12

Votes, strategies, and institutions: an introduction to the theory of collective choice

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Like markets, voting transforms individual preferences into a collective choice – a choice attributable to all of certain actors but to no one of them. Although such a choice reflects votes and votes reflect preferences, preferences alone do not determine votes and votes alone do not determine the final choice. A new body of political theory – the theory of "social choice" or "collective choice" – has shown that voting does not transform individual preferences into a collective choice so automatically or straightforwardly as some political analysts may have thought: institutional details and strategic decisions play starring roles, and the transformation sometimes works in unexpected, puzzling, and arguably anomalous ways. This theory is especially promising for the study of Congress, a voting body chosen by voting, rich in procedural complexity and strategic opportunity, and long the object of empirical scrutiny.

What follows is less a comprehensive survey of the rapidly growing literature on collective-choice theory or its applications than an attempt to make some of the most theoretically important and analytically useful aspects of it intuitive, transparent, accessible, and – dare I hope? – exciting. Besides fundamentals, I have emphasized aspects directly related to congressional scholarship, including the other essays in Part III.

MAJORITY RULE AND TWO-ALTERNATIVE COLLECTIVE CHOICE

In a broad sense, *majority rule* is rule by majorities, a form of government in which any majority can do what it pleases. In a narrower sense, majority rule – *simple*-majority rule, to be exact – is a particular voting rule for choosing between two feasible alternatives, which may be candidates for an office, passage and defeat of a motion, or whatnot. Given any voting rule or other collective-choice process, let us say that an alternative x is *collectively preferred* to another alternative y, or that x

beats y, if the rule chooses x when x and y alone are feasible; and that x and y are *collectively indifferent*, or *tied*, if neither beats the other. According to simple-majority rule, x beats y if the number of votes for x exceeds the number for y, and x ties y if these numbers are the same.

Simple-majority rule holds a distinguished position among voting rules. When a choice is made between two alternatives by voting, simple-majority rule is almost invariably used. We tend instinctively to identify pairwise collective choice with simple-majority choice and political democracy with majoritarianism. Why?

Characteristic properties of simple majority rule

The following answer is adapted from K. O. May (1952). Suppose n voters, Messrs. $1, 2, \ldots, n$, are to make a collective choice between two alternatives, represented by the numbers 1 and -1. Let 0 represent a tie. A *vote combination*, representing a possible way Messrs. $1, 2, \ldots, n$ can vote or abstain, is an n-fold combination, or *vector*, $(x_1, \ldots, x_n)$ in which each x_i is 1 (a vote for 1 by Mr. i), -1 (a vote for -1), or 0 (abstention by Mr. i). *Any* two-alternative voting rule may be represented by a function f such that:

(F) f is a function of vote combinations, and for every vote combination $(x_1, \ldots, x_n)$, $f(x_1, \ldots, x_n)$ is equal to 0, 1, or -1.

What further properties might we want or expect f to possess? These three have strong claims on our intuitions:

(A) $f(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n) \equiv f(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$ (Anonymity).

(N) $f(-x_1, \ldots, -x_n) \equiv -f(x_1, \ldots, x_n)$ (Neutrality).

(T) If $f(x_1, \ldots, x_n) = 0$ and $(y_1, \ldots, y_n)$ is the result of replacing one or more 0's by 1 in $(x_1, \ldots, x_n)$, leaving all else the same, then $f(y_1, \ldots, y_n) = 1$ (Fragility of Ties).

Property (A) says f treats voters equally: if any two voters switch votes, the collective choice remains the same. Property (N) says f treats alternatives equally: if every vote is reversed, so that 1's become -1's and -1's become 1's, the choice is thereby reversed (or remains the same if it was 0). Property (T) says ties are easily broken: if there is a tie and some erstwhile abstainers then vote for 1, other things remaining unchanged, that is enough to break the tie in 1's favor. A voting rule that lacked any of these properties would have a built-in bias against some voters or alternatives, thereby allowing elections to be decided to some degree by factors other than votes, or it would be gratuitously indecisive, allowing ties to persist in the face of new information that plainly tilted the balance in one direction.

Simple-majority rule obviously has all these properties. It holds its distinguished position because it is the *only* two-alternative voting rule with all these properties. To prove this, we must assume that f satisfies (F)–(T) and show (a) that if $(x_1,\ldots,x_n)$ is a vote combination containing just as many 1's as -1's, then $f(x_1,\ldots,x_n) = 0$; (b) that if $(x_1,\ldots,x_n)$ contains more 1's than -1's, then $f(x_1,\ldots,x_n) = 1$; and (c) that if $(x_1,\ldots,x_n)$ contains fewer 1's than -1's, then $f(x_1,\ldots,x_n) = -1$.

Proof of (a). Because $(x_1,\ldots,x_n)$ has just as many 1's as -1's, $(-x_1,\ldots,-x_n)$ is the result of switching the first 1 vote with the first -1 vote, the second 1 vote with the second -1 vote, etc. By (A), such switching leaves the choice unaffected. So

$$\begin{aligned} f(x_1,\ldots,x_n) &= f(-x_1,\ldots,-x_n) \\ &= -f(x_1,\ldots,x_n) && \text{by (N)} \\ &= 0 \end{aligned}$$

since only 0 is its own negative.

Proof of (b). Suppose $(x_1,\ldots,x_n)$ contains k more 1's than -1's. Let $(x'_1,\ldots,x'_n)$ be the result of replacing the first k 1's in $(x_1,\ldots,x_n)$ by 0. Then $(x'_1,\ldots,x'_n)$ has just as many 1's as -1's, and so, by (a),

$$f(x'_1,\ldots,x'_n) = 0.$$

Thus, by (T), since $(x_1,\ldots,x_n)$ is the result of replacing some 0's in $(x'_1,\ldots,x'_n)$ by 1,

$$f(x_1,\ldots,x_n) = 1.$$

Proof of (c). Since $(x_1,\ldots,x_n)$ has fewer 1's than -1's, $(-x_1,\ldots,-x_n)$ has *more* 1's than -1's. By (b), then,

$$\begin{aligned} 1 &= f(-x_1,\ldots,-x_n) \\ &= -f(x_1,\ldots,x_n) && \text{by (N)} \end{aligned}$$

so

$$f(x_1,\ldots,x_n) = -1 \qquad \text{Q.E.D.}$$

Other two-alternative voting rules

If (F)–(T) are uniquely satisfied by simple-majority rule, which of these conditions are violated by other familiar two-alternative collective-choice rules?

Condition (T) is violated by survey procedures designed to reckon the collective preference between alternatives (candidates, policies, soft drinks, radio stations) by polling a sample rather than an entire population. Let f represent such a procedure, and suppose $f(0, x_2,\ldots,x_n) = 0$

and Mr. 1 does not belong to the sample. Then, contrary to (T), $f(1, x_2,\ldots,x_n) = 0$ as well.

