sion consistency condition and thus avoid concentrations of power, we still do not achieve fairness. So, in a quite different way, we are back where we began. Nothing has been gained except an elegant formalism that avoids anthropomorphizing society.

5.G. The Absence of Meaning

The main thrust of Arrow's theorem and all the associated literature is that there is an unresolvable tension between logicality and fairness. To guarantee an ordering or a consistent path, independent choice requires that there be some sort of concentration of power (dictators, oligarchies, or collegia of vetoers) in sharp conflict with democratic ideals. Even the weakest sort of consistency ($\beta+$ or $*PI$) involves a conflict with unanimity, which is also an elementary condition of fairness.

These conflicts have been investigated in great detail, especially in the last decade; but no adequate resolution of the tension has been discovered, and it appears quite unlikely that any will be. The unavoidable inference is, therefore, that, so long as a society preserves democratic institutions, its members can expect that some of their social choices will be unordered or inconsistent. And when this is true, no meaningful choice can be made. If y is in fact chosen—given the mechanism of choice and the profile of individual valuations—then to say that x is best or right or more desired is probably false. But it would also be equally false to say that y is best or right or most desired. And in that sense, the choice lacks meaning. The consequence of this defect will be explored in the ensuing chapters.

6

The Manipulation of Social Choices: Strategic Voting

The possibility of a lack of meaning in the outcome is a serious problem for social judgment and social choice. It forces us to doubt that the content of "social welfare" or the "public interest" can ever be discovered by amalgamating individual value judgments. It even leads us to suspect that no such thing as the "public interest" exists, aside from the subjective (and hence dubious) claims of self-proclaimed saviors.

Serious as are such (probably irresolvable) epistemological and ontological questions, it seems to me that the simple practical consequences for social choices are much worse. These consequences are either that power is concentrated in society or that any system of voting can be manipulated to produce outcomes advantageous to the manipulators or at least different from outcomes in the absence of manipulation. If we assume that society discourages the concentration of power, then at least two methods of manipulation are always available, no matter what method of voting is used: First, those in control of procedures can manipulate the agenda (by, for example, restricting alternatives or by arranging the order in which they are brought up). Second, those not in control can still manipulate outcomes by false revelations of values.

Both methods assume a static world, with the number of participants and alternatives fixed. Hence sometimes a profile of preferences cannot be manipulated by anyone. For example, if all participants are unanimous, no one has a motive to manipulate; and, given a voting system satisfying the Pareto condition, it is impossible to manipulate anyway. But in a dynamic world, where the sets of participants, N, or of alternatives, X, can vary, some kind of manipulation is probably possible with any profile. For example, in an X with m elements, even if everyone prefers x_i to the $m - 1$ other alternatives in X, still when X is enlarged to X' containing some x_{m+1}th element, unanimity may vanish and the potential for manipulation may appear.

6.A. The Elements of Manipulation

By limiting the discussion initially to the static case with fixed N and X, it is easy to illustrate the possibilities of manipulation with simple majority voting and the amendment procedure, although in principle similar possibilities exist—even in the static case—for *all* procedures. Suppose there are four alternatives, $X = (w, x, y, z)$, z is the status quo, and three roughly equal factions have this profile:

Faction 1: $w\ x\ y\ z$

Faction 2: $x\ y\ z\ w$

Faction 3: $y\ z\ w\ x$

In the abstract, this profile produces a cycle $w\,P\,x\,P\,y\,P\,z\,P\,w$; so all alternatives tie. But, given the amendment procedures, appropriate control of the agenda produces particular (though different) winners. Suppose faction 3 can control the agenda (perhaps by controlling an agenda-making body like the Rules Committee of the U.S. House of Representatives). Then, by posing its first choice, y, as the original motion with x and w as amendments, faction 3 can make y win in a decision process that proceeds in this way:

Step 1: x vs. w; w wins (with 1 and 3)

Step 2: w vs. y; y wins (with 2 and 3)

Step 3: y vs. z; y wins (unanimously)

Hence $C(w, x, y, z) = y$. Suppose, however, faction 2 controls the agenda-making body. Then by posing x as the original alternative with y and w as amendments, faction 2 can make x win:

Step 1: w vs. y; y wins (with 2 and 3)

Step 2: x vs. y; x wins (with 1 and 2)

Step 3: x vs. z; x wins (with 1 and 2)

So $C(w, x, y, z) = x$. Faction 1 cannot make w, its first choice, win because w loses to z, faction 1's least-desired choice. But faction 1 can, of course, make either x or y win, both of which faction 1 prefers to z. So every faction can produce a preferred choice by control of the agenda. Whether x, y, or even z wins thus depends not only on participants' preferences, but also on the chance of which faction controls the agenda.

It is true that, if the procedure included an ultimate fourth step in which the survivor at the penultimate step was placed against each of the others, then that survivor would also lose—and z would win by default.[1] This is not necessarily a desirable outcome, however, because z loses unanimously to y and by two-thirds to x. The more elaborate procedure renders manipulation slightly more difficult, but it may lead to a *worse* outcome—and, of course, manipulation by false revelation of preferences is not prevented at all.

Let us look, then, at strategic voting, which is available even to those less powerful people who do not control the agenda. Suppose faction 3 does make y the original motion and presumptive winner. Can factions 1 and 2 do anything in response? Indeed, members of faction 1 can, for example, vote strategically as if their valuations were $x\ w\ y\ z$ (instead of $w\ x\ y\ z$, as they truly are). Then x would be the Condorcet winner:

Step 1: x vs. w; x wins (with 1 and 2 because 1 votes strategically)

Step 2: x vs. y; x wins (with 1 and 2)

Step 3: x vs. z; x wins (with 1 and 2)

Hence $C(w, x, y, z) = x$. Although faction 2 cannot alone stop y, members of 2 can, of course, urge faction 1 to do so.

Conversely, if faction 2 succeeds in making x the original motion and presumptive winner, members of faction 3 can thwart 2 by voting strategically, as if 3's valuations were $w\ y\ z\ x$ (instead of the true valuation, $y\ z\ w\ x$). The process will then be:

Step 1: w vs. y; w wins (with 1 and 3 because 3 votes strategically)

Step 2: w vs. x; w wins (with 1 and 3)

Step 3: w vs. z; z wins (with 1 and 3 because 3 reverts to the true valuation, $y\ z\ w\ x$)

So $C(w, x, y, z) = z$, which is faction 3's second choice and preferable to its last choice, x. Nevertheless, 3 cannot guarantee victory for z because faction 1, observing 3's dissimulation at step 1, can also dissimulate at step 2, pretending to hold $x\ w\ y\ z$. So x will survive (with support from 1 and 2) and ultimately beat z (again with 1 and 2). Faction 1 will thus get its second choice, x, rather than its last choice, z, which would be chosen if 1 did not strategically counter 3's strategic voting.

The social choice depends, therefore, not only on the values of participants, but also on whether *any* of them falsely reveal those values, and, if any do, on *which ones* do so.

It may be thought that strategic voting is mainly characteristic of majoritarian procedures. But it is just as easy with positional methods such as the Borda count. Then, for the same profile,

Faction 1: $w\ x\ y\ z$

Faction 2: $x\ y\ z\ w$

Faction 3: $y\ z\ w\ x$

the Borda sums are: $w = 4$, $x = 5$, $y = 6$, $z = 3$; so y wins. But members of faction 1, preferring x to y, have only to rearrange their revealed order to $x\ w\ z\ y$, in which case $x = 6$, $y = 5$, $z = 4$, $w = 3$, and x wins.

Agenda control and strategic voting are often possible in the static world, with N and X fixed. In the dynamic world, with both sets variable, the possibilities of manipulation are enormously increased. Indeed, in the dynamic world, it is probably *always* possible to manipulate, provided participants are eager enough to change outcomes and hence willing to expend the energy necessary to create new alternatives or to introduce new participants.

Suppose, for example, there are two alternatives, a and b, and the status quo, z, and two parties, with this profile:

Party 1: $a\ z\ b$

Party 2: $b\ z\ a$

Party 1 has an absolute majority, 60 percent of the participants. Then with the amendment procedure, the social choice is, for certain, a. What can party 2 do to improve? Party 2 can either bring in more than 20 percent additional participants as supporters or bring in new alternatives that split party 1. Typically, the latter is the easier path. It amounts to a dynamic extension of agenda control, one that even allows losers to manipulate winners.

As in Display 6-1, let party 2 propose a new alternative, c, which splits party 1. In D^1, there is a cycle in $a\ b\ c$, so the status quo, z, will win if c is the original motion. In D^2, c is the Condorcet winner. Really clever leaders of party 2 can occasionally find some c that generates D^2, but many quite ordinary politicians can invent some c that generates D^1.

In lieu of expanding X with c, party 2 can instead trade votes, in effect commingling X with some $X' = (d, f, z)$ containing an alternative d favored more than a by group A of party 1, as in Display 6-2.

Assume party 2 would rather win on b than on f. Since members of group A of party 1 would rather win on d than on a, party 2 can offer to vote for d in choosing from X', in return for which members of group A of party 1 promise to vote for b from X. So $C(X) = b$ and $C(X') = d$. Defining $X^* = X \cup X'$, then vote-trading is simply coordinated strategic voting in X^*.

Altogether, then, in the static world of fixed N and X, there are many profiles permitting manipulation by agenda control and strategic voting. Even if such profiles do not exist in the static world, often, if not always, a dynamic development does permit manipulation. The dynamic version of agenda control is expansion of X with divisive alternatives, and the dynamic version of strategic voting is vote-trading. So it seems manipulation is almost always possible.

Is this a really general result? The answer is affirmative, as I will show for strategic voting in the rest of this chapter and for agenda control in the next chapter.

6.B. The Universality of Strategic Voting

Is strategic voting possible in *any* voting system, given an appropriate profile of individual values? Ever since Duncan Black pointed out the possibility in his original essay on the paradox of voting, social choice theorists have conjectured that the possibility of strategic voting is an inherent feature of voting methods.[2] Recently this conjecture has been proved, independently, by Allan Gibbard and Mark Satterthwaite.[3] Given an appropriate profile of preferences, any voting method can be manipulated strategically. That is, assuming there are "true" preference orders for voters, then there are occasions on which some voters can achieve a desired outcome by voting contrary to their true preferences.[4]

Display 6-1

Splitting the Larger Party with a New Alternative

D on $X = (a, b, z)$	
Party 1 (60%):	$a\ z\ b$
Party 2 (40%):	$b\ z\ a$

Note. Alternative a wins.

A new alternative c in $X' = (a, b, c, z)$ is proposed such that the result is either D^1 or D^2 on X'.

	D^1 on X'	D^2 on X'
Party 1 (30%):	$c\ a\ z\ b$	$c\ a\ z\ b$
Party $1'$ (30%):	$a\ z\ b\ c$	$a\ z\ c\ b$
Party 2 (40%):	$b\ z\ c\ a$	$b\ c\ z\ a$

Number of Votes for the Alternative in the Row When Placed in Contest Against the Alternative in the Column

For D^1

	a	b	c	z
a	—	60	30	60
b	40	—	70	40
c	70	30	—	30
z	40	60	70	—

So a, b, c, and z cycle and z wins.

For D^2

	a	b	c	z	
a	—	60	30	60	
b	40	—	40	40	
c	70	60	—	70	(Condorcet winner)
z	40	60	30	—	

Gibbard remarked and Satterthwaite proved that this result is, in effect, an application of Arrow's theorem. Like Arrow's theorem, it applies only to cases where there are three or more alternatives. (Simple majority voting on *two* alternatives is not manipulable, though the reduction of many alternatives to two is.) Still the Gibbard–Satterthwaite result is narrower than Arrow's theorem. The theorem on manipulation applies only to voting, not to amalgamation by other means such as markets. Since it does apply only to voting, it specifically does not allow for ties or for the choice of more than one winner.[5]

As Gibbard pointed out, however, voting is without meaning unless it produces a unique outcome. Consequently, the proof that strategic voting inheres in all methods assumes that the method must lead to a unique choice. Similarly, it is quite easy to devise strategy-proof methods that use a chance device. (Gibbard's example is good: Everyone marks a ballot with a first choice among alternatives, and a decision is made by randomly picking one ballot. Then all are motivated to select their true first choices. If a voter selects otherwise, then, if his or her ballot is picked, the social choice will be less favorable than if the voter had selected honestly.) Although many such devices can be imagined, they all violate Arrow's independence condition in that, for a given profile, the social choice might be x with one chance selection and y with another. It seems appropriate, therefore, to exclude methods of amalgamation that use chance.[6] What remains are nontrivial voting methods, and these are all subject to manipulation by strategic voting.[7] The conclusion is, therefore, that any ordinal method of voting can be manipulated by individuals.[8] Furthermore, as will be shown in section 6.D, even demand-revealing voting, which was devised to preclude manipulation, is easily manipulable by minority coalitions. It seems, therefore, that some potential for manipulation is inescapable.

Display 6-2

Vote-trading on New and Old Alternatives

D on $X = (a, b, z)$

Party 1, Group A (20%):	$a\ z\ b$
Party 1, Group B (40%):	$a\ z\ b$
Party 2 (40%):	$b\ z\ a$

Note. Alternative a wins.

D on $X' = (d, f, z)$

Party 1, Group A (20%):	$d\ z\ f$
Party 1, Group B (40%):	$f\ z\ d$
Party 2 (40%):	$f\ z\ d$

Note. Alternative f wins.

Vote trade: Party 2 promises to vote for d and group A of party 1 promises to vote for b. The result is:

Contest

a		vs.	b
Party 1, Group B	40%	Party 1, Group A	20%
		Party 2	40%
Total	40%	Total	60%

Note. Alternative b wins.

Contest

d		vs.	f
Party 1, Group A	20%	Party 1, Group B	40%
Party 2	40%		
Total	60%	Total	40%

Note. Alternative d wins.

6.C. Examples of Strategic Voting

Some writers have suggested that strategic voting is so difficult for most people that very little of it occurs. Evidence for or against this proposition is hard to come by because to know whether people vote strategically, one must know how their true values differ (if at all) from the values they reveal. The observer knows for certain only what is revealed, so half of the data for comparison are unavailable. Nevertheless, where one can make good guesses about true values, it appears that quite a lot of strategic voting takes place.

