AGENDAS AND THE CONTROL OF POLITICAL OUTCOMES

PETER C. ORDESHOOK
THOMAS SCHWARTZ
*University of Texas
Austin*

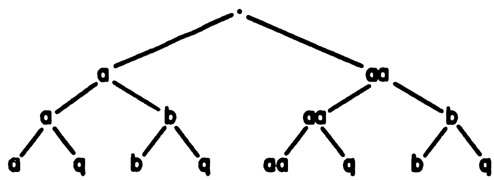
A considerable theoretical literature argues that if everyone votes sincerely, then an agenda setter has near dictatorial influence on final outcomes, whereas if everyone votes strategically, then an agenda setter's power is considerably reduced. This literature assumes that all feasible agendas are of a special type called amendment agendas. *But actual legislative and committee agendas—notably those found in Congress—often are not of this type. Once we expand the domain of feasible agendas to include all types allowed by common parliamentary practice, the influence of agendas on legislative outcomes expands, even with strategic voting. Besides showing with counterexamples that previous results do not extend to a more realistic domain of agendas, we prove some theorems that specify the limits (such as they are) of an agenda setter's power.*

Beginning with Farquharson (1969), a growing literature has investigated the interplay between agenda structure and voting strategy in legislatures and committees by examining an agenda setter's ability to manipulate outcomes and the ability of strategic voters to offset an agenda setter's power. The results of these investigations appear to support the following picture. If voters act *sincerely* rather than *strategically*—if at every vote they express their true preferences between alternatives instead of looking ahead to final consequences—an agenda setter is a near dictator: the set of outcomes achievable by different agenda forms is comparatively large (McKelvey 1979, Miller 1980, Schwartz 1986). But if voters act strategically, an agenda setter's power is not so great: the set of achievable outcomes is generally smaller (Banks 1985, Miller 1980, Shepsle and Weingast 1984).

Some have taken such results to measure the extent to which legislative decisions are at the mercy of elites who control agendas (Riker 1982). But even if no individual or small group controls agendas—even if the agenda-setting process is open and democratic—the power of a *hypothetical* agenda setter is important for political analysis. It is important because it reflects the degree to which outcomes depend on procedural structure rather than votes, hence the degree to which the real legislative game occurs at the agenda-setting stage rather than the voting stage.

The utility of these results is impaired, however, by the unrealistic assumption that feasible agendas are all of a special type called *amendment agendas.* Such agendas work as follows: a sequence of alternatives is given, and a vote is taken between the first two, after which the winner is pitted against the third alternative, then the winner in this second

Figure 1. A Four-Alternative Amendment Agenda



ballot against the fourth alternative, and so on. Suppose, for example, that just three motions are on the floor of the U.S. House or Senate: a bill, an amendment to the bill, and an amendment to the amendment. Then there are four possible outcomes:

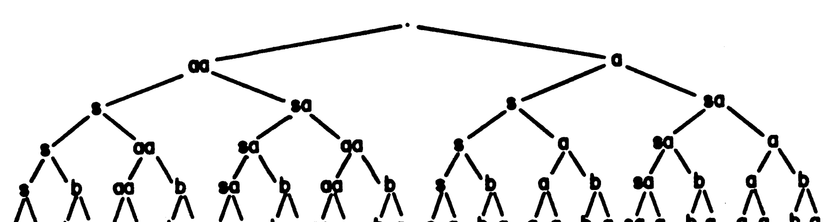
q: the status quo ante
b: the bill unchanged
a: the bill changed by the original amendment
aa: the bill changed by the amended amendment

For these alternatives, congressional procedure requires an amendment agenda in which the first vote is between *a* and *aa* (whether to amend the amendment), the second between the winner and *b* (whether to amend the bill), and the third between the winner and *q* (whether to pass the final form of the bill). We represent this agenda by the tree in Figure 1.

Not all realistic agendas are of this sort, however. Suppose, as in Riker's (1958) account of the 1953 Agricultural Appropriations Act, that the following motions are introduced in the U.S. House: a bill, an amendment to the bill, an amendment to the amendment, a substitute amendment, and an amendment to the substitute. Rule XIV of the House requires that members perfect the amendment and the substitute amendment before deciding whether to substitute and then whether to amend the bill. Hence, the agenda, shown in Figure 2, first pairs the bill perfected by the original amendment (*a*) with the bill perfected by the amended amendment (*aa*). But then, unlike an amendment agenda, this one does not pair the winner of the first ballot with anything in the second stage. Instead, the bill perfected by the substitute amendment (*s*) is paired with the bill perfected by the amended substitute amendment (*sa*). The next vote pairs the winners in these first two votes, and the winner in this vote is then paired with *b*. The final vote pits the status quo (*q*) against the survivor of all previous votes—or this vote is postponed until other sections of the bill have been considered (see Bach 1981, Sullivan 1984).

The difference between the agendas in Figures 1 and 2 is subtle. In Figure 1 the winner at one stage enters the balloting in the next, but in Figure 2 *s* and *sa* are paired in the second ballot regardless of which alternative wins in the first. Are such differences in form merely stylistic, or do they affect the power of agendas and agenda setters and the efficacy of strategic voting?

Figure 2. A Congressional Agenda with Substitutes



Our principal conclusion is that by focusing on amendment agendas, the literature conveys an inaccurate picture. Quite a variety of agendas is available in Congress and other institutions, and few of the theorems about amendment agendas apply generally. The critical assumption of previous results concerning the sensitivity of outcomes to agendas is not that voters are strategic or sincere but that all feasible agendas are amendment agendas. One might suppose that results about amendment agendas fill in part of the map, applying to an important class of procedures. The assumption commonly made, however, is not that the actual agenda is of the amendment type nor that the set of feasible agendas contains amendment agendas but that this set contains *only* amendment agendas, and that assumption is rarely true.

After surveying common agenda forms in the first section, we draw some structural distinctions among agendas in the second section, then define strategic and sincere voting in the third section. The fourth section is a review of previous results based on the amendment-agenda assumption. In the fifth section we show with counterexamples that these results no longer hold when the amendment-agenda assumption is relaxed to allow for other agenda forms. In the sixth section we state several theorems (proved in the Appendix) about the actual limits of a hypothetical agenda setter's power under sincere and strategic voting.

Examples of Alternative Agenda Forms

Figure 2 depicts a common type of congressional agenda, but even more complex agendas are possible (see Bach 1981). Suppose the following eight motions are introduced: a bill, an amendment to perfect the bill, an amendment to perfect the amendment, a substitute amendment, a substitute bill, an amendment to perfect the substitute bill, a substitute for the substitute bill, and an amendment to perfect the latter substitute. These motions give rise to nine outcomes:

- *q:* the status quo
- *b:* the bill unchanged
- *a:* the bill perfected by the amendment
- *aa:* the bill perfected by the amended amendment
- *sa:* the bill perfected by the substitute amendment
- *sb:* the substitute bill
- *asb:* the amended substitute bill
- *ssb:* the substitute for the substitute bill
- *assb:* the amended substitute for the substitute bill

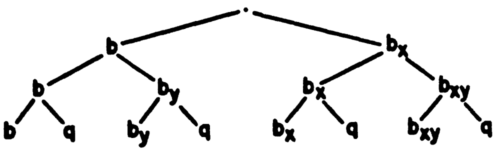
The voting order is as follows (Sullivan 1984):

1. *a* vs. *aa*
2. winner of 1 vs. *sa*
3. winner of 2 vs. *b*
4. *sb* vs. *asb*
5. *ssb* vs. *assb*
6. winner of 4 vs. winner of 5
7. winner of 3 vs. winner of 6
8. winner of 7 vs. *q*

We recoil from drawing the tree for this agenda, but note that this and the previous example both fail to qualify as amendment agendas because of *discontinuities*—steps at which the winner of the previous vote does not enter the balloting but is set aside for later consideration. In the last example, the alternatives paired in the fourth and fifth ballots are independent of all previous winners.

The existence of discontinuities is not the only way agendas in Congress and other institutions can differ from amendment agendas. Notice that the agendas in Figures 1 and 2 share two properties: First, they are *symmetric*—that is, each tree splits into two subtrees that are alike except for the interchange of alternatives (*a* and *aa*) occupying their top nodes;

Figure 3. A Two-Period Congressional Agenda



each of these subtrees splits in the same way, and so on down the agenda. Second, each is a *one-period* agenda—that is, all motions are on the floor before any are voted on. In a multiperiod agenda, by contrast, some motions are voted on before others are recognized. For example, suppose that, in addition to b and q, we also have

b_x: the bill perfected by amendment x
b_y: the bill perfected by amendment y
b_{xy}: the bill perfected by both amendments

In this case standard parliamentary procedure requires a two-period agenda in which x is voted up or down before y is recognized. Depicted in Figure 3, this agenda requires three comparisons between alternatives:

1. b vs. b_x
2. $\left\{ \begin{array}{l} b \text{ vs. } b_y \text{ if } b \text{ wins at 1} \\ b_x \text{ vs. } b_{xy} \text{ if } b_x \text{ wins at 1} \end{array} \right.$
3. winner of 2 vs. q

Besides being multiperiod, the agenda in Figure 3 differs from those in Figures 1 and 2 in that it is not symmetric. The tree branches into two subtrees that are dissimilar even apart from the interchange of b and b_x: one contains b_y where the other contains b_{xy}. This is because either b_y or b_{xy} enters the balloting at the second round, but *which* of them does so depends on which alternative—b or b_x—wins at the first round. We call the agenda in Figure 3 a *first-order* agenda because it involves only first-order amendments (amendments to the original motion) and because these amendments are compatible (one outcome can contain all of them). Every first-order agenda is continuous, but it is not symmetric unless it involves just one amendment.

