The Information Basis of Multivariate Poverty Assessments*

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Abstract

Measures of multivariate well-being, such as poverty or inequality, are scalar functions of matrices of several attributes, m, associated with a number of individual or households, N. This entails inevitable "aggregation" and summarization over individuals as well as attributes. There is no escape from this. Such aggregation, in turn, implies a set of weights attached to each individual, and some normative decision on how they relate. The aggregation over the attributes also forces decisions about the weight to be given to each attribute and the relation between the attributes as, perhaps, substitutes or complements. We argue in favor of information theory aggregation methods which are explicit about such normative choices, and help place other methods in this realistic context. According to axiomatically well developed measures of divergence in information theory, our measures are "ideal" and other methods are therefore sub-optimal. The advocacy of the latter must be accompanied by well argued positions in support of special properties and other considerations which may be compelling in a given context or application.

1 Introduction

Evaluation of household or individual well being is now widely accepted as a multiattribute exercise. Far less agreement exists on such matters as which attributes to include, how such attributes are related and/or contribute to overall

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well being, and what criteria to employ for complete (i.e., index based) ranking of well-being situations. Some degree of robustness may be sought through weak uniform rankings of states, as by Stochastic Dominance and related criteria. A useful starting point, both for the believers and non-believers in the multidimensional approach, is to see the traditional univariate assessments in the multiattribute setting: It is as though a weight of one is attached to a single attribute, typically income or consumption, and zero weights given to all other real and potential factors! Univariate approaches do not avoid, rather, they imposes very strong a priori values.

Given a matrix X of attributes, with typical element x_{ij} , for units $i = 1, 2, ...N$ and attributes $j = 1, 2, ...m$, any scalar measure of well being $f(X)$ is a function $f(.) : R^N \text{x} R^m \rightarrow R$. It is evident, and inescapable, that $f(.)$ aggregates over both individuals and attributes. In so doing, it must assign weights to both individuals and to each attribute. In addition, every $f(.)$ implies a certain relation between individuals as well as attributes. There are only two choices before us: make these functional characteristics explicit, or allow them to implicitly derive from other considerations. Viewed this way, axiomatic characterization of 'ideal' poverty (and other) measures does well to explicate the properties of $f(.)$ with respect to individual weights and relations, but not the aggregation over attributes. Similarly, axiomatic characterization of ideal aggregation measures may produce welfare theoretic features that may not be desired. There is no minimalist set of axioms commanding universal acceptance which may produce even a family of functions $f(.)$. Additional, more restrictive and less acceptable properties must be imposed to justify any one measure $f(.)$.

A deeper understanding of indices, be they of poverty or inequality, makes clear that all indices are functions of the distribution of the desired attribute(s). Put another way, any index is a function of the moments of the distribution of the attributes. As such, all indices omit more or less information relative to the full distribution. Only one function, the characteristic (or moment generating function) is equivalent to the whole distribution. Entropy comes close, see Ebrahimi, Maasoumi and Soofi (1999), since two entropies are equal if, and only if, the two underlying distributions are the same. This property of entropy and other information measures of welfare seems to be poorly appreciated by economists. For instance, there exists no better or more complete measure of 'divergence' between a given income distribution and the uniform (rectangular) distribution. Put another way, there cannot exist a more complete and more fully informed measure of equality/inequality than entropy. Only if we additionally restrict such indices can we justify other measures[1]. Many of these additional restrictions and properties are sensible. But they are almost never consensus properties. This comment generally applies to the whole edifice of welfare function-welfare theoretic assessments and the restrictions that de-

[1] Of course, there are many entropies, including Shannon's which underlies Theil's inequality measures, and Generalized Entropy, which underpins the GE measures of inequality and Atkinson's family. Maasoumi (1993) emphasizes the axiomatic properties that justify different entropies and metrics, which are the same, alas with different names, that support different measures of inequality and poverty.

rive from it, such as ‘individualistic’, ‘utilitarian’, and ‘welfarist’ Social Welfare Function (SWF) basis for the discussion of indices. While the latter provides the most disciplined and elegant formalism for analysis, it does not have a claim to producing the most complete and most ‘informed’ indices, as we shall see.

The literature on multidimensional poverty recognizes three broad classes of measures; see Deutsch and Silber (2005): The fuzzy set approach, the information theory approach (Maasoumi), and the axiomatic approach to poverty measures (e.g., Bourguignon and Chakravarty (2003) and Tsui (2002)). As argued above, all three must produce aggregate measures of well-being, or what we may term "individual representation functions". In the end, poverty measures derive from this aggregate and the distribution of the constituent attributes. All measures classify certain members of the population as "poor", and may assess the intensity of their poverty (such as the expected shortfall). In this paper we adopt the information theory perspective to assess the different aggregation methods, explicit or implicit, and examine who is classified as poor in the axiomatic and the information theory approaches.

A brief description of the Information Theory (IT) approach is as follows: Employing information functions and related entropies, divergence/distance between distributions is a well defined concept in IT. Following Maasoumi (1986), we find individual level aggregate welfare functions whose distributions are the least divergent from the distributions of the constituent welfare attributes. This provides a method of optimal aggregation in the multidimensional welfare context that is able to subsume all existing implicit aggregators in this field, but also suggest new ones. The second step is then to measure "poverty" in the distribution of this aggregate function of well-being. All of the existing univariate poverty measures present as candidates. The IT approach also opens new vistas in terms of the definition and concept of "the poverty line" in the multidimensional context. Several definitions and approaches emerge which go beyond the existing methods.

We conclude with an empirical example and some remarks concerning implementation and practical issues. One issue concerns the identification of truly distinct dimensions/ attributes. This highlights, again, the statistical role played by any chosen index and its ability to utilize "information" in different dimensions. This is both instructive, and illuminating in terms of the "information completeness" of an index alluded to above, but is not entirely unique to the multidimensional context, merely aggravated by it. Since we only consider three dimension of "income", "education" and "health" in our application to Indonesian data, in this paper we do not deal with the clustering techniques that also use consistent IT method for dimension reduction based on the "similarity" of the attribute distributions. We merely report several robust measures of dependence between our chosen attributes to shed light on their relations.

1.1 Multivariate Poverty Measures

Poverty analysis is concerned with the lower part of the distribution of well-being. In particular, the measurement of poverty generally involves three steps: first, selecting an appropriate indicator to represent individuals' well-being; second, choosing a poverty line which identifies the 'lower part' of the distribution to be the object of study, and hence to categorise people as poor and non-poor; finally, selecting a functional form to aggregate individuals.

The monetary approach to poverty uses income or consumption expenditure (Y_i) as the indicator of well-being, identifies the poor as those with insufficient income to attain minimum basic needs (z), and aggregates their shortfall to a minimum level into a poverty index (Sen, 1976). The poverty headcount, poverty gap, and severity of poverty are the most common indices used in the literature, all belonging to the family of Foster-Greer-Thorbecke (FGT) poverty measures (Foster *et al.*, 1984).

If individual i consumes M goods x_{ij}, $j = 1, 2, ...M$, his well-being indicator is $Y_i = \sum_{j=1}^{M} r_j x_{ij}$ where r_j is the market price for good j. The poverty line is determined as $z = \sum_{j=1}^{m} r_j x_{ij0}$ where x_{ij_0} belongs to the set of basic needs and $m \in M$. The FGT index can be expressed alternatively as

$$
\begin{aligned}
FGT_\alpha &= \frac{1}{n}\sum_{i=1}^{n}\Big[\max\Big(\frac{z-Y_i}{z},0\Big)\Big]^\alpha && (1)\\
&= \frac{1}{n}\sum_{i=1}^{n}\big(1-\frac{x_{ij}}{z_j}\big)^\alpha L(x_{ij}\le z_j) && (2)\\
&= \frac{1}{n}\sum_{x_{ij}\le z_j}\big(1-\frac{x_{ij}}{z_j}\big)^\alpha && (3)
\end{aligned}
$$

where l is an indicator function and α is a parameter indicating the sensitivity of the index to the distribution among poor - the higher its value, the more sensitive. For $\alpha = 0, FGT$ is the headcount, for $\alpha = 1$ it is the poverty gap, and for $\alpha = 2$ it represents the severity of poverty.