In Plurality Voting

One of the simplest kinds of strategic voting occurs in plurality systems, where supporters of third parties vote for their second choice in order to defeat the major party candidate they like the least. (See the examples in section 4.D.) Since this kind of strategic voting is one motive force behind Duverger's law ("The simple majority, single ballot system favors the two-party system.") and since Duverger's law, properly interpreted, seems almost invariably true, strategic voting of this sort must be very common in single-member district systems.[9]

In An Open Primary

One complicated strategy, sometimes recommended but, so far as I know, never used by enough voters to make a difference, is this: Given an open primary (wherein voters can participate in the primary election of any party they select at the time of the primary), given for party R two candidates, r_1 and r_2, and for party D one candidate, d, given that the chance r_1 beats d in the general election is 0.9 and that r_2 beats d is 0.4, then one recommended strategy for members of D is to vote for r_2 in the R primary, so that, if the prior estimates of probabilities are accurate, d will beat r_2 in the general election. (Similar possibilities exist in any double election or runoff system.)

This strategy assumes a profile somewhat like this, where R_i is a faction in R,

R_1	(15%)	$r_1\ d\ r_2$
R_2	(10%)	$r_1\ r_2\ d$
R_3	(35%)	$r_2\ r_1\ d$
D	(40%)	$d\ r_1\ r_2$

and it anticipates voting thus:

Step 1: r_1 vs. r_2; r_2 wins (with R_3 and D or 75%)

Step 2: r_2 vs. d; d wins (with R_1 and D or 55%)

Something quite like this was recommended by Democratic activists in the Wisconsin Republican senatorial primary of 1956, where r_1 was the incumbent Senator Wiley, an Eisenhower Republican, and r_2 was Representative Glenn Davis, who was supported by the faction of Senator Joseph McCarthy. Not surprisingly, ordinary Democrats rejected this complicated and risky strategy, although a large number of them voted in the Republican primary for Senator Wiley, who won the primary with just less than 50 percent (there was a trivial third candidate) and who won the general election by a large majority.

That ordinary voters reject such a strategy (which may be followed by a few party activists) is sometimes cited as evidence that most people cannot vote strategically. In the instance cited, however, many Democratic voters apparently voted for Senator Wiley, that is for r_1 rather than for their true preference, d, which is also strategic voting—and, furthermore, probably more in accord with their desired result.

Assume, as in Display 6-3, appropriate probabilities of election outcomes and the following (possible but imaginary) cardinal utilities for the election:

	d (Maier)	r_1 (Wiley)	r_2 (Davis)
Democratic activists	1.0	0.2	0.0
Ordinary Democratic voters in Republican primary	1.0	0.8	0.0

Then *all* Democrats (activist and ordinary voters alike) who voted in the Republican primary voted strategically, although, having different values, they voted differently. In Display 6-3 is a calculation, based on imaginary prior expectations of outcomes, to show that both strategies are reasonable for different persons. If so, then—far from showing that strategic voting is rare—an election like this is in fact evidence that it is quite common. Many Democrats did vote for their probable second choice (Wiley) in order to eliminate for certain their probable third choice (Davis). Those activists who interpreted the outcome as evidence of nonstrategic voting merely attributed their own intense interest in electing a Democratic senator to ordinary voters who were probably instead intensely interested in rejecting McCarthyism.

The rationale for different choices by different kinds of Democrats among alternative strategies is set forth in Display 6-3. The rationales depend on the notions of prior probabilities and expected utility. There are three possible outcomes: $X = O_1$ (d wins), $X = O_2$ (r_1 wins), $X = O_3$ (r_2 wins). There are three possible actions for Democratic voters, set forth in column 1 of the Display: $A = a_1$ (vote for d [Maier] in the Democratic primary), $A = a_2$ (vote for r_1 [Wiley] in the Republican primary), $A = a_3$ (vote for r_2 [Davis] in the Republican primary). Each action, a_i, leads to a different arrangement of expectations about outcomes, the calculation of which is set forth in columns 2 and 3. The following assumptions underlie these probabilities.

For column 2:

> If Democrats vote in the Democratic primary, then in the uninvaded Republican primary r_1 (Wiley) would win with probability 0.3 and r_2 (Davis) with 0.7. (Presumably Davis had the lead over Wiley among Republicans.)
>
> If Democrats invade the primary to vote for Wiley, however, r_1 (Wiley) would win with probability 0.7 and r_2 (Davis) with 0.3. (Presumably strategically voting Democrats could render Wiley the most probable winner.)
>
> If Democrats vote for Davis (a highly improbably event), then r_1 (Wiley) would win with probability 0.1 and r_2 (Davis) with 0.9. (Presumably Democrats could render Davis an almost certain winner.)

In column 3, the assumptions are that, in general elections in head-to-head contests, the probabilities of victory are

r_1 (Wiley)	vs.	d (Maier)	
$p(r_1) = 0.9$		$p(d) = 0.1$	
0.9	+	0.1	= 1

r_2 (Davis)	vs.	d (Maier)	
$p(r_2) = 0.4$		$p(d) = 0.6$	
0.4	+	0.6	= 1

Display 6-3

Imaginary Expected Utility of Alternative Actions for Democrats in the 1956 Wisconsin Senatorial Primary

(1) Alternative actions for Democrats	(2) Voters' probabilities for outcome of Republican primary, given action in column 1	(3) Voters' probabilities for outcomes of general election	(4) Voters' probabilities for final outcomes for each action (Col. 2 · Col. 3)	(5) Activist Democrats: Utility for outcome	(6) Activist Democrats: Expected utility for outcome and alternative	(7) Ordinary Democrats: Utility for outcome	(8) Ordinary Democrats: Expected utility for outcome and alternative
a_1 Vote in Democratic primary	Wiley wins .3	Wiley wins .9	.27 Wiley wins	0.2	.054	0.8	.216
		Maier wins .1	.03				
	Davis wins .7	Maier wins .6	.42				
			.45 Maier wins	1.0	.450	1.0	.450
		Davis wins .4	.28 Davis wins	0.0	.000	0.0	.000
			1.00		.504 for a_1		.666 for a_1
a_2 Vote in Republican primary for Wiley	Wiley wins .7	Wiley wins .9	.63 Wiley wins	0.2	.126	0.8	.504
		Maier wins .1	.07				
	Davis wins .3	Maier wins .6	.18				
			.25 Maier wins	1.0	.250	1.0	.250
		Davis wins .4	.12 Davis wins	0.0	.000	0.0	.000
			1.00		.376 for a_2		*.754 for a_2
a_3 Vote in Republican primary for Davis	Wiley wins .1	Wiley wins .9	.09 Wiley wins	0.2	.002	0.8	.072
		Maier wins .1	.01				
	Davis wins .9	Maier wins .6	.54				
			.55 Maier wins	1.0	.550	1.0	.550
		Davis wins .4	.36 Davis wins	0.0	.000	0.0	.000
			1.00		*.552 for a_3		.622 for a_3

*Action with highest expected utility for category of voter.

The results in column 4 are several outcomes: O_1 (Wiley wins), O_2 (Maier wins), and O_3 (Davis wins). These are calculated by multiplying the probabilities along each path from a_i to the three possible outcomes associated with it. For example, along the top path (0.3)(0.9) = .27. Notice that, for each a_i, there is some probability for each outcome, O_i, and the sum of the probabilities for the outcomes is 1. For example, for a_1, $p_1(O_1) = .27$, $p_1(O_2) = .45$, and $p_1(O_3) = .28$ and $.27 + .45 + .28 = 1.0$.

It is not enough, however, to know the varying probabilities. Action a_3 gives the highest probability of d winning:

	Probability of O_2 (Maier wins)
a_1 (vote in Democratic primary)	.45
a_2 (vote for Wiley)	.25
a_3 (vote for Davis)	.55

Still, this action may not lead to the highest satisfaction. In order to estimate the anticipated satisfaction from the choice of an alternative action, one can multiply the probability of an outcome, given some action, a_i, times the value or cardinal utility ascribed to it by the Von Neumann–Morgenstern experiment, as previously imagined. Summing the result for all O_i for a particular a_i gives its expected utility.[10] Thus, *expected utility* is the measure of the satisfaction one anticipates from an action before one takes it, given, of course, all the assumptions one makes about the probable effects of the action on the occurrence of alternative outcomes.

The usefulness of the notion of expected utility is simply that one can calculate the expected utility of alternative actions and then select the action that provides the highest expected utility. In the instant example, the expected utilities of the several actions for activist and ordinary Democrats are easy to compare (the asterisks indicate the largest utility in the column):

	Expected utility	
Action	For activist Democrats	For ordinary Democrats
a_1	.504	.666
a_2	.376	.754*
a_3	.552*	.622

Clearly, neither should in this example choose the "naive" strategy of voting for Maier, the first choice. Activists should choose a_3, voting strategically for r_2 (Davis); ordinary Democrats should choose a_2, voting strategically for r_1 (Wiley). These are, of course, relatively easy and straightforward strategic choices, and a large but indeterminate number of Democrats did in fact vote strategically.

In Union Elections

There are instances of highly complicated strategic voting by select groups of voters. One such example, analyzed by Howard Rosenthal, involves voting by proportional representation in union elections in France.[11] Strategic voting is possible—and perhaps frequent—in all systems of proportional representation. A voter may truly order the parties or candidates x_i, $i = 1, \ldots, n$, like this, $x_j P x_k P x_h$, but still vote for x_k rather than for x_j because, for example, of a belief that x_k will be more likely to enter a governing coalition. The strategy used by union "militants" is, however, much more complex and is designed to discomfit the leaders of opposing unions. Rosenthal cites evidence that this strategy and a defense against it are used in a number of French labor unions.

Voting is for lists, offered by competing unions, for membership on committees in firms. Each union may offer a list, with names ordered as its leaders choose (typically with the most important leaders highest), of as many candidates as there are seats on the committee, though to apply the strategy here discussed the union must offer more candidates than it expects to win. Even small unions often offer a full list, presumably to demonstrate their legitimacy. Each voter can vote for one list, although voters can strike out names on it, thereby losing votes. The votes for a candidate are the number of votes for the list less the number of times the candidate's name is stricken. The votes for the list are the average of the votes for its candidates. Seats are then assigned to unions by the highest-average method (see Chapter 2, note 2), the first seat assigned to a union being in turn assigned to the candidate on its list with the most votes.

If leaders of union A anticipate some "wasted votes"—that is, if they expect to win k seats whether they get v or $v - \epsilon$ votes—, then they can have a few of A's militants vote for union B's list, striking out as many of the top names on B's list as A's leaders expect B to elect. Assuming the members of union B vote sincerely for the entire B list, A's militants will thus have reduced the vote for B's main leaders, thereby electing presumably less effective spokespersons for B. But the leaders of union B have a defense: They can arrange for B's militants to strike out all but the top names on B's list, those whom B's leaders hope to elect. If the militants of both unions vote strategically, then that union succeeds which has the greatest absolute number of militants engaged in the strategy.

This seems to me a remarkable instance of strategic voting, requiring a high degree of both information and discipline. That it occurs as frequently as it apparently does is strong evidence that strategic voting is widespread.

In the U.S. House of Representatives

Since the three foregoing examples of strategic voting do not display the process in complete detail, I will conclude with an example from the U.S. House of Representatives, where it is easy to see the motives for and the complexity of strategic voting and where many observers were convinced that some members gained a lot by voting contrary to their true and well-known tastes.[12]

This example involves a bill in 1956 to authorize grants to help school districts with construction costs. Because schools were then becoming overcrowded with the children of the post war baby boom, there was an obvious argument for this bill, especially since it involved only construction, not maintenance, and thus partially avoided the divisive issue of funds for sectarian schools, an issue that had stymied federal aid to education for the previous eight years. Consequently, there was probably a substantial majority in favor of the authorization, even though it meant a sharp break from the tradition of local financing. Nevertheless, the question was complicated because school desegregation also entered in. An amendment, offered by Adam Clayton Powell, a black representative from Harlem, provided that grants could be given only to states with schools "open to all children without regard to race in conformity with requirements of the United States Supreme Court decisions"—a reference to the 1954 decision in *Brown* vs. *Board of Education,* holding racially segregated schools unconstitutional.

There were, thus, three alternatives before the House:

a. The bill as amended by Powell's amendment

b. The original unamended bill

c. The status quo—that is, no action

With three alternatives instead of just two (that is, instead of just alternatives b and c), some who favored the status quo were able to vote strategically to generate a cycle. And, under the non-neutral amendment procedure, if a cycle exists, the status quo wins.

Since there were only two roll calls on the three alternatives, there is not enough information to specify complete preference orderings. But there is enough data from other roll calls in 1956 and 1957 and from the debate to show that some people probably voted strategically and that there were enough of them to generate a cycle.

The two roll calls were (1) a vote on the Powell amendment, which passed, and (2) a vote on final passage of the bill, which failed. These votes can be summarized as in Display 6-4.[13]

From the data, it is easy to calculate the social choices for alternative a vs. alternative b and for alternative a vs. alternative c. The vote on the Powell amendment pits a against b so that yea means a vote that $C(a, b) = a$, while nay means a vote that $C(a, b) = b$. Since the yeas won, $C(a, b) = a$ (by 229 to 197).

Similarly, the vote on final passage pits a against c so that yea is a vote that $C(a, c) = a$, and nay is a vote that $C(a, c) = c$. Since the nays won, $C(a, c) = c$ (by 227 to 199).

To establish the choice between b and c, we can infer, first of all, that those who voted yea on final passage preferred b to c. There are three ways that a (the amended bill) stands ahead of c (the status quo):

> $a\,b\,c$ and $b\,a\,c$: These orderings mean that school aid, with or without the Powell amendment, is preferred to no aid.
>
> $a\,c\,b$: This ordering means that school aid is preferred to the status quo only if the Powell amendment is attached.

