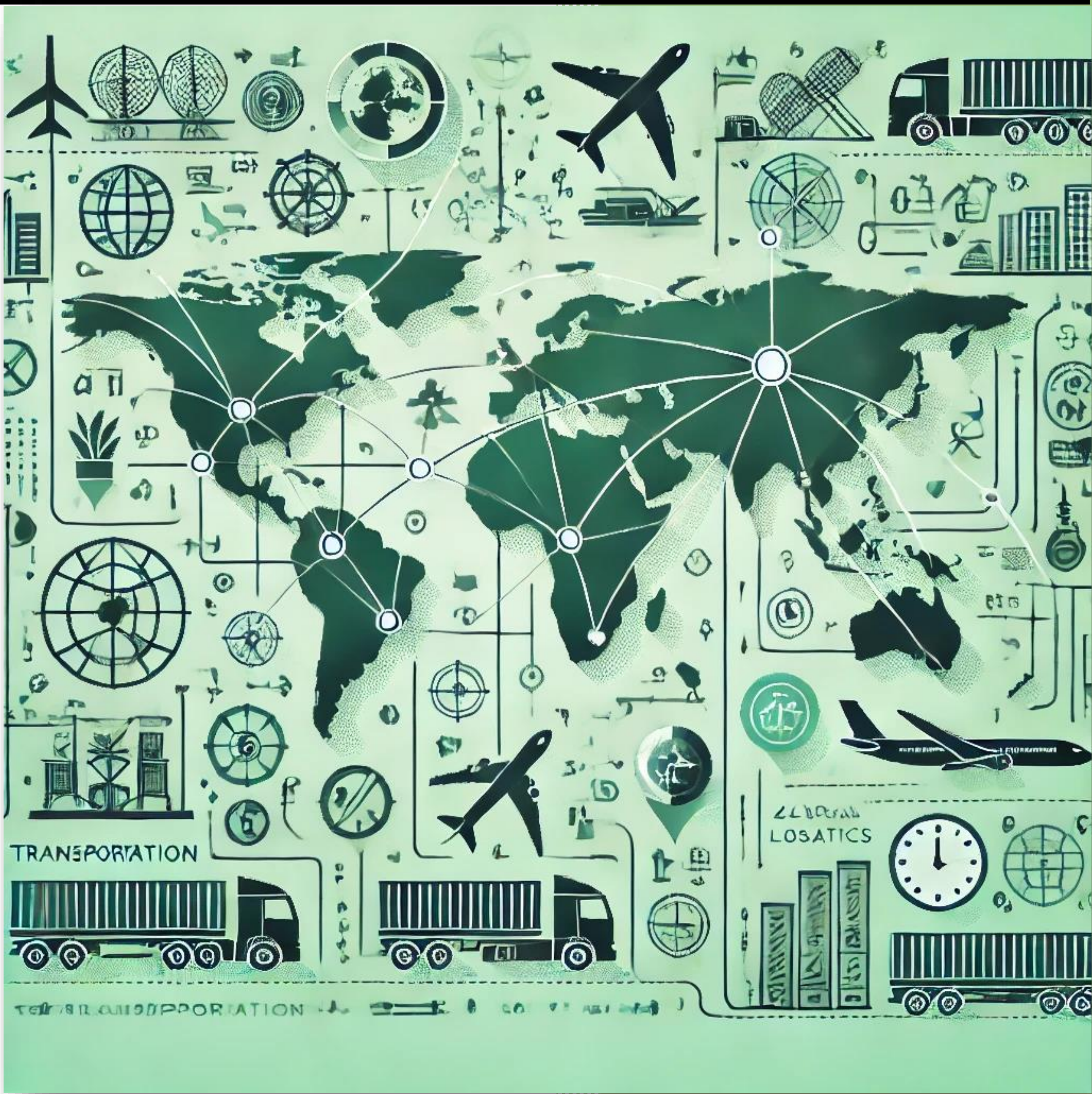
Transportation Models



Created by :

Sarah Emad Abd El-Aziz
Haidy Soliman Atta
Amr Osama Abd El-Fattah
Omar Mohamed Abd El-Razek
Mahmoud Ahmed Mahmoud
Mahmoud Hamdy Ahmed

Supervised by :-
Dr.Sorya Wahba

2024-2025

Chapter One

Formulation Of Transportation Problem

Introduction:

Transportation is a key part of the economy and logistics, helping move goods and resources efficiently between different locations. It plays a major role in global trade and business by ensuring that raw materials, semi-finished goods, and final products reach their destinations smoothly. A well-planned transportation system supports economic growth, increases business competitiveness, and reduces supply chain costs. Additionally, efficient transportation helps reduce traffic congestion, save fuel, and lower environmental pollution, making it an important factor in sustainable development.

