time and highly developed for her age. Nowadays fourteen would not even be considered marginally more respectable than eleven, but in Victorian times Lorina would have no longer been counted as a girl but as a young woman. Whomever Dodgson courted, it did not matter one bit to Mrs. Liddell, who had different plans for all of her daughters. She forbade any liaison and that may have been what Dodgson had entered into his diary.

There are those who reject the theory of the repressed pedophile in favor of the view of Dodgson as a dangerous womanizer. According to this opinion, his alleged penchant toward little girls was only a diversionary maneuver. For example, there are claims that he was romantically involved with—horror of horrors—the dean's wife. More benign assessments of Dodgson's psyche have it that he never really grew up, remaining a big child throughout his life.

Alice ended up marrying Reginald Hargreaves, a young man who was better known as a cricket player than as a student. The wedding took place at Westminster Abbey and was quite a society event. The couple had three sons, the middle one of which was named Leopold. (Prince Leopold, in turn, who was to become the Duke of Albany, named his daughter Alice.) Two of the sons were killed in action during the First World War. When Alice and her surviving son hit on hard times, she put Dodgson's manuscript up for sale with Sotheby's, hoping to raise 4,000 pounds. It fetched 15,400, an unbelievably high price at the time. In 1932, the centenary of Dodgson's birth, the eighty-year-old Alice received an honorary doctorate in literature from Columbia University. She died two years later.

CHAPTER NINE
THE FOUNDING FATHERS

With this we leave the irksome subject of voting and elections for a while to consider a different field of mathematical conundrums that plague democracies throughout the world. It is the problem of allocating seats in a parliament. Everybody would like to assign the number of delegates that a geographical region or a political party sends to the legislature in a fair and equitable manner. Unfortunately we shall see that the questions that will be raised are quite a bit as annoying, perplexing, and sometimes counterintuitive as the problems and paradoxes that occur when voting for a proposal or electing a leader.

* * *

"The House of Representatives shall be composed of members chosen every second year by the people. . . . The number of representatives shall not exceed one for every thirty thousand . . ." Such says the Constitution of the United States of September 17, 1787. The Founding Fathers decided to allocate the seats of Congress to the member states by dividing the number of each state's voters by a number that is at least as large as 30,000. The Constitution also specified that "each State shall have at least one representative."

Other countries chose different allocation methods. The Swiss constitution, for example, is more specific about its legislature. Article 149 reads as follows: "The National Assembly consists of 200 representatives of the people. The seats are allocated according to the population of the cantons." The alpine republic's statements are more specific, but—as we shall see—they are also more problematic.

A confederation consisting of states of various sizes must allocate a different number of delegates to each of them. Of course each state or canton would like to have as big a say as possible and therefore wants to have a large delegation. A method must be found to allocate the seats in the legislature in a fair and transparent manner, without giving rise to arguments.

The American Constitution's requirements of at least 30,000 citizens per representative and of at least one representative for each state was meant to guarantee that not too much power was given to large member states. Thus a small state with 15,000 inhabitants would be entitled to one representative, while a state with ten times as many inhabitants would get five representatives at most, and maybe less.

The Constitution's wording leaves much leeway, however. Since the total number of seats is not specified, it allows for a wide range of numbers of lawmakers. With a population of 280 million, the House would consist of 5,600 congressmen if the divisor were 50,000 citizens, or of 560 if the divisor were 500,000. Both numbers are constitutionally admissible and no amendment would be required . . . except to the Capitol building.

Switzerland, a federation originally of three cantons whose number grew to twenty-five over the centuries, is famous for the precision of its watches and was equally precise when it came to their parliament. The Swiss constitution specified that in each canton's delegation to parliament there should be one representative per 20,000 inhabitants—neither more nor less. With a about 2.2 million Swiss citizens, the National Assembly started out with 111 representatives in 1848, and successively grew, in parallel with the increase in population, to 198 seats in 1928. In order to keep the number of representatives from rising any further, the number of constituents that were needed to send one representative to the capital was increased to 22,000 in 1931 and to 24,000 in 1950. But then the orderly Swiss had enough of adding and removing chairs to and from the chamber every few years, and in 1962 they fixed the number of representatives at 200.

