5

Spatial Models of Majority Rule

The story line to this point has emphasized a trade-off in group decision making between the coherence of group choices on the one hand and the fairness of the method of decision making on the other. If we consider a limited domain of circumstances, then we may be able to avoid the pain of this trade-off. Put somewhat differently, if individual preferences happen to arrange themselves in particular ways—that reflect a consensus of a specific sort—then group decisions (certainly those made by majority rule) work out quite nicely. In the previous chapter, I described single-peaked preferences as one kind of consensus that facilitated coherence in majority-rule decision making. In this chapter I want to give an intuitive geometric characterization of this condition.

Frankly, however, all this gets pretty boring pretty quickly. The author, and perhaps some of the readers, may enjoy technical riffs and philosophical discourses, but most readers are more impatient and anxious to see some payoff. I think this chapter constitutes an important investment. Once I give single-peakedness a geometric representation I will be able to apply it to some interesting political situations—namely, two-party electoral competition and legislative committee decision making.

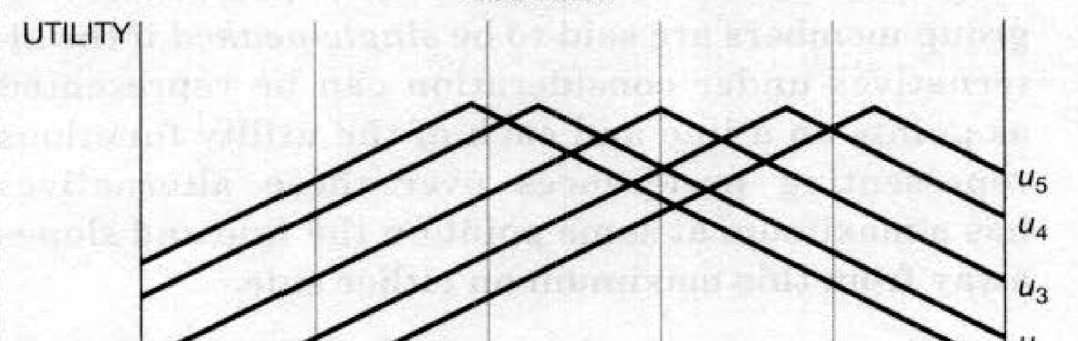
SPATIAL FORMULATION

The Simple Geometry of Majority Rule

Suppose a group's problem is, in effect, to pick a point on a line: the group must select some single numerical parameter. For example, a bank's board of directors must decide each week on the week's interest rate for thirty-year home mortgages. In effect, the relevant interest rates are points on a line, one endpoint being 0 percent and the other being some positive number, say 10 percent. This interval is written as [0, 10]. In this and other circumstances, I want the reader to imagine a group of individuals each of whom has a most-preferred point on the line and preferences that decline as points further away in either direction are taken up.

In Figure 5.1 the preferences of the five-person board of bank directors, $G = \{1, 2, 3, 4, 5\}$, are displayed. The board is meeting on Monday morning to decide the interest rate to

FIGURE 5.1



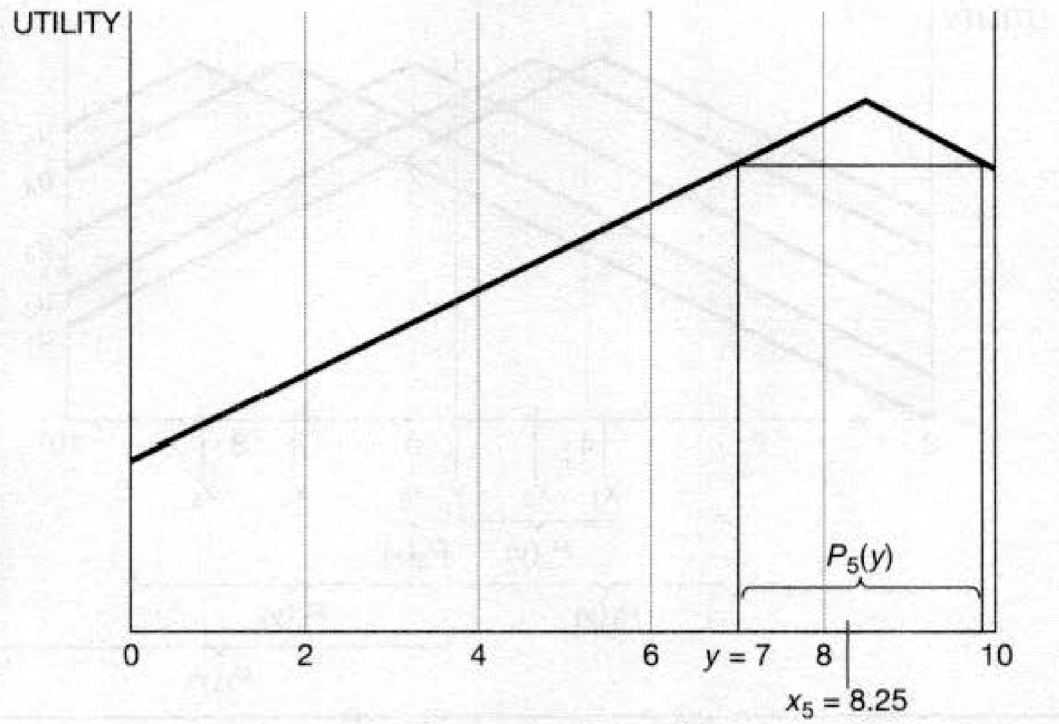
charge for home mortgages this coming week. Each individual $i \in G$ has a most-preferred point (also called *bliss point* or *ideal point*), labeled x_i, located on the [0, 10] interval (drawn as the horizontal axis), representing his or her most-preferred interest rate.[1] Thus, director 1 has a most-preferred interest rate (x_1) of just less than 4 percent, director 2's (x_2) is just more than 4 percent, and so on. On the vertical axis I have written the label "utility" to measure preferences. For each individual I have graphed a *utility function*, which represents the director's preferences for various interest rate levels in the [0, 10] interval. Naturally, the utility function, labeled u_i for Mr. or Ms. i, is highest for i's most-preferred alternative, x_i, and declines as more distant points are considered. Thus, Ms. 5 most prefers an interest rate a little higher than 8 percent, with her preference declining either for higher or lower rates. For obvious reasons (just look at the graphs) the preferences of these individuals are *single-peaked*, which is defined as follows:

> **Single-Peakedness Condition. The preferences of group members are said to be *single-peaked* if the alternatives under consideration can be represented as points on a line and each of the utility functions representing preferences over these alternatives has a maximum at some point on the line and slopes away from this maximum on either side.**

Is there any connection between this definition of single-peakedness and Black's definition given in the previous chapter? That is, do utility functions with a single peak, as displayed in Figure 5.1, have anything to do with all voters agreeing that some alternative is "not worst"? You bet! Take any three interest rates displayed in Figure 5.1—say, 3 per-

[1] Recall that "$i \in G$" means "the element i in the set G," where i stands for any one of the five bank directors.

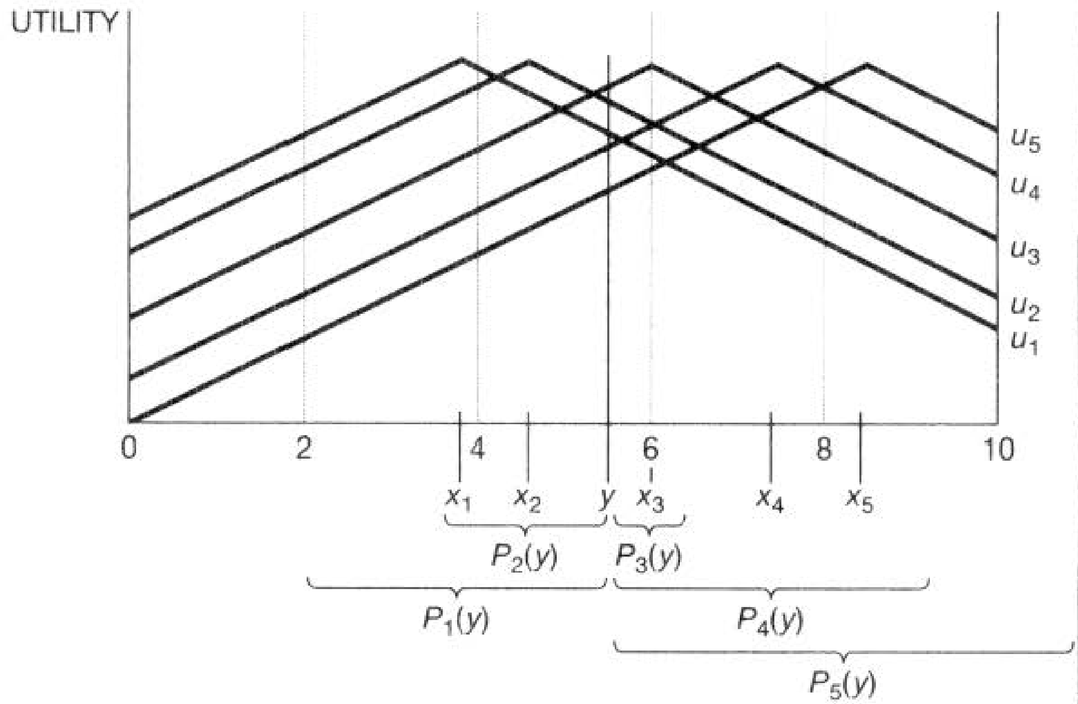
FIGURE 5.2



cent, 5 percent, and 9 percent. It is pretty easy to see that 5 percent is not the worst among these three rates for any of the five members of the group. Indeed, for any three interest rate levels the reader chooses, one of those is not worst for any of the five bankers. That's what single-peakedness means!

In order to develop some tools that will be used in subsequent analysis, let's look at one of these individual bankers in isolation (by which we *really* mean let's look at an isolated utility function). In Figure 5.2 we show the most mean-spirited of the bank's directors, Ms. 5, who most prefers a fairly high interest rate: $x_5 = 8.25\%$. Consider an alternative rate, $y = 7\%$. The set of points Ms. 5 prefers to y is described by the set labeled $P_5(y)$ in Figure 5.2. This is Ms. 5's preferred-to-y set: if y were on offer, then $P_5(y)$ describes all the points she would prefer to it, given her preferences. As the figure shows, $P_5(y)$ is computed by determining the utility level for y

FIGURE 5.3



and then identifying all the interest rates on the horizontal axis with utility levels greater than the utility for y.[2]

In Figure 5.3 I display the preferred-to-y sets of all five bank directors (note that y, in this figure, is just below 6 percent). Notice that these sets overlap to some degree—there are points in common to $P_4(y)$ and $P_5(y)$, for example. This means that there are specific points that *both* Mr. 4 and Ms. 5 prefer to y.[3]

Of great interest to us is the set of points a majority prefers to y. This is called the *majority winset of y*, written as $W(y)$. We define it as follows. Let M be the set of majorities in our group of bankers, G; it is the collection of three-person coalitions (there are ten such coalitions), four-person coali-

[2] The endpoints of the preferred-to-y set are included even though, technically speaking, the group member ranks these endpoints at the *same* utility level as y.

[3] In set-theoretic notation, we can write these common points as the *intersection* of the two preferred-to-y sets: $P_4(y) \cap P_5(y)$. ($\cap$ is the intersection symbol, so that $A \cap B$ means "the points in both set A and set B.")

DISPLAY 5.1
THE MAJORITY COALITIONS OF G={1,2,3,4,5}

Size of Coalition	Coalitions
3	{1,2,3}, {1,2,4}, {1,2,5},
	{1,3,4}, {1,3,5}, {1,4,5}
	{2,3,4}, {2,3,5}, {2,4,5}
	{3,4,5}
4	{1,2,3,4}, {1,2,3,5}, {1,2,4,5}
	{1,3,4,5}, {2,3,4,5}
5	{1,2,3,4,5}

tions (there are five of these), and the coalition-of-the-whole. So, there are sixteen different majority coalitions in M; they are listed in Display 5.1. For each of these sixteen majority coalitions, consider the common intersection of preferred-to-y sets (if there is any); these are the points that this particular majority prefers to y. Thus, the members of the majority {3,4,5} in Figure 5.3 share points each prefers to y. Determine this set for each of the majority coalitions. Then take the union of these sixteen sets. This is $W(y)$.[4]

It is now rather straightforward to describe the coherent choices of groups. If some alternative, x, has an empty winset (written: $W(x) = \varnothing$, where $\varnothing$ means "empty" in set notation), then it is a clear candidate for the group choice. Why? Simply because $W(x) = \varnothing$ means there is no other alternative that any of the sixteen majority coalitions prefers to x. It's hard to deny choosing x if there is nothing any majority agrees on in its

[4] In Figure 5.3 it turns out that members of only *one* of the sixteen majorities, {3,4,5}, has overlapping $P_i(y)$ sets. Members of the remaining fifteen majorities (listed in Display 5.1) cannot agree on any points they jointly prefer to y. For any one of those fifteen, say, {1,2,4}, some members prefer only points to the left of y while others prefer only points to the right of y. As a group they cannot agree. Thus $W(y) = P_3(y) \cap P_4(y) \cap P_5(y)$.

place. On the other hand, if the winset of x is not empty ($W(x) \neq \varnothing$), then it is hard to justify the choice of x. How can you choose x when some majority of the group clearly wants some other specific alternative? And if the winset is nonempty for *every* alternative, we have a problem: the group's preferences are incoherent, since some majority prefers something to every alternative available.

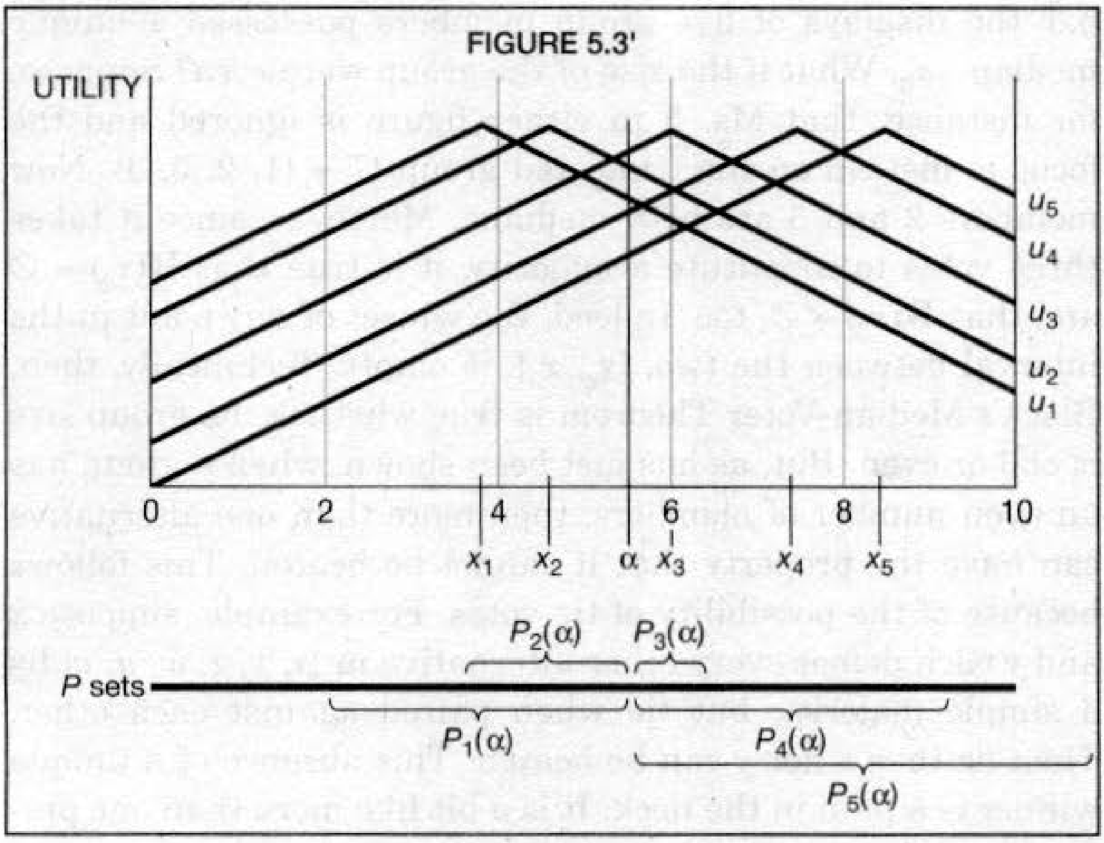
The question of the moment is whether, or in what circumstances, an x possessing an empty winset exists. If any complete and transitive preferences may be held by the individuals in G—Arrow's "universal domain" condition—then, as we have seen, the answer is "not necessarily." Why? Because under Arrow's condition U, it is possible for majority preferences to cycle, in which case $W(x) = \varnothing$ for *no* alternative. But if preferences are restricted, then a different answer is possible.