Display 6-4

Voting on Aid to Education

	Final passage		
	Yea	Nay	Total
Powell Amendment			
Yea	132	97	229
Nay	67	130	197
Total	199	227	426

Nothing in the ideological circumstance of 1956 rendered the latter ordering likely, especially since the Powell amendment won. So I conclude that the 199 who voted for final passage had either *a b c* or *b a c* and thus all preferred *b* to *c*. In addition, some members, mostly Southern Democrats, almost certainly ordered *b c a*, which means that they wanted school aid with desegregation but would forgo the aid if desegregation were required. Eighteen Democrats—15 from the South and border—who voted against final passage of the bill with the Powell amendment and who were still in the House in 1957 when another school construction bill without the Powell amendment came up—voted in 1957 in support of school construction. Hence, it appears that at least 18 of those who voted for *c* against *a* nevertheless preferred *b* to *c*. So I conclude that, since 199 had *a b a* or *b a c* and at least 18 had *b c a*, $C(b, c) = b$ (by an imputed vote of 217 to 209). This leads to a cycle: *a* beats *b*, *b* beats *c*, and *c* beats *a*.

Although one cannot know for sure the motives or true tastes of the 97 members (all Republicans) who voted yea on Powell and nay on final passage, a substantial amount of evidence indicates that they voted strategically. Certainly the Democratic leadership believed they did. Representative Bolling of Missouri read into the *Congressional Record* a message from former President Truman:

> *The Powell amendment raises some very difficult questions. I have no doubt that it was put forward in good faith to protect the rights of our citizens. However, it has been seized upon by the House Republican leadership, which has always been opposed to Federal aid to education, as a means of defeating Federal aid and gaining political advantage at the same time. I think it would be most unfortunate if the Congress should fall into the trap which the Republican leadership has thus set. This is what would happen if the House were to adopt the Powell amendment. The result would be that no Federal legislation would be passed at all, and the loser would be our children of every race and creed in every State in the Union.*[14]

Truman is suggesting that there are four positions on the subject:

1. Pro school aid: Mostly Northern Democrats and some Republicans who recognize that passage of the Powell amendment would alienate Southern supporters of the bill, which will thus fail. Consequently, this group, even though it might ideologically prefer *a* to *b*, orders the bill ahead of the Powell amendment, *b a c*, because it wants the bill.
2. Pro Powell amendment: Mostly Northern urban Democrats with a substantial number of blacks in their districts, who, though favoring school aid, clearly favor *a* more than anything else. This group orders *a b c*.

3. Segregationist Democrats, mostly Southern, who would like school aid but will reject it if it entails desegregation. This group orders *b c a*.
4. Anti–school aid Republicans, whose ordering is *c a b* if they oppose segregation, *c b a* if they favor segregation.

Truman's assertion is that the true position of members in category 4 is *c b a*, but they are pretending to hold *c a b* in order to defeat the bill. He assumes that, if members in 4 voted the tastes he has imputed to them, the process would go like this:

> Step 1: *a* vs. *b*; *b* wins with 1 (*b a c*), 3 (*b c a*), and 4 (*c b a*)
>
> Step 2: *b* vs. *c*; *b* wins with 1 (*b a c*), 2 (*a b c*), and 3 (*b c a*)

However, since people in group 4 are going to vote strategically, the process will actually go like this:

> Step 1: *a* vs. *b*; *a* wins with 2 (*a b c*) and 4 (*c a b*, which Truman believes is strategic)
>
> Step 2: *a* vs. *c*; *c* wins with 3 (*b c a*) and 4 (*c a b* or *c b a*)

Just because Truman and Bolling—intense partisans—attribute tastes to their enemies, it does not follow that the attribution is correct. Conceivably, every one of the 97 Republicans believed in the order *c a b*, in which case they were unfairly maligned by Truman because they voted according to their true values. If so, the cycle, which clearly existed, was entirely accidental and owed nothing to strategic voting. There is some support for this interpretation because Republicans had, in an earlier tradition, supported blacks' aspirations more than had Democrats, and in 1956 President Eisenhower was attempting to revive that tradition. On the other hand, there is some evidence this cycle was contrived:

> Item: Several members in category 4 gave speeches favoring both integration and school aid, which implies either *a b c* (like group 2) or *b a c* (like group 1). Yet they actually voted for *c* against *a*. This suggests that they were influenced to vote as part of a unified maneuver.
>
> Item: At least 12 members in category 4 soon afterward voted against various civil rights provisions of other bills, which suggests that their true opinions were, as Truman believed, *c b a*.

That some strategic voting occurred seems extremely likely. Whether it was enough to generate the cycle, we cannot know. Only 32 of the 97 had to vote strategically to contrive the cycle, however, and I think it likely that at least 32 did so.

6.D. The Consequences of Strategic Voting

Strategic voting, if successful produces an outcome different from the "true amalgamation," however defined, of the values of the members of the group.[15] Of course, just what the "true amalgamation" is depends, as was shown in Chapter 4, on the voting system in use. But, given an agreed-upon constitution for voting, there is some specific outcome produced by voting in accord with one's true tastes. That specific outcome—the "true amalgamation"—is precisely what strategic voting displaces.

Whether this displacement is good, bad, or indifferent depends on one's standards. For example, in the election of 1912 (see section 4.D), if Taft voters had voted strategically, Roosevelt, who was, possibly, the social choice by both the Condorcet criterion and the Borda method, would have been elected. If one believes those methods better than plurality voting, then strategic voting would have produced a social gain; still the Wilson supporters and the Taft supporters, who did *not* vote strategically, probably would not have thought so. In the case of the Powell amendment, construction would probably have been authorized in the absence of strategic voting. That would have been a social gain for the majority that wanted new schools more than anything else, including of course, those who would have used the schools to deny civil rights. On the other hand, it would have been a social loss for the majority that opposed federal invasion of school financing or did not want to perpetuate racism in schools. It is hard to say whether one majority is "better" than another.

In general I see no way to evaluate the consequences of strategic voting. In majority decisions between pairs of alternatives, strategic voting (given perfect information) leads to the selection of the Condorcet winner—if one exists—or of some alternative in the top cycle. Sincere voting in this procedure, however, may lead to a less-desired choice.[16] But this is just one procedure; in others—for example, positional methods—sincere voting may lead to "better" results. All we know in general is that the outcome of strategic voting differs from the outcome of sincere voting.[17]

6.E. Vote-trading

The most extreme version of strategic voting is vote-trading, which is coordinated strategic voting on two or more issues. Like strategic voting by individuals or ideologically unified groups, vote-trading requires that traders vote contrary to their true tastes on some issues. That is, when trading on motions x and y, person 1 votes sincerely on x and strategically on y while person 2 votes strategically on x and sincerely on y. Like strategic voting also, vote-trading can improve the outcome for individuals and possibly for a majority as well. And finally, again like strategic voting, vote-trading can also produce the worst possible outcome, wherein each trader—and everybody else—is worse off.

It is not always possible to trade votes, as I have already pointed out. For example, if the same absolute majority passes two motions, no trade is available. For a trade to occur, there must be, for two voters (or groups of voters) and two issues, a situation in which person (or group) 1 is

> Pivotal on the winning side on issue x (pivotal in the sense that, by changing sides, person 1 changes the outcome on x)
>
> On the losing side on issue y and more interested in obtaining his or her (or their) preferred outcome on issue y than on issue x

and conversely for person (or group) 2. When these conditions are met, person 1 can change sides on x and thus help person 2 get what he or she (or the group) most wants and person 2 can change sides on y and thus help person 1 get what he or she (or the group) most wants. Both persons (or groups) will then be better off. These conditions are, however, restrictive, so trading is not always possible.

When vote-trading can occur, it vastly expands the possibilities of strategic voting. On a motion, x, there are exactly two positions: (1) support, which will be written x, and (2) opposition, or "not x," which will be written $\bar{x}$. Under simple majority voting on a pair of alternatives, x and $\bar{x}$, the choice between the alternatives is ambiguous only in case of a tie. If either x or $\bar{x}$ wins, that choice cannot be upset by strategic voting simply because one is better for a majority than the other. But if another motion, y, with possible choices, y and $\bar{y}$, is joined with the consideration of x, there are four possible choices: $x\,y$, $x\,\bar{y}$, $\bar{x}\,y$, and $\bar{x}\,\bar{y}$. And with four alternatives strategic voting is, of course, possible.

Let us introduce a notation and assume simple majority voting:

$q_x = C(x, \bar{x})$, so q is the choice between passing and defeating x, assuming sincere voting.

$Q^* = q_1, q_2, \ldots, q_m$, for m motions, so Q^* is the social choice from sincere voting on each motion separately.

$Q = C(x\,y \ldots, x\,\bar{y} \ldots, \ldots, \bar{x}\,y \ldots, \ldots, \bar{x}\,\bar{y} \ldots)$ for m motions, so Q is the social choice among passing all, passing some and defeating others, and defeating all.

One can then point out that, although q is stable in the sense that it cannot be overturned by strategic voting, Q^* may not be the same as Q because of vote-trading and Q is not stable because it may be overturned by vote-trading. Of course, Q is most obviously unstable if the relation, P, of *social* preference is cyclic—if, that is, $xy\,P\,\bar{x}y\,P\,x\bar{y}\,P\,\bar{x}\bar{y}\,P\,xy$.[18] But Q is also, in a limited way, unstable, even if Q is a Condorcet winner.

Consider, first, a cyclic situation, with three voters and two motions, as in Display 6-5. Clearly, if voters vote sincerely, $C(x, \bar{x}) = \bar{x}$ (because voters 2 and 3 prefer $\bar{x}$) and $C(y, \bar{y}) = \bar{y}$ (because voters 1 and 3 prefer $\bar{y}$) and q_x and q_y are individually stable. Hence $Q^* = \bar{x}\bar{y}$, but Q^* need not be the social choice because Q is unstable in the cycle: $xy\,P\,\bar{x}\bar{y}\,P\,\bar{x}y\,P\,x\bar{y}\,P\,xy$. Indeed $C(xy, x\bar{y}, \bar{x}y, \bar{x}\bar{y}) = (xy, x\bar{y}, \bar{x}y, \bar{x}\bar{y}) = Q$, which means that the outcome may be *any* of the possible ones. Voters 1 and 2 can trade, so each gets his second best, xy. (That is, 1 votes sincerely for x and strategically for y, while 2 votes strategically for x and sincerely for y.) But voter 3 can perhaps upset this outcome, which she most dislikes, by trading with 2. If 2 agrees to abandon his trade with 1, to continue to vote sincerely for y, and to change his strategic vote for x to a sincere vote for $\bar{x}$, then 3 can vote sincerely for $\bar{x}$ and strategically for y, thus producing $Q = \bar{x}y$, which both like better than $Q = xy$. And so on around the cycle. What outcome will eventually occur is just a matter of where trading happens to stop. So anything can happen and Q is wholly unstable.

It is a remarkable fact that Q is unstable even if there is no cycle. In an example in Display 6-6, adapted from Thomas Schwartz, under sincere voting $Q^* = \bar{x}\bar{y}$ and furthermore $\bar{x}\bar{y}\,P\,x\bar{y}\,P\,\bar{x}y\,P\,xy$ so there is no cycle.[19] But Q may be unstable. Voters 1 and 3 may trade in an attempt to produce $Q = xy$. That is, 1 promises to vote sincerely for x and strategically for y while 3 promises to vote strategically for x and sincerely for y. Assuming 2 votes sincerely for x and 4 votes sincerely for y, then the outcome is $Q = xy$ (because 1, 2, and 3 voted for x and 1, 3, and 4 voted for y). Of course this can be upset by a trade of 2 and 4 to return to $Q = \bar{x}\bar{y}$, which both prefer to xy (that is, 2, 4, and 5 vote for $\bar{x}$ and $\bar{y}$).

Display 6-5

A Cyclic Situation for Vote-Trading

In each column is the ordinal ordering for that voter of outcomes on x and y. The most highly valued outcome is at the top of the column, the least highly valued at the bottom.

Voter 1	Voter 2	Voter 3
$x\bar{y}$	$\bar{x}y$	$\bar{x}\bar{y}$
xy	xy	$\bar{x}y$
$\bar{x}\bar{y}$	$\bar{x}\bar{y}$	$x\bar{y}$
$\bar{x}y$	$x\bar{y}$	xy

By sincere voting, the outcome is $\bar{x}\bar{y}$ because voters 2 and 3 vote for $\bar{x}$ and voters 1 and 3 vote for $\bar{y}$. Since $\bar{x}\bar{y}$ is less preferred than xy by 1 and 2, they can agree to vote for x (strategically for 2) and y (strategically for 1) to produce xy.

Display 6-6

The Instability of Vote-Trading Even with a Condorcet Winner

Voter 1	Voter 2	Voter 3	Voter 4	Voter 5
$x\bar{y}$	$x\bar{y}$	$\bar{x}y$	$\bar{x}y$	$\bar{x}\bar{y}$
xy	$\bar{x}\bar{y}$	xy	$\bar{x}\bar{y}$	$x\bar{y}$
$\bar{x}\bar{y}$	xy	$\bar{x}\bar{y}$	xy	$\bar{x}y$
$\bar{x}y$	$\bar{x}y$	$x\bar{y}$	$x\bar{y}$	xy

The Condorcet winner is $\bar{x}$ (by 3, 4, and 5) and $\bar{y}$ (by 1, 3, and 4). But 1 and 3, both of whom prefer xy to $\bar{x}\bar{y}$, may trade to produce xy.

Depending on where the trading stops, the Condorcet winner may lose. In general, however, if there is a Condorcet winner in $(xy, x\bar{y}, \bar{x}y, \bar{x}\bar{y})$, and if voters have perfect information, then members of the majority supporting the Condorcet winner in Q can, by individual *sophisticated voting* (that is, by voting in anticipation of votes and trades by others) force the choice of the Condorcet winner.[20] And, of course, that same majority can also enforce the Condorcet winner by a coalition. Still, if information is imperfect and trading is costly, it may be hard for the majority to counter minority trades. And if so, even the Condorcet winner may be strategically upset.