The agendas examined thus far are *uniform,* by which we mean that all branches in their trees have the same length. But even uniform agendas do not exhaust the realistic types. Consider a *sequential-elimination* agenda, in which a new motion is introduced and voting proceeds to the next stage only if the current lmotion fails. Illustrated by Figure 4, such agendas are neither uniform nor symmetric. In Farquharson's celebrated narrative (1969), Pliny sought to discourage a sequential-elimination agenda in favor of the plurality vote. Millennia later, such agendas are observed when parliamentary bodies such as the U.S. Senate vote on personnel questions or when members are obliged to take some action (to adopt a budget, say) and so must keep voting on proposed actions until one passes.

Other examples of nonuniform and hence nonsymmetric agendas include those with a succession of "killer amendments," as in Enelow and Koehler's discussion of the Panama Canal Treaty (1980). Opponents of the treaty introduced a series of seemingly innocuous amendments solely to make the treaty's wording differ in some way from the text that had been negotiated with Panama. If any one of these motions had passed, the

Figure 4. A Sequential-Elimination Agenda

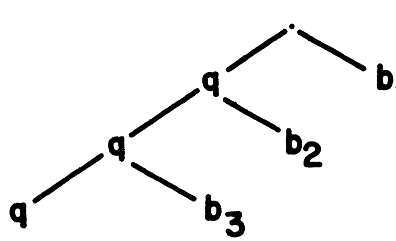
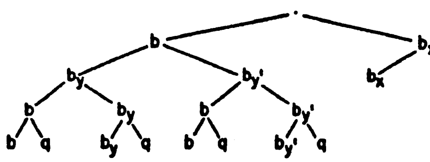


Figure 5. A Hybrid Agenda



others would not have been introduced. We can illustrate this situation with two first-order amendments and, for good measure, a second-order amendment. The agenda, depicted in Figure 5, comprises five alternative outcomes:

q: the status quo
b: the bill
b_x: the bill amended by x
b_y: the bill amended by y in its original form
$b_{y'}$: the bill amended by the amended version of y

Neither symmetric nor uniform nor continuous, this hybrid agenda suggests that our examples only begin to uncover the various forms of agendas that should interest political scientists. At any rate, they show that amendment agendas do not exhaust the possibilities.

Our ability to formulate nonamendment agendas raises the question, how prevalent are amendment agendas in practice? Although two-motion agendas can be of the sequential-elimination type, the most commonly observed two-motion agendas involve an initial motion and one amendment, and they are of the amendment variety. But consider agendas based on three motions: a bill, b, and two amendments, x and y, where x is recognized before y. If the moving of y depends on whether x passes, we cannot have an amendment agenda. So suppose y is moved regardless of the action taken on x. Then if y is a second-order amendment (Figure 6a) or a complete substitute for b (Figure 6b), the agenda is of the amendment type. But if y is a first-order perfecting amendment (Figure 6c), the agenda is not an amendment agenda. Hence, an agenda based on three motions is an amendment agenda only if (1) the third motion is made and recognized regardless of whether the second passes *and* (2) the second motion is either a second-order amendment or a complete substitute.

First-order agendas, like that of Figure 6c, are arguably the most common in most committees. The impression, however, that amendment agendas are especially prevalent might arise from a confusion between *motions* and *alternatives*. For example, suppose there is an initial motion, to pass bill b, and two first-order perfecting amendments, x and y. If the second motion is recognized regardless of the action taken on the first, then the usual language of committee deliberations, which makes reference to motions rather than final outcomes, leads to the impression that the agenda is that of Figure 6d, an amendment agenda. However, because x and y are compatible motions rather than alternative outcomes, the correct agenda is the one shown in Figure 6c, which is not an amendment agenda.

A Classification of Agendas

In this section we formally define several important categories of agendas, most of them informally introduced in the

Figure 6. Contrasting Agendas

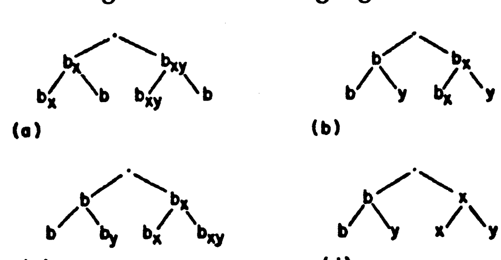
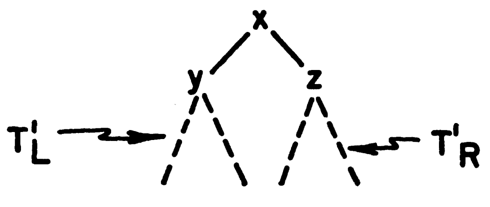


Figure 7. Agenda T′



previous section. We begin by letting X be a set of alternative outcomes, letting x, y, z, a, b, and so forth denote elements of X, and letting α, β, and so forth denote finite, nonempty subsets of X. We represent agendas by binary trees:

DEFINITION. *An* agenda *on α is a finite binary tree T such that (1) every node (vertex) of T is occupied by some member of α, (2) every member of α occupies some bottom node of T, and (3) $y \neq z$ for every subtree T' of T of the form shown in Figure 7.*

Condition (1) requires that every fork in the tree represent a choice between two alternatives and (as a mathematical convenience, which we disregard in most figures) that even the top node be occupied by some alternative (an arbitrary one will do). Condition (2) ensures that there is some path to every outcome in α, and (3) rules out "nonchoices" of the form y *vs.* y.

If an agenda T' has more than one node, then it has at least three *subtrees:* T' itself and its *left* and *right principal subtrees,* T'_L and T'_R, shown in Figure 7. Note that a principal subtree with more than one node splits into its own principal subtrees.

Now consider the two agendas in Figure 8. They are the same except for the left-right order of subtrees and alternatives at various nodes. Because they embody the same sequence of comparisons and must yield the same outcome, we call them *equivalent.* Here is a recursive definition of this relation:

DEFINITION. *If the Agenda T_1 has but one node, then T_1 is* equivalent *to T_2 if and only if $T_1 = T_2$. If T_1 has more than one node, then T_1 is equivalent to T_2 if and only if one principal subtree of T_1 is equivalent to one principal subtree of T_2, while the other principal subtree of T_1 is equivalent to the remaining principal subtree of T_2.*

To develop our classification of agendas, we begin with a type exemplified by the agendas in Figures 1 and 2:

DEFINITION. *A* symmetric *agenda on α is an agenda T on α such that, for every subtree T' of T of the form shown in Figure 7. T'_L is equivalent to the result of substituting y for z in T'_R, and T'_R is equivalent to the result of substituting z for y in T'_L.*

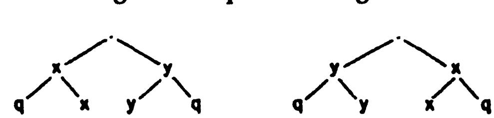
All symmetric agendas have three other important properties. First, they are nonrepetitive, a property commonly required by legislative rules. This means that they prevent an alternative once rejected from coming up again for a vote.

DEFINITION. *A* nonrepetitive *agenda on α is an agenda T on α such that, for every subtree T' of T of the form shown in Figure 7, y does not occur in T'_R and z does not occur in T'_L.*

Second, symmetric agendas on α are *complete* in the sense that they guarantee every alternative in α its day in court: whether an alternative gets to be voted on at a given stage does not depend on the choice made at the previous stage.

DEFINITION. *A* complete *agenda on α is an agenda on α such that, for every subtree T' of the form shown in Figure*

Figure 8. Equivalent Agendas



7, T'_L contains every alternative occurring in T'_R with the possible exception of z, and T'_R contains every alternative occurring in T'_L with the possible exception of y.

Figures 1 and 2 depict complete agendas; Figures 3–5, incomplete ones. Although all symmetric agendas are complete, not all complete agendas are symmetric. For example, the following agenda is complete but not symmetric: x and y are paired; if x wins, it is paired with z and the winner with w; but if y wins, z and w are paired and the winner paired with y.

The third property that symmetric agendas satisfy is uniformity:

DEFINITION. *A* uniform *agenda on α is one whose branches are all of the same length.*

Figures 1 and 2 depict symmetric, hence uniform, agendas; Figure 3, a uniform, nonsymmetric agenda; and Figures 4 and 5, nonuniform agendas.

The following theorem (proved in the Appendix) summarizes the properties of symmetric agendas:

THEOREM 1. *All symmetric agendas are nonrepetitive, complete, and uniform.*

A fifth type of agenda, introduced in the previous section, does not encompass all symmetric agendas:

DEFINITION. *A* continuous *agenda on α is an agenda T on α such that, for every subtree T′ of T of the form shown in Figure 7, $x = y$ or $x = z$.*

Figure 1 depicts an agenda that is symmetric and continuous; Figure 2, symmetric but not continuous; Figures 3 and 4, continuous but not symmetric; and Figure 5, neither.

