Implications from the Disequilibrium of Majority Rule for the Study of Institutions

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While contemporary political science (as, for example, in such subjects as political socialization, studies of public opinion, etc.) tends to emphasize the study of values and tastes (because of an assumption that political outcomes–like market outcomes–are determined by the amalgamation of individual preferences), the older tradition of political science emphasized the study of institutions. The line of research in political theory followed during the last generation has involved seeking an equilibrium of tastes; but it has revealed that such an equilibrium exists only rarely, if at all. The inference then is that prudence in research directs the science of politics toward the investigation of empirical regularities in institutions, which, though congealed tastes, are "unstable constants" amenable to scientific investigation.

Social scientists are now, and probably always have been, of divided opinion about the degree to which institutions as well as personal values, opinions, and tastes affect the content of social decisions. (I use the words "values," "opinions," and "tastes" interchangeably, not because they mean exactly the same thing, but because the processes by which they can influence decisions are identical.) It is clear that the values of at least some members of society do ineradicably influence these decisions. Even when it is claimed that God, or the law of nature, or the sovereign people, or the working class, or some other abstract non-human entity determines outcomes, it is still true that some members of society, say, priests, judges, parliamentmen, dictators, etc. must interpret what the abstraction directs. In the most extreme cases, the voice the people hear and the words spoken come immediately from the Delphic priestess, not from the god who is said to inspire her. That being so, we can never leave out the influence of some person's values and tastes on social decisions. Even if the priestess is unintelligible, the priestly interpreter tells the supplicants what to do.

On the other hand, we cannot leave out the force of institutions. The people whose values and tastes are influential live in a world of conventions about both language and values themselves. These conventions are in turn condensed into institutions, which are simply rules about behavior, especially about making decisions. Even the priestess in her frenzy probably behaves according to rules and, for certain, her interpreter is constrained by specifiable conventions. So interpersonal rules, that is, institutions, must affect social outcomes just as much as personal values.

Ambiguity arises, however, when we attempt to assess the relative significance of these two kinds of forces. Very probably, both are necessary and neither is alone a sufficient condition for outcomes. If so, a full statement of social causation must include them both. But, nevertheless, it is often believed to be convenient and practically useful to assume that one force (either the personal or the impersonal) is constant, while the other is variable and thus in some sense marginal and "more significant" than the other. With this assumption, if the institutions are constant, then one can predict outcomes from tastes, or, if tastes are constant, then one can predict outcomes from institutions. It is of course true that this easy predictability is an illusion–but it is an illusion by which many scholars are hoodwinked because in quiet times the institutions are constant and only tastes are in dispute, while in turbulent times the institutions are in flux and only human greed seems constant. One fundamental and unsolved problem of social science is to penetrate the illusion and to learn to take both values and institutions into account. In the last generation we have made some small progress in this direction, if only to acquire a bit of sophistication about the problem, and the purpose of this article is to chronicle this progress.

Methodological Traditions in Political Science

Political science draws almost equally on traditions that overemphasize institutions and traditions that overemphasize tastes, which is perhaps why political scientists seem so eclectic as compared to, say, sociologists (whose tradi-

tion is almost exclusively institutional) or to economists (whose recent tradition, at least, stresses tastes).

The emphasis on institutions is our classical heritage. Aristotle collected and described 150 constitutions because he believed that constitutions determined both social outcomes and individual character. Even Plato, who initially argued for the rule of men rather than the rule of law, nevertheless devoted most of the *Republic* to a description of the institutions necessary to produce the kind of people he wanted for rulers, thereby implying that the institutions were primary and the particular rulers merely intermediaries between the institutions and the outcome. The notion that the quality of men's character is controlled by the laws under which they live, a notion that comes to us from ancient Greece by way of Roman law and eighteenth-century philosophers like Montesquieu, is, consequently, very much a part of contemporary political science. For example, it is said, with astonishing variety, that both the welfare state and restraints on government make people free, productive, contented, and self-respecting; or, for another example, it is said that incompatible varieties of economic institutions such as capitalism and socialism make people better off both morally and economically. Doubtless, the most extreme and absurd of the modern versions of classical institutionalism is Marxism, a picture of society in which economic institutions determine not only individual character but also the whole course of human history.

The emphasis on taste and values, on the other hand, is our Christian heritage. Because Christianity based the social order on personal decisions about faith and love and because it rejected the Judaic system of rules and forms (which came close to classical institutionalism), Christian theologians—at least from the Middle Ages onward—insisted that the quality of social outcomes depended almost entirely on the moral quality of rulers: Christian kings make good decisions; pagan or irreligious kings do not. Even in this secular century, some writers directly in the Christian tradition have described society in exactly this way. For example, T. S. Eliot, responding to the Munich crisis with *The Idea of a Christian Society* (1940, p. 34), saw only a change in *beliefs* as a way out of the world crisis: A community of Christians in a Christian state governed by rulers who accept Christianity, "not simply as their own faith, ... but as the system under which they are to govern." It would be hard to find a more complete conviction that social outcomes (and personal character) are determined by what people believe.

Owing, however, to the lack of interest in theology among twentieth-century intellectuals, the contemporary force associating individual values and social outcomes is wholly secular, though probably derived (as is, for example, extreme methodological individualism) from Christian modes of thought. In the ideology of democracy, which may well be a kind of secularized Christian theology, that form of government is often, though I believe quite inaccurately, defined as the rule of the people—by which it is meant that the people's values solely determine public decisions. For reasons that I have discussed at length elsewhere, this picture of democracy is internally inconsistent and cannot be sustained (Riker, 1978, 1980). At most democracy involves a popular veto on rulers, not a popular rule. Nevertheless inconsistencies and inaccuracies do not deter most ideologues, so that it is probably the case that nowadays the most widely accepted interpretation of democracy is that it is a device to combine individual values into decisions of government. Furthermore, this understanding, which went by the name of Benthamite "radicalism" in nineteenth-century England and of "popular sovereignty" in nineteenth-century America, is today believed by huge numbers of people (incorrectly, of course) to describe what actually happens in democratic governments.

While this supposed political description is mere ideology, it is nevertheless an important part of some contemporary political science and contributes greatly to the scientific emphasis on tastes and values, an emphasis expressed, for example, in the great amount of research on public opinion (which concerns the nature of tastes and values), political socialization (which concerns the creation of tastes and values), and representation (which concerns the incorporation of tastes and values in public decisions).

Great as is the contribution of democratic ideology to an emphasis on tastes and values, there is an even greater contribution, I believe, from the example of microeconomics. The theory of price in a competitive market—one of the few well-tested and verified theories in all of social science—is a theory in which institutions (i.e., the market) are held constant, while tastes determine outcomes. The theory takes this form: Given an auction market for a continuously divisible commodity with several buyers and several sellers whose tastes are constant over the period of the auction, then the price of the commodity is jointly and completely determined in a particular, describable way by the sum of the buyers' desires to buy and the sum of the sellers' desires to sell.

(These desires are, of course, tastes and values.) Furthermore, as long as tastes are constant the price so determined is a Pareto-optimal equilibrium in the sense that no *pair* of traders would agree to depart from it because any departure in favor of one trader would hurt another.

