Physik. It was titled "*Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme*" (On formally undecidable theorems in Principia Mathematica and related systems) and showed that statements exist that are true in a mathematical system but cannot be proven within its axioms. (Gödel's paper referred to Alfred North Whitehead and Bertrand Russell's monumental work *Principia Mathematica*.) Gödel's Incompleteness Theorem, as it was henceforth called, proved that under certain circumstances an axiomatic system cannot be both consistent and complete, thus putting to rest all attempts to set mathematics on an axiomatic basis. Thereby it did to mathematics what Arrow's Impossibility Theorem did to social choice theory. An aggregation mechanism, based on a handful of axioms, cannot fulfill a few reasonable requirements and be democratic. (To the general malaise one may add that Werner Heisenberg had done something similar to physics four years earlier, in 1927, with his Uncertainty Principle.)

There is a well-known anecdote about the eminent logician that may have some bearing on what we just discussed. Gödel left Austria during the Second World War and found a new home at the Institute for Advanced Study in Princeton. In 1948, when he decided that he would not return to his fatherland, he applied for American citizenship. His colleagues Albert Einstein and Oskar Morgenstern, naturalized Americans themselves, accompanied the otherworldly mathematician to the immigration office for the crucial interview. On the way they coached him on the American Constitution. Gödel, who had studied it the night before, spent the drive to the bureau arguing that the venerated document left the door open to a dictatorship. Knowing that this would not be the kind of argument the immigration official would like, Einstein and Morgenstern persuaded him not to insist on that point when questioned. Luckily Gödel heeded the warning and was duly awarded citizenship. It is not documented what the loophole was, that Gödel thought he had discovered. Was it maybe, just maybe, the idea that the democratic, one-man-one-vote majority election system that all Americans hold dear could lead to cycles, thence to a revolution and finally to a dictatorship?

CHAPTER TWELVE

THE QUOTARIANS

We return to the frustrating subject of apportionment. In the preceding chapter I recounted that Kenneth Arrow proved that any election method that satisfies reasonable conditions of rationality—like avoiding cycles—is either imposed or dictatorial, and that Allan Gibbard and Mark Satterthwaite showed that any democratic election method can be manipulated. This chapter will, unfortunately, be the bearer of further bad tidings: a fair and true allocation of seats in Congress is also a mathematical impossibility.

With the size of the House fixed at 435 in 1912, the Alabama Paradox no longer loomed. And after the inclusion of Alaska and Hawaii in 1959 no new states were likely to join the Union, so the New State Paradox also no longer posed a problem. But the population keeps growing; hence, the Population Paradox is here to stay. And of course, all the inequities that occur when rounding seats up or down remained.

Even though Congress did its best to plaster over the differences whenever possible, the problem never really went away. According to the 1950 census, California would have gained a seat, to the detriment of Kansas, if the Webster-Willcox method, a.k.a. W-W method, or Cornell method, had been used. In 1960, North Dakota would have lost one of its two seats to Massachusetts and ten years later, in 1970, Kentucky and Colorado would have gained one seat each, to the detriment of South Dakota and Montana, had the Cornell method been in force. From time to time challenges were raised. Following the 1980 census, Indiana, which stood to gain an eleventh seat under W-W, raised a ruckus. The House actually considered changing the apportionment method—New Mexico would have been the loser—but the proposal never got off the ground.

After the 1990 census, tempers flared especially high. This time the Huntington-Hill method, a.k.a H-H method, or Harvard method, caused the state of Montana to lose out by having to forfeit one of its two seats in the House. Montana did not like this at all and sued the government, specifically the Department of Commerce, which administers the apportionment of seats. Unfortunately, the W-W method would not have let Montana keep its second seat either, so the state had to dig further in order to build a case for its second seat. Lawyers did get lucky eventually, disinterring the Dean method (see chapter 10), which heretofore had never been used or even seriously considered. It had one advantage, however. It would have awarded the additional seat to Montana. (As a consequence, Washington's seats would have been reduced from nine to eight.)

