Performance of MGCFA and (M)ANOVA in small samples under full and partial measurement invariance

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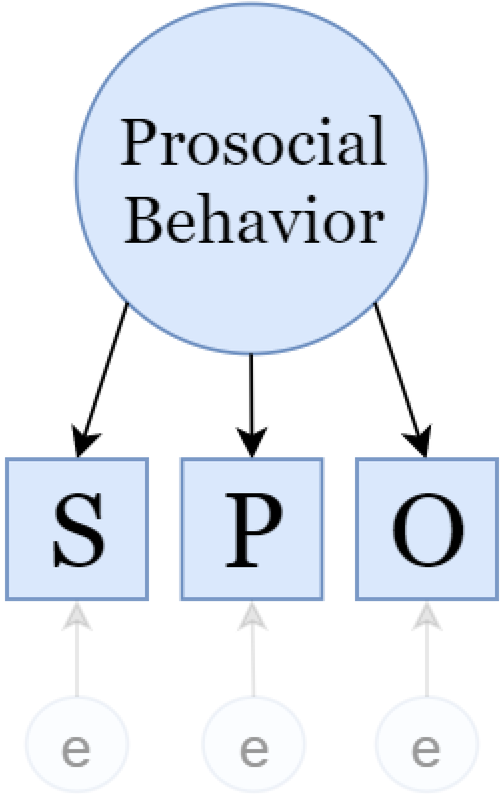
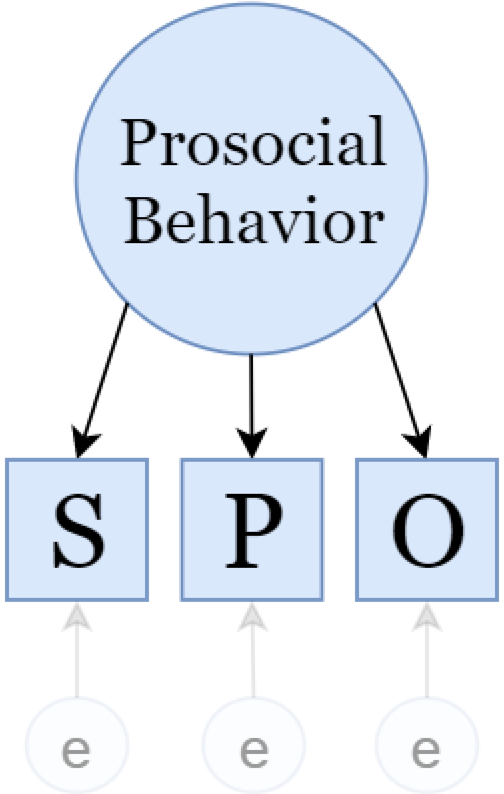
Constructs



1. Substantive → define and conceptualize construct
2. Structural → investigate psychometric properties
3. External → check convergence, divergence, prediction

Martí-Vilar et al. (2019)
Hussey et al. (2020)

Constructs



1. Substantive → define and conceptualize construct

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Martí-Vilar et al. (2019)
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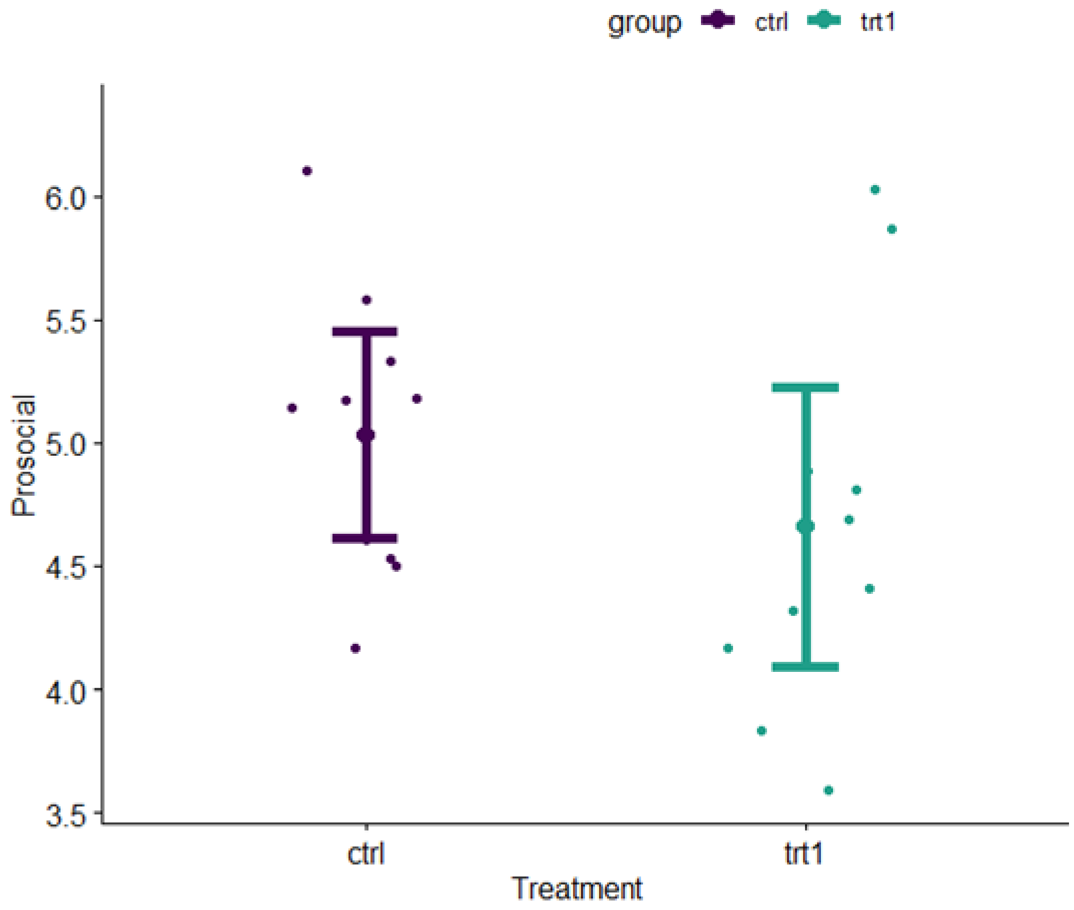
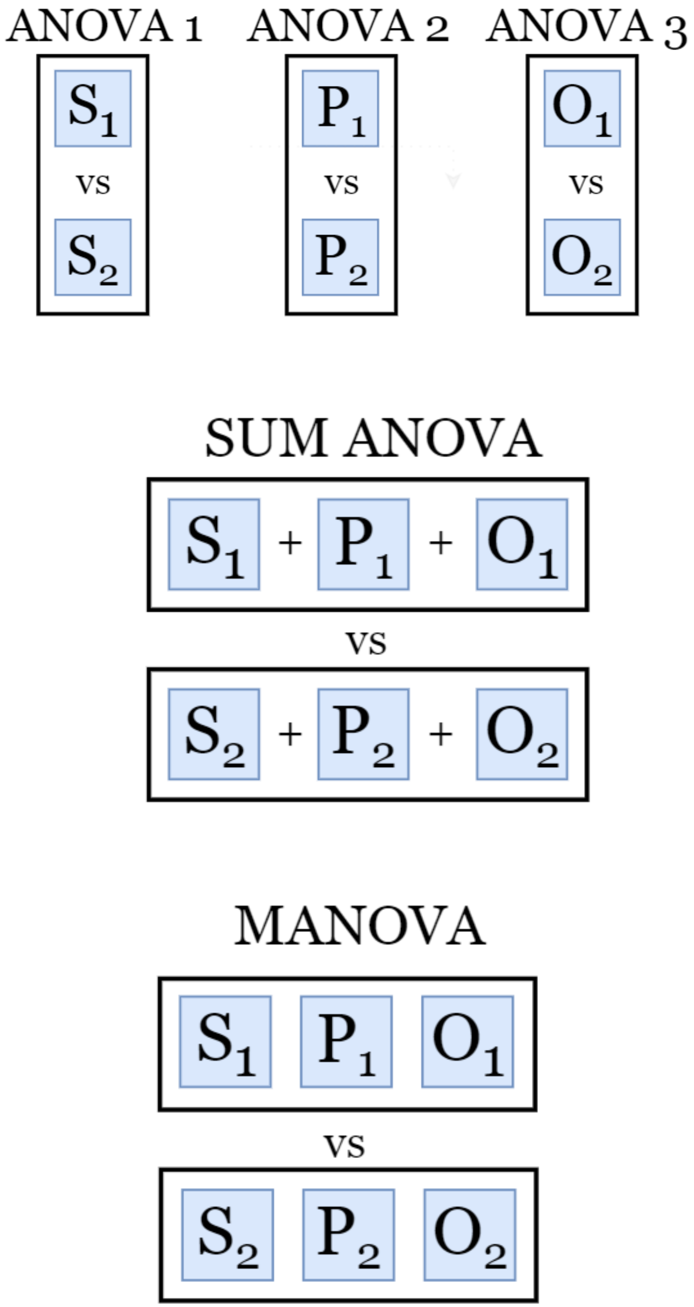
(M)ANOVA



group ctrl trt1
6.0
5.5
5.0
4.5
4.0
3.5
Prosocial
ctrl
trt1
Treatment



ANOVA 1
ANOVA 2
ANOVA 3
S1 vs S2
P1 vs P2
O1 vs O2
SUM ANOVA
S1 + P1 + O1
vs
S2 + P2 + O2
MANOVA
S1 P1 O1
vs
S2 P2 O2

(M)ANOVA

Absence of measurement error

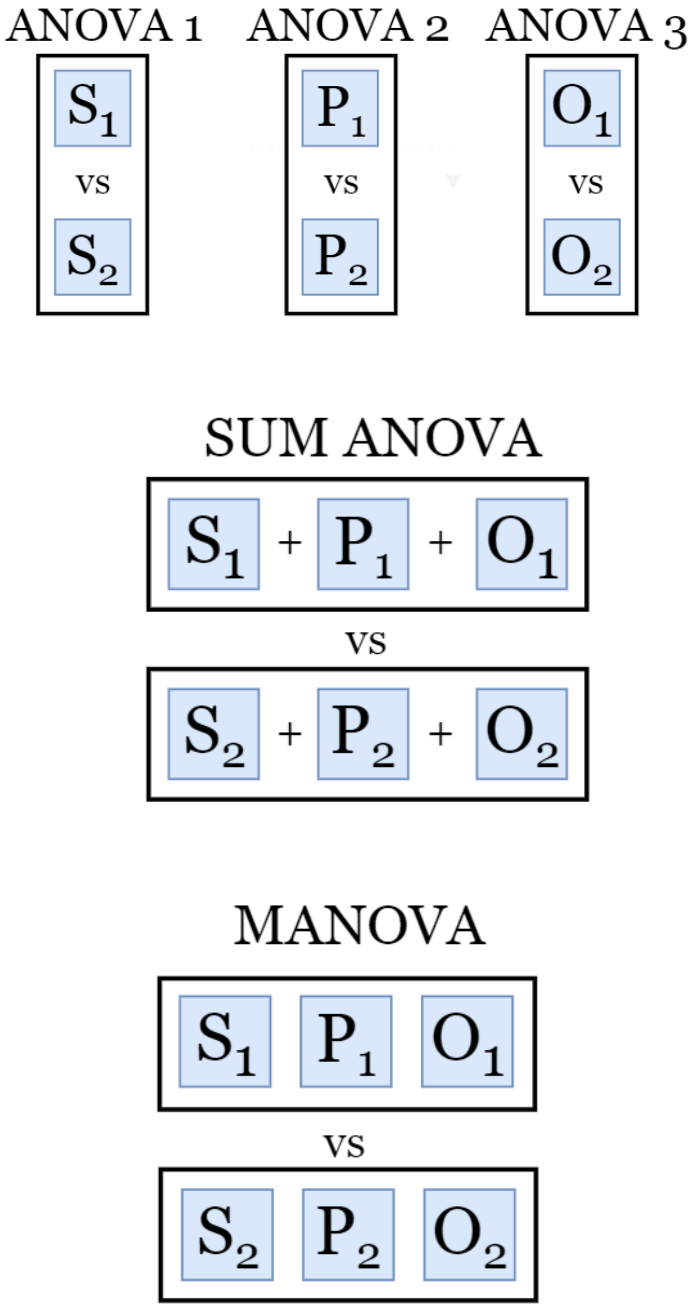
Multiple comparison problem → Type I errors