> **Black's Median-Voter Theorem. If members of group *G* have single-peaked preferences, then the ideal point of the median voter has an empty winset.**

One such group consisting of individuals with single-peaked preferences is pictured in Figure 5.3'. The median voter ideal point in this group is x_3 of Mr. 3.[5] The claim of Black's Theorem (the same Duncan Black, by the way, as in the previous chapter) is that $W(x_3) = \varnothing$, and that x_3 is the majority choice.

We can prove this theorem using the example of the five bank board members. Consider any arbitrary point in the feasible set of interest rates, [0, 10], to the left of x_3—say the point labeled α in Figure 5.3'. Notice that α is preferred to x_3 by members 1 and 2, since x_3 is not in either $P_1(\alpha)$ or $P_2(\alpha)$, but x_3 is preferred to α by members 3, 4 and 5. Thus, x_3 is

[5] The median of a set ordered from left to right is the point such that at least half the points are at or to its right and at least half the points are at or to its left.



majority-preferred to α. But α is any arbitrary point to the left of x_3. For any such point, we know at the very least that members 3, 4, and 5 will prefer x_3 to it. (It is possible that some of the remaining members will share this preference, too.) Next, consider any arbitrary point to the right of x_3 (not pictured). Members 4 and 5 may prefer it to x_3, but members 1, 2, and 3 hold the opposite preference, so that x_3 is majority-preferred. The argument is exactly the same as with α above, since we selected an arbitrary alternative to the right of x_3. To sum up, we now know that the ideal point of the median voter is preferred by a majority to *any* arbitrary point to the right or to the left of it, that is, to all remaining points. Hence, it has an empty winset and is the majority choice.

Before complicating this key result, I should mention that there are three hidden assumptions, and probably more besides, that warrant some discussion. First, in the example, the group G of bankers is *odd* in number. Thus, in Figures 5.3 and

5.3' the displays of five group members possessed a unique median—x_3. What if the size of the group were *even*? Suppose, for instance, that Ms. 5 in either figure is ignored and the focus is instead on the truncated group $G' = \{1, 2, 3, 4\}$. Now members 2 and 3 are both medians. Moreover, since it takes three votes to constitute a majority, it is true that $W(x_2) = \varnothing$ and that $W(x_3) = \varnothing$, too. Indeed, the winset of *any* point in the interval between the two, $[x_2, x_3]$, is empty. Technically, then, Black's Median-Voter Theorem is true whether the group size is odd or even. But, as has just been shown, when a group has an even number of members, then more than one alternative can have the property that it cannot be beaten. This follows because of the possibility of tie votes. For example, suppose x and y each defeat every other alternative in $\{x, y, z, w, u, v\}$ by a simple majority, but tie when paired against each other. Then neither x nor y can be beaten. This absence of a unique winner is a pain in the neck. It is a bit like more than one pretender to the throne, or more than one person claiming to be king of the mountain. It is for this reason that groups establish some procedure for breaking ties *well in advance* of any substantive deliberations or, better yet, that they make sure that the group is odd in number.[6]

Second, *full participation* is assumed. Everyone with the franchise is assumed to exercise it. Of course, in any particular instance of group choice this need not happen. If bank board members 4 and 5 oversleep one week, then Ms. 2 becomes the median voter of the now reduced three-person board; if members 1 and 3 are out of town the following week, then Mr. 4 becomes the median. In each of these cases, as well

[6] For example, the U.S. Constitution *requires* the Senate to have an even number but establishes a tiebreaking procedure. The vice president of the United States, sitting as the president of the Senate, is allowed to vote only in case of a tie. Likewise, the standing rules of the House of Representatives, which has an odd number, provide a tiebreaking rule, asserting that a motion *fails* if it obtains no more yeas than nays—it fails on a tie.

as in the case of the full board, the median-voter result applies. *Who* the median is, however, depends upon who the participants in the group are. We may forecast the group decision if we make *assumptions* about participation (for example: assume everyone votes), or we may make forecasts *contingent* on participation (for example, if 4 and 5 oversleep, then we predict x_2 will be the group's choice; but if everyone votes, then x_3 is the predicted choice).

Third, it is assumed that those exercising the franchise do so *sincerely*. But as we have seen in earlier chapters, group members will have occasion and incentive to misrepresent their preferences and not reveal them honestly. This is a subject of great interest that we take up on its own in Chapter 6.

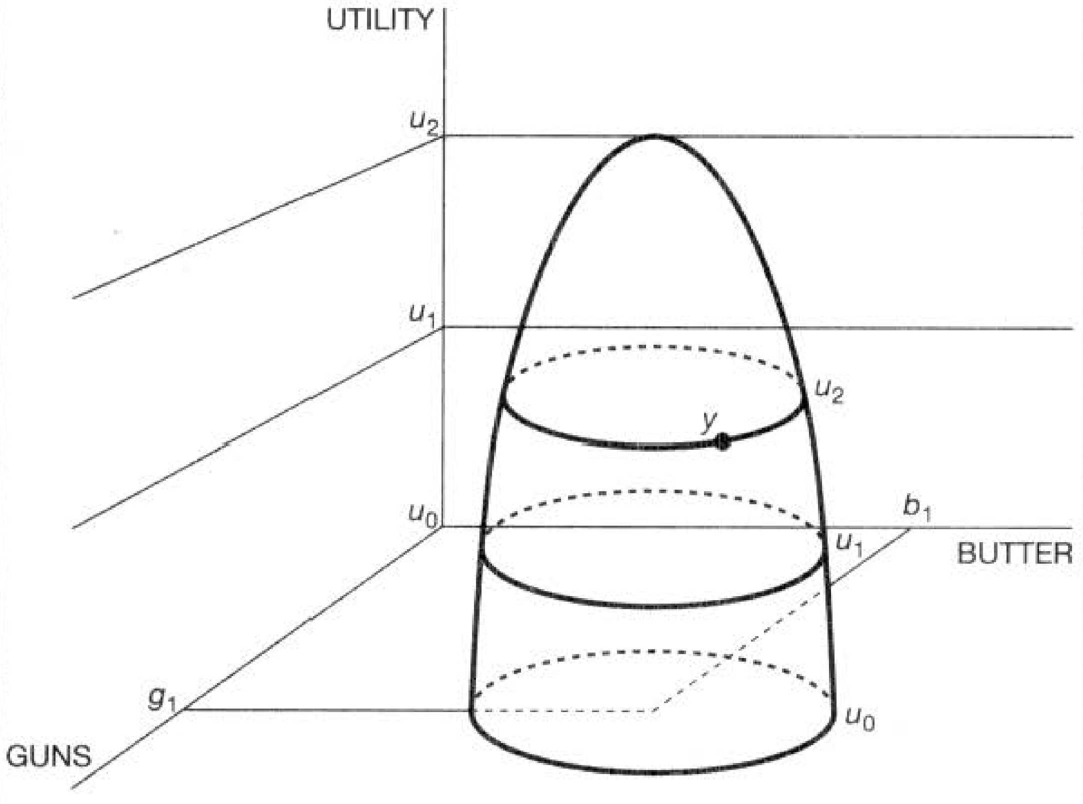
The (Slightly) More Complicated Geometry of Majority Rule

One-dimensional models of choice with the single-peakedness condition permit rather sophisticated ways to think about real politics. They generate very crisp expectations about how politics in these settings gets played out. But many social situations cannot be reduced to one-dimensional affairs.

Recall the game of "divide the dollars." If the game were played by a group of three individuals, then it is necessary to have *two* dimensions in which to represent outcomes. The first dimension gives the amount that player 1 receives, while the second dimension gives the amount that player 2 receives. (Subtract the sum of these two numbers from the total number of dollars to be divided and you get the amount that player 3 receives.[7]) I hope I convinced the reader earlier that games of division, like "divide the dollars," are commonplace in political life. So it must be conceded that as crisp and as sophisti-

[7] Generally, when dividing a fixed pie among n categories (or people), we need only n–1 dimensions to display all outcomes.

FIGURE 5.4



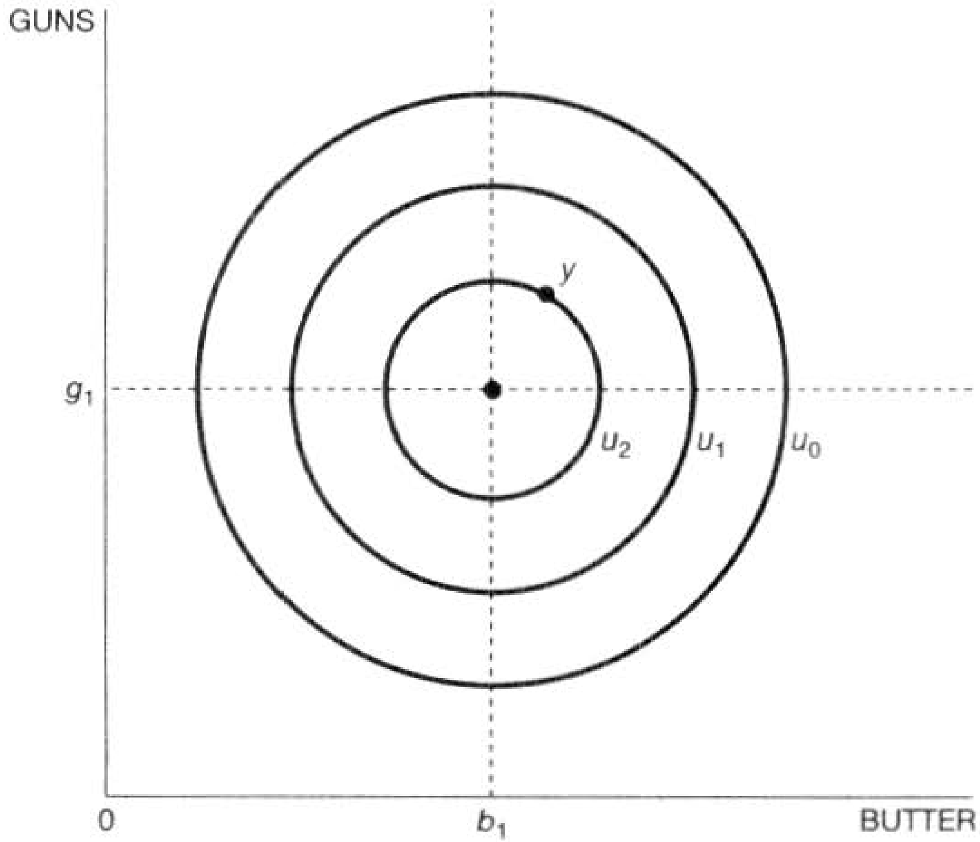
cated as the one-dimensional models are, they are *special cases* of a more general multidimensional arrangement. We need to see what this more general arrangement is like.

Most of what needs to be said is covered by focusing on a two-dimensional circumstance, like that pictured in Figure 5.4. (There are actually *three* dimensions in this figure, but this will be clarified shortly.) Let's consider a problem in budgeting, in which a group of legislators, perhaps an appropriations committee, must decide how to divide expenditures between "guns" and "butter" (symbolizing the competition between defense and other domestic programs). Outcomes, then, are described by two numbers: dollars spent on butter and dollars spent on guns. The set of outcomes, or simply the "policy space," is two-dimensional, and this is the domain over which preferences are expressed.[8] The third dimension of Figure 5.4, marked "utility," permits us to draw three-dimensional graphs of legislator preferences. As earlier, a legislator is assumed to have an ideal point in the policy space. His preference function, or utility function, is at a maximum over this point. It is assumed further (in most of the applications) that preferences decline with "distance" from the legislator's ideal point. A typical legislator, with a typical ideal point and preference function, is displayed in Figure 5.4. The legislator's preference function is a "hump" that reaches its highest utility level just over his ideal point in which b_1 dollars are spent on butter and g_1 dollars on guns. This ideal point, (b_1, g_1), is located in the plane of the butter-guns policy space.

A more convenient way to represent precisely this same information, however, is given in Figure 5.5. In this figure the reader is looking down directly onto the plane of the butter-guns policy space. It is as though you are hovering in a helicopter above the peak of the preference hump in Figure 5.4. The location of our typical legislator's ideal point is exactly the same as in Figure 5.4. But instead of adding a third dimension (coming out of the page toward you) in order to graph his preference function, we instead overlay "slices" of his utility function onto the policy space, producing the set of nested circles called *indifference curves*. Each circle is a slice of the policy hump in Figure 5.4. It is a locus of policy outcomes among which the legislator is indifferent (since all the points on a circle lie on the same slice and hence at the same height on the utility function of Figure 5.4). Since distance from an ideal point is a measure of preference, points on a circle centered on

[8] If the size of the budget were fixed in advance, then by the argument in the previous paragraph and footnote, only *one* dimension would be needed, say, dollars for defense; once this is established, dollars for domestic programs is strictly determined—it's what is left over. When the budget is *not* determined in advance, then we need *both* dimensions to display all outcomes.

FIGURE 5.5

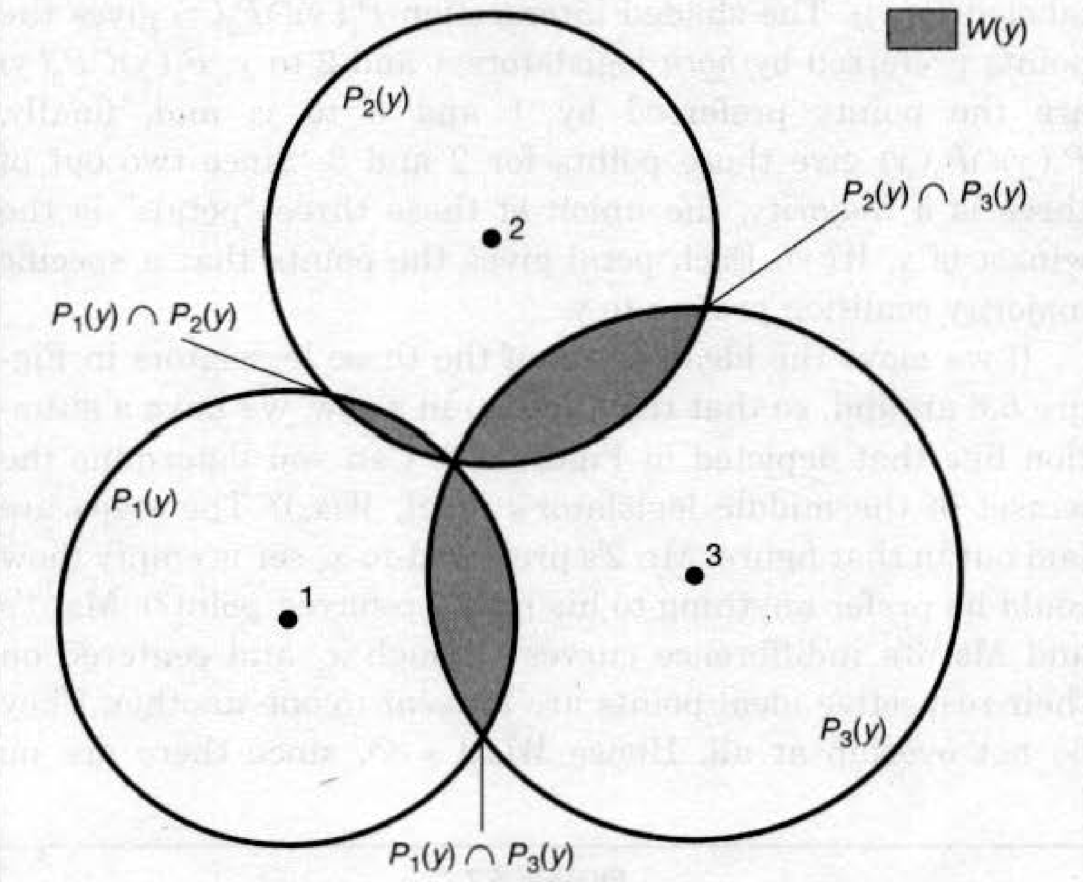


her ideal, being equidistant from that ideal, are equally preferred by her. The logic is the same in comparing a point on one circle to that on another. A legislator prefers a point on a circle with a *smaller* radius to one on a circle with a larger radius, because this means the former point is closer to her ideal than is the latter point.[9]