The jurists built a case around the Dean method, the suit was heard in the District Court of Montana in 1991 and, lo and behold, the state was successful. A panel of three judges, with one judge dissenting, found that the method of equal proportions resulted in an unjustified deviation from the ideal of equal representation. But now the government was unhappy and the verdict was appealed to the Supreme Court. The justices in Washington DC unanimously reversed the judgment of the District Court. Justice John Paul Stevens delivered the court's opinion. He ended with the words

> the decision to adopt the method of equal proportions was made by Congress after decades of experience, experimentation, and debate about the substance of the constitutional requirement. Independent scholars supported both the basic decision to adopt a regular procedure to be followed after each census, and the particular decision to use the method of equal proportions. For a half-century the results of that method have been accepted by the States and the Nation. That history supports our conclusion that Congress had ample power to enact the statutory procedure in 1941 and to apply the method of equal proportions after the 1990 census.

To the great relief of the state of Washington, the Harvard method was vindicated once again.

After the 2000 census something surprising happened. Nobody complained. How come, did all states suddenly see the wisdom of H-H? Certainly not. But it just so happened that the W-W method would have given the exact same allocation as the H-H method for all fifty states. So nobody got upset, or at least no state could blame the method for its losses.

In spite of a lull in the disputes between proponents and opponents of the various methods, an uneasy feeling persisted. Even though the method of Huntington-Hill was tacitly accepted, mostly by default, nobody was

quite at ease with the lack of theoretical underpinnings for the apportionment of congressional seats. It would be nice to justify the preference of one method over another by appealing to something more than to judicial expedience and political convenience. In this vacuous atmosphere academics felt called upon to establish the foundations for a rigorous theory of apportionment. Two mathematicians, Michel L. Balinski and H. Peyton Young, answered the call.

Balinski, professor of mathematics at the City University of New York, had invited Young to interview for the position of assistant professor at the graduate school. The two hit it off immediately. Young was very impressed by the fact that Balinski had just finishing an interview with French television journalists when he drove up to his house. Apparently a person could combine mathematical research with influencing policy. When Young joined the Graduate Center of the City University of New York as a junior faculty member he and Balinski decided to collaborate. Even though both men had obtained their doctorates in mathematics, their training, teaching, and consulting work was nothing if not interdisciplinary. The broad experience stood them in good stead for what they were about to do.

One of Balinski's areas of expertise was integer programming, a branch of operations research. Developed before, during, and after World War II, operations research originated in the military where logistics, storage, scheduling, and optimization were prime considerations. But it soon acquired enormous importance in many other fields, for example in engineering, economics, and business management. While game theory, developed at the same time, was mainly of theoretical interest, operations research was immediately applied to practical problems. Whenever something needed to be maximized or minimized—optimized for short—and resources were constrained, operations research offered the tools to do so.

Optimization problems arise in everyday life all around us. Business, household, school, engineering, and traffic are just some of the areas where we constantly strive to maximize something: income, grade point average, profits, strength, speed, pleasure, and so on. On other occasions one may want to minimize variables like expenditures, effort, the distance one has to walk, and so forth. One of the techniques from operations research's toolbox is linear programming. Whenever one wants to optimize something subject to constraints—like maximizing investments subject to a budget constraint—linear programming is the tool to use.

The solution to a linear programming problem consists of real numbers—for example, the amounts of different compounds to be used in order to create an alloy of maximal strength. Unfortunately, far more intricate problems arise when the variables must be integers. Think of the fiendishly difficult Diophantine equations, which allow only integer solutions. It is very easy to give as many solutions as you like to the equation $x^3 + y^3 = z^3$, as long as the variables can be any real numbers. But if x, y, and z must be nonzero integers, solving the equation is impossible, as Fermat tried, and Andrew Wiles managed, to prove.