Condition (N) is violated by any *special*-majority rule (two-thirds, three-quarters, unanimity, etc.). Suppose f represents two-thirds majority rule: 1 beats -1 (the status quo or default alternative) if two-thirds of nonabstainers vote for 1; otherwise, -1 beats 1. And suppose $(x_1,\ldots,x_n)$ is a vote combination in which the voters are evenly divided. Then, since 1 does not command a two-thirds majority in $(x_1,\ldots,x_n)$, $f(x_1,\ldots,x_n) = -1$. But 1 does not command a two-thirds majority in $(-x_1,\ldots,-x_n)$ either, and so, contrary to (N), $f(-x_1,\ldots,-x_n) = -1$ as well. Special-majority rules have a built-in conservative bias – a bias in favor of the status quo, or collective inaction. This is true, in particular, of the unanimity rule prescribed by classical social-contract theory for constitutional choices.

Condition (A) is violated by any rule that weights the votes of different voters differently. Let f represent the rule by which stockholders choose one of two slates of corporate directors, and suppose Mr. 1 owns a majority of shares. Then $f(1, -1, x_3,\ldots,x_n) = 1$ but, contrary to (A), $f(-1, 1, x_3,\ldots,x_n) = -1$.

Condition (A) is violated as well by the standard Anglo-American procedure for choosing one of two parties to control a legislature: voters are partitioned into districts, each of which elects a representative belonging to one of two parties (1 and -1), and the party winning a majority of votes in each of a majority of districts wins control of the legislature. Let f represent such a procedure, and suppose there are three districts, one comprising Messrs. 1, 2, and 3, another comprising Messrs. 4, 5, and 6, and the third comprising Messrs. 7, 8, and 9. Then:

$$f(1, 1, -1, 1, -1, -1, 1, -1, -1) = -1$$

but

$$f(1, 1, -1, 1, 1, -1, -1, -1, -1) = 1.$$

That violates (A) because the second vote combination can be got from the first just by switching the votes of Messrs. 5 and 7.

This example shows that a system of equal-size districts (the "One Man, One Vote Rule") does not ensure that voters count equally. It also shows that elections by single-member districts can choose a ruling party opposed by a majority: $f(1, 1, -1, 1, 1, -1, -1, -1, -1) = 1$, although $(1, 1, -1, 1, 1, -1, -1, -1, -1)$ contains a majority of -1's. That often happens in the states. It happened nationwide in 1984: the Republicans won a scant majority of votes for the U.S. House of Representatives, but the Democrats kept a clear majority of seats. (For in-depth studies of two-alternative collective choice, see Murakami 1968 and Fishburn 1973, pp. 13–68.)

PARADOXES OF COLLECTIVE CHOICE

When we jump from two alternatives to three or more, there occurs a phenomenon, widely regarded as a paradox, that has lain at the center of theoretical speculation since the marquis de Condorcet discovered it near the end of the eighteenth century. As generalized in varied ways, beginning with Kenneth Arrow's seminal contribution (1952, 1963), this phenomenon is often described as a kind of indeterminateness, incoherence, or collective irrationality.

The classical voting paradox

Suppose three voters, any majority of whom can do what they please, are to choose among three alternatives, which they rank in order of preference as follows:

Mr. 1	Mr. 2	Mr. 3
x	y	z
y	z	x
z	x	y

A majority (Messrs. 1 and 3) prefer x to y, another majority (Messrs. 1 and 2) prefer y to z, and a third majority (Messrs. 2 and 3) prefer z to x: under majority rule, the collective preference is *cyclic*, and every feasible alternative is *unstable* in the sense that another one beats it. Such is the *classical voting paradox*, discovered by Condorcet (for historical details, see Black 1958, pt. 2). Its theoretical significance is fourfold:

1. Instability makes it hard to predict what will be chosen: every possible prediction is opposed by some majority, and it is majorities generally (we have assumed) that rule.
2. The *latent* instability in this example – the power and incentive of some groups to overturn any feasible choice – may occasion a great deal of *manifest* instability – the continual overturning of choices by dint of successive realignments. True, policy change is sometimes a good thing, and a political system that allows change in the face of strong opposition thereby institutionalizes those realignments and exercises in reform that would otherwise occur in a more violent way. But continual change can be costly and can prevent any government program from being carried out.
3. If, perhaps because of May's theorem, we take majority preference as the measure of better-to-worse in point of social welfare, then there is

no social welfare optimum – no socially best alternative – in the example.
4. Because social preference can be cyclic, social choice does not always meet the minimum condition of "rationality" customarily required of individual choice: unlike a "rational" individual, a majoritarian government cannot always make a choice to which it prefers no alternative. In other words, majoritarian governments cannot in general be modeled as maximizers of anything.

How general is this phenomenon of cyclic collective preference, or unstable collective choice? For one thing, it is not peculiar to the three-voter case. Here is a four-voter example:

Mr. 1	Mr. 2	Mr. 3	Mr. 4
x_1	x_2	x_3	x_4
x_2	x_3	x_4	x_1
x_3	x_4	x_1	x_2
x_4	x_1	x_2	x_3

For *any* number of voters *greater* than four, Figure 12.1 (in which m is a bare majority) shows how to construct a majority-preference cycle among just three alternatives.

The phenomenon is yet more general. Cycles do not just occur under majority rule. Consider any collective-choice rule satisfying this condition:

> If all but one voter prefer x to y while he prefers y to x, then x beats y (Virtual Unanimity).

Let any number n of voters have the preference orderings of a set of n alternatives shown in Figure 12.2. When $j < n$, every voter prefers x_j to x_{j+1} unless x_{j+1} is the first alternative in his ordering, that is, all voters but Mr. $j + 1$ prefer x_j to x_{j+1}. And all but Mr. 1 prefer x_n to x_1. So by Virtual Unanimity, x_1 beats x_2, x_2 beats $x_3, \ldots, x_{n-1}$ beats x_n, and x_n beats x_1 – another cycle.

Although many collective-choice processes besides majority rule satisfy Virtual Unanimity, not all do. Any voting rule that gives individual veto power to one or more voters, such as the rule used in the U.N. Security Council and the rule for amending the Articles of Confederation, obviously violates Virtual Unanimity. So does any constitution that protects individual rights: if the choice of x in preference to y would violate Mr. 1's right against self-incrimination, for example, then y beats x under the U.S. Constitution even if Messrs. $2, \ldots, n$ all prefer x to y.