No measurement model

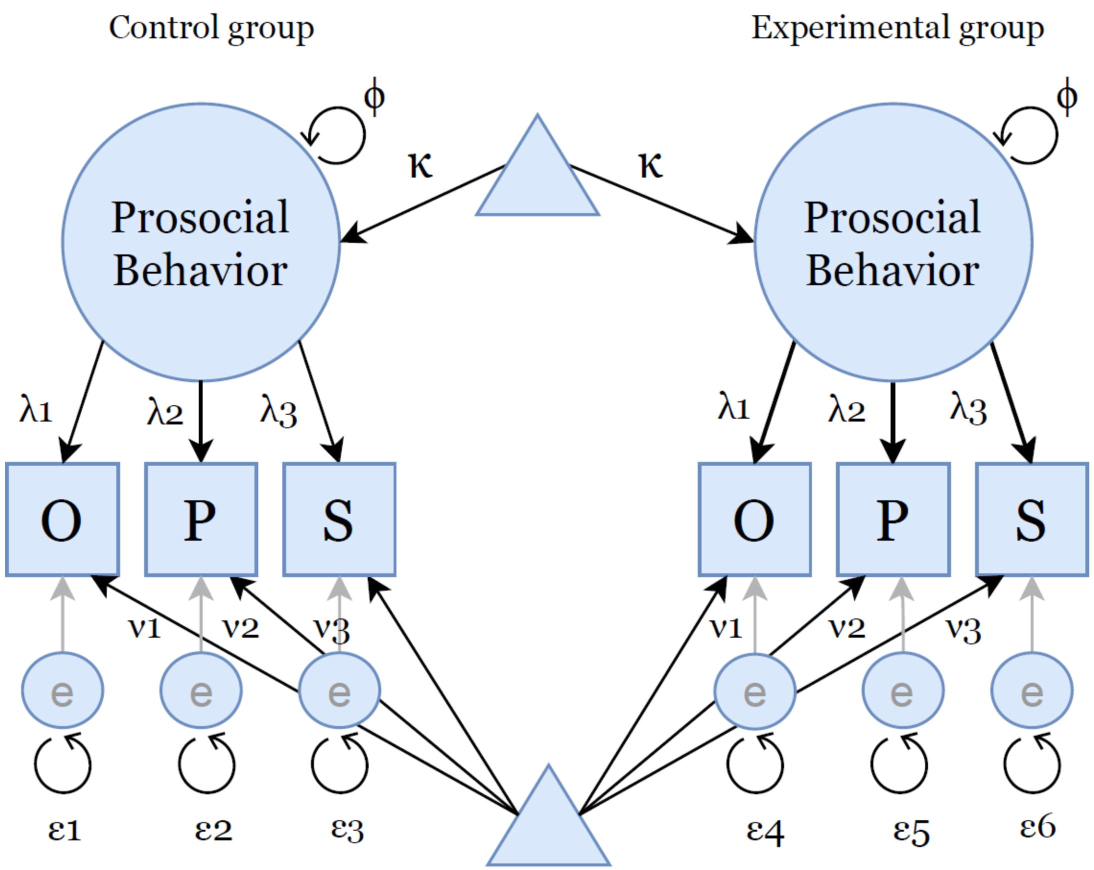
→ Used for constructs

→ Often many different constructs

→ Psychometric properties constructs not checked



Measurement Invariance

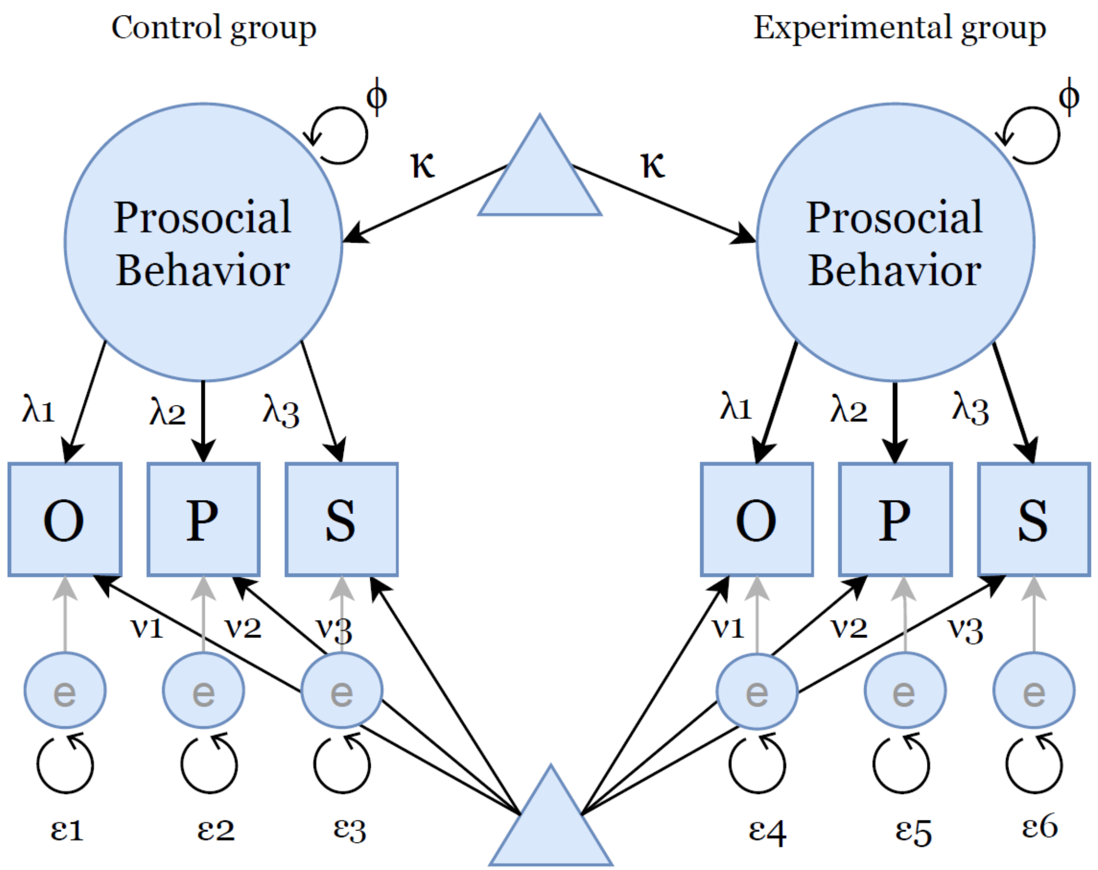


E(Y|η,control) = E(Y|η,exp) = E(Y|η)

Levels of invariance:

1. Configural
2. Metric (loadings; weak)
3. Scalar (intercepts; strong)

Measurement Invariance



E(Y|η,control) = E(Y|η,exp) = E(Y|η)

Levels of invariance:
1. Configural ✅
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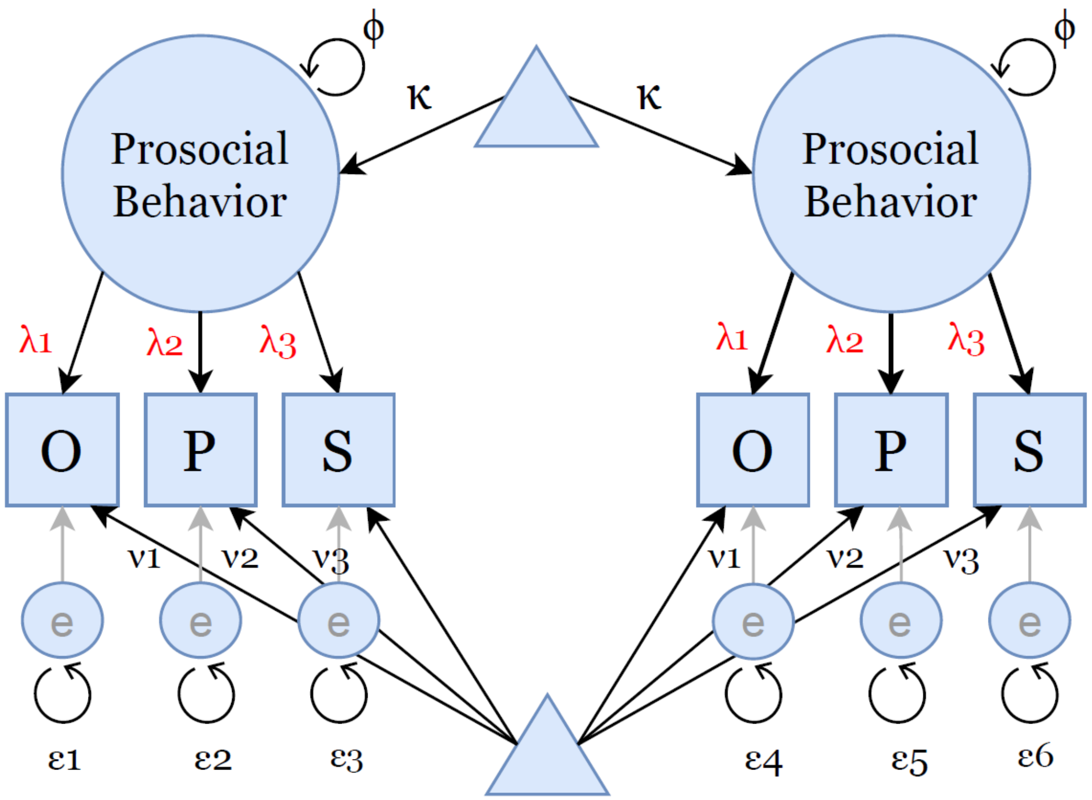
Measurement Invariance



E(Y|η,control) = E(Y|η,exp) = E(Y|η)

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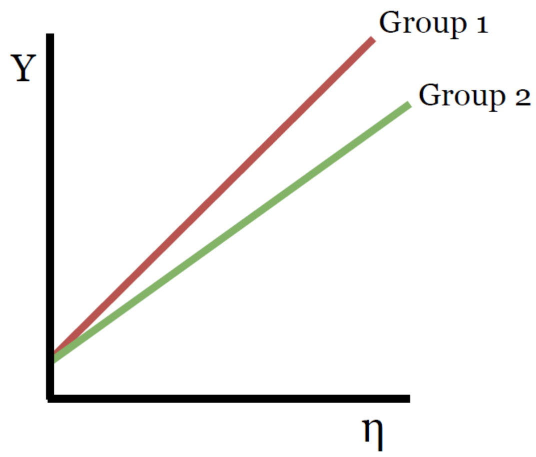
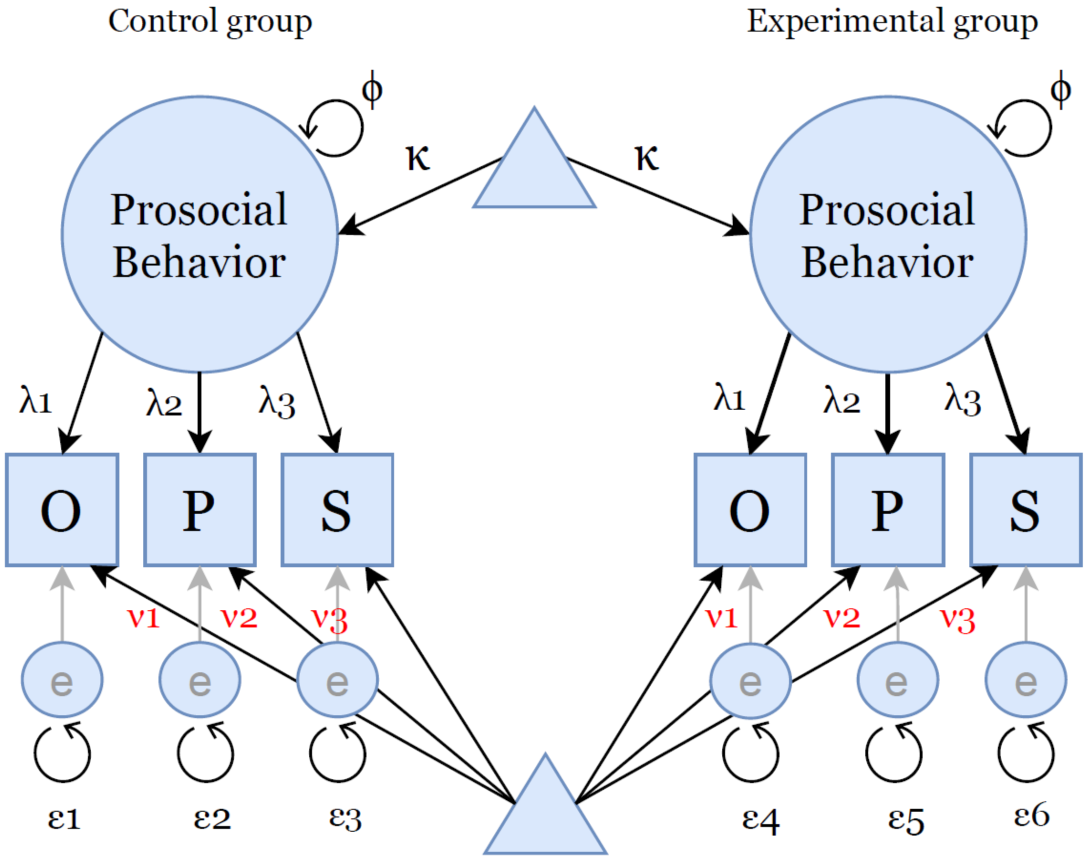


Measurement Invariance



E(Y|η,control) = E(Y|η,exp) = E(Y|η)

Levels of invariance:

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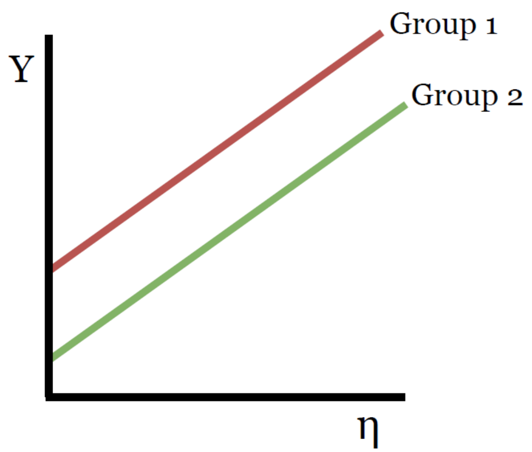
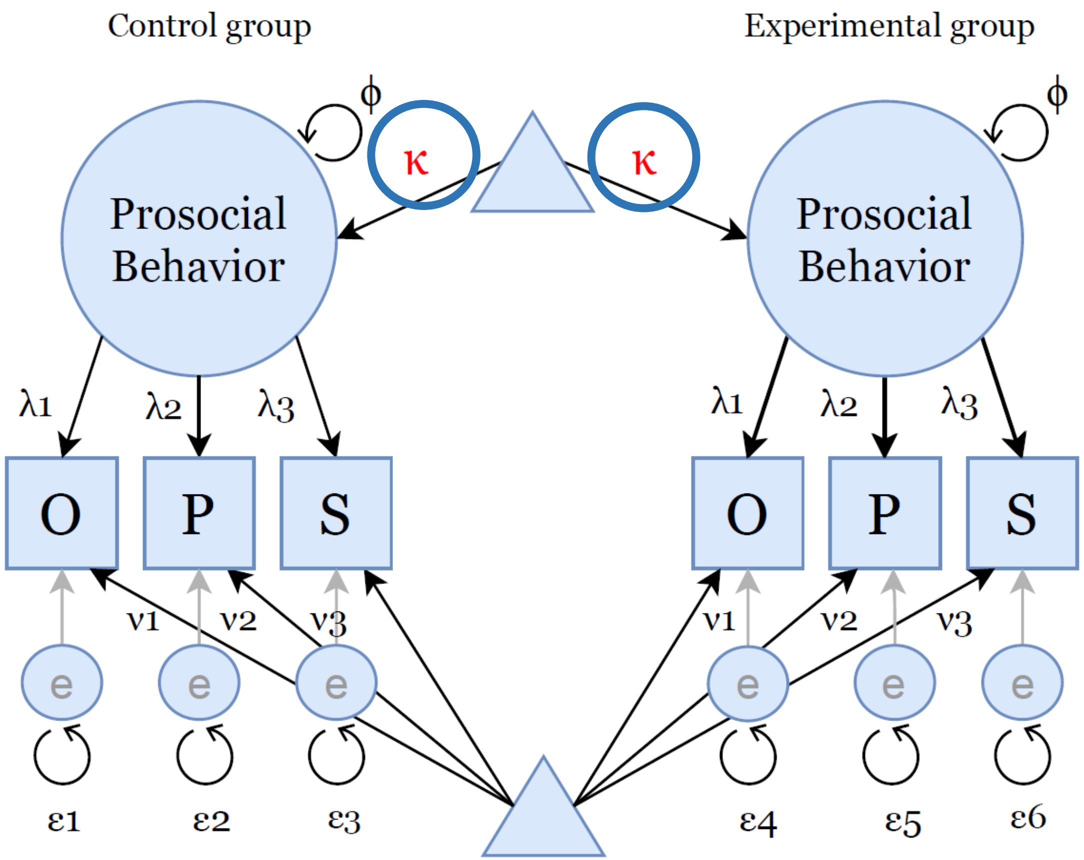