Something similar occurs when decision variables must be integers. For example, when an airline decides how many aircraft to buy, how many legs to fly, or how many crews to schedule, the solutions cannot be just any real numbers, they must be integers. After all, the airline cannot buy 17.6 aircraft, fly from Memphis to Dallas 2.4 times a day, and have 0.8 pilots on standby. While it is no longer difficult to find solutions to linear programming problems—George Dantzig solved that task with his Simplex algorithm in 1947—the assignment becomes vastly more complicated when solutions must be whole numbers. Tools to handle decision problems that allow only whole-number solutions do exist, however, and Balinski became one of the world's foremost experts in integer programming. This experience, together with his interdisciplinary worldview came in handy when studying the allocation of seats in Congress.

When Balinski and Young met, Balinski was in a sad mood. He had been working intensively on transforming his work on integer programming into a textbook when, one day, a fire destroyed his office and consumed all his notes and books. One can well imagine the pain and frustration that Balinski felt when he realized his labors had come to naught. The idea of resurrecting all that he had already done was extremely distasteful. Balinski began to look for a different challenge.

So when the chairman of the math department at the Graduate Center of CUNY asked for a volunteer to teach a one-semester course to two hundred freshmen, Balinski volunteered. It was an experiment the Graduate Center was going to do with undergraduate students. These were students who were not majoring in science or mathematics, and the class was likely to be the only math course they would ever attend. It was to give

them a taste of the importance of mathematics. Nobody on the faculty was very keen to take on the task, but for Balinski, still smarting from the loss of his material on integer programming, this was just what he needed.

The course would be an opportunity to convey the practical importance of mathematics without the burden of having to teach a specific body of techniques. What he now required was a problem that would be accepted as significant by the students. After reflecting for a while, Balinski hit upon the perfect subject: congressional appointments. It was ideal for two reasons. First, a constitutional problem would immediately be recognized as important by the students. Second, "almost everybody is prepared to suggest a solution to the problem," Balinski recalled many years later. "It often turns out to be bad, sparking debate and confrontation, which are ideal for the classroom."

Balinski began preparing for the course. While reviewing what had been done and written on the subject, it quickly became apparent to him that the H-H method, which was still being used, was suspect, despite the solemn endorsement by the National Academy of Sciences. True to his profession as a mathematician, he decided that the axiomatic approach should be used to sort out what method was in fact the most equitable. He recruited Peyton Young to work with him on the subject.

Their collaboration eventually resulted in the widely acclaimed book *Fair Representation: Meeting the Ideal of One Man, One Vote*. Published by Yale University Press in 1982, it was the first serious scientific study of apportionment since the time when the Founding Fathers signed the Declaration of Independence—and that includes the two reports to the National Academy of Sciences. It is worth noting that the monograph was reprinted as a second edition in 2001 with identical pagination. Not many scientific texts get reprinted after twenty years, maintaining the original material except for the correction of typos. By the way, the monograph may be the only mathematics book that ever got a multipage review in a U.S. Supreme Court decision.

The stated aim of the authors was to apply mathematical reasoning to a question of public policy "similar to the axiomatic approach used in mathematics, where the object is to discover the logical consequences of certain general principles." At first blush, the book seems easy enough to read, with many historical excursions and numerical examples, but its seeming simplicity belies its seriousness. Even though no more mathematics is required to follow the authors' reasoning than simple arithmetic, the problems are nothing if not challenging, and the arguments nothing if not sophisticated. But if you expect the book to give an answer to the question, which apportioning method is best, be forewarned: there is none.

As Kenneth Arrow had done in his work, Balinski and Young started their search for a good method by stating the requirements it should fulfill. The first requirement is proportionality: a state with three times the population of another state should have three times as many representatives. By the same token, if one state grows faster than another, this should be reflected in an increase in representation. If the requirement of proportionality is violated, we are faced with the dreaded Population Paradox. Since the Alabama Paradox and the New State Paradox no longer threaten, this is the one remaining obstacle.

Some allocation methods violate the requirement of proportionality. In chapter 9 we saw how Hamilton's method of allocating seats according to the greatest fractional remainder can give a paradoxical result. Although Virginia's population grew more than Maine's, both in absolute numbers as in percent, it was Maine that gained a seat at Virginia's expense.