Majority who prefer x to y

Majority who prefer y to z

1	. . .	$m-2$	$m-1$	m	$m+1$	. . .	n
z		z					
x	. . .	x	x	x			
y		y	y	y	y		y
			z	z	z	. . .	z
					x		x

Majority who prefer z to x

Figure 12.1

1	2	. . .	i	. . .	n
x_1	x_2	. . .	x_i	. . .	x_n
x_2	x_3		x_{i+1}		x_1
.	.		.		.
.	.		.		.
.	.		.		.
x_{i-1}	x_i		x_n		x_{i-2}
x_i	.		x_1		x_{i-1}
.	.		.		.
.	.		.		.
.	x_n		.		.
x_n	x_1	. . .	x_{i-1}	. . .	x_{n-1}

Figure 12.2

Impossibility theorems

The problem is more general still. Several so-called *impossibility theorems*, beginning with that of Schwartz (1970), show that collective-preference cycles and therewith unstable collective choices can occur under any voting rule (or collective-choice process) meeting mild conditions of "reasonableness" – conditions so mild they are satisfied alike by brutal tyrannies, corrupt oligarchies, and ideal constitutional democracies. These theorems are variations on Arrow's celebrated impossibility theorem. Arrow's theorem does not show, however, that cycles can occur; its conclusion, as you will see, is weaker than that.

Arrow (1963) considered a rule whereby Messrs. $1, 2, \ldots, n$ make

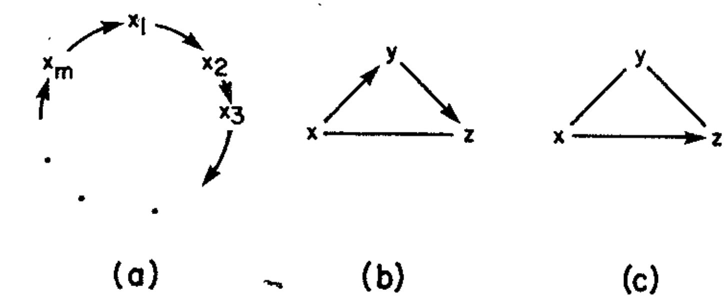


Figure 12.3

collective choices from finite subsets of a given "universal set" of alternatives and assumed:

(1) The universal set contains three or more alternatives.

(2) The rule applies to all possible combinations of individual preference orderings (orderings by Messrs. $1, 2, \ldots, n$) of the universal set (Unrestricted Domain).

(3) No individual is so powerful that, for *every* pair of alternatives x and y, x beats y under *every* combination of preference orderings in which he prefers x to y while everyone else prefers y to x (Nondictatorship, slightly modified).

(4) x beats y whenever *everyone* prefers x to y (pairwise Pareto Principle).

(5) The collective preference between two alternatives never depends on individual preferences regarding *other* alternatives (pairwise version of Independence of Irrelevant Alternatives).

Arrow deduced that, for some combination of individual preference orderings, the relation of collective preference or indifference is *nontransitive*, which means that at least one of the three cases depicted in Figure 12.3 (in which "→" represents collective preference and "—" collective indifference) arises. Only (a) is a cycle. Only in case (a) does some set of alternatives lack a stable choice. But Arrow did not show that (a) can arise, only that at least one of the three cases can arise – maybe just (c).

To put the theorem another way, Arrow assumed (1)–(5) plus:

(6) If x beats y and y beats or ties z (i.e., z does not beat y), then x beats z (Transitivity, or Collective Rationality),

and deduced a contradiction, demonstrating that no collective-choice rule can satisfy (1)–(6). Any violation of (6) must be a case in which (i) x beats y, y beats z, and z beats x; or (ii) x beats y, y beats z, and x ties z; or (iii) x beats y, y ties z, and z beats x; or (iv) x beats x, y ties z, and x ties z. But (i) has the form of (a) (a cycle), (ii) and (iii) the form of (b), and (iv) the form of (c). So every violation of (6) is a case of type (a), (b), or (c). Note that (i) is not just a cycle but a *tricycle*, one comprising three alternatives.

An especially simple if not terribly appealing addition to (1)–(5) that lets us deduce that a cycle can arise is the following:

(7) No two alternatives are ever tied (Pairwise Resoluteness).

For (7) rules out (ii)–(iv), which involve ties, ensuring that any violation of (6) must be of type (i), a tricycle.

The theorem of Schwartz (1970), which shows that a cycle can arise, replaces (1) and (7) with:

(1′) The universal set contains at least n alternatives (n being the number of voters).

(7′) If all but one individual prefer x to y, then x does *not tie* y (either x beats y or y beats x) (Minimum Resoluteness).

In other words, Arrow's inconsistency survives when we replace (1) by (1′), add (7′), and replace (6) (Transitivity) by the weaker:

(6′) It never happens that x_1 beats x_2, x_2 beats $x_3, \ldots, x_{m-1}$ beats x_m, and x_m beats x_1 (Acyclicity).

I will prove Arrow's theorem by assuming (1)–(6) and deducing three consequences, the third of which contradicts (3) (Nondictatorship). First, two definitions: A *profile* is a combination of n preference orderings, one for each individual, of the universal set of alternatives. A group g of individuals is *decisive* for x versus y if, and only if, x beats y under *every* profile in which the members of g all prefer x to y while the other individuals all prefer y to x. According to (4), the group of all n voters is decisive for every pair of alternatives. According to (3), no single individual (or, to be unnecessarily exact, no single individual's unit set) is decisive for every pair of alternatives (which is not to say that no individual is decisive for *any* pair).

Consequence 1. Suppose g is a group of individuals and p a profile in which the members of g all prefer x to y, everyone else prefers y to x, and *x beats y*. Then g is decisive for x versus y.

Proof. What must be shown is that x beats y for *every* profile q in which the g's all prefer x to y while everyone else prefers y to x. But this follows from (5) (Independence) since x beats y under p and Messrs.

$1, 2, \ldots, n$ have the same preferences between x and y in q as they have in p. Q.E.D.

Consequence 2. Suppose Mr. i is decisive for x versus y. Then (a) Mr. i is decisive for x versus any alternative z different from x; (b) Mr. i is decisive for any z different from y versus y; and (c) Mr. i is decisive for any alternative z versus any other alternative w.

Proof. (a) Trivial if $z = y$. Otherwise, there is a profile of the form:

Mr. i	Everyone else
x	y
y	z
z	x

By (2), the collective-choice rule applies to this profile. Since Mr. i is decisive for x versus y, x beats y, and by (4) (Pareto Principle), y beats z, so by (6) (Transitivity), x beats z, and thus, by Consequence 1, Mr. i is decisive for x versus z.

(b) Trivial if $z = x$. Otherwise, there is a profile of the form:

Mr. i	Everyone else
z	y
x	z
y	x

Since Mr. i is decisive for x versus y, x beats y, and by (4), z beats x, so by (6), z beats y, and thus, by Consequence 1, Mr. i is decisive for z versus y.

(c) By (1), the universal set contains an alternative t different from x and z. By (a), since Mr. i is decisive for x versus y, he also is decisive for x versus t, whence he is decisive for z versus t by (b), and thus, by (a), he is decisive for z versus w. Q.E.D.

Consequence 3. Someone is decisive for every pair of alternatives (contrary to (3)).