Since this theory admits prediction of an equilibrium and since the actual occurrences of numerous predicted equilibria have been verified, the prestige of this theory is higher, I believe, than that of any other theory in the social sciences. Indeed, it seems to me that this success alone (i.e., predictions from the theory of price) elevates the science of economics above all other social sciences in popular esteem among intellectuals, and renders economists believable even when they write of totally different subjects such as macroeconomics and social welfare, about both of which subjects their theories are as unverified as most others in social science. The scientific and intellectual success of price theorists in discovering equilibria has, of course, led many other social scientists to emulate them. We see, therefore, searches for equilibria of tastes in all branches of social science, not least of all political science.

The Search for General Equilibria

As I noted at the beginning of the previous section, political science draws eclectically both on traditions that overemphasize institutions and on traditions that overemphasize tastes. But, in the last 30 years or so, it seems to me that the traditions overemphasizing tastes have predominated. The study of constitutions, which characterized political science in the first half of the century, has latterly given way to the study of political culture, political behavior, and public opinion, all of which concern values and tastes. Simultaneously, with a kind of unspoken intellectual coordination, political theorists have analyzed the conditions for equilibria in abstract majority voting systems, which are in fact the conditions for an equilibrium of values. This development in political theory, which is described in this and subsequent sections, has by now revealed precisely what kind of equilibria can be expected, thereby allowing us to understand, with much more sophistication than was previously possible, the relation of values and institutions in structuring political outcomes.

The beginning of the search for conditions of equilibria is Duncan Black's rediscovery (in the mid-1940s) of the paradox of voting. Before that time scholars had indeed often discussed the equity and effect of voting systems, especially in disputes over proportional representation, methods of nomination, and forms of ballots. But, so far as I have been able to discover from a desultory survey, hardly anyone had recognized that the supposed defects might be based on individual tastes rather than structures of systems. If based on tastes, the defects are irremediable because, given an appropriate distribution of tastes, even a perfected system of voting might produce imperfect results. This is precisely the inference one draws from an analysis of the paradox and it is perhaps owing to an unspoken, even unrecognized, repugnance at this deduction that, when the paradox was initially discovered by Condorcet and rediscovered by Lewis Carroll, E. J. Nanson, and E. V. Huntington, it was nevertheless ignored (or perhaps repressed) by political scientists (see Black, 1958, pp. 156–238; Riker, 1961). Once Black brought the paradox irrepressibly to scholarly attention and showed that the disequilibria inherent in it depended not on the institutions of voting but on the distributions of taste, the search for conditions of equilibrium seemed an intellectual necessity.

To discuss this question, one needs an abstract society of these elements:

1. Alternatives: $\{a_1, a_2, \ldots, a_n\}$. If $n = 2$, equilibrium is certain because a_1 beats or ties a_2 or vice versa. Problems of equilibrium arise, however, when $n \geqslant 3$.

2. Voters: $\{1, 2, \ldots, m\}$. When $m = 1$, the problem is trivial, so it should be that $m \geqslant 2$.

3. Preference: Assuming that voters can compare alternatives and value some of them more than others, there are binary relations of preference, P_i (where $i = 1, 2, \ldots, m$), of indifference, I_i, and of the two combined, R_i, expressing a voter's estimate of the relative value of any pair of alternatives. (One writes "a_j R_i a_k" to mean "voter i prefers a_j to a_k or is indifferent between them.") Conventionally, R is assumed to be reflexive (a_j R_i a_j), connected (either a_j R_i a_k or a_k $R_i a_j$), and transitive (if a_h $R_i a_j$ and a_j R_i a_k, then a_h R_i a_k), so that by R (or P or I) a voter orders any triplet of alternatives from best to worst: a_h R_i a_j R_i a_k or a_h a_j a_k. [Notationally, a_h P_i a_j I_i a_k is written: a_h $(a_j$ $a_k)$.] Perhaps the attribution of the ability to order places unwarranted confidence in the fragile human ability to concentrate. By the assumption of the transitivity

of R, however, we give voting on values a chance at equilibrium with the best of human participation. If, then, voting fails, it fails the easiest possible test.

4. Outcomes from voting: Given a society of m voters faced with n alternatives, there are $(n!)^m$ possible profiles of preference D, that is possible ways the members of the group can individually order the alternatives. (That is, there are $n!$ possible orders of n alternatives and each of the m voters can select one of those orders.) The operation on a profile, D, of majority voting on pairs of alternatives yields an outcome relation, M, where "a_h M a_k" means: "(the number of i such that a_h P_i a_k) $\geqslant$ (the number of i such that a_k P_i a_h)," assuming, of course, that no i such that a_h I_i a_k participates in the voting. M may be, but need not be, a transitive relation so that M may yield either some one of the $n!$ orders of alternatives or intransitive cycles like a_h M a_j M a_k M a_h.

To consider the problems involved in the summation of preferences, observe the profiles, D^1 to D^4, where $n = m = 3$:

D^1	D^2
1. $a_h\ a_j\ a_k$	1. $a_h\ a_j\ a_k$
2. $a_h\ a_k\ a_j$	2. $a_j\ a_h\ a_k$
3. $a_j\ a_k\ a_h$	3. $a_k\ a_h\ a_j$
a_h M a_j M a_k	a_h M a_j M a_k

D^3	D^3
1. $a_h\ a_j\ a_k$	1. $a_h\ a_k\ a_j$
2. $a_j\ a_k\ a_h$	2. $a_j\ a_h\ a_k$
3. $a_k\ a_h\ a_j$	3. $a_k\ a_j\ a_h$
a_h M a_j M a_k M a_h	a_h M a_k M a_j M a_h
"forward cycle"	"backward cycle"

In profiles D^1 and D^2 there is a decisive winner by M in the sense that a_h beats each of $n - 1$ other alternatives. But in profiles D^3 and D^4, which are examples of the paradox of voting, there is no decisive winner because in, say, D^3, a_h will win if the sequence of voting is a_j versus a_k and then a_j versus a_h, a_j will win if the sequence is a_k versus a_h and then a_j versus a_k, etc., or no alternative will win if a round robin is conducted. The absence of a decisive winner is particularly disconcerting because, after assuming that each voter can order his or her values, it turns out that the group of voters cannot order them. Indeed, the people are coherent but the group is incoherent.

It is precisely the absence of a decisive winner that constitutes disequilibrium and the paradox of voting shows that disequilibrium can occur with majority voting, the relation M. Furthermore, the possibility of this kind of disequilibrium is present in *any* fair voting method, so it is not the institution of M, but the distribution of tastes, that is at fault (Arrow, 1963). One is consequently driven to ask what properties distinguish profiles with an equilibrium outcome (like D^1 and D^2) from profiles without one, like D^3 and D^4. (This question has usually been posed with respect to the relation M, so I will restrict my discussion to it. But, by reason of Arrow's theorem, we know that similar questions could be raised about any voting method, say, positional methods like plurality voting or approval voting.)