For decades, many scholars favored a multidimensional perspective to poverty where 'human deprivation is visualized not through income as an intermediary of basic needs but in terms of shortfalls from the minimum levels of basic needs themselves' (Tsui 2002, p. 70). This quote voices two common arguments against the traditional income method. The first questions the assumption of the existence of known prices and markets for all relevant determinants of deprivation. Even if market prices do exist, one can challenge the view that these are somehow 'right'. From a normative perspective, market prices are just as arbitrary as any other weights chosen by the user (Tsui, 2002). In truth, the latter have the advantage that they allow for a clear understanding of the effects of the weighting scheme.

More interestingly, the monetary approach relies on the implicit assumption of perfect substitutability between attributes. Rather, for poverty or deprivation analysis, some would argue that each attribute is to be considered 'essential' in the sense that a person who does not achieve a minimum threshold in one dimension should be seen as poor, irrespective of how much he or she has of the other attributes (Tsui 2002, Bourguignon and Chakravarty 2003). According to this view, substitution between two attributes is only relevant for individuals who are below the minimum level in all dimensions. The idea of essentiality of attributes is consistent with the *union* approach of poverty (Atkinson 2003; Duclos et al. 2003) and is expressed through the **Strong Poverty Focus Axiom** (see below). We will argue that one should also accept an *intermediate* position which allows for some degree of substitution between attributes even if some are above the threshold. This intermediate view is reflected in the **Weak version of the poverty focus axiom** which is satisfied by some of the information theory indices proposed below.

Rejecting the "market price approach", Tsui derives a set of multidimensional poverty measures following an axiomatic approach which incorporates strong poverty focus axiom, similar in spirit to his work on multidimensional inequality (1995, 1999). Specifically, Tsui extends standard univariate axioms of unidimensional poverty indices, while presenting new axioms taylored to the multivariate poverty context.

Consider the 1xm vector *(*z) of poverty lines for each j attribute. Let define a *multidimensional poverty index* as a mapping from the matrix X and the vector z to a real valued number.

$$P(X,z) = G[f(x_{i1},...,x_{im});z] : M(n) \rightarrow \Re \tag{4}$$

Axioms are imposed on the poverty index *P(X,z)* directly, rather than to some social evaluation function (as in Tsui 1999) but these properties will constrain the family of individual functions *f(x)* and aggregate function *G(.)*. These are

- **Continuity** $P(X;z)$ is a continuous function of X for any vector z.
- **Symmetry** with respect to individuals. $P(X;z) = P(\Pi X;z)$, where Π is an nxn permutation matrix.
- **Replication Invariance** $P(X;z) = P(X^r;z)$ where X^r is an r-time replication of X.
- **Monotonicity** $P(X;z) \leq P(Y;z)$ whenever X is derived from Y by increasing any one attribute with respect to which a person is poor.
- **Subgroup consistency** For any n and m such that X_1 and Y_1 are nxm matrices and X_2 and Y_2 are lxm, with $X^T := [X_1^T.X_2^T]$ and $Y^T := [Y_1^T, Y_2^T]$, $P(X;z) > P(Y;z)$ whenever $P(X_1;z) > P(Y_1;z)$ and $P(X_2;z) = P(Y_2;z)$.
- **Strong Poverty Focus**. If any attribute such that $x_{ij} \geq z_j$ changes, $P(X;z)$ does not change. This property leads us to not only ignore *individuals above* the poverty minimum threshold in all relevant attributes, but

also *attributes above* the minimum level of individuals who do not achieve the minimum in other attributes. Alternatively, **Weak Poverty Focus** makes the poverty index independent of the attribute levels of non-poor individuals only (Bourguignon and Chakravarty, 2003). In other words, some interplay between attributes above and below the poverty threshold is allowed. Tsui does not consider this weaker version.

- **Ratio-Scale Invariance**[2] For any $X \in D$ and $z \in Z$, $P(X\Lambda; z\Lambda) = P(X; z)$ where $\Lambda := diag(\lambda)$ and $\lambda \geq 0$).

The above axioms will restrict the $G(.)$ to be increasing and continuous and the $f(.)$ to be continuous and non-increasing in attributes.

- **Poverty Criteria Invariance**. If $z \neq z'$ then $P(X; z) \leq P(Y; z) \Leftrightarrow P(X; z') \leq P(Y; z')$ whenever $X(z) = X(z')$ and $Y(z) = Y(z')$.

  This axiom ensures that there is no dramatic change in the evaluation of poverty for changes in the poverty threshold not affecting the number of poor. In other words, the ordering of distributions does not change, even if the measurement itself might change.

- **Poverty Non-increasing Minimal Transfer** with respect to a majorization criteria[3]. $P(Y; z) \leq P(X; z)$ where $Y = BX$ and B is a bistochastic matrix or Pigou-Dalton transfer matrix, and the transfer is among the poor. In order words, the poverty index must be sensitive to the dispersion of the attributes among the poor.

which restricts $f(.)$ to be convex

Define a "basic-rearrangements increasing transfer" as a transfer between individuals p and q such that the resulting distribution has the same attribute marginal distribution but higher correlation between them.

- **Poverty-Nondecreasing Rearrangement**. If Y is derived from X by a finite sequence of basic-rearrangements increasing transfers among the poor with no one becoming non-poor due to the transfer, then $P(X; z) \leq P(Y; z)$. In other words, more correlation between attributes among the poor increases (or leaves unchanged) the measurement of poverty.

The last axiom restricts $f(.)$ to be L-superadditive or, if differentiable, its cross-partial derivatives with respect to attributes must be non-negative (i.e. $\frac{\partial f^2}{\partial x_{il} \partial x_{im}} \geq 0$)

The resulting multidimensional poverty measures are

$$P_1(X; z) = \frac{1}{n} \sum_{i=1}^{n} \left[\prod_{j=1}^{m} \left(\frac{z_j}{min(x_{ij}, z_j)} \right)^{\delta_j} - 1 \right] \tag{5}$$

[2] Tsui presents also measures satisfying instead the Translation Invariance axiom

[3] This refers to distributional majorization criteria, multidimensional extensions of the Pigou-Dalton Principle, Uniform Majorization or Uniform Pigou-Dalton Majorization, see Kolm 1977

with $\delta_j \geq 0, j = 1, 2, ..., m$, and chosen to maintain convexity of the functions, and.

$$P_2(X; z) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m} \delta_j \ln \left[\frac{z_j}{min(x_{ij}, z_j)} \right] \tag{6}$$

with $\delta_j \geq 0, j = 1, 2, ..., m$

To better understand the difference between Tsui's poverty index and the traditional income poverty measure we disentangle the index into the implicit individual poverty or shortfall function, and the aggregator function across individuals (or poverty index).

The implicit individual poverty function:

$$p_i = \prod_{j=1}^{m} \left[\frac{z_j}{\min(x_{ij}; z_j)} \right]^{\delta_j} - 1 \tag{7}$$

or

$$p_i = \sum_{j=1}^{m} \delta_j \ln \left[\frac{z_j}{\min(x_{ij}; z_j)} \right] \tag{8}$$

Notice that $p_i = 0$ for those who are above the poverty line in *all* dimensions. We can think of δ_j as the contribution that the relative shortfall in attribute j makes to the individual poverty.