Proof. By (4), the group of all individuals is decisive for every pair. Hence, there must exist a *minimum decisive group*: a group g that is decisive for some pair, say x versus y, while no smaller group is decisive for any pair. By (4), g cannot be empty; say Mr. i belongs to g. By (1) and (2), the collective-choice rule applies to some profile of the following form:

Mr. i	$g-i$	Everyone else
x	z	y
y	x	z
z	y	x

where $g-i$ comprises everyone in g but Mr. i. By Consequence 1, if z beat y, $g-i$ would be decisive for z versus y. But that is impossible because no group smaller than g is decisive for any pair. So z does not beat y: y beats or ties z. But because g is decisive for x versus y, x beats y. Hence, by (6) (Transitivity), x beats z, and thus, by Consequence 1, Mr. i is decisive for x versus z. By Consequence 2, therefore, he is decisive for every pair. Q.E.D.

The theorem of Schwartz (1970), which shows that collective preference can be cyclic, not just nontransitive, says that (1′), (2)–(5), (6′), and (7′) are jointly inconsistent. To prove this, let us first see which consequences of Arrow's conditions follow from the revised set of assumptions. Consequence 1 does because its proof did not invoke (6) (Transitivity). Consequence 2(a) did invoke (6) at one point: we showed that x beats y and y beats z and inferred, by (6), that x beats z. The weaker (6′) (Acyclicity) just lets us infer that z does not beat x. But since Mr. i alone prefers x to z, (7′) (Minimum Resoluteness) tells us that either x beats z or z beats x. It follows that x beats z, as required. Similarly for Consequence 2(b). And Consequence 2(c) was deduced from 2(a) and 2(b) without further use of (6). So Consequence 2 still holds.

But in the previous subsection we saw that there must be a cycle under some profile, assuming (1), (2), and Virtual Unanimity. Since the existence of a cycle contradicts (6′), it suffices to deduce Virtual Unanimity. Suppose, then, that all but one individual, Mr. i, prefer x to y, while he prefers y to x; to deduce that x beats y. But by (7′), *either* x beats y or y beats x. And by Consequence 1, if y beat x then Mr. i would be decisive for y versus x, and so, by Consequence 2, he would be decisive for every pair of alternatives, contrary to (3). Hence, y does not beat x, and thus x beats y. Q.E.D.

I conclude this section with a glimpse at more advanced results. The theorem of Schwartz (1970) actually was more general than the one just proved: it used a drastically weakened version of (5) and allowed (although it did not require) interpersonal comparisons of preference intensity. Mas Collel and Sonnenschein (1972) proved that cycles can arise after assuming that $n \geqslant 4$, strengthening (3) a bit, and replacing (7′) with:

(7″) If x beats or ties y and if one individual changes his relative ranking of x and y in x's favor (all else remaining the same), then x beats y – so that a single voter can break any tie (Positive Responsiveness).

Although dropping (1′) was an improvement, (7″) is quite strong: it is satisfied by simple-majority rule but not by sample surveys – as I explained earlier in connection with the kindred condition (T). This

limitation is inessential, however: assuming that $n \geqslant 5$, Schwartz (1982*b*) showed that (7″) can be replaced with:

(7‴) If x beats or ties y and if a coalition comprising a *fifth of all individuals* switches from a preference for y over x to a preference for x over y (all else remaining the same), then x beats y.

What you have seen so far is not that collective choices are *always* unstable, or even that they are *ever* unstable, but only that they *can* be unstable, depending on individual preferences and the feasible set (the set of feasible alternatives): the actual profile might yield no cycle; if it does, the actual feasible set might not contain that cycle; and if it does, it might also contain a stable alternative foreign to the cycle. But according to the theorem of Schwartz (1982*b*), based on (7‴), the cycle can be so constructed that it precisely exhausts *any given set*, finite or infinite, of three or more alternatives. The instability revealed by this theorem still depends, however, on individual preferences: for all the theorem tells us, the actual profile might never produce an unstable choice. (On impossibility theorems, see Arrow 1963; Plott 1976; Schwartz 1985*a*, 1985*b*; and Sen 1983.)

STABILITY AND INSTABILITY

We do know something, however, about the conditions under which instabilities actually occur, or are likely to occur, not just about their institutional possibility.

Stability in one dimension

Following Hotelling (1929), let the feasible set comprise all points along a line, which we might think of as the liberal-conservative continuum. Suppose every voter has an "ideal point" on this line and likes alternatives less and less the farther they lie from his ideal point. Given this "one-dimensional spatial model," we can show that a stable alternative exists under majority rule. We can even identify it.

A *median* of voters' ideal points is an ideal point m such that no majority of voters have their ideal points to the left of m or to its right. There must exist one or two median ideal points. Hotelling proved that such a median, as well as any point between two such medians, must be stable under majority rule: no majority of voters prefer any point to it.

Proof. Let m be such a point and r any point to the right of m. Then the voters with ideal points at or to the left of m prefer m to r. These voters are at least half the electorate inasmuch as no majority have their ideal points to the right of m. Since at least half the electorate prefer m to r, no majority prefer r to m. Similarly, if l is any point to the left of m, then no majority prefer l to m. So m is stable. Q.E.D.

Although Hotelling assumed that the feasible set comprises *all* points on a line, we can assume instead that there are finitely many feasible alternatives, each occupying a point on the line. By reasoning much like Hotelling's, Black (1948, 1958) showed that any median of voters' *favorite feasible alternatives* is stable.

The basic idea behind Hotelling's result has been applied by Downs (1957, esp. p. 115) to elections with two candidates (or parties) each of whom must decide what position to occupy on the liberal-conservative continuum, with the sole goal of winning. To simplify a bit, suppose there is a unique median, m, of voters' ideal points. If l is a point to the left of r and closer than r to m, those voters with ideal points at or to the left of m prefer l to r. Since they are a majority, l beats r. Similarly, r beats l if r is closer to m. So each candidate has an incentive to take a position closer than his opponent's to m. Therefore, candidate positions will tend to converge to m, providing voters with an echo, not a choice.

Instability in more than one dimension

The tidy stability property of majority voting for points along a line cannot be extended to points in a space of two or more dimensions, a fact revealed by a number of spatial instability theorems but most simply by that of Davis, De Groot, and Hinich (1972): Suppose the feasible alternatives are all the points in a two-dimensional space (or coordinate system) in which each voter has an ideal point and likes alternatives less and less the farther they lie from that point. The two dimensions might represent the social and economic liberal-conservative continua. A *median in all directions* is a point m such that, for every line through m, no majority of voters have their ideal points on one side of this line. The theorem says that a point is stable under majority rule only if it is a median in all directions.

Proof. Suppose x is *not* a median in all directions: there is a line L through x and a majority M of voters with ideal points on one side of L. Then parallel to L there must be a line L' that lies strictly between L and the ideal points of M. Drop a perpendicular from L' to x, as in Figure 12.4. The point at which the perpendicular intersects L' is closer than x to M's ideal points. So the members of M all prefer that point to x: x is not stable. Q.E.D.