Notice the point labeled y in Figure 5.5. The circle through

[9] In this simplest of multidimensional setups, in which the policy space is two-dimensional and preference is measured by distance, indifference curves will be circles centered on the legislator's ideal point. In more than two dimensions, the indifference "contours" will be spheres or (in four or more dimensions) hyperspheres. A second sort of complication, which applies in the (simplest) two-dimensional as well as higher-dimensional situations, is to allow preferences to be related to distance, but in a more complicated way. One dimension of policy may be "more important" to a legislator than another dimension. Thus, movement away from her ideal point along one dimension will have a greater impact on utility than an identical movement

FIGURE 5.6



y centered on our legislator's ideal point, as we just determined above, contains all the points of legislator indifference to y. This means that all the points *inside* the circle, being closer to her ideal, are actually *preferred* by her to y. That is, we can call the points inside the circle our legislator's preferred-to-y set, a natural generalization of that same concept in the one-dimensional development earlier in this chapter. Figure 5.6 displays three legislator ideal points and each legislator's indifference curve through y (the curve plus all

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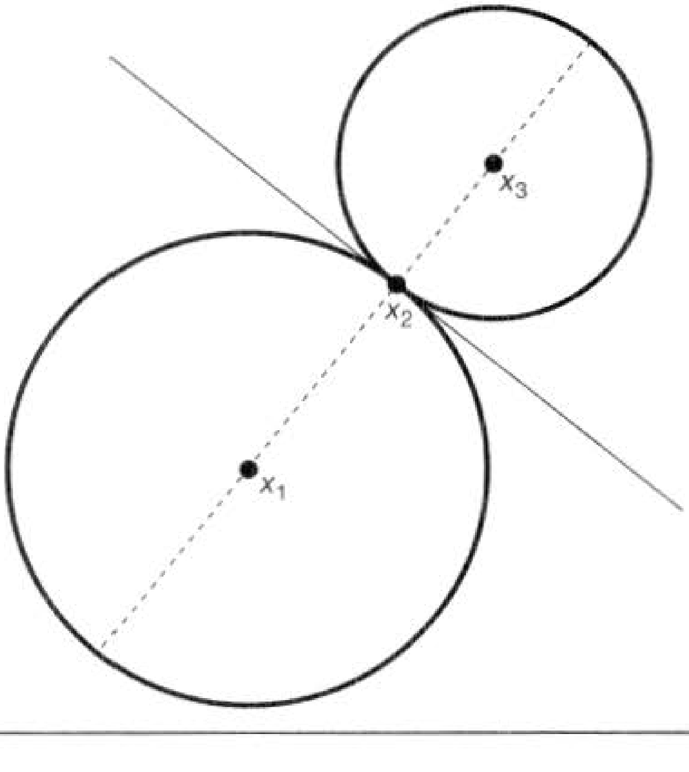
along the other dimension. Put differently, preference is said in this instance to decline with *weighted* distance from the ideal (where the weights reflect the salience of each dimension to the legislator). In this instance, indifference contours will no longer be circles, but will be *ellipses* instead. I will stick with the most basic formulation.

points inside it comprising each legislator's preferred-to-y set, labeled $P_i(y)$). The shaded intersection $P_1(y) \cap P_2(y)$ gives the points preferred by *both* legislators 1 and 2 to y; $P_1(y) \cap P_3(y)$ are the points preferred by 1 and 3 to y; and, finally, $P_2(y) \cap P_3(y)$ give those points for 2 and 3. Since two out of three is a majority, the union of these three "petals" is the winset of y, $W(y)$. Each petal gives the points that a specific majority coalition prefers to y.

If we move the ideal points of the three legislators in Figure 5.6 around, so that they line up in a row, we have a situation like that depicted in Figure 5.7. Can you determine the winset of the middle legislator's ideal, $W(x_2)$? The steps are laid out in that figure. Mr. 2's preferred-to-x_2 set is empty (how could he prefer anything to his most-preferred point?). Ms. 1's and Ms. 3's indifference curves through x_2 and centered on their respective ideal points are *tangent* to one another. They do not overlap at all. Hence $W(x_2) = \varnothing$, since there are no

FIGURE 5.7



points preferred to x_2 by majority {1,2}, {1,3}, {2,3}, or {1,2,3}. That is, the members of no majority coalition have preferred-to-x_2 sets that intersect. Thus, x_2 is the majority choice.

Another look at Figure 5.7 should show why this happened. Consider the bold line through x_2 perpendicular to the dashed line. On the x_1 side of this bold line, for any selected point off the dashed line, x_2 is closer to x_3 than the selected point is. So a majority, {2,3}, prefers x_2 to any such point. From a precisely parallel argument, a majority, {1,2}, prefers x_2 to any point off the dashed line on the x_3 side of the bold line. So, the only points that remain are those *on* the dashed line. That is, even though the group choice problem is *actually* two-dimensional, individual preferences line up so as to make the problem *effectively* one-dimensional. On this line individual legislators have single-peaked preferences (the reader should convince herself of this), with x_2 the median ideal point. Hence, Black's Median-Voter Theorem applies, which is precisely what Figure 5.7 demonstrates.

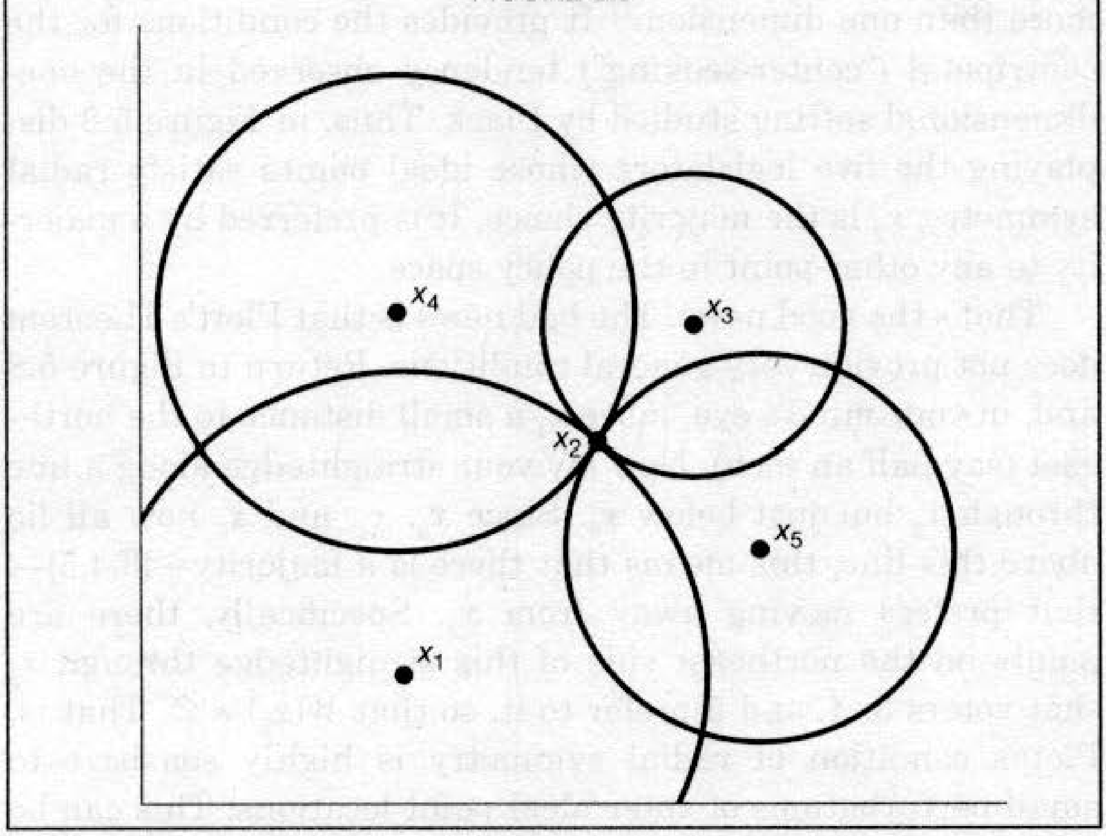
Having the legislator ideal points line up is pretty convenient, isn't it? Pretty unlikely, too. Certainly it seems more unlikely than arbitrary configurations such as that in Figure 5.6. Thus, while we have tools like Black's Median-Voter Theorem with which to analyze majority rule in one-dimensional settings, it is probably fair to say that many interesting political circumstances are genuinely multidimensional. Can we say anything about the prospects for a majority choice in multiple dimensions? The answer is yes, but the news is not very good.

The highly unlikely distribution of individual preferences in Figure 5.7 provides a basis for generalization. What allows the ideal point of Mr. 2 to emerge as the majority choice is the fact that the ideals of the others are "symmetrically" distributed about 2's ideal. From Mr. 2's ideal, any movement away from it is obviously opposed by Mr. 2 himself; but it is also always opposed by at least one of the other guys. In fact, as we saw when considering the bold line through x_2 perpendicular to the dashed line containing all three ideal points, any point on Ms. 3's side of that line is less preferred than x_2 by 1 and 2, and any point on Ms. 1's side of the line is less preferred than x_2 by 2 and 3.

Now let's add two more voters to the picture that are symmetrical in precisely the same way (Figure 5.8). Voters 2, 4, and 5 lie on a line, just as 1, 2, and 3 do. It is still the case that $W(x_2) = \varnothing$, because any departure from x_2 is opposed by at least three of the five voters. You may test this proposition out for yourself by laying a straightedge through x_2 at any angle. There are always two voters who would like to move to some point on one side of the straightedge, two who would like to move to points on the other side of the straightedge, and one (Mr. 2) perfectly content to stay at x_2. Since no majority favors moving in any direction (there are always three votes against), the winset of x_2 is empty. Something about distributing voters symmetrically around a common point seems to be producing a coherent majority choice.

Indeed, we can be very specific here. Let us consider a set of *m* voters (where *m* is any number, which we will take to be odd to simplify the presentation), whose ideal points are x_1, x_2, . . . , x_n. These *m* ideal points are in a multidimensional policy space, like the one pictured in Figure 5.8 (although the results presented below apply to policy spaces of more than two dimensions as well). These ideal points are distributed in a *radially symmetric* fashion if the following conditions hold: (1) There is a distinguished ideal point, labeled x^*; (2) the $n-1$ remaining ideal points can be divided into pairs (since n is odd, $n-1$ is even and this is possible); and (3) the two ideal points in any pair, say x_i and x_j, plus x^* all lie on a line with x^* "between" x_i and x;yj. In Figure 5.8, x_2 is the distinguished point, x_1–x_3 and x_4–x_5 are the pairs of remaining ideal points, and x_2 lies on a line "between" the ideal points in each pair. Notice that radial symmetry does not require the two ideal

FIGURE 5.8



points of a pair to be equidistant from the distinguished point (x_3 is closer to x_2 than x_1 is); they must simply line up.

The economist Charles Plott noticed that radial symmetry of ideal points captured in higher dimensions a property that single-peaked preferences possess in one-dimensional policy spaces. In a famous paper in 1967,[10] he established the following result:

> **Plott's Theorem. If voters possess distance-based spatial preferences, and if their ideal points are distributed in a radially symmetric fashion with x^* the distinguished ideal point, and the number of voters is odd, then W(x^*) = ∅.**

[10] Charles R. Plott, "A Notion of Equilibrium and Its Possibility under Majority Rule," *American Economic Review* 57 (1967): 787–806.

Plott's Theorem generalizes Black's Median-Voter Theorem to more than one dimension.[11] It provides the conditions for the centripetal ("center-seeking") tendency observed in the one-dimensional setting studied by Black. Thus, in Figure 5.8 displaying the five legislators whose ideal points satisfy radial symmetry, x_2 is the majority choice. It is preferred by a majority to any other point in the policy space.

That's the good news. The bad news is that Plott's Theorem does not provide very general conditions. Return to Figure 5.8 and, in your mind's eye, move x_4 a small distance to the northeast (say half an inch). Now lay your straightedge along a line through x_2 but just below x_5. Since x_3, x_4, and x_5 now all lie above this line, this means that there is a majority—{3,4,5}—that prefers moving away from x_2. Specifically, there are points on the northeast side of this straightedge through x_2 that voters 3, 4, and 5 prefer to it, so that $W(x_2) \neq \varnothing$. That is, Plott's condition of radial symmetry is highly sensitive to small perturbations of voter ideal point locations. This can be put in an especially dramatic form. Imagine the ideal points of 1,000,000 voters radially distributed around the ideal point of the 1,000,001st voter. So, $W(x_{1,000,001}) = \varnothing$, in accord with Plott's Theorem. Now suppose two new voters move into the community, *and their ideal points are not radially symmetric about* $\mathrm{x}_{1,000,001}$. This small perturbation in the voting situation—after all, how much effect can the introduction of two new voters have in a voting population of more than one million?—completely destroys the previous equilibrium.[12]

[11] One result is said to "generalize" another when the latter is a special case of the former. Thus, Black's median-voter result is the one-dimensional version of Plott's Theorem, in which the median voter's ideal is the distinguished point and pairs of voter ideal points, one from each side of the median, are distributed around it in a radially symmetric fashion.

[12] The sensitivity is not quite so severe when the number of voters is even. In this case the distinguished point is *not* a voter ideal point. Some shifts in voter ideal points are possible without disturbing the empty winset property of this distinguished point.

If departures from radial symmetry were relatively unusual events, then this sensitivity to ideal point distributions in Plott's Theorem would not really be bad news. But, as the reader may grasp intuitively, the requirement of radial symmetry is actually quite restrictive; one would not expect groups "naturally" to have their preferences distributed in so elegant and uniform a manner as this. So departures from this condition take on a greater significance. In what is one of the most remarkable theoretical statements in this entire field, Richard McKelvey demonstrated exactly how significant these departures from radial symmetry are.

> **McKelvey's Chaos Theorem.[13] In multidimensional spatial settings, except in the case of a rare distribution of ideal points (like radial symmetry) that hardly ever occurs naturally, there will be no majority rule empty-winset point. Instead there will be chaos—no Condorcet winner, anything can happen, and whoever controls the order of voting can determine the final outcome.**

I started out by seeking ways to restrict Arrow's universal-domain condition to see if there were narrower domains in which majority rule worked tolerably well. In one-dimensional choice situations, we saw that single-peakedness is sufficient. In multidimensional situations, a radially symmetric distribution of ideal points is sufficient. But small departures from the latter throw everything into chaos. No point is the "king of the mountain" in the sense that it is preferred by a majority to all contenders, so it is difficult to justify any particular group choice (since for any proposed choice there is some alternative a majority prefers to it). This, in turn, means that there will always be majority cycles.

[13] Richard D. McKelvey, "Intransitivities in Multidimensional Voting Models," *Journal of Economic Theory* 12 (1976): 472–82.

Indeed, McKelvey establishes that all the points are in one great big cycle. What this means, practically speaking, is that the situation is ripe for manipulation by whoever controls the agenda. What McKelvey shows is this: Pick any two points in the policy space—call them s (starting point) and t (terminating point). Then there is a sequence of points— $z_1, z_2, \ldots,$ z;zk (for some finite number, k) such that $z_1 P_G s$, $z_2 P_G z_1$, $z_3 P_G z_2, \ldots,$ z_k $P_{G\ k-1}$, and t P_G z_k. That is, from any starting point, there is a sequence of votes by which a majority will move the outcome to *any* terminal point (including, say, the ideal point of the agenda setter).