In this context, the **Transportation Problem (TP)** is a well-known optimization problem in **Linear Programming (LP)**. It focuses on finding the most cost-effective way to transport goods from multiple supply locations (such as factories or warehouses) to multiple demand locations (such as stores or distribution centers). The main goal is to minimize transportation costs while making sure that all supply and demand requirements are met. This problem is widely used in industries like retail, healthcare, and manufacturing, where efficient transportation is essential for smooth operations and customer satisfaction. Beyond cost savings, the **Transportation Problem** is also critical in emergency situations, such as disaster relief, military operations, and the delivery of perishable goods, where fast and optimized transportation is necessary.

From a mathematical perspective, the **Transportation Problem** is modeled as a **network flow problem**. Supply locations (such as factories and warehouses) are connected to demand locations (such as distribution centers and retail outlets) through transportation routes, each with a specific cost. The challenge is to determine the best way to allocate shipments to minimize total costs while ensuring that supply and demand constraints are met. This problem has broad applications in supply chain management, logistics, and transportation planning.

Definition:

The Transportation Problem is a special type of Linear Programming Problem (LPP) that deals with determining the most efficient way to transport goods from multiple sources (e.g., factories or warehouses) to multiple destinations (e.g., retail stores or distribution centers) while minimizing the total transportation cost. The problem aims to allocate supply from each source to various destinations in such a way that demand requirements are met while optimizing cost.

Components of the Transportation Problem:

1. **Supply Points:** These are the origins (e.g., factories, suppliers) that provide goods or services. Each supply point has a limited quantity of goods available for distribution.
2. **Demand Points:** These are the destinations (e.g., warehouses, customers) that require a certain amount of goods.
3. **Transportation Costs:** Each route between a supply and demand point has an associated cost per unit transported.
4. **Decision Variables:** Represent the quantity of goods transported from each source to each destination.
5. **Objective Function:** The function to be minimized, which represents the total transportation cost.
6. **Constraints:**
    - **Supply Constraints:** Ensure that the total goods transported from each source do not exceed the available supply.
    - **Demand Constraints:** Ensure that the total goods received at each destination meet the required demand.
    - **Non-negativity Constraints:** Ensure that the transported quantities are non-negative.

Mathematical Formulation

Let:

- x_{ij} be the number of units transported from source i to destination j.
- c_{ij} be the transportation cost per unit from i to j.
- S_i be the supply available at source i.
- D_j be the demand required at destination j.

The Objective Function:

$$\text{Minimize} \quad Z = \sum_i \sum_j c_{ij} x_{ij}$$

where:

- Z represents the total transportation cost.
- c_{ij} is the cost per unit transported from source i to destination j.
- x_{ij} is the number of units transported.

Subject to:

$$\sum_j x_{ij} \leq s_i, \quad \forall i$$

$$\sum_i x_{ij} \geq d_j, \quad \forall j$$

$$x_{ij} \geq 0, \quad \forall i, j$$

Examples Of Transportation Problems

Example 1: Factory to Warehouses

A company has three factories **(A, B, C)** and two warehouses **(X, Y).** The supply and demand are as follows:

Factory Supply Table:

Factory	Supply
A	30
B	40
C	20

Processing Warehouse Demand Table:

Warehouse	Demand
X	50
Y	40

The transportation cost per unit between each factory and warehouse is given in the table:

Transportation Cost Table:

	X	Y
A	8	6
B	5	9
C	6	4

Example 2: Supplier to Retail Stores

A supplier needs to distribute products to three retail stores **(R1, R2, R3).** The available stock and required demand are:

Supplier Supply Table:

Supplier	Supply
S1	50
S2	30

Processing Retail-Store Demand Table:

Retail Store	Demand
R1	40
R2	20
R3	20

The transportation costs per unit between each supplier and retail-sore is given in the table:

Transportation Cost Table:

	R1	R2	R3
S1	3	7	5
S2	6	4	2

Example 3: Distribution Center to Supermarkets

A distribution center needs to deliver goods to four supermarkets **(M1, M2, M3, M4)**. The available stock and required demand are:

Distribution-Center Supply Table:

Distribution Center	Supply
D1	100

Processing Supermarket Demand Table:

Supermarket	Demand
M1	20
M2	30
M3	25
M4	25

The transportation cost per unit between the distribution center and each supermarket is given in the table:

Transportation Cost Table:

	M1	M2	M3	M4
D1	4	2	5	3

Example 4: Farms to Food Processing Plants

A company sources raw agricultural products from three farms (F1, F2, F3) and transports them to two food processing plants (P1, P2). The available supply at each farm and the demand at each plant are as follows:

Farm Supply Table:

Farm	Supply
F1	50
F2	60
F3	40

Processing Plant Demand Table:

Processing Plant	Demand
P1	80
P2	70

The transportation cost per unit between each farm and processing plant is given in the table below:

Transportation Cost Table:

Farm	P1	P2
F1	4	6
F2	7	5
F3	3	8

Example 5: Shipping Ports to Distribution Hubs

A logistics company manages the transport of goods from three major shipping ports (S1, S2, S3) to four distribution hubs (H1, H2, H3, H4). The supply at each port and the demand at each hub are as follows:

Port Supply Table:

Port	Supply
S1	100
S2	80
S3	60

Distribution Hub Demand Table:

Hub	Demand
H1	50
H2	60
H3	70
H4	60

The transportation cost per unit between each port and hub is given in the table below:

Transportation Cost Table:

Port	H1	H2	H3	H4
S1	5	3	6	7
S2	4	8	2	5
S3	7	6	3	4

Conclusion:

All the provided examples represent **real-world balanced transportation problems**, where goods need to be transported from multiple sources to multiple destinations while minimizing costs.

Observation On The Examples:

- A transportation problem is **balanced** if the total supply equals the total demand.
- In a balanced problem, no additional adjustments (such as dummy sources or destinations) are needed.
- The goal in each example is to determine the optimal allocation of shipments to meet demand while **minimizing transportation costs**.
- These problems can be solved using **North-West Corner Method, Least Cost Method, or Vogel's Approximation Method**, each providing a different approach to finding an optimal initial solution.

Chapter Two
Northwest Corner Cell Method

Introduction

In the realm of Operations Research, numerous methods exist to tackle transportation problems, but one approach stands out for its simplicity and effectiveness: the Northwest Corner Cell Method. This technique serves as a compass, guiding us toward the optimal solution, thereby minimizing total cost and enhancing transportation efficiency. In this introduction, we will delve into the application of the Northwest Corner Cell Method and explore how it can facilitate the resolution of transportation problems with ease and speed.

Advantages of the Northwest Corner Cell Method

- Simple: The Northwest Corner Cell Method is simple and easy understand.

- Fast: The Northwest Corner Cell Method can be applied quickly.

- Cost Reduction: The Northwest Corner Cell Method aims to reduce the total transportation cost.

- Easy to Apply: The Northwest Corner Cell Method can be applied to simple transportation problems.

- Time-Saving: The Northwest Corner Cell Method can save time in solving transportation problems.

Disadvantages of the Northwest Corner Cell Method

- Does Not Guarantee Optimal Solution: The Northwest Corner Cell Method does not always guarantee the optimal solution.

- Depends on Cell Order: The order of cells in the table can affect the result of the Northwest Corner Cell Method.

- Not Suitable for Complex Transportation Problems: The Northwest Corner Cell Method is not suitable for complex transportation problems that require more complex solutions.

- Does Not Consider Other Constraints: The Northwest Corner Cell Method does not consider other constraints that may affect the transportation problem.

- Does Not Provide Flexible Solutions: The Northwest Corner Cell Method does not provide flexible solutions that can adapt to changes in the transportation problem.

Method of Solution

Step 1: Initialize the Table

Initialize the transportation table with cells containing transportation costs between sources and destinations. Determine the demand quantities and available quantities.