Measurement Invariance

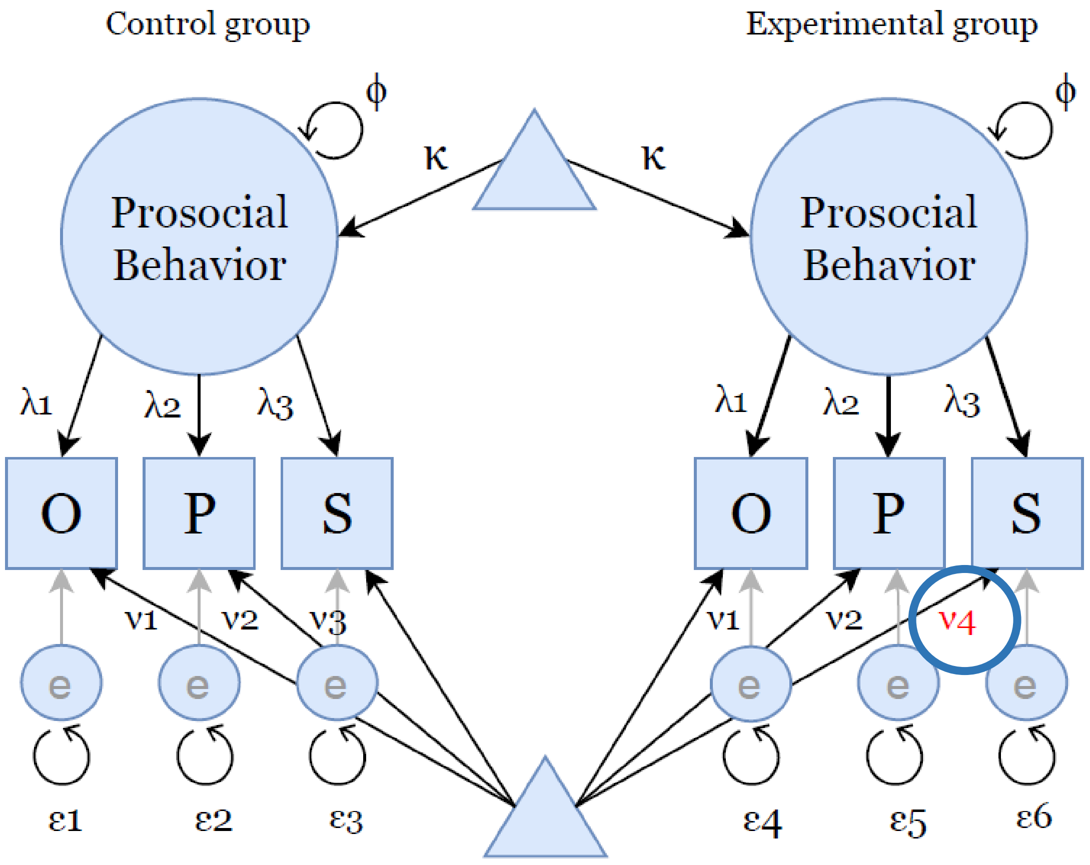


$E(Y|\eta,\text{control}) = E(Y|\eta,\text{exp}) = E(Y|\eta)$

Levels of invariance:

1. Configural ✅
2. Metric (loadings; weak) ✅
3. Scalar (intercepts; strong) ✅

Measurement Invariance



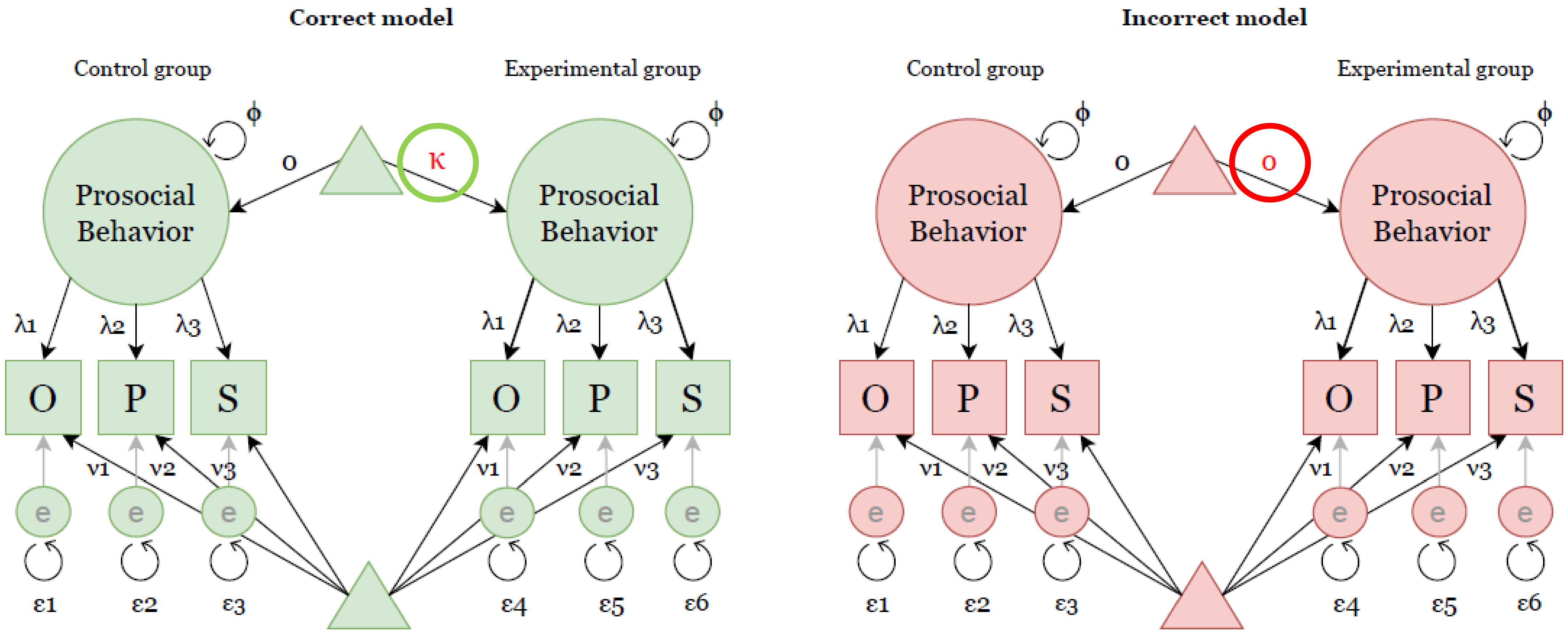
$E(Y|\eta,\text{control}) = E(Y|\eta,\text{exp}) = E(Y|\eta)$

Levels of invariance:

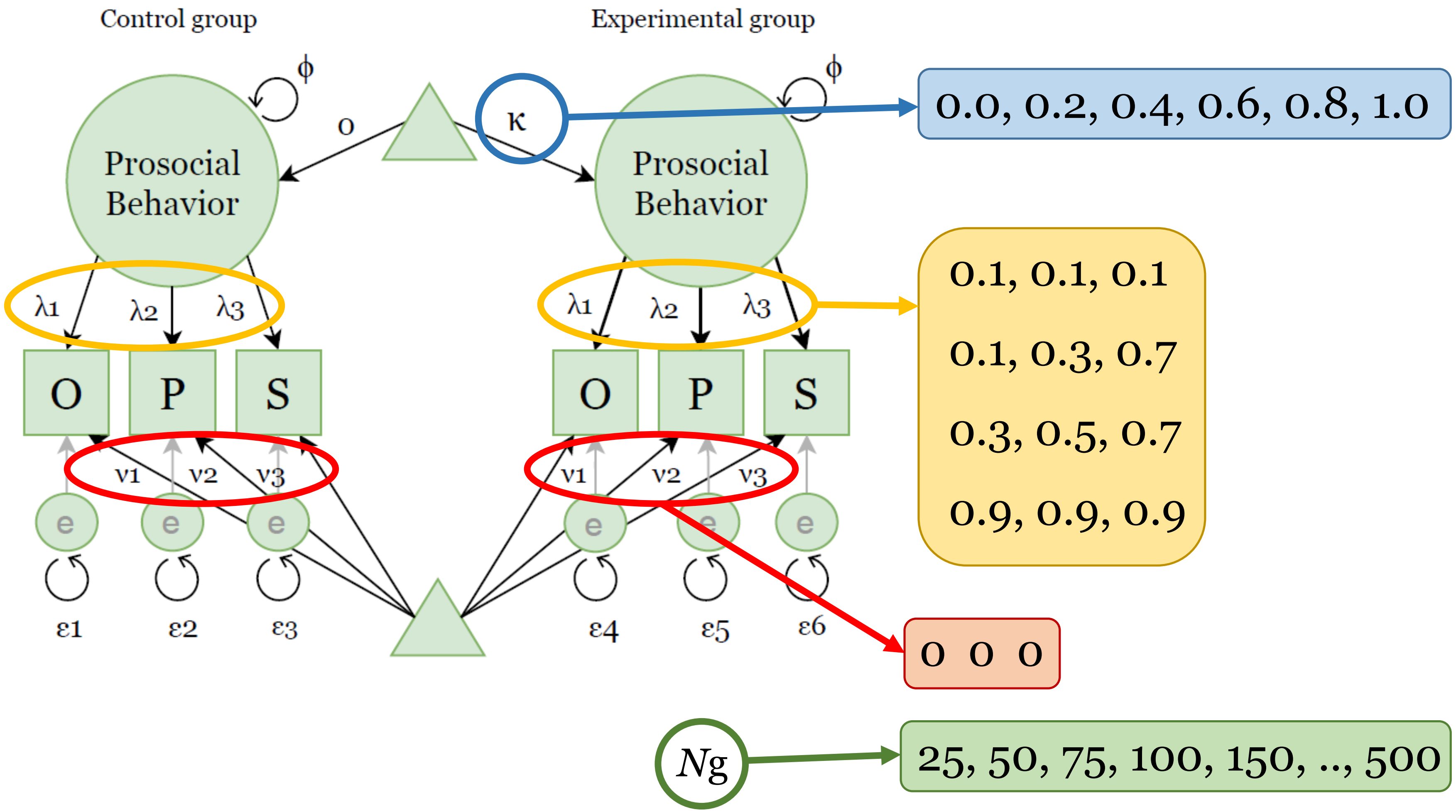
1. Configural ✅
2. Metric (loadings; weak) ✅
3. Scalar (intercepts; strong) ✅

↖ *Partial invariance*

Study 1: Full Invariance

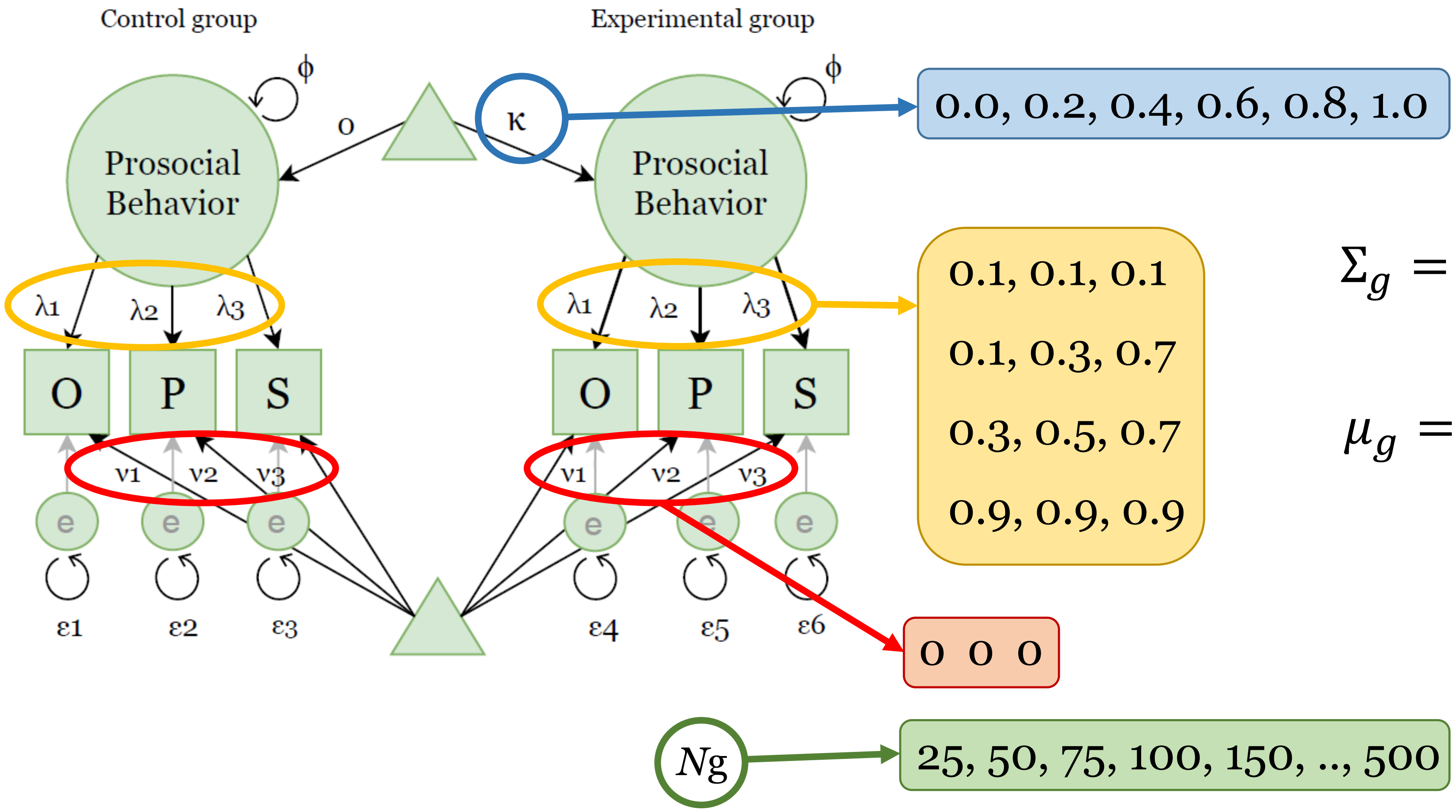


Study 1: conditions



$$Y_{1gj} = \nu_{y1g} + \lambda_{y1g}\eta_{gj} + \epsilon_{y1gj} \qquad \eta \sim N(\kappa_{g,} \phi_g) \qquad \epsilon \sim MVN(0_, \Theta_g)$$