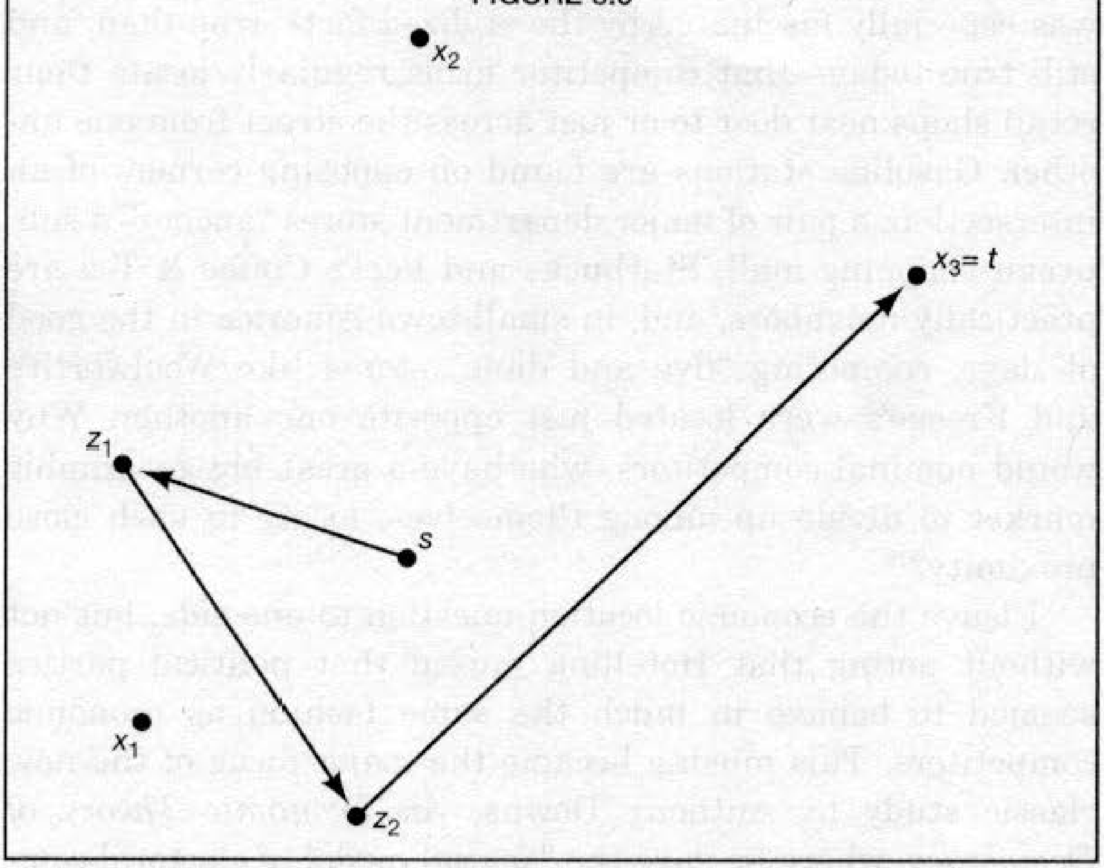
This is illustrated in Figure 5.9 for a three-person legislature. The ideal points of the three legislators are x_1, x_2, and x_3. The point s is the status quo ante. If Mr. 3 were the agenda setter empowered to make motions and order them in a voting sequence, then he could, in a small number of steps—in fact, in only *three* steps—drive the outcome to x_3, his ideal point. First he proposes z_1, which both Mr. 1 and Ms. 2 prefer to s. So $z_1 P_G s$. Then he proposes z_2, which both he and Mr. 1 prefer to z_1; so $z_2 P_G z_1$. Then, in the final step, he proposes his ideal point, x_3, which both he and Ms. 2 prefer to z_2. Voila! He has driven the legislative process, by artfully choosing the alternatives upon which to vote, to a terminal outcome located at his ideal policy: $t = x_3$.[14]

APPLICATIONS

Applications of the spatial model are so plentiful and rich that it is hard to know where to start. I begin at the beginning, so

[14] It should be noted that the members of each majority coalition in this example blindly vote their preferences, like lambs following the judas goat to slaughter. The legislators seem like putty in the hands of the wily agenda setter, Mr. 3. In the next chapter, we will endow "followers" with some sophistication by which they might be able to control their "leader."

FIGURE 5.9



to speak, with Downs's model of electoral competition. Anthony Downs was one of the first scholars to use the spatial model for political analysis. This application also demonstrates both the strengths and weaknesses of the simplifying assumption that the political world can be modeled as one-dimensional. Then we will turn our spotlight on institutional analysis, looking at both a one-dimensional and multidimensional analysis of legislative politics.

Spatial Elections

The real origins of the spatial model are found in a famous paper written in 1929 by Harold Hotelling.[15] An economist in-

[15] "Stability in Competition," *Economic Journal* 39 (1929): 41–57.

terested in the locational decisions made by firms, Hotelling was especially fascinated by the stylized fact—true then, and still true today—that competitor firms regularly locate their retail shops next door to or just across the street from one another. Gasoline stations are found on opposing corners of an intersection, a pair of major department stores "anchor" a suburban shopping mall, Starbucks and Peet's Coffee & Tea are practically neighbors, and, in small-town America in the good ol' days, competing "five and dime" stores like Woolworth's and Kresge's were located just opposite one another. Why would nominal competitors, who have a great big geographic market to divide up among themselves, locate in such close proximity?[16]

I leave the economic location question to one side, but not without noting that Hotelling mused that political parties seemed to behave in much the same fashion as economic competitors. This musing became the major focus of the now classic study by Anthony Downs, *An Economic Theory of Democracy*, where he gave the "spatial model of electoral competition" its fullest development and exposure.[17]

The "spatial" part of Downs's spatial model consists of a one-dimensional ideological continuum, [0, 100]. The continuum is scaled by the proportion of economic activity left in the hands of the private sector, so that the left endpoint reflects a fully socialized economy, while the right endpoint is identified with a totally private-enterprise economy. While political competition in real life consists of taking positions on and articulating visions about a host of political issues, Downs supposes that, when all is said and done, political debate boils down to ideology—do you want some good, service, or purpose provided

[16] Even more spectacular in many cities is a small stretch of a major highway along which dozens of automobile dealerships locate.

[17] Anthony Downs, *An Economic Theory of Democracy* (New York: Harper & Row, 1957).

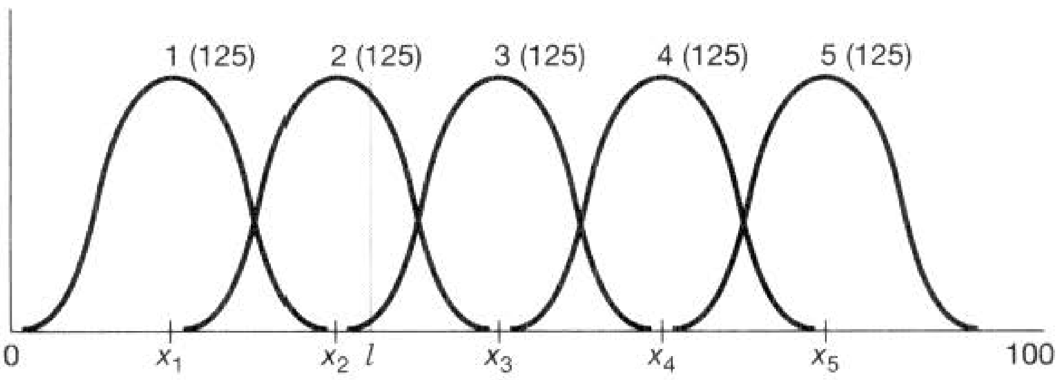
by government or by the private sector? Political competition, then, is a contest between politicians intent upon capturing control of government by appealing to voters with offers of alternative plans, platforms, programs—indeed visions. These appeals are identified with different points on the left-right ideological continuum.

As a first approximation for the hurly-burly of campaigning, electioneering, and voting, this is not a bad one. Politicians are conceived of as single-minded seekers of election. They are graduates, so to speak, of the Vince Lombardi School of Politics, whose motto is, "Winning isn't everything; it's the only thing."[18] Downs assumed politicians seek to maximize *votes*, although in variations on his model, politicians alternatively maximize their vote plurality (the difference between their vote and that of their closest competitor) or their probability of winning. In any event, most early spatial models of electoral competition took votes to be the coin of the realm, regarded politicians as focused exclusively on winning elections, and suggested that they did so by promising policies, platforms, and programs that attracted voters. In this spatial context, a candidate is represented by some location on the ideological continuum, some point in the [0, 100] interval. This is his or her political position.

Voters, Downs assumed, were single-mindedly interested in policy: the goods and services produced by government (or left to the private sector); the form and content of government regulation of the private sector; the distribution of tax, unem-

[18] For those too young to remember, Vince Lombardi was the legendary football coach of the Green Bay Packers and, at the end of his career, the Washington Redskins. To Lombardi, nothing was more important than winning. While not exactly an uplifting imperative, Lombardi's maxim is a pretty good first approximation for what it takes to succeed in the national politics of most countries, the business world, and the National Football League. (By the way, though associated with Lombardi, it is not clear he really ever said this!)

FIGURE 5.10



ployment, and inflation burdens; government policies on social issues like abortion and divorce; and matters of war and peace. Voters care mightily about these matters and base their assessments of candidates accordingly. Voters, however, are heterogeneous in their tastes so, just as there are left-wing and right-wing politicians, there are left-wing and right-wing voters. Specifically, each voter is identified with some point in the [0, 100] ideological space—the voter's ideal point—and his or her preferences are assumed to decline for points more and more distant from this ideal. That is, the set of voters may be represented by single-peaked preferences. Figure 5.10 displays an electorate of 625 voters (actually, five different voter "types" with 125 voters of each type). A voter of type i (i = 1,2,3,4,5) has ideal point x_i, and preferences declining in distance from x_i.

The most famous version of the Downsian model involves two-candidate competition. The question Downs asks is: Given a distribution of voters like that in Figure 5.10, where will two single-minded seekers of election locate themselves? We can gain some insight into this question by fixing the position of one of the candidates. Let's fix the position of L, the leftie candidate, at l as shown in the figure. What position, r, should R, the rightie candidate, adopt so as to maximize his votes? To answer this question we need a rule of calculation. The Downsian rule is that each voter votes for the candidate whose location is closest to his or her ideal point.[19]

We can now answer the questions posed in the previous paragraph. Candidate R should snuggle up infinitesimally close to the right-hand side of L. That way, R gets all the votes to the right of l and, since l is to the left of the midpoint of the voter distribution, that means that R gets more than half of all the votes.[20] That is, R gets the 375 votes from voters of types 3, 4, and 5; L gets the 250 votes of types 1 and 2. Put more generally, L's location divides the electorate into two groups: those with ideals less than l and those with ideals greater than l. R's optimal response is an r just next to l on the side of the larger group. We have thus figured out how R will respond to any move made by L. L thus knows that her position will divide the electorate into two groups and she will get the *smaller* group. Given that she, too, wants to maximize votes, she should try to make this smaller group as large as possible. She can do this, the reader may have guessed, by setting l equal to the ideal point of the median voter, since the groups to the left and right would then be equal in size. If l and r just straddle the ideal point of the median voter, x_3, then each location is optimal against the other's and the election ends in a virtual tie.

We draw precisely the same conclusion if we fix R's position first and let L respond optimally. For any r chosen by R, L will set l just next to r on the side of the larger group. Under

[19] For any two candidate positions, say, α and β in [0, 100], where α is to the left of β, the midpoint is $(\alpha + \beta)/2$. The candidate located at α receives the votes of all voters with ideals to the left of this midpoint, whereas the candidate located at β gets the votes of all voters with ideals to the right of this midpoint. Voters at the midpoint are indifferent between the two candidates, since their positions are an identical distance from these voters' ideals; these voters flip coins to decide for whom to vote.

[20] If l happened to be to the right of the midpoint of the voter distribution, then R would maximize his votes by squeezing up against L on its left side, thereby getting a majority of the votes.

these circumstances, the best R can do is to "move to the median."

Finally, suppose L and R must announce their policy platforms simultaneously. Once a policy is announced, if a candidate is "stuck" with the position for the duration of the campaign, then he or she is likely to worry that his or her position is vulnerable. A position is vulnerable if the opponent's position lies between it and the median of the voter distribution, since, by the Downsian rule of calculation, the opponent will then get more than half the votes.[21] The only position that *cannot* be vulnerable is one that actually is at the median ideal. If, on the other hand, candidates are not stuck with their announced positions but can revise their policy platforms during the course of a campaign, one of two patterns will be observed. If both initial announcements are on the same side of the median ideal, then there will be a "leapfrogging" converging pattern as the vulnerable position (as just defined) leapfrogs over her opponent's position in order to be closer to the median, that position in turn is leapfrogged over by the now vulnerable opponent, and so on until there is no more leapfrogging to do—namely when both positions have converged upon the median. If, on the other hand, initial announcements are on opposite sides of the median ideal point, then there will be a homing in on the median from each side as the one more distant moves closer.[22]

In all of these circumstances, each a slightly different modeling assumption about the sequence in which various events take place in the course of a campaign, there is a common convergence on the ideal point of the median voter. And this *centripetal* tendency is precisely what is predicted by Black's

[21] Indeed, a position is vulnerable if the opponent's position is *closer* to the median. The position closest to the median wins more than half the votes.

[22] Of course, if politicians are perfectly informed about voter preferences, they won't need to proceed tentatively toward the median, whether by leapfrogging or homing in; instead, they can move directly to the median.

Median-Voter Theorem. In effect, Downs's model provides a rationale for why majoritarian politics is centripetal.[23]

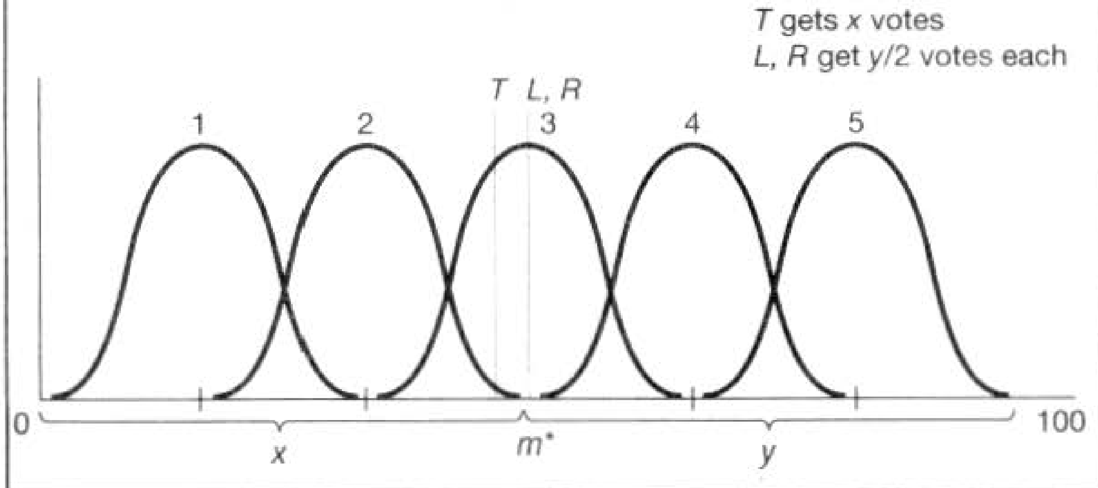
The logic of Black's theorem, as elaborated in the electoral context by Downs, reminds me of those occasions when someone says something very intelligent and quite obvious (once it is said!), causing you to reflect, "Now why didn't I think of that?" Downs was motivated by the fact that so many foreign observers of American life had, since practically the beginning of the Republic, noted how similar America's political parties were: "Tweedledum and Tweedledee," empty bottles differing only in their labels.[24] More recently, observers of British politics have begun to notice that the losing party ultimately transforms itself to look at least a bit like its more successful opponent. Thus, in the postwar period, British Tories have accepted a good deal of the welfare state championed by the Labour Party, whereas, in the latter part of the twentieth century, Labour has trimmed its more socialist sails in order to look to voters a bit more like Margaret Thatcher's and John Major's Conservative Party. In the twenty-first century, both parties struggle toward the center, neither bearing the extremist trappings that once described them.

The centripetal forces Downs identified are certainly plau-

[23] Notice that the rationale is not that the middle is "where the votes are." Certainly this may be true; in many circumstances the middle of the spectrum is where most persons' preferences lie, with the numbers getting smaller as one moves toward the more extremist tails of the distribution. But go back to Figure 5.10 and suppose that the extremists are the more plentiful. That is, suppose types 1 and 5 have 250 voters each, types 2 and 4 have 62 voters each, and type 3 consists of a single voter. Will the Downsian logic recounted above be any different here? No. The centripetal pull is the same, even though the "center" is *least populated* with voters!

[24] It might interest the reader to know that Downs's book originated as a doctoral dissertation in economics at Stanford University, where a member of Downs's dissertaion committee was Kenneth Arrow. So, Downs had both cycles and instability à la Arrow's Theorem on one side and their opposite—stylized facts about stable party configurations—on the other. His research sought to make sense of these seemingly incompatible matters. Single-peakedness did the job.