In short, fractional remainders—varying between 0 and 1—do not reflect the states' relative sizes. Hence they are an inappropriate device to determine which states should receive additional seats. Any method that is based in some way or another on fractional remainders suffers from the defect of the Population Paradox. So Hamilton's method is out.

If remainder methods are not appropriate, what method is? Let's pull the rabbit out of the hat and explain later: it's any one of the divisor methods. Recall that for this method an appropriate number is chosen—the divisor—such that, when dividing each state's population by the divisor and then rounding up or down, the appropriate number of seats is allocated. If the number of seats turns out to be too large or too small, a larger or a smaller divisor is chosen and the process is repeated. The number that results when a state's population is divided by the divisor is called the state's quota.

As we saw in chapter 9, divisor methods abound. The only differences between them are the cutoff points for rounding up or down. Any set of cutoff points works! For the five traditional methods they are: Adams

method always rounds up; Jefferson's method always rounds down; Webster's rounds at the midpoint (for example, at 1.5, 2.5); Hill's at the geometric mean (for example, $\sqrt{[1 \times 2]} = 1.414$, $\sqrt{[2 \times 3]} = 2.449$); and Dean's at the harmonic mean, which is defined as the product of two numbers, divided by their average (for example, $1 \times 2/0.5[1 + 2] = 1.333$, $2 \times 3/0.5[2 + 3] = 2.4$). Balinski and Young called the fraction where the rounding takes place a signpost. Each method has its own signpost sequence. As soon as the quota passes a method's next signpost, the quota is rounded up and the state gains a seat.

TABLE 12.1
Signposts

	Quota rounded to					
	2 seats	*3 seats*	*4 seats*	*5 seats*	*6 seats*	*Rounding scheme*
	at or beyond:					
Adams	1.000	2.000	3.000	4.000	5.000	always up
Dean	1.333	2.400	3.429	4.444	5.454	at harmonic mean
Hill	1.414	2.449	3.464	4.472	5.477	at geometric mean
Webster	1.500	2.500	3.500	4.500	5.500	at arithmetic mean
Jefferson	2.000	3.000	4.000	5.000	6.000	always down

As an example, consider the four imaginary states IO, HJ, MU, and NK. Dividing the populations of the four states by the divisor 50,000, we get the following apportionments after rounding:

TABLE 12.2
Rounding schemes

	Population	*Quota*	*Adams*	*Dean*	*Hill*	*Webster*	*Jefferson*
IO	361,250	7.225	8	7	7	7	7
HJ	222,750	4.455	5	5	4	4	4
MU	324,100	6.482	7	7	7	6	6
NK	836,250	16.725	17	17	17	17	16

Why do these methods avoid the Population Paradox? Say a state grows, thereby passing a signpost and gaining a seat. Say another state grows more quickly. Obviously, it moves forward toward the next seat. It may not grow sufficiently to pass its own next signpost, thus not gaining an additional seat, but it could never move back past the previous signpost. This is in stark contrast to remainder methods, where states move both forward and backward beyond the point where rounding takes place. Divisor methods would never cause a faster-growing state to lose a seat, if a slower-growing state gains one. Conclusion: no Population Paradox. End of story.

With this argument, the two mathematicians showed that divisor methods of whatever variant avoid the Population Paradox. Somewhat surprisingly it turns out that divisor methods—all of them—also avoid both the Alabama Paradox and the New State Paradox. That this is the case can be shown by the signpost argument again. Balinski and Young had been seeking a way to avoid the Population Paradox. They got the annulment of the two other paradoxes as an extra benefit.

But they did more than just show that divisor methods are good at steering clear of paradoxes. They proved rigorously that divisor methods are the *only* techniques that avoid the Population Paradox. Any method of apportionment that is not based on divisors will, unfailingly, fall victim to the Population Paradox. I will not reproduce the proof here. Suffice it to say—as the authors did when they consigned the proof to the appendix—"that is the part played by mathematics."