Measurement equation



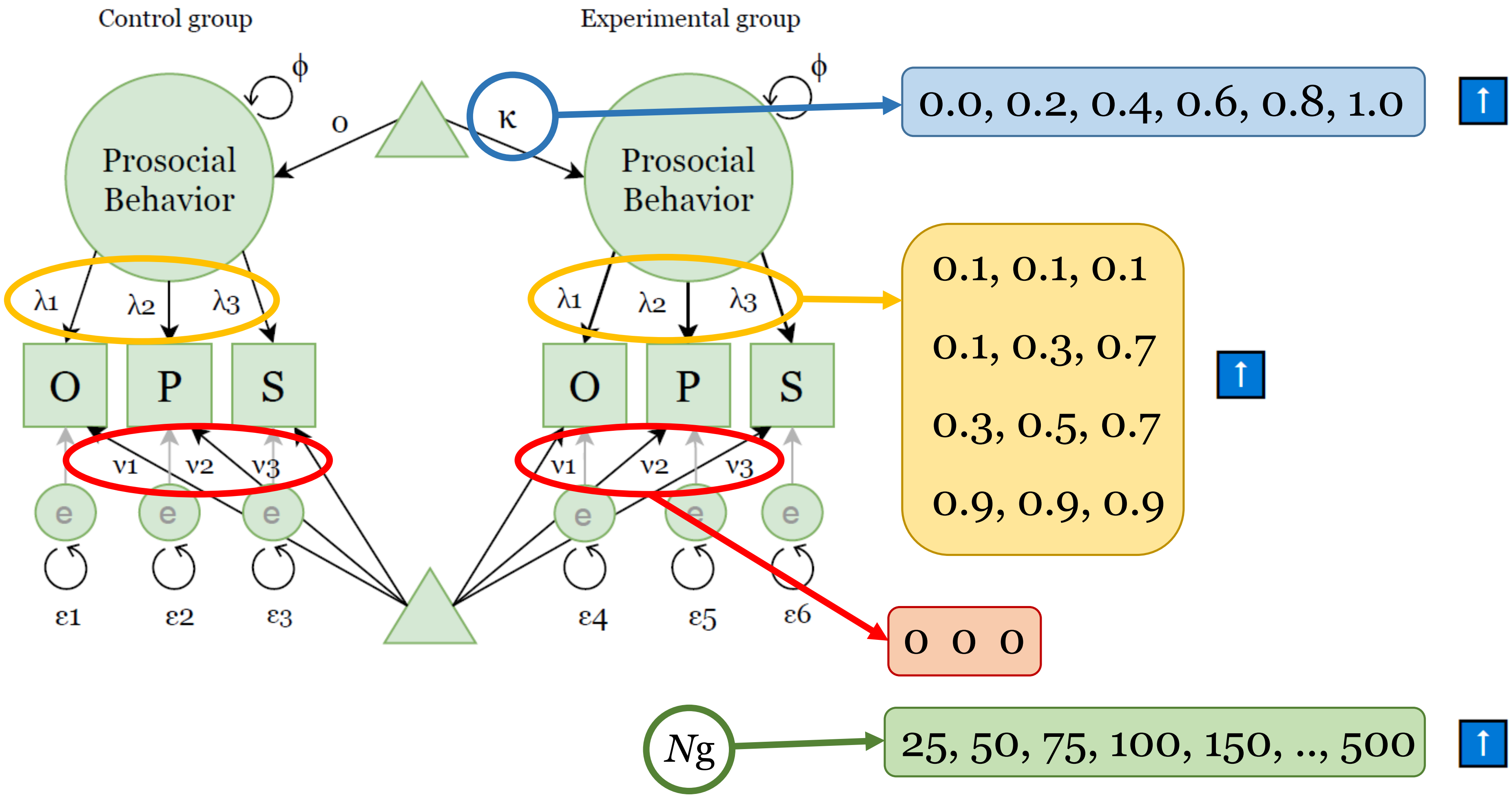
Covariance matrix

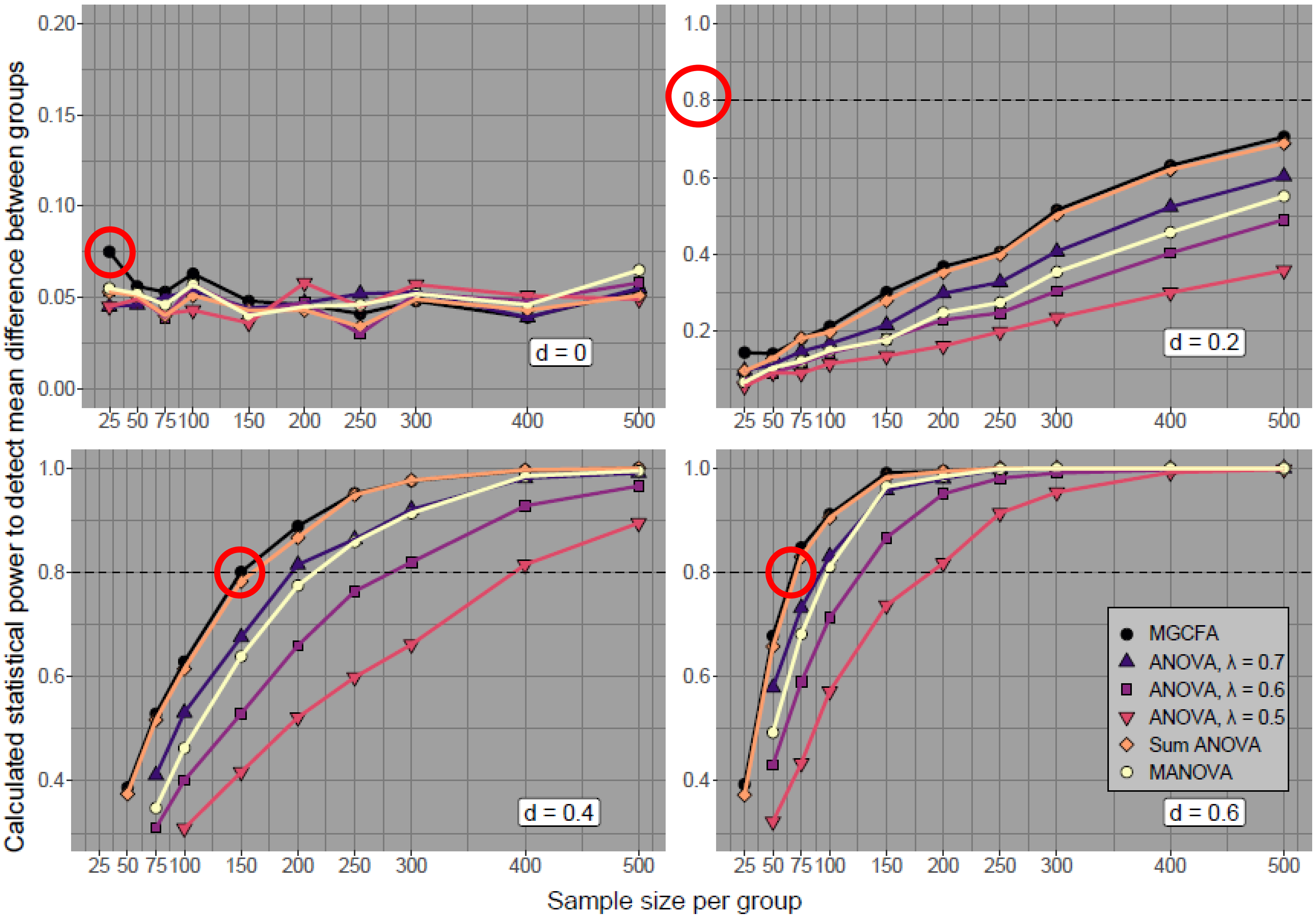
$$\Sigma_g = \lambda_g \varphi_g \lambda_g + \Theta_g$$

$$\mu_g = \nu_g + \lambda_g \kappa_g$$

Mean vector

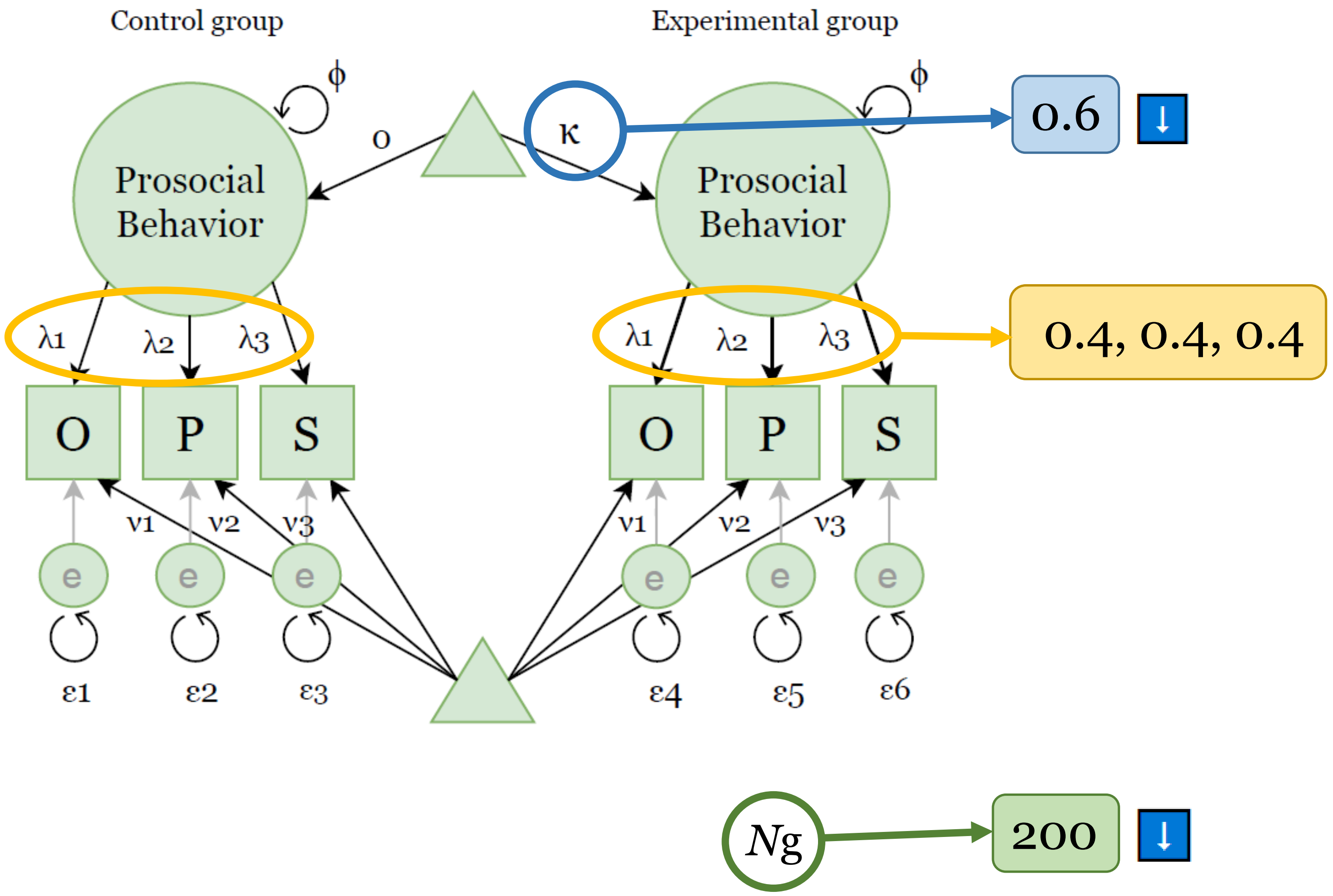
Study 1: results





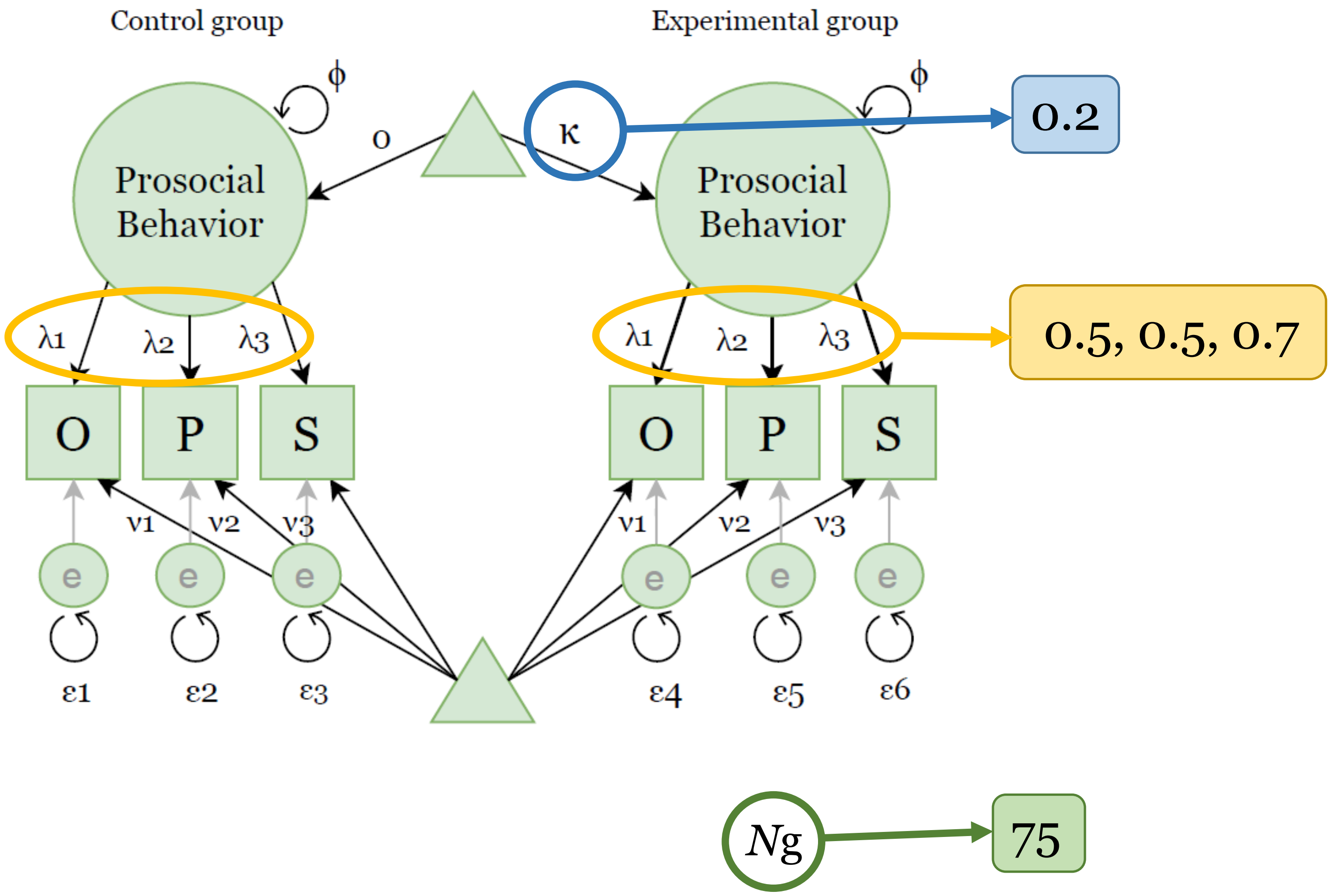
Calculated statistical power to detect mean difference between groups
Sample size per group
d = 0
d = 0.2
d = 0.4
d = 0.6
MGCFA
ANOVA, λ = 0.7
ANOVA, λ = 0.6
ANOVA, λ = 0.5
Sum ANOVA
MANOVA

