Public Choice (2008) 136: 353–377
DOI 10.1007/s11127-008-9300-3

Parliamentary procedure: principal forms and political effects

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Received: 11 June 2007 / Accepted: 11 March 2008 / Published online: 24 June 2008

Abstract Legislative agendas depend on parliamentary procedure, of which there are two main versions and a noteworthy variant: Anglo-American, Euro-Latin, and Mex-Italian. Theorems proved here identify the output of each procedure under strategic and sincere voting. Surprises abound: Strategic voting makes Anglo-American procedure more restrictive than Euro-Latin, and either sort of voting makes Anglo-American procedure more conservative, or stabilizing. Although sincere voting makes Mex-Italian procedure equivalent to the similar-looking Euro-Latin, strategic voting makes it equivalent rather to Anglo-American. Also strategic voting reduces the effects of agenda manipulation only under Mex-Italian procedure. These findings raise new research questions.

Keywords Agendas · Parliamentary procedure · Social choice · Legislative voting

1 Introduction

Political scientists have lately come to appreciate something legislative strategists have long understood: that legislative choices depend on legislative agendas—on those ad hoc rules that dictate the content and order of pairwise votes and make some votes contingent on the outcomes of others. But the allowable agendas depend on the standing rules of parliamentary procedure, and of these there are two main versions and at least one noteworthy variant. The version in force in most of Europe and Latin America calls for agendas that prescribe up-or-down votes on a sequence of rival draft laws until one of them passes or else the status quo has survived every vote. By contrast, in Britain, the United States, and most other Anglophone countries, agendas pit rival draft laws against one another in a series of amending votes, then the survivor against the status quo in a final vote. The variant, found in Mexico and Italy, extends Euro-Latin procedure by requiring any otherwise successful

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draft law to pass a second, seemingly redundant up-or-down vote lest the status quo prevail after all.

What difference can these procedural differences make in politics and policy? To answer this question I define the categories of agendas allowed by the three procedures, and assuming no ties I look for the *output set*, or set of achievable legislative choices, corresponding to each procedural category. That set depends, of course, on the feasible alternatives, on legislative preferences, and on how legislators translate their preferences into votes—sincerely, strategically, or otherwise. Once that last factor is fixed, if two of our three procedures always have identical output sets, they are equivalent in content however unalike in form. If instead the output set of one procedure is generally more inclusive than that of another—if the former always includes the latter but is sometimes bigger—then agenda setters and procedural politics have greater influence under the one procedure than under the other. Even if two procedures are not so simply related, we might be able to examine them via their output sets to see which gives greater play to the choice of agendas, which makes it easier to upset the status quo, and which meets such social-choice requirements as Condorcet's and Pareto's. Of course we cannot fully appreciate the effects of parliamentary procedure in abstraction from the wider game of elections, party politics, and constitutional bill-to-law processes, but then we shall never fully understand the wider game until we have given parliamentary procedure the sort of rigorous if abstract scrutiny enjoyed by election rules for over two hundred years.

Although Anglo-American procedure allows greater diversity of agenda structure than Euro-Latin procedure, we shall find that under strategic voting it allows considerably *less* diversity of outcomes—less inclusive output sets and therefore less opportunity for agenda manipulation. Sincere voting cancels that relationship, but under either sort of voting Euro-Latin procedure is the less conservative, or less stable: it makes it easier to upset the status quo. One might expect strategic voting to offset the influence of strategic agenda setting by making output sets generally less inclusive than does sincere voting. But that is true only under odd-looking Mexican-Italian procedure. One might also expect Mex-Italian and Euro-Latin procedure to be equivalent, and under sincere voting they are: that seemingly redundant extra vote really is redundant. But strategic voting undoes this equivalence. More surprising, it makes Mex-Italian and Anglo-American procedure equivalent. These results hold for any number of feasible alternatives, but their practical import shows up best when we assume the very small numbers most often observed and tabulate all possible output sets based on those numbers. Then the contrast between procedures is striking. Under strategic voting, for example, Anglo-American procedure almost always yields one-member output sets, but Euro-Latin procedure is far less resolute. Again, under sincere voting Anglo-American procedure often preserves the status quo but Euro-Latin procedure hardly ever does.

After introducing the three versions of parliamentary procedure by way of examples (Sect. 2), I define generic agendas and the three procedure-specific categories of agendas (Sect. 3), then sincere and strategic voting outcomes (Sect. 4). Next come the chief findings of this paper, theorems that identify the output sets for all procedures, in terms of majority preference, under strategic (Sect. 5) and sincere voting (Sect. 6). Proofs are in an appendix. These results allow us to show what happens in the small-number cases mentioned earlier (Sect. 7), to compare output sets (Sect. 8), and to show how each procedure fares against social-choice requirements (Sect. 9). I end with open questions (Sect. 10).

The influence of agendas was celebrated in the abstract by Black (1958), with historical examples by Farquharson (1969) and Riker (1958, 1982), and with an experiment by Plott and Levine (1976). It was Rasch (2000) above all who called attention to the two

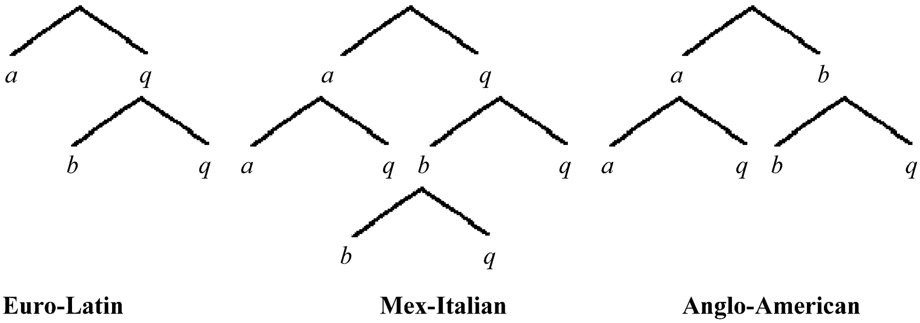

Fig. 1 Simple agendas of each type

main versions of parliamentary procedure, but his formulation was faulty in detail, as I shall explain (Sect. 10). Ordeshook and Schwartz (1987) found that output sets are considerably more inclusive than earlier research on agenda control had contended (Miller 1980; Shepsle and Weingast 1984; Banks 1985), but only for unconstrained legislative voting: while lifting a restriction (to "amendment agendas") imposed by earlier authors but not by any version of parliamentary procedure, they followed earlier authors in ignoring restrictions actually imposed by parliamentary procedure, e.g., to final votes against the status quo. Farquharson (1969) first represented agendas by binary trees, as I do here, and distinguished sincere from strategic (or "naïve" from "sophisticated") voting. McKelvey and Niemi (1978) and Moulin (1979) then corrected his treatment of strategic voting, bringing it in line with extensive-form game solutions. Ordeshook and Schwartz (1987) further remedied a shortcoming in Farquharson's trees and a related fault in his treatment of sincere voting, or so they thought, but in his magisterial treatise on agendas Miller (1995) took Farquharson's side; a second appendix addresses the controversy.

2 Procedural differences by way of examples

If we let a and b be rival draft laws and q the status quo, the trees of Fig. 1 illustrate the agendas allowed by our three versions of parliamentary procedure. The Euro-Latin agenda requires an up-or-down vote on a: a vs. q. If a wins, voting ends. Otherwise, q survives that vote, and a similar vote is then taken on b. The Mex-Italian variant requires new legislation (a or b) to pass a second up-or-down vote lest q prevail. In the Anglo-American case, a vote on whether to amend pits an original proposal (maybe b) against the amended variant (a), and a vote on enactment then pits the first-vote winner against q.

I referred to a, b, and q as rival draft laws and the status quo, but that calls for three qualifications. First, in some countries a and b are likely to be rival draft *articles* or *sections* or other parts of some larger bill, but one whose several parts can stand alone as laws: a version of one part might pass while those of another all fail. Second, when I call them draft laws for short, what I mean, more fully, is that each is a possible outcome that would alter q by the adoption of a certain draft law *alone and unchanged*. That makes them *mutually exclusive alternatives* even if their texts are logically compatible. Third, q is not necessarily the changeless continuation of all present circumstances but the state of affairs that would prevail, possibly with considerable change, absent new legislation. So if a city's budget must be enacted by a certain date then q might include the collection of garbage before but not after that date.

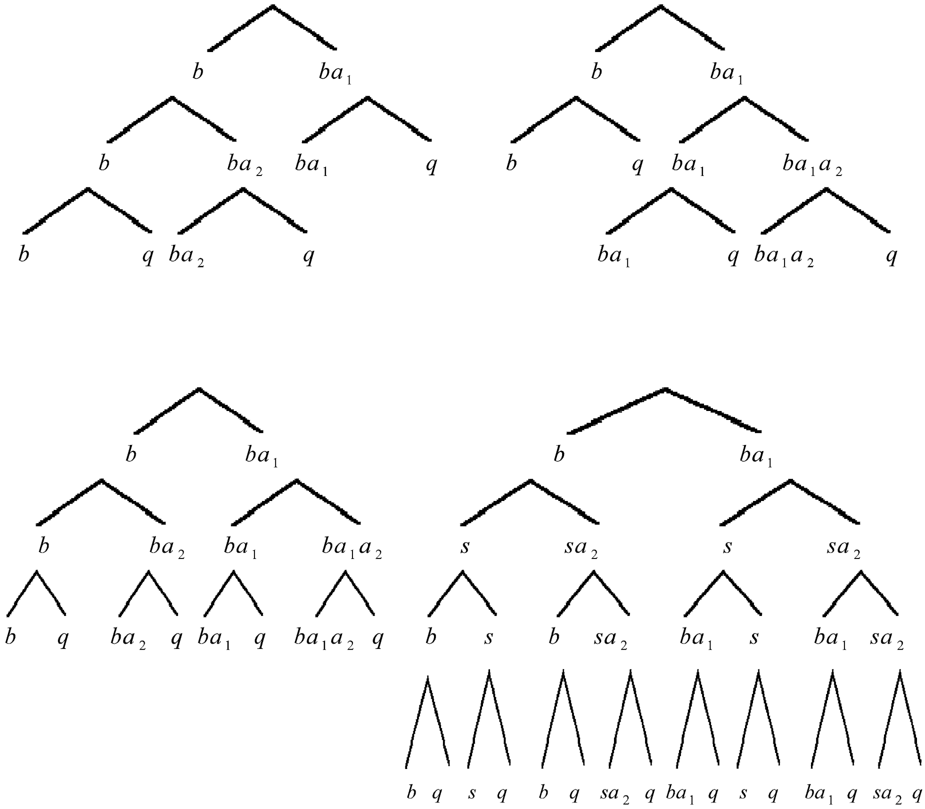

Fig. 2 Some Anglo-American agendas that are not "amendment agendas"

It is obvious how to extend the Euro-Latin and Mex-Italian agendas of Fig. 1 to encompass more draft laws. Such agendas are determined by simple sequences. But as a class, Anglo-American agendas are not. Yes, some of them, misleadingly named "amendment agendas" (by Black 1958), take a sequence of alternatives and pit the first of them against the second, the winner (whichever it be) against the third, etc. But they constitute a special case.

