Beating the Banker: Game Theoretic Applications in Deal or No Deal

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ECON 355: Game Theory

May 2007

1 Introduction

From the show's introduction in December 2005, NBC has had tremendous success withthe game show Deal or No Deal. The game show, which was created in the Netherlands, pits acontestant against the mysterious banker. The contestant is vying for the case with $1 million,while the banker's stated goal is to get the contestant to go home with as little money as possible. This creates an on-screen rivalry that makes for good TV, and for an even better economic study. Deal or No Deal is ripe with game theoretic applications due to this natural competition. Thispaper will seek to explain the game theory within the show. It will first explore how the bankercomes up with an offer, and then how contestants reach a decision on whether to say "Deal!" or "No Deal!".

Deal or No Deal is a unique game show in that it takes virtually no skill on the part of thecontestant. As long as the contestant can count to 26 (the number of cases), he can play the game. But this lack of skill requirement allows us, as economists, to study how people make decisions in a situation where all contestants are virtually all equal. In situations such as placing wagers forFinal Jeopardy!, contestants need to factor in their confidence in their own abilities. In Deal or NoDeal, the only factor is the amount of risk that particular contestant is willing to take on. This provides a unique opportunity on how contestants behave and how they respond to similar offersfrom the Banker. First, I will explain the rules of the game, followed by a brief review of some ofthe previous litreature. From there, I will examine the Banker's behavior and then the contestants' behavior.

1.1 Deal or No Deal Rules

Prior to the start of each game, a third party (meaning not Howie Mandel or the Banker) assigns each of the 26 monetary values to a case. From this point forward, no one involved inplaying the game knows where each dollar amount is placed. In other words, the Banker isn't basing his offer on the contestant's case value, and Mandel isn't either encouraging or discouraging playing further based on the case value. The values placed in the cases are listed in the table below.

\$0.01	\$1,000
\$1	\$5,000
\$5	\$10,000
\$10	\$25,000
\$25	\$50,000
\$50	\$75,000
\$75	\$100,000
\$100	\$200,000
\$200	\$300,000
\$300	\$400,000
\$400	\$500,000
\$500	\$750,000
\$750	\$1,000,000

The contestant picks one case, which contains the amount of money he will take home, unless he takes a deal before the end of the game. He then picks six cases to open to find out which values arenot in his case. After six cases, the Banker offers the contestant a sum of money to end the game. If the contestant rejects the offer, he must open five cases before getting the next offer. The game continues, with the number of cases being opened in each round decreasing my one; so round threehas four cases opened, round four has three cases open, and so on. At the end, if the contestanthas not taken a deal, he takes home the amount of money in the case he picked at the beginningof the game. The rest of this paper will examine the economic and game theoretic applications of this game.

1.2 Literature Review

There are two main articles on Deal or No Deal and decision making under risk. They are outlined below. Roos and Sarafidis (2006) looked at the Australian version of the show to examine how decisions were made by contestants. They explore the utility derived from the remaining casesand the offer. The authors look at 399 contestants and analyze the decisions made by them. They then generate a function for the Banker's offer. The simple model, run for each round, suggests that the offer is a function of the expected value and the standard deviation of the remaining cases. They get statistically significant variables and a very high R^2.

Mulino et al. (2006) also looked at the Australian version to see how contestants reacted torisk. They find that the contestant's characteristics, such as age and gender, affect how decisions were made under risk. Interestingly, they find and wealth does not affect change the contestant's risk aversion.

They also generate a decision making model using the Australian data.

2 The Banker

The Banker is a strategic player in some respects, but not in others. According to Roos andSarafidis (2006), the Banker uses a set formula to determine the offer a contestant is given in eachround. We, therefore, cannot say that the banker decides how much to offer a contestant in each round, or that he makes offer decisions strategically. In other words, he does not decide whether to offer “high” or offer “low” like one might see in many games. The Banker’s stated goal is to send the contestant home with as little money as possible. His unstated goal is to create compelling TV. As we will see, bank offers can be determined by a formula which takes into account the expected valueof the cases, but also the round of play. The following equation describes the Banker’s offer:

$$b = \beta_0 + \beta_1 EV + \beta_3 \sigma + \beta_3 R \tag{1}$$

where b is the bank offer, EV is the expected value, σ is the standard deviation, and R is the round just played.