Study 1: results



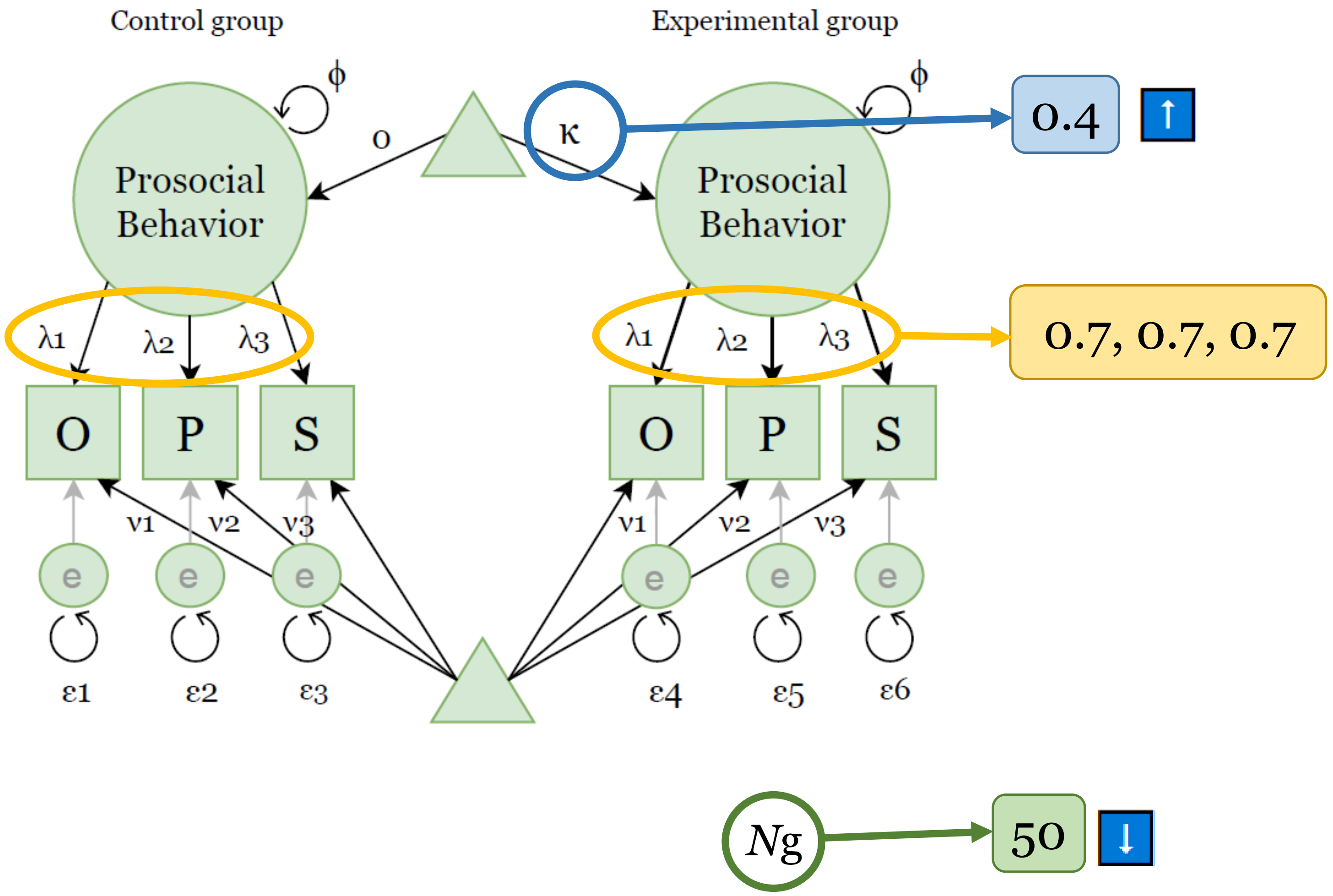
$r = 0.16$

Study 1: results



$r = 0.33$

Study 1: results



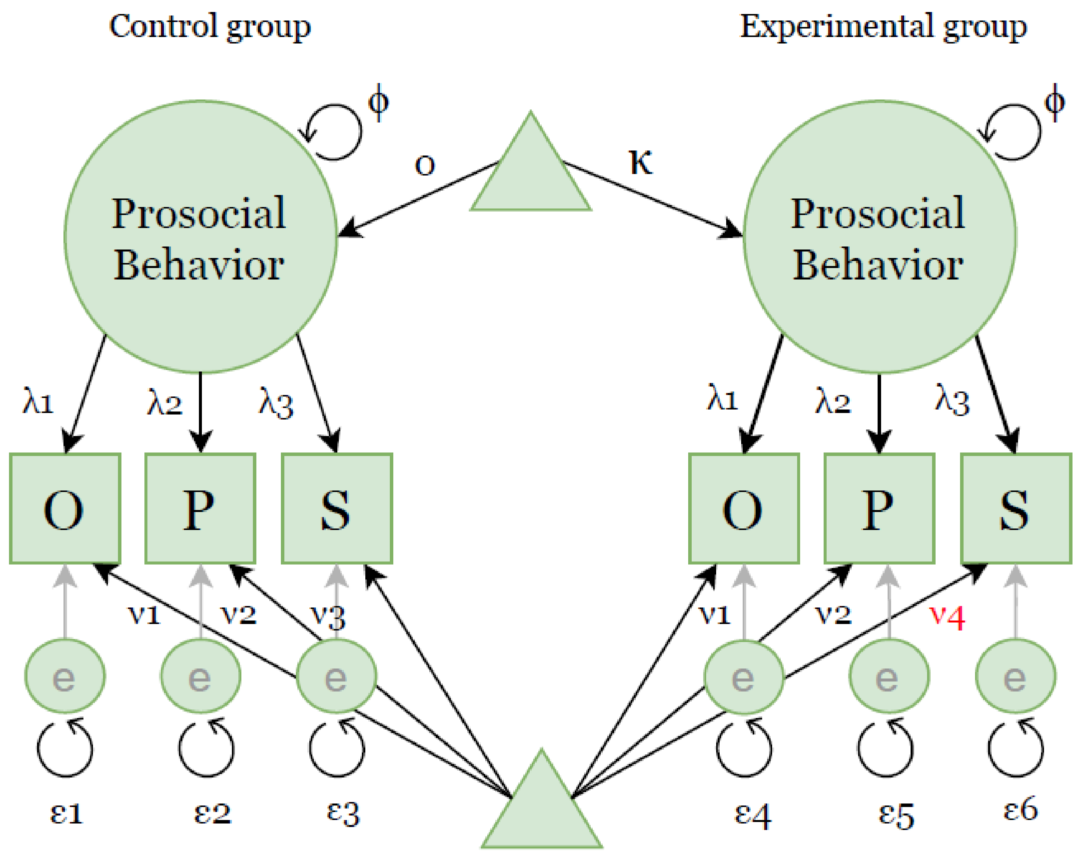
$r = 0.49$

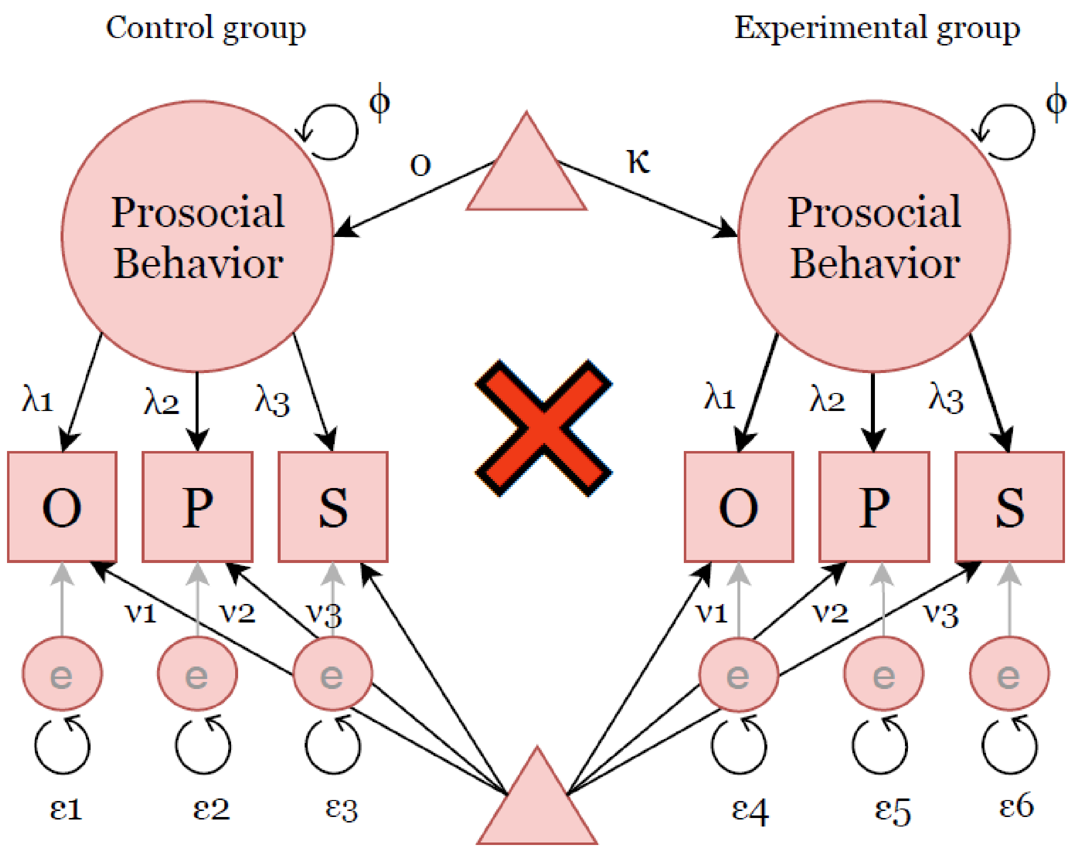
Study 2: Partial invariance

Statistical power to detect intercept invariance?

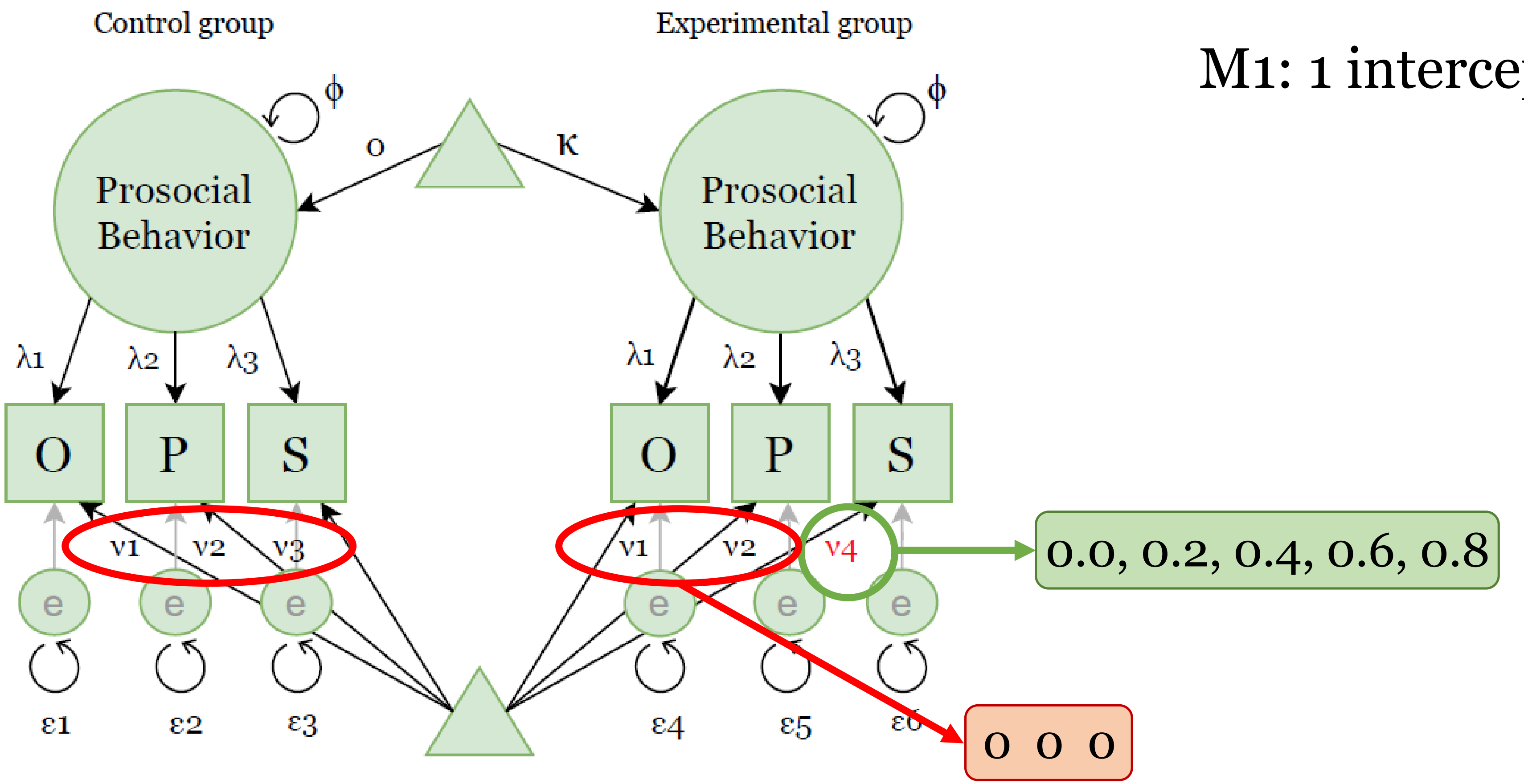
Accuracy of estimated group mean differences vis-à-vis ANOVA?

