets, namely the so-called price 'tick'. The expression 'tick' denotes a change in the price of an asset from one trade to the next. Should the later trade be made at a higher price than the earlier trade, we call this an 'uptick' trade since the price went up. If, on the other hand, the later trade is is made at a lower price than the previous trade, that trade is known as a 'downtick' trade because the price went down. This information is given by $sign(\Delta p_t)$ where '$sign$' returns the signs of the corresponding elements of the price change from time $t-1$ to time t expressed as $sign(\Delta p_t) = sign(p_t - p_{t-1})$ which assumes values 1, 0, or -1 if the sign of Δp_t is positive, zero, or negative, respectively. Price tick information is used to regulate financial markets. For instance, 'shorting' a stock, can only be executed on an uptick (i.e., before you begin shorting a stock the last trade *must* be an uptick). Price ticks also form the basis for widely watched market indicators. For instance, the so-called 'tick indicator' is a market indicator that measures how many stocks are moving up or down in price, and this tick indicator is computed based on the last trade in each stock. The Wall Street Journal publishes a daily short tick indicator table. Potential asymmetry in tick behaviour is of direct interest to traders.

For what follows we consider one stock, namely, daily returns for General Electric from January 3, 1969 through December 31, 1998. Data was obtained from the Center for Research in Security Prices at the Graduate School of Business in the University of Chicago. Figure 4 plots the histogram of price ticks during this period.

For this data, the relative frequencies of -1, 0, and 1 are 0.449, 0.061, and 0.491, respectively, as can be observed from Figure 4. We apply the proposed test for asymmetry to this discrete process using 399 bootstrap replications, and obtain a P-value of 0.018 indicating that we would reject the null hypotheses of symmetry of ticks for this stock during this

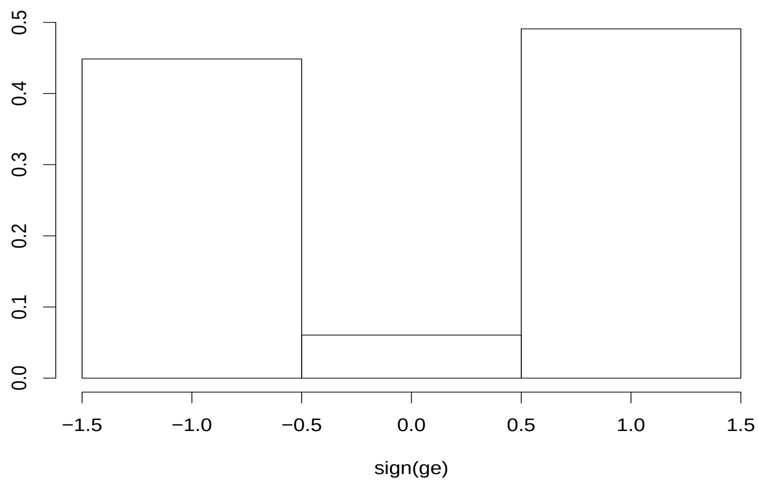

Figure 4: Price ticks for General Electric stock, 1969-1-03 to 1998-12-31.

period at the 5% level. This simple application highlights the fact that our proposed test is versatile and is equally adept at detecting asymmetry present in both continuous and discrete processes.

6 Conclusion

We examined a simple robust entropy-based test for asymmetry along with a resampling method for obtaining its null distribution. The test admits both discrete and continuous processes. Finite-sample performance is examined and the test is correctly sized possessing power that increases with both the sample size and degree of departure from the null. An application to the updated Nelson & Plosser (1982) data indicates power gains relative to tests deployed in Belaire-Franch & Peiro (2003).

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