The implicit Poverty index is:

$$P(X; z) = \frac{1}{n} \sum_{i=1}^{n} p_i \tag{9}$$

In other words, the FGT version chosen is the poverty gap, which is the first moment of the discrete (empirical) distribution of p_i.

In a closely related paper, Bourguignon and Chakravarty (2003) impose similar axioms but two, and present a distinct family of multidimensional poverty indices. Their indices also fall into the union approach to poverty, but replace subgroup consistency with the separability axiom, and allow for correlation increasing transfers to have either an increasing or decreasing effect on the evaluation of poverty depending on the nature of the attributes involved. In other words, they accept both 'Poverty-Nondecreasing Rearrangement' and 'Poverty-Nonincreasing Rearrangement'. The resulting poverty index is of the following general form, similar to CES:

$$P_\theta(X; z) = \frac{1}{n} \sum_{i=1}^{n} \left[\sum_{j=1}^{m} w_j \left[\max\left(1 - \frac{x_{ij}}{z_j}; 0\right) \right]^{\theta} \right]^{\alpha/\theta} \tag{10}$$

Disentangling the components of (10), we observe that the implicit individual poverty function or 'shortfall from threshold levels' is:

$$p_i = \left[\sum_{j=1}^{m} w_j \left[\max\left(1 - \frac{x_{ij}}{z_j}; 0\right) \right]^{\theta} \right]^{1/\theta} \tag{11}$$

where parameters are set so that p_i is increasing and convex. w_j are positive weights attached to each j attribute, whereas θ sets the level of substitutability between shortfalls. The higher the θ , the lower the degree of substitutability. When θ tends to infinity relative deprivations are non-substitutes; when $\theta = 1$ shortfalls are perfect substitutes. Under both situations, poverty will be defined unidimensionally, in the first case by the attribute deprivation with the highest value, in the second, as a simple weighted sum of attributes. Note that the second option shares some resemblances with the standard income poverty approach whenever the weights are determined using market prices. Convexity of attributes - i.e. concavity in the space of deprivations - will restrict the parameter to be $\theta \geq 1$.

The implicit Poverty index is the α^{th} moment of the p_i distribution:

$$P(X; z) = \frac{1}{n} \sum_{i=1}^{n} (p_i)^{\alpha} = FGT_{\alpha} \tag{12}$$

The Bourguignon and Chakravarty proposal has the advantage of making explicit the role of the parameters involved in the measure, such as weights, substitution levels between attributes, and a parameter related to the weight to be attached to poverty gaps at different levels of the distribution. Interestingly, the effect of increasing correlation on the poverty index is dependent on the specific relative magnitude of the θ and α paramters. The poverty measure is also broader than Tsui's in allowing for a more general formulation of the "welfare function" (G) across individuals.[4]

2 An Information Theoretic Analysis of the aggregation functions and Poverty Measures

The issue of aggregation of attributes in many dimensions has an information theoretic interpretation and solution which reveals the information content of each poverty aggregator function. In the context of multidimensional measurement of inequality, Maasoumi (1986) proposed functionals for p_i ($f(.)$ in (4) above) which would summarize the information in all the attributes in an efficient manner. This "efficiency" refers to completeness of information being incorporated in any summary or aggregate function. As has been noted above, poverty measures are (moment) functions of the distribution of $p_i, i = 1, 2, ...n$. Every attribute j has a distribution as well, $x_j = (x_{1j}, x_{2j}, ...x_{nj})$. Naturally, the distribution of p_i is derived from , and follows the m distributions $x_j, j = 1, 2, ...m$. In objective, empirical science, the distribution of a variable contains all the information about that variable that is or can be accessed and inferred objectively. Given this truism, one must select functional forms for

[4]Bourguignon and Chakravarty also present an interesting case where θ depends on the poverty level, so that the substitution between shortfalls changes according to how far the individual is from the poverty line.

the aggregator functions p_i that would make its distribution the closest to the distributions of its constituent members, x_js. This ideal can be achieved by solving an information theory inverse problem, based on distributional divergences/distances, which produces 'optimal' functions for p_i.

The basic measure of 'divergence' between two distributions is the difference between their entropies, or the so called 'relative entropy'. Let S_i denote the "summary" or aggregate function for individual i, based on his/her m attributes $(x_{i1}, x_{i2}, ...x_{im})$. Then consider a weighted average of the 'relative entropy' divergences between $(S_1, S_2, ...S_n)$ and each $x_j = (x_{1j}, x_{2j}, ...x_{nj})$, as follows:

$$D_\theta(S, X; w) = \sum_{j=1}^{m} w_j \{ \sum_{i=1}^{n} S_i \big((S_i/x_{ij})^{-\theta} - 1 \big) / \theta(\theta - 1) \} \tag{13}$$

where w_js are the weights attached to the Generalized Entropy divergence from each attribute. Minimizing $D_\theta(.)$ with respect to S_i such that $\sum S_i = 1$, produces the following 'optimal' Information Theory (IT) aggregation functions:

$$S_i \quad \propto \quad \Big(\sum_{j}^{m} w_j x_{ij}^{\theta} \Big)^{1/\theta} \text{ when } \theta \neq 0 \tag{14}$$

$$S_i \quad \propto \quad \prod_j x_{ij}^{w_j} \text{ when } \theta = 0 \tag{15}$$

The function $D_\theta(.)$ is linear in the mutual divergences since it is merely a weighted sum or average. One could just as easily consider hyperbolic means of the mutual divergences. Also, the solution functions will be the same if we considered normalized attributes, such as x_{ij}/μ_j,where $\mu_j = E(x_j)$, or $x_{ij}/\sum_{i=1}^{n} x_{ij}$ which are the attribute "shares" (see Maasoumi (1986)). Note that the standard consumer theory requirement of convexity of indifference curves in the attribute space will demand θ to be less or equal to one. In the context of poverty indices, one might consider the relative deprivation functions, $q_{ij} = 1 - x_{ij}/z_j$, in place of x_{ij}. In this case, the convexity requirement is the opposite $\theta \geq 1$. See below for this alternative.

Will we show here that both Tsui and Bourguignon-Chakravarty indices can be included within one of two approaches to IT indices of poverty. And, as such, these satisfy the axioms advocated by them, as well as being based on aggregator functions which are 'information efficient' based either on the attribute quantity possessed or on relative poverty gaps ($q_{ij} = 1 - x_{ij}/z_j$). But the IT approach opens the way to more general measures of poverty, including more complex moments than the average/mean functions ($\frac{1}{n}\sum_{i=1}^{n}$) favored in the axiomatic approach.

Another point worth emphasizing is that the first version of IT indices are not limited to observing the "strong focus" axiom. This means that our indices can allow for substitution, that is compensation, from an attribute that exceeds its poverty level to another that falls short of it. The individual does not have to be poor in all dimensions to be either found to be poor or non-poor in the multidimensioned context. We think that Weak Focus is, indeed, a very attractive feature of multidimensional approach which deserves to be examined in many real life situations.

In the empirical part we compare these different approaches for the same data and case study, for a range of 'substitution parameters' and weights.

2.1 Aggregate Poverty Line Approach to IT Indices of Poverty.

Case A. Let us define an "aggregate poverty line" S_z that is consistent with the IT aggregator functions S_i derived above:

$$S_z = \Big(\sum_j^m w_j z_j^\theta\Big)^{\frac{1}{\theta}} \text{ when } \theta \neq 0 \tag{16}$$

and the generalized geometric mean for $\theta = 0$.

A two step approach is to:

1. Define the multi-attribute relative deprivation function as

$$p_i = \max\Big[(S_z - S_i)/S_z; 0\Big] = \max\Big[1 - S_i/S_z; 0\Big] \tag{17}$$

2. Define the following IT multi-attribute poverty measures:

$$P_\alpha(S; z) = \frac{1}{n}\sum_{i=1}^n \Big[\max(1 - S_i/S_z; 0)\Big]^\alpha = \frac{1}{n}\sum_{i=1}^n p_i^\alpha \tag{18}$$

   This is the α^{th} moment FGT poverty index based on the distribution of $S = (S_1, S_2, ...S_n)$.