Study 2: Partial invariance – model 1



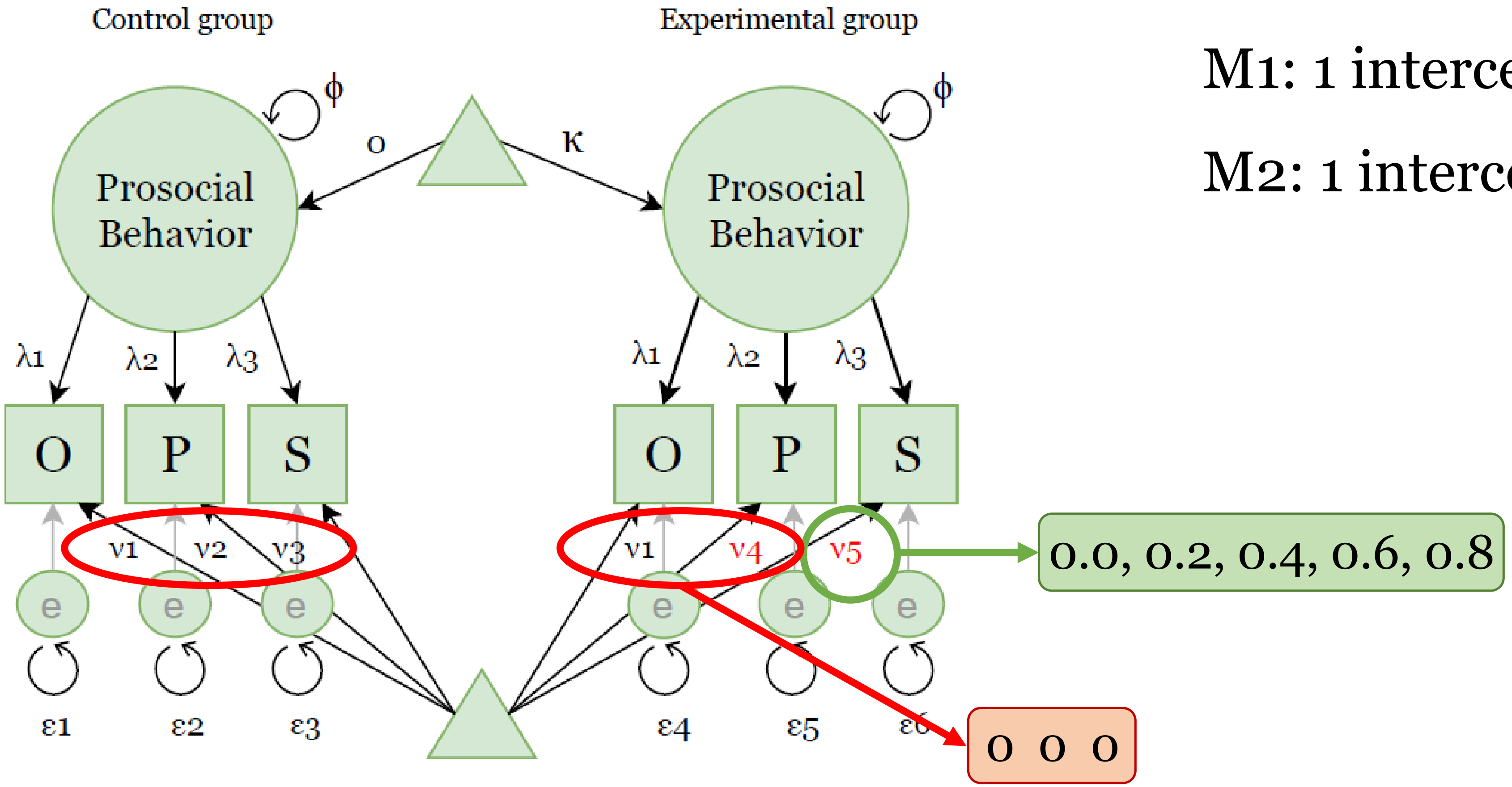


Study 2: Partial invariance – model 1



M1: 1 intercept biased, 1 free to vary

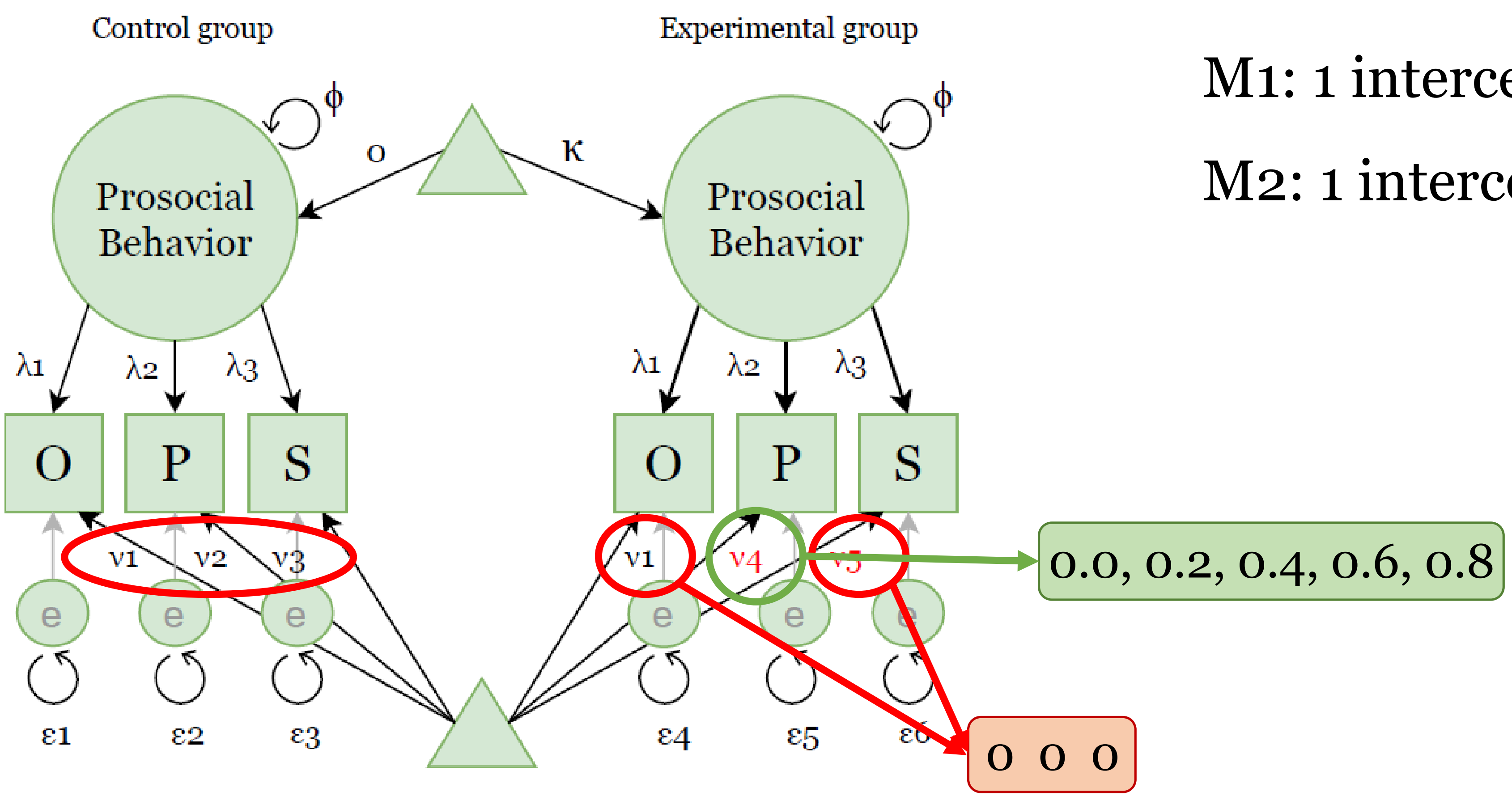
Study 2: Partial invariance – model 2



M1: 1 intercept biased, 1 free to vary

M2: 1 intercept biased, 2 free to vary

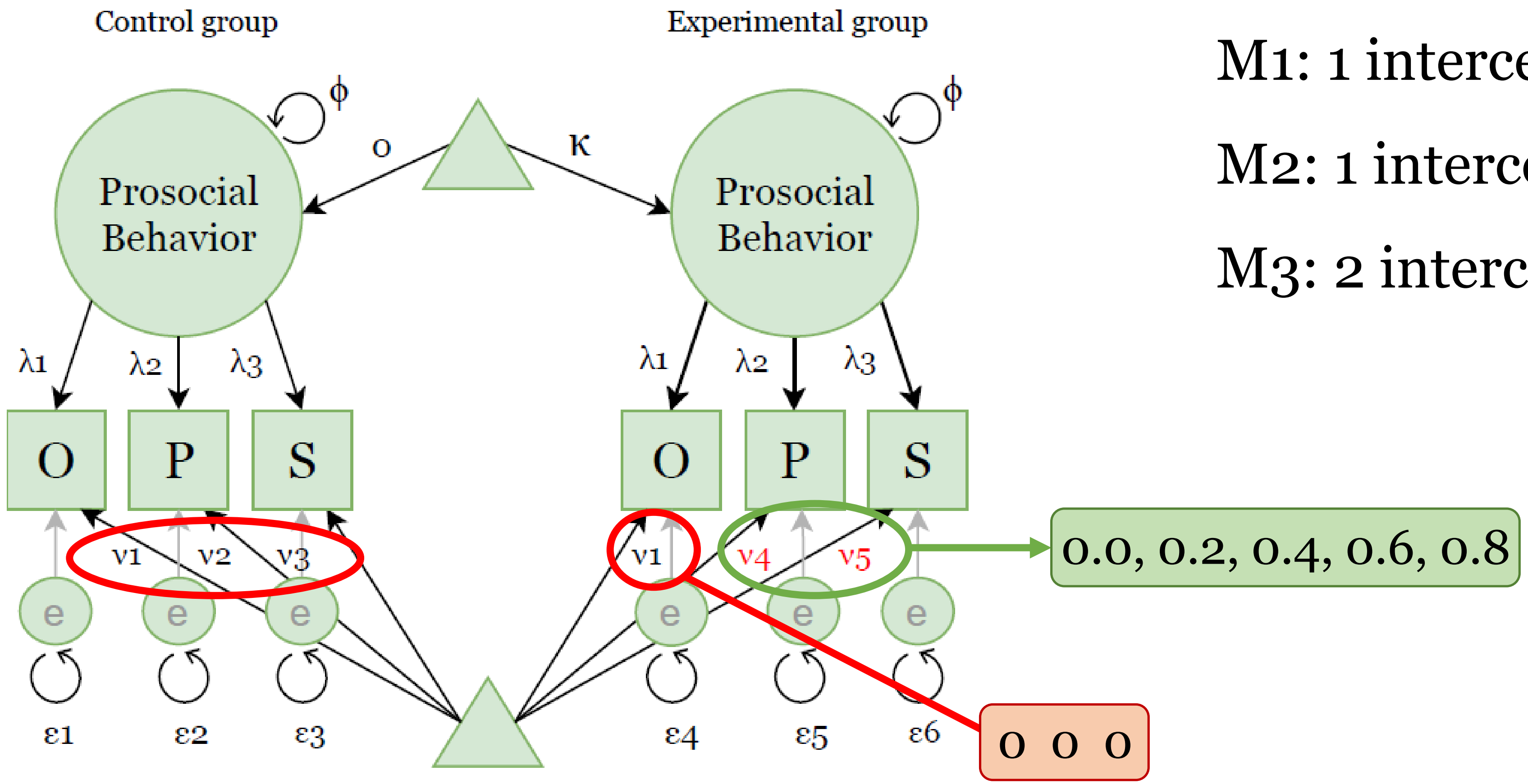
Study 2: Partial invariance – model 2



M1: 1 intercept biased, 1 free to vary

M2: 1 intercept biased, 2 free to vary

Study 2: Partial invariance – model 3

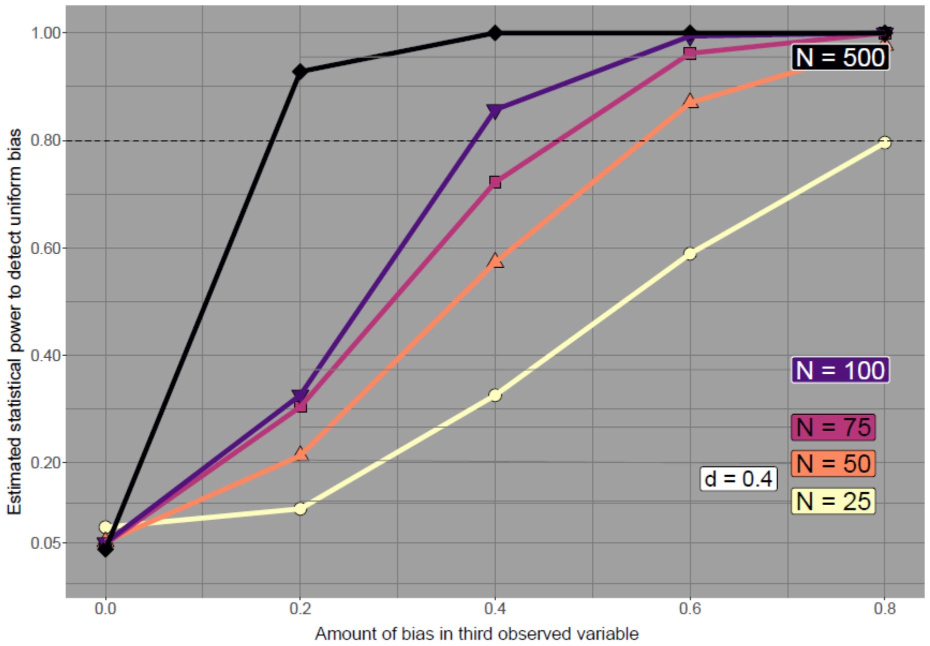


M1: 1 intercept biased, 1 free to vary

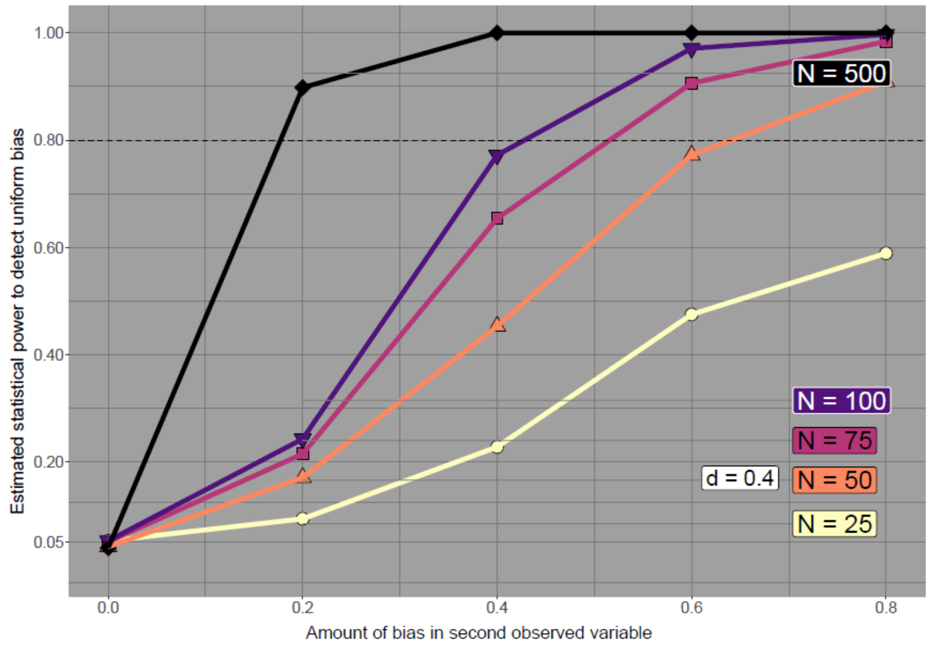
M2: 1 intercept biased, 2 free to vary

M3: 2 intercepts biased, 2 free to vary

M1: v_3 biased, v_3 free



M2: v_2 biased, v_2 & v_3 free

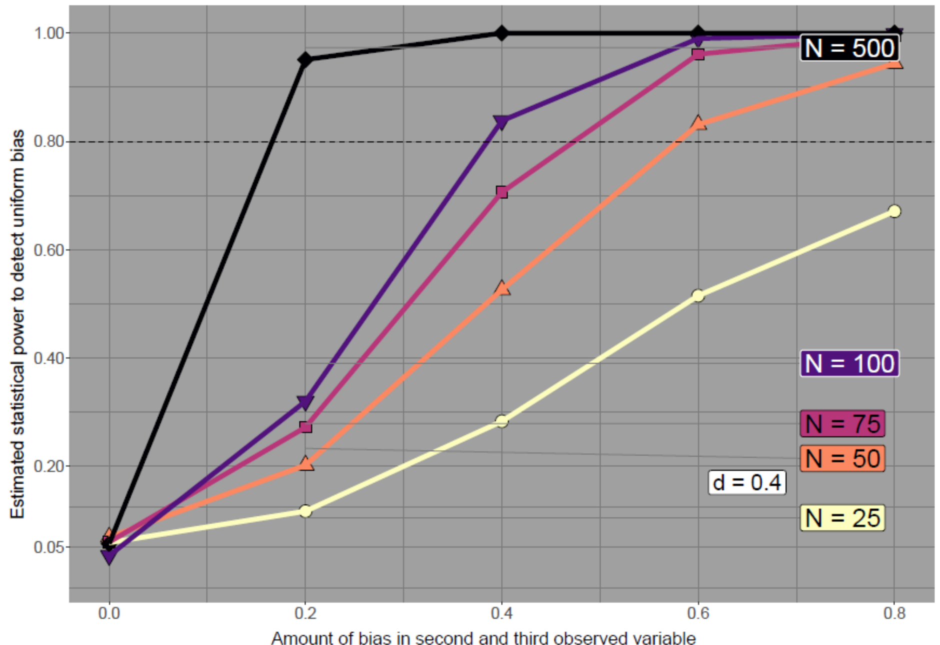


$\lambda = [0.8, 0.7, 0.6]$

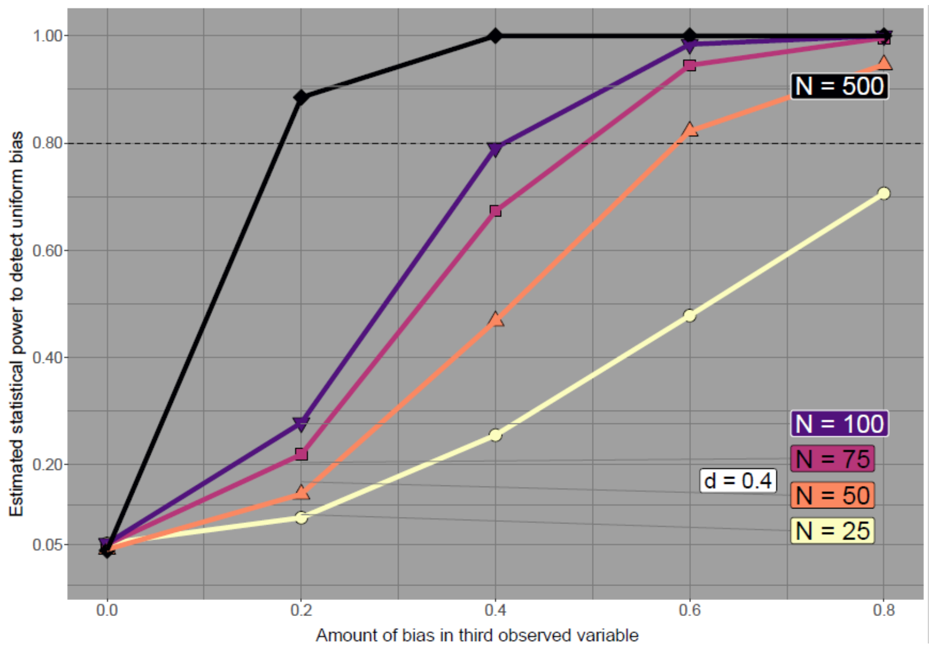
$M_r = 0.49$

$d = 0.4$