FIGURE 5.11



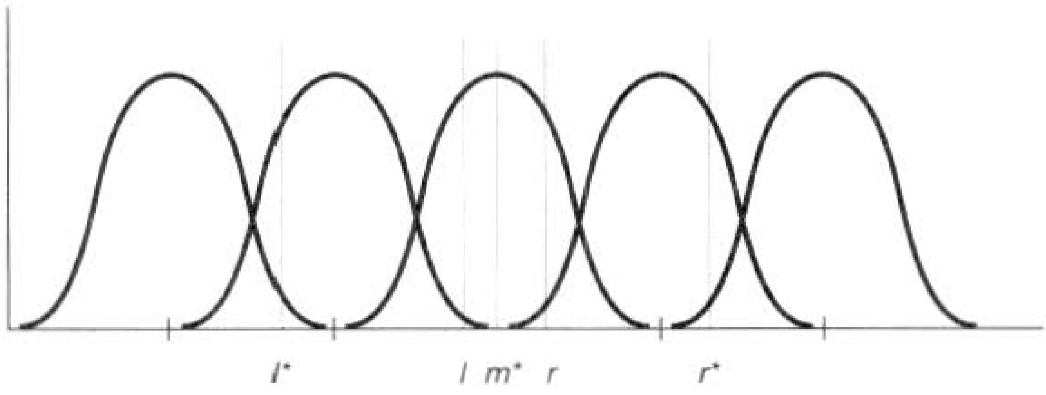
sible, yet it is clear that parties do not converge all the time. Why might this be? Downs's spatial model is quite user-friendly as a "discovery tool," so we can vary some of its assumptions and see what happens. Suppose, for example, we do not foreordain that there are two candidates. What if Leftie (L) and Rightie (R) are not the only two kids on the block? There is a third candidate—call her Trey (T)—who may enter the race if she thinks she has a chance. Well, if L and R locate at the median (call it m^*)—$l = r = m^*$—and if, when there are more than two candidates, the one with the *most* votes (not necessarily a majority) wins the election, then T certainly does have a chance. She can locate close on one side or the other of the median, win nearly all the votes on that side, and thus defeat L and R, who end up splitting the remaining votes (Figure 5.11). On the other hand, if the positions of L and R are sufficiently widely dispersed, then T can enter *between* them at some position t. She will get the votes of voters whose ideal points lie in the interval $[(l + t)/2, (t + r)/2]$. The left boundary of this interval is the midpoint between the positions of L and T, whereas the right boundary is the midpoint between the positions of T and R. By the same Downsian rule of calculation, L gets all the voters in the interval $[0, (l + t)/2]$, and R gets all the voters in $[(t + r)/2, 100]$. If there are more voters in the first interval than in the second or in the third, then T wins. So, when there is the possibility of entry, L and R can locate neither too closely together nor too far apart.

In fact, there may be a set of *entry-deterrence* locations for T and R, with these two getting roughly the same number of votes, and no third candidate able to locate in any place that would give her a victory (thereby discouraging her from entering at all). The point here is that when we broaden Downs's initial model to take account of some factor he had omitted—the possibility of entry by a third candidate—we discover that there may be occasions and circumstances in which the established parties (T and R) are ill-advised to converge toward the median.

Research has, in fact, been conducted on precisely the issue of Downsian candidate competition with (prospective) entry.[25] As noted, it is clearly one extension of the original Downsian assumptions that produces the possibility of nonconvergent candidate locations. But there are other possibilities. Candidates, for instance, may have their own policy preferences, ones often known to the voters. Thus, suppose T and R have their own policy ideal points at l^* and r^*, respectively (shown in Figure 5.12). They may declare policy programs at other locations, say, $l \neq l^*$ and $r \neq r^*$. But why should the voters believe these policy declarations? It's not necessary to be altogether cynical to believe that once one of them wins, she or he will be sorely tempted to implement her or his preferred policy (l^* or r^*), not declared policy (l or r); politicians cannot be trusted to do what they say when they have preferences of their own. Effectively, then, candidates once again will not converge, this time because there is no point in doing so (they

[25] The interested reader may consult Kenneth A. Shepsle, *Models of Multiparty Electoral Competition* (Chur, Switzerland: Harwood, 1991), for a summary of much of this research.

FIGURE 5.12



won't be believed by voters), even if they were willing to implement what they promised. The "commitment technology" is simply not up to the task.

What if it were? What if candidates had policy preferences, as in the previous paragraph, but had available to them means of making promises stick. Perhaps all they need to say is "Cross my heart," and the voters will believe them. Perhaps voters believe policy promises because they know that politicians know that a reputation for deception and misrepresentation is a serious electoral obstacle in future electoral campaigns. So, for any of a number of reasons, suppose that candidates' promises are credible on the one hand, but that candidates still care about what policies are implemented on the other hand. What will the candidates do in this circumstance? In a lovely paper, Randall Calvert[26] demonstrates that, just as in the case where candidates didn't care a whit about policy, these two candidates will converge to the median voter's ideal. Referring again to Figure 5.12, L wants an outcome closest to l^* and R wants the final policy to be closest to r^*. If these two points happen to be equidistant from m^*,

[26] Randall Calvert, "Robustness of Multidimensional Voting Models: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science* 29 (1985): 69–95.

and if each candidate (credibly) announced his or her ideal policy, respectively, then the election would end in a tie (and, presumably, the winner would be determined by something like a flip of a fair coin). But L, by moving just a tad toward the center, could win the election outright at a very small cost to herself in terms of policy. But this would be terrible for R—not that he lost the election but that a policy near l^* is so awful. He could avoid all this by moving in toward the center a bit more than L had, which, in turn, encourages L to move in a bit more, and so on. In the end, *even though both candidates had policy preferences and, in fact, did not care at all about who won the election but only about what policy would be implemented,* they converge to the median voter's ideal anyhow.

Needless to say, we could play with Downs's model in a variety of interesting ways. Many have.[27] What has been shown in this section is that the stripped-down spatial model of Downs, with competition on an ideological dimension between two election-oriented candidates, leads to policy convergence. The policy that emerges from the competitive forces captured by this model is the ideal point of the median voter. This result, to the casual observer, describes what often happens in real elections, as candidates try to smooth down their more extremist edges in order to curry favor with voters in the center of things. Thus, once Bill Clinton vanquished his liberal opponents within the Democratic Party in 1992 (Jesse Jackson and Mario Cuomo), he headed toward the ideological center, running in the general election as a more conservative

[27] For two early summaries of the extensive literature the interested reader may turn to James M. Enelow and Melvin J. Hinich, *The Spatial Theory of Voting* (New York: Cambridge University Press, 1984); and James M. Enelow and Melvin J. Hinich, eds., *Advances in the Spatial Theory of Voting* (New York: Cambridge University Press, 1990). A more recent review is Torun Dewan and Kenneth A. Shepsle, "Economic Models of Elections," *Annual Review of Political Science*, forthcoming 2011.

"new Democrat." The incumbent president, George H. W. Bush, on the other hand, tried to shed some of his hardline conservative attributes, also moving toward the center as he compromised on his "no new taxes" pledge. In 2000, the Republican, George W. Bush ran as a "compassionate conservative" against the centrist Democrat Al Gore. In 2008, the Democrat, Barack Obama moderated his views in the general election, after securing the nomination in a close contest with Hillary Clinton; the Republican Party nominated its most moderate candidate, John McCain. In many other elections one sees a similar dynamic—partisan candidates of the left and the right hedging, qualifying, and compromising in order to appear more centrist.

This convergence is not always complete, however. Sometimes a candidate applies brakes on convergence for fear of alienating his or her base, or even stimulating a third-party entrant. Thus, civil rights activists, unions, and government workers—elements of the Democratic base—made it virtually impossible for Walter Mondale to converge toward the center as a candidate in the 1984 presidential election. Elements of the conservative movement kept Ronald Reagan ideologically true in that same election. Third-party candidates entered the presidential races of 1968, 1980, and 1992 (George Wallace, John Anderson, and Ross Perot, respectively), sometimes because the candidates were thought to have converged too much,[28] sometimes because they were thought to have stayed too close to their more extremist supporters.[29] Thus, both too much convergence and too little convergence may provide the impetus for a third-party challenge.

[28] In 1968, Wallace entered on the right, thinking "there's not a dime's worth of difference" between the Democrat Hubert Humphrey and the Republican Richard Nixon.

[29] Both Anderson and Perot sought to capture the center, which they believed had been conceded by Carter and Reagan in 1980 and Clinton and Bush in 1992, respectively.

I have clearly only scratched the surface of Downs's spatial model of party competition and only covered some of the many mechanisms and rationales according to which competitors converge toward the median voter's ideal policy on the one hand, or maintain distinctive policy profiles on the other. This, in sum, suggests the richness of Downs's approach.

Electoral phenomena, however, are not the only focuses of the spatial model. A twin enterprise, a kind of "elections writ small," has employed the spatial model to study the selection of policy in legislative settings. I turn to those now, and examine both one-dimensional and multidimensional versions of the spatial model.

Spatial Models of Legislatures

There will be a much more thorough look at legislatures in Part IV, so here I am interested primarily in seeing what the spatial model can do. It turns out to be quite a powerful analytical tool for representing the ways in which preference-based (rational) behavior and structural features of institutions interact to produce final outcomes. It suggests that legislative outcomes depend in essential ways not only on what legislators want but also on how they conduct business in the legislature.

To keep things as simple as possible, the legislature is taken to be a set of n individuals, where n is an odd number, and where everyone casts a vote. It makes decisions by majority rule. The most elementary situation, one that is examined first, is the unidimensional case in which the legislature must choose a point on a line. Each legislator, i, has an ideal point x_i, and single-peaked preferences. The median voter is legislator m with ideal point x_m. We know in this circumstance that x_m can defeat any other point on the dimension in a majority contest (Black's Theorem). Perhaps more amazing is the fact that the median preferences prevail in a comparison between *any* two alternatives, so that if m prefers x to y then so does a majority for *any* x and y.[30]

In addition to the preferences of the median legislator, x_m, two other distinguishing features of the situation are important. Whenever a legislature faces a decision-making opportunity, there is always a *status quo* in place, labeled x^0. This is the current policy at the time of legislative choice. It remains in place if the legislature chooses not to change it.[31] The second feature of interest common to most legislatures is a division-of-labor arrangement known as a *committee system*. In such a system, a committee is a subset of the n legislators (momentarily, we describe some of its specific powers). The median ideal point of the committee members is labeled x_c. Just as majority preferences in the entire legislature are identical to the preferences of the legislature's median voter, majority preferences inside a committee are a copy of the preferences of the committee's median member. Because of these identities, much of our analysis need only consider x^0, x_m, and x_c. In what follows, then, I put the spatial model through

[30] This may be proved as follows. Suppose x_m, x, and y are all points in the dimension, and that $x_m \leq x \leq y$. Legislator m clearly prefers x to y. But then so does every legislator to the left of m. Together these legislators constitute a majority, so x is preferred by a majority to y. Likewise, by the same reasoning, if $y \leq x \leq x_m$, then both m and a majority (all the legislators to the right of m) prefer x to y. So, it has been shown that whenever x and y are on the *same* side of the median, a majority always agrees with the preferences of the median voter. Suppose, then, that $x \leq x_m \leq y$, and that m prefers x to y. Consider the legislator just to the left of m. Her ideal is closer to x and farther from y than was m's ideal; so if m prefers x to y, surely she does, too. But then, so do all the other voters to the left of m's ideal, and once again they jointly comprise a majority. This establishes that a majority *always* agrees with the preferences of the median voter.

[31] In some circumstances, the status quo policy does *not* remain in place, unless the legislature takes positive action to keep it in place. If the legislature fails to do anything, then the status quo reverts to some specific policy (known, naturally enough, as the *reversion point*). This is true, for instance, in statutes that possess *sunset provisions*, an example explored further in Case 5.1.

its paces in examining the making of policy choices by an n-member legislature possessing a committee system.

Three decision-making regimes, or institutional arrangements, are identified. The first is ***pure majority rule***. There is a status quo, and any legislator can offer a motion to change it. A motion, once proposed, is pitted against the status quo. If it wins it becomes the new status quo; if it loses it goes to the place where all losing proposals go (a sort of elephants' burial ground). The floor is once again open for some new motion (against the old status quo, if it survived, or the new status quo, if the previous proposal prevailed). This procedure of motion making and voting continues until no member of the legislature wishes to make a new motion.[32]

The second regime is the ***closed-rule committee system***. In this system, a (previously appointed) committee first gets to decide whether the legislature will consider changes in the status quo; that is, it has *gatekeeping agenda power* and can decide whether to open the gates to enable policy change or not. Second, if the gates are opened, only the committee gets to make a proposal (*monopoly proposal power*). Third, the parent legislature may vote the committee's proposal either up or down. If it passes, then it becomes the new status quo; if it fails, then the old status quo prevails. The proposal is closed to amendments. Hence, the proposal is said to be considered under a *closed rule*, and the committee is said to offer its parent body a *take-it-or-leave-it* proposal.

The third regime is the ***open-rule committee system***. This system is identical to the one described in the previous paragraph, except for the third feature. Under an open rule, once the committee has made a proposal, the parent legislature may open the floor to *amendments* to the committee's pro-

[32] A variation on this "stopping rule" is to allow a motion to be in order at any time to close the floor to new motions (in effect, a motion to take a final vote and then to adjourn the legislature, at least on the subject matter at hand).

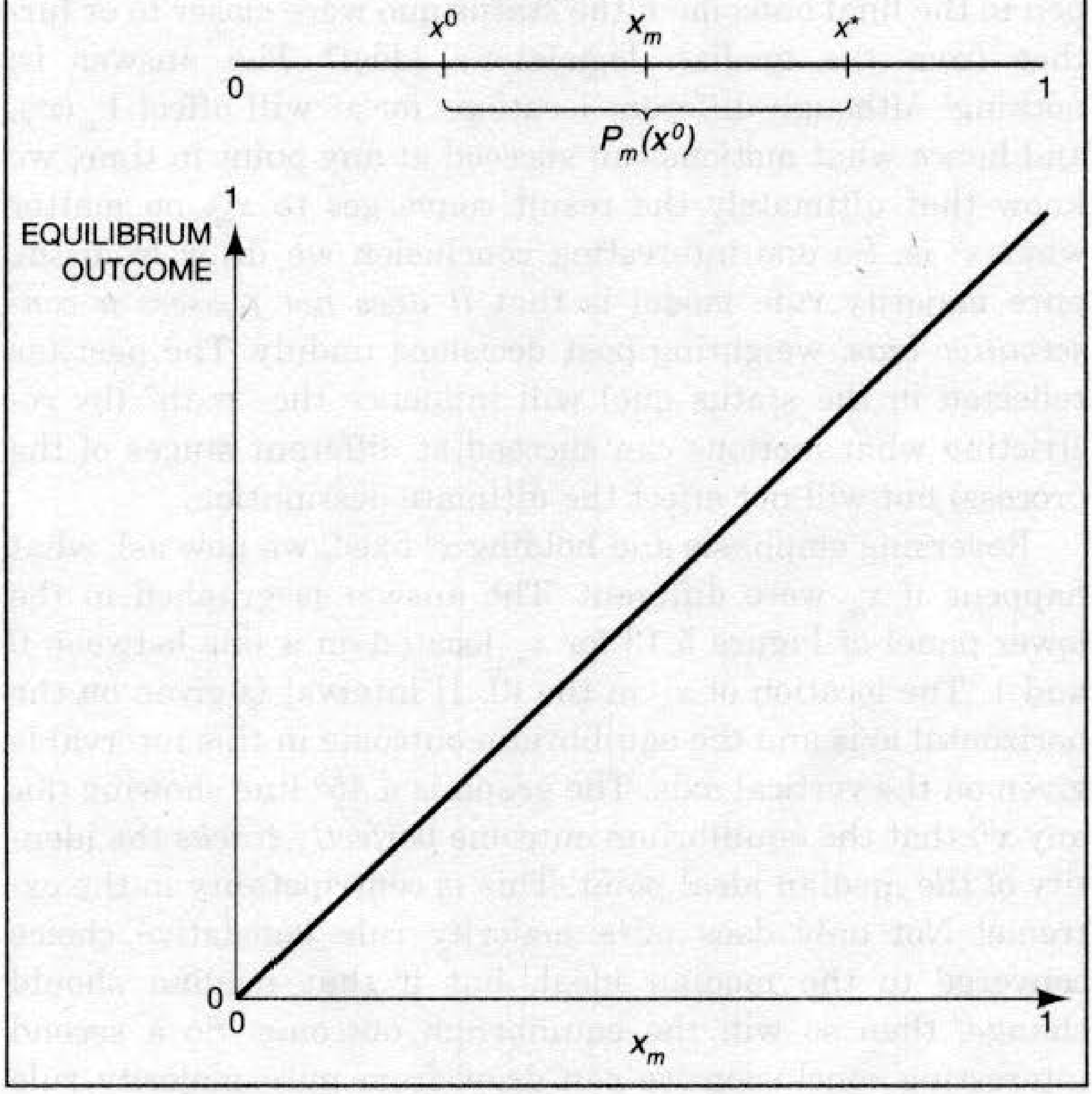
posal. Once the committee has opened the gates and made a proposal, it concedes its monopoly access to the agenda.

Each of these systems is explored in both the one-dimensional and the multidimensional setting in order to determine whether there is anything regular or routine that we can expect from these alternative majority-rule regimes. Some brief comparative observations on these regimes are offered, leaving a full-blown consideration for chapters 11 and 12, where institutions are taken up more systematically.

