

# Complexity of Select

$$T_S(n) = T_S(\lceil \frac{n}{5} \rceil) + T_S(\frac{7}{10}n + 6) + \Theta(n)$$

Max size of  
S or G

COMPUTE  
THE MEDIAN  
OF  
MEDIANs

MEDIANs  
OF  
CHUNKs

RECURSIVE  
STEP on  
EITHER S  
OR G

TO GET  
THE  
MEDIANs  
OF  
CHUNKs



Solve  $T_S(n)$  by using **Substitution Method**

1) GUESS  $T_S(n)$  complexity (A.K.A. **INDUCTIVE PROVE**)

Let us GUESS

$$T_S(n) \in O(n)$$

SELECT A REPRESENTATIVE OF  $O(n)$   
(e.g.,  $c \cdot n$ )



WE ALSO NEED A REPRESENTATIVE  
FOR THE TERM  $\Theta(n)$  in  $T_S(n)$  eq  
(e.g.,  $c \cdot n$ )

2- ASSUME THAT  $T_S(m) \leq c \cdot m \quad \forall m < n$

So, LET US PROVE THAT

$$T_S(n) \leq c \cdot n$$



$$\begin{aligned}
T_S(n) &\leq T_S\left(\left\lceil \frac{n}{5} \right\rceil\right) + T_S\left(\frac{7}{10}n + 6\right) + C' \cdot n \\
\text{if } n > 20 & \quad | \quad \text{if } n > 5 \quad \text{AND} \quad \frac{7}{10}n + 6 < n \\
&\leq C \cdot \left\lceil \frac{n}{5} \right\rceil + C \left(\frac{7}{10}n + 6\right) + C' \cdot n \\
&| \\
&\leq C \cdot \left(\frac{n}{5} + 2\right) + \quad || \quad + \quad || \\
&| \\
&\leq C \cdot \frac{9}{10}n + C \cdot 7 + C' \cdot n \\
&|
\end{aligned}$$

$$\frac{7}{10}n + 6 < n$$

$$\Leftrightarrow$$

$$20 = \frac{10 \cdot 6^2}{2} < \frac{3}{10}n \Leftrightarrow n > 20$$

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IF  $c \geq 20c'$

$$T_S(n) \leq \frac{9}{10}c \cdot n + \frac{1}{20}cn + 7c$$



$$T_5(n) \leq \frac{19}{20} cn + c7 \leq cn$$

$$\text{IF } n \geq 140$$

So, By induction,  $T_5(n) \leq cn$

$\forall n \geq 140$  AND  $\forall c \geq 20c'$ . Thus,  $\overset{T_5(n)}{\uparrow}$   
 $O(n)$