During the late 1960s, a systematic answer was developed, based on the observation that, for the 3! orders of a triplet of alternatives,

1. $a_h\ a_j\ a_k$
2. $a_h\ a_k\ a_j$
3. $a_j\ a_h\ a_k$
4. $a_j\ a_k\ a_h$
5. $a_k\ a_h\ a_j$
6. $a_k\ a_j\ a_h$,

numbers 1, 4, and 5 constitute D^3 and result in the forward cycle and numbers 2, 3, and 6 constitute D^4 and result in the backward cycle. Hence D^3 and D^4 exhaust the ways in which intransitive triples can occur. Any conditions on orderings by individuals such that either $a_h\ a_j\ a_k\ a_h$ or $a_h\ a_k\ a_j\ a_h$ are rendered impossible by M is thus a guarantee of equilibrium. One such condition is, for example, that, for any number of voters, some alternative in a triple is never in first place in a voter's order (as a_k is not in D^1) or some is never in last place (as a_h is not in D^2) or never in the middle place (as a_h is not in D^1)—this is the condition of "value restriction" (Sen, 1966). Or another condition is "extremal restriction," which is that, for any number of voters and for some order $a_j\ a_h\ a_k$ in D, if another order has a_k first, then this other order must have a_j last (as in D^2, $a_j\ a_h\ a_k$ is voter 2's order and $a_k\ a_h\ a_j$ is voter 3's) (Sen and Pattanaik, 1969). An exhaustive list of similar conditions is set forth in Fishburn (1973).

In addition to their completeness, the merit of these conditions on profiles is their clear revelation that equilibrium depends entirely on the accident of a non-cyclical set of voters' preferences. The defect of these conditions is,

on the other hand, their failure to indicate the likelihood that tastes might or might not be cyclical. Lacking that indication, they do not admit assessment of the practical significance of disequilibrium. Fortunately there exist less complete but intuitively more vivid geometric or topological conditions for equilibrium that do allow practical interpretation.

Historically, the first such condition, single-peakedness, was devised by Black even before Arrow's theorem was formulated (Black, 1948). If alternatives are arranged on a horizontal axis and the voters' ordinal valuation is measured on a vertical axis, then a voter's ordering may be represented as a curve on the positive quadrant connecting the voter's valuation of alternatives (see Figure 1). Such a preference curve is single-peaked if, as it flows from left to right, it is always rising, always falling, or rising to a peak or a plateau and then falling. By an appropriate arrangement of alternatives on the horizontal axis, any ordering of alternatives may be expressed as a single-peaked curve. If, however, three or more preference curves are drawn above a particular arrangement of alternatives on the horizontal axis, it may happen that all curves cannot be single-peaked. (See Figure 2, in which the reader may verify that, no matter how the three alternatives are ordered on the horizontal axis, at least one of the three curves from D^4—a cyclical profile—must fail to be single-peaked. In the particular ordering of Figure 2, voter 3's curve fails; but, were the ordering on the horizontal axis to be, say, $a_h\ a_k\ a_j$, then voter 2's curve would fail.)

Black's discovery was that, if any ordering on the horizontal axis exists such that all voters' preference curves are single-peaked, then an equilibrium exists in the sense that one alternative can beat or tie $(n - 1)$ others. Moreover the winning alternative(s) can be specified: Identifying the alternative beneath the peak of voter i's curve as O_i (for "optimum for i") and numbering the optima so that O_1 is at the far left and O_m at the far right, then, if m is odd, $O_{(m+1)/2}$ wins and, if m is even, $O_{m/2}$ and $O_{(m+1)/2}$ tie. For proof, when m is odd, note that, if a curve is rising from a_h to a_j, then the voter prefers a_j to a_h and, conversely, when the curve is falling, the voter prefers a_h to