Each attribute's poverty line, z_j plays a role in defining a multi-attribute poverty line, S_z, which incorporates the same weights for, and relationship between, the attributes as considered for each individual/unit. All of the axioms which support FGT are applied to individual summary functions of well being, S_i. All other univariate poverty indices are applicable to the summary distribution.

Notice that the above general formulation allows for the possibility of some substitution between attributes above and below the poverty thresholds provided the individual is poor in at least one dimension. This will be consistent with the Weak Poverty Focus axiom.

If, instead, one prefers to highlight the 'essentiality' of each component and support a Strong version of the focus axiom (*union approach*), one has only to replace x_{ij} by the expression $\min(x_{ij}, z_j)$ [5],[6]. In fact, when $\theta = 0$, and for $w_j = -\delta_j$, the implicit p_i in (17) is equivalent to Tsui's individual poverty function. In general, as we presented our measure is non-negative and normalized to be less than one. Tsui's P_1 index is also non-negative but unbounded. This has the disadvantage that the upper bound is dependent on values and units chosen for each poverty line z_j. One interpretation is that our IT measures include a normalized version of Tsui's when $\theta = 0$.

Case B. A similar but somewhat different version of this approach may also be considered. Consider following as described above, but without the consistent derivation of the S_z. Suppose a multidimensional poverty line is chosen directly from the distribution $S = (S_1, S_2, ...S_n)$, as though it were a target univariate distribution. Suitable candidates for this line would be the so called "relative" poverty lines, such as the lower quantiles, or a percentage of the median of the distribution. Indeed, this has been suggested by D'Ambrosio et al (2004), and Miceli (1999) who seems to have been the first to apply the Maasoumi (1986) approach to poverty, with application to the Swiss data.

2.2 Component Poverty Line Approach to IT Indices of Poverty

Consider obtaining summary functions of $q_{ij} = 1 - x_{ij}/z_j$ in place of x $_{ij}$. q_{ij} can be interpreted as 'shortfalls to threshold', as in Bourguignon and Chakravarty, where for poor persons$0 \leq q_{ij} \leq 1$ and 'rich' $q_{ij} \leq 0$. The optimal IT functionals will be the same as given above. Then the second two step IT indices of poverty are similarly derived as follows:

1. Let the relative deprivation function be

$$S_{q_i} = \Big[\sum_j^m w_j q_{ij}^{\theta} \Big]^{1/\theta} \text{for } \theta \neq 0 \text{ and for all } j, q_{ij} \geq 0 \text{ i.e. } x_{ij} \leq z_j \tag{19}$$

[5]An intersection approach to poverty could be also obtained if the sample is restricted to individuals with all attributes below their threshold

[6]To clarify the difference between Weak and Strong versions consider the individual poverty functions when only two attributes are included. For individuals who are poor in both dimensions, both the weak and the strong version would lead to

$$p_i = 1 - \left[\frac{x_{i1}}{z_1}\right]^{w_1} \left[\frac{x_{i2}}{z_2}\right]^{w_2}$$

But for persons who are poor only in one dimension - say, x_1 the weak version would be

$$p_i = max \left[1 - \left(\frac{x_{i1}}{z_1}\right)^{w_1} \left(\frac{x_{i2}}{z_2}\right)^{w_2}; 0 \right]$$

which is different from the strong version

$$p_i = \left(\frac{z_1}{x_{i1}}\right)^{-w_1} - 1$$

So that individual poverty function is

$$\begin{aligned} p_i &= \Big[\sum_{j}^{m} w_j q_{ij}^{\theta}\Big]^{1/\theta} \text{ for all } j, x_{ij} \leq z_j & (20) \\ &= \Big[\sum_{j}^{m} w_j \max(q_{ij};0)^{\theta}\Big]^{1/\theta} & (21) \end{aligned}$$

In other words, the 'Strong focus axiom' and 'union' definition of poverty are imposed. This step obtains an aggregate of relative deprivations which allocates weights to each, and allows trade offs between these relative deprivations in various attributes. Again, this is only for attributes that are below the poverty threshold. Weak Focus poverty axiom is not invoked in the second IT approach[7].

2. Define the multiattribute poverty measure

$$P_\alpha(S_q;z) = \frac{1}{n}\sum_{i=1}^{n}\Big(S_{qi}\Big)^{\alpha} \tag{22}$$

This is the α^{th} moment of the distribution of $S_q = (S_{q1}, S_{q2}, ...S_{qn})$.

Here there is no explicit 'aggregate poverty line'. To be explicit, the second IT approach index for two dimensions, and for someone who is poor in both dimensions is as follows:

$$P_\alpha(S;z) = \frac{1}{n}\sum_{i=1}^{n}\left[w_1\Big(1-(x_{i1}/z_1)\Big)^{\theta} + w_2\Big(1-(x_{i2}/z_2)\Big)^{\theta}\right]^{\alpha/\theta} \tag{23}$$

which is the same as Bourguignon-Chackavarty poverty index.

3 Empirical Results

This section presents an application of the proposed poverty measures to data from Indonesia. The exercise highlights the inevitability of making value judgements when comparing any two multivariate distributions.

We compare three-dimensional distributions of Indonesians' expenditure, health status, and level of education for three different regions. These are Java, Sumatra, and 'Other' regions, which contain 60%, 20% and 20% of the total Indonesian population, respectively. The exercise is meant to be merely illustrative and, for this reason, we choose to represent well-being by only three

[7] The reason why Weak Focus cannot be invoked by the second approach is that $q_{ij} < 0$ when the individual possesses more than the poverty line level of that attribute. For even θ this implies that the farther away (richer) the person is the higher his value of q_{ij}, that is,his 'deprivation'. Clearly, an undesirable property

attributes. Naturally, results can be extended to more dimensions. The choice of dimensions was made given the wide agreement on their fundamental role as both means and ends - particularly in the case of education and health (Anand and Sen, 2000).

Data comes from the 2000 Indonesian Family Life Survey (IFLS) conducted by RAND, UCLA and the Demographic Institute of the University of Indonesia. The IFLS is a continuing longitudinal socioeconomic and health survey, representing 83% of the Indonesian population living in 13 provinces (out of 26). It collects data on individual respondents, their families, their households, the communities in which they live, and the health and education facilities they use (Strauss, 2004). The IFLS was previously conducted in 1993, 1997, and 1998, but data on health status is publicly available only for 2000.

Approximately 10,400 households and 39,000 individuals were interviewed in 2000. We will restrict the study to individuals with complete information on all relevant variables, omitting just over 1% of the sample.

The indicators used are *real per capita expenditure, level of hemoglobin (Hb), and years of education achieved by the head of household*. Nominal per capita expenditure data is adjusted using a temporal deflator (Tornquist CPI, base year Dec 2000) and a spatial deflator (regional poverty lines) (Strauss, 2004). Individuals' hemoglobin levels are expressed in grams per deciliter (g/dl). Low levels of hemoglobin indicate deficiency of iron in the blood where '...[i]ron deficiency is thought to be the most common nutritional deficiency in the world today" (Thomas et al, 2003, p. 4) [8]. Given that normal values of Hb depend on sex and age, we adjusted individual values to transform them into equivalent adult levels [9].