A final category, included in all five preceding ones, is that of amendment agendas:

DEFINITION. *An* amendment agenda *is an agenda that is equivalent to $A(x_1, \ldots, x_n)$ for some $x_1, \ldots, x_n$, where $A(x_1, \ldots, x_n)$ is defined by the following recursion: $A(x_1)$ is the 1-node tree consisting of x_1. $A(x_1, x_2, \ldots, x_{k+1})$ is the tree T in which x_1 occupies the top node and $T_L = A(x_1, x_3, \ldots, x_{k+1})$ while $T_R = A(x_2, x_3, \ldots, x_{k+1})$.*

The intuition here is that an amendment agenda is determined by a sequence $(x_1, \ldots, x_n)$ such that x_1 is first pitted against x_2. If x_1 wins, we move into the subtree T'_L, which is determined by the same sequence, less x_2; but if x_2 wins, we move into T'_R, which is determined by $(x_2, x_3, \ldots, x_n)$.

Amendment agendas are nonrepetitive, complete, uniform, and continuous. All symmetric agendas have the first three properties, but

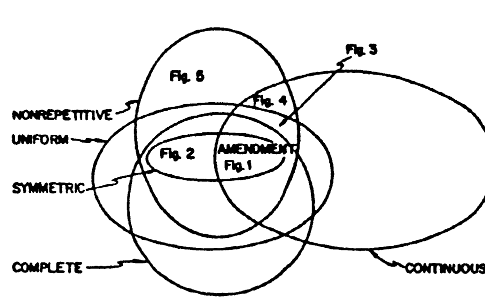
THEOREM 2. *All and only amendment agendas are symmetric and continuous.*

The Venn diagram in Figure 9 summarizes our classification, Theorems 1 and 2, and the easily proved fact that all complete and nonrepetitive agendas are uniform. (Although we have not supplied examples for every cell, it is easy to do so.)

Sincere and Strategic Voting

To define the choices from a given agenda under sincere and strategic voting,

Figure 9. Classification of Agendas



let P be a "social preference" relation on X. We assume that P is *asymmetric* (if $x\,P\,y$ then not $y\,P\,x$) and *connected in* X (if $x,y \in X$ and $x \neq y$ then $x\,P\,y$ or $y\,P\,x$). Under majority rule, $x\,P\,y$ if more people prefer x to y than prefer y to x. For the sake of generality, however, we do not require that P be the majority-preference relation. Our one restrictive assumption (common to most of the literature) is connexity, which rules out ties. Although some tie-breaking rules, such as the rule letting a chairman cast the deciding vote, can be thought of as built into P, others cannot because they depend on the positions occupied by tied alternatives in an agenda.

We can now define the *sincere* and *strategic choices*, $SIN(T)$ and $Strat(T)$, from an arbitrary agenda T. If T contains just one alternative, x, then $SIN(T) = STRAT(T) = x$. Otherwise, let x occupy the top node of T_L and y the top node of T_R. Then

$$SIN(T) = \begin{cases} SIN(T_L) \text{ if } x\,P\,y \\ SIN(T_R) \text{ if } y\,P\,x \end{cases}$$

and

$$STRAT(T) = \begin{cases} STRAT(T_L) \text{ if } STRAT(T_L) \\ \quad P\,STRAT(T_R) \\ STRAT(T_R) \text{ otherwise.} \end{cases}$$

These recursive definitions define the sincere and strategic choices from a given complex tree in terms of choices from its less complex components. The definition of $STRAT(T)$ is equivalent to the definitions of Moulin (1979) and McKelvey and Niemi (1978).

Choice Sets and Theorems about Amendment Agendas

This section reviews what is already known about the sincere and strategic choices from amendment agendas. Since P is connected, it is obvious that every α must contain either a unique dominant alternative or a unique dominant cycle, where

DEFINITION. x *is a* dominant alternative *in* α *if and only if* $x \in \alpha$ *and* $x\,P\,y$ *for all* $y \neq x$ *in* α. β *is a* dominant cycle *in* α *if and only if, for some* $x_1, \ldots, x_n$, $\beta = \{x_1, \ldots, x_n\} \subseteq \alpha$, $x_1\,P\,x_2 p \cdots P\,x_n\,P\,x_1$, *and* $x_i\,P\,y$ *for all* $x_i \in \beta$ *and all* $y \in \alpha - \beta$.

Now let

$$G(\alpha) = \{x \in \alpha \mid x \text{ is a dominant alternative in } \alpha \text{ or } x \text{ belongs to the dominant cycle in } \alpha\}.$$

Miller (1980) proves:

$$G(\alpha) = \{x \mid x = SIN(T) \text{ for some amendment agenda } T \text{ on } \alpha\}. \tag{1}$$

More generally:

$$G(\alpha) = \{x \mid x = SIN(T) \text{ for some symmetric agenda } T \text{ on } \alpha\}. \tag{2}$$

The latter result follows from a still more general result by Schwartz (1986, chap. 6), which allows ties and nonbinary choices. Thus, if a dominant alternative exists among the alternatives appearing on an amendment or other symmetric agenda, that alternative will be chosen. If, on the other hand, a dominant cycle exists, then some outcome in the cycle must be chosen, and any outcome can be chosen with some admissable agenda.

A third resullt follows from McKelvey's proof (1979) that for majority rule and the usual spatial context (Euclidean or, more generally, differentiable and convex preferences), if there is no undominated point, the entire space is a cycle ($x \in X$ is undominated if $y\,P\,x$ for no $y \in X$): If x and y are *any* two points in X, there is an amendment agenda T on some finite subset of X such that x is the first alternative in T (x occupies the top nodes of T and T_L) and $y = SIN(T)$. (Although the relation, M, of majority preference is not connected, we can interpret P as the result of adding some tie-breaking rule to M.)

For strategic voting, Miller's definitions (1980) of the covering relation and the uncovered set are crucial. Stated for an arbitrary subset Y of X,

> DEFINITION. *x covers y in Y if and only if $x, y \in Y$, $x \, P \, y$, and for all $z \in Y$, if $y \, P \, z$ then $x \, P \, z$. And*
>
> $U(Y) = \{x \in Y \mid \text{nothing covers } x \in Y\}$ (*the* uncovered set *of* Y).

Miller proves:

> $STRAT(T) \in U(\alpha)$ if T is an amendment agenda on T. (3)
>
> The significance of this result derives from these properties of U:
>
> $U(\alpha) \subseteq G(\alpha)$ and $U(\alpha) \subseteq$ Pareto set of α.

Since the sincere choice from an amendment agenda on α can be anything in $G(\chi)$—often a large subset of α that includes Pareto inefficient outcomes—and since $U(\alpha)$ and hence any strategic choice from α is necessarily in the Pareto set, strategic voting has been interpreted as a means whereby voters can limit the power of an agenda setter.

Banks (1985) shows that strategic voting can limit outcomes further. His result requires three definitions:

> DEFINITION. *A vector* $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ *is a* chain *in α if and only if $x_1 \in \alpha$ for all i and $x_i \, P \, x_j$ whenever $i < j$. A* maximal chain *in α is a chain $\mathbf{x}$ in α such that there is no longer chain in α containing every member of $\mathbf{x}$. And*
>
> $B(\alpha) = \{x \mid x = \text{first element of some maximal chain in } \alpha\}$.

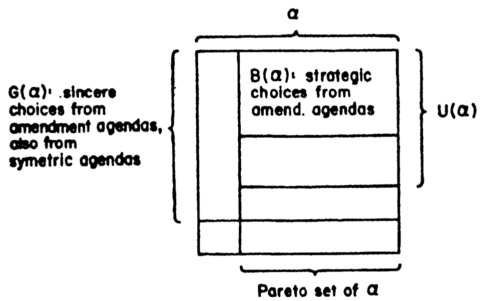
Banks proves:

> $B(\alpha) = \{x \mid x = STRAT(T($ for some amendment agenda T on $\alpha\} \subseteq U(\alpha)$. (4)

We summarize results (1)–(4) in Figure 10. Notice that these results relate the final outcome only to the other alternatives on the agenda: they say nothing about alternatives in X but outside α. Hence, they do not say whether McKelvey's theorem holds for strategic voting—whether agendas can "lead anywhere" in a Euclidean space when voters act strategically. Shepsle and Weingast (1984), however, extend (3) for simple majority rule in the spatial context to show that if x_1 is the first alternative in an amendment agenda T on α and if x_j is chosen over x_k whenever $j > k$ and x_j and x_k tie, then x_1 does not cover $STRAT(T)$ in X. That is, the only points that can be reached, under strategic voting and an amendment agenda, from some initial alternative x_1 are points that x_1 does not cover. McKelvey (1986), moreover, offers a model of endogenous amendment-agenda formation in which outcomes outside the uncovered set of the entire space cannot be reached. Strategic voting, then, *appears* to limit the power of an agenda setter regardless of whether our focus is on a fixed, finite set of alternatives that must enter the agenda or an entire issue space and hence regardless of whether the agenda setter can control only the order of voting or the set of feasible alternatives as well. We say "appears" because the assumption that all permissible agendas are of the amendment type is essential to these results.

