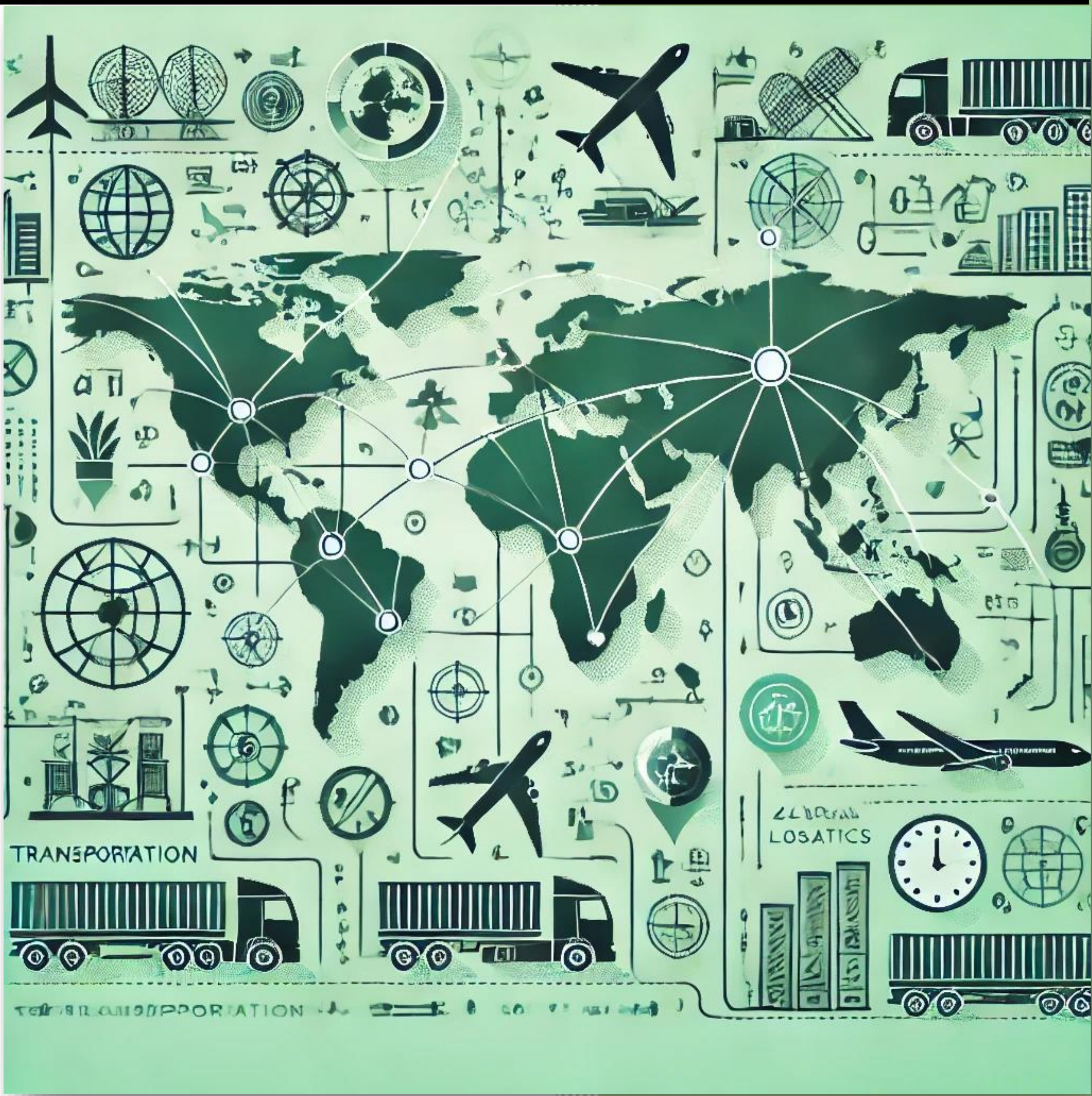
Transportation Models



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2024-2025

Chapter One

Formulation Of Transportation Problem

Introduction:

Transportation is a key part of the economy and logistics, helping move goods and resources efficiently between different locations. It plays a major role in global trade and business by ensuring that raw materials, semi-finished goods, and final products reach their destinations smoothly. A well-planned transportation system supports economic growth, increases business competitiveness, and reduces supply chain costs. Additionally, efficient transportation helps reduce traffic congestion, save fuel, and lower environmental pollution, making it an important factor in sustainable development.