3.1 Poverty measurements

Computing poverty involves choosing a cut-off point for each indicator. To allow for sensitivity to different poverty lines we use two values representing reasonable boundaries for alternative thresholds. These can be also be related to poverty and extreme poverty lines, as in the traditional poverty literature. In particular, for per capita expenditure we utilise Strauss (2004)'s values of Rp. 100,000 and Rp. 150,000, respectively [10]; for hemoglobin 12 g/dl and 13 g/dl

[8] Low levels of Hb are linked to susceptibility to diseases, fatigue, and lower levels of productivity. Reflects the combination of a diet that is high in animal proteins (primary source of iron) and greater absorption capacity (which is reduced by disease insults, presence of worms, loss of blood and diets high on rice). More generally, low levels related to iron deficiency. See WHO (2001) and Thomas (2001)

[9] We use threshold values from WHO report (2001) to compute the table of equivalence (Table 6, chapter 7). Normal levels of hemoglobin also vary with long exposure to altitudes - which we ignore for our calculations but given our sample of Indonesia in this survey it shouldn't be problematic. Also, studies show that in US individuals from African extractions tend to have normal lower values. A thorough assessment of anemia for Indonesian population should consider both these issues.

[10] see Strauss, 2004, chapter 3. In December 2000, the exchange rate for the Rupiah was Rp.9,480 / 1 US dollar.

[11]; and for education 4 and 6 years of schooling.

Table 1 presents measurements of poverty for each attribute, using the FGT index for values of $\alpha = 0, 1, 2$ [12]. With few exceptions, the poverty levels in expenditure and education are invariant to the version of FGT used, with the highest poverty in the Other regions, followed by Java and then Sumatra. In the case of the health indicator, on the other hand, Sumatra reports the highest values of poverty whereas Others the lowest.

[T1]

Employing multidimensional poverty indices involves, necessarily, a significant loss of information. Depending on how the aggregation is done – in terms of functional form, indicator variables, and parameter values – the results will vary in terms of cardinal values and, in some cases, the ordinal rankings of the distributions. Table 2 shows the resulting measurements using the two approaches presented in the previous section and alternative values for the parameters. We utilize two weighting schemes (equal weighting and giving half the importance to expenditure), and distinct values for the substitution level θ (from -3 to 1 in the first approach and from 0 to 3 in the second)[13]. As in the previous table, we use the three standard α values of FGT measures.

[T2]

The shading of cells indicates the ranking of the distributions, with the darkest being the highest poverty level in each combination of index and parameters.

We first compare the results with those obtained from the univariate poverty analysis. Java ranks second in each of the unidimensional measures. However, when aggregating the different dimensions there appears to be some compensation between attributes such that Java is poorest by all measures. In other words, Java has the highest level of multidimensional poverty and extreme poverty for all combinations of weights and parameter values calculated here.

Comparison of Sumatra and the Other regions is less straightforward. All the poverty headcount measures suggest that Sumatra is poorer than the Other regions. This is true for all poverty lines, IT approaches, weighting schemes and values of substitution parameters between attributes. However, once we move to poverty measures that are sensitive to the distribution among the poor, the ranking becomes ambiguous.

Using the strong version of the first IT poverty approach, the order between these regions will depend on the value chosen for the substitution level between attributes, θ. For higher values of θ, Sumatra presents higher poverty than the Other regions, irrespective of the value of α chosen. The opposite is true for negative values of θ.

In contrast, in the weak version of the first approach and the second IT approach, the level of substitution between attributes will not affect the ranking between Sumatra and the Others. It will depend exclusively on the specific α chosen. If we ignore the distribution of relative deprivations between the

[11] From the WHO report, a male adult is considered anemic, possibly suffering from iron deficiency, if his Hb is below 13 g/dl

[12] In the Annex we include a table with basic statistics for variables employed.

[13] This is to comply with the convexity requirement in the space of attributes

Table 1. Univariate poverty measurement by regions.
Indonesia, 2000.

	JAVA			SUMATRA			REST		
POVERTY	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
Expenditure	0.333	0.100	0.042	0.311	0.093	0.040	0.372	0.119	0.051
Hemoglobin	0.251	0.024	0.005	0.298	0.031	0.006	0.232	0.023	0.005
Education	0.388	0.262	0.202	0.378	0.220	0.154	0.493	0.346	0.273
EXTREME POVERTY									
Expenditure	0.139	0.032	0.012	0.127	0.032	0.013	0.166	0.041	0.015
Hemoglobin	0.111	0.011	0.003	0.140	0.015	0.004	0.098	0.011	0.003
Education	0.309	0.207	0.157	0.259	0.154	0.108	0.402	0.281	0.217

Source: authors' calculations

Table 2. Multivariate poverty measurement by regions.
Indonesia, 2000.

POVERTY	Java			Sumatra			Others		
IT - first approach WEAK FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	59.84	33.71	25.64	34.99	16.90	11.62	27.94	17.45	13.62
θ = - 1	59.43	28.92	20.09	34.84	13.94	8.616	27.81	15.09	10.80
θ = 0	58.72	18.11	8.12	34.17	8.68	3.46	27.32	9.45	4.39
θ = 1/3	57.97	13.18	4.23	33.57	6.49	1.89	27.01	6.83	2.27
θ = 1	54.32	5.80	0.88	31.34	3.17	0.479	25.46	2.91	0.46
weights {1/2, 1/4, 1/4}									
θ = - 3	60.86	33.28	25.19	35.22	16.59	11.34	28.23	17.25	13.40
θ = - 1	59.93	26.90	17.94	34.75	12.73	7.520	27.93	14.10	9.69
θ = 0	58.61	14.68	5.44	33.61	6.97	2.30	27.21	7.69	2.95
θ = 1/3	57.64	10.06	2.50	33.05	4.93	1.12	26.91	5.24	1.35
θ = 1	53.31	4.21	0.48	30.20	2.30	0.267	25.05	2.13	0.25
IT - first approach STRONG FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	83.80	33.91	25.67	51.46	17.06	11.64	36.34	17.52	13.63
θ = - 1	83.80	29.65	20.30	51.46	14.49	8.75	36.34	15.38	10.89
θ = 0	63.91	8.49	2.42	44.58	5.82	1.56	25.20	3.87	1.15
θ = 1/3	63.91	7.03	1.55	44.58	4.88	1.03	25.20	3.16	0.73
θ = 1	83.80	9.56	1.82	51.46	5.06	0.87	36.34	4.76	0.95
weights {1/2, 1/4, 1/4}									
θ = - 3	83.80	33.52	25.22	51.46	16.78	11.37	36.34	17.34	13.41
θ = - 1	83.80	27.68	18.15	51.46	13.31	7.65	36.34	14.41	9.78
θ = 0	63.91	6.95	1.58	44.58	4.73	1.02	25.20	3.16	0.75
θ = 1/3	63.91	5.59	0.94	44.58	3.84	0.62	25.20	2.50	0.44
θ = 1	83.80	7.31	1.03	51.46	3.89	0.50	36.34	3.63	0.54
IT - second approach STRONG FOCUS (also BC)									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = 1	83.80	12.91	3.33	51.46	6.83	1.60	36.34	6.50	1.74
θ = 2	83.80	21.24	9.00	51.46	11.07	4.21	36.34	10.73	4.74
θ = 3	83.80	25.41	12.90	51.46	13.21	6.01	36.34	12.84	6.80
weights {1/2, 1/4, 1/4}									
θ = 1	83.80	10.15	2.01	51.46	5.39	0.98	36.34	5.10	1.06
θ = 2	83.80	18.61	6.83	51.46	9.71	3.20	36.34	9.39	3.60
θ = 3	83.80	23.25	10.73	51.46	12.09	5.00	36.34	11.74	5.65

Table 2. Multivariate poverty measurement by regions. (cont)
Indonesia, 2000.