Were the set of feasible agendas restricted to amendment agendas, then strategic voting also would limit the *length* of agendas required to secure reachable outcomes. With sincere voting, if an alternative y is required to enter the voting in the first stage of an amendment agenda T on α, then there exists an amendment agenda on part or all of α by which any given outcome $x \in G(\alpha)$ can be

Figure 10. Summary of Amendment-Agenda Results



reached. With strategic voting, on the other hand, Miller (1980) shows that if $x \in G(\alpha)$ can be reached at all, then there exists a one- or two-stage amendment agenda on a subset of α that begins with y and yields x. Shepsle and Weingast (1984) establish a parallel result for spatial preferences. They show that all the outcomes that can be reached from x_1 (namely, the outcomes that do not cover x_1) can be reached by some one- or two-stage agenda (which may, however, contain an outcome foreign to the original agenda).

Counterexamples to Extensions

What we want to show now is that the results we reviewed in the previous section hold only for institutions in which the feasible agendas are limited to amendment agendas. That is, the apparent limits that strategic voting sets on a hypothetical agenda setter's power depend critically on the set of permitted agendas, and if we properly extend a setter's strategies to include all agendas that we might actually observe in a legislature, strategic voting is not the constraint it was thought to be.

Example 1. To see that things can be different when we turn from amendment agendas to other agenda forms, we note first that *agendas can be found from which the sincere choice, SIN(T), need not belong even to G(α).* Suppose *every* voter ranks the five alternatives shown in Figure 3 as follows: b_y, b_{xy}, b_x, b, q. Since this order is unanimous, we may suppose that P orders the alternatives the same way. Hence, if everyone votes sincerely in the agenda—call it T—of Figure 3, then b_x defeats b, after which b_{xy} defeats b_x and then q, so $SIN(T) = b_{xy}$. But b_y is the dominant alternative, so $SIN(T) \notin G(\alpha) = \{b_y\}$.

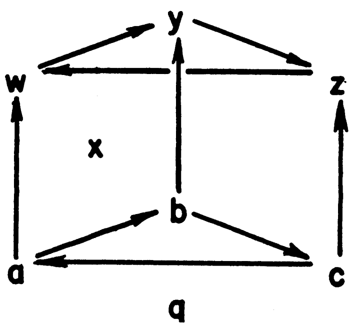
Example 2. Although Example 1 shows that outcomes outside $G(\alpha)$ can prevail under sincere voting, notice that strategic voting in this example yields b_y, which, because it is dominant, is perforce the sole element of $U(\alpha)$ and $B(\alpha)$. But suppose the following eight alternatives are on the floor of the U.S. House:

- *w:* an unamended bill
- *x:* the bill perfected by an amendment
- *y:* the bill perfected by an amended amendment
- *z:* the bill perfected by a substitute amendment
- *a:* a substitute bill unchanged
- *b:* a substitute for the substitute bill
- *c:* the substitute substitute perfected by an amendment
- *q:* the status quo

Then the agenda is as follows (Sullivan 1984):

1. x vs. y (whether to amend the amendment)
2. the winner at 1 vs. z (whether to substitute for the [perfected] amendment)
3. the winner at 2 vs. w (whether to amend the bill)
4. b vs. c (whether to amend the substitute substitute bill)
5. the winner at 4 vs. a (whether to substitute for the [perfected] substitute bill)
6. the winner at 5 vs. the winner at 3

Figure 11. Social Preference for Example 2



(whether to replace the [perfected] bill by the surviving substitute bill)
7. the winner at 6 vs. q (whether to pass the bill's surviving form)

Suppose the social preference under majority rule takes the form shown in Figure 11, where, for those alternatives not paired by an arrow, their height on the page represents P. Because the agenda tree is especially complex, we do not reproduce it here, but note that $STRAT(T) = x$ even though $x \notin B(\alpha)$. That is, chains with x as the first element include (x), (x,b), (x,b,c), (x,b,c,q), and so forth, but none of these chains is maximal since we can set w, z, or y at the top of any of them and still have a chain. Thus, *whereas an amendment agenda over the alternatives in this example must yield a point in $B(\alpha)$ with strategic voting, the agenda prescribed by Rule XIV yields an outcome outside $B(\alpha)$*. This shows that Banks's theorem about amendment agendas does not extend to other agenda forms, even realistic symmetric ones.

Example 3. Although in Example 2, $x = STRAT(T)$ is not in $B(\alpha)$, x *is* in the uncovered set, $U(\alpha)$: there is no alternative that defeats x while also defeating everything x defeats. Nevertheless, we can construct a counterexample to the proposition that the strategic choice from a realistic agenda—again a symmetric agenda permitted by Rule XIV—must belong to the uncovered set. There are seven alternatives:

- *c:* an unamended bill
- *a:* the bill perfected by an amendment
- *b:* the bill perfected by a substitute amendment
- *x:* a substitute bill unamended
- *y:* the substitute bill perfected by an amendment
- *z:* the substitute bill perfected by a substitute amendment
- *q:* the status quo

Agendas with both a perfecting amendment to a bill and a substitute bill may be rare in the House. Bach (1981), however, illustrates a similar agenda with the case of Senate Bill 7, the Veterans' Health Care Amendments of 1979. The required congressional agenda is the following:

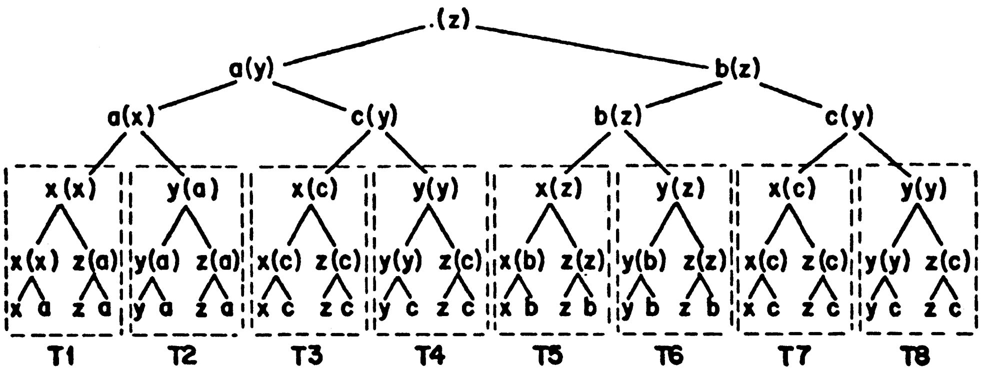
1. a vs. b (whether to replace the amendment with a substitute amendment)
2. winner at 1 vs. c (whether to amend the bill)
3. x vs. y (whether to amend the substitute bill)
4. winner at 3 vs. z (whether to replace the [perfected] substitute by the substitute to the substitute)
5. winner at 2 vs. winner at 4 (whether to replace the [perfected] bill by the surviving substitute)
6. winner at 5 vs. q (whether to pass the surviving motion)

Suppose a three-member committee holds the following preference orders:

Member 1: $a\ c\ z\ b\ y\ x\ q$
Member 2: $b\ y\ c\ x\ a\ z\ q$
Member 3: $x\ z\ a\ y\ c\ b\ q$

The majority-preference relation is as follows, where, aside from the exceptions noted by arrows, the left-to-right order of

Figure 12. Agenda for Example 3



the motions corresponds to the majority preference:

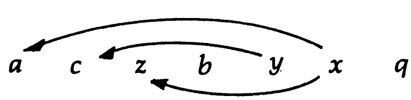
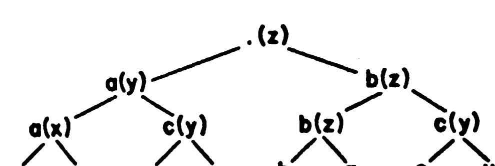


Figure 12 portrays the agenda and, in parentheses, shows the strategic choice from each subtree (to save space we omit q from the tree since it is defeated by everything else). So $STRAT(T) = z$. But $z \notin U(\alpha)$: a covers z because a beats z and also beats the three alternatives (b, y, and q) that z beats.

To see why a covered alternative is chosen, consider the eight subtrees, $T1$–$T8$, in Figure 12. Each is equivalent to a mini-amendment agenda (e.g., in $T1$, x is paired with z, then the winner with a), so from result (3), the strategic choice from each is uncovered in that subtree.

Figure 13. Reduced Agenda

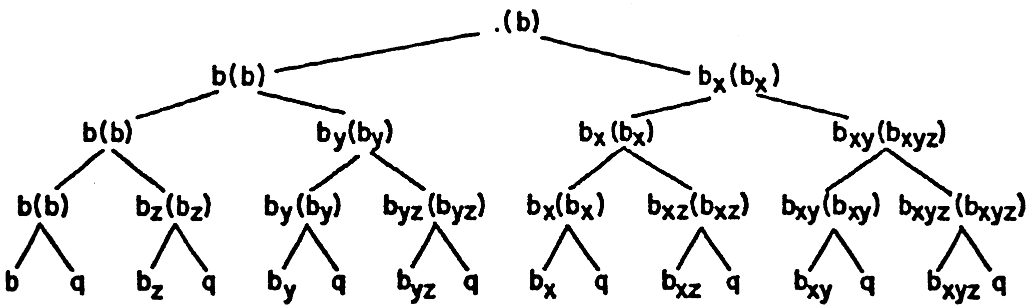


Since no information about outcomes is lost if we replace a subtree by its strategic choice, we redraw the agenda in Figure 13 to extend each branch down just to where subtrees become amendment agendas (for $T5$ we substitute b for z since z beats b and will beat b in the corresponding comparison with the strategic choice from $T6$).