In this context, the **Transportation Problem (TP)** is a well-known optimization problem in **Linear Programming (LP)**. It focuses on finding the most cost-effective way to transport goods from multiple supply locations (such as factories or warehouses) to multiple demand locations (such as stores or distribution centers). The main goal is to minimize transportation costs while making sure that all supply and demand requirements are met. This problem is widely used in industries like retail, healthcare, and manufacturing, where efficient transportation is essential for smooth operations and customer satisfaction. Beyond cost savings, the **Transportation Problem** is also critical in emergency situations, such as disaster relief, military operations, and the delivery of perishable goods, where fast and optimized transportation is necessary.

From a mathematical perspective, the **Transportation Problem** is modeled as a **network flow problem**. Supply locations (such as factories and warehouses) are connected to demand locations (such as distribution centers and retail outlets) through transportation routes, each with a specific cost. The challenge is to determine the best way to allocate shipments to minimize total costs while ensuring that supply and demand constraints are met. This problem has broad applications in supply chain management, logistics, and transportation planning.

Definition:

The Transportation Problem is a special type of Linear Programming Problem (LPP) that deals with determining the most efficient way to transport goods from multiple sources (e.g., factories or warehouses) to multiple destinations (e.g., retail stores or distribution centers) while minimizing the total transportation cost. The problem aims to allocate supply from each source to various destinations in such a way that demand requirements are met while optimizing cost.

Components of the Transportation Problem:

1. **Supply Points:** These are the origins (e.g., factories, suppliers) that provide goods or services. Each supply point has a limited quantity of goods available for distribution.
2. **Demand Points:** These are the destinations (e.g., warehouses, customers) that require a certain amount of goods.
3. **Transportation Costs:** Each route between a supply and demand point has an associated cost per unit transported.
4. **Decision Variables:** Represent the quantity of goods transported from each source to each destination.
5. **Objective Function:** The function to be minimized, which represents the total transportation cost.
6. **Constraints:**
   - **Supply Constraints:** Ensure that the total goods transported from each source do not exceed the available supply.
   - **Demand Constraints:** Ensure that the total goods received at each destination meet the required demand.
   - **Non-negativity Constraints:** Ensure that the transported quantities are non-negative.

Mathematical Formulation

Let:

- x_{ij} be the number of units transported from source i to destination j.
- c_{ij} be the transportation cost per unit from i to j.
- S_i be the supply available at source i.
- D_j be the demand required at destination j.

The Objective Function:

$$\text{Minimize} \quad Z = \sum_i \sum_j c_{ij} x_{ij}$$

where:

- Z represents the total transportation cost.
- c_{ij} is the cost per unit transported from source i to destination j.
- x_{ij} is the number of units transported.

Subject to:

$$\sum_j x_{ij} \leq s_i, \quad \forall i$$

$$\sum_i x_{ij} \geq d_j, \quad \forall j$$

$$x_{ij} \geq 0, \quad \forall i, j$$

Examples Of Transportation Problems

Example 1: Factory to Warehouses

A company has three factories **(A, B, C)** and two warehouses **(X, Y).** The supply and demand are as follows:

Factory Supply Table:

Factory	Supply
A	30
B	40
C	20

Processing Warehouse Demand Table:

Warehouse	Demand
X	50
Y	40

The transportation cost per unit between each factory and warehouse is given in the table:

Transportation Cost Table:

	X	Y
A	8	6
B	5	9
C	6	4

Example 2: Supplier to Retail Stores

A supplier needs to distribute products to three retail stores **(R1, R2, R3).** The available stock and required demand are:

Supplier Supply Table:

Supplier	Supply
S1	50
S2	30

Processing Retail-Store Demand Table:

Retail Store	Demand
R1	40
R2	20
R3	20

The transportation costs per unit between each supplier and retail-sore is given in the table:

Transportation Cost Table:

	R1	R2	R3
S1	3	7	5
S2	6	4	2

Example 3: Distribution Center to Supermarkets

A distribution center needs to deliver goods to four supermarkets **(M1, M2, M3, M4)**. The available stock and required demand are:

Distribution-Center Supply Table:

Distribution Center	Supply
D1	100

Processing Supermarket Demand Table:

Supermarket	Demand
M1	20
M2	30
M3	25
M4	25

The transportation cost per unit between the distribution center and each supermarket is given in the table:

Transportation Cost Table:

	M1	M2	M3	M4
D1	4	2	5	3

Example 4: Farms to Food Processing Plants

A company sources raw agricultural products from three farms (F1, F2, F3) and transports them to two food processing plants (P1, P2). The available supply at each farm and the demand at each plant are as follows:

Farm Supply Table:

Farm	Supply
F1	50
F2	60
F3	40

Processing Plant Demand Table:

Processing Plant	Demand
P1	80
P2	70

The transportation cost per unit between each farm and processing plant is given in the table below:

Transportation Cost Table:

Farm	P1	P2
F1	4	6
F2	7	5
F3	3	8

Example 5: Shipping Ports to Distribution Hubs

A logistics company manages the transport of goods from three major shipping ports (S1, S2, S3) to four distribution hubs (H1, H2, H3, H4). The supply at each port and the demand at each hub are as follows:

Port Supply Table:

Port	Supply
S1	100
S2	80
S3	60

Distribution Hub Demand Table:

Hub	Demand
H1	50
H2	60
H3	70
H4	60

The transportation cost per unit between each port and hub is given in the table below:

Transportation Cost Table:

Port	H1	H2	H3	H4
S1	5	3	6	7
S2	4	8	2	5
S3	7	6	3	4

Conclusion:

All the provided examples represent **real-world balanced transportation problems**, where goods need to be transported from multiple sources to multiple destinations while minimizing costs.

Observation On The Examples:

- A transportation problem is **balanced** if the total supply equals the total demand.
- In a balanced problem, no additional adjustments (such as dummy sources or destinations) are needed.
- The goal in each example is to determine the optimal allocation of shipments to meet demand while **minimizing transportation costs**.
- These problems can be solved using **North-West Corner Method, Least Cost Method, or Vogel's Approximation Method**, each providing a different approach to finding an optimal initial solution.

Chapter Two
Northwest Corner Cell Method

Introduction

In the realm of Operations Research, numerous methods exist to tackle transportation problems, but one approach stands out for its simplicity and effectiveness: the Northwest Corner Cell Method. This technique serves as a compass, guiding us toward the optimal solution, thereby minimizing total cost and enhancing transportation efficiency. In this introduction, we will delve into the application of the Northwest Corner Cell Method and explore how it can facilitate the resolution of transportation problems with ease and speed.

Advantages of the Northwest Corner Cell Method

- Simple: The Northwest Corner Cell Method is simple and easy understand.

- Fast: The Northwest Corner Cell Method can be applied quickly.

- Cost Reduction: The Northwest Corner Cell Method aims to reduce the total transportation cost.

- Easy to Apply: The Northwest Corner Cell Method can be applied to simple transportation problems.

- Time-Saving: The Northwest Corner Cell Method can save time in solving transportation problems.

Disadvantages of the Northwest Corner Cell Method

- Does Not Guarantee Optimal Solution: The Northwest Corner Cell Method does not always guarantee the optimal solution.

- Depends on Cell Order: The order of cells in the table can affect the result of the Northwest Corner Cell Method.

- Not Suitable for Complex Transportation Problems: The Northwest Corner Cell Method is not suitable for complex transportation problems that require more complex solutions.

- Does Not Consider Other Constraints: The Northwest Corner Cell Method does not consider other constraints that may affect the transportation problem.

- Does Not Provide Flexible Solutions: The Northwest Corner Cell Method does not provide flexible solutions that can adapt to changes in the transportation problem.

Method of Solution

Step 1: Initialize the Table

Initialize the transportation table with cells containing transportation costs between sources and destinations. Determine the demand quantities and available quantities.

Step 2: Allocate Quantities to Cells

Start allocating transportation quantities to cells in the Northwest Corner (top-left corner) of the table. Allocate the maximum possible quantity to the first cell.

Step 3: Allocate Remaining Quantities

After allocating quantities to cells in the Northwest Corner, allocate remaining quantities to other cells in the table. Allocate remaining quantities to cells that follow in the horizontal or vertical direction.

Step 4: Repeat the Process

Repeat the process until all quantities are allocated. Update the table after each allocation.

