4.2 Constructing a Tournament Schedule - revisited

Let us consider once more the construction of a tournament schedule for n players. The problem is described in Section 3.4. This time, we use divide-and-conquer. The following solution is taken from [Aho83]. We consider the design of a round robin tournament schedule for $n = 2^k$ players for an integer k > 1. The divide-and-conquer approach constructs a schedule for one-half of the players. This schedule is designed by a recursive application of the algorithm by finding a schedule for one half of these players and so on. When we get down to two players, we have the base case and we simply pair them up.

Suppose there are eight players. The schedule for players 1 through 4 fills the upper left corner (4 rows by 3 columns) of the schedule being constructed. The lower left corner (4 rows by 3 columns) of the schedule must match the high numbered players (5 through 8) against one another. This sub-schedule is obtained by adding 4 to each entry in the upper left.

Now we have a partial solution to the problem. All that remains is to have lower-numbered players play high-numbered players; or equivalently, fill the top-right and bottom-right sections of the schedule. For the top-right section, this is easily accomplished by having players 1 through 4 play 5 through 8, respectively, on day 4 and cyclically permuting 5 through 8 on subsequent days. Similarly, for the bottom-right section, we have players 5 through 8 play 1 through 4, respectively, on day 4 and cyclically permuting 1 through 4 on subsequent days. The process is illustrated in Figure 4.3. This process can be generalized to construct a schedule for 2^k players for any k.

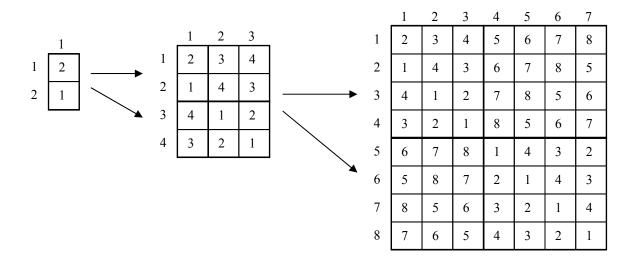


Figure 4.3 Using divide-and-conquer to construct a tournament schedule for 8 players.

Exercise 4.1 Write, with proper explanation, recurrence equations for the running time T(n) of the preceding algorithm.