The Anglo-American agenda of Fig. 1 is an amendment agenda, but it is based on a single amendment. What if there are two? Then if one of them is of second degree—an amendment to the other—we still have an amendment agenda: amended bill vs. amended-amended bill (whether to amend the amendment), winner vs. bill (whether to amend the bill), winner vs. q (whether to enact). Or if we have only first-degree amendments but the first is a perfecting amendment and the second a complete substitute, introduced regardless of the action taken on the first, then we again have an amendment agenda: bill vs. perfected variant, winner versus substitute, winner vs. q. But those examples are somewhat exotic. Take the more common case of two first-degree perfecting amendments. For bill b, singly amended variants ba_1 and ba_2, and doubly amended ba_1a_2, the agenda is one of the first three of Fig. 2. Not one of these is an "amendment agenda": not one follows b and ba_1 with a third alternative, automatically pitted against the winner of the first vote (whichever it be). Or suppose we have bill b, complete substitute s, and perfecting amendments a_1 to b and a_2

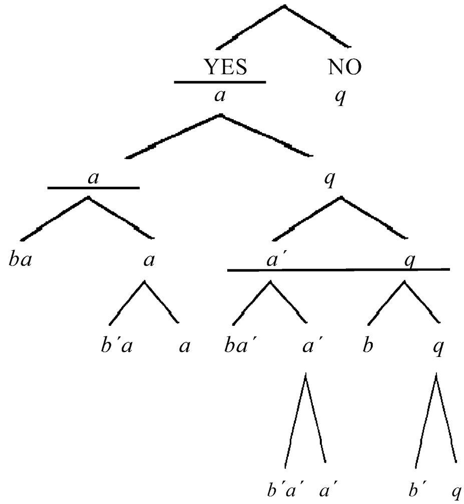

Fig. 3 Excluded agenda: compound, with procedural motion

to s, with s and a_2 introduced regardless of earlier decisions. Then the agenda is the fourth of Fig. 2, again not an amendment agenda, this time because the first-vote winner does not even appear at the second vote. In discrediting the "amendment agenda" assumption, which had governed previous deductive work on agenda control, Ordeshook and Schwartz (1987) cited a number of more elaborate agendas, all drawn from actual legislative histories in the U.S. Congress, which flout that assumption.

The range of agendas I mean to discuss is not limited to *pre-set* agendas—to agendas whose draft laws have all formally been introduced before voting begins. For example, in the first three agendas of Fig. 2, because a_1 and a_2 are both perfecting amendments to b, most legislative chambers governed by Anglo-American procedure (the U.S. Senate is an exception) would ban recognition of a_2 until a_1 had passed or been defeated. However, pre-set agendas make it easier to vote strategically—to look ahead to bottoms of trees and reason back to best choices.

My range of agendas does leave out *procedural motions* and *compound agendas*. The former include recommittal, ending debate, division of the question, and the like. Excluding them limits voting to choices between alternative outcomes. Compound agendas append new agendas to bottom nodes of old ones. That does more than lengthen the old ones. If a new agenda follows a bottom node occupied by a draft law b, that agenda must treat b (not q) as the new status quo, to be combined with (and altered by) any new draft law. These restrictions are stronger than they look. Farquharson's (1969) famous example has a procedural motion, and Plott and Levine's (1976) a compound agenda. If, as typically happens under Euro-Latin and Mex-Italian procedure, legislators vote on separate articles, then any agenda that encompasses the several articles of one bill is perforce compound, and any preliminary vote called "first reading" or "vote in general" is a procedural vote on whether to activate that compound agenda.

To illustrate, assume Euro-Latin procedure, and suppose a bill has two articles, a and b, to be voted on separately, with rivals a' to a and b' to b. In Fig. 3, the tree between the horizontal lines represents the first-article agenda. Considered by itself, the second-article agenda would look the same except with b in place of a. But the agenda governing both articles is the compound one represented by the entire tree below the upper line. The "yes or no" tree above that line represents one of those preliminary votes. I count it as a procedural vote to allow more voting, not a substantive vote on *ab*, because "yes" does not foreclose any outcome, because *ab* does not appear on the first-article agenda, and because no legislator would regard a "yes" vote as a vote for *ab* specifically: it is merely a vote to enact some bill or other among the eight possibilities.

3 Agendas and procedure-specific categories defined

Like earlier authors, beginning with Farquharson (1969), I have represented agendas by *finite binary trees*, each a finite set of *nodes* related as *predecessor to successors* so that four conditions are met: (1) No sequence of successive nodes begins and ends with the same node. (2) One node has no predecessor. (3) Every other node has exactly one predecessor. (4) Every node that has a successor has exactly two. I shall need some names for parts of trees. The node that has no predecessor is the *top node*, and those that have no successors are *bottom nodes*. A *branch* is any sequence of successive nodes from top to bottom. A *subtree* of a tree T is any of those trees-within-a-tree headed by a node of T. Unless T has but one node (a possibility I find it useful to allow), it has two *principal subtrees*, headed by the successors of T's top node.

An agenda is not a bare tree but one whose nodes are occupied by alternatives, among which the status quo plays a special role. So the definitions and theorems to follow will treat of a set A, whose members I call *alternatives*, and a particular alternative q. Denote alternatives by x, y, etc., and finite, nonempty sets of them by α, β, etc.

This definition is meant to capture all possible agendas—in effect all binary decision trees—without regard to specific procedures or the special role of q:

An *agenda on* α is a finite binary tree together with a function that assigns members of α to its nodes so that

(1) every member of α occupies (is assigned to) one or more nodes,
(2) every node is occupied by (is assigned) one member of α,
(3) no alternative occupies both successors of the same node, and
(4) any alternative that occupies a nonbottom node on a branch also occupies a successor either of that node or of some lower node on the same branch.

It will prove convenient to require, as clause (2) does, that even top nodes be occupied by alternatives; any alternative will do. Clause (3) bans nonchoices: x vs. x. Clause (4) ensures that if x wins a vote before the end then x remains a live option until later defeated or finally chosen. Instead of (4), Ordeshook and Schwartz (1987) mistakenly required only that x occupy some bottom node of the subtree it heads. (In the fourth agenda of Fig. 2, that weaker requirement would be met even if b were replaced by some new alternative, say x, in the leftmost subtree headed by s. But then, if b won the first vote and s the second, b would not remain a live option even though undefeated.)

Now we can define the categories of agendas allowed by the three versions of parliamentary procedure:

T is a *Euro-Latin agenda on* α if, and only if, T is an agenda on α and T has the first form shown in Fig. 4, where $x_1, \ldots, x_n$ are $n \geq 1$ in number and do not include q.

T is a *Mex-Italian agenda on* α if, and only if, T is an agenda on α and T has the second form shown in Fig. 4, where $x_1, \ldots, x_n$ are $n \geq 1$ in number and do not include q.

T is an *Anglo-American agenda on* α if, and only if, α has two or more members including q, T is an agenda on α, and T meets three conditions:

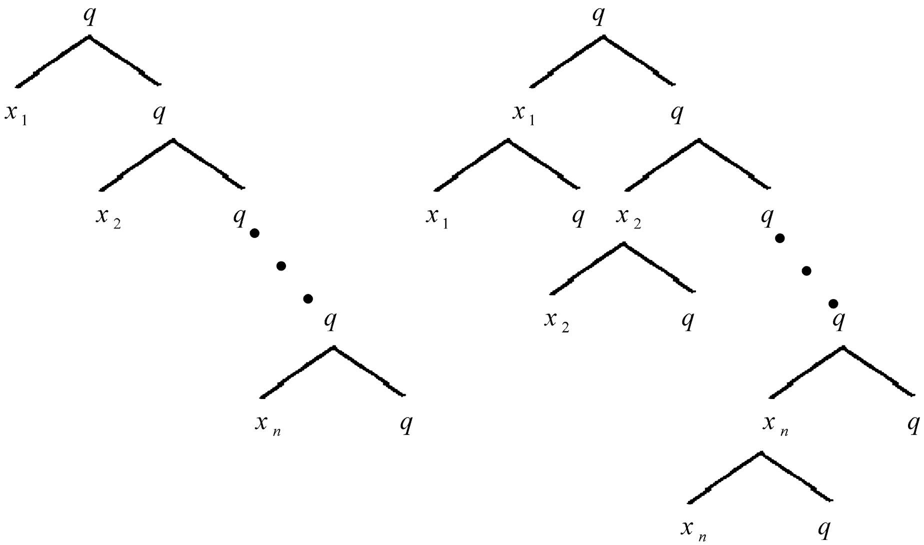

Fig. 4 General forms of Euro-Latin and Mex-Italian agendas

(1) For every nonbottom node of T, either both of its successors are bottom nodes or neither is, and if both are then one is occupied by q.
(2) q occupies only bottom nodes of T.
(3) No alternative that occupies one successor of a node of T ever occurs in the subtree headed by the other successor.