Although it states a necessary condition for *stability*, the result just proved really is an *in*stability theorem because the condition is so severe: however we rotate a line at a stable point, we will never find a majority of

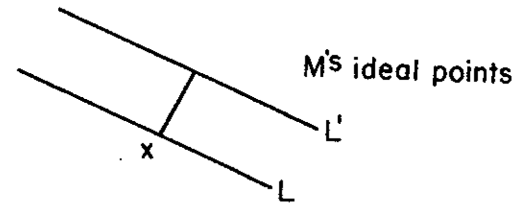


Figure 12.4.

voters' ideal points on one side. This requires quite a balanced spread of ideal points, ruling out almost any preferential clustering of voters in terms of ideology, geography, economic interest, party loyalty, or plain chance.

Other spatial instability theorems are stronger in some ways, although less simple to prove. Plott's (1967) celebrated condition of pairwise symmetry, which requires a stable point to be a seesaw fulcrum on which every ideal point different from it is balanced with one other ideal point, is more severe than that of being a median in all directions. Although it is only a sufficient condition for stability, Enelow and Hinich (1983) have turned it into a necessary condition as well by tacking on some qualifications. McKelvey (1976) proved that if (as is likely) no point in a multidimensional space containing voters' ideal points beats all other points under majority rule, then there exists not only a cycle but one that eats up the entire space. Schofield (1978) and Cohen (1979) have proved interesting variations on this theme.

The spatial instability theorems do not show that the potential instability revealed by the impossibility theorems is always or even often realized. For their assumptions are much stronger than those used in the impossibility theorems: individuals are assumed to have ideal points, or points of satiation, precluding the interpretation of spatial dimensions as economic goods; the collective-choice rule is assumed to be majoritarian (an assumption that can be weakened somewhat for certain theorems); and what is most questionable, the feasible set is assumed to be the whole space, or at least (for some theorems) a convex subset thereof. (A set of points is *convex* if, for any two points in the set, the line segment joining them also is in the set.)

Although political analysts have considerable latitude in identifying "the" feasible set in any given case (Schwartz 1986; sec. 10.1), the feasible sets commonly identified are finite, and they are constrained in innumerable ways. Often a feasible set comprises the candidates for some office, often the motions voted on in some legislature or committee. In either case it is finite. True, the set of all possible platforms a candidate might conceivably adopt and the set of all possible motions a legislature or committee might conceivably pass are infinite. But even if these sets could be construed as *feasible* sets, they cannot be convex because platforms and motions must be formulated in English and there are only countably many English sentences (an infinite number smaller than the infinite number of points on a line).

Instability and vote trading

Conditions for instability are also specified by a set of theorems based not on the spatial representation of alternatives and preferences but on vote trading. Independently discovered by Kadane (1972), Oppenheimer (1972), and Bernholz (1973), then generalized by Schwartz (1977), this type of theorem asserts that, given certain assumptions, any collective choice is unstable for which *vote trading* is essential.

An *outcome* is a vector $(x_1, \ldots, x_k)$ consisting of one position on each of k *issues*. Let $m = (m_1, \ldots, m_k)$ be the *no-trade outcome* – the outcome that would be chosen in the absence of any vote trading. We need two assumptions:

(8) If x_i is a position on the ith issue other than m_i, then m beats $(m_1, \ldots, x_i, \ldots, m_k)$.

This says that if a position on a given issue that would be defeated in the absence of vote trading is combined in an outcome with only the no-trade positions on all other issues, then that outcome is beaten by the pure no-trade outcome.

(9) Suppose $x(a_i)$ and $x(b_i)$ are two outcomes differing only in that a_i is the ith component of $x(a_i)$, and b_i the ith component of $x(b_i)$, and suppose $y(a_i)$ and $y(b_i)$ are two other outcomes differing in the same way. Then if $x(a_i)$ beats $x(b_i)$, $y(a_i)$ beats $y(b_i)$ (Separability).

This says that the collective preference on any issue is independent of the positions chosen on other issues.

It follows from (8) and (9) that any outcome other than m, hence any outcome for which vote trading is essential, must be unstable: some outcome beats it.

Proof. Let $x = (x_1, \ldots, x_i, \ldots, x_k)$ where $x_i \neq m_i$. By assumption (8),

$$m = (m_1, \ldots, m_i, \ldots, m_k) \text{ beats } (m_1, \ldots, x_i, \ldots, m_k),$$

whence it follows, by (9), that

$$(x_1, \ldots, m_i, \ldots, x_k) \text{ beats } (x_1, \ldots, x_i, \ldots, x_k) = x \qquad \text{Q.E.D.}$$

Although stated in terms of vote trading, this theorem really is more general than such language implies. For the effect of vote trading can be achieved by making a "package" motion to begin with – one that combines positions from different issues – and virtually all legislative motions are packages of some sort.

Like the spatial instability theorems, this one rests on highly restrictive assumptions. Although majority rule was not assumed, (8) required a unique no-trade outcome (no ties), whereas (9) required separability of collective preference – which means, for example, that a committee of banquet planners who collectively prefer red wine to white with meat must also prefer red wine to white with fish. And implicit in the statement of the theorem was the assumption that every outcome – every combination of positions on the k issues – is feasible (since otherwise the outcome that beats x might not be feasible).

To be sure, nothing prevents us from interpreting each issue as a set of *feasible positions*. But a *combination* of feasible positions (an outcome) need not itself be feasible. A position on one issue that exhausts a given budget and a position on a second issue that exhausts that budget are each feasible (affordable), but their combination is not. Constitutionally, lengthy confinement and hanging may be singly feasible as penalties for a crime but not jointly feasible (it may be "cruel and unusual" to lock someone up for 30 years and then hang him).

The Universal Instability Theorem of Schwartz (1981; 1985; 1986, sec. 11.2) generalizes this last theorem by dropping the separability and no-tie assumptions and allowing the feasible set to be *any set of outcomes whatever*. Details are beyond the scope of this survey.

STRATEGY

Although collective choices depend on preferences and institutions (voting procedures), democratic institutions are not algorithms into which we can simply plug preferences and reckon final outcomes. For collective choices also depend on strategic maneuvers of various sorts.

Strategic voting

Voting rules transform votes into choices, but preferences alone do not always determine votes. Voters sometimes have an incentive to misrepresent their true preferences – to vote *strategically* rather than *sincerely*, thereby "manipulating" the voting rule – in order to secure a collective choice they prefer to that which would otherwise have been made. Three examples follow.

Example 1. In a congressional primary, a candidate with a majority of votes wins, but if none has a majority, a runoff is held between the top two candidates (the two with the most votes). There are eight voters (or equal-size groups) with the following preference orderings of three candidates:

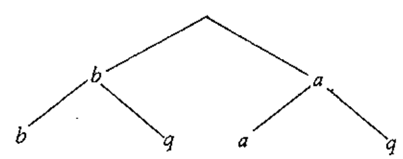
Mr. 1	Mr. 2	Mr. 3	Mr. 4	Mr. 5	Mr. 6	Mr. 7	Mr. 8
x	x	x	y	y	z	z	z
z	z	y	x	x	y	y	y
y	y	z	z	z	x	x	x

If everyone votes sincerely, Messrs. 1–3 will vote for x, Messrs. 4 and 5 for y, and Messrs. 6–8 for z, so that no candidate has a majority and x and z enter a runoff, which x wins. Since Messrs. 6–8 prefer y to x, however, they have an incentive to misrepresent their true preference and vote strategically for y. If even one of them does so, y rather than z will join x in the runoff, and since a majority prefer y to x, y will win.