The greater precision of the Swiss constitution did not put order in the house. To the contrary, it created problems. First, there is its wording: the constitution states facts where one would have hoped for a method of seat allocation. Second, and more importantly, the alleged facts are no facts at all. They are incompatible. Allocating a fixed number of seats, be it 200 or any other number, exactly according to the populations of the cantons is impossible for a very simple reason. Since a canton's parliamentary delegation cannot include 3.7 or 16.2 members, 200 representatives can never be allocated exactly according to the cantons' populations. And herein lies the problem that has plagued the United States, Switzerland, and many other countries for the past few centuries. Any allocation

that is exactly proportional to the populations contains fractional parts. These remainders must also be allocated. But how?

An instinctive reaction of almost everybody would be to round the seats to the nearest integer. But this does not work at all, as the following example readily shows.

TABLE 9.1
Three-state union with 1,000 citizens, 100 seats to be allocated

State	*Population*	*Percentage*	*"Raw" seats*	*Rounded seats*
Louisibama	506	50.6	50.6	51
Calyoming	307	30.7	30.7	31
Tennemont	187	18.7	18.7	19
Total	1,000	100.0	100.0	101

There are only 100 seats in the chamber, but the rounding method allocated 101 representatives to the three states.

But first things first. Since the American Constitution was so poor on details, dissent soon arose. The Founding Fathers wanted to create as large a legislature as possible, in order to decrease to a minimum the danger of corruption. (After all, it is easier to corrupt a small group of people than a large one.) According to the census of 1790, and after Vermont and Kentucky had been admitted to the Union, the population of the United States stood at 3,615,920. Using the Constitution's magic number 30,000 as a divisor, that would result in a Congress with 120 seats. Alexander Hamilton, the first secretary of the treasury, suggested a two-step procedure to allocate the congressmen to the individual states: at first, each state gets the number of seats rounded down. The seats left over would then be distributed according to the largest remainders.

Doing the arithmetic and rounding the numbers down, 112 seats were allocated in the first step. Accordingly, Congress presented a bill on March 26, 1792, that would give the eight states who had the largest fractional remainders one additional seat each. For example, Connecticut, which had 236,841 voters, would arithmetically have been due 7.895 seats (236,841 divided by 30,000). The state would have received seven seats in the first round, and since it belonged to the eight states with the largest fractional remainders, it would have received an additional seat in the second round, for a grand total of eight.

Before signing the bill into law, George Washington conferred with his

TABLE 9.2
Census of 1790. 120 seats to be allocated, 30,000 citizens per representative

		Seats allocated		
State	*Population*	*"Raw"*	*Initial*	*Final*
Connecticut*	236,841	7.895	7	8
Delaware*	55,540	1.851	1	2
Georgia	70,835	2.361	2	2
Kentucky	68,705	2.290	2	2
Maryland	278,514	9.284	9	9
Massachusetts*	475,327	15.844	15	16
New Hampshire*	141,822	4.727	4	5
New Jersey*	179,570	5.986	5	6
New York	331,589	11.053	11	11
North Carolina*	353,523	11.784	11	12
Pennsylvania	432,879	14.419	14	14
Rhode Island	68,446	2.282	2	2
South Carolina*	206,236	6.875	6	7
Vermont*	85,533	2.851	2	3
Virginia	630,560	21.019	21	21
Total	3,615,920	120.531	112	120

*States receiving an additional seat after the initial allocation.

closest advisors. One of them was Thomas Jefferson, whom we already met in chapter 6 when, as ambassador to France, he was a frequent guest at Sophie de Condorcet's salon in Paris. Jefferson was the author of the Declaration of Independence, Washington's secretary of state, and would later become the United States' third president.

Jefferson did not like the bill one bit. He hailed from Virginia, a state whose delegation, though by far the largest, would not be granted any rounding up. In his negative opinion of the bill he was joined by his fellow Virginian Edmund Randolph, the attorney general. On the other side of the argument stood Alexander Hamilton and General Henry Knox, the secretary of war. They argued for adoption of the bill. Knox came from a state that was a candidate for rounding up: Massachusetts with a fractional remainder of 0.844. The only player who seemed to have been quite selfless was Hamilton, whose home state, New York, stood to lose 0.053 seats by the system he advocated.