Even demand-revealing voting, which was invented to render strategic voting impossible by individuals, is subject to manipulation by coalitions in a way comparable to vote-trading.[21] (See Chapter 3, note 18.) Since votes are cast in terms of money, vote-trading is by exchange of money. An example is set forth in Display 6-7, where voter 1, for example, can conspire with voter 3 to upset the "honestly revealed" victory for y.[22] The method is intended to prevent this because each voter is supposedly motivated by the tax to vote his or her true tastes. If voter 1, for example, increases his reported valuation of x to \$7 to make x win, his tax would be \$5 or his marginal contribution to victory for x. (That is, z, the winner without 1's vote, is valued by others at \$5 and x is valued by others at zero. To make x win, voter 1 must bid more than \$5. Hence the tax is \$5 − 0 = \$5, or 1's marginal contribution.) Since \$5 is all that x is worth to 1, it is apparently pointless for him to exaggerate his valuation of x. So, with truthful valuation, each voter gets a net value of \$1 from the choice of y, as in Part A of the Display.

But if x were to win, voter 1 could make \$5, part of which he would use to bribe voter 3, say, to make x win. Thus it is not pointless to trade votes. If voter 1 gives voter 3 a bribe of \$2.50 to report a valuation of, say, \$4 for x (which eliminates the tax), then x wins, each makes \$2.50 by vote-trading, and both are indeed better off. Other coalitions are possible (for example, to make z win), but it is always possible for at least two voters to be better off from the choice of x or z than from the choice of the "true" winner, y.

In this example, vote-trading makes a majority better off. This is, however, an unusual (even bizarre) method of voting, and the improvement may be untypical of ordinary methods. Nevertheless, under ordinary procedures, vote-trading can sometimes improve outcomes for a majority. Schwartz offers a good example, which is set forth in Display 6-8.[23]

Without trading, voters 1 and 3 choose $\bar{x}$ and voters 1 and 2 choose $\bar{y}$, so $Q^* = \bar{x}\bar{y}$. Yet $Q = xy$ is the Condorcet winner (preferred by 2 and 3 to Q^*) and the social order is transitive: $xy\ P\ \bar{x}\bar{y}\ P\ \bar{x}y\ P\ x\bar{y}$. The Condorcet winner cannot be reached by sincere voting; but, if voters 2 and 3 trade (with 2 voting strategically [yea] on y and 3 strategically [yea] on x), then both x and y pass and a majority is clearly better off.

On the other hand, vote-trading can produce outcomes that are universally undesirable—outcomes, indeed, that everyone is strongly motivated to reach but that make everyone worse off than sincere voting. This occurs when issues are voted on serially and when the costs for losers are large—a common situation in most legislatures. The root of this disaster is that vote-trading imposes an external cost on nontraders.

If we think of an external cost as the cost a person suffers through no action of his or her own (like the cost to the pedestrian who suffers from auto exhaust), then the nontrading voter (who is changed from a winner to a loser by others' trading) innocently suffers a cost externally imposed. Conceivably the gains a member makes on trades might outweigh any external costs. But just as conceivably, the sum of the external costs may exceed the gains from trade. If that happens to everybody, then everybody is worse off from trading.

This kind of outcome seems especially characteristic of contemporary legislatures, which so often spend more than they tax, thereby generating inflation that hurts everybody. They do so because by vote-trading they create small benefits such as subsidies, public works, and fixed prices; yet the inflation that the excess cost of all these benefits creates is suffered by everybody.

Steven Brams and I have worked out a highly simplified example of such self-destructive trading, using cardinal utilities, three voters (i = 1, 2, 3), and three pairs of issues (x and y, w and z, and t and v) that are voted on serially so that trades made on prior issues cannot be undone at the time of later trades. A different member is left out of the trading on each pair of issues. The detail is set forth in Display 6-9.[24] Considering just motions x and y, $Q^* = xy$ and the sum of the utilities for each voter, U_i, from sincere voting will be:

$$U_1(x, y) = 1 + 1 \quad = 2$$

$$U_2(x, y) = 1 + (-2) = -1$$

$$U_3(x, y) = (-2) + 1 = -1$$

Display 6-7

Strategic Manipulation with Demand-Revealing Voting

A. Truthful Voting (Without Vote-Trading)

Voters	Valuation of alternatives			Other voters	Sum of valuations without i's valuation			Tax* on i	Net benefit from choice of y (Valuation less tax)
	x	y	z		x	y	z		
$i = 1$	$5	2	0	2, 3	$0	4	5	$1	$2 − 1 = $1
$i = 2$	0	2	0	1, 3	5	4	5	1	2 − 1 = 1
$i = 3$	0	2	5	1, 2	5	4	0	1	2 − 1 = 1
Sum	$5	6	5						

Note. Alternative y wins.

*The tax is i's marginal contribution to making the winning alternative (here y) at least tie with any other. Thus, without voter 1, z wins. So 1 pays a tax of [(valuation of z without 1's valuation) less (valuation of y without 1's valuation)] or $5 − 4 = $1.

B. Strategic Voting (with Vote-Trading)

Voters	Valuation of alternatives (False by voter 3)			Other voters	Sum of valuations without i's valuation			Tax** on i	True benefit from choice of x	Bribe by voter 1	Net benefit
	x	y	z		x	y	z				
$i = 1$	$5	2	0	2, 3	$4	4	4	0	$5	$−2.50	$2.50
$i = 2$	0	2	0	1, 3	9	4	4	0	0	0	0
$i = 3$	4	2	4	1, 2	5	4	0	0	0	+2.50	2.50
Sum	$9	6	4								

Note. Alternative x wins.

**Voter 1 is not assessed a tax because, without 1's valuation, x and z tie. Voters 2 and 3 are not assessed because x wins without 2's or 3's valuation.

Display 6-8

Vote-Trading Improves the Outcome for a Majority

Voter 1	Voter 2	Voter 3
$\bar{x}\bar{y}$	$x\bar{y}$	$\bar{x}y$
xy	xy	xy
$\bar{x}y$	$\bar{x}\bar{y}$	$\bar{x}\bar{y}$
$x\bar{y}$	$\bar{x}y$	$x\bar{y}$

Without trading, $\bar{x}\bar{y}$ wins. If 2 and 3 trade, xy wins and xy is better than $\bar{x}\bar{y}$ for both 2 and 3.

Of course, 2 and 3 can trade, 2 voting (strategically) for $\bar{x}$ and (sincerely) for $\bar{y}$ and 3 voting (sincerely) for $\bar{x}$ and (strategically) for $\bar{y}$. Then $Q = \bar{x}\bar{y}$ and the sum of the utilities will be:

$$U_1(\bar{x}\bar{y}) = (-2) + (-2) = -4$$

$$U_2(\bar{x}\bar{y}) = 2 + (-1) \qquad = 1$$

$$U_3(\bar{x}\bar{y}) = 2 + (-1) \qquad = 1$$

Thus, 2 and 3 have a net gain of two units and 1 a net loss of six units. Considering just this pair, x and y, voters 2 and 3 have a powerful motive to trade and so presumably they do so. As x and y are settled and voters go on to w and z, voters 1 and 2 now trade and 3 suffers an external cost. So also with t and v: 1 and 3 trade and 2 suffers. The net effect of all the trading is as follows:

Voters	Outcomes $\bar{x}\bar{y}$	$\bar{w}\bar{z}$	$\bar{t}\bar{v}$	Total
1	-4	$+1$	$+1$	-2
2	$+1$	$+1$	-4	-2
3	$+1$	-4	$+1$	-2

Display 6-9

The Paradox of Vote-Trading: Cardinal Utilities for Outcomes on Motions

Voters	Preferred outcome	Cardinal utility: Motion wins	Cardinal utility: Motion loses	Preferred outcome	Cardinal utility: Motion wins	Cardinal utility: Motion loses
		Motion x			Motion y	
1*	x	1	-2	y	1	-2
2	x	1	-1	$\bar{y}$	-2	2
3	$\bar{x}$	-2	2	y	1	-1
		Motion w			Motion z	
1	w	1	-1	$\bar{z}$	-2	2
2	$\bar{w}$	-2	2	z	1	-1
3*	w	1	-2	z	1	-2
		Motion t			Motion v	
1	$\bar{t}$	-2	2	v	1	-1
2*	t	1	-2	v	1	-2
3	t	1	-1	$\bar{v}$	-2	2

*Nontrader on the pair of motions.

Had these voters been able to foresee the sorry consequences, they would perhaps have exercised greater self-control, avoided trading, and produced:

Voters	Outcomes xy	wz	tv	Total
1	$+2$	-1	-1	0
2	-1	-1	$+2$	0
3	-1	$+2$	-1	0

They would all have been better off with sincere voting than with strategic voting. But, given that these are serial events, voters would not dare pass up a possible trade. Suppose a voter, trying for the sincere outcome, refused to trade. Then 1, for example, would refuse trades on w and z and t and v. If 2 and 3 trade on x and y, however, the result is

	Outcomes			
Voters	$\bar{x}\bar{y}$	wz	tv	Total
1	−4	−1	−1	−6
2	+1	−1	+2	+2
3	+1	+2	−1	+2

So 1 would be worse off for not trading, when others traded, than if he too had traded at every chance. Hence 1 has a strong motive to trade, even though he knows it will hurt. Of course, 1 might persuade others to agree not to trade; but, as in any cartel, agreement is difficult to enforce because great advantages accrue to those who break the agreement. Since the positions of the players are identical over the three pairs of issues, universal trading is to be expected, *even though everyone knows it will make everyone worse off.*

There is, of course, one way to avoid this devastating outcome. Some pair, say 2 and 3, can form a coalition to trade when they can with each other and vote sincerely otherwise. Then the members of the pair each gain two units and the outsider loses six. Coalitions of this sort are possible if the voters have perfect information about utilities and about the actual issues that will arise, and if the costs of organization are less than the possible net gains.[25] But in the real world of legislatures, future issues are unknown and, even if guessed at, the payoff to other members on these potential future issues is doubtless obscure. Hence when members trade on x and y, they may not know that w and z will arise. Moreover, in legislatures of more than a few members, party discipline—that is, coalitional cohesion—over time is difficult and expensive to maintain. In decentralized governments like the United States it is impossible. Since members of a coalition (party) depend for reelection almost entirely on people in their home districts rather than on other members of the legislature, leaders of a legislative coalition can neither reward nor punish its members. Hence while people may not hurt themselves by vote-trading in the small-group laboratory, they are quite likely to do so in the real world.

To summarize the discussion of vote-trading, I want to make these observations:

1. Vote-trading enormously expands the potential for strategic voting in legislatures.
2. One manner in which vote-trading works is by generating cyclic situations where none existed before.
3. Vote-trading is also possible regardless of whether Q is a Condorcet winner; vote-trading may even produce another outcome when there is a Condorcet winner, although with perfect information and costless trading the Condorcet winner can always be chosen.
4. Cases exist in which vote-trading improves Q for a minority and even for a majority. Possibly more frequently in the real world, cases also exist in which vote-trading makes more people worse off than are made better off. And there are doubtless occasions in which vote-trading makes *everybody worse off.*

6.F. The Ineradicability of Strategic Voting

Strategic voting is an ineradicable possibility in all voting systems. It is not possible for me to estimate how frequently chances to vote strategically occur; but I am sure they are frequent in popular elections and, by reason of vote-trading, almost always present in legislatures. And even if parties are disciplined in order to prevent vote-trading, still vote-trading doubtless occurs inside cabinets and committees.

The factual question of whether people take advantage of these opportunities is difficult to answer. To identify strategic voting requires that we know both the voter's true values and the voter's actual expression of the values in a vote. From direct observation we can know only the latter. We must infer the former from other and softer evidence. Nevertheless, considering the real-world examples I have offered, it does seem likely that strategic voting occurs quite frequently.

If it does, I conclude that the meaning of social choices is quite obscure. They may consist of the amalgamation of the true tastes of the majority (however "majority" and "amalgamation" are defined), or they may consist simply of the tastes of some people (whether a majority or not) who are skillful or lucky manipulators. If we assume social choices are often the latter, they may consist of what the manipulators truly want, or they may be an accidental amalgamation of what the manipulators (perhaps unintentionally) happened to produce. Furthermore, since we can by observation know only expressed values (never true values), we can never be sure, for any particular social choice, which of these possible interpretations is correct.

Many philosophers, partially aware of these difficulties, have argued that strategic voting and vote-trading ought to be prohibited. This is shortsighted, of course, because sometimes strategic voting reveals Condorcet winners that would otherwise lose.[26] But since strategic voting does often produce "worse" situations, it is frequently condemned. When it was pointed out to Borda that strategic voting could distort outcomes, he replied, "My scheme is intended only for honest men."[27] Unfortunately, we have no test for honesty. If a man *tells* me his vote reflects his true tastes, how can I prove he perjures himself?

Dodgson (Lewis Carroll) had a clearer view than Borda. Seeing that strategic voting "makes an election more a game of skill than a real test of the wishes of electors," his solution was that "all should know the rules by which this game may be won."[28] Thereby he displayed the correct belief that strategic voting *cannot* be eliminated. What he did not recognize, but what we now know, is that strategic voting renders the meaning of *all* social choices obscure. I emphasize *all* because we never can know, on any particular outcome, how much or what kind of strategic voting occurred.

7

The Manipulation of Social Choices: Control of the Agenda

Strategic voting is usually unobservable, and its incidence is therefore not calculable. In contrast, agenda control, another kind of manipulation, is often obvious, and its incidence can in principle be estimated. This incidence is surely very high, for the consequences of agenda control are apparent in some degree in the content of almost all social choice. The significance, variety, and pervasiveness of agenda control are partially revealed simply by a categorization of the kinds of controls exercised. One category is control by formal leaders.

In any organized decision-making body, committee, legislature, or whole government, one function of leaders is to guide the operation of the body. For a body as ad hoc as a jury, the judge instructs, setting limits to the agenda; for a committee, a chairperson or agenda committee guides the order of business; for a legislature, partial control resides in chairpersons (speakers), party and coalition leaders, agenda committees (such as rules committees), executives (premiers and presidents) who propose bills; and so on for nations, corporations, universities, and other bodies. Despite institutional variations, the leaders in every such body must select alternatives among which decisions will be made, and they must select the procedures for coming to a choice. Typically, of course, leaders work within customs, constitutions, and other constraints; typically, also, they have some freedom of choice.