PURE MAJORITY RULE I start with a legislative choice in a one-dimensional spatial setting. In the pure majority-rule regime there are no committees, so we need to know only the locations of x_m and x^0. A typical situation is given in the top panel of Figure 5.13. From the median ideal and the status quo, we determine the median legislator's preferred-to-x^0 set, $P_m(x^0)$.[33] Since we've established that the median's preferences are the same as a majority's preferences, this interval is the set of motions that would prevail over x^0 in a majority contest. So, if someone is recognized and makes a motion *outside* this set, it will go down in flames, whereas any motion *inside* this set will be victorious and become the new status quo. It is evident that the political process defined this way will produce outcomes that either leave the status quo unchanged or move it closer to x_m (since every point in $P_m(x^0)$ is closer than x^0 to x_m). As the process of motion making and voting is repeated, the winning alternative will ultimately converge on x_m. Moreover, it will not depart once it reaches x_m (since, as shown above, the status quo cannot move further from x_m in any vote). So, just as in the Downsian model of electoral competition, there is a centripetal tendency in the pure majority-rule legislative regime.

[33] Since we assume that legislative preferences are distance-based, we know that legislator m prefers to x^0 all points closer than it to x_m. This determines the set $P_m(x^0)$ pictured in Figure 5.13.

FIGURE 5.13



It is for this reason that we think of pure majority-rule legislative choice as an "election writ small."

The great utility of these spatial models of legislative choice is they permit the analyst to do what in economics is called *comparative statics*—we can ask "what if" questions. Having derived an equilibrium outcome from our basic setup, as I did in the previous paragraph, we may now ask how that equilibrium changes as relevant parameters change. We have already seen that there are really only two relevant parameters—x^0 and x_m. Holding the latter fixed, we first ask what happens if the former changes—that is, what would happen to the final outcome if the status quo were closer to or further from the median legislator's ideal? The answer is: nothing! Although different locations for x^0 will affect $P_m(x^0)$, and hence what motions can succeed at any point in time, we know that ultimately the result converges to x_m, no matter what x^0 is. So one interesting conclusion we draw from the pure majority rule model is that *it does not possess a conservative bias*, weighting past decisions unduly. The past (as reflected in the status quo) will influence the "path" (by restricting what motions can succeed at different stages of the process) but will not affect the ultimate destination.

Reversing emphasis and holding x^0 fixed, we now ask what happens if x_m were different. The answer is graphed in the lower panel of Figure 5.13 for x_m located on a line between 0 and 1. The location of x_m in the [0, 1] interval is given on the horizontal axis and the equilibrium outcome in this interval is given on the vertical axis. The graph is a 45° line showing (for any x^0) that the equilibrium outcome *perfectly tracks* the identity of the median ideal point. This is centripetality in the extreme! Not only does pure majority rule legislative choice converge to the median ideal, but if that median should change, then so will the equilibrium outcome. So a second interesting conclusion we can draw from pure majority rule is that it is perfectly responsive to central tendencies: The median legislator's ideal is, by definition, the central point in the distribution of preferences; pure majority rule produces an outcome at this point; and, were this point to change (as a result, say, of an election), the legislative outcome would "track" it.

CLOSED-RULE COMMITTEE SYSTEM Most legislatures are not pure majority-rule institutions. Even town meetings and other approximations of pure majority rule about which observers occasionally wax romantic require some mechanism to determine the content of agenda items and the order in which they will be taken up. Some legislatures establish a single agenda committee to decide these matters. However, most legislatures (certainly in the United States) employ a division-of-labor committee system that divides up agenda power by policy area. Subsets of legislators have disproportionate influence over the agenda in specific policy jurisdictions. The committee serves, in its jurisdiction, as an agenda agent for its parent legislature.

I will have much more to say about these things in Part IV. For now, I need focus only on the fact that what distinguishes the closed-rule regime from pure majority rule is that there is, in addition to x^0 and x_m, a third parameter of interest, namely the median ideal point of an agenda-setting committee, x_c.[34] Many of the conclusions drawn about this regime depend on the relative locations of x^0, x_c, and x_m.

The decision-making procedure, as suggested earlier, is for the committee either to make no proposal at all, in which case x^0 remains in place, or to make a motion to change the status quo, which the parent body must accept or reject as is. What will such a committee do? To answer this question, we once again determine $P_m(x^0)$, as in the top panel of Figure 5.13. This is a set whose boundary points are x^0 itself and x^*; it contains the only points a legislative majority prefers to x^0. The committee, as personified by its median voter, c, treats these points as its "opportunity set," picking its favorite as the motion it makes (if it makes any motion at all). We look at three orderings of the relevant parameters (there are six orderings in all, but the omitted ones are simply mirror images of the ones we consider):

[34] Since the one-dimensional model is being elaborated here, we are concerned with the median ideal of only a single committee. In multidimensional contexts, where there are many jurisdictions into which the dimensions of the policy space are arranged, we will need to know the policy preferences of different committees, each responsible for its own bundle of policy dimensions. More on this will be developed in Part IV.

CASE 1 ($x^0 \leq x_m \leq x_c$). Here the median legislator is between the status quo and the median committee member. In this case x^* is the *right* boundary of $P_m(x^0)$, just as shown in Figure 5.13. If $x_c \leq x^*$, then the committee will propose its median ideal point, which then will be approved by a legislative majority (since it lies inside $P_m(x^0)$). If, on the other hand, $x^* \leq x_c$, then the best the committee can do is to propose x^*, which is approved by a legislative majority.[35] In either case, both committee and parent legislature wish to move away from the status quo *in the same direction*. The final outcome will move x^0 in that direction, further than the median voter would want, but not always as far as the committee median wants.

CASE 2 ($x^0 \leq x_c \leq x_m$). Here the median committee member is between the status quo and the median of the whole legislature. In this case $x_c \in P_m(x^0)$ automatically. So the committee can get majority legislative approval for x_c, the committee's best outcome.

CASE 3 ($x_m \leq x^0 \leq x_c$). In this last setting the status quo is between the two medians. This is a particularly interesting case because committee and legislative majority are at loggerheads. The committee wishes to move right, while a majority of the parent legislature wants to move left. The committee's gatekeeping authority pays off for it in a big way here because it will choose simply to keep the gates closed.[36]

[35] Actually, a legislative majority is indifferent between x^0 and x^*. I assume that an indifferent voter votes *for* the motion on the floor. (Alternatively, the committee could propose a point just to the left of x^*, which secures a majority outright.)

[36] The committee could move a proposal (some point to the right of x^0), but it would be defeated. So it might as well not bother and simply keep the gates closed (especially if the bother were at all costly). On the other hand, if out-

So, the first thing we learn about the closed-rule regime is that only a very limited number of things can happen—three things, in particular. If x_c is interior to the legislative median's preferred-to-x^0 set, then the outcome is x_c. If it is not, then either of the two endpoints of $P_m(x^0)$ are possible—x^0 if committee and legislative median are at loggerheads; x^* otherwise. In the pure majority rule regime, in contrast, only *one* thing can happen: x_m, something that *never* happens under the closed-rule regime (unless, by coincidence, x_m=x_c or x_m=x^0). This suggests that endowing a privileged group with agenda power is not without its consequences: *agenda power discourages centripetal outcomes as it tugs the process in the direction of the privileged group.*

A variety of comparative statics exercises exists that one might do. I focus on one: For a fixed legislative median and committee median, what happens as x^0 changes? (Whoever asks this question doesn't literally mean that the status quo suddenly changes. Rather, it is a question of what would happen if the status quo were more or less extreme.) In case 1, for example, if x^0 were further to the left, then $P_m(x^0)$ would get bigger (x^* moves to the right). At some point it contains x_c (if it doesn't already). So, as x^0 moves away from the chamber median, there will be a discontinuity when x_c jumps from being outside m's preferred-to-x^0 set to inside that set. Put crudely, the worse the status quo, from m's perspective, the more likely c can get her way.[37] The same pattern prevails as x^0 moves to the right. At first it moves toward x_m, so m's preferred-to-x^0 set

side interests took heart in the fact that the committee was at least putting up a good fight and rewarded the committee accordingly, then the committee might wish to "bother" (though the result would be unchanged—x^0 would stay in place).

[37] The classic statement of this result, plus a derivation of some of the political consequences of it, is found in Thomas Romer and Howard Rosenthal, "Political Resource Allocation, Controlled Agendas, and the Status Quo," *Public Choice* 33 (1978): 27–43.

contracts. Once it "passes" x_m, the preferred-to set begins expanding again.[38]

Case 5.1
Sunset Provisions and Zero-Based Budgeting

In the 1970s public policy analysts developed two ideas as an attempt to counter rising budget pressures. The first idea was called a sunset provision. The identified problem was the persistence of expenditures that might have outlived their usefulness. It seemed that once a project was on the books, it never went away. With a sunset provision as part of the enabling legislation, the project would have to be renewed after a specified time period in order to extend its life. In other words, the sun would automatically set on a project unless the legislature took further action.

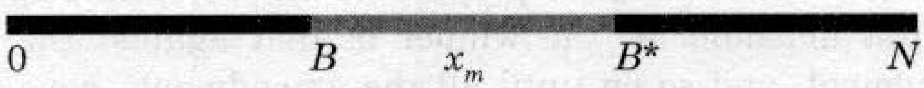
The second idea was called zero-based budgeting. Also associated with the problem of expenditures that were growing out of control, this concept required bureaucratic agencies to justify a project budget "from zero," rather than merely justifying the *growth* in proposed expenditures over the previous year's budget. It was alleged that this procedure would reduce accumulating and persisting inefficiencies in agency budgets.

For our purposes, sunset provisions and zero-based budgeting are similar because they create situations in which the status quo alternative to a proposal is zero. Consider the case of zero-based budgeting for an agency. The legislative median is at x_m and last year's agency budget is at B. Now we let the agency make a proposal for next year's

[38] The reader might try to see what happens as x^0 changes in cases 2 and 3 above.

budget, a proposal which the legislature may accept or reject by majority rule. Under ordinary procedures, we assume that legislative rejection of the agency proposal results in last year's budget, B, continuing in place next year. Under zero-based budgeting, on the other hand, we assume that legislative rejection leads to a zeroing out of the agency budget altogether.

At first glance, it would seem that the zero-based budgeting procedure is pretty tough on the agency. That's the whole idea, since this method was designed to limit an agency's power in budget negotiations. However, zero-based budgeting actually *increases* agency discretion. This is seen in the figure below. If, under ordinary procedures, B is the reversion outcome (the outcome if the legislature rejects the agency proposal), then as long as the agency proposes a budget in the gray region, the legislature will approve it (any such proposal is in $P_m(B)$); that is, under ordinary procedures, the agency could extract a budget as large as B^*. If, on the other hand, the zero-based budgeting procedure were in effect, then the agency could get a budget as large as N (since N is in $P_m(0)$). In their pure form, both sunset provisions and zero-based budgeting provide perverse incentives. Their whole rationale was to *discipline* "out-of-control" agencies and budgets, not *empower* them. (The reader might explore the consequences of a zero-based budgeting regime in which agency proposals may be amended by the legislature.)



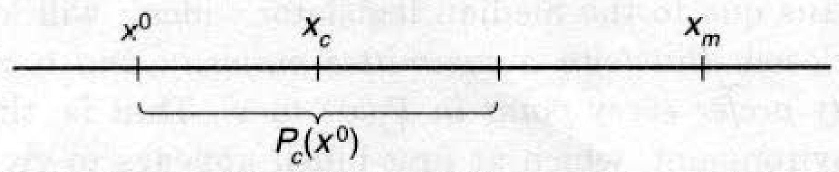
In concluding this brief treatment of the closed-rule regime, let me reemphasize the fact that the key parameters are x^0, x_c, and x_m. An electoral earthquake that fails to change relationships among these parameters will not change policy outcomes (a fact that may puzzle those not equipped with the theory developed here). If, for instance, a legislative election caused massive turnover in incumbents but did so symmetrically so as to leave x_m unchanged, then "the more things change, the more they stay the same." Likewise, if before an election legislators c and m are at loggerheads, as defined in case 3 above, then electoral change, no matter how massive, that leaves the (possibly newly determined) c and m at loggerheads will simply maintain the status quo ante. The institutional impediments implicit in the closed-rule regime stand in stark contrast to the hypersensitivity of pure majority rule.

OPEN-RULE COMMITTEE SYSTEM We've seen thus far that although there is an entire continuum of possible final outcomes, only one thing (x_m) can occur under the pure majority-rule regime, and only one of three things (x_c, x^0, or x^*) can possibly happen under the closed-rule regime. In the following treatment of the open-rule regime, it will be seen that only two possibilities exist. Either the gates remain closed and x^0 prevails or the gates are opened and x_m is the final outcome. *Nothing else is possible.* We will consider all the cases as we did in the previous regime.

In the open-rule regime the committee once again has the first move. If it makes no motion, then x^0 persists. If it makes a motion, then that motion is open to amendment (hence the term *open* rule). It is assumed here that alternative amendments continue to be offered until no legislator wishes to offer another. So the committee proposal is initially pitted against the first amendment, the winner of that against the next amendment, and so on until all the amendments have been taken up; the survivor of that sequence is then pitted against the status quo (this last vote is often called the "vote on final passage").[39]

[39] An alternative procedure would be to allow, after a committee motion, an amendment that is directly voted on. The winner stays on the floor and is

FIGURE 5.14



This procedure looks very much like the pure majority rule regime, *except that the committee has the first move.* Once it opens the gates, we're in the world of pure majority rule. This means that once a proposal is made, it will be amended and amended again, with successful amendments converging the process toward x_m. Indeed, it doesn't even matter what the initial committee proposal is. The reality is:

$$\begin{array}{lcl} \text{open the gates} & \Rightarrow & x_m \\ \text{keep gates closed} & \Rightarrow & x^0 \end{array}$$

The committee decision is really pretty simple. If the committee median voter, Ms. c, prefers x_m to x^0, then she makes a motion (any motion); if Ms. c prefers x^0 to x_m, then the committee keeps the gates closed. Thus, all we need to inspect is Ms. c's preferred-to-x^0 set, $P_c(x^0)$, to see whether x_m is in it or not.

Recall the three possible cases in the preceding section. For the parameter ordering of case 1 ($x^0 \leq x_m \leq x_c$), the committee clearly prefers x_m to x^0, so it will open the gates. For case 3 ($x_m \leq x^0 \leq x_c$), the committee clearly has the opposite preference, so it will keep the gates closed. It is the case 2 ordering ($x^0 \leq x_c \leq x_m$) that is the interesting one. If c's ideal policy is less than halfway between x^0 and x_m, then she keeps the gates closed; if it is more than halfway, then she makes a motion.

The first of these case 2 situations is shown in Figure 5.14. What makes this especially interesting is that it represents a

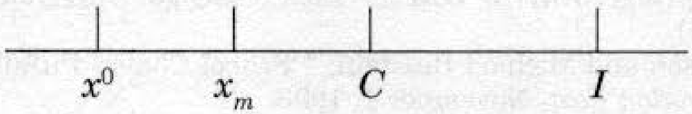
subject to another amendment. The process continues until no more amendments are forthcoming, after which there is a final vote between the alternative left standing on the floor and the status quo.

very frustrating situation. The committee, because it prefers the status quo to the median legislator's ideal, will keep the gates closed. *But both a committee majority and a chamber majority prefer every point in $P_c(x^0)$ to x^0.* That is, the open-rule environment, which at first blush appears to give a legislative majority potent authority, in fact penalizes both committee and legislative majorities. It gives the chamber *too much* authority—the right to amend whenever it wants. Its strength is its weakness, because it cannot promise *not* to use its authority; yet, it would be better off if it could credibly promise not to amend some proposal in $P_c(x^0)$ made by the committee (for, if it could precommit in this fashion, then the committee would be prepared to open the gates).

Case 5.2
The Importance of Compromise and Strategic Thinking

The discussion of the closed-rule and open-rule regimes addresses the general question of how politicians and interested others think about legislative possibilities. In one-dimensional situations, as we have seen, politicians locate a proposal on the policy dimension relative to their own preferences and the status quo that will otherwise prevail if the proposed change is rejected. When faced with a choice between the proposal and the status quo, the politician votes for the alternative closer to his or her ideal. With the open-rule regime, once a proposal is made, the dynamic of amendment activity leads inexorably to a unique outcome—the ideal point of the median legislator. With the closed rule, a proposal wins only if it is closer than the status quo to the median voter's ideal.