The naysayers did have a legitimate constitutional argument. As the astute Randolph pointed out, the bill would award all states whose delegations were to be rounded up more than one representative per 30,000

citizens. This would be unconstitutional. For example, New Hampshire would have received one representative per 28,364 citizens (141,822 divided by 5). Washington was still unsure. It was April 4. In two days the bill would automatically become law, even without his signature.

The morning of April 5 arrived. It was the last day a veto could be cast, and Washington had to make a decision. He called Jefferson to his office, before the secretary of state had even had a chance to eat his breakfast. The president was upset. Opinions seemed to be divided not because the apportionment of seats was problematic but because Northern states were pitched against the Southern states. The president did not want to take sides. Jefferson, secretly rejoicing, put Washington's mind at ease. A letter to Congress was drafted in which the president announced his veto against the apportionment bill. "I have maturely considered the Act passed by the two Houses . . . and I return it . . ." he wrote in his message to Congress. One of the reasons he gives for his veto is that "the bill has allotted to eight of the States more than one [seat] for thirty thousand [voters]." It was the first veto in the history of the United States, and only one of two that George Washington would ever cast. Be it noted, by the way, that George Washington's home state was Virginia.

So back to square one it was. On April 10, Congress threw out the vetoed bill and adopted a method of apportionment suggested by Thomas Jefferson. It consisted, first, of determining the preferred size of the House and, second, of finding a ratio that—when results are rounded down—gives exactly the required House size. Thus, the trick was to adjust the divisor according to the size of the House. As we saw above, using Jefferson's method with a divisor of 30,000 would have resulted in a House with 112 representatives. In order to obtain a House with 120 seats, a divisor of about 28,500 would be needed. (Actually, any divisor between 28,356 and 28,511 would have worked.) But this number leaves a very bad feeling. Even though, on average, it satisfies the constitutional requirement of one congressman for at least 30,000 voters (3.6 million voters for 120 seats) for the United States as a whole, it violates the requirement for some states. After all, it was the unconstitutionality of a divisor lower than 30,000 for each individual state that had prompted Washington to cast his veto. To circumvent problems, Congress decided that the House would comprise 105 members. With the number of seats set, it was determined that one representative per 33,000 citizens would do the trick. Now everything seemed in order.

Jefferson's method is referred to as a "divisor method." It remained in force for fifty years, until 1830. In the meantime the Union grew from fifteen states to twenty-four and population increased to nearly 12 million. To accommodate the increasing population of the growing Union, the number of seats in the House grew from 104 to 240. But the principle remained the same: decide on the size of the House, find an appropriate divisor and round down.

Not everyone was happy with the system, however. The small states started getting restive. They noticed that something was amiss. The big brothers, like Virginia, always seemed to be getting more than their fair share. It soon became apparent that Jefferson's method—which had given Virginia an advantage in 1790 but otherwise seemed fair enough—put small states at a disadvantage. Delaware, for example, got rounded down four times, with "raw" numbers of seats of 1.61, 1.78, 1.68, and 1.52. The state of New York, with quotas of 9.63, 16.66, 26.20, 32.50, and 38.59, got rounded up every single time.

One reason lay in the fact that rounding 3.5 representatives down to 3 hurts more than rounding 30.5 down to 30. Thus a small state would require more citizens for each representative.

TABLE 9.3
Jefferson's round-DOWN method

Union of two states, total population 340,000, 33 seats in the House of Representatives. The ratio of citizens per representative (the "divisor") that is needed to achieve this House size, using the round-down method, has been determined as 10,000.

State	*Population*	*"Raw" seats*	*Seats*	*Ratio*
Massaware	305,000	30.5	30	10.167
Louisylvania	35,000	3.5	3	11,667
Total	340,000	34.0	33	10,303

The smaller state requires about 15 percent more citizens for each representative than the larger state (11,667 versus 10,167).

There is another reason why small states get booted most of the time. It is mathematical and a bit subtler. Let me first give a numerical example. (Note that "raw" seats of 26.20 and 1.68 in table 9.4 correspond to a true situation in New York and Delaware, mentioned above.)

TABLE 9.4
Jefferson's round-DOWN method

With a population of 10 million and 100 seats in the House, the initial divisor is 100,000. Twenty-eight seats are to be allocated to Neware and Delayork. In order to apply Jefferson's method, the divisor is reduced from 100,000 to 97,000.