The general definition of agendas allows one-node agendas and agendas devoid of q—a convenience when we define sincere and strategic outcomes—but the three definitions of procedure-specific agendas do not. In the definition of Anglo-American agendas, clause (1) ensures that every survivor of amending votes eventually faces q in an up-or-down vote, and clause (2) that such a vote always comes last. Clause (3) makes T *nonrepetitive*: an alternative once rejected is never reconsidered, so if x defeats y then y does not appear in the subtree beneath x. I could not add (3) to the general definition of agendas because Mex-Italian agendas are perforce repetitive: if x_1 defeats q at the first vote then both must reappear at the second. But the clause "$x_1, \dots, x_n$ are $n \geq 1$ in number and do not include q" bans other sorts of repetition in the Mex-Italian case, and all repetition in the Euro-Latin case.

When I need an adjective to mark distinctions I shall refer to agendas in the general sense as *generic agendas*, and to the "procedure" that allows them as *generic procedure*.

4 The effects of strategic and sincere voting

To find the strategic and sincere legislative choices from agendas, we need to know which alternatives are preferred to which by a majority. Let P be a binary relation on A and assume: P is *asymmetric* (never $xPyPx$) and *connected in A* (either xPy or yPx whenever $x \neq y$). I shall interpret P as majority preference and read xPy as *x beats y*. Connexity bans ties.

To illustrate and motivate the definitions below, take the Anglo-American agenda of Fig. 5, and suppose $xPyPqPx$—a cycle. At each pairwise vote, the majority-preferred alternative is underlined. *Sincere* legislators compare alternatives according to their *contents*,

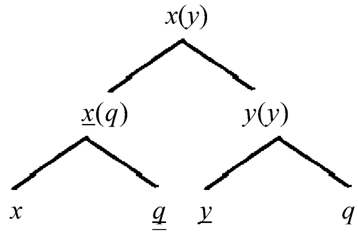

Fig. 5 Sincere and strategic outcomes illustrated

strategic legislators according to their *consequences*. At a vote between two alternatives, a sincere legislator votes for the one he prefers. So if every legislator is sincere, a majority will vote for x at the first vote. Having won, x will then face q at the second vote and lose: qPx. That makes doubly underlined q the final choice under sincere voting.

But strategic legislators look ahead and reason back. At the first vote their choices reflect their preferences between the *strategic equivalents* of x and y—the final consequences of choosing x at the first vote and likewise y. If x won the first vote, it would then lose to q at the second, making q the strategic equivalent of x at the first vote: to choose x at first is tantamount to choosing q in the end. But if y won the first vote it would go on to win the second as well (yPq), making y its own strategic equivalent at the first vote. Because a majority prefer y to q, y is the ultimate victor, the strategic equivalent (we say) of the whole agenda. Strategic equivalents are in parentheses, with that of the whole agenda atop the tree.

To generalize, here are the recursive definitions of $SN(T)$ and $ST(T)$, the sincere and strategic choices from any given agenda T:

If T has but one node, occupied by x, then $SN(T) = ST(T) = x$.

Otherwise, let the principal subtrees of T be T_1, headed by x, and T_2 , headed by y. Then

$$SN(T) = \begin{cases} SN(T_1), & \text{if } xPy, \\ SN(T_2), & \text{otherwise;} \end{cases}$$

and

$$SN(T) = \begin{cases} ST(T_1), & \text{if } ST(T_1)\ PST(T_2), \\ ST(T_2), & \text{otherwise.} \end{cases}$$

See how the general definition of "agenda" helped simplify these definitions by letting us consider one-node agendas not necessarily occupied by q. In the tree—call it T—of Fig. 4, exactly one branch consists solely of underlined alternatives; the bottom one is $SN(T)$. And the strategic equivalent if each node is the ST-value of the subtree it heads, so that of T is $ST(T)$.

With these definitions in hand, we can now define the *output sets*, for any α, corresponding to both types of voting and to each of the four categories of agendas. I begin with strategic voting:

$$G^{ST}(\alpha) = \{x | x = ST(T) \text{ for some agenda } T \text{ on } \alpha\}.$$

$$E^{ST}(\alpha) = \{x | x = ST(T) \text{ for some Euro-Latin agenda } T \text{ on } \alpha\}.$$

$$A^{ST}(\alpha) = \{x | x = ST(T) \text{ for some Anglo-American agenda } T \text{ on } \alpha\}.$$

$$M^{ST}(\alpha) = \{x | x = ST(T) \text{ for some Mex-Italian agenda } T \text{ on } \alpha\}.$$

For the four output sets under sincere voting the definitions are the same except that *SN* replaces *ST* in definienda and definientia. See how output sets are cross-classified by procedure and behavior:

	Generic	Euro-Latin	Anglo-American	Mex-Italian
Strategic	$G^{ST}(\alpha)$	$E^{ST}(\alpha)$	$A^{ST}(\alpha)$	$M^{ST}(\alpha)$
Sincere	$G^{SN}(\alpha)$	$E^{SN}(\alpha)$	$A^{SN}(\alpha)$	$M^{SN}(\alpha)$

Note the mnemonic names of the eight sets (or more exactly, of the functions that choose them).

5 Output sets under strategic voting

Assuming (as I have here) that P is asymmetric and connected in A, Ordeshook and Schwartz (1987) identified the output sets for "generic procedure" under strategic voting:

Theorem 1 *$x \in G^{ST}(\alpha)$ if, and only if, x is the Condorcet winner in α or x belongs to the top cycle in α,*

where x is the *Condorcet winner* in α if, and only if, $x \in \alpha$ and xPy for all $y \neq x$ in α,
and β is the *top cycle* in α if, and only if, $\beta \subseteq \alpha$, β is a P-cycle ($\beta = \{x_1, \dots, x_n\}$ and $x_1 P x_2 P \cdots P x_n P x_1$ for some $x_1, \dots, x_n$), and yPz for every y in β and z in $\alpha - \beta$.

Thanks to connexity (no ties), every α has either one Condorcet winner or one top cycle but not both. Theorem 1 was directed against earlier research (Miller 1980; Shepsle and Weingast 1984; Banks 1985) based on the "amendment agenda" assumption. Absent a Condorcet winner, that research had located the output of otherwise unconstrained strategic voting in certain subsets of the top cycle, notably the so-called uncovered set.

Couched in these terms, the condition for membership in $G^{ST}(\alpha)$ is hard to compare with those for other output sets. An equivalent condition makes comparison easier:

Theorem 2 *$x \in G^{ST}(\alpha)$ if, and only if, x is the first component of some path on α,*

where a *path on* α is any nonrepeating sequence $(x_1, \dots, x_n)$ of all and only members of α such that $x_1 P x_2 P \cdots P x_n$ ("nonrepeating" means $x_i = x_j$ only if $i = j$).

So an alternative is the Condorcet winner or a member of the top cycle in α just in case it is first in some path on α—first in some nonrepeating sequence coextensive with α whose successive components are beaten by their immediate predecessors.

We now have a membership condition comparable to those for Euro-Latin and Anglo-American procedure under strategic voting:

Theorem 3 *$x \in E^{ST}(\alpha)$ if, and only if, α has two or more members including q, and x is first in some path on α in which qPy for every y below q in the path.*

Theorem 4 *$x \in A^{ST}(\alpha)$ if, and only if, α has two or more members including q, and x is first in some path on α in which qPy for every y below q in the path while zPq for every z above q.*

Under strategic voting, moreover, Mex-Italian procedure is equivalent to Anglo-American:

Theorem 5 $M^{ST}(\alpha) = A^{ST}(\alpha)$.

To be chosen by strategic voting from some generic agenda on α, an alternative has only to be first in some path on α. But for Euro-Latin, Anglo-American, and Mex-Italian agendas, the path must be one in which q beats every later entry, and in the Anglo-American and Mex-Italian cases the path must, in addition, be one in which every earlier entry beats q.

Theorems 2–4 state membership conditions for $G^{ST}(\alpha)$, $E^{ST}(\alpha)$, and $A^{ST}(\alpha)$ in terms of *paths*, but we obtain equivalent conditions when we doubly relax the path requirement on sequences, first by allowing repetitions, then by requiring only that each alternative beyond the first be beaten by some earlier entry or other, not necessarily its immediate predecessor:

Theorem 6 *Theorems 2–4 remain true if we replace the term "path" with "quasi-path,"*

where a *quasi-path on* α is any sequence $(x_1, \ldots, x_n)$ of all and only members of α such that, for every x_j other than x_1, there exists an $i < j$ for which $x_i P x_j$.

6 Output sets under sincere voting

For "generic procedure" under sincere voting, Ordeshook and Schwartz (1987) proved:

Theorem 7 $x \in G^{SN}(\alpha)$ *if, and only if,* $x \in \alpha$ *and either* $\alpha = \{x\}$ *or* xPy *for some* y *in* α.

To be chosen by sincere voting from *some* agenda on α it is enough that an alternative beat something or other in α—a condition met by all but at most one member of α.

In the procedure-specific cases the membership conditions are tougher:

Theorem 8 $x \in E^{SN}(\alpha)$ *if, and only if,* α *has two or more members, including* x *and* q, *and either* $x = q$ *and* q *is the Condorcet winner in* α *or else* xPq.

Theorem 9 $x \in A^{SN}(\alpha)$ *if, and only if,* α *has two or more members, including* x *and* q, *and either* $\alpha = \{x, y\}$ *and* xPy *for some* y, *or* $x = qPyPz$ *for some* y, z *in* α, *or* xPq *and* $xPz \neq q$ *for some* z *in* α.

Theorem 10 $M^{SN}(\alpha) = E^{SN}(\alpha)$.

Under Euro-Latin or Mex-Italian procedure, to be chosen by sincere voting, q has to be the Condorcet winner—it has to beat everything else—but any other alternative has merely to beat q. Under Anglo-American procedure, q need not beat everything else so long as it beats something that in turn beats some third thing (if there is one), but any other alternative has to beat both q and some third thing (unless there is none).