Example 2. A three-member legislature or committee have the following "voting paradox" preference orderings of a bill b, an amended version a, and the status quo q:

Mr. 1	Mr. 2	Mr. 3
b	a	q
a	q	b
q	b	a

There are two votes, or *divisions*: the first between b and a, the second between the winner in that contest and q. This agenda order is represented by the following agenda tree (or "extensive form" of the "voting game"):



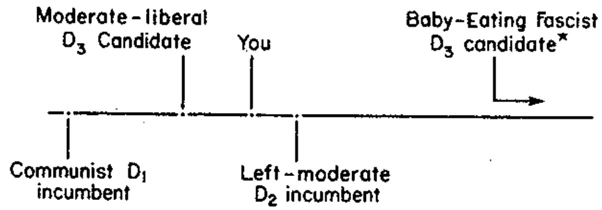
The first level of this tree represents the vote (division) between b and a; the second, the vote between q and either b or a – whichever wins in the first vote. If everyone votes sincerely, b wins at the first division (since it beats a) and q at the second, so that q is the final choice. But if Mr. 1, who likes b best but prefers a to q, votes strategically for a rather than b at the first division, a wins the first vote and goes on to win the second.

If one can play this game, so can all. Suppose everyone votes strategically. Then the three voters will not necessarily compare b with a at the first division. Instead, they will look ahead to the consequence of choosing b and the consequence of choosing a. If b were chosen at the first division, q would be chosen at the second, and if a were chosen at the first division, it would win at the second as well. So a voter will vote for b at the first division if he prefers q to a, and for a if he prefers a to q. Let us summarize this by saying that q is the *strategic equivalent* of b, and a of itself, at the first division. Because a beats q, a would win in the end if everyone voted strategically.

In general, if there are k divisions, the strategic equivalent of an alternative x at the $(k-1)$th division is the collectively preferred of the two alternatives that would be compared at the kth division if x were chosen at the $(k-1)$th, the strategic equivalent of x at the $(k-2)$th division is the collectively preferred of the strategic equivalents of the two alternatives that would be compared at the $(k-1)$th division if x were chosen at the $(k-2)$th, and so on up to the first division. When everyone votes strategically, then, the final choice is the collectively preferred of the strategic equivalents of the two alternatives compared at the first division. We can find it by reading the agenda tree from the bottom up. To do so, we need only know the collective preference, not the individual orderings. On the other hand, the assumption of uniform strategic voting requires voters to have a great deal more knowledge than they may actually have in many cases. (On agendas and strategic voting, see Farquharson 1970; McKelvey and Niemi 1978.)

Example 3. There are three legislative districts, D_1, D_2, and D_3. It is certain that D_1 will reelect its popular incumbent, a Communist, and that D_2 will reelect its left-of-center moderate incumbent. In D_3, your district, there are two candidates, a moderate liberal and a Baby-Eating Fascist, celebrated for having ridiculed Ghengis Khan as a mealy-mouthed pinko. Your own ideal point lies between those of the left-moderate D_2 incumbent and the moderate-liberal D_3 candidate, and it is slightly closer to that of the D_2 incumbent, as shown in Figure 12.5. If the moderate-liberal wins D_3, his ideal position becomes the legislative median and, therefore, wins in any legislative vote. But if the Fascist wins D_3, the left-moderate D_2 incumbent's ideal position becomes the legislative median, hence the winning position in the legislature. Therefore, since you prefer the left-moderate position to the moderate-liberal position, you have an incentive to vote strategically for the Fascist, whom you loathe, rather than the moderate liberal, whom you prefer.

Vote trading, too, is a kind of strategic voting. By contrast with the examples just above, it is *cooperative* rather than *individual* strategic voting.



* Unfortunately, limitations of space make it impossible to display the position of the Baby-Eating Fascist without making the rest of the diagram submicroscopic.

Figure 12.5

The extent of manipulability

Is the possibility of manipulating an outcome by individual strategic voting peculiar to certain voting rules? Can a voting rule be devised that always elicits truthful statements of preferences – a rule under which no one can, by misrepresenting his preferences (by voting strategically), thereby secure a collective choice he prefers to that which would have been reached had he voted sincerely?

It seems not. To see why, let all profiles comprise only *linear* orderings – ones in which no two alternatives are ranked at the same level. Consider any voting rule applicable to all finite, nonempty subsets of a given universal set of alternatives and satisfying Arrow's conditions (1) ($\geqslant 3$ alternatives), (2) (Unrestricted Domain), and (3) (Nondictatorship), plus three more:

(4′) If x belongs to a set A of two or more alternatives and *everyone* prefers *every other* member of A to x, then the collective choice from A remains unchanged when x is deleted from A.

(7*) The rule prescribes a unique choice from every finite, nonempty set of alternatives (i.e., there never exist two or more permissible choices) (Resoluteness).

(10) Suppose x_1 is the collective choice from A under a profile p_1, and x_2 the choice under a profile p_2 that differs from p_1 only in Mr. i's preference ordering. Then x_2 is not preferred to x_1 according to Mr. i's p_1 ordering, and x_1 is not preferred to x_2 according to Mr. i's p_2 ordering (Nonmanipulability).

Condition (4′) is obviously a bit stronger than (4) (Pareto Principle). Because (7*) rules out multiple permissible choices in general, it does so in particular for two-member feasible sets, which is to say that it rules out all *ties*. Thus, (7*) is even stronger than (7). Condition (10) says that the

voting rule is *nonmanipulable*: no individual can, by misrepresenting his preferences (as his p_2 ordering rather than his "true" p_1 ordering, say), thereby secure a choice (x_2) he prefers to that which would otherwise have been reached (x_1).

These conditions are inconsistent: any voting rule satisfying (1)–(3), (4′), and (7*) must be manipulable.

We can prove this by deducing Arrow's conditions (1)–(6), which we know to be inconsistent, from (1)–(3), (4′), (7*), and (10). Since (1)–(3) are among Arrow's conditions and (4) obviously follows from (4′), it suffices to deduce (5) (Independence) and (6) (Transitivity).

To deduce (5), suppose x beats y, and let Messrs. $1, 2, \ldots, n$ change their preference orderings any way you please but *without* altering their preferences *between x and y*. What must be shown is that x still beats y after the change. Suppose on the contrary that y beats x after the change ((7*) rules out ties). Let Mr. i be the first voter (in order from Mr. 1 to Mr. n) whose change of preference ordering reverses the collective preference. Then by (10), y cannot be preferred to x according to Mr. i's original ordering and x cannot be preferred to y according to his new ordering. Hence, since the relative positions of x and y are the same in both orderings, neither alternative is preferred to the other according to Mr. i's original ordering. But that is impossible since preference orderings are linear and $x \neq y$. Q.E.D.

