

Solving the Steiner tree problem by an approximation algorithm

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1 Introduction

An approximation algorithm for finding a Steiner tree of a connected, undirected distance graph was implemented. This algorithm has approximation ratio of $2 - \frac{2}{l}$, which means it gives us a Steiner tree with total distance of all edges at most $2 - \frac{2}{l}$ times that of the optimal tree, where l is the number of leaves in the optimal tree. The worst case time complexity of this algorithm is $\mathcal{O}(|V| \log |V| + |E|)$, where E is the set of edges and V is the set of vertices.

2 Steiner tree problem

Given an undirected distance graph $G = (V, E, d)$ and a set S , where E is the set of edges in G , V is the set of vertices in G , d is a distance function which maps E into the set of nonnegative numbers and $S \subseteq V$ is a subset of V . Let Q be any subset of vertices in a connected subgraph G' of G . We shall say that G' spans Q . A spanning tree of G is tree subgraph of G that spans V . The minimal spanning tree of G is a spanning tree of G such that the total distance of on its edges is minimal among all spanning trees. The Steiner tree for a given G and S is the tree that spans S , and the minimal Steiner tree is the one among all Steiner trees for G and S that has minimal total distance. The problem for finding a minimal Steiner tree for any given G and S has been proved to be NP-Complete[2].

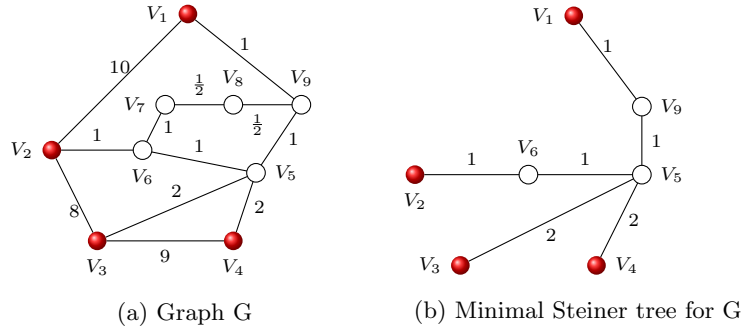


Figure 1: Example of Steiner tree problem with terminal set $S = \{V_1, V_2, V_3, V_4\}$

3 The approximation algorithm

The algorithm is a modified version of Kou, Markowsky and Berman[3], which gives a better running time and also because of reducing the problem to shortest path and minimum spanning tree it is simpler. Given $G = (V, E, d)$ a connected, undirected distance graph, and the set of terminals $S \subseteq V$. Then the graph $G'_1 = (S, E'_1, d'_1)$ can be constructed as follows.

For every vertex $s \in S$ let $N(s)$ be the set of vertices in V which are closer to s than to any other vertex in S (in computational geometry $N(s)$ is called the Voronoi region of vertex s). We can partition V according to $N(s)$ by considering $\{N(s); s \in S\}$, in other words every vertex $v \in V$ belongs uniquely to one of the

partitions $N(s)$ with s as its center, where v is closer to s than any other vertex $q \in S$, $q \neq s$. Now, the graph $G'_1 = (S, E'_1, d'_1)$ is defined by

$$E'_1 = \{(s, t); s, t \in S \text{ and there is an edge } (u, v) \in E \text{ with } u \in N(s), v \in N(t)\} \quad (1)$$

and

$$d'_1(s, t) = \min\{d_1(s, u) + d(u, v) + d_1(v, t); (u, v) \in E, u \in N(s), v \in N(t)\}. \quad (2)$$

In equation 2 $d_1(v_i, v_j)$ is equal to the distance of shortest path from v_i to v_j for $v_i, v_j \in V$ in graph G .

$d'_1(s, t)$ in equation 2 is the length of shortest path in G restricted to $N(s) \cup N(t)$. And the edge $(u', v') \in E'_1$ corresponds to the bridge edge $(u, v) \in E$ which acts like a bridge that goes from region $N(s)$ to region $N(t)$. Now the algorithm is as follow :

1. Construct the graph $G'_1 = (S, E'_1, d'_1)$
2. Find a minimum spanning tree G_2 of G'_1 .
3. Construct a subgraph G_3 of G by replacing each edge in G_2 by its corresponding shortest path in G .
4. Find a minimum spanning tree G_4 of G_3 .
5. Construct a Steiner tree G_5 from G_4 by deleting edges in G_4 , if necessary, so that no leaves in G_5 from the set $V - S$

Algorithm 1

Step 1 of Algorithm 1 dominates the computational time. This step requires the solution of a single shortest path problem which takes $\mathcal{O}(|V| \log |V| + |E|)$ by using Dijkstra algorithm and Fibonacci heaps[4].