EXTREME POVERTY	Java			Sumatra			Others		
IT - first approach WEAK FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	41.25	26.88	20.28	21.27	12.05	8.37	20.96	14.32	11.00
θ = - 1	41.25	23.27	16.16	21.33	10.11	6.41	20.96	12.50	8.83
θ = 0	39.93	12.60	5.17	20.57	5.36	2.02	20.54	6.83	2.84
θ = 1/3	37.52	7.76	2.04	19.07	3.36	0.84	19.53	4.20	1.12
θ = 1	24.89	1.64	0.18	12.68	0.88	0.10	13.15	0.86	0.10
weights {1/2, 1/4, 1/4}									
θ = - 3	41.29	26.74	20.13	21.26	11.96	8.29	20.98	14.25	10.93
θ = - 1	41.26	21.84	14.59	21.32	9.36	5.70	20.97	11.77	7.99
θ = 0	38.93	9.88	3.26	19.75	4.15	1.27	20.08	5.38	1.81
θ = 1/3	36.21	5.52	1.07	18.36	2.37	0.45	18.87	3.01	0.60
θ = 1	21.35	1.01	0.08	11.07	0.57	0.05	11.44	0.54	0.05
IT - first approach STRONG FOCUS									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = - 3	57.76	25.92	19.03	32.96	11.54	7.73	26.61	13.82	10.35
θ = - 1	57.76	23.89	16.39	32.96	10.55	6.55	26.61	12.74	8.94
θ = 0	37.87	4.72	1.16	26.08	3.07	0.71	15.47	2.22	0.56
θ = 1/3	37.87	2.41	0.26	26.08	1.62	0.17	15.47	1.11	0.13
θ = 1	57.76	5.88	0.93	32.96	2.81	0.41	26.61	3.06	0.51
weights {1/2, 1/4, 1/4}									
θ = - 3	57.76	26.83	20.15	32.96	12.04	8.30	26.61	14.29	10.94
θ = - 1	57.76	26.35	19.51	32.96	11.79	7.98	26.61	14.03	10.60
θ = 0	37.87	3.77	0.74	26.08	2.45	0.45	15.47	1.78	0.36
θ = 1/3	37.87	2.81	0.37	26.08	1.86	0.24	15.47	1.30	0.18
θ = 1	37.87	3.17	0.47	26.08	2.13	0.31	15.47	1.46	0.23
IT - second approach STRONG FOCUS (also BC)									
	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2	α = 0	α = 1	α = 2
weights equal									
θ = 1	57.76	9.61	2.40	32.96	4.53	1.04	26.61	5.04	1.30
θ = 2	57.76	16.31	6.91	32.96	7.61	2.91	26.61	8.56	3.74
θ = 3	57.76	19.57	9.95	32.96	9.12	4.18	26.61	10.27	5.38
weights {1/2, 1/4, 1/4}									
θ = 1	57.76	7.39	1.41	32.96	3.51	0.61	26.61	3.87	0.76
θ = 2	57.76	14.24	5.24	32.96	6.66	2.21	26.61	7.47	2.83
θ = 3	57.76	17.87	8.27	32.96	8.33	3.48	26.61	9.37	4.47

poor ($\alpha = 0$) Sumatra has higher poverty measurement than Java. Once we incorporate some sensitivity to the disparities between the poor, Other has higher poverty values than Sumatra.

All these results are robust to the two weighting strategies employed here. We expect that only very extreme a priori weight systems may produce results that are closer to the unidimensional poverty values. As expected, the measured poverty rates increase as the substitutability between attributes decreases. At the extreme, when there is no substitution, multidimensional poverty rates will equal the unidimensional poverty rate for the component of the index with the highest poverty. For all Indonesian regions this is education. Recall that higher substitution between attributes corresponds to *high* values of θ in the first IT approach and to *low* values of θ in the component poverty line approach (based on shortfalls). Finally, within the Aggregate Poverty Line approach we can observe the implications of using the Weak versus the Strong Poverty Focus Axiom. In our data, poverty rates are sensitive to this choice, but the ranking of regions is not affected. As expected, for each combination of (w_j, θ, α) the Weak Poverty Focus Axiom yields lower measurements the the strong version. This is due to the fact that the former allows for some degree of substitution (compensation) between attributes for those who are poor in one dimension and not in some other such that they end up being above the multidimensional poverty threshold. This example shows that employing the Weak Poverty Focus Axiom can be seen as intermediate case between union and intersection approaches.

4 Conclusions

We have presented the Information Theory approach to multidimensional poverty measurement in a connected way that allows both new measures and a deeper interpretation of the existing methods, primarily based on the axiomatic approaches. The IT approach emphasizes clarity in aggregation choices that, it is argued, are inevitable in any multidimensional setting. The univariate methods are not exempt from this. By making aggregation issues explicit, the IT methods are also able to reveal the meaning and the working of the multidimensional context when one allows 'compensation' to an individual/household from the above threshold attributes for those attributes that fall short. We feel it is essential to have an accommodation for this possibility since, otherwise, the case for a 'multidimensional' approach to poverty and welfare may not exceed far beyond adding up, or averaging, over many dimensions. Future work will focus on differential substitution levels between individual categories, and attribute levels. These nonlinearities require deeper and careful analysis in each case study and empirical setting.

We have shown where, and under which conditions, our IT measures are identical to the index families proposed earlier in the literature, and have new IT indices when some of those conditions are relaxed. The Indonesian case study brings out some of these issues, but not all. The CDF graphs are merely indicative (but not statistically definitive) of a great degree of robustness in our ranking of poverty status of different regions of the country at a particular point of time. Nevertheless, some degree of fragility of numerical conclusions was observed relative to the degree of substitution between attributes, and 'inequality aversion' within the group classified as poor, as well as allowance for compensation from higher-than-threshold attributes. The size of the group which is not poor in all dimensions deserves a deeper examination and may itself characterize economies and societies in meaningful ways. We defer these issues to future research.

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6 Annex

The following presents basic summary statistics and the figures show their respective distribution, using Kernel approximation.

Table A. 1. Summary Statistics by regions.
Indonesia, 2000.

Variable	Obs	Mean	Std. Dev.	Min	Max
Java					
Real per capita expenditure	20174	284,930	335,997	20,348	19,500,000
Hemoglobin (g/dl)	20174	13.9	1.7	2.8	25.8
Education of head of hh	20110	6.4	4.7	0.0	22.0
Sumatra					
Real per capita expenditure	7213	276,867	326,213	10,409	7,058,715
Hemoglobin (g/dl)	7213	13.7	1.8	3.2	22.3
Education of head of hh	7191	6.6	4.2	0.0	19.0
Others					
Real per capita expenditure	7280	245,122	255,346	21,833	4,395,996
Hemoglobin (g/dl)	7280	14.0	1.7	2.7	35.7
Education of head of hh	7249	5.6	4.8	0.0	19.0

Source: authors' calculation from IFLS 2000.