Step 5: Calculate Total Cost

Calculate the total transportation cost by summing the transportation costs of each cell. Analyze the result and ensure that the total cost is minimized.

Mathematical rule :

Total Cost = ∑(Allocated Units × Cost per Unit)

These are the steps of the Northwest Corner Cell Method for solving transportation problems. This method is simple and effective, but it may not always guarantee the optimal solution.

Illustrative Example:

Given three sources O1, O2, and O3 and four destinations D1, D2, D3, and D4. For the sources O1, O2, and O3, the supply is 300, 400, and 500 respectively. The destinations D1, D2, D3, and D4 have demands of 250, 350, 400, and 200 respectively.

Solution

Initial Table

Source/Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	250	350	400	200	

After First Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	1	7	4	50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	0	350	400	200	

After Second Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	0	300	400	200	

After Third Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	5	9	100
O3	8	3	3	2	500
Demand	0	0	400	200	

After Fourth Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	3	2	500
Demand	0	0	300	200	

After Fifth Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	300	2	200
Demand	0	0	0	200	

Final Table (All Demand Fulfilled)

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	300	200	0
Demand	0	0	0	0	

Final Calculation:

Now just multiply the allocated values with their respective costs and sum them:

(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400

Thus, the initial basic feasible solution = 4400.

Examples of Northwest corner method

Example 1: Solving a Transportation Problem

Consider a transportation problem with three sources (O1, O2, O3) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	250	350	400	200	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 250 units from O1 to D1.

- Step 2: Allocate 50 units from O1 to D2.

- Step 3: Allocate 300 units from O2 to D2.

- Step 4: Allocate 100 units from O2 to D3.

- Step 5: Allocate 300 units from O3 to D3.

- Step 6: Allocate 200 units from O3 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	-	-	0
O2	-	300	100	-	0
O3	-	-	300	200	0
Demand	0	0	0	0	

Total Cost Calculation :

(250 × 3) + (50 × 1) + (300 × 6) + (100 × 5) + (300 × 3) + (200 × 2)

= 4400 dollars

Example 2: Another Transportation Problem

Now, consider another transportation problem with four sources (O1, O2, O3, O4) and three destinations (D1, D2, D3).

Source/Destination	D1	D2	D3	Supply
O1	2	3	1	180
O2	5	4	8	200
O3	5	6	3	150
O4	7	2	4	170
Demand	220	150	330	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 180 units from O1 to D1.

- Step 2: Allocate 40 units from O2 to D1.

- Step 3: Allocate 150 units from O2 to D2.

- Step 4: Allocate 10 units from O2 to D3.

- Step 5: Allocate 150 units from O3 to D3.

- Step 6: Allocate 170 units from O4 to D3.

Final Allocation Table :

Source/Destination	D1	D2	D3	Supply
O1	180	-	-	0
O2	40	150	10	0
O3	-	-	150	0
O4	-	-	170	0
Demand	0	0	0	

Total Cost Calculation :

(180 × 2) + (40 × 5) + (150 × 4) + (10 × 8) + (150 × 3) + (170 × 4)

= 360 + 200 + 600 + 80 + 450 + 680

= 2370 dollars

Example 3: A Moderate Difficulty Transportation Problem

Consider a transportation problem with four sources (O1, O2, O3, O4) and four destinations (D1, D2, D3, D4). The supply and demand are balanced.

Source/Destination	D1	D2	D3	D4	Supply
O1	4	3	8	6	150
O2	2	7	5	9	250
O3	6	5	7	3	200
O4	3	6	9	4	180
Demand	100	200	220	260	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 100 units from O1 to D1.

- Step 2: Allocate 50 units from O1 to D2.

- Step 3: Allocate 150 units from O2 to D2.

- Step 4: Allocate 100 units from O2 to D3.

- Step 5: Allocate 120 units from O3 to D3.

- Step 6: Allocate 80 units from O3 to D4.

- Step 7: Allocate 180 units from O4 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	100	50	-	-	0
O2	-	150	100	-	0
O3	-	-	120	80	0
O4	-	-	-	180	0
Demand	0	0	0	0	

Total Cost Calculation :

$(100 \times 4) + (50 \times 3) + (150 \times 7) + (100 \times 5) + (120 \times 7) + (80 \times 3) + (180 \times 4)$

$= 3900$ dollars

Example 4: Solving a Transportation Problem

Consider a transportation problem with three sources (O1, O2, O3) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	5	8	6	7	300
O2	3	7	4	6	500
O3	6	5	7	3	400
Demand	250	400	300	250	

Solution steps using the Northwest Corner Method:

-Step 1 : Allocate 250 units from O1 to D1.

