Introduction to Information Retrieval

http://informationretrieval.org

IIR 14: Vector Classification

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Overview

Outline

Naive Bayes classification rule

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \left[\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c) \right]$$

- Each conditional parameter $\log \hat{P}(t_k|c)$ is a weight that indicates how good an indicator t_k is for c.
- The prior $\log \hat{P}(c)$ is a weight that indicates the relative frequency of c.
- The sum of log prior and term weights is then a measure of how much evidence there is for the document being in the class.
- We select the class with the most evidence.

Parameter estimation

- Prior:

$$\hat{P}(c) = \frac{N_c}{N}$$

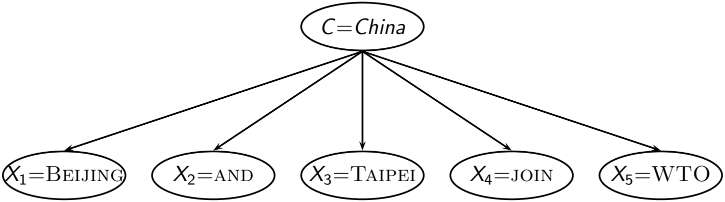
  where N_c is the number of docs in class c and N the total number of docs

- Conditional probabilities:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)}$$

  where T_{ct} is the number of tokens of t in training documents from class c (includes multiple occurrences)

Add-one smoothing to avoid zeros



- In this example:

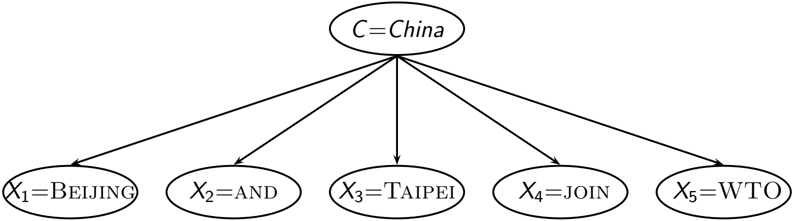
  $P(China|d) \propto P(China)P(\text{Beijing}|China)P(\text{and}|China)P(\text{Taipei}|China)P(\text{join}|China)P(\text{WT}$

- If there are no occurrences of WTO in documents in class China, we get a zero estimate for the corresponding parameter:

$$\hat{P}(\text{WTO}|China) = \frac{T_{China,\text{WTO}}}{\sum_{t' \in V} T_{China,t'}} = 0$$

- With this estimate: $[d$ contains $\text{WTO}] \rightarrow [P(China|d) = 0]$.
- We must smooth to get a better estimate $P(China|d) > 0$.

Naive Bayes Independence Assumption



$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$

- Generate a class with probability $P(c)$
- Generate each of the words (in their respective positions), conditional on the class, but independent of each other, with probability $P(t_k|c)$

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- Eliminating features is called feature selection.

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- Such an incorrect generalization from an accidental property of the training set is called overfitting.
- Feature selection reduces overfitting and improves the accuracy of the classifier.

Basic feature selection algorithm

```
SelectFeatures(𝔻, c, k)
V ← ExtractVocabulary(𝔻)
L ← []
for each t ∈ V
do A(t, c) ← ComputeFeatureUtility(𝔻, t, c)
    Append(L, ⟨A(t, c), t⟩)
return FeaturesWithLargestValues(L, k)
```

Basic feature selection algorithm

$\textsc{SelectFeatures}(\mathbb{D}, c, k)$

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return $\textsc{FeaturesWithLargestValues}(L, k)$

How do we compute A, the feature utility?

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- Frequency – select the most frequent terms
- Mutual information – select the terms with the highest mutual information
- Mutual information is also called information gain in this context.