Often leaders are constrained by procedures that allow the whole body itself to change leaders' selections. This means, simply, that in voting bodies there is a way for backbenchers to seize control and become leaders themselves. Thus, party leaders may endorse candidates, but the final decision on nominations is made in primary elections or conventions. A chairperson may rule on whether a motion is in order, but the ruling is often subject to appeal to the floor.

Despite these restrictions, however, leaders' control of agenda is ordinarily not challenged. One reason is that most bodies have customary criteria of fairness, and most leaders abide by them. For example, they accept for consideration alternatives they oppose when it is apparent some members are in favor of them. They use customary routines, bending them perhaps, but seldom devising new routines blatantly designed to get their own way. In the absence of extreme "unfairness," therefore, leaders' control is seldom challenged. But there is a subtler reason for this lack of conflict: To challenge leaders on agenda and procedures is to challenge their leadership itself. So, unless there is an intent to dethrone or displace them, leaders usually have some, though constrained, control of the content of the set, X, of alternatives and of the process by which a choice is made from it.

Thus one category of methods of agenda control is defined in terms of leaders' opportunities to dominate. There is another category, however, that admits control by ordinary members: They can generate new alternatives. Leaders officially define X, but the definition is in terms of what the surrounding culture suggests, and ordinary members can determine the content of that culture. This method of control is, of course, open to leaders also; but typically it is the method used by nonleaders, mainly because it is the only one easily open to them. Furthermore, nonleaders are often losers on recent agenda and so have a compelling motive to generate a new agenda on which they have a chance, at least, to win.

To summarize: Agenda control consists on the one hand of chosen leaders actually leading and on the other hand of nonleaders developing new alternatives. This chapter consists of a brief survey of the infinite variety subsumed in these two categories.

7.A. The Universality of Agenda Control

If, given a profile of preference, there is some outcome potentially in a strong equilibrium (in the sense that, for the set Y of all possible alternatives, some alternative is a first-place choice for a majority of voters); and if voters have enough information to know that such an outcome could be arrived at, then it is probably difficult to manipulate an agenda. (Later in this chapter I will sometimes use "equilibrium" in a weaker sense to mean merely a Condorcet winner from a set, X (which is a subset of Y) such that this winner, under sincere voting, can beat each other alternative in X individually, but not all others in X simultaneously.)

Since the conditions for a strong equilibrium are difficult to fulfill, manipulation of the agenda is not usually precluded because of the distribution of tastes. For example, with majority voting, there is often a Condorcet winner without an absolute majority of first-place preferences in $X = (a, b, c)$, as in this profile:

D	
1	$a\,b\,c$
2	$b\,a\,c$
3	$c\,a\,b$

This condition, of course, admits strategic voting. Although I do not know of any way, by individual manipulation, to beat an absolute majority in the static case (where X is fixed) with majority voting, it is easy enough to do so in the dynamic case (where X is not fixed), by vote-trading. One can enlarge X and thereby transform D^1 into D^2:

D^1		
1	2	3
ab	ab	$\bar{a}b$
$\bar{a}\bar{b}$	$\bar{a}b$	$\bar{a}\bar{b}$
.	.	.
.	.	.
.	.	.

D^2		
1	2	3
abc	$ab\bar{c}$	$\bar{a}bc$
$\bar{a}\bar{b}c$	$\bar{a}b\bar{c}$	$\bar{a}\bar{b}\bar{c}$
.	.	.
.	.	.
.	.	.

Although in D^1 ab seems invulnerable, in D^2 persons 2 and 3 can trade to produce $\bar{a}b\bar{c}$. Likewise, in Borda voting even an absolute majority is vulnerable to agenda control, as in profile D,

D	
D_1–D_3:	$a\,b\,c$
D_4, D_5:	$b\,a\,c$

with these point sums: $a = 8$, $b = 7$, $c = 0$. Here a wins with 8 points and beats each other alternative in pairwise contests with 3 or 5 points.

If an additional alternative, d, is added to X to make it X', and if D becomes D',

D'	
D'_1–D'_3:	$a\,b\,d\,c$
D'_4, D'_5:	$b\,d\,a\,c$

with these point sums—$a = 11$, $b = 12$, $c = 0$, $d = 7$—then the winner is changed from a to b.

Equilibrium is thus a highly restrictive notion, probably rarely satisfied in nature. Of course, it is satisfied if X has just two members and no additions are allowed, (this is why simple majority voting is attractive). But such an X is usually the result of narrowing down a larger set, Y, not itself in equilibrium. Even if a given profile over a larger Y does have an equilibrium outcome, leaders still can manipulate the agenda to prevent its adoption if voters do not know this outcome exists. This is one reason why ordinary members like, and leaders dislike, straw votes, opinion polls, and test votes. Such trial runs often reveal equilibria, if they exist, and, once revealed, they cannot be prevented. Fortunately for leaders it is often difficult to conduct trial runs, and thus leaders may procure a nonequilibrium outcome simply because voters never discover that an equilibrium potentially existed.

Since, therefore, the conditions that preclude manipulation (that is, equilibrium *and* information about it) are difficult to fulfill, manipulation must usually be possible. Furthermore, it is possible under *any* method of voting. We know that no fair method of voting can satisfy the logical criterion of path independence. This criterion (see section 5.F) is the requirement that, regardless of how alternatives are divided up for a serial decision process, the same choice will result from all possible divisions.[1] Since no nonoligarchic method of voting satisfies this criterion, in any democratic procedure that operates by successive reduction of alternatives manipulation of the agenda is possible simply by choosing one division of alternatives into subsets for selection rather than another division. Furthermore, even for positional methods that operate on the entire set X simultaneously, expansion of the number of alternatives to form X' permits inconsistencies in subset expansion that also violate path independence. Consequently, just as the inability of any fair method of voting to guarantee social acyclicity permits strategic voting, so the inability of any fair method to guarantee path independence permits agenda manipulation. Since manipulation can occur under *any* method of voting and *most* profiles of preference, we can assume that it does occur sometimes.

7.B. Examples of Agenda Control

From the earliest recorded history of voting bodies we know that some kind of manipulation has been standard practice. In the Athenian assembly, choice by lot was often substituted for choice by election because, with nonsecret voting, great men controlled their clients' choices. This crude manipulation was common until the end of the nineteenth century in popular elections and is still the basis of party discipline (as it is euphemistically called) in legislatures.

Much more interesting than mere physical control of the voter is control of the agenda in such a way that voters are constrained to vote as the manipulator wishes. This kind of control, though ancient and universal, is not always easy to observe. So I will here set forth two examples of conscious control of the agenda, one by Pliny the Younger in the Roman Senate, the other by Charles Plott and Michael Levine in a contemporary laboratory.

In the Roman Senate

Pliny the Younger tells of an instance of both kinds of manipulation, and Robin Farquharson used that tale as the continuing example throughout his *Theory of Voting*. Pliny presided over a case in the Roman Senate concerning the fate of the freedmen of a consul found killed in his house. The consul's slaves had already been condemned to death, and traditional law required execution of the freedmen also; but in this case there was reason for leniency because the consul may have been a suicide, or, if he was murdered, the freedmen may have acted from "obedience" rather than from "malice."

There were three factions in the Senate: A, for acquittal, which was a plurality but not a majority; B, for banishment; and C, for condemnation to death. Pliny favored acquittal. If he had put the question in the customary way, pairwise on guilt or innocence, B and C would have voted together, acquittal would have been eliminated, and then, in choice between banishment and death, A and B would have voted together and the ultimate outcome would have been banishment. Pliny foresaw that outcome and chose, therefore, to put the three alternatives simultaneously, requiring each senator to vote for just one, in the hope that A's plurality would result in acquittal. As it happened, the leader of faction C foresaw this consequence and strategically aligned his party with B so that, evidently, banishment prevailed.[2] Thus, the presiding officer's attempt at controlling the agenda was foiled by the strategic voting of an opponent—convincing evidence that knowledge about manipulation is nearly as old as voting bodies themselves.

Pliny did not recognize the frequency of manipulation, partly because the Senate had just been reactivated under Emperor Trajan and partly because no science of politics then existed by which knowledge of such behavior could be systematized. Today it is different. A substantial part of the commentary on legislatures by political scientists concerns techniques for controlling agendas by presiding officers (for example, speakers, of whom the American prototype was Speaker "Czar" Reed, and party leaders and premiers, who preside in fact if not in name), by agenda committees (such as rules committees, cabinets, and even more substantive bill-writing committees), and by extra-legislative bodies that write bills and select topics for legislation (for example, specialized bodies like the Office of Management and Budget in the United States and the Treasury in Great Britain and diffuse bodies like the bureaucracy, parties, and lobbies).

It would be burdensome here to repeat or even summarize that huge literature. It is enough to point out its immediate relevance to this discussion. Nevertheless, I will describe in some detail an experiment on the control of an agenda, which is especially interesting because it throws light on a difficult epistemological question that often arises in reflection on natural events.

In a Contemporary Laboratory

In reflecting on the example from Pliny, it might be argued that banishment would have resulted in any case because it was the Condorcet winner and that the assertion about manipulation is merely *my* attribution of intent based on *my* interpretation of the motion and the voting. A similar difficulty arose in section 6.C, where the observer could not be absolutely certain that the cyclic outcome on school construction was in fact contrived. Since all historical interpretation of manipulation requires an attribution of motives, an attribution that cannot be directly validated, it is always possible to assert that apparent manipulation is the product not of actors' conscious intent, but of observers' overactive imagination. If, however, one can display instances in which experimenters consciously and successfully used different agendas on identical profiles of preference to produce different outcomes, then one has more confidence in historical

interpretations. Such an experiment was conducted almost exactly for these reasons by Charles Plott and Michael Levine.[3]

They came to the problem out of a practical interest in influencing the composition of a new fleet of aircraft for a flying club. One of them, who was chairman of the agenda committee, had interests somewhat different from the interests of most members. While nearly everybody wanted a fleet composed basically of moderate-cost (type E) or more expensive (type F) four-seat planes, he wanted in addition a secondary fleet of two (or at least one) moderate-cost six-seat planes (type C) and no type A, an expensive six-seat plane. After carefully estimating the tastes of members based on earlier discussions and questionnaires, Plott and Levine devised an agenda that serially narrowed consideration in such a way that A's were eliminated and C's survived: The chosen fleet was seven planes—five E's and F's and two C's. They believed that their agenda had an influence because a questionnaire subsequently sent to the members at the meeting revealed that the Condorcet winner was a fleet of six E's and F's, and one A.

Nevertheless, in the absence of certain knowledge about the predecision profile of preferences, they could not be absolutely assured that they had influenced the outcome. To settle their doubts, they then carried through a data-generating experiment in which they used different agendas for partitioning a set of five (wholly abstract) alternatives, $X = (1, 2, 3, 4, 5)$, for committees with exactly the same induced preferences. The goal of the experiment was to show that they could use four agendas to produce four different social choices from X in four identically motivated human groups. The experiment was successful enough to show fairly conclusively that conscious manipulation could change outcomes.

The agendas consisted of partitioning X into two complementary subsets, S and $\overline{S}$, of partitioning each of these into complementary subsets, etc., until a single alternative was reached. For example, one agenda might be:

Step 1: Choose between $\{2, 4\}$ and $\{1, 3, 5\}$.

Step 2a: If $\{2, 4\}$ is chosen, choose between $\{2\}$ and $\{4\}$.

Step 2b: If $\{1, 3, 5\}$ is chosen, choose between $\{1, 5\}$ and $\{3\}$. If step 2a is used or if $\{3\}$ is chosen at step 2b, the process is complete. Otherwise go on to step 3.

Step 3: If $\{1, 5\}$ is chosen at step 2b, choose between $\{1\}$ and $\{5\}$.

This agenda and another are depicted schematically in Figure 7-1. Altogether, with five alternatives, 105 distinct partitioning agenda are possible

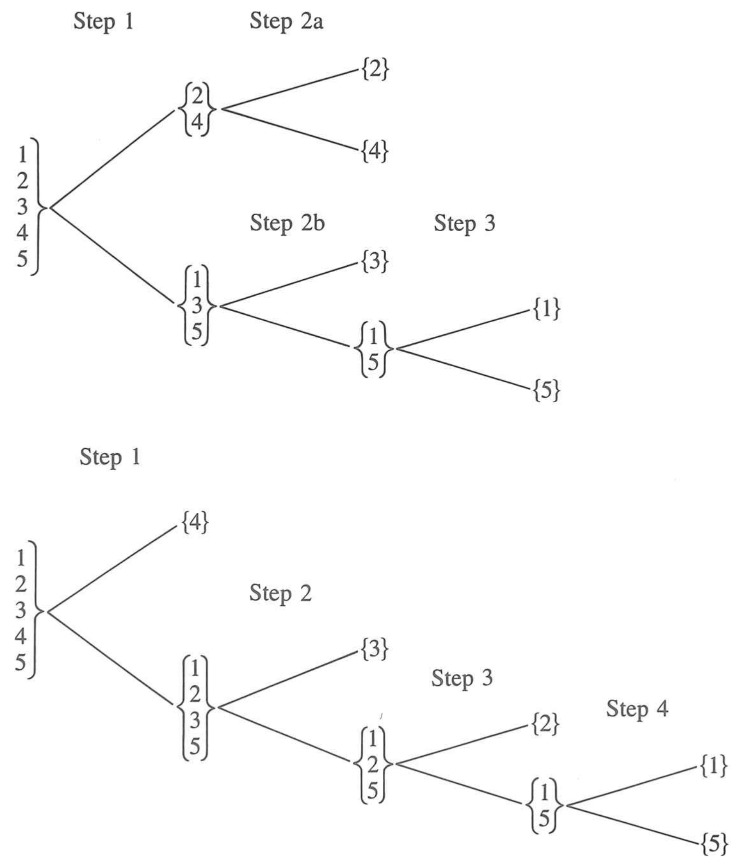

Figure 7-1 Two possible agendas.

(not counting 30 others that could be constructed for the amendment procedure).

