Problem Set 1

Due Time:

- Friday, March 2 (**12th Esfand**) , **23:59 PM**

Notes:

- Solution should be submitted in : ***http://Courses.iust.ac.ir/***(Where you may get this problem set)
- If you can't solve a problem, **Don't copy** It's answer.
- Format of file name : **ProblemSet1_[Student ID].zip/rar**
- All problems will be solved in our next class, on **Saturday, 12-13:15 . Don't miss it.**

Additional Resources:

- **CLRS** 3rd – Chapter 2,3,4
- **Foundations of Algorithms** 3rdEdition – Chapter 1

Exercises:

1. Observe that the INSERTION-SORT procedure, uses a linear search to scan (backward) through the sorted subarray A[1 .. j -1] to find proper place of A[j]. Can we use a binary search instead to improve the overall worst-case running time of insertion sort to $\theta(n \lg n)$?<u>Why</u> ?**(2.3-6)**

---

2. Is $2^{n+1} = O(2^n)$?Is$2^{2n} = O(2^n)$?(<u>Prove</u>)**(3.1-4)**
3. <u>Prove</u> that the running time of an algorithm is $\theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$.**(3.1-6)**
4. Is the function $\lceil \lg n \rceil !$ polynomially bounded? Is the function $\lceil \lg \lg n \rceil !$ polynomiallybounded?(<u>Prove</u>)**(3.2-4 **)**
   (We say that a function f(n) is ***Polynomially*** bounded if $f(n) = O(n^k)$ for some constant k)

5. Which is asymptotically larger: $\lg(\lg^* n)$ or $\lg^*(\lg n)$? (Show with an example)**(3.2-5)**
6. Let f(n) and g(n) be asymptotically positive functions. Prove or disprove each of the following conjectures :**(3-4)**
   a. $f(n) + g(n) = \theta(\min(f(n), g(n)))$.
   b. $f(n) = O((f(n))^2)$.
   c. $f(n) = \theta(f(n/2))$.
   d. $f(n) + o(f(n)) = \theta(f(n))$.

---

7. Use the master method, to show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \theta(n^{\log_3 4})$. Then show that a substitution proof with the assumption $T(n) \leq c\, n^{\log_3 4}$ fails and show how to subtract off a lower-order term to make a substitution proof work. **(4.3-7 *)**
8. Use the master method, to show that the solution to the recurrence $T(n) = 4T(n/2) + n^2$ is $T(n) = \theta(n^2)$. Then show that a substitution proof with the assumption $T(n) \leq c\, n^2$ fails and show how to subtract off a lower-order term to make a substitution proof work. **(4.3-8 *)**

Emad Aghajani