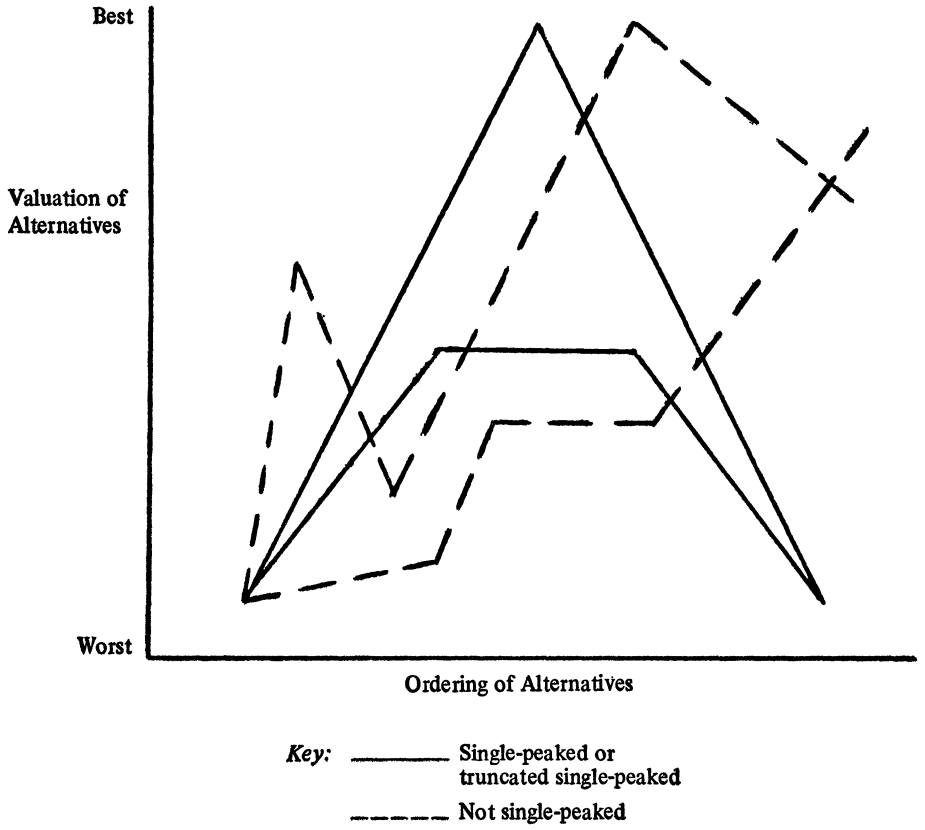

Figure 1. Preference Curves

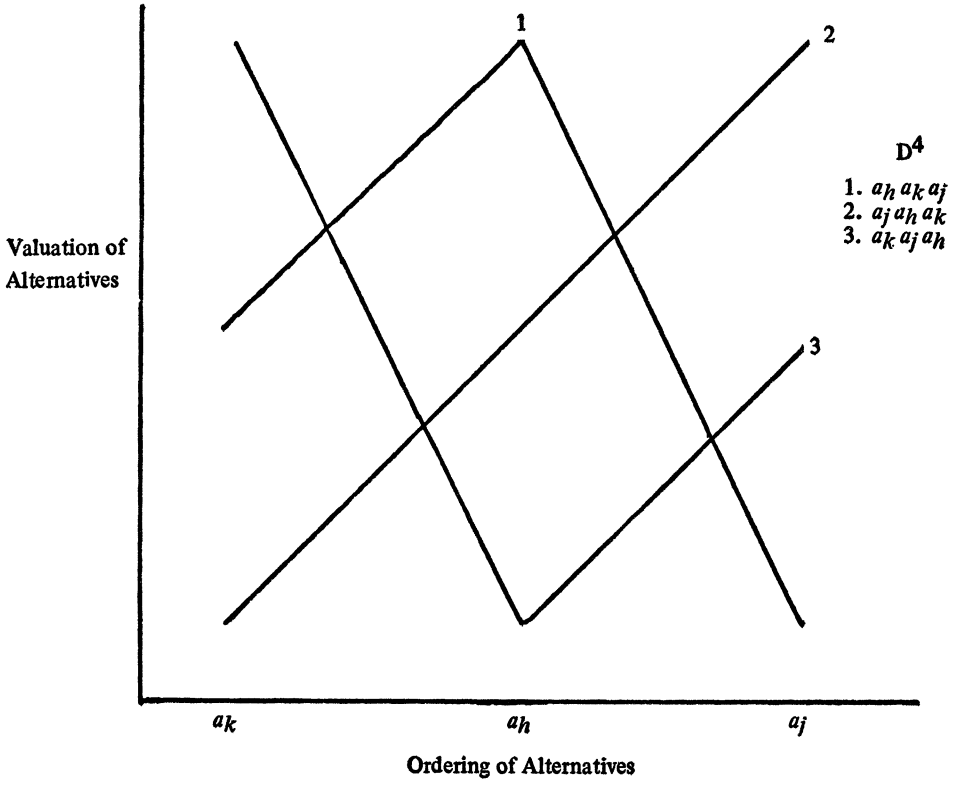

Figure 2. Non Single-Peakedness

a_j. Placing some a_k to the left of $O_{(m+1)/2}$ against $O_{(m+1)/2}$ in a vote, note that $O_{(m+1)/2}$ wins because over half the curves are rising between a_k and $O_{(m+1)/2}$, specifically all those curves with optima numbered $O_{(m+1)/2}$ to O_m (which is a majority itself) and all curves with optima lying between a_k and $O_{(m+1)/2}$. Similarly, placing some $a_{k'}$ to the right of $O_{(m+1)/2}$ against it, note that $O_{(m+1)/2}$ wins because over half the curves are falling between $O_{(m+1)/2}$ and $d_{k'}$, specifically all those from O_1 to $O_{(m+1)/2}$ (a bare majority) and those with optima between $O_{(m+1)/2}$ and $a_{k'}$. A similar argument establishes that O_m and $O_{(m+1)/2}$ tie when n is even.

It should be noted that this equilibrium at the median optimum is characterized by a balancing of opposites, a feature found in all other geometrically defined equilibria of voting. There are an equal number of voters on either side of the median, which is why it is the equilibrium. Suppose one subtracts (or adds) two voters whose optima are on opposite sides of the median, then the equality is unaffected and the equilibrium is characterized in some fundamental way by a pairing of opposites.

As a condition of equilibrium, single-peakedness (like the previously mentioned conditions, all of which were, however, discovered later) guarantees that neither a_h M a_j M a_k M a_h nor a_h M a_k M a_j M a_h occurs. It has the additional merit, moreover, of revealing a rationale for the existence of equilibria: That all curves are single-peaked means that all voters judge the alternatives consistently with respect to one issue, namely, that measured by the dimension on the horizontal axis. They may, of course, disagree about the best position on the issue, but they do agree that this single issue is the relevant basis for judgment.

This is why this condition has an intuitively obvious application to political campaigns, as in Downs' proposition that party platforms in a two-party system converge to the values of the median voter (1957, pp. 114–25). While Downs derived this argument from an economic model of the spatial location of firms, still his argument for equilibrium at the median voter's optimum assumes single-peakedness and is indeed invalid without it. This application suggests just how restrictive the condition is in practice because it seldom appears to be satis-

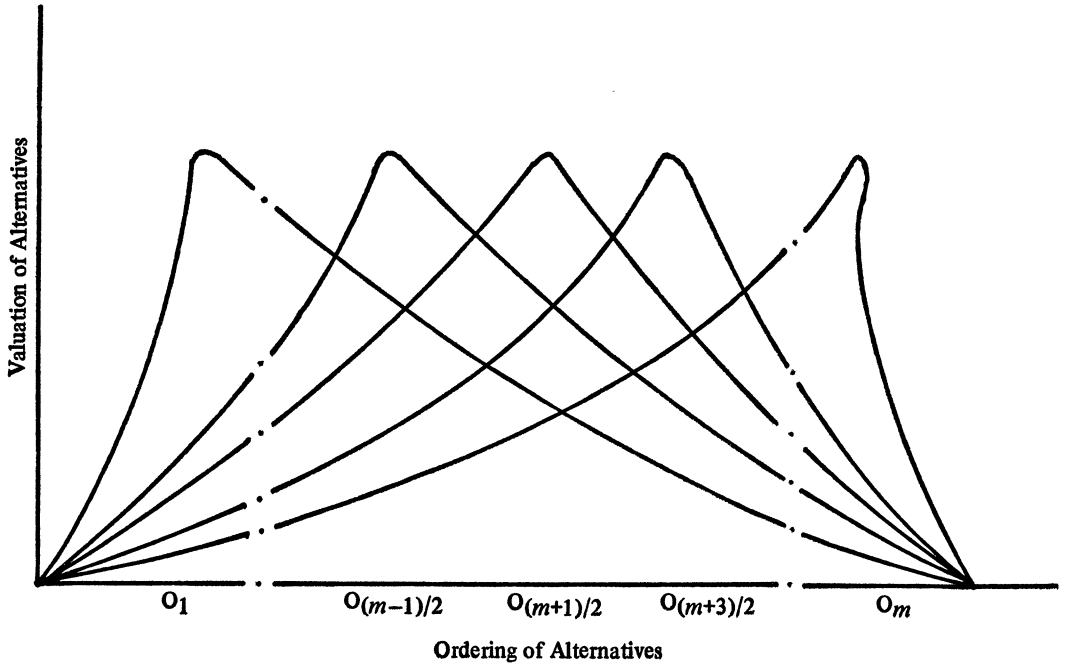

Figure 3. Voter's Optima

fied in the real world (Robertson, 1976). Despite the frequent journalistic use of dichotomies, e.g., "left-right," "Catholic-Protestant," "Fleming-Walloon," etc.—all of which are extremes on one issue dimension—still, scholarly efforts to describe real politics on one dimension seem always to break down. Indeed, once Downs set forth his model, it seemed so inadequate that other theorists soon developed an *n*-dimensional analogue (Davis and Hinich, 1966; Davis, Hinich, and Ordeshook, 1970).

Given the intuition that the one-dimensional model is inadequate for description, the appropriate next step is to search for equilibria in two-dimensional and ultimately in *n*-dimensional issue spaces. Black and Newing (1951) started the search with three voters in a two-dimensional model in which the peaks of Figures 1, 2, and 3 have become the humps of Figure 4. The vertical axis in that figure is, like the vertical axes in previous figures, for the ordinal measure of valuations. The two horizontal axes are issue dimensions or bases of judgment that define a plane (rather than a line) on which alternatives are located. Voter *i*'s optimum (O_i) lies in that plane directly beneath the highest point of the hump. Curves *e* and *f*, which are reflections into the $x_1\ x_2$–plane of two levels of preference, are indifference curves in the sense that voter *i* prefers all alternatives in the open space between *e* and *f* to any alternative on *e* but *i* is indifferent among all alternatives on, say, *e*.

To search for an equilibrium we need look only at the indifference curves in the $x_1\ x_2$–plane, as in Figure 5. Between two voters, with optima at O_1 and O_2, is a "contract curve" which connects points of tangencies of the voters' indifference curves. All the points on which voters 1 and 2 might agree lie on the contract curve. To see why, consider point a_k which lies, for each voter, on the outer of the two sets of indifference curves displayed. By definition, all points in the open shaded space are preferred by *both* voters to a_k. By successive reduction of the shaded area, one arrives at some point on the contract curve. When there are three voters, however, agreement is less easy to arrive at. Observe in Figure 6 that, while voters 1 and 2 might agree by majority vote on a_k, still a_j M a_k by voters 2 and 3. Nevertheless a_h M a_j (by 1 and 3) and a_k M a_h (by 1 and 2), so a cycle exists and there is no equilibrium.