Failure to recognize these dynamics can lead to disappointment for principled—that is, stubborn—lobbyists. Advocates of proposed legislation must take into consideration the preferences of the decision maker(s), the rules of procedure in effect, and the relative location of the status quo. An unwillingness to compromise in light of these strategic realities can keep the status quo in place, even though the possibility exists to defeat it with results satisfactory to the lobbyist. This is illustrated in the figure below, where the legislative median is x_m, the status quo is x^0, and a powerful lobbyist's ideal policy is I.



Without going into any of the specifics concerning the way powerful lobbyists exercise their power, suppose that lobbyist I is in a position to undermine any change in the status quo if it finds the change not to its liking (perhaps by "bribing" influential legislators—that is, contributing to their campaign committees). Under the closed rule, the best I could hope for is the compromise point, C—a policy just a little bit closer than x^0 to x_m. If the lobbyist stubbornly refuses to accept C by seeking something more extreme, it loses. Under the open rule, it must be prepared to accept x_m, for this is where the process of amendment will drive the final result. In either of these cases, the lobbyist must be able to anticipate the best deal it can cut and settle for it. In particular, it must be especially sensitive to the fact that "the best deal it can cut" depends upon the procedural rules for amendments. Even though it is powerful enough to *undermine* proposed changes in x^0, it cannot *impose* its own will. It needs a little help from its (legislative) friends.

Some observers have cited the absence of such strategic thinking as a reason for the failure of the Equal Rights

Amendment* and Proposition 174 in California, which would have implemented school choice as a voucher system.† Unwilling to compromise, lobbyists unwittingly kept their proposals further from the status quo than the compromise point required by the strategic realities. Politicians or voters voted against their proposals when more moderate versions very probably would have passed. The importance of such strategic thinking is obvious after the fact but not always in the heat of battle.

* Jane Mansbridge, *Why We Lost the ERA* (Chicago: University of Chicago Press, 1986).

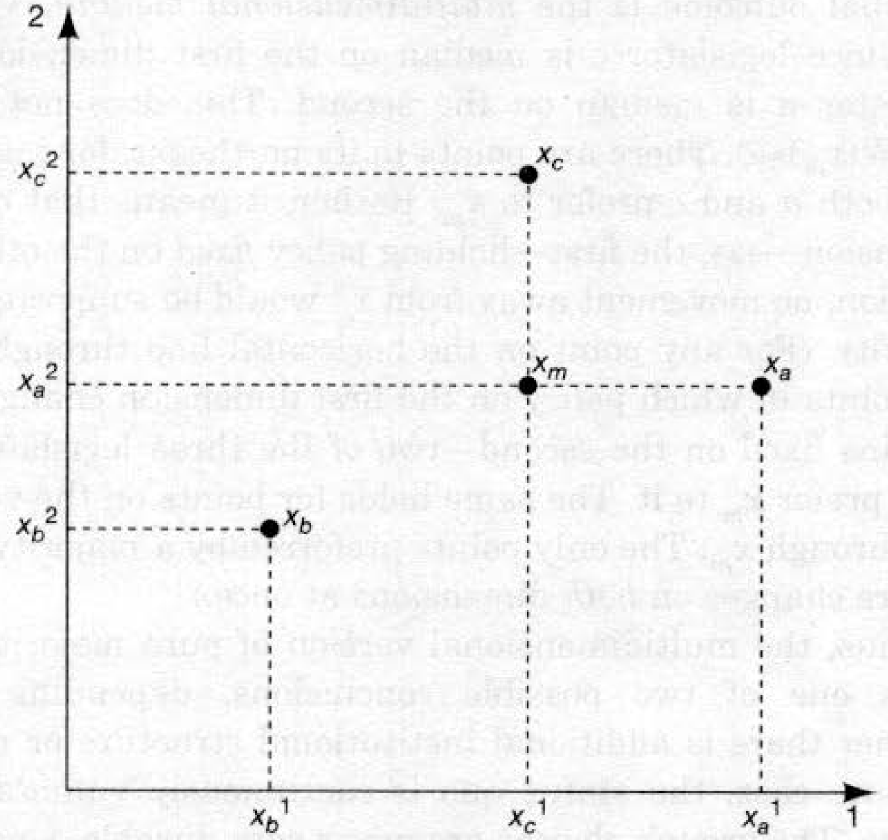
† Jack Anderson and Michael Binstein, " 'School Choice' Pitfalls in California," *Washington Post*, November 1, 1993.

MULTIDIMENSIONAL EXTENSIONS Once multiple dimensions come into play, matters get a bit more dicey. In a pure majority-rule regime, the results of the McKelvey Chaos Theorem loom large. Putting to one side the highly unlikely circumstance that legislator preferences are distributed in a radial symmetric manner, we know that $W(x) \neq \varnothing$ for any x in the policy space. Anything can be beaten. In particular, any status quo, x^0, has a nonempty winset, $W(x^0)$. As long as a motion is made from that set, the status quo will be replaced. But then $x^1 \in W(x^0)$, in turn, has a nonempty winset of its own, $W(x^1)$. A motion $x^2 \in W(x^1)$ will replace x^1. Under the assumptions made about legislative voting,[40] an existing status quo is continually replaced.

Suppose the setup is altered ever so slightly. The condition from pure majority rule still holds that anyone is free to make a motion to change the status quo. But let's assume that decision making takes place *one dimension at a time* in some preset order. The first person recognized to make a motion on the

[40] Namely, that everyone votes his or her preference rather than voting strategically (which is taken up in the next chapter).

FIGURE 5.15



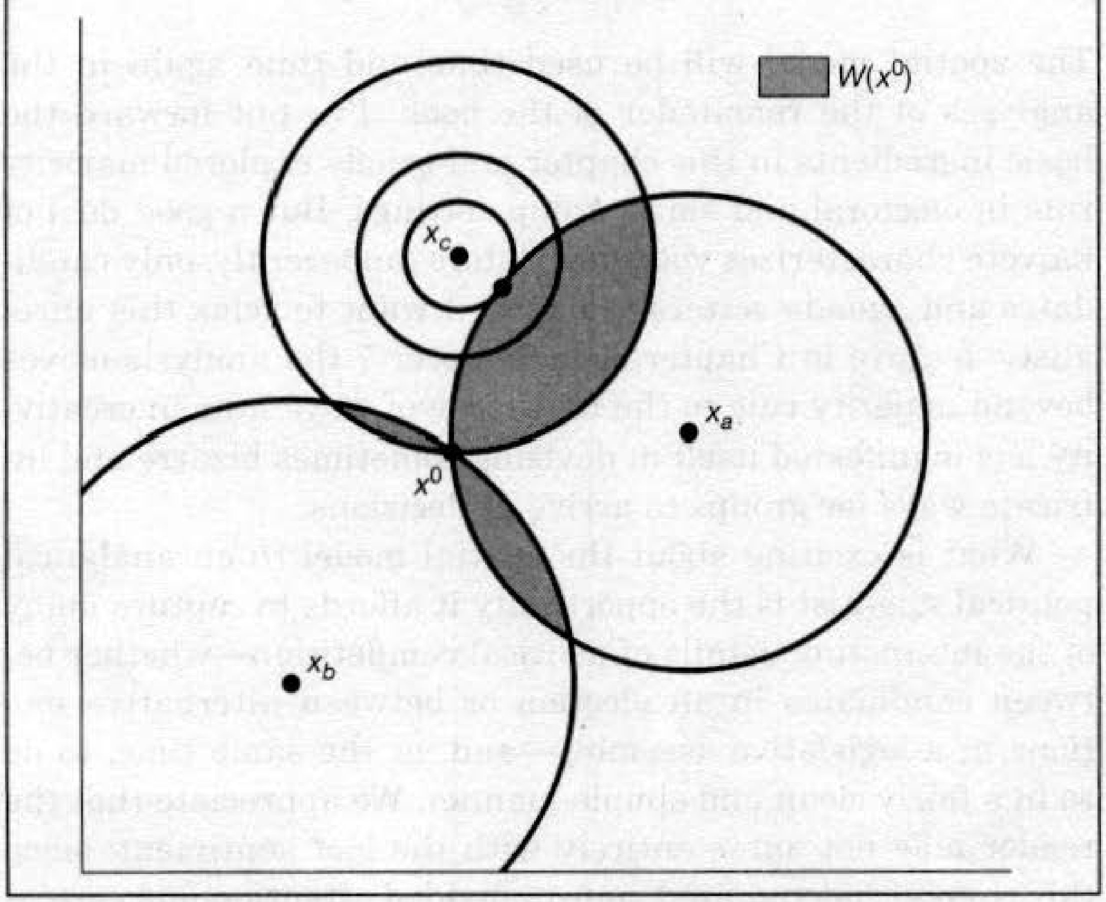
initially designated dimension states his or her amendment to x^0; this amendment can only change x^0 on the dimension currently under consideration. The group continues to focus on amending the status quo on this dimension until no more amendments are offered. Once it completes its task, the group turns its attention to the next dimension. It continues in this manner until there is no dimension left on which any legislator wishes to alter the status quo level.

It is easy to see in Figure 5.15 that this multidimensional version of pure majority rule mimics the result of the one-dimensional setting. There are three legislators with ideal points $x_a=(x_a^1, x_a^2)$, $x_b=(x_b^1, x_b^2)$, and $x_c=(x_c^1, x_c^2)$. For any status quo (not pictured), $x_0=(x_0^1, x_0^2)$, motions are entertained, first on dimension 1 and then on dimension 2. At the end of the day, when all motion-making and voting are said and done, the final outcome is the *multidimensional median*, $\boldsymbol{x}_m=(x_c^{\,1}, x_a^{\,2})$, since legislator c is median on the first dimension and legislator a is median on the second. This does not mean that $W(\boldsymbol{x}_m)=\varnothing$. There are points to its northeast, for instance, that both a and c prefer to $\boldsymbol{x}_m$. Rather, it means that on any dimension—say, the first—holding policy *fixed* on the other dimension, no movement away from $x_c^{\,1}$ would be supported by a majority. (For any point on the horizontal line through $\boldsymbol{x}_m$—the points in which policy on the first dimension changes but remains fixed on the second—two of the three legislators always prefer $\boldsymbol{x}_m$ to it. The same holds for points on the vertical line through $\boldsymbol{x}_m$. The only points preferred by a majority to $\boldsymbol{x}_m$ require changes on *both* dimensions at once.)

Thus, the multidimensional version of pure majority rule yields one of two possible conclusions, depending upon whether there is additional institutional structure or not. In the pure case, the status quo is continuously vulnerable to change. The group's choices are never very durable, since it is always in someone's interest to introduce a motion to change it, and it is always in some majority's interest to comply. In the case of institutional structure in the form of dimension-by-dimension decision making, the result is both predictable and centripetal. The median ideal point on each dimension prevails under the procedure described above (although it need not be the same median voter on each dimension, of course).

Since I will take up the multidimensional versions of the open-rule and closed-rule regimes in the chapter on legislatures in Part IV, the discussion will be especially brief on this subject now. Imagine, in Figure 5.16 (a reproduction of the spatial positions in Figure 5.13), that Ms. c is an agenda setter and the status quo is x^0. If her proposals are subject to amendment by the parent legislature, then we are back to the wild-and-woolly open-rule majority system. Under a closed rule, however, she can make a take-it-or-leave-it proposal, one that

FIGURE 5.16



is *not* subject to amendment but only to an up-or-down vote. The petal-shaped shaded regions comprise *c*'s opportunity set—it is $W(x^0)$. The circles centered on x_c are various of c's indifference contours. Her objective is to move the final policy outcome onto the indifference contour of smallest radius (hence closest to her ideal point) that still lies in $W(x^0)$. The point in this figure (a big black dot!) at the tangency between one of the petals of $W(x^0)$ and the smallest indifference curve of Ms. *c* is the proposal she will make, which a majority (*a* and *c*) will then support. With the closed rule, then, a monopoly agenda setter has considerable power, though constrained by majority preferences.

CONCLUSION

The spatial model will be used time and time again in the analyses of the remainder of the book. I've put forward the basic ingredients in this chapter and briefly explored majority rule in electoral and small group settings. But a good deal of naïveté characterizes voter/legislators (apparently, only candidates and agenda setters are wily). I want to relax this unrealistic feature in Chapter 6. In Chapter 7 the analysis moves beyond majority rule to the multitude of ways human creativity has manifested itself in devising sometimes bizarre and intricate ways for groups to arrive at decisions.

What is exciting about the spatial model to an analytical political scientist is the opportunity it affords to capture many of the interesting details of political competition—whether between candidates in an election or between alternative motions in a legislative assembly—and, at the same time, to do so in a fairly clean and simple manner. We appreciate that the reader may not agree entirely with the last sentiment, since this chapter has required your undivided attention and careful reading. Nevertheless, political scientists over the past fifty or so years have found the model to serve as the principal building block for the analysis of political rivalries of all stripes.

A single chapter in a book, of course, can only portray the spatial model at its simplest. But even the simple formulation possesses nonobvious implications. In the context of one-dimensional pure majority rule with single-peaked preferences, for example, whether in two-party electoral competition or legislative policy choice, the magnetic attraction of the median participant's ideal point is powerful. Majoritarian politics is subjected to centripetal forces, producing outcomes that observers describe with words such as "compromise," "moderate," or "centrist." At the very least, then, the simple spatial model provides a rationale or explanation for the inexorable movement of majority-rule competition toward the center of participant preferences. Its surprise value lies in the fact that this centripetal dynamic is *not* because "that's where the votes are." As we demonstrated earlier, movement toward the median voter's ideal is an equilibrium tendency in a pure majority rule arrangement *even if there are very few voters at the center of things.*

In the legislative realm, where rules of procedure and agenda-setting committees constrain the operation of pure majority rule, there were other surprises. At least in the world of one-dimensional politics, only a limited number of things are possible. A committee system operating under the open rule can produce only one of two possible results. It can, by "closing the gates" and not permitting a motion to be proposed, keep policy at the status quo. Or, if it should make a motion, the sequence of amendments permitted under the open rule will drive the outcome to the median legislator's ideal. These are the only possibilities. Under a closed rule, there are three possibilities. If the gates are kept closed (possibly because the committee and legislature are at loggerheads, with the status quo between their respective median ideals), the status quo remains intact. If the committee median's ideal lies between the status quo and the median legislator's ideal, then the committee's ideal will be proposed and will pass. Finally, if the median legislator's ideal lies between the status quo and the committee's median ideal, then the outcome is the point closest to the committee's median that leaves the median legislator of the full legislature just indifferent between it and the status quo. The details are found in this chapter. The surprise, however, is found in the conclusions: first, that only a small number of items are possible under various legislative procedural regimes and, second, that these small numbers of things differ from regime to regime. Put differently, *institutional arrangements—the political ways of doing business—matter profoundly for the outcomes that emerge from the political process.*

The spatial model also allows us to begin to assemble explanations for why convergence to the center is not always complete. The centripetal tendency is always present, to be sure, but there may be countervailing tendencies as well. In the context of Downs's model of elections, for example, we noted that a politician may fear he will lose his extremist support (to abstention or to a third-party entrant) if he converges too much toward his opponent. In the legislative arena, powerful agenda setters may, through their control of motions and amendments, prevent the process from converging on the median legislator's ideal policy, either because the agenda setter can propose and get passed something she likes better or because she chooses to keep the gates closed.

Thus, the great advantages of the spatial model are its (relative) simplicity, its analytical power, and the "surprises" it produces. Not only do we begin to understand things that we may have long appreciated in an intuitive fashion (like the tendency toward moderation in majority-rule systems), we develop a sophisticated understanding of new things. While surely not a perfect explanatory tool, it's a pretty good start.

One of the matters that I touched on only briefly and unsystematically was strategic thinking. Many applications, especially early in the history of spatial modeling, assume that voters and legislators are "honest" in their voting behavior. When confronted with two alternatives, they simply vote for their favorite, doing so without regard for subsequent consequences. Yet there are many circumstances in which a rational person will think things through in a more sophisticated fashion, sometimes coming to the conclusion that, in a particular voting opportunity, she should *not* vote for her favorite alternative. This is *sophisticated* or *strategic* behavior, the subject of the next chapter.

EXPERIMENTAL CORNER
Group Choice under Majority Rule

Imagine a group that must make a decision. For example, suppose a group of five individuals must choose a point in a two-dimensional space, one that looks like that in Figure 5.8, where each individual values the points in this space differently. Ms. 1 most likes points in the southwestern part of the space; Mr. 3 favors points to the northeast; others prefer points in other regions of the space. In short, there is conflict among the preferences of the members of this group. There are many theories of group choice that attempt to analyze and explain situations like this. In a classic paper, Morris Fiorina and Charles Plott set out to assess these theories by running experiments with real, live groups (mainly of college students, but others have experimented with university employees, business people, townspeople, etc.).[a] In this "Experimental Corner," I will focus on one of their experiments. This is actually a slight variation that I run with my own students.