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- For example, if a term's occurrence is independent of the class (same proportion of docs within/without class contain the term), then MI is 0.
- Definition:

$$I(U;C) = \sum_{e_t \in \{1,0\}} \sum_{e_c \in \{1,0\}} P(U=e_t, C=e_c) \log_2 \frac{P(U=e_t, C=e_c)}{P(U=e_t)P(C=e_c)}$$

How to compute MI values

- Based on maximum likelihood estimates, the formula we actually use is:

$$\begin{aligned} I(U;C) &= \frac{N_{11}}{N}\log_2\frac{NN_{11}}{N_{1.}N_{.1}} + \frac{N_{01}}{N}\log_2\frac{NN_{01}}{N_{0.}N_{.1}} \\ &\quad + \frac{N_{10}}{N}\log_2\frac{NN_{10}}{N_{1.}N_{.0}} + \frac{N_{00}}{N}\log_2\frac{NN_{00}}{N_{0.}N_{.0}} \end{aligned}$$

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\end{aligned}
$$

- N_{10}: number of documents that contain t ($e_t = 1$) and are not in c ($e_c = 0$); N_{11}: number of documents that contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{01}: number of documents that do not contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{00}: number of documents that do not contain t ($e_t = 1$) and are not in c ($e_c = 1$); $N = N_{00} + N_{01} + N_{10} + N_{11}$.

MI example for *poultry*/EXPORT in Reuters

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Plug these values into formula:

$$\begin{aligned} I(U;C) &= \frac{49}{801{,}948}\log_2\frac{801{,}948\cdot 49}{(49+27{,}652)(49+141)} \\ &+ \frac{141}{801{,}948}\log_2\frac{801{,}948\cdot 141}{(141+774{,}106)(49+141)} \\ &+ \frac{27{,}652}{801{,}948}\log_2\frac{801{,}948\cdot 27{,}652}{(49+27{,}652)(27{,}652+774{,}106)} \\ &+ \frac{774{,}106}{801{,}948}\log_2\frac{801{,}948\cdot 774{,}106}{(141+774{,}106)(27{,}652+774{,}106)} \\ &\approx 0.000105 \end{aligned}$$

MI feature selection on Reuters

UK

LONDON	0.1925
UK	0.0755
BRITISH	0.0596
STG	0.0555
BRITAIN	0.0469
PLC	0.0357
ENGLAND	0.0238
PENCE	0.0212
POUNDS	0.0149
ENGLISH	0.0126

China

CHINA	0.0997
CHINESE	0.0523
BEIJING	0.0444
YUAN	0.0344
SHANGHAI	0.0292
HONG	0.0198
KONG	0.0195
XINHUA	0.0155
PROVINCE	0.0117
TAIWAN	0.0108

poultry

POULTRY	0.0013
MEAT	0.0008
CHICKEN	0.0006
AGRICULTURE	0.0005
AVIAN	0.0004
BROILER	0.0003
VETERINARY	0.0003
BIRDS	0.0003
INSPECTION	0.0003
PATHOGENIC	0.0003

coffee

COFFEE	0.0111
BAGS	0.0042
GROWERS	0.0025
KG	0.0019
COLOMBIA	0.0018
BRAZIL	0.0016
EXPORT	0.0014
EXPORTERS	0.0013
EXPORTS	0.0013
CROP	0.0012

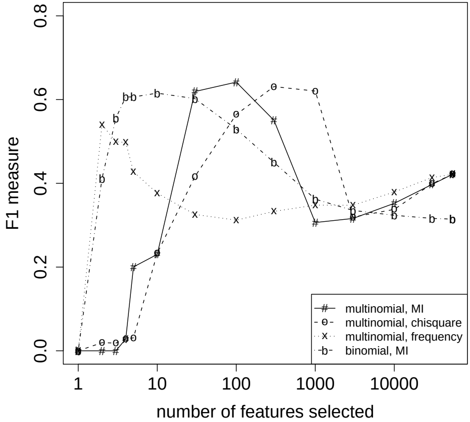
elections

ELECTION	0.0519
ELECTIONS	0.0342
POLLS	0.0339
VOTERS	0.0315
PARTY	0.0303
VOTE	0.0299
POLL	0.0225
CANDIDATE	0.0202
CAMPAIGN	0.0202
DEMOCRATIC	0.0198

sports

SOCCER	0.0681
CUP	0.0515
MATCH	0.0441
MATCHES	0.0408
PLAYED	0.0388
LEAGUE	0.0386
BEAT	0.0301
GAME	0.0299
GAMES	0.0284
TEAM	0.0264

Evaluation of feature selection



Feature selection for Naive Bayes

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- In general, feature selection is necessary for Naive Bayes to get decent performance.
- Also true for most other learning methods in text classification: you need feature selection for optimal performance.