Pearson Correlation Coefficients (sign 0.05)

Sumatra

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0675*	1.0000	
education	0.2112*	0.0900*	1.0000

Java

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0719*	1.0000	
education	0.3296*	0.0870*	1.0000

Rest

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0566*	1.0000	
education	0.3263*	0.0609*	1.0000

Spearman Correlation Coefficients (sign 0.05)

Sumatra

	expenditure	hemoglobi	education
expenditur	1.0000		
hemoglobi	0.1236*	1.0000	
education	0.3711*	0.0929*	1.0000

Java

	expenditure	hemoglobi	education
expenditur	1.0000		
hemoglobi	0.1035*	1.0000	
education	0.4195*	0.0894*	1.0000

Rest

	expenditure	hemoglobi	education
expenditur	1.0000		
hemoglobi	0.0676*	1.0000	
education	0.4015*	0.0572*	1.0000

Kendall Correlation Coefficients (sign 0.05)

Sumatra

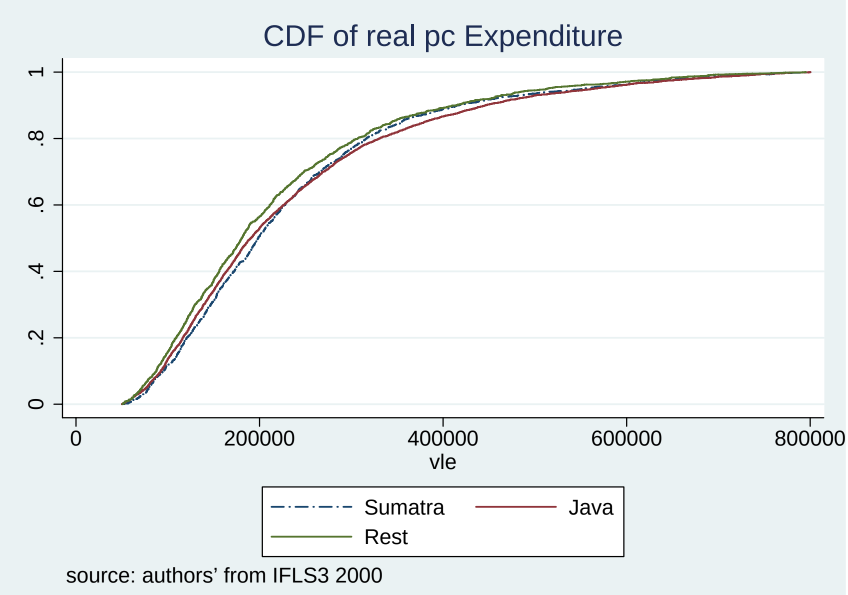
	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0825*	1.0000	
education	0.2497*	0.0612*	1.0000

Java

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0689*	1.0000	
education	0.2831*	0.0589*	1.0000

Rest

	expenditure	hemoglobin	education
expenditure	1.0000		
hemoglobin	0.0451*	1.0000	
education	0.2719*	0.0378*	1.0000



CDF of real pc Expenditure
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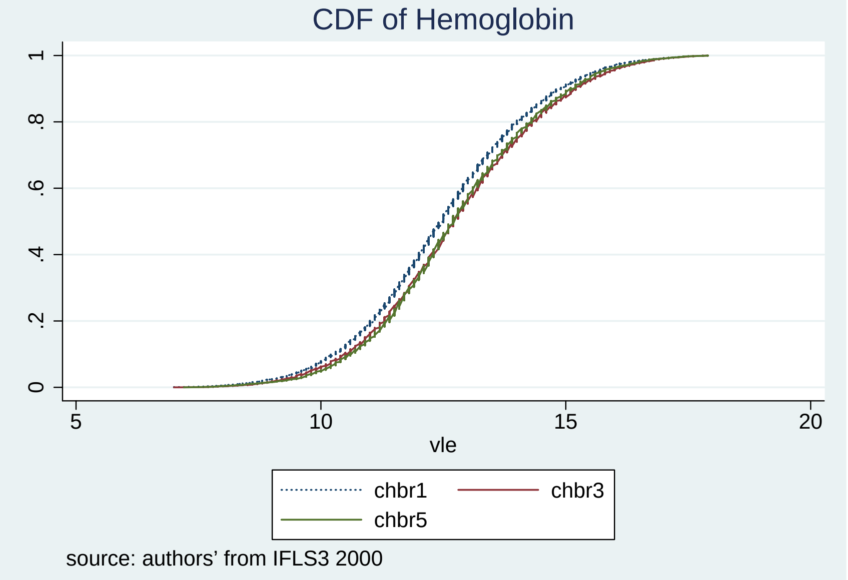
Rest
source: authors' from IFLS3 2000



CDF of Hemoglobin
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chbr3
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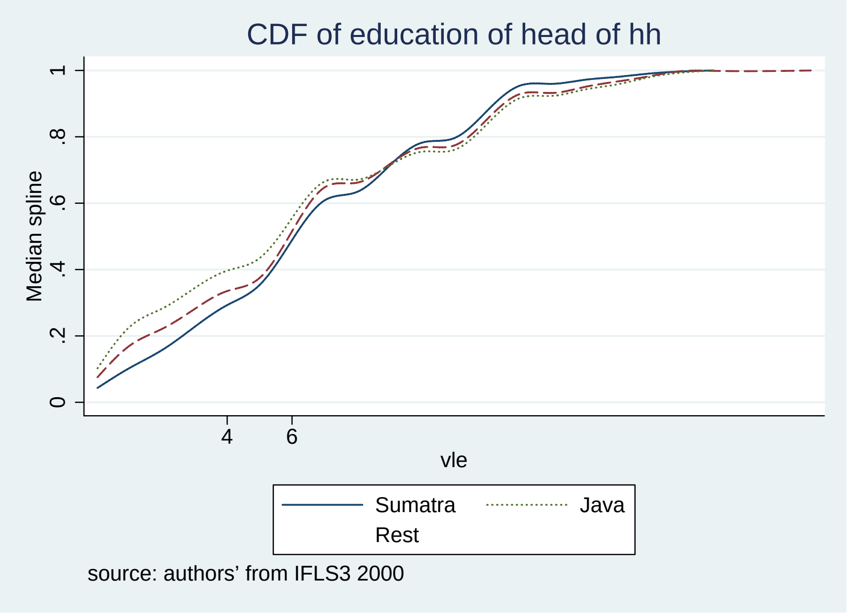
source: authors' from IFLS3 2000