The expected value is expected to have a positive effect on the Banker’s offer. It is logical that as the expected value in the cases rises, so will the offer. When all 26 cases are unopened, there isan expected value of $131,477.54. As the contestant starts to open cases, taking low number will increase the expected value of the remaining cases and will push the offer higher. Likewise, takingout high numbers will decrease the expected value and will lower the offer.

The standard deviation could go either way. Roos and Sarafidis (2006) found that the standard deviation had a negative coefficient for the first two rounds, and then it had a positive coefficient forthe other rounds. Therefore, it is hard to say what will be its effect when all rounds are regressed together.

The round is expected to have a positive effect on the offer. The Banker wants to keep contestants in the game for as long as possible to keep the show interesting. Therefore, he won’t make a particularly intriguing offer for at least a few rounds. In the first season (which featured 38 contestants), no contestant took a deal before the sixth round. The offer as a percentage of the expected value rises with each round. The next table shows summary statistics for offers in each round.

Round	1	2	3	4	5	6	7	8	9
Observations	38	38	38	38	38	37	28	17	6
Average Offer (%)	11	22	36	50	63	74	87	94	97
Minimum (%)	6	11	17	12	36	30	56	60	67
Maximum (%)	17	42	76	86	91	99	110	113	112

A clear pattern in the offers is evident from the above table. Starting at an average of 11percent in the first round, the Banker gradually increase the offers until they are almost 100 percent in the last round. This pattern allows the Banker to achieve both goals. By starting the offers quite low, there is little chance that the contestant will take the deal. In fact, no one took a deal before round six in season one. This gradual increasing of the deals also fulfills the Banker's other goal. In the later rounds, the Banker can get many contestants to take a deal - and thus send them home with less money than if they got the highest valued case on their board - by offering close to, or over, 100 percent of the expected value. In fact, in many cases, when a contestant picks a case with a large monetary value, their offer will drop while the percentage will increase.

The regression results show that all three variables exhibit the direction expected and are very statistically significant. The next table summarizes the results.

Variable	Coefficient	t-Stat
Intercept	-63468.42	-9.58
Expected Value	0.9952	18.749
Standard Deviation	-0.2231	-5.794
Round	10847.53	11.847

These results allow us to calculate an estimated bank offer based on the expected value of the remaining cases, the standard deviation and the current round. The R^2 is 0.85, which means that a very high percentage of the Banker's offer is included in this model. The expected value and the round number had the expected coefficient direction. Standard deviation had a negative coefficient.Therefore, as the risk of continuing increases, the offer goes down. This seems somewhat odd, but like the other variables, it is highly significant. This regression is not a perfect predictor of the Banker's offer, but it does give a fairly clear picture of what the Banker considers when making an offer.

3 The Contestant

The goal of the contestant is to take as much money home as possible. This goal is reached through a combination of luck and good decision making. There is relatively little skill needed to win large amounts of money, just the ability to decide when it is time to end the game and take the offer. This lends itself nicely to applying game theory because it takes strategy but not skill to do well. It also means that a player's move is determined by their risk tolerance and not by their confidence in their skill set.

The contestant's decision making rule is based on the utility that the contestant receives from the offer amount, the expected value of the remaining cases, and of the risk of continuing. Inequation

form, the two scenarios take the following form, where EV is the expected value of theremaining cases, r is the risk of continuing, and O is the offer amount.
To Take the deal:

$$u(EV) + u(r) \leq u(O) \tag{2}$$

To continue playing:

$$u(EV) + u(r) \geq u(O) \tag{3}$$

One way to measure risk is by using the standard deviation of the remaining cases. In other words, there is a greater risk involved with continuing when the remaining cases are very far apart than when they are close together. Two sets of cases can have the same expected value, yet have much different risk levels associated with not taking the deal. A large standard deviation will mean that there is more risk of getting a substantially lower pay-out than if there is a small standard deviation. Take, for example, a set of cases with \$100,000 and \$200,000 and another set with \$0.01 and 300,000. These two sets of cases have almost the same expected value (\$150,000), but the deal is much more enticing for the second group because of the much higher standard deviation.