Once it has been established that divisor methods are the only apportionment methods that should be considered, the question arises what the differences are, if any, among the rounding schemes of Adams, Dean, Hill, Webster, and Jefferson. In terms of shunning paradoxes, they are equally suitable. But we demand more from a good method than just an avoidance of ridiculous outcomes. An appropriate technique for the allocation of seats in Congress should fulfill additional requirements. The next requirement of a good allocation method is that it not be biased toward certain states.

What is meant by *bias* is a systematic tendency to favor either large states or small states. The stress is on the word *systematic*, because in any given year it is unavoidable that some states get a little more representation than they would be due, and others get a little less. As we saw in chapter 9, Jefferson's method was abandoned precisely because it consistently favored the large states. A method is considered unbiased "if the class of large states has the same chance of being favored as the class of small states," Young and Balinski declared. The true test of an unbiased

method is that over the long run the advantages and disadvantages average out.

So which divisor methods—Adams, Dean, Hill, Webster, Jefferson—are unbiased, in addition to being immune to all the paradoxes? Balinski and Young attacked this question from two perspectives: historical and theoretical. They devised an index to account for the cumulative bias between 1790 and 2000 and found that Adams had a bias-index of about fifteen toward small states, Jefferson had a bias-index of about fifteen toward large states, and Dean and Hill favored small states to the lesser rate of about three and five. But the hands-down winner on the historical test was Webster's method with a cumulative bias-index of only about 0.5 toward small states.

This is not surprising. With cutoff points at 1.5, 2.5, 3.5, . . . Webster's method gives each state a 50 percent chance of being rounded up or down every time. Obviously, this averages out in the long run. In contrast, Adams's, Dean's, and Hill's cutoffs are always below 0.5—see the signposts in the table above—which favors small states for a few reasons. First, rounding up from 2.4 to 3.0 is "worth" more than rounding up from 32.4 to 33.0. Second, the cutoff points become larger, increasing toward 0.5, the larger the state. Finally, as was explained in chapter 9, rounding up necessitates an increase in the numerator, which, in turn, means that states are penalized for each seat they already have. This, in turn, implies that larger states are penalized more. Finally, Jefferson's method rounds only down, which hurts the small states for the converse of all the reasons listed above.

From among all methods that use divisors, the Webster method (which became the Webster-Willcox method after the Cornell professor Walter F. Willcox got involved) is the only one that is practically unbiased. This fact is borne out both from an empirical-historical as well as from a theoretical perspective. Balinski and Young could not hide their surprise that nobody had noticed this until then, and that Hill's method (which became the Huntington-Hill method after the Harvard professor Edward V. Huntington got involved) had been officially sanctioned by all relevant institutions. "It seems amazing therefore that Hill's method could have been chosen in 1941 . . . and that Webster's method was discarded. A peculiar combination of professional rivalry, scientific error, and political accident seems to have decided the issue." Webster had the correct insight but Willcox lacked the mathematical wherewithal to prove the point. Recall that Willcox was a statistician, a member of a social sciences department, and as such not considered a serious interlocutor by Huntington who was a mathematician and behaved as such.

Do you remember that the National Academy of Sciences had supported Huntington-Hill's method because it lay in the middle with respect to favoring small and large states? Not a very sound argument, Balinski and Young thought when summing up their thinking on the matter: "In the end, Huntington's claim bolstered by the muddle over the 'middle,' provided the scientific excuse, and straight party-line interests provided the votes." One might add that the Supreme Court also had the wool pulled over its eyes. Even though Balinski and Young's book is extensively cited in the judges' opinion in the Montana versus the Department of Commerce case, there is no mention of its conclusion.

We demanded that a suitable apportionment method be unbiased and immune to paradoxes. Webster's method fulfills both requirements and we could lean back and relax. But there is another item on Balinski and Young's wish list. A state should receive no more and no less than its fair share of seats. This sounds pretty basic, but what do they mean by "fair share"? Let us start with the "raw seats" of a state, that is, the pro-rated number of seats, fractions and all, that the state would have received, were it not for the fact that there can only be an integer number of representatives. The fair share, sometimes called the quota, is defined as the number of raw seats, rounded up or down by no more than a half.