7 Small numbers

The agendas most often observed are small, created by one, two, or three motions. Add the status quo and you still have four or fewer alternatives. To make Theorems 1–10 a bit more concrete, then, assume that x, y, z, and q are four distinct alternatives, and look at

Table 1 Output sets for three and four alternatives

	P on α	$G^{ST}(\alpha)$	$E^{ST}(\alpha)$	$A^{ST}(\alpha)$	$G^{SN}(\alpha)$	$E^{SN}(\alpha)$	$A^{SN}(\alpha)$
1	$xPyPq$, xPq	x	x	x	x, y	x, y	x
2	$xPqPy$, xPy	x	x	x	x, q	x	x
3	$qPxPy$, qPy	q	q	q	q, x	q	q
4	$xPqPyPx$	x, y, q	x, y	x	x, y, q	x	q
5	$xPyPzPq$, xPz, yPq, xPq	x	x	x	x, y, z	x, y, z	x, y
6	$xPyPqPz$, xPq, yPz, xPz	x	x	x	x, y, q	x, y	x, y
7	$xPqPyPz$, xPy, qPz, xPz	x	x	x	x, y, q	x	x, q
8	$qPxPyPz$, qPy, xPz, qPz	q	q	q	x, y, q	q	q
9	$xPyPqPx$, xPz, yPz, qPz	x, y, q	x, y	y	x, y, q	y	y, q
10	$xPyPzPx$, xPq, yPq, zPq	x, y, z	x, y, z	x, y, z	x, y, z	x, y, z	x, y, z
11	zPx, zPy, zPq, $xPqPyPx$	z	z	z	x, y, z, q	z, x	z, q
12	qPx, qPy, qPz, $xPyPzPx$	q	q	q	x, y, z, q	q	q
13	$xPyPzPqPx$, xPz, yPq	x, y, z, q	x, y	y	x, y, z, q	y, z	q, y
14	$xPyPzPqPx$, zPx, yPq	x, y, z, q	x, y, z	y	x, y, z, q	y, z	q, z, y
15	$xPyPzPqPx$, xPz, qPy	x, y, z, q	x, y, z	z	x, y, z, q	z	q
16	$xPyPzPqPx$, zPx, qPy	x, y, z, q	x, y, z	z	x, y, z, q	z	q, z

Table 1. It shows the output sets corresponding to generic, Euro-Latin, and Anglo-American procedure, under strategic and sincere voting, for all possible patterns of majority preference on sets of three and four alternatives. One might object that some P-patterns are missing. For example, the table has transitive P with $xPyPzPq$ (line 5) but not $zPxPyPq$. But those two cases are alike in form: we can turn either case into the other by relabeling alternatives other than q.

The first three columns illustrate how membership conditions become stronger from G^{ST} to E^{ST} to A^{ST}. Indeed, assuming strategic voting on small numbers of alternatives, Anglo-American procedure is highly resolute compared with generic and even Euro-Latin procedure: $A^{ST}(\alpha)$ is always a subset, sometimes proper, of $E^{ST}(\alpha)$, and a singleton in all but one case. Under sincere voting the picture is murkier. Even when $A^{SN}(\alpha)$ and $E^{SN}(\alpha)$ are alike in size they are not always alike in membership; sometimes they have no member in common. The one striking pattern in the sincere case is that $A^{SN}(\alpha)$ contains q much more often than $E^{SN}(\alpha)$.

8 Comparison of output

The theorems and table together allow us to compare output sets of all sizes. Assume strategic voting. Then generic, Euro-Latin, and Anglo-American procedure are progressively more restrictive. For these two inclusions obviously follow from Theorems 2–4:

$$A^{ST}(\alpha) \subseteq E^{ST}(\alpha) \subseteq G^{ST}(\alpha),$$

but line 14 of Table 1 shows that neither inclusion can be reversed.

In the sincere case, the membership conditions of Theorems 8 and 9 are obviously stronger than that of Theorem 7. So:

$$E^{SN}(\alpha) \subseteq G^{SN}(\alpha) \quad \text{and} \quad A^{SN}(\alpha) \subseteq G^{SN}(\alpha),$$

reflecting the fact that $G^{SN}(\alpha)$ is virtually all-inclusive. But line 2 of Table 1 shows that neither inclusion can be reversed. And in the sincere case there is no subset relationship at all between Euro-Latin and Anglo-American output sets. Line 4 shows, indeed, that $E^{SN}(\alpha)$ and $A^{SN}(\alpha)$ do not always share a member.

As for subset relationships between sincere and strategic output sets, Theorem 2–4 and 7–9 reveal four:

$$G^{ST}(\alpha) \subseteq G^{SN}(\alpha), \qquad E^{ST}(\alpha) \subseteq G^{SN}(\alpha), \qquad A^{ST}(\alpha) \subseteq G^{SN}(\alpha), \quad \text{and}$$
$$A^{ST}(\alpha) \subseteq E^{SN}(\alpha).$$

But the first three are uninteresting, and beyond these four there are no more. In line 9 of Table 1, neither of the other strategic output sets is included in $E^{SN}(\alpha)$, and none at all is included in $A^{SN}(\alpha)$. In line 5, moreover, none of the sincere output sets is included in any of the strategic ones.

Offhand one would expect the switch from sincere to strategic voting to contract output sets—to reduce the options open to strategic agenda setters. That is true under "generic procedure"—if only because $G^{SN}(\alpha)$ is so inclusive. It is not true under Euro-Latin or Anglo-American procedure. Oddly, perhaps, Euro-Latin output under sincere voting always includes Anglo-American output under strategic voting: $A^{ST}(\alpha) \subseteq E^{SN}(\alpha)$. But Mex-Italian procedure is equivalent to Anglo-American under strategic voting and to Euro-Latin under sincere voting. So the odd-looking inclusion is tantamount to saying that *Mex-Italian* output under sincere voting always includes *Mex-Italian* output under strategic voting: $M^{ST}(\alpha) \subseteq M^{SN}(\alpha)$. Maybe, then, it is not so odd after all. Still it is at least a bit surprising that the oddest of our three versions of parliamentary procedure is the one that ensures the least odd connection between sincere and strategic voting.

9 Social-choice requirements

Our findings let us see how well the three prevailing versions of parliamentary procedure fare against some familiar requirements on social-choice processes.

Condorcet requirement. This calls for the exclusive choice of the Condorcet winner when one exits: if x is the Condorcet winner in α then $\{x\}$ should be the output set for α. It is obvious from Theorems 1–4 that all our procedures meet this requirement under strategic voting. But line 5 of Table 1 shows that none of them does under sincere voting.

A weaker requirement is that any Condorcet winner is a potential choice, depending on the specific agenda: if x is the Condorcet winner in α then x belongs to the output set for α. It is obvious from Theorems 7–10 that all our procedures meet this weaker requirement even under sincere voting.

Pareto efficiency. This requires that every choice be Pareto efficient: it x belongs to the output set for α then nothing in α should be unanimously preferred to x. To apply this requirement we must look beyond P at individual preferences. Suppose a legislature can be partitioned into three minority factions, with the following preference orderings of five-fold α:

Faction 1 prefers w to x to y to z to q.
Faction 2 prefers y to z to w to x to q.
Faction 3 prefers z to w to x to y to q.

Then $zPxPyPzPwPx$, wPy, and x, y, z, and w all bear P to q. It follows that (x, y, z, w, q) is a path on α in which final q is beaten by everything else. So x satisfies the membership conditions of Theorems 2–5 and 7–10: x belongs to all eight output sets. But x is not Pareto efficient: every legislator prefers w to x. Therefore, *none* of our procedures meets the requirement, under sincere or strategic voting.

Top-cycle inclusion. A problem with the Condorcet Requirement is that there need not exist a Condorcet winner, but a natural extension requires that every allowable choice from α (every member of α's output set) belong to the top cycle in α when α has no Condorcet winner. For G^{ST}, that is part of what Theorem 1 says, and as we know, $E^{ST}(\alpha)$, $A^{ST}(\alpha)$, and $M^{ST}(\alpha)$ are subsets of $G^{ST}(\alpha)$. So every procedure meets this requirement under strategic voting. But under sincere voting none of them does. For suppose $\alpha = \{x, y, z, w, v, q\}$, $\{x, y, z\}$ is a top cycle in α, wPv, and wPq. Then w, which is foreign to the top cycle, belongs to $G^{SN}(\alpha)$, $E^{SN}(\alpha)$, $A^{SN}(\alpha)$, and $M^{SN}(\alpha)$.

For connected P, several refinements have been offered over the years of the top cycle and with it the Top-Cycle Requirement (Miller 1980; Fishburn 1977; Shepsle and Weingast 1984; Banks 1985; Dutta 1988; Schwartz 1990). However, none of our four strategic output sets generally is included in any of those refinements because all the latter are perforce Pareto efficient (proved at cited sources), but as we saw, none of the former are.

When are those sets the *whole* top cycle? $G^{ST}(\alpha)$ always is (by Theorem 1), and $E^{ST}(\alpha)$, $A^{ST}(\alpha)$, and $M^{ST}(\alpha)$ are too whenever q lies outside the top cycle:

Theorem 11 *If β is the top cycle in α and $q \in \alpha - \beta$ then $\beta = E^{ST}(\alpha) = A^{ST}(\alpha) = M^{ST}(\alpha)$.*

But when q belongs to the top cycle, all three output sets contract the top cycle to exclude q, and $A^{ST}(\alpha)$ always excludes more than q, but as lines 13 and 14 of Table 1 show, $E^{ST}(\alpha)$ might or might not exclude more:

Theorem 12 *If β is the top cycle in α and $q \in \beta$ then $E^{ST}(\alpha)$ is a subset and $A^{ST}(\alpha)$ and $M^{ST}(\alpha)$ are proper subsets of $\beta - \{q\}$.*

Conservatism, or policy stability. This requires respect for the status quo, and more specifically that no draft law pass which does not beat q. Set aside "generic procedure": it flouts that requirement but was never meant to take account of which alternative is the status quo. The three "real" procedures were. Of them, it is obvious from Theorems 4, 5, and 8–10 that all three meet the requirement under sincere voting, and Anglo-American and Mex-Italian do under strategic voting. But line 4 of Table 1 shows that Euro-Latin procedure fails under strategic voting: sometimes it allows the choice of draft laws beaten by q.