How a person interprets and handles the risk associated with the standard deviation depends on how much they like risk. Someone who is risk averse will be less willing to play on when facing a case set with a high standard deviation than someone who is risk seeking. The contestant's risk tolerance can be measured by the deal they take. As was stated above, the offers are a gradually increasing percentage of the expected value. This means that the Banker is essentially looking for the contestant's certainty equivalent. A certainty equivalent, also known as α, is the amount where a contestant is indifferent between the certain amount (the Banker's offer) and the risky amount (the expected value in the remaining cases). A contestant who takes a deal less than the expected value ($\alpha < 1$) is said to be risk averse, whereas a contestant who takes a deal greater than the expected value ($\alpha > 1$) is said to be risk seeking.

In the round where the contestant takes the deal, the average certainty equivalent is 0.9. This means that the average contestant values a sure \$0.90 to every risky \$1.00. Or, in other words, it takes a deal worth \$0.90 for every \$1.00 of the expected value to make the contestant quit. The following table summarizes all 38 contestant's certainty equivalents.

	Certainty Equivalent
Average	0.90
Maximum	1.13
Minimum	0.49

The certainty equivalent is not a perfect measure of the contestant's risk tolerance, but it does give an idea of how much risk a contestant is willing to take on. It does show the level at which a contestant is no longer willing to risk getting less money. While many other factors can also contribute to the contestant's decision - such as audience encouragement, their supporters' encouragement, etc. - the certainty equivalent can give an idea of the contestant's tolerance of risk.

4 Game Theoretic Applications

The contestant makes the first move in the game. He picks the required number of cases for that round, which generates an expected value of the remaining cases at the end of the round. This process generates one expected value out of a range, and thus is represented by a straight line with dotted rays above and below. The Banker then uses that information, as outlined above, to generate an offer. Once again, this is a single offer, so it is represented by a straight line with dotted rays above and below. Next, the contestant plays again, by either saying "deal" or "no deal." If the contestant says "deal," the game stops. If the contestant says "no deal," the game continues. It will continue in that downward direction until a deal is taken. The following two figures show an extensive form of the game. Figure 1 shows what an entire game would look like without any deals being taken. Figure 2 shows contestant Tia Robertson's game. She took a deal after the sixth round.