M3: v_2 & v_3 biased, v_2 & $v_3$free



M2: v_3 biased, v_2 & v_3 free



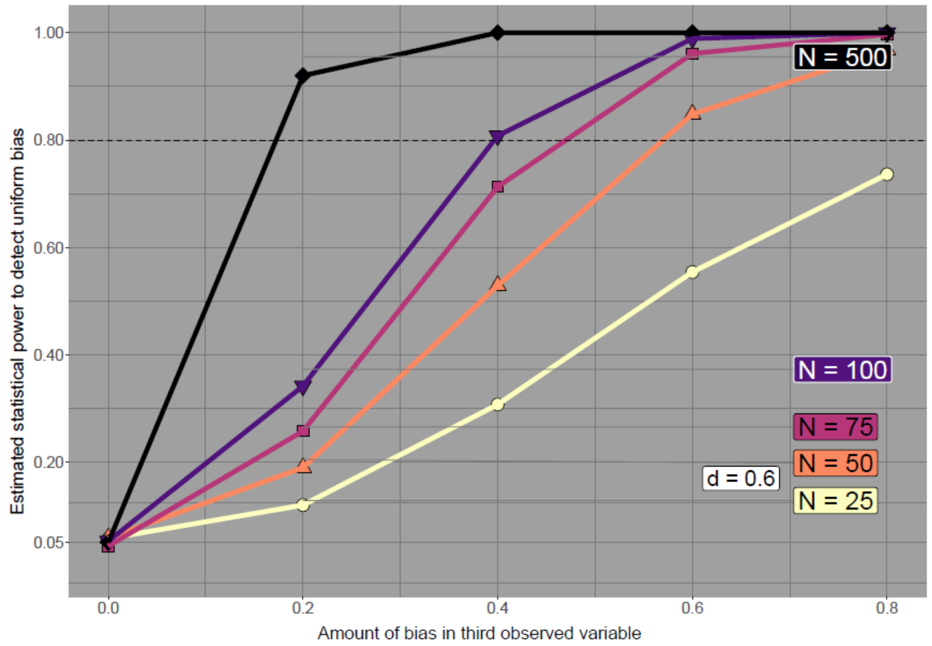
Power 💪

N ⬆️

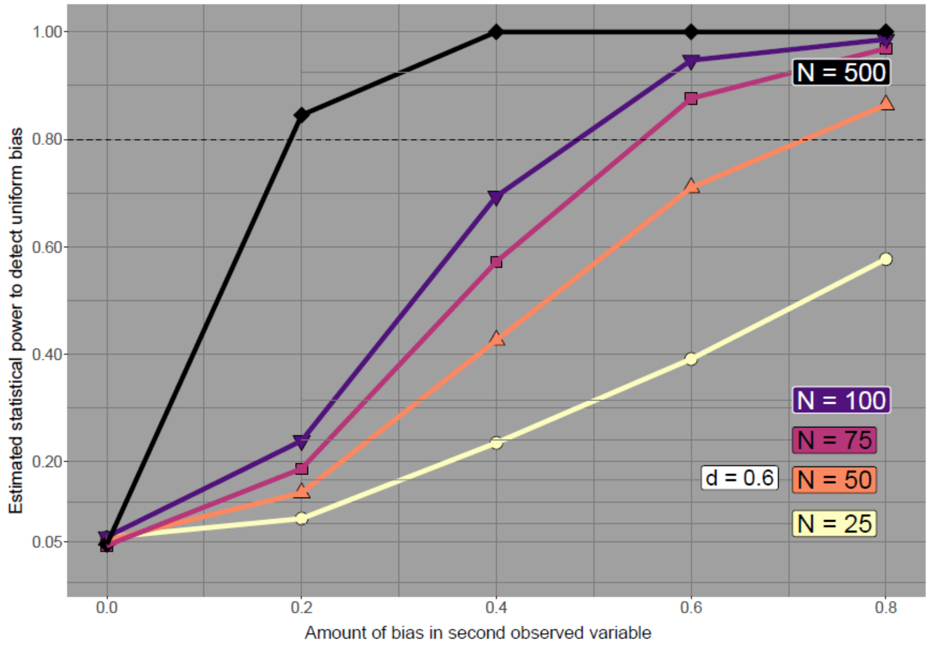
υ ⬆️

d

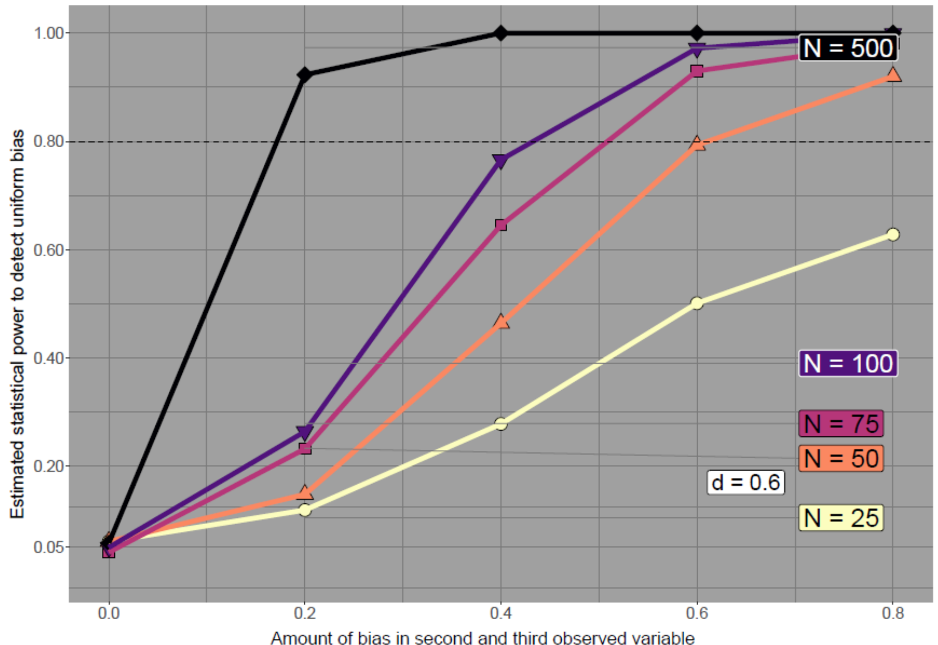
M1: v_3 biased, v_3 free



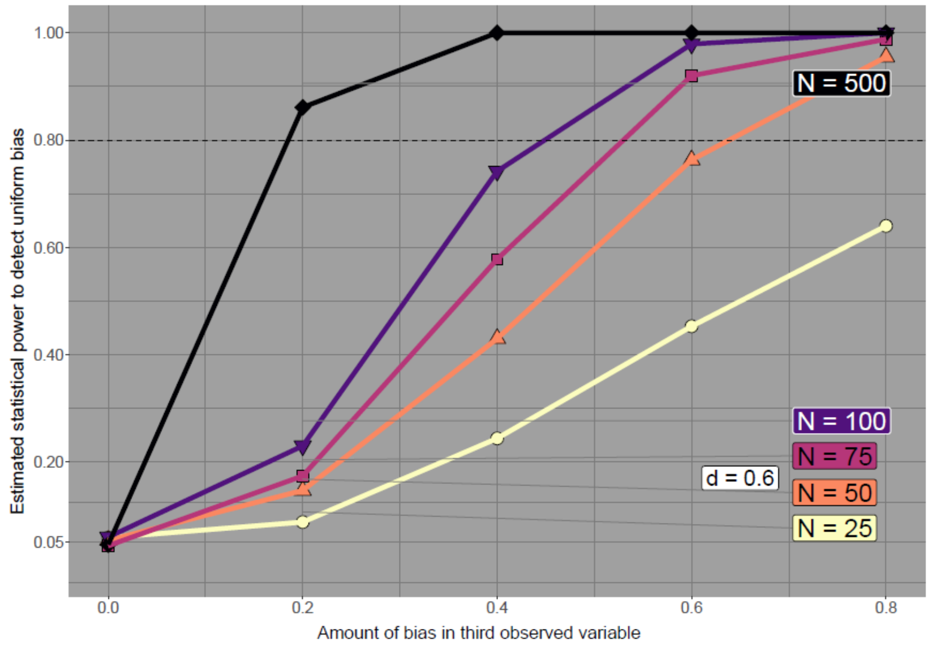
M2: v_2 biased, v_2 & v_3 free



M3: v_2 & v_3 biased, v_2 & v_3 free



M2: v_3 biased, v_2 & v_3 free



$\lambda = [0.8, 0.7, 0.6]$

$M_r = 0.49$

$d = 0.6$

Power 💪

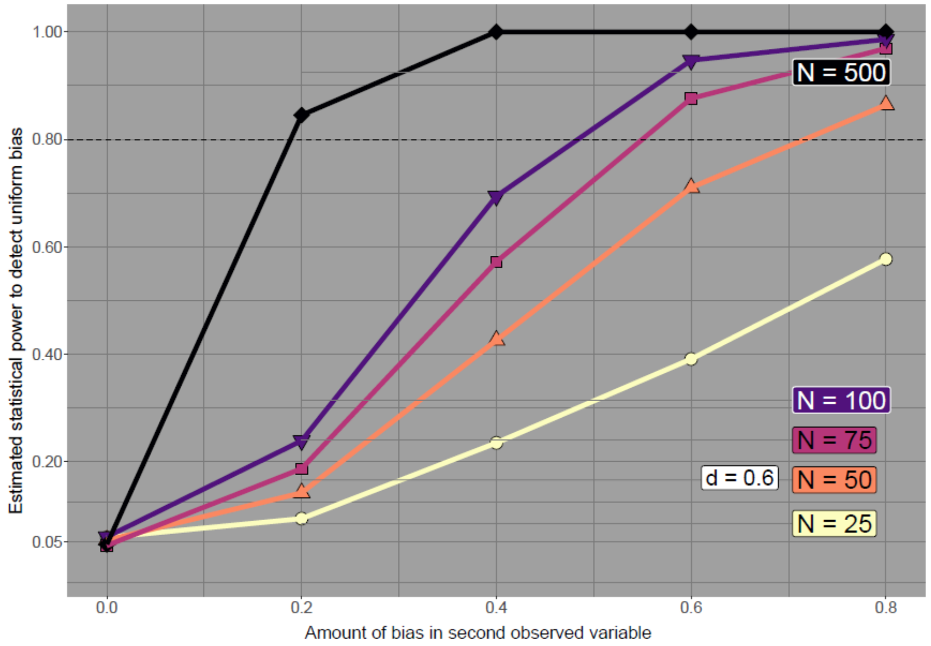
N ↑

υ ↑

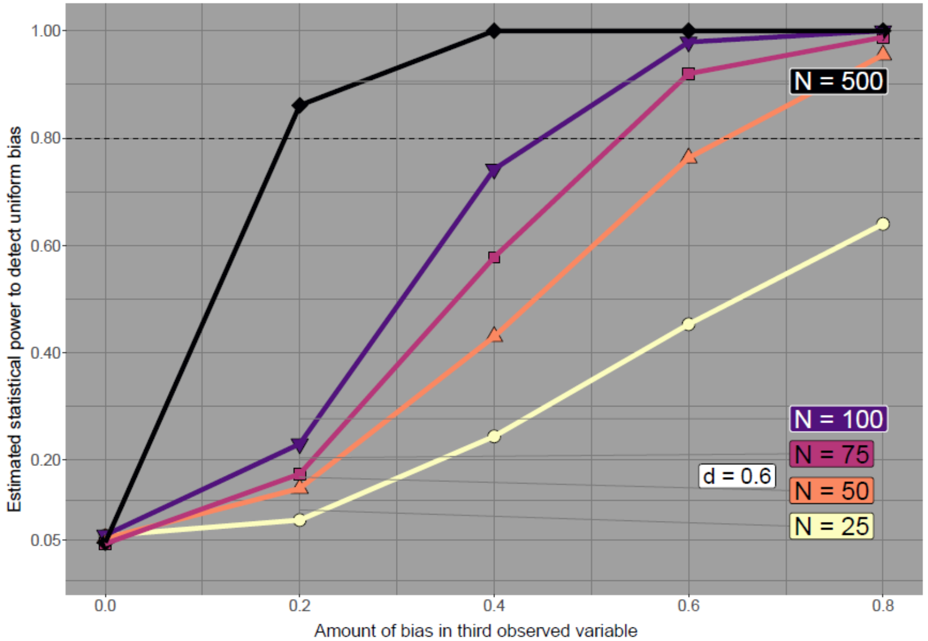
d ↓

λ ↑

M2: ν_2 biased, ν_2 & ν_3 free



M2: ν_3 biased, ν_2 & ν_3 free



$\lambda = [0.8, 0.7, 0.6]$

$M_r = 0.49$

$d = 0.6$

Power 💪

N ⬆

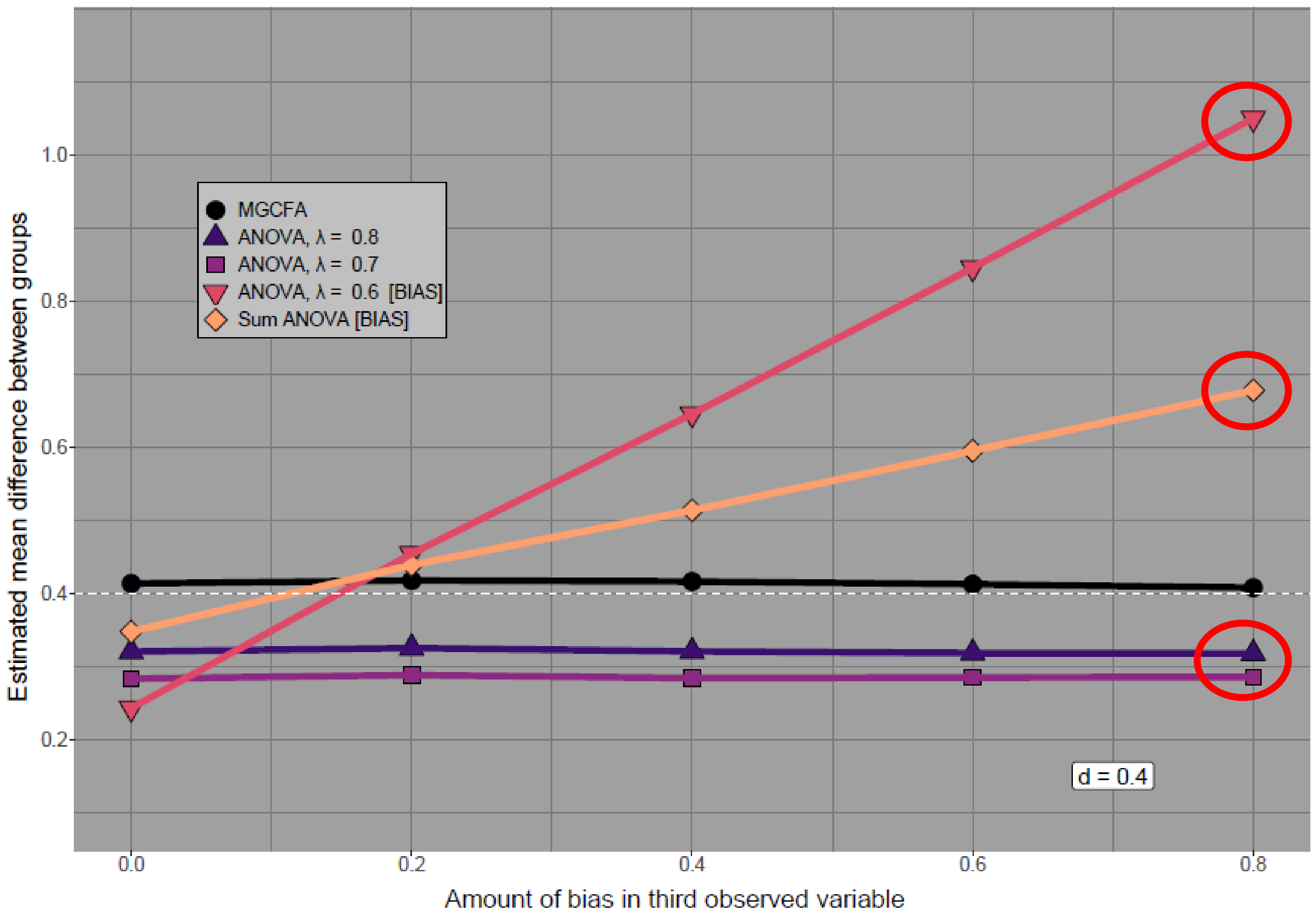
υ ⬆

d ⬇

λ ⬆

M1: 1 intercept biased, 1 free to vary

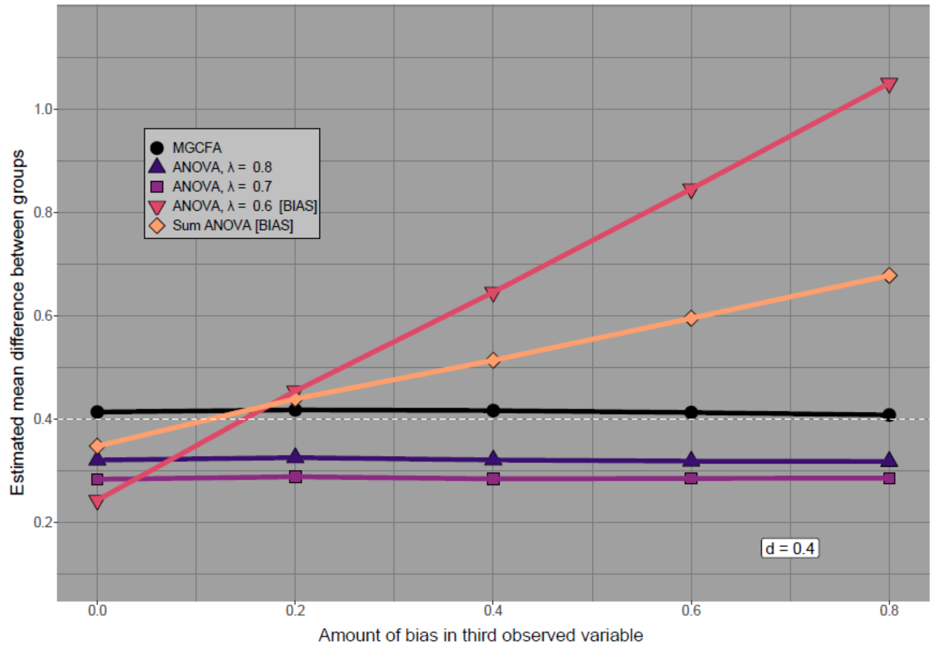
$\lambda = [0.8, 0.7, 0.6]$ $M_r = 0.49$ $d = 0.4$ $N_g = 50$



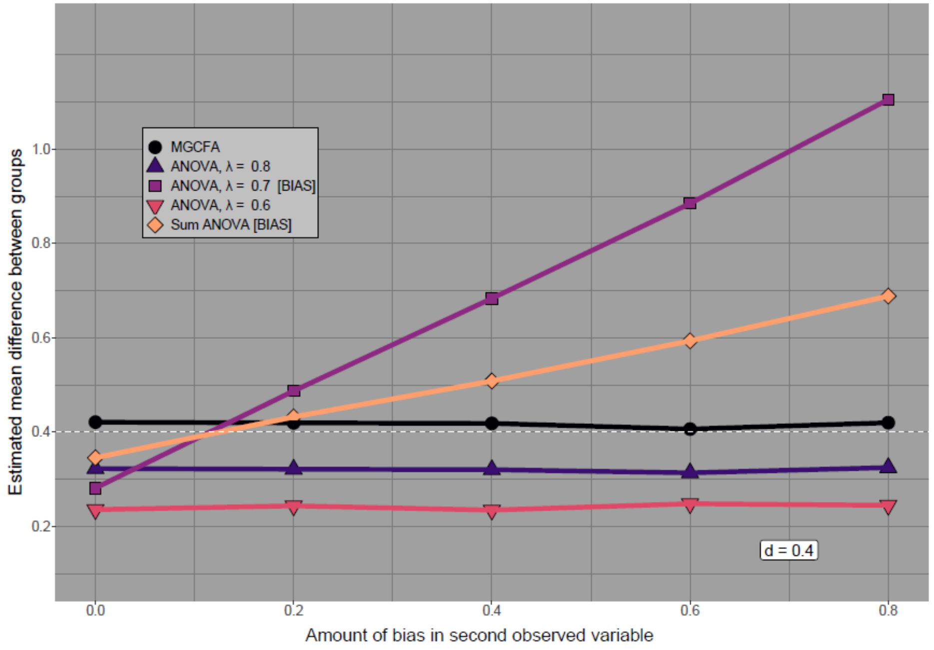