Outline

Recall vector space representation

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- Normalize vectors (documents) to unit length
- How can we do classification in this space?

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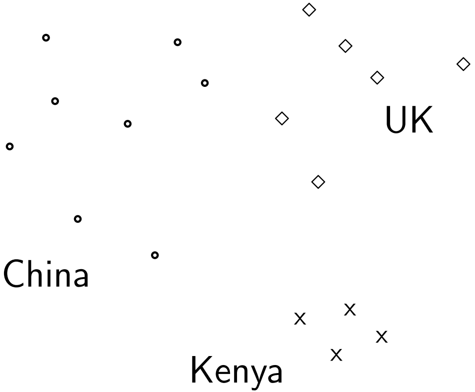
- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space



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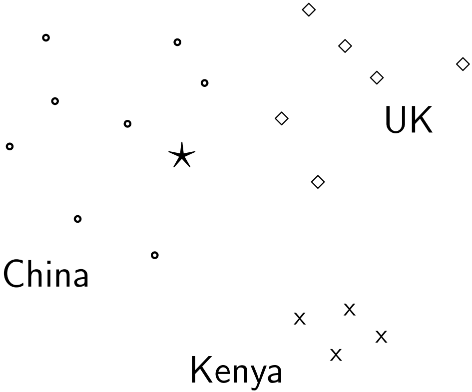
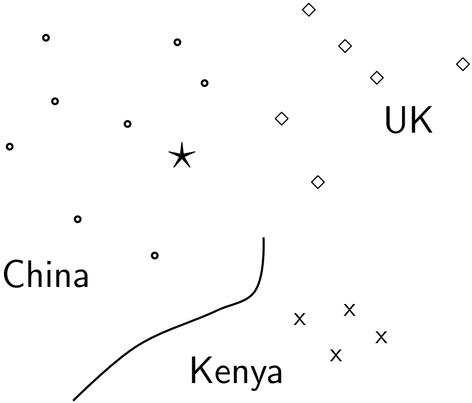


Should the document $\star$ be assigned to *China*, *UK* or *Kenya*?

Classes in the vector space



Find separators between the classes

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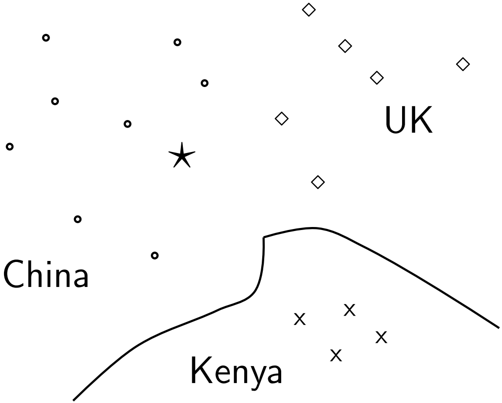

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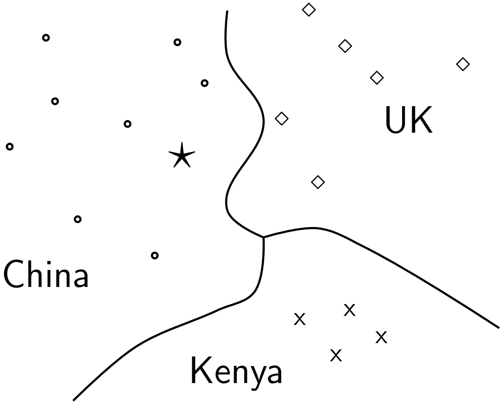