There is, however, some chance for equilibria in this situation: If O_3 were to lie on the contract curve between O_1 and O_2, say, at a_k, then $O_3 = a_k$ would be a median between O_1 and O_2 and hence preferred by some pair (either 1 and 3 or 2 and 3) to *any* other point in the plane. In general, if one voter's optima lies on a contract curve between two others, then there is an equilibrium outcome. Note

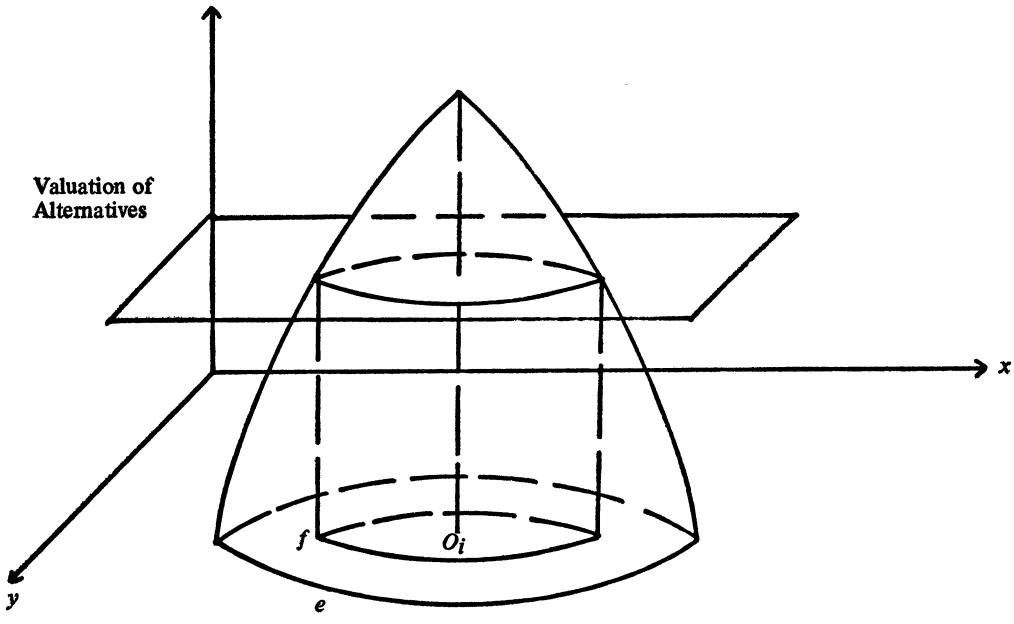

Figure 4. Single-Humped Valuation of Alternatives in Two Dimensions (x,y)