This failure makes Euro-Latin procedure stand alone as anti-conservative, or instability-prone, but the truth is even more extreme than that. Look again at that anti-Paretian example of five-fold α and three preference orderings, but reverse the positions of w and q. We now have: $zPxPyPzPqPx$, qPy, and x, y, and q all bear P to w. It follows that (x, y, z, q, w) is a path on α in which q beats the one alternative below it. So $x \in E^{ST}(\alpha)$ although qPx, contrary to Conservatism. But in this case q does more than beat x, whose choice is allowed by Euro-Latin procedure: q is *unanimously* preferred to x.

Assuming sincere voting, Euro-Latin procedure now meets the Conservative requirement, but still it is less conservative than Anglo-American procedure in a more subtle sense. Start with a question: Is it easier for an alternative x to be chosen under Euro-Latin or Anglo-American procedure? If $x = q$ then x has to be the Condorcet winner to be chosen (by sincere voting) under Euro-Latin procedure, but under Anglo-American procedure

x need only beat something that beats something else (when there is something else). On the other hand, if $x \neq q$ then x need only beat q to be chosen under Euro-Latin procedure but must, in addition, beat some third alternative (when there is one) to be chosen under Anglo-American procedure. That may seem odd: whether it is easier to choose x under one procedure or the other depends on whether $x = q$. But a rationale quickly emerges: under sincere voting Anglo-American procedure makes it both easier to preserve and also harder to replace the status quo—a double mark of greater conservatism, or policy stability.

Anti-manipulability. A final familiar requirement is that there should be scant opportunity, ideally none, for the strategic manipulation of outcomes. That includes agenda control: outcomes should depend as much as possible on the list of proposed draft laws and on legislative preferences rather than on the initial choice, cunning or capricious, among the procedurally allowable agendas. This means that output sets should have but one member, or failing that, comparatively few members. Table 1 shows that no procedure always yields one-member output sets under either strategic or sincere voting. But $A^{ST}(\alpha)$ is generally less inclusive than $E^{ST}(\alpha)$ or $G^{ST}(\alpha)$, making Anglo-American procedure—or equivalently Mex-Italian procedure—the least susceptible to agenda control under strategic voting. Sincere voting erases such simple relationships but preserves the greater conservatism of Anglo-American procedure, giving its manipulators fewer opportunities for policy innovation, more opportunities for policy preservation.

Agenda control has come to mean several things. Besides (1) the power to pick any allowable agenda on the set α of feasible alternatives, there is (2) the power to pick any draft law from α for an *up-or-down vote* against q (Romer and Rosenthal 1978), and (3) the power to pick any allowable agenda on any *subset* of α that contains q. Under A^{ST}, A^{SN}, and E^{SN}, every chosen outcome beats q unless it is q itself. That makes (3) tantamount to the simpler (2), which strictly includes (1). But anti-conservative E^{ST} allows the choice of outcomes beaten by q, making (3) greater than (1) or (2).

Besides agenda control there is vote manipulation, or insincere strategic voting. As Table 1 shows, sincere and strategic outcomes can differ under all our procedures, so none is immune from vote manipulation. But instead of seeking a procedure that minimizes strategic agenda setting or strategic voting, one might seek one that allows strategic voters to offset the power of strategic agenda setters. A procedure does that unequivocally only if its output set is generally less inclusive under strategic than sincere voting—only if the switch from sincere to strategic voting never expands and sometimes contracts an agenda setter's options. As we saw, neither Euro-Latin nor Anglo-American procedure has this property. Only the odd-looking Mex-Italian procedure has.

10 Open questions

Compound agendas. How would Theorems 2–10 change if we allowed compound agendas? As I explained in Sect. 2, a compound agenda appends one or more new agendas to some bottom nodes of an old one. If one of those nodes has a draft law a, then the new agenda makes a the new status quo, to be combined with (and altered by) the draft laws beneath it—yielding ba and $b'a$ in Fig. 3. Such composite alternatives are theoretically troublesome: they might not belong to a given α, and even if they do they can fill only certain agenda positions. Do we need them? They are not needed to distinguish one outcome from another, e.g., b from ba: the positions of those outcomes in the tree do that. Composite alternatives are needed, if at all, to allow certain combinations of preferences, e.g., bPb' and $b'aPba$. But often it is plausible to postulate the contrary: xPy iff $xzPyz$ for all x, y, z. That lets us dispense with composite alternatives—writing b rather than ba, for example. It is then much easier to define the compound agendas and corresponding output sets of our three procedural categories and to ask whether and how the expansion of those categories to include compound agendas expands output sets.

Ties allowed. What if we allow ties? The assumption that P is connected barred ties, and that helped keep things manageably simple while introducing a complex subject. But eventually one must drop the assumption. Two questions arise. One is how to resolve ties and how to incorporate this resolution in revised definitions of *ST* and *SN*. Some methods of resolving ties, such as letting the presiding officer decide, can be built into P. The most common method cannot: at each vote, parliamentary procedure dictates which of the two alternatives is the default choice. That alternative is q when q is one of the two, and at other times it is the alternative being amended, which is the one that first occurred higher in the tree unless both are new. To accommodate this method, we can alter our definition of binary trees and agendas to distinguish a *left* from a *right successor* of any nonbottom node and to assign any default choice always to a right successor, then redefine *ST* and *SN* so that ties—comparisons undecided by P—are always resolved to the right. The hard question posed by allowing ties is how to generalize the membership conditions for output sets stated in Theorems 1–10.

Embedding in wider games. An agenda is a simplified extensive game form; add P and you have a game, solvable by reckoning *ST*. What happens when we widen such a game by allowing more players or moves—players other than members of one legislative chamber, moves other than votes on legislation?

One sort of widening would add votes on *procedural motions*—"table," "recommit," "divide the question," "end debate," etc.—to the substantive votes discussed above. Of special importance are those initial procedural votes, discussed in Sect. 2 and illustrated in Fig. 3, that open the rest of the agenda to voting; they are variously styled "votes in general," "first readings," and "discharge votes." Assuming strategic voting, such votes salubriously restrict Euro-Latin procedure by blocking the choice of draft laws beaten by q. One problem is how to represent procedural choices on an agenda. The ultimate consequences of all votes are still draft laws and q, but we cannot use them to label the options in a procedural vote as we have been doing for substantive votes, and that blocks our definition of *SN*. Another problem is that procedural votes cannot always be decided by looking to P because some of them require qualified majorities.

A very different wider game is the constitutional bill-to-law game in which one legislative chamber interacts with another chamber or with a president who can veto bills or with a chamber-dependent government that can set the agenda and dissolve the chamber. For presidential systems, Alemán and Schwartz (2006) show how veto powers interact in different ways with different forms of parliamentary procedure. Parliamentary systems are harder to capture because they draw no clear line between votes on policy and votes on personnel: votes on government survival affect legislation, and some votes on legislation amount to votes of confidence.

A third way of widening the game is to add partisanship, especially majority-party control of votes and procedures. A perfectly disciplined majority is not constrained by procedure. But Rohde (1991) has found that partisan voting, or the appearance of discipline, varies with intra-party agreement, and Cox and McCubbins (2005) that it often rests less on discipline than on agenda control, specifically the power to bar draft laws from final-passage votes when they are supported by a chamber majority but only a minority of the majority party. An examination of parliamentary procedure might reveal more tools for majority-party control. For example, certain nonfinal votes too might help or hurt the majority party by altering final drafts. Again, because Euro-Latin procedure allows outcomes beaten by q, it thereby allows a party that can force as well as bar final votes to secure legislation favored by a majority of the majority party but only a minority of the chamber.

A more challenging game to describe and analyze adds agenda setting to voting: legislators build an agenda in steps by making motions recognized by a presiding officer and seconded from the floor. Each successive motion creates an agenda that incorporates and extends the previous one. Outcomes are whole agendas, and the game is solved by reckoning *ST* for each of those agendas, then reasoning back up the wider game tree to find best choices by potential motion-makers and seconders and by the presiding officer. Given α, the agenda-setting game ends once that set is exhausted or no one seeks recognition. The obvious question is whether a presiding officer can achieve more, fewer, or the same outcomes as an agenda dictator.

The first conceptual problem is how to specify the steps of agenda enlargement allowed by each procedure. That is easy enough in the Euro-Latin and Mex-Italian cases, which require only one-alternative additions to a growing sequence. It is quite daunting in the Anglo-American case, where one-motion additions can (as in Fig. 2) effect a variety of complex changes in a given agenda tree. Complications ramify when an agenda is not preset—when votes intervene between motions. Sometimes the process is directed in part by committees, presiding officers, ministries, sister chambers, or even presidents. And sometimes it is constrained by standing rules based on the content or style of legislative proposals, e.g., that more extreme draft laws precede more moderate ones on Euro-Latin agendas (Rasch 2000), or that certain categories of amendment precede others (higher before lower degree, perfecting before substitute of same degree) on Anglo-American agendas (Sullivan 1984); how much that matters depends on how hard it is to rewrite legislative proposals to fit desired levels of moderation or desired categories of amendment.

Procedural geography. Where is each procedure used? Rasch (2000) classified the countries of continental Europe, the British Isles, and Anglophone North America according to whether they use Euro-Latin or Anglo-American procedure (not his labels). He found that the Anglophone countries except Ireland use Anglo-American procedure while the European countries except Sweden, Finland, and Switzerland use Euro-Latin procedure. These findings are suspect. Rasch offered no exact definitions, only toy examples and rough descriptions, and in Eastern Europe he relied entirely on local political scientists to apply those descriptions to their countries. Worse, some of those descriptions are wrong. For example, Rasch equated Anglo-American agendas with "amendment agendas" (then puzzlingly cited Ordeshook and Schwartz 1987 to the contrary). Again, he classified Italian procedure as Euro-Latin. Yet again, he insisted that in Britain the survivor of amending votes need not face a final-passage vote, a vote against the status quo. His point was not that such a vote might be blocked by loss of a quorum or a procedural vote. It was that a bill can pass without a passing vote. Whatever could he have had in mind?