Begin with an odd number of group members. Each is given experimental preferences by the experimenters. For a five-member group as portrayed in Figure 5.8, member **1** has an ideal or most-preferred point located at x_1, member **2** at x_2, and so on. Their preferences are *Euclidean*, which means that they prefer the final outcome to be a point closer to their ideal point than one further away. Thus, **3**, for example, is indifferent among all the points on a circle centered on her ideal point (because they are equidistant from x_3) and prefers any point on a circle of smaller radius to one on a larger radius (because the former is closer to x_3

[a] Morris P. Fiorina and Charles R. Plott, "Committee Decisions under Majority Rule: An Experimental Study," *American Political Science Review* 72 (1978): 575–98.

than the latter is). In the experiment, the horizontal and vertical dimensions range from zero to 100. A status quo, x^0, is set at (100, 100) in the extreme northeast of the two-dimensional space. Subjects are paid cash depending on the final outcome—a higher payment for a final outcome closer to their ideal point and a lower payment for an outcome further away. Each subject knows his or her preferences and payment schedule but not those of other subjects.

The procedure for decision making is as follows. A subject is randomly selected by the experimenter to make a proposal. For example, if **1** is recognized, she might say, "I move we change the status quo from (100, 100) to (25, 50)." This motion is put to a majority vote. If a majority approves, then (25, 50) becomes the new status quo; if it fails (with ties constituting failure), then (100, 100) remains the status quo. A new person is recognized to make another motion. This process is repeated, with subjects recognized randomly to make proposals. At any time there is a *privileged motion*: Any subject can move to end the session. This is voted on immediately, and if it passes, the session ends and each subject is paid the value to him or her of the status quo prevailing at that time. Otherwise, motions continue to be made until no subject wants to make a new proposal. (Even if subjects still have motions to make, if a time limit is reached, then the session automatically ends.)

What happens? My experience in running this experiment in classes over nearly two decades is that the spatial theory of majority rule works quite well. To illustrate, return to Figure 5.8, where preferences as displayed there satisfy the Plott Theorem (distance-based preferences, an odd number of subjects, ideal points distributed in a radially symmetric fashion). The theory of majority rule implies that the final outcome should be the ideal point of subject 2, since x_2 can defeat any other point in a majority comparison. As long as someone proposes this point, it will become the new status quo and then it cannot be dislodged by any subsequent proposal. In actual experiments, subjects start off tentatively, partly because they are only just figuring out the setting and partly because they don't know the preferences of other subjects. But once they gain familiarity, things move along at a rapid clip. Subjects seek recognition to make motions; motions are made and approved that move the status quo from the extreme northeast point into the "center of things." Each successive motion, as the theory would predict, moves the status quo closer to x_2, the point the Plott Theorem identifies as the final outcome (the equilibrium of the majority-rule process). Logically, there is always a majority that will prefer a point closer to x_2 to one further away, so motions that move a status quo closer to x_2 should prevail over a more distant status quo, and motions that move the status quo further away should fail. This typically happens in the experiments I've run, though occasionally a mistake is made. But even if this should happen, it is quickly corrected by a new motion. There are intermittent efforts to bring the proceedings to a close (especially when a status quo lands close to some subject's ideal point), but typically a majority rejects this until x_2 is reached. Once x_2 is reached, any subsequent motion is defeated and, ultimately, subjects tire of more motions and finally approve a motion to end the session. A typical experimental session draws to a close quite rapidly; rarely is it ended because it reaches the time limit.

If, however, a distribution of ideal points is not like that of Figure 5.8, violating radial symmetry instead, then majority rule does not have an equilibrium like x_2. The McKelvy Theorem applies. What happens in this experimental condition? My experience in running the experiment in this context is that McKelvey's Theorem captures the situation. Majorities approve proposals moving the status quo around but never settling on any specific point.

More often than in the previous setting, the time limit determines when the session will end. The final outcome is somewhere in the middle of the distribution of preferences, but it does not hone in on a specific point as it did when the Plott conditions were satisfied. It is not exactly "chaos," but outcomes do not display the same regularity they do when preferences are distributed in a radially symmetric fashion.

As a final exercise, I have added a new twist to the original Fiorina-Plott experimental design. In some sessions I tell subjects not only their own ideal points but also those of the other subjects. Does this make a difference? Theory says it shouldn't—either there is a Plott equilibrium point or there is the wandering around of the McKelvey Theorem. The additional information does not alter these facts. And, experimentally, the results are about the same as in the limited-information context, though the chatter among subjects during the experiments does reveal envy and competitiveness when some approved status quo looks to benefit the person sitting across the table.

At least in this rather carefully controlled setting, pure majority rule (with only the most elementary of institutional features) pretty much works the way the theorems of Plott and McKelvy suggest. Indeed, against a large number of theoretical competitors, Plott and Fiorina conclude that this theory of majority rule is superior.

Problems and Discussion Questions

1. Suppose that a society consists of three individuals, 1, 2 and 3, who must choose one among three proposed budgets, x, y, and z. Their preferences over these three possible budgets are as follows: xP_1yP_1z, yP_2zP_2x, and zP_3xP_3y.

- Write down the majority preference relation for this profile of preferences (e.g., indicate which alternatives would beat which in two-way contests).
- Does Black's Median-Voter Theorem support a prediction about which policy will be chosen if the group uses simple majority rule? Why or why not?
- Suppose that the group is going to use a voting agenda $v = (y, x, z)$ to select the budget, where this notation means that the group first votes over y and x, and then votes over the winner of this contest and z, where the winner of this second vote is chosen as the budget. If each individual votes sincerely at each stage of the agenda, what would the outcome be? What would the outcome be if $v' = (z, x, y)$ or $v'' = (z, y, x)$?

2. The Senate Finance committee, in debating a health care reform bill, contains some members who demanded a full government-run insurance scheme ("the public option"), others who were strongly opposed, and a third group who favored a compromise health care cooperative, a kind of government-affiliated nonprofit. Both those in favor of a full public option and those opposed agree that the "co-op" is the second-best compromise outcome. What can you say about this group's preferences? Could you predict the outcome of a vote if you knew the number of members who held each of the three preference profiles?

Some of the committee's more liberal members start to take political flak from interest groups strongly in favor of a full public option, who convince them that it would be better to vote against the "co-op"—and wait for a more propitious time—if the full public option is unachievable. What can you say about this new set of preferences? Could you always predict the outcome of a vote if you knew the number of members who held each preference profile?

3. A famous example of preferences which violate single-peakedness was first presented in Verba et al.'s "Public Opinion and the War in Vietnam."[a] This study found that most individuals identified a specific policy as their ideal point on a one-dimensional policy space varying from a reduction or end to U.S. engagement at one end, continuation of U.S. engagement in the middle, and expansion of U.S. commitment at the other end. Moreover, most stated that policies further away from their ideal points were increasingly less desirable. However, there was a small segment of the population who favored either a U.S. withdrawal or an expansion of the U.S. commitment to a continuation of the present policy. What property of a society's preference profile is not respected in this example? How widespread must non-single-peakedness get before group transitivity is violated? Illustrate this graphically, and then explain whether preferences over policy in the Vietnam War might better be represented using two or more issue dimensions. What might those dimensions include?

4. A seven-member governance committee of a corporation is charged with allocating funds for end-of-year bonuses, and a special subcommittee is appointed to research and then propose to the full governance committee a total value of all bonuses. The corporation's rules state that the subcommittee may bring a proposal before the full committee under a predetermined amendment procedure (either an open or closed rule). All subcommittee and committee decisions are made using majority rule; the minimum the committee may allocate is $0 and the maximum is $12,000. The preferences of each members of the seven-member governance committee over alternative total amounts to allocate for the bonuses are single-peaked and symmetric about each member's ideal points, and are arrayed as follows:

[a] Sidney Verba et al., 1967, "Public Opinion and the War in Vietnam," *American Political Science Review* 61, no. 2 (1967): 317–33.

Bobby and Emma: 0; Amy: 1500; Cathy: 6000; Frank: 7500; Geri: 10,000; and David: 12,000.

Answer the following questions, illustrating pictorially where helpful.

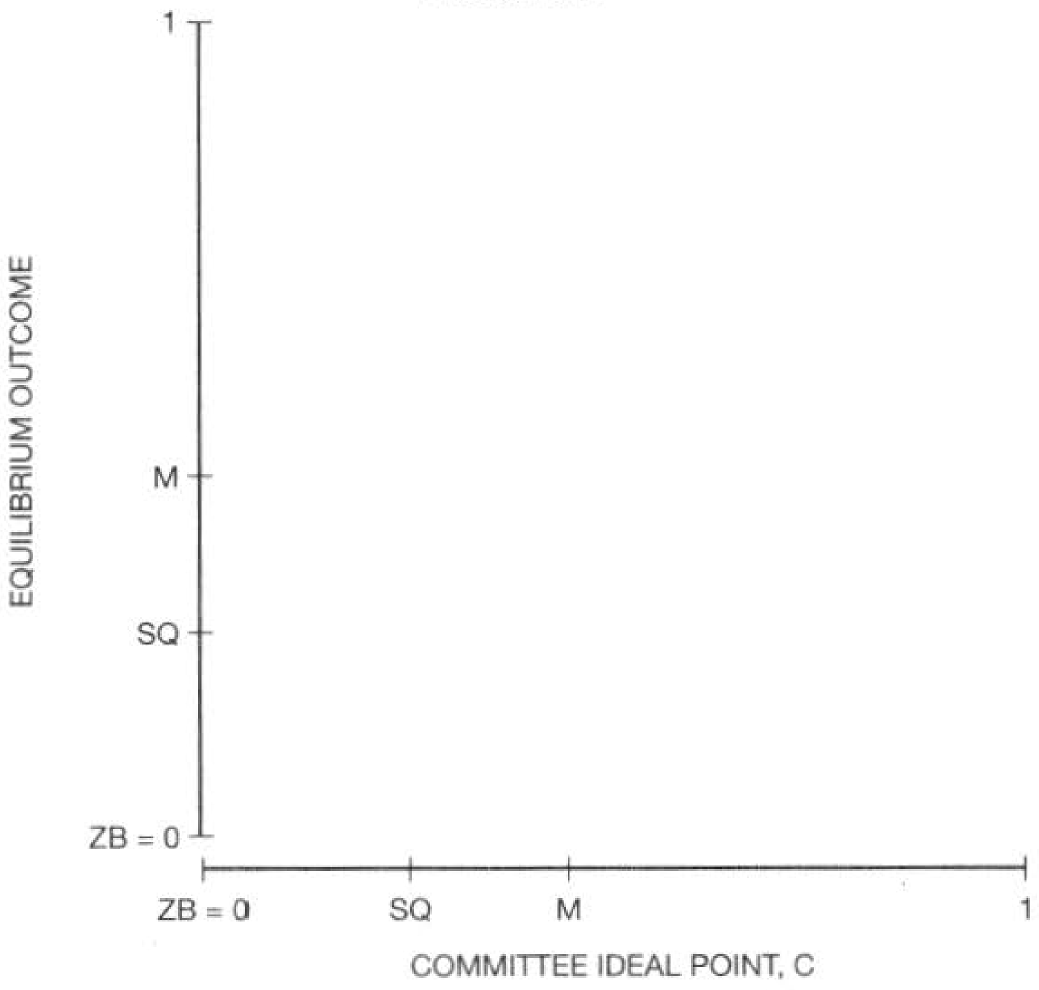
- The subcommittee is composed of Frank, Geri, and David. What is the subcommittee's most-preferred level of funding?
- Last year the governance committee allocated $3000 for bonuses, and under committee rules this is the status quo level of funding (it will be the decision this year, too, if the full committee makes no change). Assuming the subcommittee brings a proposal to the full committee under a closed rule, what is the subcommittee's proposal and what will be the outcome? Why?
- Now assume the subcommittee brings its proposal to the full committee under an open rule. Will the subcommittee "open the gates" and bring a proposal to the floor? If so, what will it propose? What will be the outcome? Explain.
- Now assume that the governance committee follows "zero-based budgeting" in which each year's allocation is initially set to zero, and the committee then proceeds to vote on any new proposals made by the subcommittee. In this case, what will the outcome of the voting be under a closed rule? Under an open rule? Comment on the implications of zero-based budgeting.

5. A legislature is going to vote on a policy that is well represented by a single-issue dimension, on a scale of zero to one. The initial policy proposal will be supplied by a committee (whose ideal point for this exercise can be any point along the x-axis) to a legislature with median ideal point M. The status

quo is currently at point *SQ*. Draw a line showing the equilibrium outcomes for any committee ideal point, when the committee's proposal is considered under a *closed rule.* On a separate set of axes, perform the same exercise when the committees proposal is considered under an *open rule*. Then, do the same exercise for both rules (using a dashed line on the two axes already created), assuming that the legislature operates on the principal of zero-based budgeting, so if no bill is passed in the current session, the policy reverts to $ZB = 0$.

What is the impact of zero-based budgeting under a closed rule? Are equilibrium outcomes closer or further from the legislature's median, on average? What about under the open rule?

FIGURE 5.A



6. A legislature with three groups similar to that shown in Figure 5.6 is attempting to reach a bargain on some issue which has two policy dimensions. The legislature uses majority rule, and the indifference contours are all circular. Suppose that groups 2 and 3 reach an agreement on some deal where 3 gets its ideal policy on the horizontal dimension and the outcome on the vertical dimension is one-third of the distance from 2's ideal point moving toward 3's. Draw the preferred-to sets for 2 and 3 and show that there are other bargains between 2 and 3 that could leave them better off. Consider the set of bargains that 2 and 3 could make that cannot be altered without making either group worse off; what does this set look like? This is called the "contract curve" for groups 2 and 3. Draw in the contract curve for all other possible majority coalitions. Are any points on these contract curves empty win-set points under majority rule?

Now, suppose the legislature operates under a unanimity rule. If the status quo is at the agreement point between 2 and 3 described above, will the three sides be able to find a bill that makes all of them better off? What if the status quo is at 2's ideal point in the horizontal dimension, and halfway between 2 and 3's ideal points in the vertical dimension? What bills have empty winsets when the unanimity rule is employed? Do the existence of these empty winsets contradict McKelvy's Chaos Theorem?

*7. Assume five members of a voting body A, B, C, D, and E. They have ideal points in a two-dimensional issue space, given by A: (1,4), B: (4,4), C: (2,2), D: (1,1), and E: (2,1), where the first coordinate gives a person's ideal policy on the X dimension and the second coordinate gives the ideal policy on the Y dimension. Assume further that each member prefers an outcome closer to his or her ideal point to one further away. All votes are taken over X-Y policy bundles, so all proposals are in the form of ordered pairs (x,y). Is there an equilibrium proposal if the body uses majority rule?

Now suppose that player B has been appointed the sole agenda setter. If the status quo is at C's ideal point, can B construct an agenda that will lead eventually to her own ideal point being approved by the entire legislative body? If so, construct such an agenda (you may assume sincere voting, and that the agenda setter can always persuade legislators indifferent between two policy bundles to vote for the bundle that she prefers). What theorem does this illustrate?

8. Perhaps the most famous political aphorism belongs to Otto von Bismarck: "Politics is the art of the possible." Comment, with reference to Black's Median-Voter Theorem, McKelvey's Chaos Theorem, and the discussion of legislative rules (especially Case 5.2).

9. This question asks you to consider the links between Arrow's Theorem and McKelvey's Chaos Theorem. Do the group preference intransitivities that motivated Arrow's ideas occur in McKelvey's multidimensional, majority-rule settings? Does the "anything can happen" message of McKelvey resonate with any of the lessons of Arrow's Theorem? Does agenda control bestow an ability to determine the outcome in the Arrowian world? In all cases? Develop some small three-person, three-outcome examples to show that the perverse outcomes of the multidimensional spatial model can, but do not always, occur in the discrete world of Arrow.

10. Anthony Downs employed the logic of unidimensional decision making first articulated by Duncan Black in order to understand politics in the largest "committee" imaginable—the electorate. He pointed out that if there were only two parties, and if politicians with extreme views had little inclination to

form parties of their own, then the platforms and campaign promises of the two parties would be very similar.

- Why is that? How does his conclusion follow from Black's Median-Voter Theorem?
- Downs takes politicians to be interested only in winning office. Does a different result (i.e., something other than convergence) arise when politicians have strong policy preferences of their own? Under what circumstances?
- Which other of Downs's assumptions (about the number of candidates, voting behavior, or the dimensionality of the issue space) might explain the apparent differences in party platforms in the United States, and elsewhere?