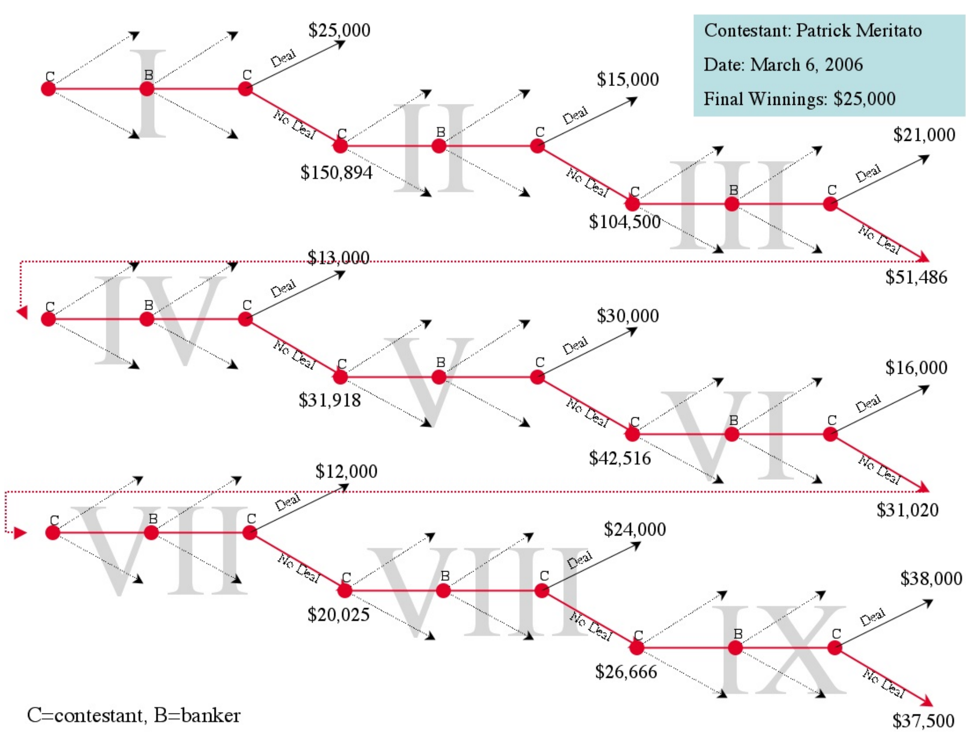

Figure 1: Patrick Meritato's Game

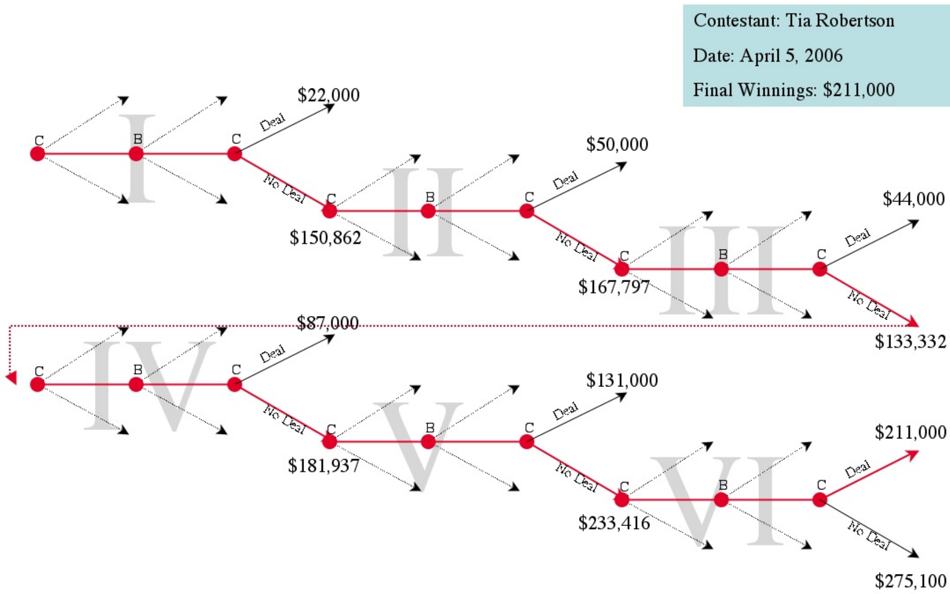

Figure 2: Tia Robertson's Game

4.1 Nash Equilibrium

The Nash equilibrium shifts throughout the game. This is due to the Banker trying to fulfill his two goals talked about earlier. The Banker has two possible actions: "offer high" and "offer low." The contestant also has two options: "say 'deal'" or "say 'no deal'." The Nash will start outat "offer low" and "say 'no deal'." Because of the Banker's strategy of gradually increasing the offer as a percentage of the expected value, his dominant strategy will shift to "offer high" as the rounds progress. This allows the Banker to keep the game going long enough to create compelling TV, while also encouraging many contestants to leave with possibly lower amounts of money than they would get by playing all the way to the end. The player makes his decision based on what the Banker does. Therefore, when the Banker is still making low offers, the contestant's dominantstrategy is "say 'no deal'." As the game progresses, the contestant's dominant strategy will dependon the player's tolerance of risk. A contestant who has a certainty equivalent less than one will have a dominant strategy of "say 'deal'" at some point in the game. A contestant who has a certainty equivalent greater than one will only switch to a strategy of "say 'deal'" if the offer is greater than the expected value, something that only happened 15 times out of 287 offers in the first season. If the contestant does not take any of the deals offered, the game ends out of equilibrium. Most contestants take a deal, as only six contestants made it to the end of the game without taking a deal.

5 Conclusion

Deal or No Deal is full of game theoretic applications. The Banker has goals, but uses a formula to determine offer amounts. The contestant responds to the offer by weighing the risk with continuing, the expected value and the offer. This process continues until the contestant reaches his certainty equivalent, or the point where he is indifferent between the risky value in the cases and the certain amount of the offer. By looking at Deal or No Deal from a game theoretic angle, we are able to gain insight into how people make decisions. These insights can help us to understand how people operate in every day life, and how risk can change people's behavior. Game shows provide a picture of how people think, and Deal or No Deal is one of the best examples of that.

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