The definitions of Sect. 3 should enhance the reliability of classifications, but their mathematical form might prove an obstacle to local informants. Rather than rely entirely on informants, I would urge the examination of authoritative procedural texts and of detailed legislative histories. Even then one faces a problem of translation. In operation, parliamentary procedure is fraught with labels whose meanings are immanent, not transcendent: the same or cognate labels have subtly different meanings from one country or procedure to another. For example, an Anglo-American "amendment" is not a draft law but a proposed change in a draft law or in another amendment. But under Euro-Latin procedure, which allows no amending votes, an "amendment" is any whole draft law different from the one first introduced. In case you still think that procedural geography must be trivial, consider all those scholars (Miller 1980; Riker 1982; Shepsle and Weingast 1984; Rasch 2000, among others) who had equated Anglo-American agendas with "amendment agendas."

Stability. Does the difference between Euro-Latin and Anglo-American procedure help explain *policy stability*, or the modesty and infrequency of changes in the status quo? Theorists from de Montesquieu (1977) to Hammond and Miller (1987) have argued that constitutionally separated powers of concurrence (bicameralism included) encourage stability. Tsebelis's (2002) nice discovery is that these requirements do not explain cross-national differences very well by themselves but they do when combined with party system—with the number and ideological diversity of coalition-government partners. I conjecture that the use of Anglo-American rather than the less conservative Euro-Latin procedure helps further to explain policy stability.

When parliamentary procedure interacts with constitutional requirements, the effects on policy stability can be subtle and somewhat surprising. Every Latin American country allows the president to veto bills passed by congress. On its face that enhances stability, maybe too much: it may encourage inter-branch deadlock (Linz 1990; Mainwaring 1993). But the constitutions of Brazil, Columbia, El Salvador, Nicaragua, Panama, Paraguay, Peru, and Venezuela all allow simple majorities to override vetoes. That prevents interbranch deadlock, but it appears to erase the stabilizing effect of the veto. But under Euro-Latin procedure it does not (Schwartz 2005; Alemán and Schwartz 2006). Our findings explain why in an interesting way. Even if the president would veto any bill that might pass congress, the effect, thanks to simple-majority override, is to turn Euro-Latin into Mex-Italian procedure: every initial positive vote on a bill must be followed by a second one, formally an override vote. Under sincere voting that makes no difference. But under strategic voting it makes the operative procedure equivalent to the comparatively conservative, or stabilizing, Anglo-American procedure.

11 Concluding reflections

Besides defining the principal forms of parliamentary procedure and locating their output, we have discovered some noteworthy equivalences and contrasts. For example, Mex-Italian procedure, that odd-looking variant of Euro-Latin procedure, is equivalent to the latter under sincere voting but to the very different looking Anglo-American procedure under strategic voting. Again, Anglo-American procedure is more conservative than Euro-Latin under both sorts of voting, and more resolute in the strategic case. Yet again, only odd-looking Mex-Italian procedure offsets strategic agenda setting with strategic voting. Not surprisingly, all three procedures have very big output sets under sincere voting. But all three remain surprisingly irresolute even under strategic voting: all three can be Pareto inefficient, and all three can choose any member of any top cycle that does not contain the status quo.

True, an examination of parliamentary procedure cannot tell us much about political outcomes until it is embedded in the wider context, sketched a bit in Sect. 10, of party structure, constitutional bill-to-law requirements, and systems of representation. But that is likewise true of election rules—and no reason not to seek clarity about the content of both. I draw this comparison because a small library can be filled with all the books and articles by social-choice theorists on the mathematical intricacies of election rules, yet there is hardly anything of the sort on parliamentary procedure. The oddity is only heightened by the fact that on the parliamentary side there are so few basic procedures to study, also by the fact that binary trees allow us to calculate strategic outcomes, an impossibility in the electoral case. If it is good to isolate election rules for study, then likewise parliamentary procedure. An interest in cells and organisms is not a reason to neglect chemistry.

Acknowledgements Thanks for interesting questions and comments to Eduardo Alemán, Gary Cox, Thad Kouser, Eric Mager, Mathew McCubbins, Dan Rodriguez, and Sebastian Saieg, but especially to Alemán for verifying details of Mexican procedure through legislative histories, and to McCubbins for verifying details of Italian procedure through correspondence with the speaker of that country's parliament.

Appendix 1: Proofs

Theorems 1 and 7 were proved by Ordeshook and Schwartz (1987). To prove Theorems 2–6 and 8–12 I need a definition and a lemma.

A sequence of alternatives containing q has the *qP-property* if qPx holds for every x below q is the sequence, and the *Pq-property* if xPq holds for every x above q.

Lemma *If x is first in a quasi-path π on α, then x is first in some path on α that has the qP-property if π has and has the Pq-property if π has.*

Proof Delete from π all but the first occurrence of every alternative that occurs more than once; let $\pi' = (x_1, \dots, x_n)$ be the result. Obviously such deletions preserve the qP- and Pq-properties, and obviously π' is still a quasi-path with $x = x_1$ first. If π' is not already a path, find the longest initial path $(x_1, \dots, x_k)$ in π', then permute π' by moving x_{k+1} back so it immediately follows the latest x_j, $j < k$, for which $x_j P x_{k+1}$. Obviously that lengthens the longest initial path. So if we repeat often enough we shall arrive at a path on α. Because no y was moved ahead of q unless yPq, and because q was moved ahead of no z unless qPz, the qP- and Pq-properties are preserved. □

Theorem 2 *$x \in G^{ST}(\alpha)$ iff x is first in some path on α.*

Proof Suppose $x \in G^{ST}(\alpha)$, i.e., $x = ST(T)$ for some agenda T on α. I shall prove by induction on the number $\#T$ of nodes of T that x is first in some path on α. Trivial if $\#T = 1$. Otherwise, T has principal subtrees T_1 and T_2 with $x = ST(T_1)PST(T_2)$ or $x = ST(T_1) = ST(T_2)$. By inductive hypothesis, $x = x_1$ is first in some path $(x_1, \dots, x_n)$ on the set of alternatives in T_1, and $ST(T_2) = y_1$ is first in some path $(y_1, \dots, y_m)$ on the T_2 set. Because x_1Py_1 or $x_1 = y_1$, the sequence $s = (x_1, \dots, x_n, y_1, \dots, y_m)$ has the property that for every component $z \neq x_1$ there is an earlier component w such that wPz. That is, s is a quasi-path, obviously on α. It follows by the Lemma that $x = x_1$ is first in some path on α.

Conversely, suppose x is first in some path $(x_1, \dots, x_n)$ on α. Then $x = x_1 P x_2 P \cdots P x_n$. Let T be the first agenda in Fig. 6. Then $x = x_1 = ST(T)$ by a straightforward induction on n. So $x \in G^{ST}(\alpha)$. □

Theorem 3 *$x \in E^{ST}(\alpha)$ iff α has two or more members and x is first in some path on α that has the qP-property.*

Proof Suppose $x \in E^{ST}(\alpha)$, i.e., $x = ST(T)$ for some Euro-Latin agenda T on α. Then T has the first form shown in Fig. 3. The proof that x is first in a path on α with the qP-property is by induction on n. Trivial if $n = 1$. Otherwise, let T^q be the principal subtree headed

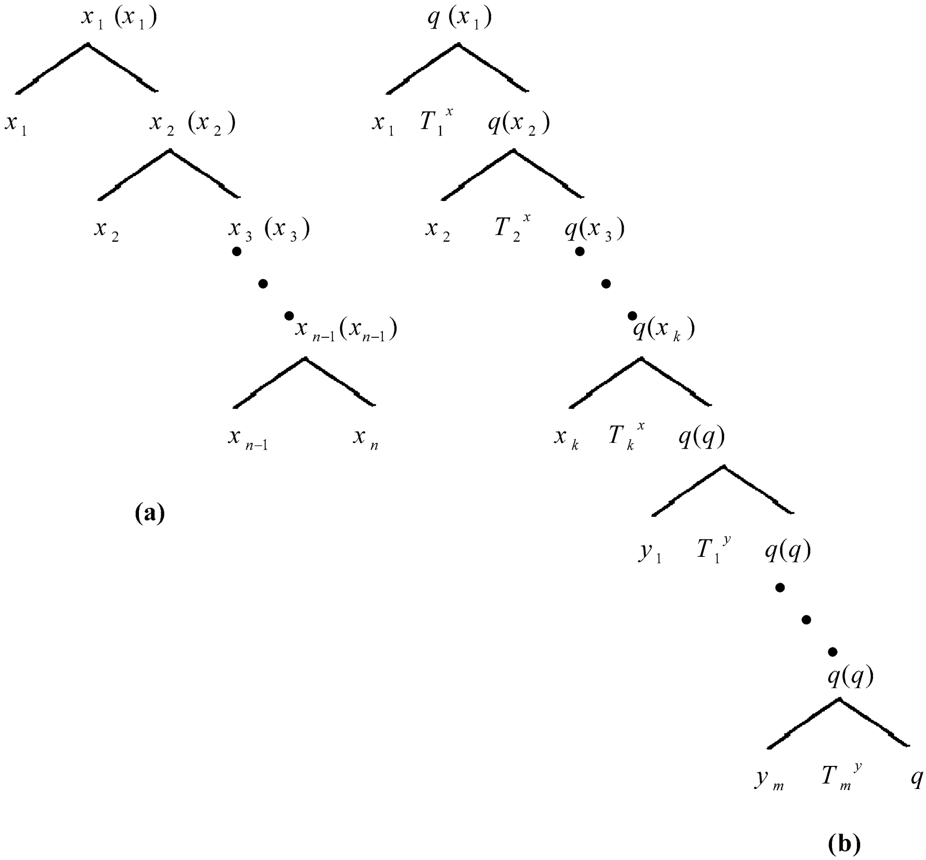

Fig. 6 Agendas constructed in proofs of Theorems 2–4

by q. Then by inductive hypothesis, some $y_1 = ST(T^q)$ is first in some path $(y_1, \ldots, y_n)$ on $\alpha - \{x_1\}$ that has the qP-property, and either $x = x_1 P y_1$ or $x = y_1 P x_1$. If $x = x_1 P y_1$ then $(x, y_1, \ldots, y_n)$ is a path that inherits the qP-property from $(y_1, \ldots, y_n)$. And if $x = y_1 P x_1$ then $(y_1, x_1, y_2 \ldots, y_n)$ is a quasi-path with that property, ensuring the existence of a suitable path by the Lemma.

