Solving the Steiner tree problem by an approximation algorithm

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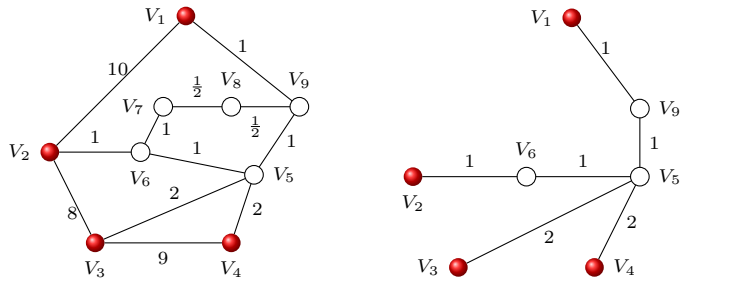
1 Introduction

An approximation algorithm for finding a Steiner tree of a connected, undirected distance graph was implemented. This algoritm has approximation ratio of $2-\frac{2}{l}$, which means it gives us a Steiner tree with total distance of all edges at most $2-\frac{2}{l}$ times that of the optimal tree, where l is the number of leaves in the optimal tree. The worst case time complexity of this algorithm is $\mathcal{O}(|V| \log |V| + |E|)$, where E is the set of edges and V is the set of vertices.

2 Steiner tree problem

Given an undirected distance graph $G = (V, E, d)$ and a set S, where E is the set of edges in G, V is the set of vertices in G, d is a distance function which maps E into the set of nonnegative numbers and $S \in V$ is a subset of V. Let Q be any subset of vertices in a connected subgraph G' of G. We shall say that G' spans Q. A spanning tree of G is tree subgraph of G that spans V. The minimal spanning tree of G is a spanning tree of G such that the total distance of on its edges is minimal among all spanning trees. The Steiner tree for a given G and S is the tree that spans S, and the minimal Steiner tree is the one among all Steiner trees for G and S that has minimal total distance. The problem for finding a minimal Steiner tree for any given G and S has been proved to be NP-Complete[2].



(a) Graph G (b) Minimal Steiner tree for G

Figure 1: Example of Steiner tree problem with terminal set $S = \{V_1, V_2, V_3, V_4\}$

3 The approximation algorithm

The algorithm is a modified version of Kou, Markowsky and Berman[3], which gives a better running time and also because of reducing the problem to shortest path and minimum spanning tree it is simpler. Given $G = (V, E, d)$ a connceted, undirected distance graph, and the set of terminals $S \subseteq V$. Then the graph $G'_1 = (S, E'_1, d'_1)$ can be cosntructed as follows.

For every vertex $s \in S$ let $N(s)$ be the set of vertices in V which are closer to s than to any other vertex in S(in computational geometry N(s) is called the Voronoi region of vertex s). We can partition V according to N(s) by considering $\{N(s); s \in S\}$, in other words every vertex $v \in V$ belongs uniquely to one of the

partitions N(s) with s as its center, where v is closer to s than any other vertex $q \in S$, $q \neq s$. Now, the graph $G'_1 = (S, E'_1, d'_1)$ is defined by

$$E'_1 = \{(s,t); s,t \in S \text{ and there is an edge } (u,v) \in E \text{ with } u \in N(s), v \in N(t)\} \quad (1)$$

and

$$d'_1(s,t) = min\{d_1(s,u) + d(u,v) + d_1(v,t); (u,v) \in E, u \in N(s), v \in N(t)\}. \quad (2)$$

In equation 2 $d_1(v_i, v_j)$ is equal to the distance of shortest path from v_i to v_j for $v_i, v_j \in V$ in graph G.

$d'_1(s,t)$ in equation 2 is the length of shortest path in G restricted to $N(s) \cup N(t)$. And the edge $(u', v') \in E'_1$ corresponds to the bridge edge $(u,v) \in E$ which acts like a bridge that goes from region N(s) to region N(t). Now the algorithm is as follow :

1. *Construct the graph* $G'_1 = (S, E'_1, d'_1)$
2. *Find a minimum spanning tree* G_2 *of* G'_1.
3. *Construct a subgraph* G_3 *of* G *by replacing each edge in* G_2 *by its corresponding shortest path in* G.
4. *Find a minimum spanning tree* G_4 *of* G_3.
5. *Construct a Steiner tree* G_5 *from* G_4 *by deleting edges in* G_4*, if necessary, so that no leaves in* G_5 *from the set* $V - S$

Algorithm 1

Step 1 of Algorithm 1 dominates the computational time. This step requires the solution of a single shortest path problem which takes $\mathcal{O}(|V| \log |V| + |E|)$ by using Dijkstra algorithm and Fibonacci heaps[4].

4 Implementation

Step 1 of Algorithm 1 requires the voronoi region $\{N(s); s \in S\}$ of each vertex $v \in V$. The partition $\{N(s); s \in S\}$ can be computed by adjoining an auxiliary vertex S_0 and edges $(s_0, s), s \in S$, with length 0 to G and then performing a single source shortest path with source s_0. This step was implemented in the program by using a min priority queue based on a priority heap.The priority queue provides $\mathcal{O}(log(n))$ time for enqueing and dequeing. For the Dijkstra algorithm the priority queue stores at most $|V|$ number of item, thus each of enqueing and dequeing need $\mathcal{O}(log(|V|))$ and totally there are $|E|$ number of such operations, which gives the running time of $\mathcal{O}(|E|\, log(|V|))$ for step 1. This step

also needs $\mathcal{O}(|E|)$ for finding G_1' edges. For finding G_1''s edges the program goes through all edges $(u, v) \in E$ and generates the triples $(s(u), s(v), d_1(s(u), u) + d(u, v) + d_1(v, s(v))$ in which $s(v) \in S$ is the center of region that v belongs to.Then the program finds the minimum distance for edge $(s(u), s(v)$ over all distances $d_1(s(u), u) + d(u, v) + d_1(v, s(v))$.