CDF of education of head of hh
Median spline
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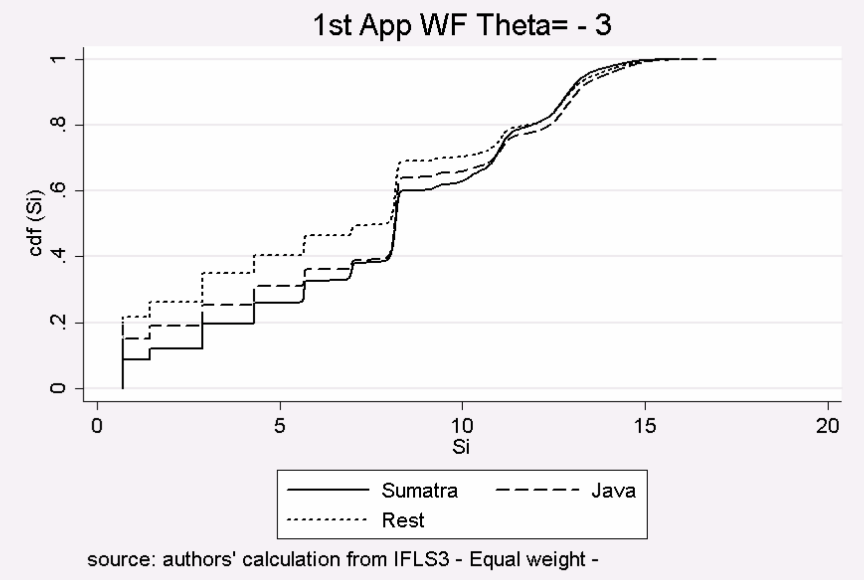
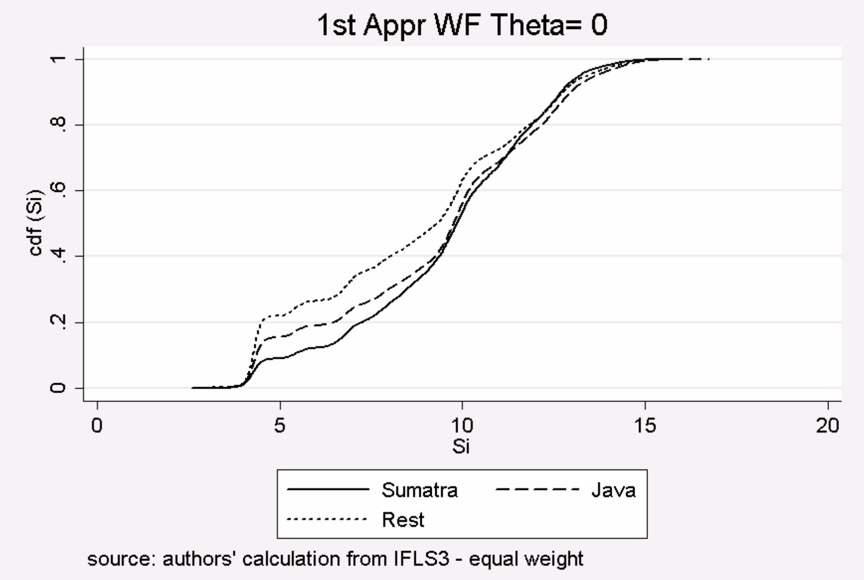
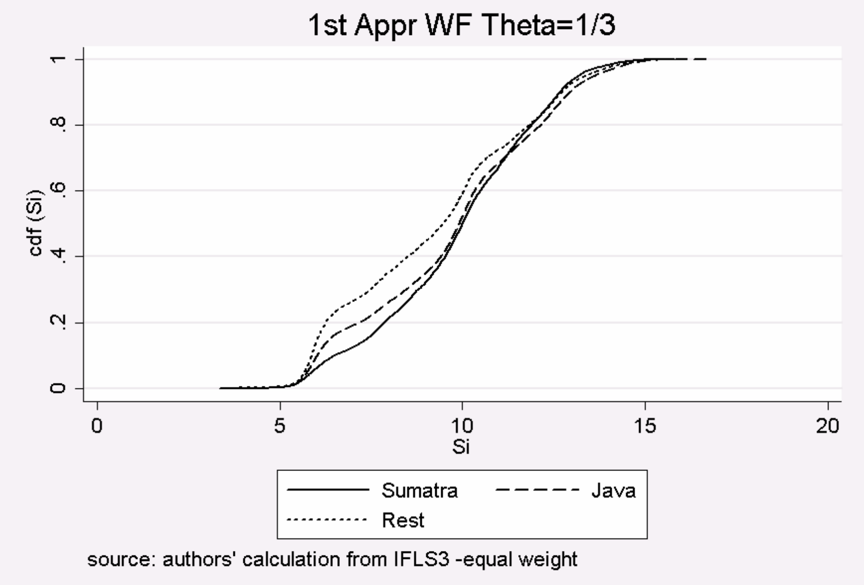
vle
Sumatra
Java
Rest
source: authors' from IFLS3 2000

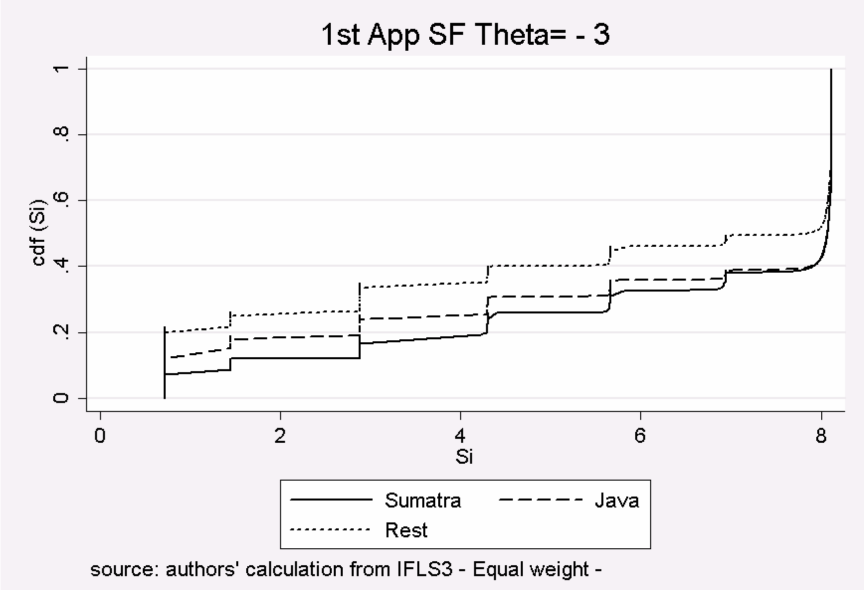
First Approach (Weak Focus)
Equal weight

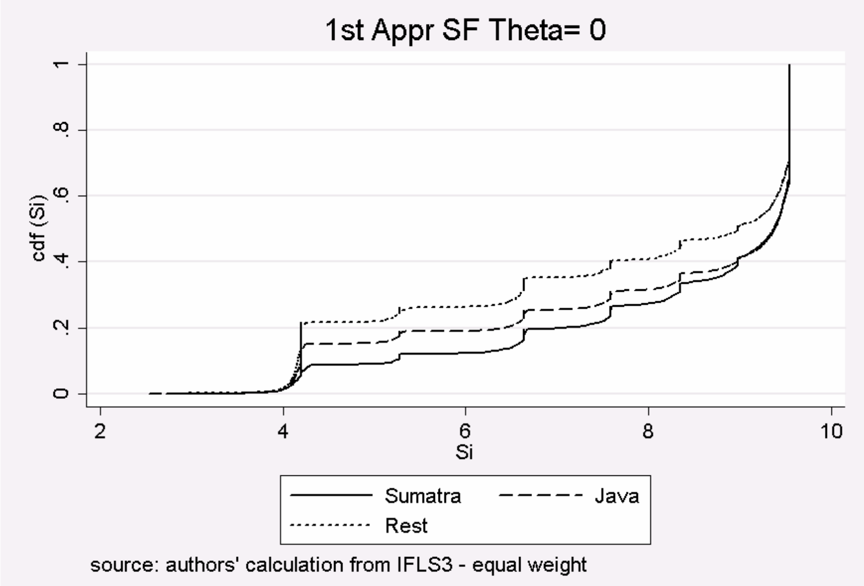


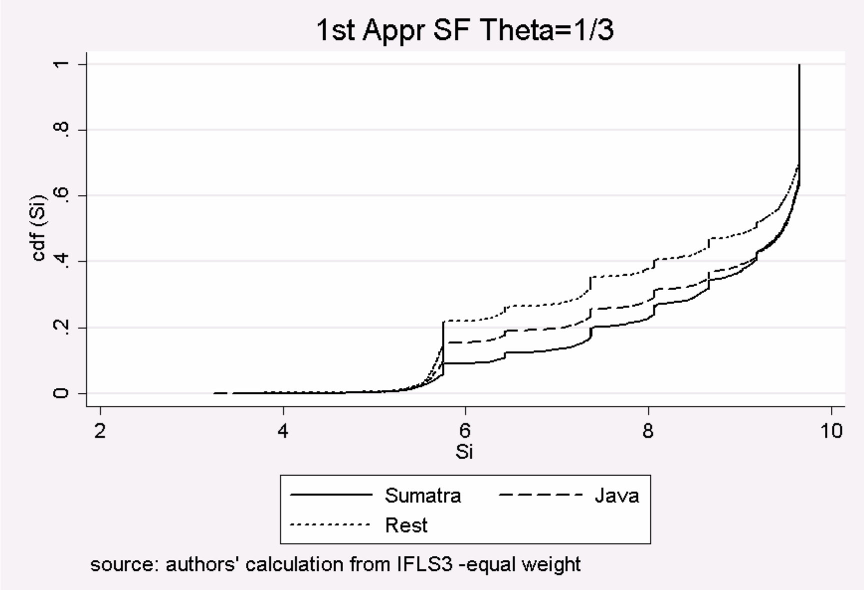




First Approach (Strong Focus)
Equal weight







Second Approach (Strong Focus)
Equal weight