-Step 2 : Allocate 50 units from O1 to D2.

-Step 3 : Allocate 350 units from O2 to D2.

-Step 4 : Allocate 150 units from O2 to D3.

-Step 5 : Allocate 150 units from O3 to D3.

-Step 6 : Allocate 250 units from O3 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	-	-	0
O2	-	350	150	-	0
O3	-	-	150	250	0
Demand	0	0	0	0	

Total Cost Calculation :

$(250 \times 5)+(50 \times 8)+(350 \times 7)+(150 \times 4)+(150 \times 7)+(250 \times 3)$
$=1250+400+2450+600+1050+750$
=6500 dollars

Example 5: Another Transportation Problem

Now, consider another transportation problem with four sources (O1, O2, O3, O4) and three destinations (D1, D2, D3).

Source/Destination	D1	D2	D3	Supply
O1	4	6	5	220
O2	7	3	2	300
O3	5	8	6	150
O4	6	4	7	180
Demand	250	200	400	

Solution steps using the Northwest Corner Method:

-Step 1 : Allocate 220 units from O1 to D1.

-Step 2 : Allocate 30 units from O2 to D1.

-Step 3 : Allocate 200 units from O2 to D2.

-Step 4 : Allocate 70 units from O2 to D3.

-Step 5 : Allocate 150 units from O3 to D3.

-Step 6 : Allocate 180 units from O4 to D3.

Final Allocation Table :

Source/Destination	D1	D2	D3	Supply
O1	220	-	-	0
O2	30	200	70	0
O3	-	-	150	0
O4	-	-	180	0
Demand	0	0	0	

Total Cost Calculation :

$(220\times4)+(30\times7)+(200\times3)+(70\times2)+(150\times6)+(180\times7)$

= 3990 dollars

Example 6: A More Challenging Transportation Problem

Consider a transportation problem with four sources (O1, O2, O3, O4) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	3	5	6	8	200
O2	4	7	3	5	300
O3	6	4	5	7	250
O4	5	3	7	4	200
Demand	150	250	200	350	

Solution Steps using Northwest Corner Method

-Step 1 : Allocate 150 units from O1 to D1.

-Step 2 : Allocate 50 units from O1 to D2.

-Step 3 : Allocate 200 units from O2 to D2.

-Step 4 : Allocate 100 units from O2 to D3.

-Step 5 : Allocate 100 units from O3 to D3.

-Step 6 : Allocate 150 units from O3 to D4.

-Step 7 : Allocate 200 units from O4 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	150	50	-	-	0
O2	-	200	100	-	0
O3	-	-	100	150	0
O4	-	-	-	200	0
Demand	0	0	0	0	

Total Cost Calculation :

$(150 \times 3)+(50 \times 5)+(200 \times 7)+(100 \times 3)+(100 \times 5)+(150 \times 7)+(200 \times 4)$

$=4750$ dollars

Chapter Three

Vogel's Approximation Method

Introduction

The transportation problem is a fundamental model in operations research that focuses on determining the most cost-effective way to transport goods or services from multiple sources to multiple destinations, while satisfying supply and demand constraints. To solve such problems efficiently, it is essential to first obtain an **Initial Basic Feasible Solution (IBFS)**, which serves as the starting point for more advanced optimization methods such as the Modified Distribution Method (MODI).

Among the various techniques used to find an IBFS, **Vogel's Approximation Method (VAM)** is one of the most effective and widely adopted heuristics. VAM aims to minimize the total transportation cost by considering the opportunity cost of not choosing the next best route. It does so by calculating a "penalty" for each row and column, which is defined as the difference between the two lowest transportation costs. The algorithm then selects the row or column with the highest penalty and allocates supply and demand to the cell with the minimum cost in that row or column. This process is repeated until all supply and demand values are allocated.

VAM is known for producing solutions that are often very close to the optimal solution, making it highly valuable in real-world applications, especially when dealing with large-scale transportation and logistics problems. Its structured and cost-sensitive approach makes it preferable over other IBFS methods such as the Northwest Corner Rule or the Least Cost Method.