	Population	*Divisor 100,000*	*Seats*	*Divisor 97,000*	*Seats*	*Ratio*
Neware	2,620,000	26.20	26	27.01	27	97,037
Delayork	168,000	1.68	1	1.73	1	168,000
. . . .		. . . .	72	. . . .	72	
Total	10,000,000		99		100	100,000

Delayork needs 73% more citizens per seat in the House.

"Neware" gets the additional seat, in spite of initially having had the lower fractional remainder (0.20 versus 0.68). The explanation for this manifestly unsatisfactory situation is that when the divisor is reduced from 100,000 to 97,000, a smaller population is needed for every seat already assigned. In the above case, each of Neware's 26 initial seats requires 3,000 fewer citizens, thus granting this large state a 27th seat. At the same time the small state of "Delayork" profits only once from the lower divisor. Another way to see this is to realize that the lower divisor increases the number of raw seats by 3.1 percent. Thus Delayork's raw seats increase from 1.68 to 1.73, while Neware's seats increase from 26.20 to 27.01, putting it a hair's breadth beyond the threshold for an additional seat. The upshot of all this is that while Neware needs less than 100,000 citizens per congressional seat, Delayork requires a whopping 168,000.

Not surprisingly, all this is patently unfair to the small states, and they finally caught on. They complained that rounding down left some of their voters unrepresented. They certainly had a valid point. Voters who make up the fractional parts of a seat effectively got rounded out of the system. The small states found an advocate for their case in the person of John Quincy Adams. This former president and elder statesman, whose home state, Massachusetts, was the second largest in the Union, no longer had any ax to grind, neither for himself nor for his state. Deeply troubled by the fact that Jefferson's method effectively disenfranchised many voters, he became a spokesman for the small states. After passing many a sleepless night, he announced that he had found a system that would end the discrimination of small states.

Adams had not looked very far for a remedy to this ill of society. In fact, he decided to advocate Jefferson's method with only a teeny difference: after performing the initial computations, the number of seats would be rounded up instead of down. In his view, this corresponded more closely to the spirit of the Constitution, since by rounding the fractional parts of a seat up to a full seat, every citizen would be represented, and then some. Of course, the method would give the small states an advantage. But this rounder-upper apparently felt that a little affirmative action would do no harm after so many years of discrimination.

TABLE 9.5
Adams's round-UP method

With a population of 10 million and 100 seats in the House, the initial divisor is 100,000. 28 seats are to be allocated to Neware and Delayork. In order to apply Adams's method, the divisor is increased from 100,000 to 104,000.

	Population	*Divisor 100,000*	*Seats*	*Divisor 104,000*	*Seats*	*Ratio*
Neware	2,668,000	26.68	27	25.65	26	102,615
Delayork	120,000	1.20	2	1.15	2	60,000
⋮		⋮	72	⋮	72	
Total	10,000,000		101		100	100,000

In spite of initially having the lower fractional remainder (0.20 versus 0.68) Delayork gets the additional seat. Neware needs 71% more citizens per seat in the House.

The shoe is now definitely on the other foot. With an increase of the divisor from 100,000 to 104,000, a larger population is required for every seat. Neware would have needed 4,000 additional citizens for each of the 27 seats. Since it does not have them, its delegation obtains only 26 seats. Delayork has citizens left over and gets rounded up to 2 seats. In the final count, Delayork needs only 60,000 citizens per representative, while huge Neware requires over 100,000.

As had to be expected, the large states would have none of that. Affirmative action was not their forte and—being the stronger side in this dispute—the rounder-downers got their way. Adams's suggestion, which is sometimes called "the method of smallest divisors," was considered by

Congress but never enacted. "I hung my harp upon my willows," Adams wrote in his memoirs, and simply gave up.

It took the rhetorical skills of Senator Daniel Webster, one of the most eloquent Americans to have ever walked the floor of the Senate, to convince Congress to adopt the course of action that reasonable people would have found most sensible had they not been so caught up in looking out for themselves. Webster, a lawyer by profession, was first catapulted to prominence by his famous defense of Dartmouth College's independence against the New Hampshire legislature. He was a spellbinding orator and his speeches are considered even today rare examples of rhetorical brilliance. Whenever he addressed the Senate, it was standing room only. Men and women traveled from afar to hear him speak, and the moment he took to the podium, all present fell silent. Reportedly, his perorations could move the most reserved of men to tears.