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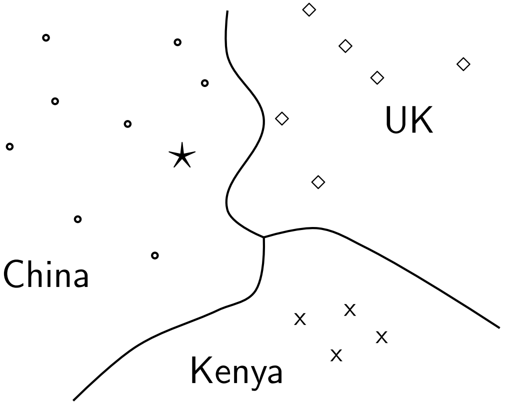
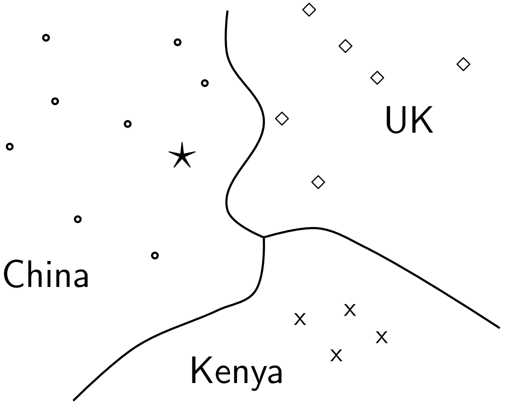


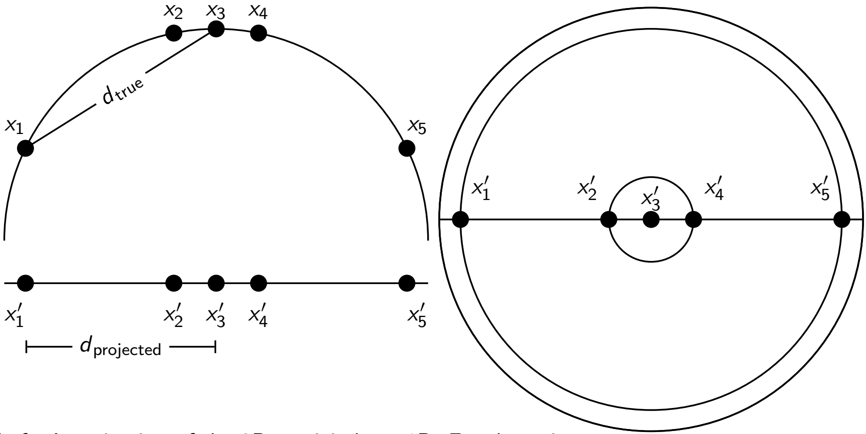
Based on these separators: $\star$ should be assigned to *China*

Classes in the vector space



How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

Aside: 2D/3D graphs can be misleading



Left: A projection of the 2D semicircle to 1D. For the points x_1, x_2, x_3, x_4, x_5 at x coordinates $-0.9, -0.2, 0, 0.2, 0.9$ the distance $|x_2x_3| \approx 0.201$ only differs by 0.5% from $|x'_2x'_3| = 0.2$; but $|x_1x_3|/|x'_1x'_3| = d_{\text{true}}/d_{\text{projected}} \approx 1.06/0.9 \approx 1.18$ is an example of a large distortion (18%) when projecting a large area. *Right:* The corresponding projection of the 3D hemisphere to 2D.

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- The principal difference between relevance feedback and text classification:
    - The training set is given as part of the input in text classification.
    - It is interactively created in relevance feedback.

Rocchio classification: Basic idea

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- Compute a centroid for each class
  - The centroid is the average of all documents in the class.
- Assign each test document to the class of its closest centroid.

Recall definition of centroid

$$\vec{\mu}(c) = \frac{1}{|D_c|} \sum_{d \in D_c} \vec{v}(d)$$

where D_c is the set of all documents that belong to class c and $\vec{v}(d)$ is the vector space representation of d.

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What can we say about the length of this centroid given that each $\vec{v}(d)$ is normalized?