Conversely, suppose α has two or more members and x is first in a path $(x_1, \ldots, x_k, q, y_1, \ldots, y_m)$, $k \geq 0 \leq m$, with the qP-property. Let T be the Euro-Latin agenda (b) of Fig. 6. Because qPy_i for all $i \leq m$, we obviously have $q = ST(T_1^y)$. So $ST(T) = q = x$ if $k = 0$. Otherwise, $ST(T_k^x) = x_k$, and because $x_1 P x_2 P \cdots P x_k$ it follows by a straightforward induction on k that $ST(T) = ST(T_1^x) = x_1 = x$. □

Theorem 4 *$x \in A^{ST}(\alpha)$ iff α has two or more members and x is first in some path on α that has the qP- and Pq-properties.*

Proof Suppose $x \in A^{ST}(\alpha)$, i.e., $x = ST(T)$ for some Anglo-American agenda T on α; to prove by induction on $\#T$ that x is first in a suitable path. Trivial if α has two members. Otherwise, T has two principal subtrees, T_1 and T_2, with $x = ST(T_1)$ and either $x = ST(T_2)$ or $xPST(T_2)$. By inductive hypothesis, x is first in a path $(x_1, \ldots, x_k, q, y_1, \ldots, y_m)$,

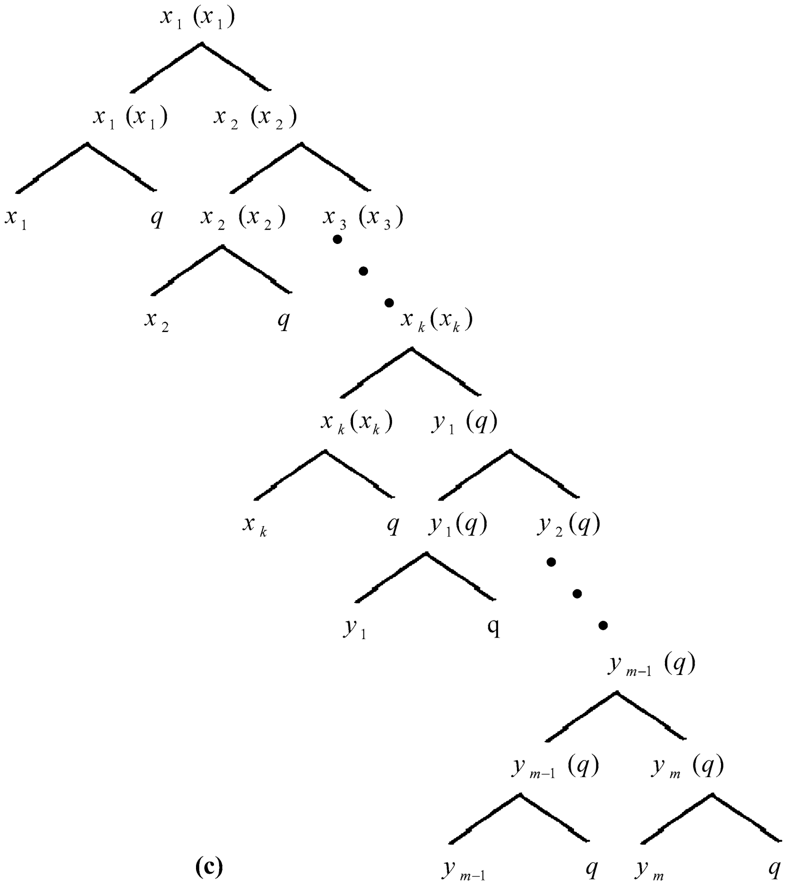

Fig. 6 (*Continued*)

$k \geq 0 \leq m$, on the set of alternatives in T_1 with $x_i PqPy_j$ for every x_i and y_j, and $ST(T_2)$ is first in a similar path $(x'_1, \ldots, x'_k, q, y'_1, \ldots, y'_{m'})$ for T_2. Then $\pi = (x_1, \ldots, x_k, x'_1, \ldots, x_{k'}$, $q, y_1, \ldots, y_m, y'_1, \ldots, y'_{m'})$ is a quasi-path with the Pq- and qP-properties. Moreover, either $ST(T) = x = x_1$ is first in π or else $k = k' = 0$ and $ST(T) = x = q$ is first. It follows by the Lemma that a suitable path exits.

Conversely, suppose α has two or more members and x is first in a path $(x_1, \ldots, x_k$, $q, y_1, \ldots, y_m)$, $k \geq 0 \leq m$, with the Pq- and qP-properties. Let T be the Anglo-American agenda (c) of Fig. 6. Because qPy_i for $i \leq m$, we obviously have $q = ST(T_1^y)$. So $ST(T) = q = x$ if $k = 0$. Otherwise, $ST(T_k^x) = x_k$, and because $x_1 P x_2 \cdots P x_k$ while $x_i P q$ for all $i \leq k$, it follows by induction on k that $ST(T) = ST(T_1^x) = x_1 = x$. □

Theorem 5 $M^{ST}(\alpha) = A^{ST}(\alpha)$.

Proof Suppose $x \in M^{ST}(\alpha)$, i.e., $x = ST(T^M)$ for some Mex-Italian agenda T^M on α. Then T^M has the form shown in Fig. 7. Now construct Anglo-American agenda T^A. By a straightforward induction on n, $ST(T^M) = ST(T^A)$, so $x \in A^{ST}(\alpha)$.

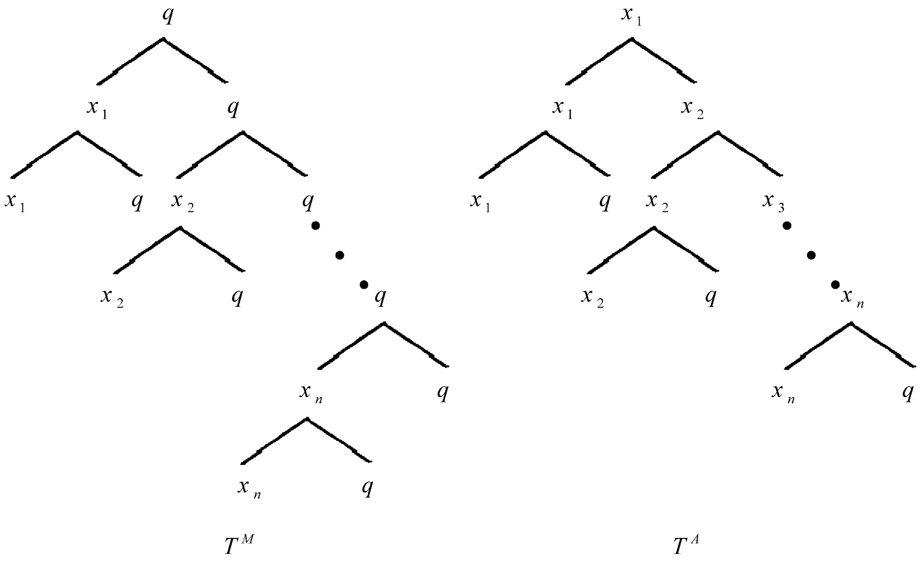

Fig. 7 Agendas to relate Mex-Italian to Anglo-American procedure

Conversely, suppose $x \in A^{ST}(\alpha)$, i.e., $x = ST(T)$ for some Anglo-American agenda T on α. If T has the form of T^A the proof is of course finished, but T need not have that form. By Theorem 4, however, x is first in some path on α that has the Pq- and qP-properties, and in the proof of Theorem 4 we showed how in that case to construct Anglo-American agenda (c) of Fig. 6, which does have the form of T^A. □

Theorem 6 *Theorems 2–4 remain true if we replace "path" with "quasi-path."*

Proof Immediate from the Lemma. □

Theorem 8 $x \in E^{SN}(\alpha)$ *iff* α *has two or more members including* q, *and either* $x = q$ *and* q *is the Condorcet winner in* α *or* xPq.

Proof Suppose $x \in E^{SN}(\alpha)$, i.e., $x = SN(T)$ for some Euro-Latin agenda T on α. Then $x, q \in \alpha$, and I shall prove by induction on $\#T$ that either $x = q$ is the Condorcet winner in α or xPq. T has principal subtrees T_1, consisting solely of x_1 (say), and T_2, headed by q. If x_1Pq then $x = SN(T) = x_1$, so xPq. Otherwise, qPx_1, so $x = SN(T_2)$. By inductive hypothesis we have either xPq again, or else $x = q$ is the Condorcet winner in $\alpha - \{x_1\}$; but qPx_1, making x the Condorcet winner in α as well.

Conversely, suppose α has two or more members including q, and either $x = q$ is the Condorcet winner in α or else xPq. Let $(q, x_1, \ldots, x_n)$ be a nonrepeating sequence of the members of α, with $x = x_1$ unless $x = q$, and look again at Euro-Latin agenda T in Fig. 4. By a straightforward induction on n, $SN(T) = q = x$ if q is the Condorcet winner in α, and $SN(T) = x_1 = x$ if instead x_1Pq. □

Theorem 9 $x \in A^{SN}(\alpha)$ *iff* α *has two or more members including* q, *and either* $\alpha = \{x, y\}$ *and* xPy *for some* y, *or* $x = qPyPz$ *for some* y, z *in* α, *or* xPq *and* $xPz \neq q$ *for some* z *in* α.