With such credits, it is somewhat surprising that the allocation method Webster came up with was very simple indeed. Again, it was the procedure that Jefferson had originally proposed, but this time with a two-sided twist. It consisted of finding a divisor for the populations of the states, such that the result, when rounded *up* or *down* to the nearest integer number, gave the desired amount of seats. The "method of major fractions," as it was thereafter called, would not avoid inequities but at least it was unbiased; it would favor large states sometimes, small states at other times. In 1842, Congress, which since 1787 comprised the Senate and the House of Representatives, adopted Webster's method.

TABLE 9.6
Webster's round-to-nearest-number method

Union of two states, total population 330,000, 33 seats in the House of Representatives. The ratio of citizens per representative that is needed to achieve this House size has been determined as 10,000.

(1)	*State*	*Population*	*"Raw" seats*	*Seats*	*Ratio*
	Coloraska	304,000	30.4	30	10,133
	Nebrado	26,000	2.6	3	8,667
	Total	330,000	33.0	33	
(2)	*State*	*Population*	*"Raw" seats*	*Seats*	*Ratio*
	Oregansas	296,000	29.6	30	9,867
	Arkanson	34,000	3.4	3	11,333
	Total	330,000	33.0	33	

Sometimes, as in (1) above, the small state needs fewer citizens per representative, at others the large state profits.

The ever so reasonable method of major fractions stayed in force for not more than ten years. Maybe it was too reasonable a procedure, because soon the squabbles started again. In 1850, before anybody had a chance to get all worked up about the results of this year's census, Senator Samuel Vinton from Ohio stepped in. His objective was to put an end to the bickering and quarrelling that, like clockwork, took place every ten years, after each census. He proposed a new method. Each state would, first of all, be allocated the number of seats rounded down. The leftover seats would then be distributed to the states with the greatest fractional remainders.

Only the proposal wasn't so new. In fact, it was precisely the one that Hamilton had proposed half a century earlier. It was also precisely the one that had been vetoed by George Washington. But a name change avoided a replay of the debacle of 1792, and the method that became known henceforth as the "Vinton method" was enacted by Congress. (Hamilton would have rejoiced at the rehabilitation of his method, had he not been killed in a duel in 1804.) To make everybody happy, the House was also increased from 233 seats to 234, a size in which Hamilton's and Webster's methods actually agreed.

The population of the United States continued to grow, and with it the size of the House was increased again and again. In 1860 the number of congressmen representing their constituents increased to 241 from 234 ten years earlier, and in 1870 the number of seats was going to be fixed at 283, a number in which Hamilton's and Webster's methods again resulted in the same apportionment. But because of political squabbles the House's size was subsequently increased to 292 seats. Now everybody was unhappy, because the final apportionment did not agree with either method.

Then something extraordinary happened. After the results of the census of 1880 became known, everybody expected the House to grow again. In order to give the congressmen the necessary ammunition for the infighting that would undoubtedly precede the next apportionment of the House, C. W. Seaton, the chief clerk of the Census Office, did some computations. Using the census results of 1880, he worked out the apportionments according to Vinton's method for all House sizes between 275 and 350. Starting with 275 representatives everything worked out just fine all the way up to 299. Whenever he added an additional seat it was picked up by some lucky state. But when he reached 300 seats, a bombshell fell in

his lap. The delegation of the state of Alabama *decreased* by one representative, from 8 to 7. In its stead, *two* states, Illinois and Texas, each got an additional seat.

Seaton was dumbfounded. The congressmen were flabbergasted. How could such a thing happen? The phenomenon became known as the Alabama Paradox.

TABLE 9.7
Alabama Paradox

299 seats are to be allocated. Total population is 49,713,370 and the appropriate divisor is 165,120.

	Alabama	*Texas*	*Illinois*
Population	1,262,505	1,591,749	3,077,871
"Raw" allocation	7.646	9.640	18.640
Seats in first round	7	9	18
Fractional part	0.646	0.640	0.640
Additional seats	1	0	0
Total seats	8	9	18

Now 300 seats are to be allocated. The appropriate divisor is 164,580.