Step 2: Allocate Quantities to Cells

Start allocating transportation quantities to cells in the Northwest Corner (top-left corner) of the table. Allocate the maximum possible quantity to the first cell.

Step 3: Allocate Remaining Quantities

After allocating quantities to cells in the Northwest Corner, allocate remaining quantities to other cells in the table. Allocate remaining quantities to cells that follow in the horizontal or vertical direction.

Step 4: Repeat the Process

Repeat the process until all quantities are allocated. Update the table after each allocation.

Step 5: Calculate Total Cost

Calculate the total transportation cost by summing the transportation costs of each cell. Analyze the result and ensure that the total cost is minimized.

Mathematical rule :

$$\textbf{Total Cost} = \sum(\textbf{Allocated Units} \times \textbf{Cost per Unit})$$

These are the steps of the Northwest Corner Cell Method for solving transportation problems. This method is simple and effective, but it may not always guarantee the optimal solution.

Illustrative Example:

Given three sources O1, O2, and O3 and four destinations D1, D2, D3, and D4. For the sources O1, O2, and O3, the supply is 300, 400, and 500 respectively. The destinations D1, D2, D3, and D4 have demands of 250, 350, 400, and 200 respectively.

Solution

Initial Table

Source/Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	250	350	400	200	

After First Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	1	7	4	50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	0	350	400	200	

After Second Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	0	300	400	200	

After Third Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	5	9	100
O3	8	3	3	2	500
Demand	0	0	400	200	

After Fourth Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	3	2	500
Demand	0	0	300	200	

After Fifth Allocation

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	300	2	200
Demand	0	0	0	200	

Final Table (All Demand Fulfilled)

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	7	4	0
O2	2	300	100	9	0
O3	8	3	300	200	0
Demand	0	0	0	0	

Final Calculation:

Now just multiply the allocated values with their respective costs and sum them:

(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400

Thus, the initial basic feasible solution = 4400.

Examples of Northwest corner method

Example 1: Solving a Transportation Problem

Consider a transportation problem with three sources (O1, O2, O3) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	250	350	400	200	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 250 units from O1 to D1.

- Step 2: Allocate 50 units from O1 to D2.

- Step 3: Allocate 300 units from O2 to D2.

- Step 4: Allocate 100 units from O2 to D3.

- Step 5: Allocate 300 units from O3 to D3.

- Step 6: Allocate 200 units from O3 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	-	-	0
O2	-	300	100	-	0
O3	-	-	300	200	0
Demand	0	0	0	0	

Total Cost Calculation :

(250 × 3) + (50 × 1) + (300 × 6) + (100 × 5) + (300 × 3) + (200 × 2)

= 4400 dollars

Example 2: Another Transportation Problem

Now, consider another transportation problem with four sources (O1, O2, O3, O4) and three destinations (D1, D2, D3).

Source/Destination	D1	D2	D3	Supply
O1	2	3	1	180
O2	5	4	8	200
O3	5	6	3	150
O4	7	2	4	170
Demand	220	150	330	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 180 units from O1 to D1.

- Step 2: Allocate 40 units from O2 to D1.

- Step 3: Allocate 150 units from O2 to D2.

- Step 4: Allocate 10 units from O2 to D3.

- Step 5: Allocate 150 units from O3 to D3.

- Step 6: Allocate 170 units from O4 to D3.

Final Allocation Table :

Source/Destination	D1	D2	D3	Supply
O1	180	-	-	0
O2	40	150	10	0
O3	-	-	150	0
O4	-	-	170	0
Demand	0	0	0	

Total Cost Calculation :

$(180 \times 2) + (40 \times 5) + (150 \times 4) + (10 \times 8) + (150 \times 3) + (170 \times 4)$

$= 360 + 200 + 600 + 80 + 450 + 680$

= 2370 dollars

Example 3: A Moderate Difficulty Transportation Problem

Consider a transportation problem with four sources (O1, O2, O3, O4) and four destinations (D1, D2, D3, D4). The supply and demand are balanced.

Source/Destination	D1	D2	D3	D4	Supply
O1	4	3	8	6	150
O2	2	7	5	9	250
O3	6	5	7	3	200
O4	3	6	9	4	180
Demand	100	200	220	260	

Solution steps using the Northwest Corner Method:

- Step 1: Allocate 100 units from O1 to D1.