Although actual subjects (college students) in each trial were different, the same profile of preferences was induced in each group by the promise of a money payment for the choice of each alternative. For example, person 1 in each group was told to expect

$$\left.\begin{array}{r}\$6.00\\ \$7.00\\ \$5.00\\ \$8.00\\ \$\ .50\end{array}\right\}\text{ if the group chose alternative }\left\{\begin{array}{l}1\\2\\3\\4\\5\end{array}\right.$$

Display 7-1

Payoffs in Dollars in the Plott-Levine Experiment

	Alternative				
Person	1	2	3	4	5
1	6.00	7.00	5.00	8.00	.50
2	6.00	7.00	5.00	8.00	.50
3	6.00	7.00	5.00	8.00	.50
4	6.00	7.00	5.00	8.00	.50
5	6.00	7.00	5.00	8.00	.50
6	6.00	7.00	5.00	8.00	.50
7	7.50	7.75	6.75	5.75	.25
8	7.50	7.75	6.75	5.75	.25
9	7.50	7.75	6.75	5.75	.25
10	7.50	7.75	6.75	5.75	.25
11	7.50	7.00	6.00	8.00	.50
12	8.00	7.50	7.00	6.00	.50
13	8.00	7.50	7.00	6.00	.50
14	8.00	7.50	7.00	6.00	.50
15	7.00	5.50	7.50	6.50	.25
16	7.00	5.50	7.50	6.50	.25
17	7.00	5.50	7.50	6.50	.25
18	7.00	5.50	7.50	6.50	.25
19	7.00	5.50	7.50	6.50	.25
20	7.00	5.50	7.50	6.50	.25
21	7.00	5.50	7.50	6.50	.25

Source. Charles Plott and Michael Levine, "A Model of Agenda Influence on Committee Decisions," *American Economic Review,* Vol. 68 (March 1978), pp. 146–160.

The entire schedule of payoffs for the 21 members is set forth in Display 7-1. Assuming each person votes sincerely, in pairwise contests, for the alternative paying more for him or her, the Condorcet winner is alternative 1, which beats 2 by 11 to 10, 3 by 14 to 7, 4 by 14 to 7, and 5 by 21 to 0. Alternatives 2, 3, and 4 cycle because 2 beats 3 by 14 to 7, 3 beats 4 by 14 to 7, and 4 beats 2 by 14 to 7. Alternative 5 always loses.

But, of course, even in pairwise contests, voters can vote strategically; so the Condorcet winner need not really win, unless all voters have perfect information and all use it to vote strategically. In the more complicated serial partitions of Figure 7-1, there is even less likelihood of choosing the Condorcet winner. So the next step in construction of the experiment is to predict the choice under any given agenda (all 105 of them), assuming that each voter knows *only* his or her own payoffs. This prediction requires assumptions about the way voters behave. Plott and Levine suggest three rules of behavior, all of which appeared to be used by some subjects:

> Rule 1: Sincere strategy. Vote for the set, S or $\overline{S}$, that contains the alternative with the highest payoff.
>
> Rule 2: Avoid-the-worst strategy. Vote for the set, S or $\overline{S}$, that does *not* contain the alternative with the lowest payoff.
>
> Rule 3: Highest-expected-value strategy. Vote for the set, S or $\overline{S}$, in which the alternatives have the highest average value.

Notice that these strategies do not necessarily lead to the same action by a voter. For example, if $S = (1, 4, 5)$ and $\overline{S} = (2, 3)$, voter 1 should choose S by rule 1 and $\overline{S}$ by rules 2 and 3. Which of these—and other—strategies are actually used is, of course, an empirical question. So Plott and Levine estimated probabilities, from data gathered in pretesting,

1. That S would be chosen over $\overline{S}$ when all three rules prescribed S or one (or two) prescribed S and the other one (or two) were ambiguous in that it (they) prescribed indifference between S and $\overline{S}$
2. That S would be chosen when one rule prescribed S, another $\overline{S}$, and the third was ambiguous
3. That S would be chosen over $\overline{S}$ when rule 1 prescribed S and 2 and 3 $\overline{S}$

Etc., through all nine possible cases.

Using this empirical data, they calculated, for each agenda, first the probability that S would be chosen over $\overline{S}$ by majority rule and then the probability that some y in X would be finally chosen. This number, $P(y/a)$, the probability y is chosen given a particular agenda, a, they called the "strength of the agenda for y." They then selected four agendas each with unit strength (that is, maximum strength), $P(y/a) = 1$, for $y = 1, 2, 3, 4$. Thus they would, if they succeeded, induce the subjects to choose four different alternatives with four different agendas and identical preference profiles.

The actual experiment was conducted in the following way. On each trial, 21 subjects assembled in a classroom were given detailed written instructions, including, for each, his or her own payoffs but no one else's. They were told that each person's reward would depend exclusively on the group choice of an alternative, that payoffs were different for different people, and that, while they should discuss which alternative to choose, they should under no circumstances reveal quantitative information about their own payoffs. A student chairman, uninformed about the purpose of the experiment, presided. He explained a typical predetermined agenda with its series of questions like "Do we want to consider further only the numbers 1 and 2 or only the numbers 3, 4, and 5?" He explained the rules of debate, closing debate, and voting, and he led the group through a practice agenda. Finally the group selected a winner according to the agenda for that trial.

The four agendas are shown in Figure 7-2. The predicted path is indicated with a boldface arrow, **→**, the predicted outcome with an asterisk, *, and the actual path (with vote record) with a sharp, #. Notice that, to predict for all contingencies, a boldface arrow is provided for each pair of sets, S and $\overline{S}$, even though it is *not* predicted that subjects would ever be in a position to choose between S and $\overline{S}$. As can be seen at the second and third stages of trial 2, the completeness of the prediction turned out to be useful.

As the figure shows, the predicted outcome was actually chosen on trials 1, 3, and 4. On trial 2, the Condorcet winner (alternative 1) was chosen, though alternative 4 was predicted. Plott and Levine attribute this failure to the fact that, early in the discussion, the chairman allowed a straw vote, which revealed that 5 was least preferred by everyone. Plott and Levine believe that this changed the agenda to that of trial 2′. Once S and $\overline{S}$ were changed by the straw vote before step 1, the path predicted by the "new" agenda was then followed. If one accepts this explanation, they succeeded completely; if not, then they succeeded three times out of four.

Richard Freedman and I repeated this experiment, but with only 11 subjects per trial, somewhat smaller payoffs, and similar agendas. We obtained similar results—three successes out of four. And our one failure had exactly the same explanation—a straw vote that enabled the group to arrive at the Condorcet winner when we did not intend that they should.

These results on controlling outcomes impress me deeply. They depend, probably, on the subjects' lack of perfect information. Given some quantitative information from the straw vote, they managed in both the experiment and its repetition to find the Condorcet winner. But in the world outside the laboratory, information is almost never perfect because

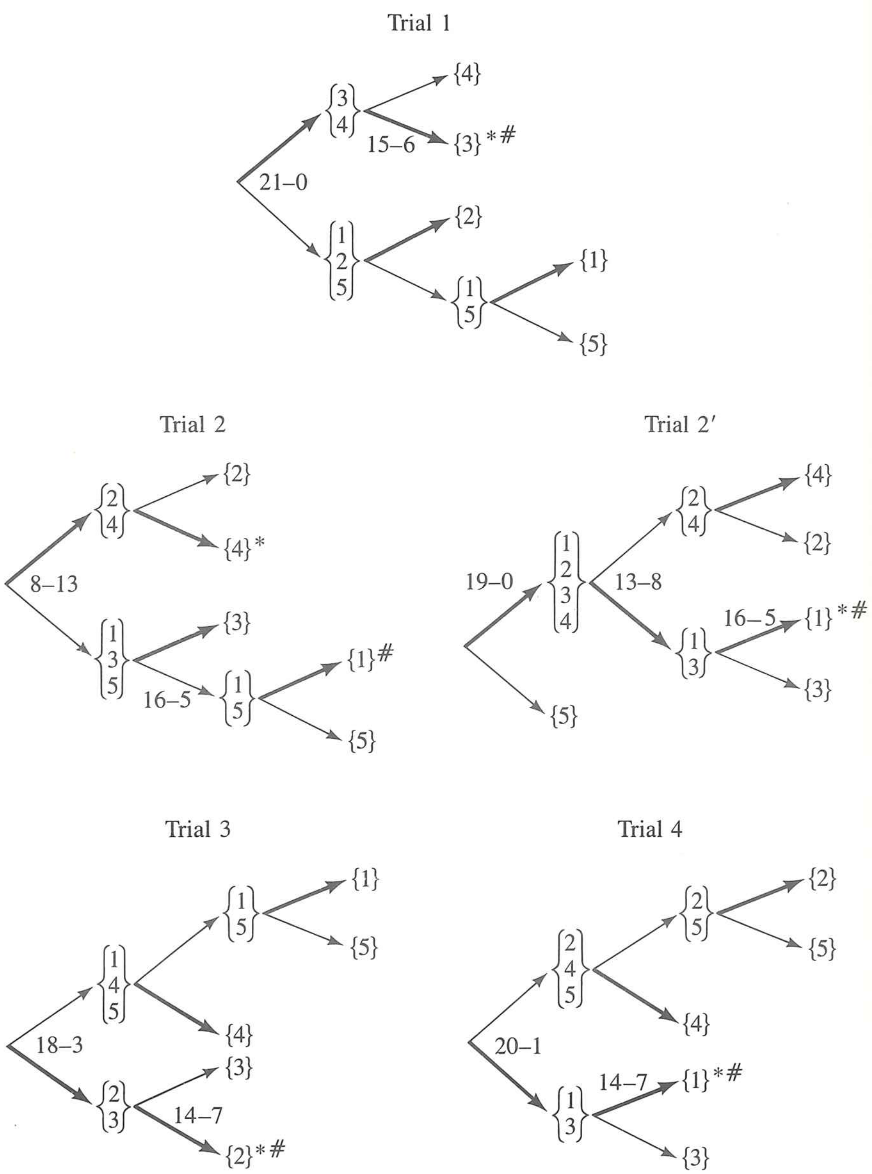

Figure 7-2 Agendas and outcomes. (Adapted from Plott and Levine, "A Model of Agenda Influence on Committee Decisions," *American Economic Review,* Vol. 68 (March 1978), pp. 146–160.)

rewards from choices are typically expressed ordinally, not cardinally, and almost never in dollars. My conclusion is, therefore, like Plott and Levine's, that it is demonstrably possible for social choices to be determined not by the true values of voters but by the manipulation by agenda-makers. If manipulation can occur so easily in the laboratory, we have good reason to believe it occurs in the larger world, where, indeed, allegations of manipulation are frequently uttered.

7.C. The Paucity of Equilibria

In the discussion in section 5.B of the frequency of profiles with and without a Condorcet winner, it was implicitly assumed that alternatives were discrete objects. If so, then the creation of new alternatives would often be difficult, especially in decision-making bodies having a rule that amendments must be germane to the main motion or in popular elections conducted under rules that favor the main parties. Although it is true that political alternatives often appear discrete, it may still be more realistic to think of them as continuous. Indeed Duncan Black's original idea that alternatives could be expressed as curves in two-dimensional space implies, by the very use of curves, that alternatives are continuous variables. Similarly, although Kenneth Arrow's theorem is proved with discrete mathematics, his verbal interpretation of alternatives as "social states," the objects of choice in his model, also implies continuous alternatives. He defined a "social state" as

> *a complete description of the* amount *of each commodity in the hands of each individual, the* amount *of labor to be supplied by each individual, the* amount *of each productive resource invested in each type of productive activity, and the* amounts *of various types of collective activity [emphasis added].*[4]

Note that every component of a social state is an amount and so continuous—apart from the lumpiness of some commodities.

Some alternatives are unquestionably continuous. A common subject of parliamentary motions is an appropriation or a tax rate, and money is for all practical purposes a continuous variable in alternative motions involving x dollars, $(x + \epsilon)$ dollars, and so on. In a less obvious way *all* sentences are embedded in a continuum. One of the significant assumptions of structural linguistics is the idea that a language can express an infinity of propositions. If so, then between a pair of sentences one can insert a third, intermediate in meaning. On a motion to declare war, the choice is not just between "war" or "peace." Amendments are possible: to declare war unless the enemy pays tribute, or unless an apology is uttered, or unless a sign of friendship is given, and so on. In that sense, propositions are continuous.

A quite separate tradition that utilizes and reinforces this belief in the continuity of alternatives is the spatial model of party competition. In Downs' model with only one dimension of judgment, voters' ideal points are arranged on an ideological scale and parties are shown to write platforms located at the median point of the scale, as in Figure 3-1, in section 3.G.[5] Since a platform is a sentence identifying a particular point on a line, this model amounts to an assertion that propositions are continuous. As this model was developed, especially for the case of n dimensions of judgment, the assumption of continuity was embedded even more deeply in it.[6]

One important feature of continuity is that it is relative to one or more dimensions. This feature is wholly realistic because survey researchers repeatedly find that voters appear to judge candidates and issues by one or more dimensions of concern. They say that they like candidate A better than candidate B "because A favors *more* [a quantitative word]" defense or inflation or employment or some constellation of such issues. Thus the very language of popular political discourse suggests that voters themselves impose the dimensions that admit continuity in the real world of political choice.

This continuity in a world of several dimensions is of utmost importance for strategic manipulation. It is this feature that allows for the easy multiplication of alternatives, that generates a wide variety of individual orderings, and that thereby creates situations of disequilibrium in which the chance of the existence of a Condorcet winner is reduced to practically zero.

Disequilibrium in Three Dimensions

Historically, the first discussion of disequilibrium was by Duncan Black and R. A. Newing, whose fundamental model is set forth in Figure 7-3, part A and part B.[7] Horizontal dimensions, x and y, are standards of judgment, and the vertical dimension is the order of preference. In the xy plane, the point, A, which is directly beneath the peak of the single-peaked hump (an analogue of the single-peaked curve), is the individual's ideal point—that is, the combination of amounts of x and y that the person prefers over any other possible combination. Since the hump is single-peaked, the farther away a point is from A on a ray, k, the less desired it is because it is beneath a lower point on the surface of the hump. Thus, $A\,P_i\,G\,P_i\,F$ in the figure. All the points at the intersection of the hump with the xy plane are an indifference curve, c, in the sense that any point on the curve is as good as any other for person i. Other indifference curves, c', can be reflected in the xy plane from parallel planes, like $x'y'$.