4 Implementation

Step 1 of Algorithm 1 requires the voronoi region $\{N(s); s \in S\}$ of each vertex $v \in V$. The partition $\{N(s); s \in S\}$ can be computed by adjoining an auxiliary vertex S_0 and edges $(s_0, s), s \in S$, with length 0 to G and then performing a single source shortest path with source s_0 . This step was implemented in the program by using a min priority queue based on a priority heap. The priority queue provides $\mathcal{O}(\log(n))$ time for enqueueing and dequeuing. For the Dijkstra algorithm the priority queue stores at most $|V|$ number of item, thus each of enqueueing and dequeuing need $\mathcal{O}(\log(|V|))$ and totally there are $|E|$ number of such operations, which gives the running time of $\mathcal{O}(|E| \log |V|)$ for step 1. This step

also needs $\mathcal{O}(|E|)$ for finding G'_1 edges. For finding G'_1 's edges the program goes through all edges $(u, v) \in E$ and generates the triples $(s(u), s(v), d_1(s(u), u) + d(u, v) + d_1(v, s(v)))$ in which $s(v) \in S$ is the center of region that v belongs to. Then the program finds the minimum distance for edge $(s(u), s(v))$ over all distances $d_1(s(u), u) + d(u, v) + d_1(v, s(v))$.

Step 2 computes the minimum spanning tree G_2 of G'_1 . The graph G'_1 has $\mathcal{O}(|E|)$ edges and hence this step can be carried out in $\mathcal{O}(|E|\log(|S|))$ by using Kruskal algorithm and Union Find data structure[4].

In step 3 the edges of G_2 have to be replaced by its corresponding edges from the shortest path computed in step 1. Consider the edge (u, v) of G_2 , this edge corresponds to the shortest path between u and v in the graph G , here all the edges of G from this shortest path are plugged back to G_2 . In step 1 the predecessor of each vertex has been computed and stored in an array, hence by means of this array the shortest path can be constructed.

Step 4 is the same as step 2 in which the minimum spanning tree of the graph from step 3 was computed. A

5 Results

Several tests have been performed to evaluate the performance of this approximation algorithm. The code is implemented in Java 1.8 and the dataset from 11th DIMACS Implementation Challenge[5] is used. All runs are performed on a machine with score 181.433863 (the score is calculated according to DIMACS Implementation Challenge benchmark code to compare the results from different machines).

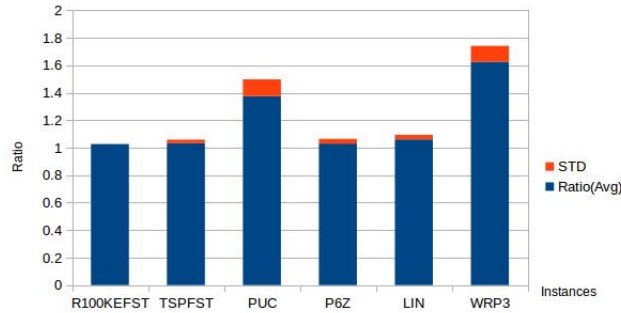
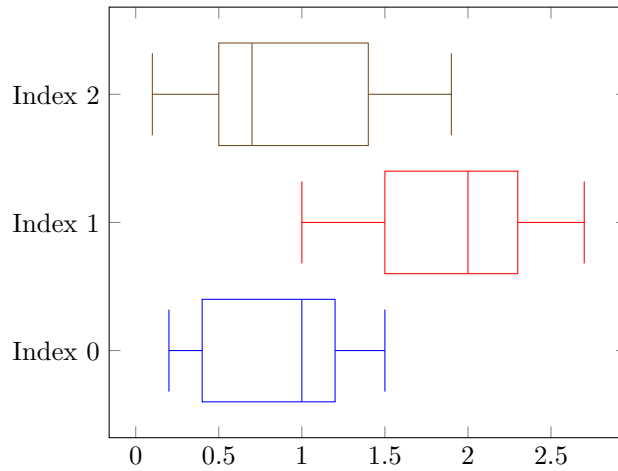


Figure 2: Comparing approximation ratio over instances of Steiner tree problem

The ratio in Figure 2 is the average (arithmetic mean) of several instances for each kind. R100KEFST, is the instance with fractional weight, Euclidean distance. TSPFST, PUC, P6Z and LIN are the instances of *SteinLib*[6] testsets.



6 Conclusions

We have presented an implementation of an approximation algorithm for Steiner tree problem. The algorithm has approximation ratio of $2 - \frac{2}{l}$, where l is the minimum number of leaves in any Steiner tree for the given graph. The algorithm running time is dominated by a single shortest path problem which is computed in our code using Dijkstra algorithm and priority queue which yields the running time of $\mathcal{O}(|E|\log|V|)$.

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