This tree is almost an amendment agenda: after a and b are paired, the winner is paired with c, and the winner in that context with y. The important exceptions are that if a wins in the first two rounds, x and a are paired (rather than a and y), whereas if b wins in those rounds, b is paired with z (rather than y). Thus, x can defeat a and then lose to y without ever being compared to z. The discontinuities permit this asymmetry in the treatment of z and a and account for the survival of a covered alternative. Although the original tree is symmetric, the strategically equivalent reduced tree, owing to the discontinuity, is not symmetric.

Example 4. It would be a mistake to infer from the preceding examples that symmetry or discontinuity is essential to agendas in which strategic voting leads outside the uncovered set. Consider an agenda involving three compatible first-order amendments, x, y, and z. Suppose P

Figure 14. Agenda for Example 4



orders the nine alternative outcomes, from left to right, thus:

$$b_{xyz}\ \ b_{xy}\ \ b\ \ b_x\ \ b_y\ \ b_z\ \ b_{yz}\ \ b_{xz}\ q$$

with the exception that $b_x\, P\, b_{xyz}$. Then b_{xy} covers b. Nevertheless, the appropriate agenda, depicted in Figure 14 with strategic equivalents in parentheses, yields b as the final outcome. Although not symmetric, this agenda is continuous, uniform, and nonrepetitive.

These examples, then, establish the amendment-agenda assumption as essential to Results (1)–(4). They establish the same for the three other propositions that we discuss in the previous. First, *strategic choices need not be Pareto-efficient.* In Example 3, the strategic choice, z, is covered but Pareto efficient. Now switch a and z in Member 3's preference ordering. Then P remains the same, but z becomes Pareto inefficient.

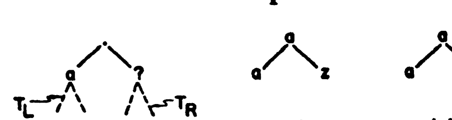
Second, *the first alternative on an agenda can cover the final strategic choice.* In Example 3, a is voted on first, and a covers z, yet z is the outcome under strategic voting.

Finally, *not every agenda can be reduced to a $\leqslant$ two-step agenda with the same first alternative and the same strategic choice.* Looking again at Example 3, notice that every one- or two-stage agenda T that begins with a and yields z as the strategic choice must have the form shown in Figure 15a. Since $z = STRAT(T)$, z must also equal $STRAT(T_L)$ or $STRAT(T_R)$. If $z = STRAT(T_L)$, then T_L must have the form shown in Figure 15b since, with two or fewer steps, this must be the last step and only a and z can be paired. But a beats z, so $z \neq STRAT(T_L)$. Suppose, then, that $z = STRAT(T_R)$. Then T_L must have the form shown in Figure 15c, where *?* is an alternative that beats a but loses to z (so that z survives as the final outcome). But the only alternative that defeats a is x, which z does not defeat. Thus, the agenda in Figure 14 cannot be reduced to one or two stages with alternative a entering the first stage and z being the final outcome.

Theorems

The preceding examples demonstrate that none of the earlier results specifying limits on a hypothetical agenda setter's power under strategic voting applies when that agenda setter's strategy set contains all agendas permitted in real legis-

Figure 15. Two-Step Reduction of Example 3



latures, such as the U.S. Congress. The results we prove in this section show how wide the true limits are, both for strategic and sincere voting. When compared with the theorems reviewed earlier, these results reveal exactly how special a case the amendment agenda really is. We begin with an unsurprising result:

THEOREM 3. *If T is any agenda on α, then $STRAT(T) \in G(\alpha)$.*

Because $U(\alpha)$ and $B(\alpha)$ are subsets of $G(\alpha)$, this theorem does not exclude the possibility that an agenda setter's power is restricted by strategic voting to these particular subsets of $G(\alpha)$. Strategic voting does restrict a setter's power, but not in the way we might think. First, consider the "kitchen sink" set:

$$K(\alpha) = \{x \in \alpha \mid \alpha = \{x\} \text{ or } x\,P\,y \text{ for some } y \in \alpha\}.$$

Thus, x is in $K(\alpha)$ so long as α consists of x alone or x defeats something or other in α. Clearly, $G(\alpha) \subseteq K(\alpha)$, and $K(\alpha)$ comprises all but at most one member of α. Not surprisingly:

THEOREM 4. *If T is an agenda on α, $SIN(T) \in K(\alpha)$.*

Like Theorem 3, this result places weak bounds on final outcomes. What is interesting, however, is that *these bounds are the strongest there are:* the G and K sets are the exact limits of an agenda setter's power under strategic and sincere voting, respectively.

To see what we mean, consider a sequential-elimination agenda, the general form of which is that of Figure 4, but with any number of nodes. Our next theorem says that *any* outcome in $K(\alpha)$—which is all of α less at most one member—can be the sincere choice from some nonrepetitive sequential-elimination agenda, hence from some nonrepetitive continuous agenda:

THEOREM 5. *If $x \in K(\alpha)$, then for some nonrepetitive sequential-elimination agenda T on α, $x = SIN(T)$.*

When the set of feasible agendas is restricted to the amendment agendas, or even the symmetric ones, sincere voting cannot lead outside the G-set, but as soon as the feasible agendas are allowed to include even nonrepetitive sequential-elimination agendas (hence, more generally, continuous agendas), sincere voting can lead practically anywhere.

We can now see one weak limit, however, that strategic voting places on an agenda setter's power: although large, $G(\alpha)$ is often a proper subset of $K(\alpha)$. But because $G(\alpha)$ might be large and some of its elements of Pareto inefficient, we want to know whether strategic voting places more severe limits on an agenda setter's power. The next five results show that tighter limits do not exist except in special cases.

THEOREM 6. *If $x \in G(\alpha)$, there exists a nonrepetitive sequential-elimination agenda T on α such that $STRAT(T) = x$.* (Moulin 1985)

Figure 16 summarizes the Theorems 3–6.

If we drop the requirement of nonrepetitiveness in Theorem 6, we can ensure that T begins with any given alternative and yields any other given alternative as the strategic choice:

THEOREM 7. *If $x \in G(\alpha)$, then there exists a sequential-elimination agenda T on α such that y is the first alternative in T and $x = STRAT(T)$.*

McKelvey's theorem about agendas leading anywhere under sincere voting can be restated for strategic voting if, instead of amendment agendas, we use sequential-elimination agendas:

THEOREM 8: *For the "standard spatial model," if X is the issue space, if the majority-preference relation M is a* *subrelation of P, and if no dominant point in X exists, then for any two points $x, y \in X$, there is a nonrepetitive sequential-elimination agenda T on some finite subset of X such that (1) x is the first alternative in T, (2) $y = STRAT(T)$, and (3) $STRAT(T'_L)$ M $STRAT(T'_R)$ or conversely for every > 2-node subtree T′ of T.*

Since P is connected but M is not, we might think of P as the result of extending M by the addition of some tie-breaking rule. Clause (3) says that no M-ties actually arise in T. The essential point of this theorem, though, is that it is not strategic voting per se that limits choices to the uncovered set or the Banks set but the assumption that an agenda setter must choose agendas of the amendment type.

Theorem 6 tells us that if all voters are strategic and if all binary agendas are admissable, then there exists some binary agenda on α that will choose any outcome in $G(\alpha)$, whereas from the results reviewed in the Choice Sets and Theorems section, we know that if agendas are limited to the amendment type, then only outcomes in $B(\alpha) \subseteq G(\alpha)$ can prevail. It is interesting, however, to consider separately the class of symmetric agendas since one-period agendas often are required to be symmetric (e.g., by House Rule XIV). Of course, our examples from the previous section tell us that such agendas can lead outside the Banks set (Example 2), outside the uncovered set (Examples 3 and 4), and outside the Pareto set (Example 3 with modified preferences for Member 3). Often, though, the rule assigned to a bill in the House limits the number of alternatives that can be voted on, whence the importance of the following theorem:

THEOREM 9. *If α has 5 or fewer members, then for any symmetric agenda T on α, $STRAT(T) \in U(\alpha)$.*

Thanks to Example 3, the five-alternative limit cannot be raised. Hence, earlier results do extend to certain nonamendment agendas if the number of alternatives is severely constrained: even under strategic voting, the power of an agenda setter is no greater with a strategy set comprising all symmetric agendas than with a strategy set limited to amendment agendas if the alternatives that can be included in the agenda are five or fewer. But what if they exceed five? Can just any outcome in $G(\alpha)$ be achieved with a symmetric agenda, as it can with a sequential-elimination agenda, or are final outcomes limited to a set lying "between" $U(\alpha)$ and $G(\alpha)$? Here is a partial answer:

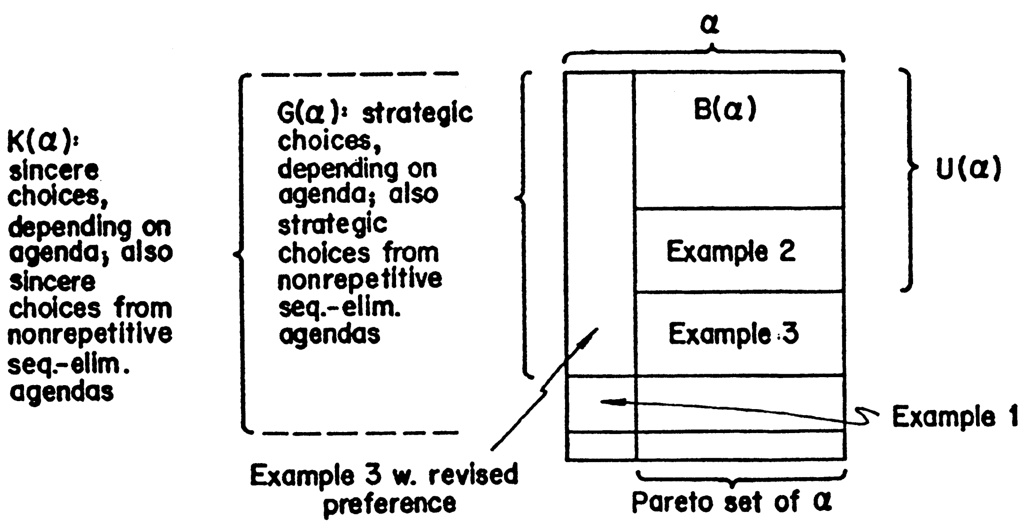
THEOREM 10. *Suppose T is a symmetric agenda on α, α has at least 4 members, and x is covered in α by all but 2 other members of α. Then $x \neq STRAT(T)$.*

Notice that if $|\alpha| \geqslant 4$ and if $x \in G(\alpha)$, then there must be at least two other members of α that do not cover x, whereas everything else in α can cover x (see the construction used to prove the theorem). Given the assumptions of Theorem 10, although $x \in G(\alpha)$, x cannot be chosen from any symmetric agenda if voters are strategic. Thus, it appears that the set of reachable outcomes when only symmetric agendas are feasible lies somewhere between what is reachable if only amendment agendas are feasible and what is reachable if all binary agendas are feasible.

Conclusions

It is tempting to interpret our analysis as supporting the idea that political outcomes are at the mercy of elites who control agendas. Certainly, our analysis supports the proposition that strategic voting is not always effective for thwarting the intent of an unconstrained agenda setter, but it is naive to interpret our results as implying a reemergence of the power of agenda setters, hypothetical or real. For

Figure 16. Summary of General Agenda Results



one thing, the set of feasible legislative agendas, even with a handful of motions, may be large. And Hammond's (1986) adaptation of agendas to the study of hierarchical decision processes suggests that the set of feasible organizational agendas may be larger still. Hence, power may lie more with those who are best able to ascertain the implications of particular agendas than with those who control procedures and agenda form.

Second, in addition to the formal constraints imposed by such measures as Rule XIV, setters are also subject to those who would upset their calculations by the introduction of new issues and dimensions (Riker 1982). And as the experiences of Plott and Levine (1976) attest, even those unschooled in agenda manipulation soon learn the importance of agenda control and seek the means to thwart that control. Third, our analysis assumes that all preferences are common knowledge. But as Ordeshook and Palfrey (1986) demonstrate, outcomes can be radically different with incomplete information. Although their analysis is limited to amendment agendas, it shows how agendas reveal different things about preferences, suggesting that a setter can manipulate the agenda to reveal preferences in particular ways.

We also should consider the ways in which motions are introduced for consideration. With the exception of Theorem 6, which concerns spatial majority-rule games, our analysis focuses on the manipulation of outcomes by deciding how some fixed set of alternatives is considered and disposed of. But agenda power has two sources: the ability to choose a procedure for selecting from some fixed set of alternatives and the ability to choose that set. And we suspect that agenda power, if it exists at all in Congress, has more to do with the second source than the first. The labeling of alternatives as *the bill, an amendment,* or *a substitute amendment* is an important strategic process, but once all motions are made and the labels attached, there may be little room for strategic maneuver.

The relevance of agendas to the study of final outcomes, then, may depend on the rules that determine the alternatives that a legislature considers. Here, however, the possible endogenicity of the process complicates matters. Some progress has been made in the study of endogenous agenda formation (McKelvey 1986), but again under the assumption that all agendas are of the amendment type. The variety of congressional agendas and the limits that might otherwise be placed on the types of agendas that a legislature can use raise new research possibilities. The labeling of motions may be an important strategy available to members because the decision about how to label a motion has more profound consequences than earlier research indicates. The sole strategic consideration in an amendment agenda is the order of the alternatives. But if, for example, two proposed amendments to a committee report are both moved and recognized as perfecting amendments rather than as a perfecting amendment and a substitute bill, the prescribed agenda must be nonsymmetric (as in Figure 2).

Hence, instead of proclaiming the preeminence of the agenda setter—hypothetical or otherwise—our analysis warns us that our subject concerns games in which participants' strategies include more than simply deciding the order in which to present motions or whether to vote sincerely on the current ballot. We are only beginning to identify and understand those strategies, but this essay sheds some light on the potential consequences of part of the strategy space of those who participate in the institutions of social choice.

Appendix

Proof of Theorem 1. Because symmetric agendas are obviously complete and uniform, we prove only that they are nonrepetitive. If T on α is symmetric and repetitive, then some subtree of T is equivalent to a tree T' in which some $x \in \alpha$ occupies the top node of T'_L, some $y \in \alpha$ occupies the top node of T'_R, but x occurs in T'_R, say at node n. By symmetry, T'_L is the same as T'_R but for the substitution of x for y. So x must occur at node n in T'_L as well. But because we can get T'_R from T'_L by substituting y for x, y must occur at node n in T'_R, which is impossible since x occurs there. Q.E.D.

Proof of Theorem 2. We first prove, by induction on n, the lemma that if $T = A(x_1, \ldots, x_n)$ is an amendment agenda, then $(x_1, \ldots, x_n)$ contains no repetitions. This is trivially true if $n = 1$. Otherwise, $T_L = A(x_1, x_3, \ldots, x_n)$ and $T_R = A(x_2, x_3, \ldots, x_n)$ are agendas. By inductive hypothesis, $(x_1, x_3, \ldots, x_n)$ and $(x_2, x_3, \ldots, x_n)$ contain no repetitions, so if $(x_1, \ldots, x_n)$ contained a repetition, we would have $x_1 = x_2$, in which case x_1 $(= x_2)$ would occupy the top nodes of both T_L and T_R, contrary to the definition of an agenda.

We prove the theorem itself by induction on the number of nodes of T, which we assume exceeds 1 (else the theorem is trivially true). Let T be an amendment agenda, so that T is equivalent to $T' = A(x_1, \ldots, x_n)$ for some $x_1, \ldots, x_n$ $(n \geqslant 2)$. Then $T'_L = A(x_1, x_3, \ldots, x_n)$ and $T'_R = A(x_2, x_3, \ldots, x_n)$, which are continuous and symmetric by inductive hypothesis. Therefore, since x_1 occupies the top node of T' and T'_L while x_2 occupies the top node of T'_R, T' is continuous. But since, by the lemma, $(x_1, x_3, \ldots, x_n)$ and $(x_2, x_3, \ldots, x_n)$ contain no repetitions, T'_L is the result of substituting x_1 for x_2 in T'_R, while T'_R is the result of substituting x_2 for x_1 in T'_L. Hence, owing to the symmetry of T'_L and T'_R, T' is symmetric.

Conversely, suppose T is continuous and symmetric. Then T is equivalent to an agenda T' in which, for some x and y, x occupies the top nodes of T' and T'_L while y occupies the top node of T'_R. By inductive hypothesis, T'_L is equivalent to $A(x, x_1, \ldots, x_n)$ and T'_R to $A(y, y_1, \ldots, y_m)$ for some $x_1, \ldots, x_n$

and $y_1, \ldots, y_m$ ($n \geqslant 0$, $m \geqslant 0$). But by symmetry, $A(y, y_1, \ldots, y_m)$ is the result of substituting y for x in $A(x, x_1, \ldots, x_n)$, and by the lemma, $(y, y_1, \ldots, y_m)$ and $(x, x_1, \ldots, x_n)$ have no repetitions. So $A(y, y_1, \ldots, y_m) = A(x, x_1, \ldots, x_n)$. Hence, T' is equivalent to $A(x, y, x_1, \ldots, x_n)$. Q.E.D.