$$\mu_g = \nu_g + \lambda_g \kappa_g$$

$$\mu = 0.8 + 0.6 \times 0.4 = 1.04$$

M1: v_3 biased, v_3 free



M2: v_2 biased, v_2 & v_3 free



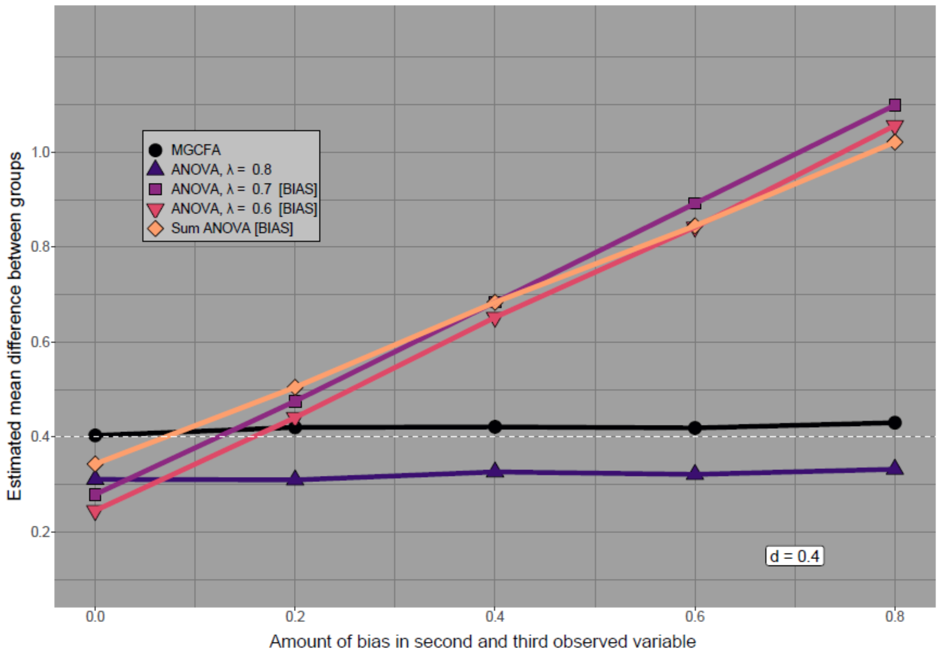
$\lambda = [0.8, 0.7, 0.6]$

$M_r = 0.49$

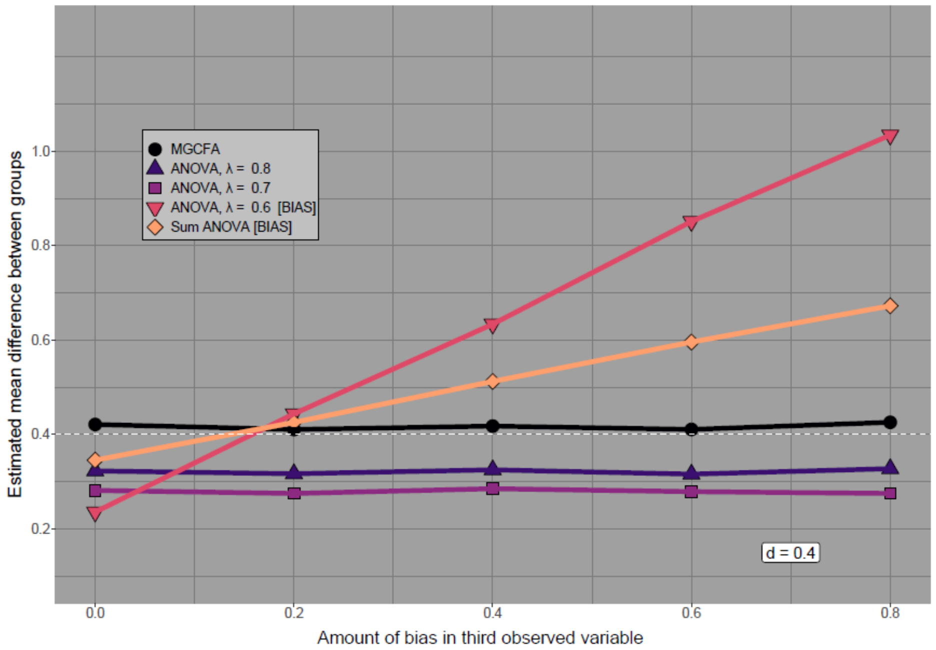
$d = 0.4$

$N_g = 50$

M3: v_2 & v_3 biased, v_2 & v_3 free

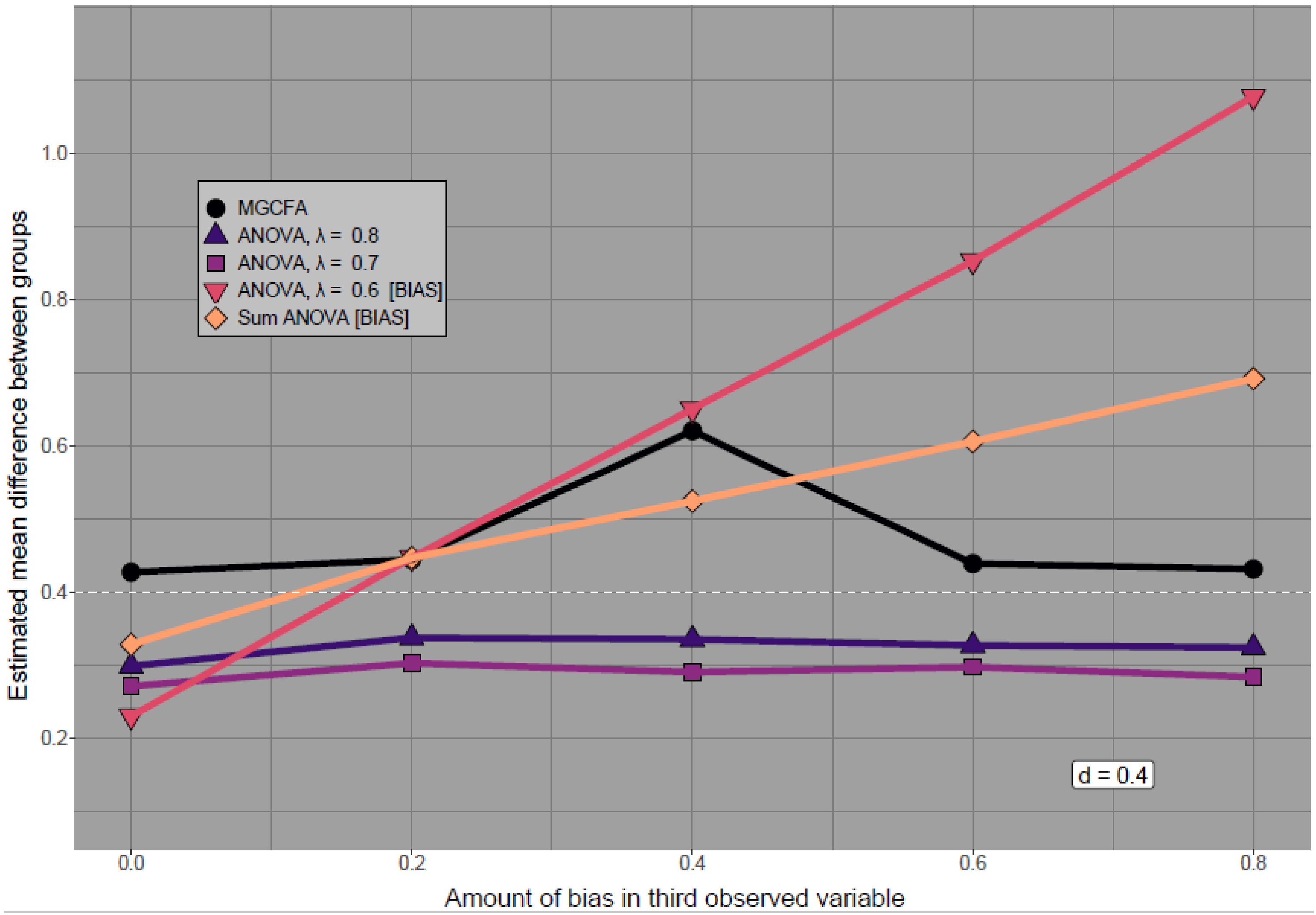


M2: v_3 biased, v_2 & v_3 free



M1: 1 intercept biased, 1 free to vary

$\lambda = [0.8, 0.7, 0.6]$ $M_r = 0.49$ $N_g = 25$



Conclusions

MGCFA w/ one factor, three indicators:

$>$ (M)ANOVA, even with small N, small κ, small λ

Under uniform bias:

- Don't use ANOVA!
- MGCFA accurately estimates latent mean difference

Next?

$N_g = 25$

Violated assumptions

indicators, # of biased items, # groups

Two-way interactions (incl./excl. uniform bias)

Thank you for your attention! 🙏

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