Advantages of Vogel's Approximation Method

1. **Provides a Better Initial Solution**
   VAM typically generates an initial solution that is closer to the optimal one compared to other methods such as the Northwest Corner Rule or the Least Cost Method. This leads to faster convergence during the optimization phase.
2. **Efficient in Cost Minimization**
   By considering penalties—the difference between the two lowest costs in each row or column—VAM prioritizes allocations that have the greatest impact on reducing the total transportation cost.
3. **Systematic and Logical Procedure**
   The method follows a structured step-by-step process, making it easy to understand, implement, and explain. Its logical basis enhances its reliability for practical applications.
4. **Reduces Computational Effort in Optimization**
   Since VAM often produces a near-optimal initial solution, it significantly reduces the number of iterations required when using optimization methods such as the MODI (Modified Distribution) Method or the Stepping Stone Method.
5. **Practical for Real-World Applications**
   VAM is highly applicable in real-life logistics and transportation scenarios, particularly where minimizing cost is a primary objective. It offers an efficient starting point for complex distribution networks.
6. **Considers Opportunity Cost**
   Unlike simpler methods, VAM accounts for the opportunity cost of choosing a specific route over others, which leads to more informed and effective decisions.
7. **Handles Both Balanced and Unbalanced Problems**
   VAM can be applied to both balanced and unbalanced transportation problems. If the total supply does not equal the total demand, the problem can be balanced by introducing a dummy row or column.
8. **Minimizes Initial Total Cost**
   The initial allocation generated by VAM tends to result in a relatively low total cost, making it a cost-effective method for initiating the transportation model.

Disadvantages of Vogel's Approximation Method

1. **Does Not Guarantee the Optimal Solution**
   VAM provides only an initial basic feasible solution. Although it is often close to optimal, it does not guarantee the minimum total cost and may still require further optimization using methods like MODI or Stepping Stone.
2. **Relatively Complex Compared to Simpler Methods**
   Compared to the Northwest Corner Rule or the Least Cost Method, VAM involves additional steps such as penalty calculation, which can make it slightly more time-consuming and complex to implement manually.
3. **Penalties May Be Misleading in Some Cases**
   The penalty calculation is based solely on cost differences and does not always reflect the true impact of an allocation on the total cost, which may lead to suboptimal initial choices.
4. **Can Be Time-Consuming for Large Problems**
   For transportation problems involving a large number of sources and destinations, the repeated calculation of penalties and updates can become tedious and computationally intensive without software tools.
5. **Ignores Supply and Demand Volatility**
   VAM focuses primarily on cost optimization and does not consider real-world factors such as fluctuating supply or demand, transportation delays, or capacity constraints.
6. **May Require Ties to Be Broken Arbitrarily**
   When penalties are equal across multiple rows or columns, the method requires an arbitrary choice, which could affect the quality of the initial solution.

Less Effective in Highly Unbalanced Cost Structures

In cases where transportation costs vary drastically, VAM may not always prioritize the most cost-effective routes due to its reliance on relative differences rather than absolute cost values

Method of Solution

Step 1: Check if the problem is balanced

This means the total supply = total demand.
If they are not equal, you need to add a dummy row or column with zero values to balance the problem.

Step 2: Calculate the Penalty

The penalty is the difference between the two smallest costs in each row and column.

How to calculate it?
In each row (or column):

- Take the two smallest numbers (the two lowest costs)
- Subtract them from each other
- This will give you the penalty for that row or column.

Step 3: Select the row or column with the highest penalty

This means we will start by distributing in the place with the largest difference in costs, in order to minimize the overall cost as much as possible.

Step 4: In the selected row or column, find the minimum cost cell

Choose the cell with the lowest cost, and allocate units based on:

- The minimum of the supply and demand values.

Step 5: Update the table

- Subtract the allocated value from both the supply and the demand.
- If the supply for a row is finished (becomes zero): cross out the row.
- If the demand for a column is finished (becomes zero): cross out the column.

Step 6: Repeat steps 2-5

- Go back and recalculate the penalties for the remaining rows and columns.
- Select the highest penalty.
- Allocate and update the table again.
- Continue until all supply and demand values are zero.