To deduce (6) (Transitivity), recall from our discussion of Arrow's Theorem that, in the absence of ties (which (7*) rules out), (6) is violated only if there is a *tricycle* under some profile. Suppose, contrary to (6), that some profile yields a tricycle:

$$\begin{tikzcd}
& y \arrow[dr] & \\
x \arrow[ur] & & z \arrow[ll]
\end{tikzcd}$$

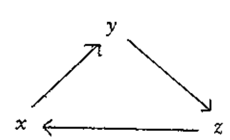
By (7*), the voting rule chooses one alternative, say x, from among the three. Change the profile by pushing y to the bottom of everyone's ordering. I will first show that x is still the collective choice. Suppose not. Let Mr. i be the first voter (in order from Mr. 1 to Mr. n) whose change of preference ordering changes the collective choice. By (10), the new choice cannot be preferred to x according to Mr. i's original ordering, and x cannot be preferred to the new choice according to Mr. i's new ordering. Hence, since nothing was raised above x in constructing Mr. i's new ordering, x cannot be preferred to the new choice according to Mr. i's original ordering either. So neither x nor the new choice is preferred to the other according to Mr. i's original ordering. But that is impossible since the new choice is different from x. Consequently, x remains the collective choice after y is pushed to the bottom of everyone's preference ordering. Thus, by (4′), x remains the collective choice under the new profile when y is deleted, that is, x beats z under the new profile. But that is impossible, by (5), since z beats x under the original profile and voter preferences between z and x are the same in both profiles. Q.E.D.

The general manipulability of voting rules was first demonstrated by Gibbard (1973) and Satterthwaite (1975). Although their theorem differs in detail from the one just proved, it too assumes (7*) (Resoluteness), an exceedingly severe condition. I know of no real-world voting rule that satisfies (7*). Interestingly, the *only* way to manipulate plurality rule, the most common election rule in English-speaking countries, is to make or break a "generalized tie" – a multiple permissible choice, proscribed by (7*). Condition (7*) rules out much of the real world. However, Schwartz (1982*a*) proved the above theorem without (7*) or any weakened version thereof – adding some mild assumptions about individual preferences between multimembered sets of alternatives. Details are beyond the scope of this paper.

Agenda manipulation

Besides preferences, institutions (or rules), and voting strategies, collective choices depend on strategies of another sort: the manipulation of *agendas*. Committee chairmen, legislative leaders, and other officials can sometimes affect collective choices by their control, partial or complete, of agendas. This can take three forms: (i) *set manipulation* – manipulating a choice by controlling what gets on the agenda (which alternatives qualify as feasible), (ii) *order manipulation* – manipulating a choice by controlling the order of voting, and (iii) *question-manipulation* – manipulating a choice by combining legislative items in a single question or dividing an item into separate questions. Let us examine each in turn.

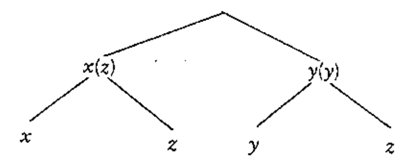
Set manipulation. Suppose you are able to decide what gets on an agenda. Obviously, then, by keeping something off, you can prevent its choice. Obviously, too, you can ensure the choice of one alternative or prevent the choice of another by placing on the agenda an alternative that would be chosen if included. Less obviously, you can ensure or prevent the choice of a given alternative by including or excluding another alternative that would not be chosen anyway. Suppose there is a cycle:



And suppose x would be chosen if all three alternatives were feasible. Then by excluding y, you can prevent x's choice since z beats x – which means that z would be chosen if x and z alone were feasible. Likewise, by including y you can ensure the choice of x rather than z.

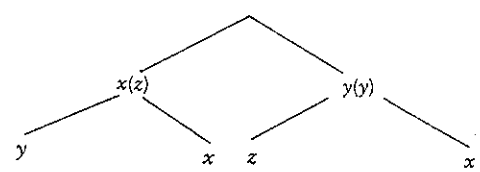
The conditions under which set manipulation is minimized are surprisingly severe (Schwartz 1976, 1985*a*, sec. 10.2). Among other things, they require Arrow's full Transitivity condition. As you just saw, the existence of a cycle gives the set manipulator considerable power.

Order manipulation. Suppose the cycle above arises under majority rule. And suppose the agenda, or voting order, pits x against y, then the winner in that contest against z, as in this agenda tree:



Strategic equivalents are listed in parentheses: at the first division, z is the strategic equivalent of x, and y the strategic equivalent of itself, because z would win at the second division if x were chosen at the first whereas y would win at the second division if chosen at the first. If everyone voted sincerely, x would win at the first division and z at the second. If everyone voted strategically, y would win at the first division (since y beats z) and go on to final victory.

Now consider a different order of voting: y against z, then the winner against x.



Under sincere voting, y would win at the first division, x at the second. Under strategic voting, z would win at the first division (z beats x) and also the second. So under sincere voting, the first agenda yields z whereas the second yields x, and under strategic voting, the first agenda yields y whereas the second yields z. He who controls the order of voting partly decides the outcome.

Question manipulation. Suppose an agriculatural price support measure and a food stamp measure are each supported by a minority and the two minorities together are a majority. Then both measures can pass by means of a vote trade. The same effect might be achieved by a legislative committee that reports both measures in a single bill (at least under a closed rule). That is one kind of question manipulation. Alternatively, the defeat of both measures might be secured (assuming no vote trades) by reporting them as separate bills or by a motion or parliamentary ruling to "divide the question" – the other kind of question manipulation.

DEMOCRATIC FAILURES

Despite the ostensibly majoritarian character of our electoral and congressional voting rules, these rules can produce choices that seem antidemocratic – contrary to the "popular will." A *Condorcet winner* is a feasible alternative that beats all others under majority rule. The classical voting paradox shows that there does not always exist a Condorcet winner. Sometimes, however, a Condorcet winner does exist but is not chosen: the actual choice is opposed by some majority although an alternative choice is unopposed. A *Pareto-efficient* feasible alternative is one to which no other feasible alternative is unanimously preferred. Sometimes the collective choice is Pareto *in*efficient: every voter would be happier with an alternative choice. I offer five examples.

Example 1. Liberal, moderate, and conservative candidates contest a congressional seat. The liberal receives the most votes, the moderate the fewest, none a majority. Under plurality rule, the liberal wins. Taken together, however, the moderate and conservative voters are a majority who prefer the moderate to the liberal, whereas the moderate and liberal voters are a majority who prefer the moderate to the conservative. So the moderate candidate, who lost, is the Condorcet winner. And if a runoff were held between the top two candidates (the liberal and conservative), the Condorcet winner would still lose. Just this pattern of voting occurred in the 1972 Chilean presidential election and, with the conservative receiving the most votes, in the 1970 U.S. senatorial election in New York State: in each case the moderate candidate – the apparent Condorcet winner – received the fewest votes. Such squeezing out of the middle also is a frequent feature of U.S. presidential primaries.