Assume there are three voters, 1, 2, and 3, and let Figure 7-3B consist of indifference curves for persons 1 and 2. Given the point, H, at the intersection of indifference curves for person 1 (ideal point at P_1) and person 2 (ideal point at P_2), any point, L, in the overlapping portion is preferred by both 1 and 2 to H because L is on an indifference curve closer for each person to P_1 and P_2 respectively. If there is any possibility of a majority decision consisting of persons 1 and 2 (out of 1, 2, and 3), that decision point must lie on a locus of points connecting the tangents of indifference curves around P_1 and P_2 as the dashed line in Figure 7-3C, often called the *contract curve* because it is where 1 and 2 can agree.

To appreciate agreement on this curve, assume persons 1 and 2 initially agree on some point, off the curve, say H in Figure 7-3C. Then, the pair can move to point K on the curve because (1) there is no cost to person 1 because H and K are on the same indifference curve for 1, and (2) there is a gain for person 2 because K is on a curve closer to P_2 than is H. Thus, when off the contract curve, one person, at least, can always be made better off with no cost to the other person. Hence, it is expected that agreements between 1 and 2 will lie on the contract curve.

To see if there is a Condorcet winner, we need to consider a third person's ideal point, P_3, in Figure 7-3D. It is apparent that no Condorcet winner exists in Figure 7-3D. For example, consider the contract curve between persons 1 and 3, or simply $P_1\,P_3$. If P_1 itself is tested, 2 and 3 vote against it, for any point on $P_2\,P_3$. If P_3, 1 and 2 defeat it for any point on $P_1\,P_2$. If it is H, then 2 and 3 prefer, say, K; and so on, so no point on $P_1\,P_3$ can get a majority. Similarly for $P_1\,P_2$ and $P_2\,P_3$. There are cases, however, in which some point might win by a majority, as in Figure 7-3E, where P_3 lies on $P_1\,P_2$. In this case P_3 is a majority winner. (Notice, for example, if P_3 is put against any point closer to P_1 on $P_1\,P_2$, then 2 and 3 will vote for P_3. Or if P_3 is put against H, then 1 and 3 will vote for P_3.) This argument assumes sincere, not strategic, voting. Even so, a Condorcet winner must be very rare indeed.

Gerald Kramer applied essentially this analysis to all the conditions for the existence of a Condorcet winner discussed in section 5.C—single-peakedness and value restrictedness, among others.[8] He assumed a multidimensional continuum and mathematically useful and humanly acceptable restrictions on utility functions. Then he showed that no Condorcet winner exists if there is even a modest divergence of tastes. These Condorcet winners cannot prevent voting paradoxes in the two-dimensional case unless there is something approaching a unanimous preference profile—for example, a majority of voters with exactly the same ideal

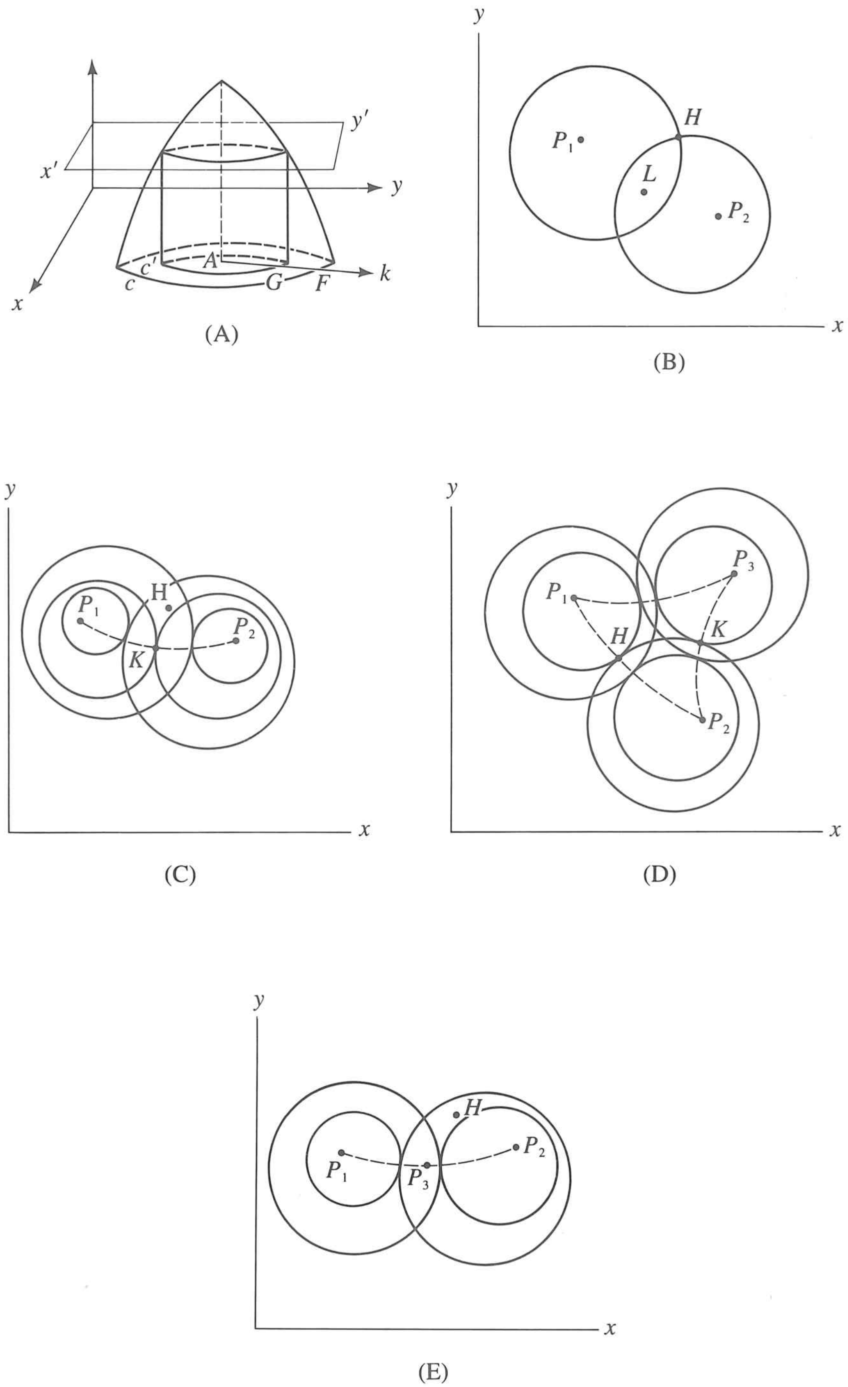

Figure 7-3 Multidimensional equilibria.

point. Continuous variables in two or more dimensions thus eliminate, for all practical purposes, the majority-rule conditions that seemed so promising in Chapter 5.

Conditions for Equilibrium

Reopening the discussion started by Black and Newing, Plott looked for a majority-rule equilibrium (of sincere voting) given continuous variables in two or more dimensions; that is, he sought a general definition of a point such as P_3 in Figure 7-3E. He found one in an extension of Black and Newing's suggestion for the three-person case.[9] A local equilibrium, μ, exists for m voters on a social choice in n dimensions, if the following sufficient conditions are satisfied:

1. Indifferent persons abstain from voting.
2. If n is odd, where n is the number of voters who are not indifferent, the equilibrium, μ, is the ideal point for one voter (or an additional even number of voters). If n is even, μ is the ideal point for no voters or an even number of voters.
3. The remaining voters can be divided into pairs, h and j, such that h and j are diametrically opposed around μ in the sense that, if they could move the equilibrium point as they wished, they would move it in exactly opposite directions in n-dimensional space. Furthermore, the contract curve between the ideal points for h and j, P_h and P_j, passes through μ.

This equilibrium may be thought of as an n-dimensional median, an analogue of the median that in one dimension is the equilibrium for single-peaked curves. It can be visualized for the five-person, two-dimension case as in Figure 7-4. The ideal points, P_{1-5}, are so marked and μ and P_3 coincide. The contract curves between P_1 and P_2 and P_4 and P_5 pass through μ. As can be seen, if some point, a, were set against μ, then a majority of 2, 3, and 5 would choose μ. As the number of voters increases, the difficulty of the pairing doubtless increases, so that majority-rule equilibria must be extremely rare in this model.

But perhaps they are not rare in the world, but only in the model, where the equilibria are "local" (that is, restricted to some defined neighborhood in the space) rather than "global" (that is, effective for the entire space), where there are only particular kinds of utility functions (such as differentiable ones), and where there is absolute majority rule. Unfortunately, one cannot escape this conclusion so easily. Much subsequent work

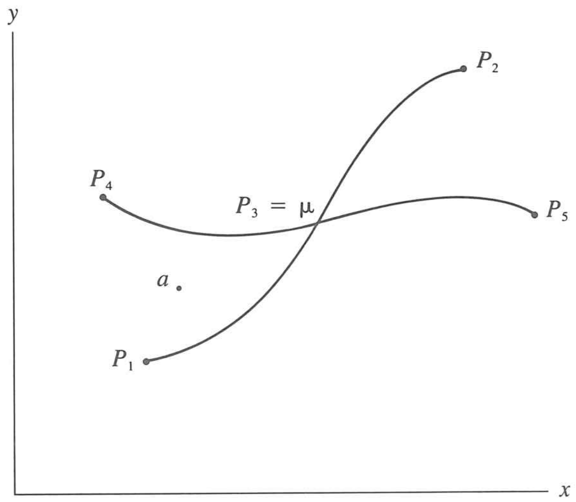

Figure 7-4 Majority-rule equilibrium, five persons, two dimensions.

has generalized the model considerably and thus made it more realistic without in any way reducing the presumption of rarity. McKelvey and Wendell, integrating and elaborating this work, show that, for a wide variety of utility curves (including nondifferentiable ones), a median, μ, at the unique intersection of contract curves among paired diametrically opposed voters is a sufficient condition for a Condorcet winner.[10] Furthermore, the existence of a median, μ, is a necessary condition for equilibrium if each utility curve by itself is single-peaked.[11] Finally, the existence of μ is a necessary and sufficient condition for a global equilibrium in the form of a Condorcet winner if utility curves are differentiable. So the present state of knowledge suggests that majority-rule equilibria must be very rare, given continuous variables in n dimensions, which, in my opinion, is a thoroughly realistic model of the world.

Complete Disequilibrium

Although cyclic outcomes "almost always" exist in this general model of majority rule, one might hope that the cycles would be confined to a small area in the space, so that there would be a "few" cycling alternatives, all of which, however, are socially preferred to everything else in the space. Then, in spite of intransitivities, the small top cycle would be socially better than the large set of worse outcomes. Unfortunately this is no way out of the social incoherence of almost universal

disequilibrium. McKelvey and Schofield have shown, in quite different ways, that this escape from instability will not work.

In a remarkable discovery, McKelvey has shown that, for a wide class of differentiable utility functions, once "transitivity breaks down, it *completely* breaks down, engulfing the whole space in a single cycle set. The slightest deviation from the conditions for a Condorcet point (for example, a slight movement of *one* voter's ideal point) brings about this possibility."[12] Hence, not only is a Condorcet winner unlikely, but also, when one does not exist, *anything* can happen. There is no "small" set of probable outcomes.

McKelvey's argument runs as follows: There is a set of points, $P(x)$, which is the set of all points that can be reached by majority rule from point x.[13] (To reach some point r in $P(x)$, let y beat x by majority rule in a pairwise contest, let z beat y similarly, and so on until r beats $r - 1$.) The set $P(x)$ can be either all points in the multidimensional space or a smaller set. If it is a smaller set, then it must have a boundary and the boundary points must satisfy the symmetry conditions of the constrained Plott equilibria, which are special cases of the symmetry conditions of the Plott equilibria.[14] Since we already know such equilibria are rare, it is likely that such a boundary is extremely rare. Hence intransitive cycles of social choice almost always exist and almost always include the *whole* space. In short, in the absence of an equilibrium, *anything* can happen.

Norman Schofield approaches the problem differently, but comes to the same result.[15] His analysis begins with the observation that, for any point, x, in a multidimensional issue space, some indifference curve for each participant passes through x. Given these indifference curves, he defines the set of points, $P_M(x)$, as the set of points in the neighborhood of x that are preferred to x by some winning coalition, M. Putting all the coalitions, M, together in the set, W, of winning coalitions, Schofield defines $P_W(x)$ as the set of points in the neighborhood of x that are preferred to x by *any* winning coalition. If there are some points, y, that cannot be included in $P_W(x)$ by some path ($y\, P_M\, z, \ldots, w\, P_M\, x$, when M is any winning coalition)—that is, if there are some points, y, that x can defeat no matter what—then there may be an equilibrium at x. If, however, all points in some arbitrary neighborhood of x can by some sequence of majority coalitions defeat x, then equilibrium at x is impossible. Furthermore, if for any particular set of individual ideals and indifference curves, there is even just one point x for which equilibrium is impossible, then every point in the entire space is included in the majority-rule cycle. Effectively, this means that, unless the individual preferences are highly similar—so that all winning coalitions are similar—social choices are certain to be cyclical.

Although this condition does not in itself indicate the likelihood of cyclical outcomes, Schofield has shown that, if the issue space has at least as many dimensions as one more than the number of persons necessary for a minimal winning coalition, then the system is, for certain, cyclical. Of course, all political issues of a distributive nature (for example, "Who gets what?") have at least as many dimensions as participants in the sense that each participant is concerned, among other things, with the amount distributed to him or her. For that huge category of issues, perhaps most of human concerns, disequilibrium is, therefore, certain.

Since McKelvey's and Schofield's theorems are similar in tone, it seems likely that majority rule is almost always in disequilibrium. The inference one therefore draws from the foregoing analysis is that, for the amalgamation of an extremely wide variety of individual value structures (that is, utility functions) by majority rule under fair procedures, intransitivities almost always exist and cycles include the whole space of political possibilities. This means that wide swings in political choices are possible and expected. Topsy-turvy revolution is as certainly predicted as incremental change.