Step 7: Calculate the total cost

After all the distributions are completed, multiply the units allocated in each cell by the cost in that cell.
Then, sum all the values to get the total cost.

Examples of Vogel's Approximation Method

Example 1:

Use Vogel's Approximation Method (VAM) to find the initial feasible solution for the following transportation problem:

	D1	D2	D3	Supply
S1	4	6	8	50
S2	3	7	5	60
S3	2	4	7	40
Demand	40	50	60	

Solution

Step 1:

- Total supply = 50 + 60 + 40 = 150
- Total demand = 40 + 50 + 60 = 150

Since the total supply equals the total demand, the problem is balanced.

Step 2:

- **Row S1**: The lowest costs are 4 and 6.
  Penalty = 6 - 4 = 2
- **Row S2**: The lowest costs are 3 and 5.
  Penalty = 5 - 3 = 2
- **Row S3**: The lowest costs are 2 and 4.
  Penalty = 4 - 2 = 2
- **Column D1**: The lowest costs are 2 and 3.
  Penalty = 3 - 2 = 1

- **Column D2**: The lowest costs are 4 and 6.
  Penalty = 6 - 4 = 2
- **Column D3**: The lowest costs are 5 and 7.
  Penalty = 7 - 5 = 2

Step 3:

The highest penalty is 2, and it appears in Rows S1, S2, S3, D2, D3.
We select Row S1.

Step 4:

In Row S1, the minimum cost is 4 (S1 → D1).
Allocate 40 units to S1 → D1 (since the demand for D1 is 40).

Step 5:

After allocating 40 units to S1 → D1, update the table:

- S1's remaining supply = 50 - 40 = 10
- D1's remaining demand = 40 - 40 = 0

Updated table:

	D1	D2	D3	Supply
S1	4	6	8	10
S2	3	7	5	60
S3	2	4	7	40
Demand	0	50	60	

Step 6:

Now that D1 has been satisfied, recalculate the penalties:

- **Row S1**: The remaining costs are 6 and 8.
  Penalty = 8 - 6 = 2
- **Row S2**: The lowest costs are 3 and 5.
  Penalty = 5 - 3 = 2
- **Row S3**: The lowest costs are 2 and 4.
  Penalty = 4 - 2 = 2
- **Column D2**: The lowest costs are 6 and 7.
  Penalty = 7 - 6 = 1
- **Column D3**: The lowest costs are 5 and 7.
  Penalty = 7 - 5 = 2

Now, choose Row S2 (since it has the highest penalty of 2).

Step 7:

In Row S2, the minimum cost is 3 (S2 → D1), but D1 is already satisfied. The next lowest cost is 5 (S2 → D3).

Allocate 60 units to S2 → D3 (since D3 has a demand of 60).

Step 8:

After allocating 60 units to S2 → D3, update the table:

- S2's remaining supply = 60 - 60 = 0
- D3's remaining demand = 60 - 60 = 0

Updated table:

	D1	D2	D3	Supply
S1	4	6	8	10
S2	3	7	5	0
S3	2	4	7	40
Demand	0	50	0	

Step 9:

Now, choose Row S3, since S2 has no remaining supply.

In Row S3, the minimum cost is 2 (S3 → D1), but D1 has no demand. We move to the next column.

In Row S3, the next minimum cost is 4 (S3 → D2).

Allocate 30 units to S3 → D2 (since D2 has a remaining demand of 50).

Step 10:

After allocating 30 units to S3 → D2, update the table:

- S3's remaining supply = 40 - 30 = 10
- D2's remaining demand = 50 - 30 = 20

Updated table:

	D1	D2	D3	Supply
S1	4	6	8	10
S2	3	7	5	0
S3	2	4	7	10
Demand	0	20	0	

Step 11:

Now allocate the remaining 10 units from S3 to D2 (D2 still has 20 demand). Allocate 10 units to S3 → D2.