Rocchio algorithm

$\text{TrainRocchio}(\mathbb{C}, \mathbb{D})$

for each $c_j \in \mathbb{C}$

do $D_j \leftarrow \{d : \langle d, c_j \rangle \in \mathbb{D}\}$

$\quad \vec{\mu}_j \leftarrow \frac{1}{|D_j|} \sum_{d \in D_j} \vec{v}(d)$

return $\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}$

$\text{ApplyRocchio}(\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}, d)$

return $\arg\min_j |\vec{\mu}_j - \vec{v}(d)|$

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$\textsc{ApplyRocchio}(\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}, d)$

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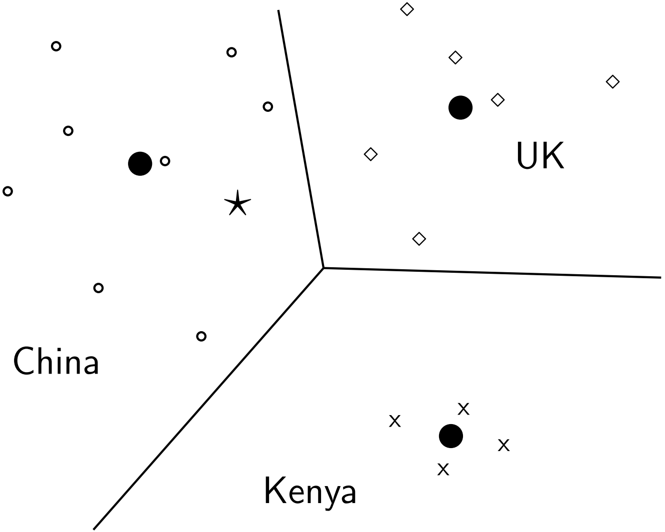
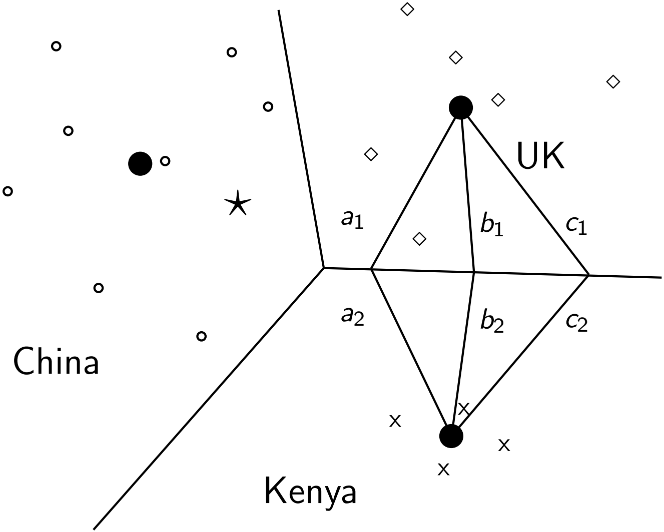
Questions?

Rocchio illustrated



Rocchio illustrated: $a_1 = a_2, b_1 = b_2, c_1 = c_2$



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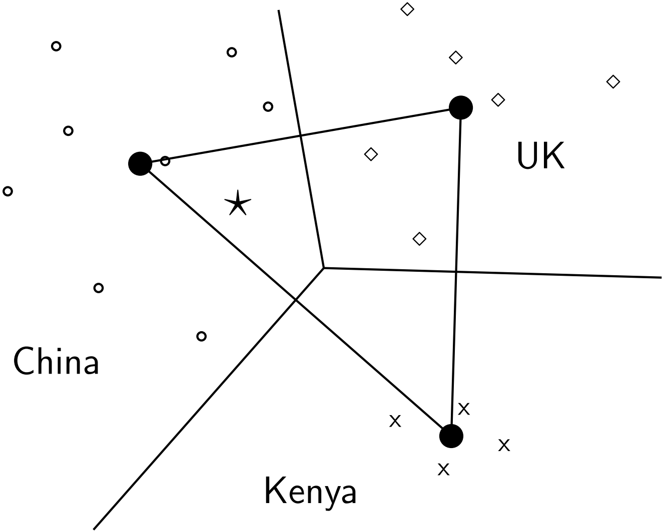

Rocchio properties

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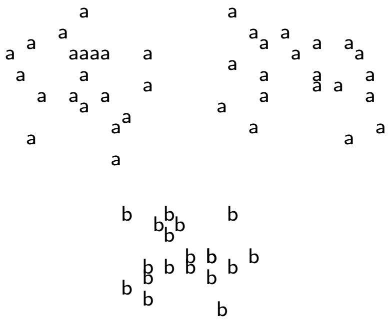
- Classification is based on similarity to / distance from centroid/prototype.
- Does not guarantee that classifications are consistent with the given training data.