- Step 2: Allocate 50 units from O1 to D2.

- Step 3: Allocate 150 units from O2 to D2.

- Step 4: Allocate 100 units from O2 to D3.

- Step 5: Allocate 120 units from O3 to D3.

- Step 6: Allocate 80 units from O3 to D4.

- Step 7: Allocate 180 units from O4 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	100	50	-	-	0
O2	-	150	100	-	0
O3	-	-	120	80	0
O4	-	-	-	180	0
Demand	0	0	0	0	

Total Cost Calculation :

$(100 \times 4) + (50 \times 3) + (150 \times 7) + (100 \times 5) + (120 \times 7) + (80 \times 3) + (180 \times 4)$

$= 3900$ dollars

Example 4: Solving a Transportation Problem

Consider a transportation problem with three sources (O1, O2, O3) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	5	8	6	7	300
O2	3	7	4	6	500
O3	6	5	7	3	400
Demand	250	400	300	250	

Solution steps using the Northwest Corner Method:

-Step 1 : Allocate 250 units from O1 to D1.

-Step 2 : Allocate 50 units from O1 to D2.

-Step 3 : Allocate 350 units from O2 to D2.

-Step 4 : Allocate 150 units from O2 to D3.

-Step 5 : Allocate 150 units from O3 to D3.

-Step 6 : Allocate 250 units from O3 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	250	50	-	-	0
O2	-	350	150	-	0
O3	-	-	150	250	0
Demand	0	0	0	0	

Total Cost Calculation :

$(250 \times 5)+(50 \times 8)+(350 \times 7)+(150 \times 4)+(150 \times 7)+(250 \times 3)$
$=1250+400+2450+600+1050+750$
$=6500$ dollars

Example 5: Another Transportation Problem

Now, consider another transportation problem with four sources (O1, O2, O3, O4) and three destinations (D1, D2, D3).

Source/Destination	D1	D2	D3	Supply
O1	4	6	5	220
O2	7	3	2	300
O3	5	8	6	150
O4	6	4	7	180
Demand	250	200	400	

Solution steps using the Northwest Corner Method:

-Step 1 : Allocate 220 units from O1 to D1.

-Step 2 : Allocate 30 units from O2 to D1.

-Step 3 : Allocate 200 units from O2 to D2.

-Step 4 : Allocate 70 units from O2 to D3.

-Step 5 : Allocate 150 units from O3 to D3.

-Step 6 : Allocate 180 units from O4 to D3.

Final Allocation Table :

Source/Destination	D1	D2	D3	Supply
O1	220	-	-	0
O2	30	200	70	0
O3	-	-	150	0
O4	-	-	180	0
Demand	0	0	0	

Total Cost Calculation :

$(220\times4)+(30\times7)+(200\times3)+(70\times2)+(150\times6)+(180\times7)$

= 3990 dollars

Example 6: A More Challenging Transportation Problem

Consider a transportation problem with four sources (O1, O2, O3, O4) and four destinations (D1, D2, D3, D4).

Source/Destination	D1	D2	D3	D4	Supply
O1	3	5	6	8	200
O2	4	7	3	5	300
O3	6	4	5	7	250
O4	5	3	7	4	200
Demand	150	250	200	350	

Solution Steps using Northwest Corner Method

-Step 1 : Allocate 150 units from O1 to D1.

-Step 2 : Allocate 50 units from O1 to D2.

-Step 3 : Allocate 200 units from O2 to D2.

-Step 4 : Allocate 100 units from O2 to D3.

-Step 5 : Allocate 100 units from O3 to D3.

-Step 6 : Allocate 150 units from O3 to D4.

-Step 7 : Allocate 200 units from O4 to D4.

Final Allocation Table :

Source/Destination	D1	D2	D3	D4	Supply
O1	150	50	-	-	0
O2	-	200	100	-	0
O3	-	-	100	150	0
O4	-	-	-	200	0
Demand	0	0	0	0	

Total Cost Calculation :

$(150 \times 3)+(50 \times 5)+(200 \times 7)+(100 \times 3)+(100 \times 5)+(150 \times 7)+(200 \times 4)$

=4750 dollars