The final updated table:

	D1	D2	D3	Supply
S1	4	6	8	10
S2	3	7	5	0
S3	2	4	7	0
Demand	0	10	0	

Final Solution:

- S1 → D1: 40 units at cost 4
- S2 → D3: 60 units at cost 5
- S3 → D2: 30 units at cost 4

Step 12:

Now, calculate the total transportation cost:

- S1 → D1: 40 × 4 = 160
- S2 → D3: 60 × 5 = 300
- S3 → D2: 30 × 4 = 120

Total cost Calculation :

160 + 300 + 120 = 580

Example 2:

Use Vogel's Approximation Method (VAM) to find the initial feasible solution for the following transportation problem:

	D1	D2	D3	Supply
S1	8	6	10	50
S2	4	9	6	60
S3	3	7	5	40
Demand	40	50	60	

Step 1:

- Total supply = 50 + 60 + 40 = 150
- Total demand = 40 + 50 + 60 = 150

The problem is balanced, so we can apply VAM directly.

Step 2:

We calculate the penalty = difference between the two smallest costs in each row and column:

- **Row S1**: [8, 6, 10] → smallest: 6, second smallest: 8 → Penalty = 8 - 6 = **2**
- **Row S2**: [4, 9, 6] → smallest: 4, second smallest: 6 → Penalty = 6 - 4 = **2**
- **Row S3**: [3, 7, 5] → smallest: 3, second smallest: 5 → Penalty = 5 - 3 = **2**
- **Column D1**: [8, 4, 3] → smallest: 3, second smallest: 4 → Penalty = 4 - 3 = **1**
- **Column D2**: [6, 9, 7] → smallest: 6, second smallest: 7 → Penalty = 7 - 6 = **1**
- **Column D3**: [10, 6, 5] → smallest: 5, second smallest: 6 → Penalty = 6 - 5 = **1**

Step 3:

Maximum penalty = 2, appears in S1, S2, S3.
Let's choose Row S1 (any of the max penalty rows is valid).

Step 4:

- In S1 → [8, 6, 10] → lowest cost is 6 at S1 → D2
- D2 demand = 50, S1 supply = 50
  → Allocate 50 units to S1 → D2

Allocation:

- S1 → D2 = 50 units
- S1 supply becomes 0
- D2 demand becomes 0

Step 5:

Since S1 is used up (supply = 0), we remove it from the table.

Updated table:

	D1	D2	D3	Supply
S2	4	9	6	60
S3	3	7	5	40
Demand	40	0	60	

Step 6:

- **Row S2**: [4, 9, 6] → 4 & 6 → Penalty = 6 - 4 = **2**
- **Row S3**: [3, 7, 5] → 3 & 5 → Penalty = 5 - 3 = **2**
- **Column D1**: [4, 3] → 3 & 4 → Penalty = 4 - 3 = **1**
- **Column D3**: [6, 5] → 5 & 6 → Penalty = 6 - 5 = **1**

(D2 is already satisfied, skip it)

Choose Row S2 (Penalty = 2)

Step 7:

Allocate to the minimum cost in S2

- Row S2: [4, –, 6] → minimum is 4 at S2 → D1
- D1 demand = 40, S2 supply = 60
  → Allocate 40 units to S2 → D1

Allocation:

- S2 → D1 = 40 units
- S2 remaining supply = 60 - 40 = 20
- D1 demand = 0

Step 8:

Remove D1 (demand satisfied)

	D2	D3	Supply
S2	9	6	20
S3	7	5	40
Demand	0	60	

Step 9: Recalculate penalties

- **Row S2**: [9, 6] → Penalty = 9 - 6 = **3**
- **Row S3**: [7, 5] → Penalty = 7 - 5 = **2**
- **Column D3**: [6, 5] → Penalty = 6 - 5 = **1**

Choose **Row S2** (Penalty = 3)

Step 10:

Allocate to minimum cost in S2

- Row S2: [9, 6] → minimum is **6** at **S2 → D3**
- D3 demand = 60, S2 supply = 20
  Allocate **20 units** to **S2 → D3**

Allocation:

- S2 → D3 = 20 units
- S2 supply = 0
- D3 demand = 60 - 20 = **40**

Step 11:

Remove S2 (supply used)

	D2	D3	Supply
S3	7	5	40
Demand	0	40	

Step 12: Only one row and one column left

- S3 → D3 = allocate 40 units

Allocation:

- S3 → D3 = 40 units
- S3 supply = 0
- D3 demand = 0

Final Allocations:

From → To	Allocation	Cost per unit	Total Cost
S1 → D2	50	6	300
S2 → D1	40	4	160
S2 → D3	20	6	120
S3 → D3	40	5	200

Total cost calculation :

300 + 160 + 120 + 200 = 780