Our ordinary experience indicates, however, that there is some stability in political life: Issues persist and similar outcomes repeat themselves. There must be more to the world, therefore, than the almost complete disequilibrium suggested by the Black and Newing, Plott, McKelvey, and Schofield analyses. How can one fit incremental change and stability into this model?

7.D. Practical Stability and Theoretical Instability

To begin with, disequilibrium does not prohibit incremental change. A theory of disequilibrium tells us only that change will occur; it says nothing about whether it will be incremental or catastrophic. Indeed, it seems likely that most of the apparent stability we see in the world, such as generation-long periods of dominance by one political party, is really incremental disequilibrium in which party composition changes slightly while always maintaining some cyclicity of values.

Theories of disequilibrium, however, do not require that change be incremental. The difference between theories of disequilibrium and theories of equilibrium is that the former admit revolution (although they do not necessarily predict it) and the latter do not. We know, however, revolution is not infrequent. Even the United States, the oldest of modern

democracies (originating in the 1770s), has had one bloody and devastating civil war. Since revolution occurs and is admissible in a theory of disequilibrium but is inadmissible in a theory of stability, it follows that some kind of theory of disequilibrium is a priori empirically superior to a theory of equilibrium. The advantage of theories of disequilibrium is that they admit both long periods of apparent stability (often indistinguishable from incremental change) and episodes of catastrophic revolution.

For another thing, these theories of disequilibrium concern values, preferences, or tastes, not constitutions and political structures. It may well be that the stability and continuity we observe in the world come mainly from institutions that, by reason of their interference with and restriction on majority rule, render majority-rule disequilibria less likely to influence and affect natural outcomes.

Indeed, Kenneth Shepsle has investigated institutions in just this way.[16] He observed that many institutions have the effect of forcing participants to approach political questions in just one dimension. Voting bodies often have single-peaked profiles on each of several dimensions but are in disequilibrium when these dimensions are combined in multidimensional issue space. Thus it follows that any institutional arrangement that forces consideration dimension by dimension may induce an equilibrium, even though an abstract general equilibrium does not exist. Many institutions—for example, committee systems in legislatures, rules restricting amendments, and agenda-setting devices—do indeed divide up the decision-making process into a set of decisions on single dimensions. Hence, institutionally engendered equilibria are often observed in the real world.

It should be noted, however, that these real-world equilibria, which depend on constitutions as much as on voters' tastes and values, are often subject to attack because they enforce an equilibrium that tastes would not allow. Hence they frustrate majorities; and if majorities are frustrated by institutions, these majorities may change the institutions. Thus, in the U.S. House of Representatives, the appropriations system developed in the period from the 1930s to the 1960s probably provided for frequent equilibria through the policy and structure of the appropriations committee. But the frustrations engendered in House members by the rules restricting outcomes led to the post-Watergate reforms that destroyed many of the restrictive rules and probably reduced sharply the frequency of equilibria.

Finally, many institutions in the real world force the reduction of the set of alternatives to exactly two—for example, the two-party system (and the method of plurality voting in single-member districts on which the two-party system seems to depend). In the choice between two alternatives, there is a majority winner. Since the individual tastes for one of the two parties appear to be fairly constant, the values thus institutionally restricted and incorporated in the political system may indeed provide for considerable stability. Thus, in the United States, one of the two parties has appeared dominant for fairly long periods (Jeffersonian Republicans from 1800 to 1825, Democrats from 1832 to 1854, Republicans from 1861 to 1930, Democrats from 1933 to 1980). This stability is more apparent than real, however, not only because it masks a large amount of incremental disequilibrium (as indicated in Chapter 2 by the fact that about 40 percent of the presidential elections have involved significant third-party candidates), but also because (as will be illustrated in Chapter 9) there is always an intense struggle, beneath the apparent stability, to induce a genuine disequilibrium of tastes.

We have, therefore, a number of good reasons, exogenous to the world of tastes and values in social choice theory, for the observed social stability. An equilibrium of tastes and values is in theory so rare as to be almost nonexistent. And I believe it is equally rare in practice. But individuals in society are more than ambulatory bundles of tastes. They also respect and are constrained by institutions that are intended to induce regularity in society. And it is the triumph of constraints over individual values that generates the stability we observe. But tastes and values cannot be denied, and they account for the instability we observe.

Although stability probably roots in institutions, Gerald Kramer has attempted to explain stability in terms of social choice equilibria, using a model of electoral competition between two parties acting in a multidimensional space of policies.[17] In this model, parties compete for voters over an infinite series of elections (or preelection time periods), offering platforms as points in the space. The voters respond by supporting the platform closest to their ideal points (using a conventional, rather restricted measurement system to determine "closeness"). In this simple majority system, one of the two platforms is chosen, the winning party enacts it, and the losing party picks out another platform (point in space) to maximize its chance of winning. Assuming it finds such a platform, the parties alternate in office.

Kramer calls the succession from winning platform to winning platform a *trajectory*. Then he defines a set for possible equilibria: the minmax set. For alternatives x and y, $n(x, y)$ is the number of votes by which y beats x. Some particular y gets the most votes against x, and that number of most votes is $v(x)$.[18] Assuming cyclicity, x will not beat all y, so $v(x) > n/2$. Nevertheless, some x, say x', comes closest to winning, in the sense that the value for $v(x')$ is the smallest of all $v(x)$—that is, it is the minmax number, n^*.[19] The difference between n^* and $n/2$ is the measure of how close a society comes to having a majority winner.

Kramer's notion of equilibrium, then, is this: If parties seek to maximize votes, then the trajectory of successive platforms leads to the set of all n^* (the set of all alternatives with the minmax number). That is, the trajectory leads, perhaps by a path on which there are backward steps, to the alternatives closest to being majority winners. When the trajectory arrives at the set of n^*, equilibrium exists. This equilibrium is not unique—except when the number of voters is infinite—but it may be a "small set" as against the "whole space" of McKelvey and Schofield.

Does Kramer's elegant model save us from devastating disequilibrium? Unfortunately, I think not because at least three of its assumptions are extremely unrealistic. One is the assumption of two parties. As was shown in Chapter 3, simple majority voting is attractive, but two and only two parties in both unrealistic and unfair. Another unrealistic assumption is that parties always maximize votes. Sometimes they do, as possibly happened for the winning party in the 1972 and 1964 presidential elections. But much more frequently one or both parties do not successfully deter factions from breaking off; Display 2-1 shows this failure in over 40 percent of the presidential elections. This failure to deter the breakoff of factions is evidence that parties do not care about maximizing votes or pluralities but rather care simply about winning with enough votes to ensure election.[20] So when old parties break up, they are, for certain, not maximizing votes. Hence the assumption of vote maximization is highly unrealistic, but crucial to the mathematical argument for the convergence of the trajectory to the set, n^*, of alternatives closest to winning.

A third unrealistic assumption is that dimensions do not change, even though the model is otherwise dynamic. If one allows platforms to change, then surely dimensions should change too. And if that happens, then trajectories have to start over. Given dynamism in dimensions, it seems probable that no trajectory would ever get very far in the convergence to n^*. In addition, trajectories are likely to be deflected by parties' occasional efforts, just mentioned as the opposite of the second assumption, to minimize (not maximize) votes toward the minimal winning size, the so-called size principle.

No one doubts that there is occasional stability in the real world. Sometimes this stability is more apparent than real, for there are cycles of similar alternatives and the disequilibrium moves from one outcome to another by increments so minute that political life seems stable even though it is not. But sometimes also the stability is real, and it is imposed by institutions not the product of preferences and values. If we consider only values, then disequilibrium seems inherent in majority rule. *Anything* can happen—incremental change or revolution.

Of course when institutional stability is imposed on what would otherwise be a disequilibrium of tastes, the imposed equilibrium is necessarily unfair. That majority which would, were it not institutionally restrained, displace the current outcome is denied the opportunity to work its will. In that sense institutional stability (such as the responsible two-party system discussed in Chapter 3) is unfair and is sure to cause frustration. Perhaps this frustration in turn ultimately brings about great changes, such as the disruption of long-standing alliances, rewriting of constitutions, and even violent revolution.

7.E. The Fragility of Equilibria: An Example of the Introduction of New Alternatives to Generate Disequilibrium

McKelvey interpreted his discovery that cycles cover the whole space to mean that a chairperson, with complete information and a taste for sophisticated voting, could, with appropriate agenda, lead the society to choose *any* alternative she or he most desired. As such, this model is, for majoritarian voting, a theoretical explanation of agenda control. It does not, however, explain the efforts of either Pliny or Plott and Levine, for they manipulated by procedures not contemplated in McKelvey's model.

It seems to me, however, that much the more significant practical consideration raised by McKelvey and Schofield is the extraordinary *fragility* of equilibria. Just a little bit of change in the situation—by strategic voting or by introducing another alternative—opens up a whole new world of political possibilities. I will conclude this chapter with an illustrative instance of just that fragility, and then in the next chapter I will set forth a general interpretation of political disequilibrium.

My concluding illustration concerns the history of the motions that ultimately became the Seventeenth Amendment (about the direct election of U.S. senators).[21] Originally this issue appeared to be one of discrete alternatives on one dimension (that is, for or against amending the Constitution). Furthermore, an equilibrium of single-peaked curves probably existed. But clever parliamentary tacticians—ordinary backbenchers, not leaders—generated other alternatives in at least two other dimensions,

one a dimension of party loyalty, the other a dimension of racism. Consequently, they destroyed the equilibrium (if it existed) and generated cycles from which no motion would be chosen. This is what the fragility of equilibria permits, and this is what makes the dynamic disequilibrium of majority rule so significant a feature of all democratic life.

Toward the end of the nineteenth century, there began to be considerable agitation for direct election of senators, agitation deriving from quite diverse sources. One powerful source was Southern white populist racism. As a means of eliminating the influence of black voters, Southern white populists had invented the device of nominating Democratic candidates in primary elections from which blacks were excluded. Since Democrats had a considerable majority in Southern states, this meant that only white officials would be elected. The primary election, however, was a dubiously valid device as applied to U.S. senators, who were supposed to be elected by state legislatures. Adoption of direct election nationally was, then, an attractive means of strengthening racist institutions locally. Another source was the progressive movement in the North, with its emphasis on direct citizen participation. A third source was a movement for good-government reform supported by middle-class reformers repelled by the corruption of state legislators in electing senators.

It would seem that a coalition of such strange bedfellows would be both an absolute majority and an irresistible force—and of course it was in the end. But for about twenty years it was successfully opposed by an *almost* immovable object—the distaste of a minority of sitting senators for submitting to the sexennial torture of a popular canvass. At first those senators simply ignored resolutions regularly passed by the House, which seemed to delight in embarrassing the Senate. But by 1902 the Senate could no longer ignore the pressure, and the proposed constitutional amendment was considered in committee in substantially the form it now has: "the Senate of the United States shall be composed of two Senators from each State elected by the people thereof, for six years." This sentence may well have been an equilibrium outcome, if not in 1902, at least by 1905 or 1906, had not a remarkably inventive parliament man, Chauncey DePew of New York, disrupted the equilibrium with the following proposed addition:

> *The qualifications of citizens entitled to vote for United States Senators and Representatives in Congress shall be uniform in all States, and Congress shall have the power to enforce this article by appropriate legislation and to provide for the registration of citizens entitled to vote, the conduct of such elections, and the certification of the result.*[22]

The DePew amendment, though it perhaps seems innocuous today, was highly devisive, for it was then interpreted as a "force bill," a bill to authorize the president to send the army into the South to register blacks and enforce their voting rights—in short a revival of Reconstruction. White supremacist Southern Democrats who were enthusiastic for the constitutional amendment without the DePew amendment would be bitterly opposed to the constitutional amendment if the DePew amendment were attached to it. And attached it would be, for at that time the one point of ideological unity for the Republican party was support for black aspirations. Hence, even those Republicans who very much wanted the constitutional amendment were obliged by party loyalty and their strong antiracist sentiments to favor the DePew amendment. The opposition to the DePew amendment consisted of Southern Democrats, of Northern Democrats who were for the most part tolerantly sympathetic to Southern racism, and of a few progressive Republicans who put electoral reform above party loyalty. In 1902 and 1911 these were simply not enough to defeat DePew.

The issue never got to the floor in 1902, but it did get there in February 1911, when there was much more support for reform. This time the progressive proponents of reform protected their Southern allies against the DePew maneuver by adding a proviso to guarantee white supremacy: "The times, places, and manner of holding elections for Senators shall be prescribed by the legislatures thereof [i.e., the states]." The opponents of direct election moved to delete this sentence. The motion to delete was identified as the Sutherland amendment, and it was a negatively stated version of the DePew amendment. There were thus three alternatives:

a. The resolution to amend the Constitution, as amended by the Sutherland amendment

b. The original resolution to amend the Constitution (including the clause to protect Southern racism)

c. The status quo

The vote on the Sutherland amendment put a against b, and the Sutherland amendment passed 50 yea to 36 nay. So the social choice was $C(a, b) = a$. The vote on passage of the amended resolution, which put a against c, was 54 yea (for a) and 34 nay (for c). Since the resolution to amend the Constitution required two-thirds yeas (here, 59), the motion failed. So the social choice was $C(a, c) = c$.

To finish out the pairings and thus to show that the introduction of a new alternative involving new dimensions generated a cycle, b and c must be compared. Presumably all 54 who voted for the amended resolution also favored the original resolution (b) over the status quo (c). So presumably also did at least 10 Southern Democrats who had originally insisted on the proviso deleted by the Sutherland amendment and who ultimately voted against the amended resolution (that is, for c against a). Together these are 64 out of either 86 or 88, easily enough to pass the constitutional amendment. So the social choice is $C(b, c) = b$. Hence follows this cycle: a beats b, b beats c, and c beats a. In the non-neutral amendment procedure, c wins when a cycle exists. Hence, a fairly small minority, probably no more than 24, won because they generated a cycle with a new alternative with new dimensions.[23] This is not, I believe, an isolated example of manipulation, but a typical instance of electoral and democratic politics, as I will show in the next chapter.[24]