Time complexity of Rocchio

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\mathrm{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert)$
testing	$\Theta(L_{\mathrm{a}} + \lvert\mathbb{C}\rvert M_{\mathrm{a}}) = \Theta(\lvert\mathbb{C}\rvert M_{\mathrm{a}})$

Rocchio cannot handle multimodal classes



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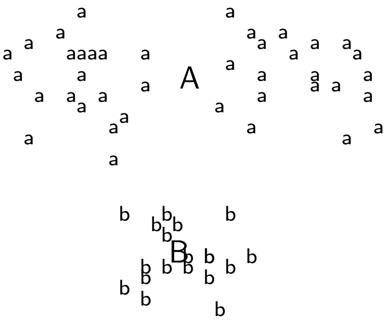

- A is centroid of the a's, B is centroid of the b's.

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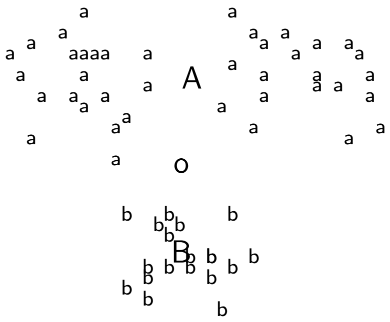

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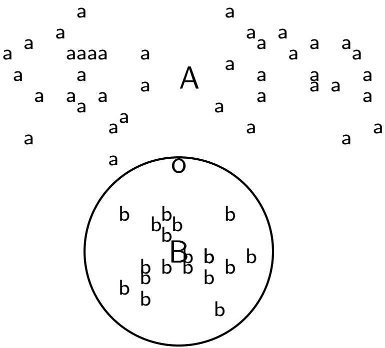

- A is centroid of the a's, B is centroid of the b's.
- The point o is closer to A than to B.
- But it is a better fit for the b class.

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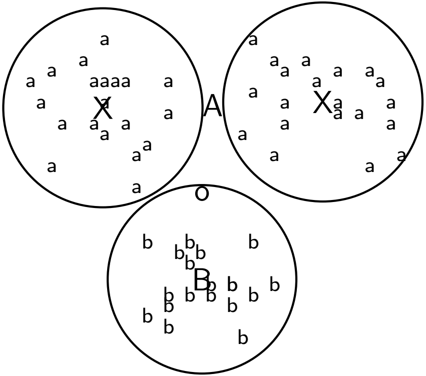

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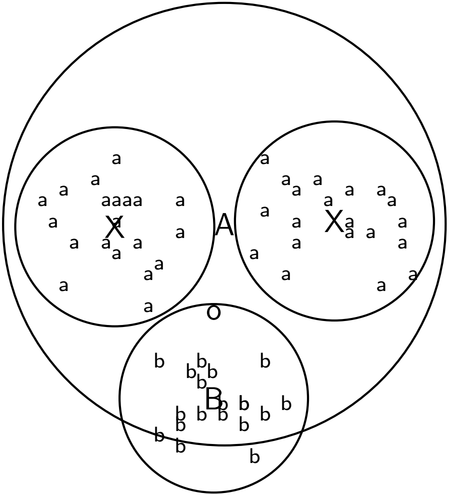

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Outline

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- Methods for finding separator: Perceptron, Rocchio, Naive Bayes – as we will explain on the next slides

Example of a linear two-class classifier

t_i	w_i	d_{1i}	d_{2i}	t_i	w_i	d_{1i}	d_{2i}
prime	0.70	0	1	dlrs	-0.71	1	1
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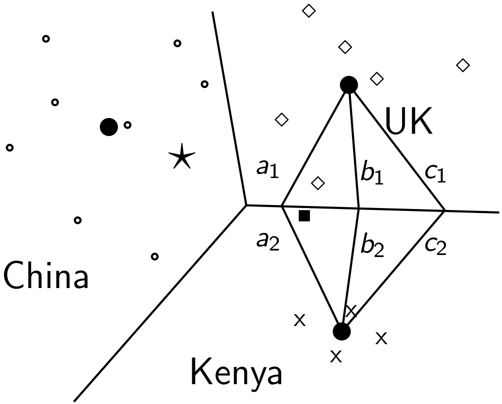
- We assign $\vec{d}_2$ "prime dlrs" to the complement class (not in *interest*) since $\vec{w}^T\vec{d}_2 = -0.01 \leq b$.