	Alabama	*Texas*	*Illinois*
Population	1,262,505	1,591,749	3,077,871
"Raw" allocation	7.671	9.672	18.701
Seats in first round	7	9	18
Fractional part	0.671	0.672	0.701
Additional seats	0	1	1
Total seats	7	10	19

Alabama loses one seat; Texas and Illinois each gain a seat.

The reason for this paradox becomes apparent when we delve a little deeper into the numbers. When the total number of seats increases from 299 to 300, the states' "raw" numbers of seats grow on average by about one-third of 1 percent. But Texas and Illinois start out with larger populations and therefore gain more in absolute numbers. Thus the number of "raw" seats grows by only 0.025 in Alabama (from 7.646 to 7.671), by 0.032 in Texas (from 9.640 to 9.672), and by 0.061 in Illinois (from 18.640 to 18.701). As a consequence, the larger states creep past Alabama.

Actually, the phenomenon had already been noted ten years earlier. Rhode Island had had two representatives in the House since 1790. But following the census of 1860, the Plantation State, to its consternation, was allowed only one congressman out of 241. Ten years later, with an increase in population and in the size of the House, Rhode Island hoped to get its second representative back. Initial calculations showed that this would, in fact, be the case if the number of congressmen were increased to 270. If the House were enlarged to 280 members, however, Rhode Island stood to lose that second congressman again. So the Alabama Paradox should have correctly been called the Rhode Island Paradox in 1870. But then the powers to be decided to set the size of the House at 292. Rhode Island got its second representative and the matter was forgotten for another ten years.

In 1880, however, Congress went into a tizzy. The Hamilton-Vinton method of apportionment, which had by now become dear to all, was in danger. Tempers ran high. One congressman accused another of "committing a classic rape on a cloud of statistics, right in the face of the House." In order to prevent the contest between the proponents of the two methods from turning even uglier, Congress decided not to decide and resolved, instead, to enlarge the House to 325 seats. With this size the congressmen did not have to take sides because Webster's and Hamilton's methods agreed and the problem could be postponed for at least another ten years. Maybe a wholly different apportionment method would be found in the meantime? Or the methods would again agree? Or the congressmen would no longer be in Congress and could let their successors worry about the Alabama Paradox.

They were right. All it took in 1890 was an increase to 356 seats and the same compromise could be forged. With a House of that size, both methods agreed, no state would lose a seat as compared to the previous apportionment. Ten years later, no such luck. When tables on the apportionment were prepared in 1901 for sizes of the House between 350 and 400, Maine's apportionment oscillated between 3 and 4 seats and Colorado would receive 3 seats for every size of the House, except at 357 seats where it would be allocated just 2. Of course, the chairman of the Select Committee on the Twelfth Census, no friend of Colorado's and Maine's, suggested fixing the size of the House precisely at 357. Tempers rose and the atmosphere again became ugly.

And more bad news was on the way. The congressmen and the pencil pushers had failed to notice another, even more serious threat to Hamilton's method of apportionment. The nation's population was continually

rising and with it the size of the House was increased. But could problems arise even with a House of constant size?

Yes, they could. Let me illustrate by example. In 1900 the populations of Virginia and Maine stood at 1,854,184 and 694,466 citizens, respectively. During the following year Virginia's population grew by 19,767 citizens (+1.06 percent), while Maine's increased by 4,648 (+0.7 percent). If an additional seat were to be allocated to one of the two states, one would think that it would have to go to Virginia. Far from it. The surprising fact is that by Hamilton's method of allocating leftover seats to the states with the largest remainders, it would have been Maine that would have received an additional seat, while Virginia would have lost one. Let us look at the numbers:

TABLE 9.8
Population Paradox

	1900	*Seats*		*1901*	*Seats*	
	Population	*raw*	*rounded*	*Population*	*raw*	*rounded*
Virginia	1,854,184	9.599*	10	1,873,951	9.509	9
Maine	694,466	3.595	3	699,114	3.548*	4
Total	74,562,608		386	76,069,522		386

*rounded up

Total population grew from 74,562,608 to 76,069,522 and the appropriate divisors were 193,167 in 1900 and 197,071 in 1901. Virginia's population grew by 19,767, while Maine's grew by only 4,648. Nevertheless, if a new House had been appointed in 1901, Virginia would have lost a seat to Maine. (The numerical values for 1901 are inferred from the population growth between 1900 and 1910. There was no separate census in 1901.)