Proof of Theorem 3. By induction on the number of nodes of T. Trivial if T has one node. Otherwise, let α_L be the set of alternatives in T_L, let α_R be the set of alternatives in T_R, and let $\alpha = \alpha_L \cup \alpha_R$. By inductive hypothesis,

$$STRAT(T_L) \in G(\alpha_L) \quad \text{(A1)}$$

$$STRAT(T_R) \in G(\alpha_R) \quad \text{(A2)}$$

Without loss of generality, suppose that

$$STRAT(T) = STRAT(T_L) \quad \text{(A3)}$$

From the definition of $G(\alpha)$, everything in $G(\alpha)$ bears P to everything in $\alpha - G(\alpha)$, while everything in $G(\alpha_L)$ bears P to everything in $\alpha_L - G(\alpha_L)$. So if there existed a $w \in G(\alpha_L) - G(\alpha)$ and a $v \in G(\alpha) \cap \alpha_L$, we would have both wPv and vPw, contrary to the asymmetry of P. Hence, either $G(\alpha_L) - G(\alpha)$ or $G(\alpha) \cap \alpha_L$ is empty. That is, either $G(\alpha_L) \subseteq G(\alpha)$ or $G(\alpha) \cap \alpha_L = \emptyset$. Similarly, either $G(\alpha_R) \subseteq G(\alpha)$ or $G(\alpha) \cap \alpha_R = \emptyset$. Consequently, if $G(\alpha_L) \nsubseteq G(\alpha)$, then $G(\alpha) \cap \alpha_L = \emptyset$, so that $G(\alpha) \cap \alpha_R \neq \emptyset$ (since otherwise $G(\alpha) = G(\alpha_L \cup \alpha_R)$ would be empty), whence $G(\alpha_R) \subseteq G(\alpha)$, and thus, by (A1) and (A2), $STRAT(T_R) \in G(\alpha)$ (because $G(\alpha) \cap \alpha_L = \emptyset$), but $STRAT(T_L) \notin G(\alpha)$, which implies that $STRAT(T_R)\ P\ STRAT(T_L)$, contrary to (A3). Hence, $G(\alpha_L) \subseteq G(\alpha)$ after all, and so by (1) and (3), $STRAT(T) \in G(\alpha)$. Q.E.D.

Proof of Theorem 4. By induction on $|\alpha|$. Trivial if $|\alpha| = 1$. Otherwise, let y occupy the top node of T_L and z the top node of T_R. Without loss of generality, suppose $y\ P\ z$. Then $SIN(T) = SIN(T_L)$. If T_L consists just of y, then $SIN(T) = SIN(T_L) = y\ P\ z$, so $SIN(T) \in K(\alpha)$. On the other hand, if T_L has more than one node, then by inductive hypothesis, $SIN(T) = SIN(T_L)\ P\ w$ for some $w \in \alpha$, and so $SIN(T) \in K(\alpha)$. Q.E.D.

Our next results require a formal definition of sequential-elimination agendas.

> A *sequential-elimination agenda* is an agenda that is equivalent to $S(x_1, \ldots, x_n)$ for some $x_1, \ldots, x_n$, where $S(x_1, \ldots, x_n)$ is defined thus: $S(x_1)$ is the 1-node tree occupied by x_1; $S(x_1, \ldots, x_n)$ is the binary tree T in which x_n occupies the top node, T_R is the 1-node tree occupied by x_1, and $T_L = S(x_2, \ldots, x_n)$.

Proof of Theorem 5. Trivial if $\alpha = \{x\}$. Otherwise, there exists a $y \in \alpha$ for which $x\ P\ y$. So there exist $y_1, \ldots, y_n$ such that (1) $\alpha - \{x\} = \{y_1, \ldots, y_n\}$, (2) $y_i \neq y_j$ unless $i = j$, and (3) $y_1 = y$. Then for the nonrepetitive sequential-elimination agenda $T = S(x, y_1, \ldots, y_n)$, $SIN(T) = SIN(T_R) = x$, because $x\ P\ y_1 = y$. Q.E.D.

Two definitions and a lemma. Before proceeding to the remaining theorems we need the following:

> DEFINITION. *A* path *on* α *is a vector* $(x_1, x_2, \ldots, x_n)$ *such that* $\alpha = \{x_1, x_2, \ldots, x_n\}$ *and for each* $x_i \neq x_1$, $x_j\ P\ x_i$ *for some* $j < i$. *It is* nonrepetitive *if* $x_i \neq x_j$ *whenever* $i \neq j$.

> LEMMA. If $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a path on α, then there exists a sequential-elimination agenda T on α such that (1) $STRAT(T) = x_1$, (2) T begins with x_n, and (3) T is nonrepetitive if $\mathbf{x}$ is nonrepetitive.

Proof. Trivial if $n = 1$. Otherwise, let $T = S(x_1, \ldots, x_n)$. Then T begins with x_n and is nonrepetitive if $\mathbf{x}$ is. For $i = 1, 2, \ldots, n-1$, let T_i be the subtree of T in which the first vote is between x_n and x_i. Then it suffices to show, by induction on $n-i$, that for every $i < n$, either $STRAT(T_i) = x_i$ (so that, in particular, $STRAT(T_1) = x_1$), or $STRAT(T_i) = x_1$, or $x_j P STRAT(T_i)$ for some $j = 1,2, \ldots, i-1$. If $x_n P x_{n-1}$, then $STRAT(T_{n-1}) = x_n$ and, since **x** is a path, either $n = 1$ or $x_j P STRAT(T_{n-1}) = x_n$ for some $j = 1,2, \ldots, n-2$. Otherwise, $x_{n-1} P x_n$, so $STRAT(T_{n-1}) = x_{n-1}$. Now suppose that $i < n-1$. If $x_i P STRAT(T_{i-1})$, then $x_o = STRAT(T_i)$. On the other hand, suppose $STRAT(T_{i+1}) P x_i$. Then $STRAT(T_i) = STRAT(T_{i+1})$. But by inductive hypothesis, either $STRAT(T_{i+1}) = x_1$, or $x_j P STRAT(T_{i+1})$ for some $j < i+1$, or else $STRAT(T_{i+1}) = x_{i+1}$, in which case, since **x** is a path, we again have $x_{i+1} = x_1$ or $x_j P x_{i+1} = STRAT(T_{i+1})$ for some $j < i+1$. Hence, since not $x_i P STRAT(T_{i+1})$, there must exist a $j < i$ such that $x_j P STRAT(T_{i+1}) = STRAT(T_i)$ unless $STRAT(T_i) = STRAT(T_{i+1}) = x_1$.Q.E.D.

Proof of Theorem 6. Trivial if $\alpha = \{x\}$. Otherwise, if $\alpha - G(\alpha) \neq \emptyset$, let $\alpha - G(\alpha) = \{y_1, \ldots, y_k\}$ where $y_i \neq y_j$ whenever $j \neq i$. We have two cases:

Case 1. $G(\alpha) = \{x\}$. Then $x P y_i$ for all i. So $(x, y_1, \ldots, y_k)$ is a nonrepetitive path on α, and the theorem follows from the lemma.

Case 2. $G(\alpha)$ is a dominant cycle in α. Then there exist $x_1, \ldots, x_m$ such that (1) $G(\alpha) = \{x_1, \ldots, x_m\}$; (2) $x_1 = x$; (3) $x_1 P x_2 P \cdots P x_m P x_1$; and (4) $x_i P y_j$, $j = 1,2, \ldots, k$; $i = 1,2, \ldots, m$. Let $(x'_1, \ldots, x'_r)$ be the result of deleting from $(x_1, \ldots, x_n)$ all but the first occurrence of each x_i that occurs more than once. Then $x'_1 = x$, and $(x'_1, \ldots, x'_r, y_1, \ldots, y_k)$ is a nonrepetitive path on α, whence the theorem follows by the lemma. Q.E.D.

Proof of Theorem 7. Trivial if α consists of a single alternative. Otherwise, if $\alpha - G(\alpha) \neq \emptyset$, let $\alpha - G(\alpha) = \{z_1, \ldots, z_k\}$. We now have the same two cases to consider as before.

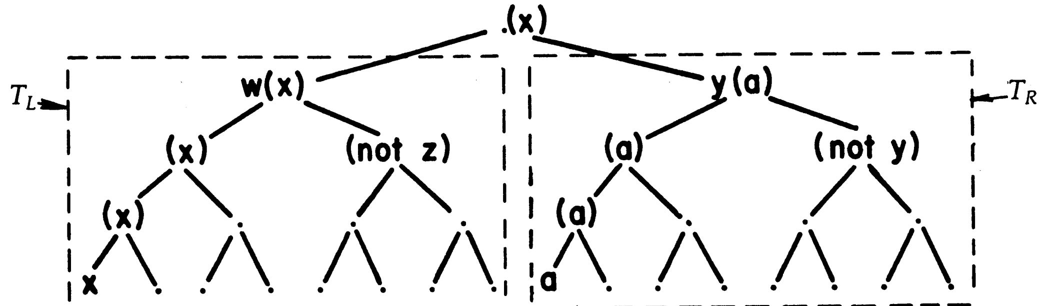
Case 1. $G(\alpha) = \{x\}$. Then $x P z_i$ for all i, and either $x = y$ or $x P y$. So $(x, z_1, \ldots, z_k, y)$ is a path, and the theorem follows by the lemma.