perceptron example: one way of finding a separator

Rocchio separators are linear classifiers that can be expressed as $\sum_i w_i x_i > \theta$



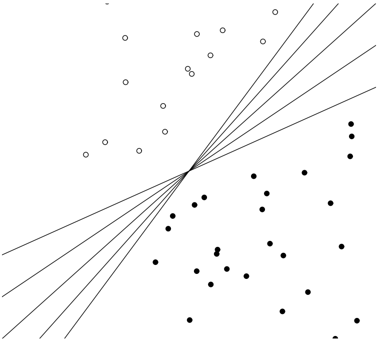
Two-class Rocchio as linear classifier

Line (or plane or hyperplane) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where the normal vector $\vec{w} = \vec{\mu}(c_1) - \vec{\mu}(c_2)$ and
$\theta = 0.5 * (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$.

Which hyperplane?



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- Perceptron: generally bad; Naive Bayes, Rocchio: ok; linear SVM: good
- Many more classification methods

Naive Bayes is also a linear classifier

We can derive the linearity of Naive Bayes from its decision rule, which chooses the category c with the largest $\hat{P}(c|d)$ where:

$$\hat{P}(c|d) \propto \hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)$$

and n_d is the number of tokens in the document that are part of the vocabulary. Denoting the complement category as $\bar{c}$, we obtain for the log odds:

$$\log \frac{\hat{P}(c|d)}{\hat{P}(\bar{c}|d)} = \log \frac{\hat{P}(c)}{\hat{P}(\bar{c})} + \sum_{1 \leq k \leq n_d} \log \frac{\hat{P}(t_k|c)}{\hat{P}(t_k|\bar{c})}$$

We choose class c if the odds are greater than 1 or, equivalently, if the log odds are greater than 0. One can show that this is a linear classifier.

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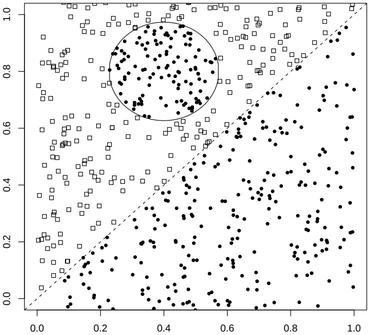
Linear classifiers: Discussion

- Many common text classifiers are linear classifiers: Naive Bayes, Rocchio, logistic regression, linear support vector machines etc.
- Each method has a different way of selecting the separating hyperplane – huge differences in performance.
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

A nonlinear problem



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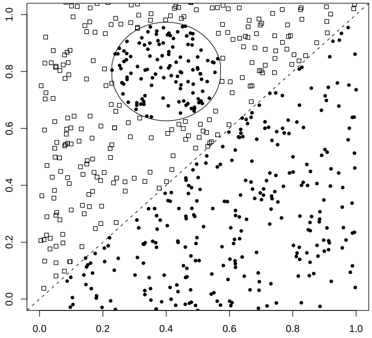

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- kNN will do well (assuming enough training data)

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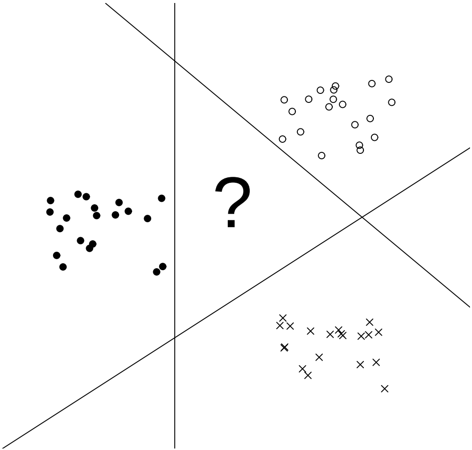
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.