Step 2 computes the minimum spanning tree G_2 of G_1'. The graph G_1' has $\mathcal{O}(|E|)$ edges and hence this step can be carried out in $\mathcal{O}(|E|log(|S|))$ by using Kruskal algorithm and Union Find data structure[4].

In step 3 the edges of G_2 have to be replaced by its corresponding edges from the shortest path computed in step 1.Consider the edge (u, v) of G_2, this edge corresponds to the shortest path between u and v in the graph G, here all the edges of G from this shortest path are plugged back to G_2. In step 1 the predecessor of each vertex has been computed and stored in an array, hence by means of this array the shortest path can be constructed.
Step 4 is the same is step 2 in which the minimum spanning tree of the graph from step 3 was computed. A

5 Results

Several tests has been performed to evaluate the performance of this aproximation algorithm. The code is implemented in Java 1.8 and the dataset from 11th DIMACS Implementation Challenge[5] is used. All runs are performed on a machine with score 181.433863(the score is calculated according to DIMACS Implementaion Challenge benchmark code to compare the results from different machines).

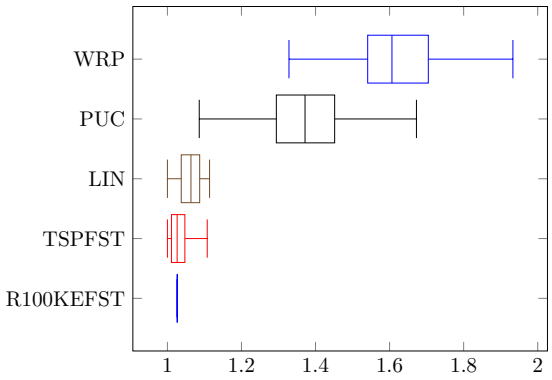


Figure 2: Comparison of approximation ratio

R100KEFST, is the instance with fractional weight, Euclidean distance and it is obtained by full Steiner tree generation for the classical Euclidean Steiner tree problem in the plane[7]. TSPFST, PUC, , LIN and WRP are the instances from *SteinLib*[8] testsets. These instances all have integer weights. Here is brief description of each of this dataset. TSPFST is a rectilinear graph with L1 distances. PUC contains 3 classes of the instance, hypercubes, code covering and

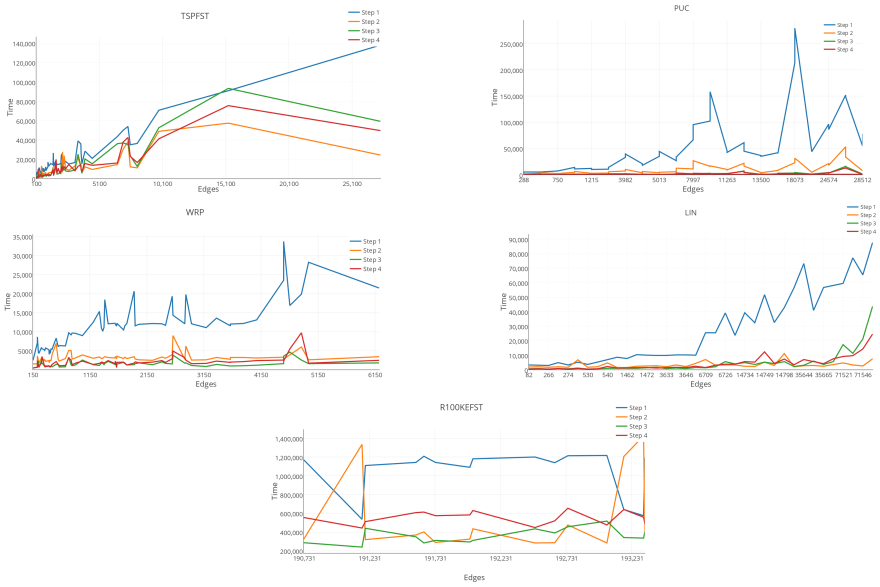


Figure 3: Running time in microsecond for of each step of the algorithm

bipartite graphs, and they are artificial instances(i.e. there are not derived from real world problem)[9]. The LIN instance is derived from VLSI grid graphs and it has L1 distances[10]. WRP instances are wire routing problems from industry.

In theory, it is claimed that step 1 of algorithm 1 dominates the whole running time. The runtime of the program was measured in microsecond and for most of the instances contribution of step 1 in running time is the most comparing to other steps. Figure 3 represtnts the running time diagram for seleceted instances.

6 Conclusions

We have presented an implementation of an approximation algorithm for Steiner tree problem. The algorithm has approximation ratio of $2 - \frac{2}{l}$,where l is the minimum number of leaves in any Steiner tree for the given graph. The algorithm running time is dominated by its first step in which a single shortest path problem is computed, in our code we used the Dijkstra algorithm and priority queue which yields the running time of $\mathcal{O}(|E|log|V|)$. From the experimental results, we observed that the most consuming step for most of the instances is the first one and this result agrees with the theory.

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