Case 2. $G(\alpha)$ is a dominant cycle in α. Then the same four conditions hold as in the proof of Theorem 6 except that the last condition (4) reads: $x_i P z_j$ for all i and j. Consequently, either $y = x = x_1$ or $y = x_i$ for some $i > 1$ or $y = z_j$ for some j. So $(x_1, \ldots, x_m, z_1, \ldots, z_k \cdot y)$ is a path in which $x_1 = x$, and the theorem follows from the lemma. Q.E.D.

Proof of Theorem 8. There exist $x_1, \ldots, x_n$ such that (1) $x_1 = x$, (2) $x_n = y$, and (3) $x_1 P x_2 P \cdots P x_n$, where P is a connected extension of the majority-preference relation (McKelvey 1979). Since $(x_1, \ldots, x_n)$ is a path, the theorem follows from the lemma. Q.E.D.

Proof of Theorem 9. Since q is not critical to Example 3 in Section 5, as few as six alternatives are required to generate a symmetric agenda that leads outside of $U(\alpha)$. To see that we cannot construct an example with fewer alternatives, let some alternative x be covered by y (so $y P x$). Then for x to prevail, we must eliminate y, say with z. But if $z P y$, then $z P x$ (since anything that defeats y defeats x), so z also must be eliminated by an alternative, say w, for which $x P w$. Otherwise, if $\alpha = \{x,y,z\}$ or if $\alpha = \{x,y,z,w\}$ with $w P x$, then $x \notin G(\alpha)$ and x cannot prevail. So we must have $\alpha = \{x,y,z,w\}$ with $x P w$ (and since y covers x, with $y P w$). But even this is not enough. For x to be the outcome, at least one principal subtree, say T_L, must have $x = STRAT(T_L)$, and x or w must equal $STRAT(T_R)$. If T is symmetric, then T_L must look like an amendment agenda over three members of α, and the only such agenda that yields x is "z vs. x, the winner against w." So the first ballot of T involves y, which enters T_R, and T_R yields w only if it is the agenda "y vs. w, the winner against z." But if the first node of T_L pits z against x and if the first node of T_R pits y against w, then T is

Figure A-1. Agenda Beginning with w and y



not symmetric. This shows that if x prevails, then α cannot have four or fewer elements.

Now let $\alpha = \{a,x,y,z,w\}$ and, as before, let $x = STRAT(T_L)$ and let a or w equal $STRAT(T_R)$. Since $STRAT(T_L) = x$, then, from the previous argument that no covered alternative can prevail in any four-alternative symmetric agenda, y cannot enter T_L. Thus, y must be voted on in the first ballot of T and enter T_R. But x cannot be paired with y in this first ballot. Since y must be defeated by something in T_R for x to prevail in T, pairing x and y first means that T_L is identical to T_R except for the interchange of x and y, in which case, whatever defeats y also defeats x. Hence, we have three cases, depending on what, besides y, is voted on in the first ballot.

Case 1. If the first ballot pairs y and w, then the agenda must look like the one shown in Figure A-1, where the outcomes in parentheses denote strategic choices. Alternative a must be the strategic choice of T_R since it is the only alternative in this subtree that does not necessarily defeat x. But if $x\ P\ a$, then $y\ P\ a$, in which case we must have $a\ P\ z$, else $a \notin G(\{x,y,z,a\})$ and a would not equal $STRAT(T_R)$ as assumed. If, without loss of generality, $a = STRAT(T_{R_L})$ as indicated, then T_{R_L} must correspond to the agenda "y vs. a, winner against z" or to "x vs. a, winner against z." But if z enters the agenda only in the last ballot, it must do so also in T_L, owing to the symmetry of T, and since $z\ P\ x$, x cannot rise in T_L to become $STRAT(T_L)$.

Case 2. If the first ballot pairs a and y, then the agenda looks like the preceding picture except that w and a reverse roles. But by the same argument, $w = STRAT(T_R)$ only if the left subtree of T_R is the agenda "y vs. w, the winner against z" or "x vs. w, the winner against z," so again z enters the agenda only in the last ballot and $x \neq STRAT(T_L)$.

Case 3. If the first ballot pairs z and y, then we must have $w = STRAT(T_R)$, since, to ensure that $y \neq STRAT(T_R)$, we must have $a\ P\ y$ and hence $a\ P\ x$. Also, to ensure that $w \in G(\{a,x,y,x\})$, we must have $w\ P\ a$. Substituting w for a in T_R in the previous figure, $w = STRAT(T_R)$ only if the left subtree of T_R is the agenda "x vs. w, the winner against a" or "y vs. w, the winner against a." But now, as in Case 1, the symmetry of T requires that a also enter the agenda in the final ballot of T_L, and since $a\ P\ x$, $x \neq STRAT(T_L)$.

Q.E.D.

Proof of Theorem 10. By induction on $|\alpha|$. Assume that $x = STRAT(T)$. Then $x \in G(\alpha)$ by Theorem 3. Since x is covered, $G(\alpha)$ is a cycle, so $x\ P\ y\ P\ w$ for some y, $w \in G(\alpha)$. Let $\{z_1, \ldots, z_n\} = \alpha - \{x,y,w\}$. Since $x\ P\ y$, y does not cover x, and since $y\ P\ w$ and anything that covers x bears P to y, w does not cover x. So $z_1, \ldots, z_n$ cover x by hypothesis of the theorem. Hence, $z_i\ P\ x$ and $z_i\ P\ y$ for all i, and thus, since $G(\alpha)$ is a dominant cycle in α, w bears P to some z_i, say $w\ P\ z_1$. Because $x = STRAT(T)$, x equals $STRAT(T_L)$ or $STRAT(T_R)$; say $x = STRAT(T_L)$. By Theorem 3, $x \in G(\alpha_L)$, where α_L is the set of alternatives in T_L. By the symmetry of T, α_L lacks just one member of α. Hence, $y,w \in \alpha_L$ (else $x \notin G(\alpha)$). So if any z_i belonged to α_L, we would have $x \neq STRAT(T_L)$ by inductive hypothesis. Thus, no z_i belongs to α_L. Consequently, $n = 1$ and $\alpha = \{x,y,w,z_1\}$. Since $|\alpha| = 5$, the theorem follows from Theorem 9. Q.E.D.

Note

This research was supported, in part, by a National Science Foundation grant to the University of Texas at Austin. We thank David Koehler, Kenneth Shepsle, Thomas Hammond, Keith Krehbiel, and Samuel Patterson for their exceptionally helpful comments and suggestions.

References

Bach, Stanley. 1981. Parliamentary Strategy and the Amendment Process: Rules and Case Studies of Congressional Action. Public Choice Workshop, Washington University, St. Louis. Photocopy.

Banks, Jeffrey. 1985. Sophisticated Voting Outcomes and Agenda Control. *Social Choice and Welfare* 1:295–306.

Enelow, James, and David Koehler. 1980. The Amendment in Legislative Strategy: Sophisticated Voting in the U.S. Congress. *The Journal of Politics* 42:396–413.

Farquharson, Robin. 1969. *Theory of Voting*. New Haven: Yale University Press.

Hammond, Thomas H. 1986. Agenda Control, Organizational Structure, and Bureaucratic Politics. *American Journal of Political Science* 30:379–420.

McKelvey, Richard D. 1979. General Conditions for Global Intransitivities in Formal Voting Models. *Econometrica*, 47:1085–1112.

McKelvey, Richard D. 1986. Covering, Dominance, and Institution Free Properties of Social Choice. *American Journal of Political Science*. Forthcoming.

McKelvey, Richard D., and Richard Niemi. 1978. A Multistage Game Representation of Sophisticated Voting for Binary Procedures. *Journal of Economic Theory* 18:1–22.

Miller, Nicholas. 1980. A New Solution Set for Tournaments and Majority Voting. *American Journal of Political Science* 24:68–96.

Moulin, Herve. 1979. Dominance-solvable Voting Schemes. *Econometrica* 47:1337–51.

Moulin, Herve. 1985. Choosing from a Tournament. Institute for Mathematics and Applications, University of Minnesota. Mimeo.

Ordeshook, Peter C., and Thomas Palfrey. 1986. Agendas, Strategic Voting, and Signaling with Incomplete Information. Pittsburgh: Carnegie-Mellon University. Typescript.

Plott, Charles, and Michael Levine. 1976. A Model of Agenda Influence on Committee Decisions. *American Economic Review* 68:146–60.

Riker, William H. 1958. The Paradox of Voting and Congressional Rules for Voting on Amendments. *American Political Science Review* 52: 349–66.

Riker, William H. 1982. *Liberalism Against Populism*. San Francisco: W. H. Freeman.

Schwartz, Thomas. 1986. *The Logic of Collective Choice*. New York: Columbia University Press.

Shepsle, Kenneth, and Barry Weingast. 1984. Uncovered Sets and Sophisticated Voting Outcomes with Implications for Agenda Control. *American Journal of Political Science* 28:49–74.

Sullivan, Terry. 1984. *Procedural Structure*. New York: Praeger.

Peter C. Ordeshook is Professor of Political Science and holds the Frank C. Erwin, Jr. Centennial Chair in State Government and Thomas Schwartz is Professor of Political Science, University of Texas, Austin, TX 78712.