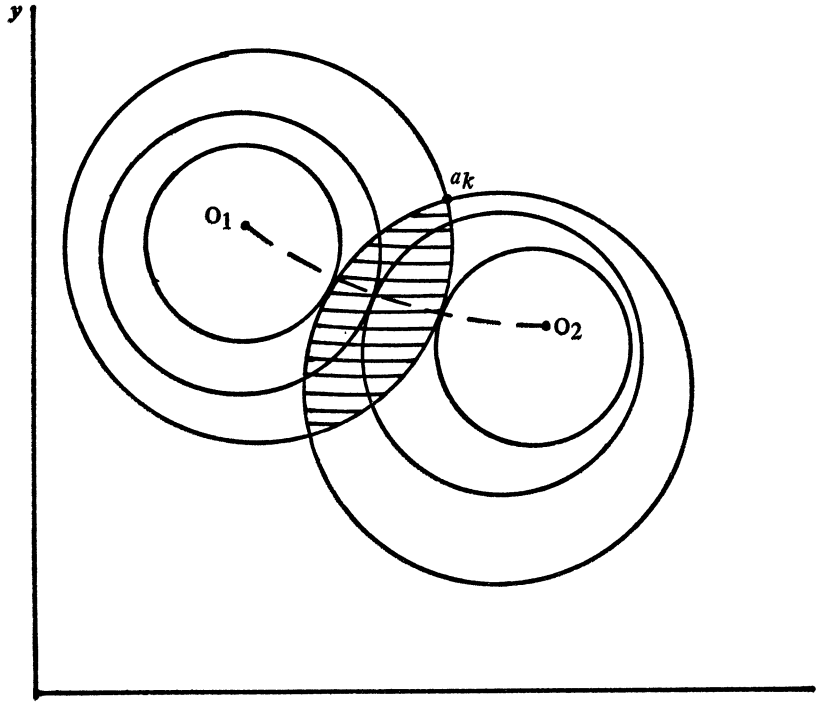

Figure 5. Indifference Curves

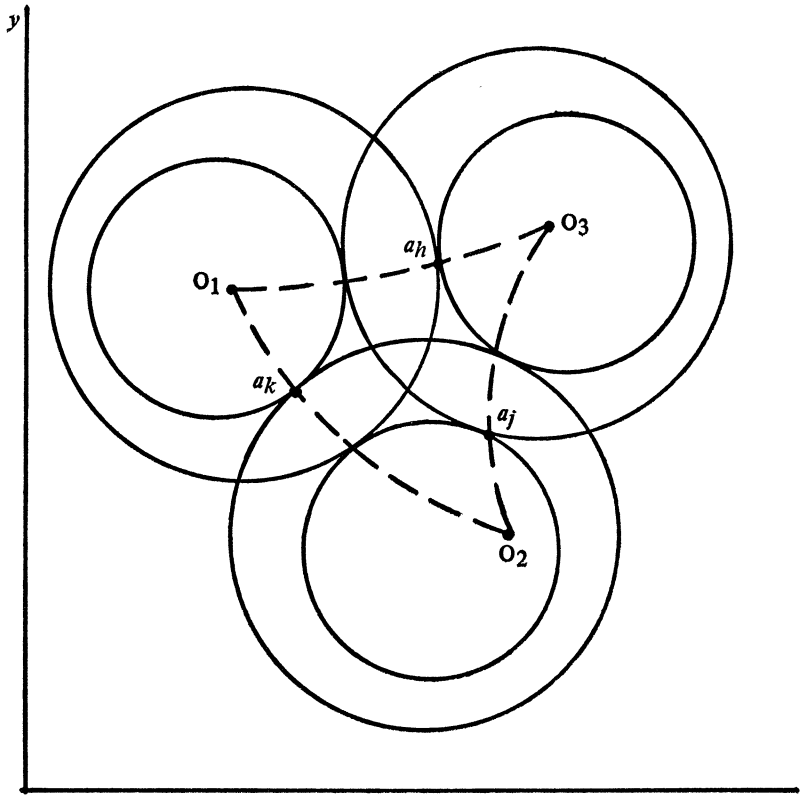

Figure 6. Contract Curves for Three Voters

that, as in the one-dimensional case, there is a balance between opposites, because a_k would remain the equilibrium if O_1 and O_2 were removed or if additional pairs of optima were added, one of each pair on the O_1 O_2 contract curve between a_k and O_1 and the other between a_k and O_2. This discovery was generalized by Charles Plott (1967). For the expanded situation we need some new notation. Since we now assume an infinite number of alternatives and continuously differentiable utility (as a measure on individual preference), we can no longer use n for the number of alternatives. Rather we use it to identify dimensions by which voters judge alternatives, 1, 2, . . . , n-dimensions. Then $U^i = U^i(x_1, x_2, \ldots, x_n)$ is the utility to the i^{th} voter of some point, a, in n-dimensional space. Let there be a status quo alternative, $\bar{a}$, that is, the alternative currently in force and let some a_j, which is a "small" distance, d, from $\bar{a}$, be placed against $\bar{a}$ in a majority vote. If a_j P_i $\bar{a}$, voter i obtains an increase in utility from a_j over $\bar{a}$. Let ΔU^i be the measure in utility of a vector in n-space from $\bar{a}$ toward some (unspecified) other point. Specifying the other point as a_j, one can say that, if $\Delta U^i\, a_j > 0$, then voter i prefers a_j, that, if $\Delta U^i\, a_j < 0$, voter i prefers $\bar{a}$, and if $\Delta U^i\, a_j = 0$, voter i is indifferent between a_j and $\bar{a}$. If there is some set of voters of size $\frac{m+1}{2}$ such that the gradient vectors, of utility for each voter, i, in the set are $\Delta U^i\, a_j \leqslant 0$, then a majority prefers $\bar{a}$ to a_j and $\bar{a}$ is a Condorcet winner or equilibrium.

A set of sufficient conditions for $\bar{a}$ to be in equilibrium are, for m odd:

1. that indifferent voters do not vote on a motion;
2. that there is at least one voter, i, for whom $\bar{a}$ provides the maximum utility;
3. that the $m-1$ remaining voters–an even number–can be divided into pairs, i and i', the interests of voters i and i' are diametrically opposed in direction and amounts of utility.

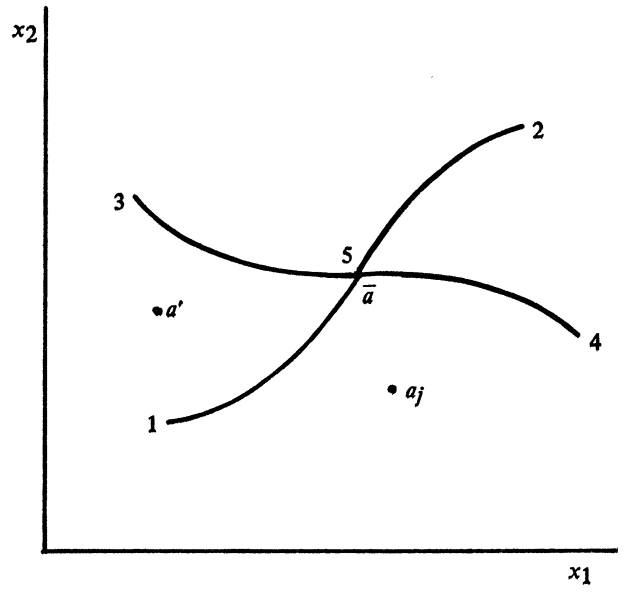

Figure 7. Equilibrium for m=5 Voters in n=2 Space

It is intuitively evident (and proved in Plott [1967]) that for m voters for which conditions 1, 2, and 3 hold, the voters would not wish to move from $\bar{a}$. One voter prefers $\bar{a}$ to anything else and for any a_j different from $\bar{a}$ one voter in each pair would prefer a_j to $\bar{a}$ and the other in the pair would prefer $\bar{a}$ to a_j. In Figure 7, where the maxima for voters 1, 2, 3, 4, 5 are at points numbered 1, 2, 3, 4, 5, $\bar{a}$ would beat a_j with a majority of 2, 3, and 5 or $\bar{a}$ would beat a' with a majority of 2, 4, and 5. Furthermore, any point $a' \neq \bar{a}$ can be beaten by some other point a_j, as in Figure 7 a_j beats a' with 2, 4, and 5.

The interesting feature of Plott's conditions for equilibrium are that the likelihood of satisfying them in the real world is extremely remote. Even if they were, by some amazing chance, to be satisfied, it would still be true that even a slight change in *one* voter's preferences would disrupt the equilibrium, because it would upset the necessary pairing of opposites. For all practical purposes, therefore, we can say that, given $m > 2$ voters and $n \geqslant 2$ dimensions of judgment with continuous alternatives, equilibrium of tastes is nonexistent.

This conclusion has been extended further by translating it into the language of game theory. When a majority prefers a_h to a_j, one says a_h *dominates* a_j and one defines a *core* as the set of undominated points, which is the same as the definition of Condorcet winners. Schloss (1973) has shown that the Plott equilibrium defines a core, given that the differentiability feature is removed; and Rubinstein (1979) has shown that, generically, cores of voting games, in which only continuous preferences are assumed, are empty.

Just how devastating is the absence of equilibria has been impressively demonstrated by Richard McKelvey, whose work has been aimed at showing that, when equilibrium breaks down, it breaks down completely (McKelvey, 1976, 1978). It has been frequently supposed that, although it is extremely unlikely that an alternative can beat $n - 1$ others, still all members of a relatively small set of k alternatives (themselves in cycle) can probably all beat the remaining $n - k$ others. If this were so, then some alternative in this "top cycle" might reasonably be regarded as a satisfactory winner and disequilibrium would simply mean the absence of a clear choice among several generally preferred outcomes. McKelvey has shown, however, that given continuous utilities, the top cycle can be expected to include all possible alternatives in an n-dimensional space. In a fashion similar to Plott's, McKelvey showed with an extremely general topological model, that the conditions for an equilibrium were

1. that the indifference contour for some voter i (not restricted as to convexity or any of the usual economic assumptions) must coincide with the frontier of the set of points that can beat some point a_h.

2. that all other voters' indifference contours can be paired up in the sense that, if some points a_j and a_k are on voter i''s indifference frontier, then they must also be on at least one other voter i''s indifference frontier.

These are, of course, very general formulations of the same kind of symmetry conditions that have been required all along, by Black, by Black and Newing, and by Plott. And they tell us about the same thing, namely, that conditions for equilibria are so restrictive as to render equilibria virtually nonexistent. Furthermore, even in the unlikely event that they should be satisfied, any single individual who is paired with another to generate equilibrium, can break it up by dissembling about his or her true preferences. Hence, not only are equilibria rare, they are also extremely fragile.

An important feature of McKelvey's conditions is that, if they are not met, then the top cycle includes all points in the policy space. This fact means that there is some way by which *any* point can beat the status quo. Suppose $\bar{a}$ is the status quo; then, if it is desired to replace $\bar{a}$ with a', there is some sequence of majority rule decisions (and often many such sequences) such that a' beats a_h, a_h beats a_j, . . . , a_k beats $\bar{a}$. Hence, any official or participant who can control the agenda can bring about the adoption of his or her desired alternative a'. But, of course, there is also a path by which $\bar{a}$ may then beat a'. So a second participant may foil the first.