The reason for this strange situation, which would henceforth be known as the Population Paradox, is that the remainders traded places. In 1900, Maine had the smaller remainder behind the decimal point (0.595) and failed to make the mark. A year later—because the nation as a whole grew faster than either of the two states—it would have been Virginia that would have had the smaller remainder (0.509) and Maine would have picked up the extra seat.

Why is the Population Paradox even more of a threat to Hamilton's method of apportionment than the Alabama Paradox? Well the latter, which appears when the size of the House increases, can be avoided if Congress decides to hold the number of its members constant. But population increase cannot be stopped and so this paradox was here to stay.

This time Congress did take a stand and Hamilton's method was abandoned in favor of Webster's. At least it did not suffer from the defect of the Alabama and the Population paradoxes. In addition, the House was enlarged to 386 seats, which ensured that no state would lose a seat. (Amazingly, it is not absolutely clear whether it really was Webster's method or Hamilton's that was used in 1901. Depending on which population data one looks at, the preliminary data that was used by the House or the final data published by the Census Bureau, one could arrive at either conclusion.)

And this was still not all. Another paradox was looming just around the corner. In 1907, Oklahoma joined the Union. The House consisted of 386 seats. This time the congressmen thought they knew exactly what needed to be done to keep everybody happy. Oklahoma's population stood at about 1 million, which corresponded to five seats in Congress. So the congressmen decided to simply add five seats to the House. Oklahoma would receive them, nobody would get hurt, and everybody would be happy.

Or so they thought. The five seats were added and to nobody's surprise, when the new total of 391 seats was reallocated, using Hamilton's method again, Oklahoma got them all. But something strange happened on the way. New York lost a seat, which Maine picked up! It was quite infuriating. For once everybody had done everything right, and then this happened. The situation was called the New State Paradox. Let us see how it came about.

TABLE 9.9
New State Paradox

	Population before incl. of Oklahoma	*Seats* *raw*	*Seats* *rounded*	*Population after incl. of Oklahoma*	*Seats* *raw*	*Seats* *rounded*
New York	7,264,183	37.606*	38	7,264,183	37.589	37
Maine	694,466	3.595	3	694,466	3.594*	4
Oklahoma	—	—	—	1,000,000	5.175	5
Total	74,562,608		386	75,562,608		391

*rounded up
After addition of the five seats—which went to Oklahoma—New York lost a seat to Maine.

Even though the populations of New York and Maine did not change, and Oklahoma was awarded exactly the five additional seats, apportionment of the remaining seats was affected. When adding Oklahoma's population to the nation's total, both New York's and Maine's and all other states' fractional seats decreased. But New York, being the largest state, lost more in absolute terms than the smaller states. As a result, Maine's fractional seat managed to inch past New York's remainder.

By the way, the number of electors each state has in the Electoral College corresponds to the number of representatives and senators that the state sends to Congress. Hence the apportionment problem spills over to presidential elections. In 2000, George W. Bush defeated Al Gore by a tally of 271 to 266 in the Electoral College (using the apportionment method that will be described in the next chapter). Had Jefferson's method been used to apportion the House after the 1990 census, Gore would have garnered 271 electoral votes and become the president.

To close this chapter let me summarize the advantages and disadvantages of each of the methods in the following table.

TABLE 9.10

Method	*Hamilton*	*Jefferson*	*Adams*	*Webster*
Bias toward:	large states	large states	small states	Neither
Paradox				
Alabama	Yes	No	No	No
Population	Yes	No	No	No
New State	Yes	No	No	No

Webster's method seems the most reasonable way of apportioning seats in the House, while Hamilton's, which falls prey to all paradoxes discovered so far, seems the odd man out. But Hamilton's has one "redeeming feature": it favors the large states. So, larger states would continue their battle against Webster and further troubles were unavoidable. (There is one other redeeming feature to Hamilton's method, but we will have to wait until chapter 12 to discover it.)