Example 2. As we saw in the first section, even if only two parties run in each congressional district, it can happen that one receives a majority of votes nationwide while the other receives a majority of votes in a majority of districts, thereby winning control of Congress, contrary to the party preference of a majority of voters (who may, however, care more about individual candidates than parties).

Example 3. A three-member legislature votes on three bills, a, b, and c, to which the members assign values, in millions of dollars, as follows:

	Mr. 1	Mr. 2	Mr. 3
a	4	4	−9
b	4	−9	4
c	−9	4	4

Since each bill benefits some majority, all three pass. As a result, everyone loses \$1 million ($4 + 4 - 9$). This outcome is Pareto inefficient: everyone prefers the defeat of all three bills to the passage of all three.

The example is similar to the extreme case of a market failure: each action benefited its participants (the winning majority) but imposed an external cost on the nonparticipant (the losing minority); and although everyone participated in some actions, the external cost he bore from the action in which he did not participate outweighed the internal benefits he received from those in which he did participate.

It is essential to the example that each bill represents a project or program that is not cost effective: the aggregate cost exceeds the aggregate benefit. It seems reasonable, however, for each winning majority to raise the scale of its project above a cost-effective level because the losing minority subsidizes the cost.

It is worth noting that, although regulation by majority-ruled government is often prescribed as the cure for market failures, majority rule always imposes external costs on losing minorities except when majorities are unanimous (whereas the *requirement* of unanimity would increase the incidence of private-sector externalities). It is worth noting, too, that a Pareto-efficient outcome can be achieved in the example by a three-man vote trade (resulting in the defeat of all three bills) or a two-man trade (resulting in the passage of one bill, and with it a much greater external cost). Party discipline also would secure the passage of just one bill.

Example 4. A legislature votes on a bill and two amendments, x and y. four outcomes are possible:

- b: the bill without amendments
- bx: the bill perfected just by x
- by: the bill perfected just by y
- bxy: the bill perfected by both amendments
- q: the status quo ante

At the first division, b is pitted against bx. If b wins, it is pitted against by at the second division, and if bx wins, it is then pitted against bxy. At the third division, the second-division winner is pitted against q. Every voter prefers *by* to *bxy* to *bx* to *b* to *q*. So *by* is the Condorcet winner, and every other alternative is Pareto inefficient. The agenda tree, with strategic equivalents in parentheses, is shown in Figure 12.6.

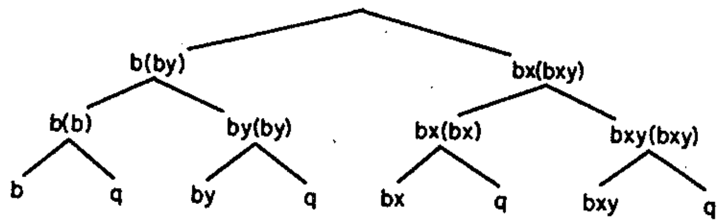


Figure 12.6

Under sincere voting, *bx* wins at the first division and *bxy* at the second and third: the Condorcet winner is rejected and a Pareto-inefficient alternative chosen.

To be sure, this particular affliction can be cured by strategic voting: *b* wins at the first division (because *by* beats *bxy*), and *by*, the Condorcet winner, wins at the second and third divisions. But consider:

Example 5. A three-member legislature is to choose among seven alternatives:

c: a bill
a: *c* perfected by an amendment
b: *c* perfected by a substitute amendment
x: a substitute bill
y: *x* perfected by an amendment
z: a substitute for *x*
q: the status quo ante

According to Rule 14 of the U.S. House of Representatives (on congressional procedures, see Sullivan 1984), the order of voting is as follows:

1st division: *a* versus *b*
2nd division: winner at 1st division versus *c*
3rd division: *x* versus *y*
4th division: winner at 3rd division versus *z*
5th division: winner at 2nd division versus winner at 4th division
6th division: winner at 5th division versus *q*

Here are the legislators' preference orderings and the relation of majority preference:

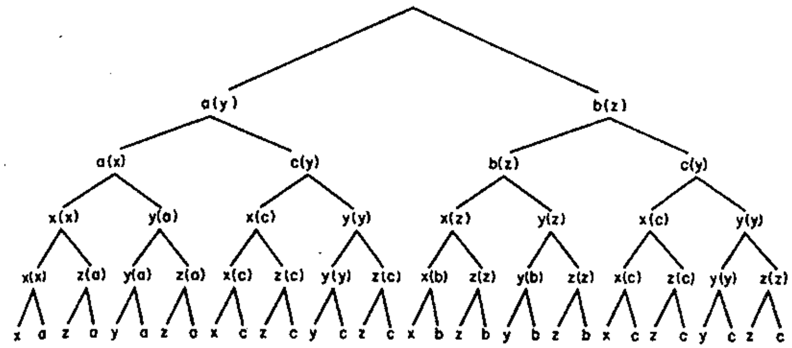


Figure 12.7

Mr. 1	Mr. 2	Mr. 3	Majority preference
a	*b*	*x*	*a*
c	*y*	*a*	*c*
z	*c*	*z*	*z*
b	*x*	*y*	*b*
y	*a*	*c*	*y*
x	*z*	*b*	*x*
q	*q*	*q*	*q*

Majority preference is depicted by an arrow and, in the absence of any arrow, by height on the page. The agenda tree, with strategic equivalents in parentheses, is shown in Figure 12.7. I omitted the representation of the sixth division because *q* is beaten by every other alternative. The final choice is *z* under strategic voting. But *z* is Pareto inefficient: everyone prefers *a* to *z*. Interestingly, *a* would have been chosen under sincere voting. (Much recent research [esp. Banks 1985, Miller 1980, Shepsle and Weingast 1984] has argued that legislative agendas are better behaved under strategic voting than this example shows, yielding outcomes that are, among other things, Pareto efficient. The mistake seems attributable to an overly restrictive formal assumption about the class of binary trees that can represent legislative agendas. On congressional agendas, see Ordeshook and Schwartz 1987.)

CONCLUSION

That the institutions of representative democracy are prone to instability and manipulation does not mean that political analysis is impossible, only that good analysis depends, to a greater degree than some may have thought, on the fine details of institutions, political alignments, and strategic opportunities. Despite producing outcomes that ostensibly flout the popular will, these institutions may be far preferable, even from a strictly democratic point of view, to the feasible alternative institutions. On the other hand, a Marxist or other critic of representative democracy might make clever use of some of the anomalies discussed above; the debate would be interesting. My own reaction is that the apparent shortcomings of voting procedures are the price of securing the Madisonian goal of "republican government," which is not to reckon the "popular will" but to prevent tyranny: the easier it is to alter and manipulate political outcomes – the greater the opportunities for realignment and strategic maneuver – the less concentration of power there will be (cf. Riker 1982, pp. 233–53).

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