A result similar to McKelvey's has been arrived at from a quite different topological model by Schofield (1978). His analysis begins with the observation that, for any point, x, in a multidimensional issue space, some indifference curve for each participant passes through x. Given these indifference curves, one can find the set of points, $P_C(x)$, which is the set of points in the neighborhood of x that are preferred to x by some winning coalition, C_k, $k = 1, 2, \ldots$. If, for the set, W, of all winning coalitions, $P_W(x)$ is such that there are some points, y, in the neighborhood of x that cannot be included in $P_W(x)$ by some path (y M_{C_i} z, . . . , $w M_{C_k}$ x, when C_i and C_k are any specific-winning coalitions), that is, if there are some points y that x can defeat no matter what, then there may be an equilibrium at x. If, however, all points in some arbitrary neighborhood of x can by some sequence of majority coalitions defeat x, then equilibrium at x is impossible. Furthermore, if for any particular set of individual ideal points and indifference curves, there is even just one point x for which equilibrium is impossible, then the system as a whole is cyclical. Effectively, this means that unless the individual preferences are highly similar—so that *all* winning coalitions are similar—social choices are certain to be cyclical. While this condition does not in itself indicate the likelihood of cyclical outcomes, Schofield has also shown that, if the issue space has at least as many dimensions as one more than the number of persons necessary for a minimal winning coalition, then the system is, for certain, cyclical. In legislatures most members probably have on every decision a dimension concerning the effect of the several alternatives on their chances of reelection and in electorates on all political platforms containing issues of a distributive nature ("who gets what") each participant is concerned, *inter alia*, with what he or she gets. Hence for these types of voting situations, there are at least as many dimensions as voters and disequilibrium is, therefore, certain.

In comparison with McKelvey's statement of the condition for a global cycle from $\bar{a}$ to $\bar{a}$, Schofield's theorem is a condition for a local cycle in an open neighborhood of $\bar{a}$. Practically, the difference is that McKelvey's condition guarantees instability by admitting what may seem like farfetched alternatives. (For example, the free soil issue that broke up the great Jeffersonian-Jacksonian agrarian coalition seemed to Democratic politicians, southerner and dough-face alike, to be an absurd irrelevancy because it was an issue more or less proscribed by the constitutional settlement. Nevertheless it broke up a seemingly overwhelming and persistent majority. This is the kind of event that we are assured is possible by McKelvey's theorem.) Schofield's theorem, while not excluding global cycles, assures us that local cycles can occur based presumably on only "slight" changes in the alternatives. (This is the kind of change we see in ordinary American politics where parties that appear extremely close ideologically turn each other out of office.) Hence, for most practical politics disequilibrium is assured.

The Significance of Disequilibrium

I now return to the philosophical question I raised in the beginning by asking what is the relevance for politics of the rarity and fragility of majority rule equilibria. And I start off by observing that the discoveries about majority rule probably apply to all methods of summing individual preferences. We know from Arrow's theorem that cycles cannot be avoided by any fair system, but we do not know from that fact

much about the likelihood or fragility of a cyclical outcome for other methods of summation. We do know, however, that other methods of voting (e.g., positional methods like plurality voting and approval voting and point counting or electoral methods like proportional representation which are intended to make minorities win) are subject to disequilibria, manipulation, agenda control, etc., in much the same way as majority rule. It seems fairly safe to conjecture, therefore, that equilibria in other voting systems are as rare and fragile as in majority rule. And this rarity and fragility are doubtless as much features of systems based upon coopted committees as of those based on popular election. Turning to non-voting methods of summation (e.g., statements of the sense of the meeting by a speaker or the selection of alternatives by a dictator), we know that the single summarizer necessarily imposes his or her own order on the outcome. Inasmuch as equilibrium is thus achieved by suppressing alternatives that might beat the single summarizer's own choice, such an equilibrium is not the product of summation, but of force. The rebellious discontent of those whose preferred alternatives are suppressed is simply evidence that the equilibrium achieved by a dictator or single summarizer is spurious. It seems to me, therefore, that what we have learned about equilibria under majority rule applies equally well to any political society whether it uses the institutions of majority rule or some other kind of voting or merely dictatorship.

And what we have learned is simply this: Disequilibrium, or the potential that the status quo be upset, is the characteristic feature of politics.

In the nineteenth century, economics was often called the "dismal science" largely because the equilibria predicted from price theory were not palatable to those who called it dismal. In what seems to me a deeper sense, however, politics is *the* dismal science because we have learned from it that there are no fundamental equilibria to predict. In the absence of such equilibria we cannot know much about the future at all, whether it is likely to be palatable or unpalatable, and in that sense our future is subject to the tricks and accidents of the way in which questions are posed and alternatives are offered and eliminated.

Yet there are some features of social decisions that we do understand and, in the short run at least, those do grant us some prevision. Although there are not likely to be equilibria based entirely on tastes, still there are outcomes of social decision processes, those outcomes do embody some people's values, and the outcomes themselves are not wholly random and unexpected. What prevents purely random embodiments of tastes is the fact that decisions are customarily made within the framework of known rules, which are what we commonly call institutions. Since institutions certainly affect the content of decisions, we can see something of the future by specifying just what these effects are and how they are produced. Thus, despite the recent enthusiasm for studying tastes (e.g., public opinion, political culture, and the like), what we learn from recent political theory is that the particular structure of an institution is at least as likely to be predictive of socially enforced values as are the preferences of the citizen body. So the sum of the recent discoveries is to re-emphasize some of the classical heritage of political science. It is important to study constitutions simply because, if there are repetitive equilibria in social decisions, these equilibria derive at least as much from institutions as from tastes and values.

The outcome, then, of the search for equilibria of tastes is the discovery that, failing such equilibria, there must be some institutional element in the regularities (or actual equilibria) we observe. We are forced back, therefore, to the eclectic stance that political scientists have typically taken. Outcomes are, of course, partially based on tastes because some person's (not necessarily a majority of people's) tastes are embodied in outcomes. But the ways the tastes and values are brought forward for consideration, eliminated, and finally selected are controlled by the institutions. And institutions may have systematic biases in them so that they regularly produce one kind of outcome rather than another. In this sense, therefore, both institutions and tastes contribute to outcomes. To offer an example, in electoral systems it not infrequently happens that the same party or coalition of parties wins election after election. Conceivably, this stability may be caused by the fact that tastes are constant, but more often it is caused by the fact that exactly those issues likely to upset the stability of tastes are kept out of the electoral process by structures like constitutions and political parties. (An excellent example of this process is the exclusion of the issues of slavery and free soil from American national politics from the 1780s to the 1840s. The hegemony during that period of the Jefferson-Jackson Democracy, which did provide a long-term equilibrium, could exist only by suppressing the slavery issue. Once it was raised, dividing rural slaveholders from rural yeomen, the Democracy and the nation were disrupted.) What results, there-

fore, is an outcome based both on tastes and on the way in which some tastes are admitted to and some not admitted to the decision process.

Consequently, we cannot study simply tastes and values, but must study institutions as well. Nevertheless, as we return to the study of institutions after a generation of preoccupation with values and preferences, we do so with a deeper appreciation of the appropriate scientific program and of our opportunities and limitations as scientists. The scientific program is to explain by the application of covering laws to in particular situations how institutions generate equilibrium by systematically excluding or including certain tastes or values. As for our opportunities and limitations, we have already learned from the devleopments chronicled here that we cannot expect to find equilibria of preferences, but we may be able to find equilibria generated from a given subset of preferences by particular institutions.

In the earlier tradition of studying constitutions, it was customary to look for the centers of power in a constitutional structure—to look, that is, for who could control which portions of the political process. This is, of course, an interesting practical question for the world, because it concerns the distribution of "power." But while such distributions are a fascinating subject for ideologues and inside dopesters, they are not of much scientific interest because the idea of power is itself an inexact and probably meaningless notion (Riker, 1965). What is instead scientifically interesting is the interaction among the several participants in a system to discover the particular kinds of outcomes that are both feasible and likely, given a particular institutional arrangement.