Proof Suppose $x \in A^{SN}(\alpha)$, i.e., $x = SN(T)$ for some Anglo-American agenda T on α. Then $x, q \in \alpha$, and I shall prove by induction on $\#T$ that x and q meet the stated condition. Trivial if $\#T = 3$. Suppose $\#T > 3$. Then T has, say, principal subtrees T^y, headed by y, and T^z, headed by $z \neq y$, with $x = ST(T) = SN(T^y)$. So yPz. If $\#T^y > 3$ then T^y comprises three or more alternatives, so the inductive hypothesis applied to T^y says that either $x = qPwPv$ for some w, v in α or else xPq and $xPw \neq q$ for some w in α. Suppose, however, that $\#T^y = 3$. Then T^y consists solely of y and q, and either qPy or yPq. But if qPy then $x = SN(T^y) = q$, so $x = qPyPz$. And if yPq then $x = SN(T^y) = y$, so xPq and $x = yPz \neq q$.

Conversely, suppose α has two or more members including q, and either $\alpha = \{x, y\}$ and xPy for some y, or $x = qPyPz$ for some y, z in α, or xPq and $xPz \neq q$ for some z in α. In the first case the theorem is trivial. Otherwise, construct Anglo-American agenda T with 3-node principal subtree T_1, headed by y and consisting solely of y and q, and T_2, headed by z and consisting of z, q, and any remaining alternatives in α. If $x = qPy$ then $q = x = SN(T_1)$ and yPz, so $x = q = SN(T)$. On the other hand, if xPq and $xPz \neq q$, let the y in T be x. Then $x = SN(T_1)$ and xPz, so $x = SN(T)$. □

Theorem 10 $M^{SN}(\alpha) = E^{SN}(\alpha)$.

Proof Let T^E be the Euro-Latin agenda on α and T^M the Mex-Italian agenda on α, both given by the sequence $(x_1, \ldots, x_n)$, as in Fig. 4. Let S^M be the principal subtree of T^M headed by q, and S^E the principal subtree of T^E headed by q. It suffices to prove by induction on n that $SN(T^M) = SN(T^E)$. If $x_1 Pq$ then obviously $x_1 = SN(T^M) = SN(T^E)$. And if qPx_1 then $SN(T^M) = SN(S^M)$ and $SN(T^E) = SN(S^E)$, but $SN(S^M) = SN(S^E)$ by inductive hypothesis, so $SN(T^M) = SN(T^E)$. □

Theorem 11 *If β is the top cycle in α and $q \in \alpha - \beta$ then $\beta = E^{ST}(\alpha) = A^{ST}(\alpha) = M^{ST}(\alpha)$.*

Proof We lately saw that $E^{ST}(\alpha) \subseteq \beta$. But $A^{ST}(\alpha) = M^{ST}(\alpha) \subseteq E^{ST}(\alpha)$. So it suffices to take any $x \in \beta$ and show that $x \in A^{ST}(\alpha)$. Because β is a P-cycle there exist $x_1, \ldots, x_n$ such that $\beta = \{x_1, \ldots, x_n\}$ and $x = x_1 P x_2 P \cdots P x_n P x_1$. Because P is connected in A we may partition $\alpha - \beta$ into $\{q\}, \{y_1, \ldots, y_m\}$, and $\{z_1, \ldots, z_k\} (m \geq 0, k \geq 0)$ so that $y_i Pq$ for every y_i and qPz_j for every z_j, and we may suppose without loss that $y_1 P \cdots P y_m$ and $z_1 P \cdots P z_k$. Then because $\beta = \{x_1, \ldots, x_n\}$ is a top cycle in α, $(x_1, \ldots, x_n, y_1, \ldots, y_m, q, z_1, \ldots, z_k)$ is a quasi-path on α with the qP- and Pq-properties. Hence $x = x \in A^{ST}(\alpha)$ by Theorems 4 and 6. □

Theorem 12 *If β is the top cycle in α and $q \in \beta$ then $E^{ST}(\alpha)$ is a subset and $A^{ST}(\alpha)$ and $M^{ST}(\alpha)$ are proper subsets of $\beta - \{q\}$.*

Proof Because β is a P-cycle containing q, there exist $x, y, \in \beta$ such that $xPqPy$. Having shown that $E^{ST}(\alpha)$ and $A^{ST}(\alpha) = M^{ST}(\alpha)$ are subsets of β, I shall prove that $q \notin E^{ST}(\alpha), q \notin A^{ST}(\alpha)$, and $y \notin A^{ST}(\alpha)$. Take any path π on α. Because xPq, if π has the qP-property then q cannot be first in π, and because qPy, if π has the Pq-property then y cannot be first in π either. Hence, by Theorems 3 and 4, $q \notin E^{ST}(\alpha), q \notin A^{ST}(\alpha)$, and $y \notin A^{ST}(\alpha)$. □

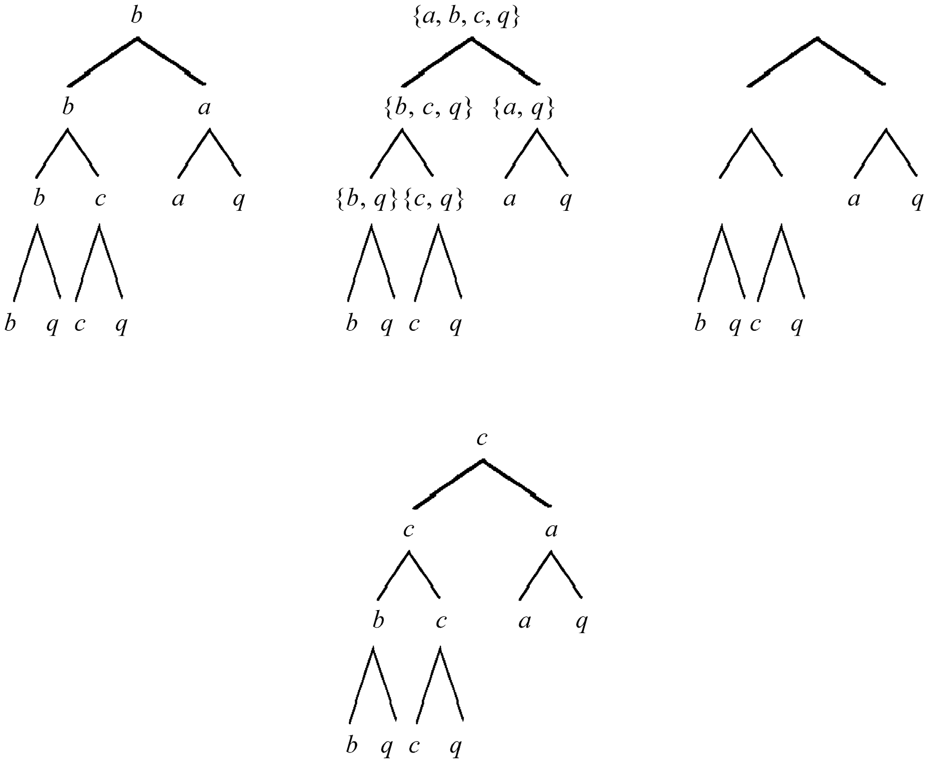

Fig. 8 The trouble with Farquharson agenda trees

Appendix 2: Controversy over agenda trees and sincere voting

My representation of agendas and my definition of $SN(T)$, the sincere voting outcome from agenda T, deviate from the original treatment of Farquharson (1969), agreeing instead with Ordeshook and Schwartz (1987). Miller (1995) opts for Farquharson's approach.

Under Anglo-American procedure, suppose we have a bill, an amendment, and a backup amendment, introduced only if the original amendment is rejected. For bill b and amended versions a and c, the agenda is the first of Fig. 8. But Farquharson and Miller would represent the same agenda by the second tree. *Alternatives* occupy only bottom nodes. *Sets* of alternatives occupy higher nodes; the set at each node comprises the alternatives at the bottom of the subtree beneath that node. Strategic voting is unaffected because it depends only on bottom nodes. But at each vote a sincere legislator can no longer compare two alternatives: they are no longer there. Instead he must compare two sets, and Farquharson and Miller say he will do so according to *lexicographic maximax*: pick the set whose best member is better, or if they are equally good then the one whose second-best is better, and so on.

To see what is wrong, note first that the Farquharson-Miller tree omits information. It does not tell us which two alternatives are compared at any but a final vote. It does show two sets, but they are redundant: the set at node N can always be found by looking to the bottom of the subtree headed by N. So their tree contains exactly the same information as the bare third tree. The cost shows up in the fourth tree. It obviously differs from the first in its information content—in which alternatives are compared at the first vote. Yet Farquharson and Miller would represent both agendas by the second tree, or equivalently the third.

Suppose you are a sincere legislator who prefers c to a to b. On my view you would initially vote for a (against b) under the first agenda but against a (for c) under the fourth. As a result, a might be chosen under the first but not the fourth. But according to Farquharson and Miller, you would vote *against a* under both agendas, even the first, where a vote *for a* would sincerely voice your preference for a over b.

What, in general, is sincere voting? In the 2000 U.S. presidential election, a voter who preferred Nader to Gore to Bush and voted for Nader rather than Gore would plainly have been sincere, not strategic. The evident principle is this: A sincere voter votes for his most preferred of the alternatives available for voting. In the legislative case we also say that a sincere legislator is short-sighted, that he does not look beyond his two voting options to the bottom of the tree, that he compares their content, not their consequences. All these formulations fit sincere voting in my sense. But a sincere legislator in the Farquharson-Miller sense need not vote for his preferred of the two alternatives available for voting. Instead, like a strategic legislator, he compares them by looking ahead to all their reachable consequences at the bottom of the tree, only he then compares them on the basis of extreme optimism rather than informed calculation.

To bring out the difference, suppose we at first have an Anglo-American agenda that pits b against a, then the winner again q. A whip needs your vote for b against a (not against q), but you love a and loathe b. Knowing that you are "sincere" in the Farquharson-Miller sense, this whip shows you a new draft law, c, written to gratify your every wish, urge, ideal, dream, and interest. He promises to bring it to a vote if b defeats a at the first vote—a costlessly reliable promise, inasmuch as everyone but you abominates c. Thus expanded, the agenda is the first of Fig. 8. If Farquharson and Miller are right, you would vote for odious b against attractive a. How cheap they make your vote!

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