Outline

How to combine hyperplanes for $>$ 2 classes?



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Any-of classification

- Combine two-class classifiers as follows for any-of classification:
  - Simply run each two-class classifier separately on the test document and assign document accordingly

One-of classification

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One-of classification

- Combine two-class classifiers as follows for one-of classification:
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Outline

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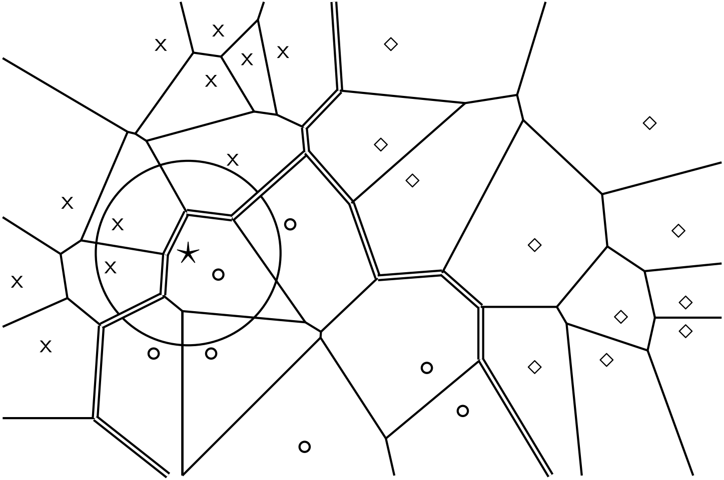
- Rationale of kNN: contiguity hypothesis
  - We expect a test document d to have the same label as the training documents located in the local region surrounding d.

Probabilistic kNN

Probabilistic version of kNN: $P(c|d) =$ fraction of k neighbors of d that are in c

kNN is based on Voronoi tessellation



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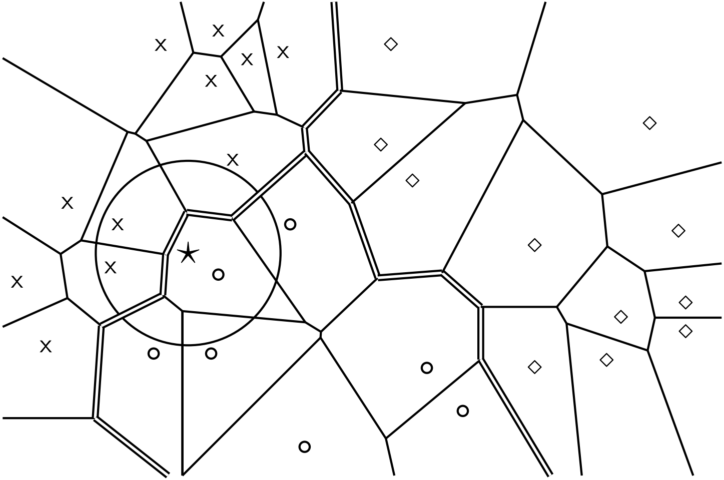

1NN, 2NN, 3NN classification decision for star?

kNN algorithm

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$\mathbb{D}' \leftarrow \text{PREPROCESS}(\mathbb{D})$

$k \leftarrow \text{SELECT-K}(\mathbb{C}, \mathbb{D}')$

return $\mathbb{D}', k$

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$S_k \leftarrow \text{COMPUTENEARESTNEIGHBORS}(\mathbb{D}', k, d)$

for each $c_j \in \mathbb{C}$

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Questions?

Time complexity of kNN

kNN with preprocessing of training set

training $\Theta(|\mathbb{D}|L_{\mathrm{ave}})$

testing $\Theta(L_{\mathrm{a}} + |\mathbb{D}|M_{\mathrm{ave}}M_{\mathrm{a}}) = \Theta(|\mathbb{D}|M_{\mathrm{ave}}M_{\mathrm{a}})$

kNN without preprocessing of training set

training $\Theta(1)$

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- But constant factor much smaller for inverted index than for linear scan.

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- kNN test time proportional to the size of the training set.
  - kNN is inefficient for very large training sets.

Is kNN a linear classifier?

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