This is the kind of study of institutions that has developed more or less unconsciously among specialists on the U.S. Congress, who are among the first political scientists to study a single institution in intensive detail. Having identified the several centers of authority in the conventional kind of constitutional analysis, they have gone on to generalize about how these centers interact in the selection of values to be incorporated into legislation. So sophisticated has this kind of inquiry become that Kenneth Shepsle, a scientist trained in both fields, has attempted to integrate it formally with the study of equilibria of preferences. Thereby he has managed to lay down an outline of what the new kind of study of institutions might typically look like (Shepsle, 1979). Shepsle distinguishes two ways in which rules and structures may impose conditions that affect the outcome of the decision-making process in a legislature, or indeed in many other kinds of decision-making bodies. One is decentralization: to divide the body into subsets which act on some issues for the whole body: e.g., committees or parties in legislatures, departments in colleges or firms or bureaucracies, etc. The other is the creation of jurisdictions: to divide up for separate consideration the dimensions of decision so that, in a policy space with m dimensions, it may be required that each dimension, $x_1, \ldots, x_m$, be considered by itself. Sometimes these two kinds of rules are combined, as when a congressional committee is given control over one feature, and only one feature, of a bill. Shepsle defines a structure-induced equilibrium as one in which, taking tastes as given, a particular arrangement of subsets of decision makers and particular assignment of jurisdictions allow for the passage of a motion that cannot be defeated by any other alternative. (Of course, an equilibrium of tastes, where, regardless of institutions, a motion can beat any other, implies an equilibrium of structure. But the converse does not hold: a structural equilibrium does not imply an equilibrium of preferences. In that sense, the notion of a structural equilibrium is narrower in meaning than an equilibrium of values or tastes.) Shepsle's main theorem is that structural equilibria exist in a committee system, provided the members' preferences can be represented by quasi-concave, continuous utility functions and the committee system operates in an m-dimensional space in such a way that each dimension is under the jurisdiction of a particular committee. (Particular assignments of jurisdiction are assumed to be protected by a germaneness rule that permits amendments in committee only on the appropriate dimension.)

The secret of this theorem is that, when social choices are made dimension by dimension, then, if an equilibrium condition (say, single-peakedness) is satisfied on one of the dimensions, some degree of stability is imposed on the whole system. Since equilibrium conditions often do exist when choice is on only one dimension, especially if the decision-making body shares cultural standards, it follows that structural equilibrium is much easier to obtain than a pure equilibrium of tastes.

But, asks Shepsle, how robust is a structural equilibrium? The answer is that, insofar as a constitutional system supplies an outcome that is not the same as outcomes that might have been obtained from simple majority rule in the system without committees, jurisdictions, etc., the losers are likely to want to change the committees and jurisdictions in the hope of winning on another day. In the end, therefore, institutions are no more than rules and rules are

themselves the product of social decisions. Consequently, the rules are also not in equilibrium. One can expect that losers on a series of decisions under a particular set of rules will attempt (often successfully) to change institutions and hence the kind of decisions produced under them. In that sense rules or institutions are just more alternatives in the policy space and the status quo of one set of rules can be supplanted with another set of rules. Thus the only difference between values and institutions is that the revelation of institutional disequilibria is probably a longer process than the revelation of disequilibria of taste.

Our new sophistication about institutions, induced by our long foray into the search for equilibria of tastes, is that institutions are probably best seen as congealed tastes. We ought, I think, to be thoroughly aware that the distinction between constitutional questions and policy questions is at most one of degree of longevity. If institutions are congealed tastes and if tastes lack equilibria, then also do institutions, except for short-run events.

It is true that we can get a lot of mileage out of relatively stable institutions. If elections are zero-sum or constant-sum, then all the restrictions embodied in game theory notions of solutions of zero- or constant-sum games and all the restrictions embodied in sociological laws like the size principle are more or less permanently imposed on outcomes. Only the abolition of zero-sum or constant-sum methods of election is likely to eliminate these restrictions. Similarly, while it is easy enough to change some prisoners' dilemmas to situations with Pareto-optimal outcomes (as for example the so-called "tragedy of the commons" was solved by the enclosure of common lands into private property), still there are other apparently intractable prisoners' dilemmas (such as arms races and the extinction of species of creatures like passenger pigeons and perhaps whales). In the former cases there are governmental organizations inclusive enough to change the institutions of the dilemma. But the latter sort of institutions are likely to last for a very long time.

Nevertheless, if the non-Pareto optimal feature of an institution is sufficiently distasteful to most participants, it is possible to reconstruct institutions. Private property in land was extended to the commons to prevent the destruction of soil, and it is not impossible to imagine private property in whales. If institutions do generate an outcome in which everyone loses, it is reasonable to expect some new and less distasteful institutions—which is to say that even the most fundamental institutions lack equilibria, although it may take generations to alter them.

The sum of our new sophistication is, therefore, that political outcomes truly are unpredictable in the long run. We may have a few pretty well-verified generalizations to guide us (for example, the size principle or Duverger's law), but for the most part we live in a world that is uncertain because it lacks equilibria.

Conclusion

And this conclusion sets the problem of political science: In the long run, outcomes are the consequence not only of institutions and tastes, but also of the political skill and artistry of those who manipulate agenda, formulate and reformulate questions, generate "false" issues, etc., in order to exploit the disequilibrium of tastes for their own advantage. And just what combination of institutions, tastes, and artistry will appear in any given political system is, it seems to me, as unpredictable as poetry. But given the short-term structural and cultural constants, there is some stability, some predictability of outcomes, and the function of the science of politics is to identify these "unstable constants."

Appendix

This paper was originally presented at the meeting of the International Political Science Association, Moscow, 1979. For that occasion the concluding remarks were as follows:

Given the location of the platform for the presentation of this paper, I should conclude with the observation that political science can exist only in an open society, that is, a society with unfettered freedom of speech. Insofar as the science involves a study of values and tastes, scientists can be accurate in their predictions only if they are able to ignore official doctrine (as for example in Marxism) about the preferences and interests of groups and classes. Official doctrine may be right or wrong, but whether it is or is not right is a subject not for official decree but rather for empirical investigation, which is possible only an open society. Moreover, insofar as the science of politics involves the study of institutions, scientists must be able to examine critically the way governmental institutions operate at the highest as well as lowest levels of government. Only thus can they study the way institutions systematically bias the selection among preferences. Of course, this means that governmental

secrecy, if it exists, prohibits scientific investigation of political structures.

Which of the two—official doctrine about preferences or governmental secrecy—is the more inhibiting for scientific inquiry probably varies from place to place. But I believe secrecy is more often a barrier. The scientist can often guess fairly well about tastes and preferences, but the way institutions work is extremely difficult to guess about. Consequently, if I am correct in believing that the study of tastes is not enough and that one must study institutions as well, then it follows that the new emphasis on institutions as a necessary part of the science of politics probably precludes this science in any society governed secretly.

Finally, there is another way in which the conclusions of this paper imply that political science can exist only in an open society. One important conclusion, indeed the most important conclusion, of the line of reasoning set forth in this paper is that, in the long run, nearly anything can happen in politics. Naturally this conclusion is a sharp contradiction of all philosophies of history (such as Marxism) that necessitate a belief in the existence of a determined course for the future. This belief is precisely what the discoveries recounted in this paper deny. So, if these discoveries are true—and mathematically they appear to be irrefutable—then a science of politics is incompatible with Marxism.

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