(By the way, why can the states not have fractional numbers of representatives? The first NAS report mentions this possibility (see chapter 10) and there seems to be nothing in the Constitution that prohibits this, and one could envisage a scheme whereby a state whose fair share is 15.368 representatives would send 16 congressmen to Washington. For the purposes of voting on a bill, the first fifteen would have one vote each, while the sixteenth would count for 0.368 votes. All the states' representatives would add up to exactly 435, and all problems of apportionment would disappear. The fractional congressperson's speaking time could be pro-rated and so could his/her office budget. On average, the House would have to accommodate no more than an extra twenty-five congresspersons.)

At first blush, the fair share requirement may sound superfluous; of course, one does not round by more than one-half. But in apportionment the unthinkable does sometimes happen. Remember that if the seats of all states, after appropriate rounding, do not add up to the desired total, the divisor method instructs us to use a higher or a lower divisor and repeat the allocation process. By the time all the seats do add up to the appropriate total, it could very well be that some states had their delegation rounded not to the next integer, but to the one beyond. These states will have received more or less than their fair share; they would be "out of quota." Have a look at the following table to see how this can happen.

TABLE 12.3
Fair share

36 seats are to be apportioned to four states.

State	*Population*	*"Raw" seats**	*Rounded seats*	*Divisor 46,000*	*Rounded seats*
AA	70,000	1.58	2	1.52	2
BB	112,000	2.52	3	2.57	3
CC	208,000	4.68	5	4.61	5
DD	1,200,000	27.23	27	26.30	26
Tot.	1,600,000	36	37		36

*(State's population/Total population)*36 = State's population/44,444

After pro-rating the seats (i.e., by dividing the populations with the divisor 44,444), the rounded seats add to 37 instead of 36. Hence a higher divisor is used (46,000) and—after rounding—the seats add to the desired number of 36. However, State DD receives 26 seats, which is "out of quota." (The correct quota, or "fair share," would be either 27 or 28 seats.)

Which apportionment methods fulfill the fair share requirement? One that surely does is Alexander Hamilton's method of largest remainders. After rounding the raw seats down, the method assigns the remaining seats to the states with the largest fractional remainders. By definition, all states stay within their quotas, and this is the redeeming feature of Hamilton's method that I mentioned at the end of chapter 9. We know, however, that the method suffers from the Population Paradox. (It also suffers from the Alabama Paradox and the New State Paradox but we no longer care about these.) Since the Population Paradox must be avoided by all means, only divisor methods may be considered. So which of them, Young and Balinski ask—Adams, Dean, Hill, Webster, Jefferson—guarantees that the states' delegations to Congress lie within their quotas?

Their answer is short . . . and depressing: not one of them! In fact, no divisor method exists that respects the fair share requirement. They state and prove the sad fact as a theorem: if there are four or more states, and the House has at least three more seats than there are states, then "there is no method that avoids the Population Paradox and always stays within the quota."

How about that? Thirty years after Arrow's Impossibility Theorem we are again left in a lurch. We posited just three humble prerequisites for a good allocation method: it should be unbiased, avoid paradoxes, and stay within quota. Is that too much to ask for? Yes, it is. Even if we accept a minute bias as unavoidable, Balinski and Young showed that any conceivable allocation method violates one or the other of the remaining, utterly reasonable requirements. Either it is susceptible to the Population Paradox or violations of the quota requirement cannot be excluded. (Incidentally, the Balinski-Young bibliography is an interesting example of how scientists with open minds may, in the midst of their careers, revise their points of view in a fundamental manner. At first, Balinski and Young vehemently fought for the quota method. Then they came to appreciate the divisor methods. Shortly thereafter, they completely withdrew their support for the quota method and from then on rooted for Webster.)

In classic understatement Balinski and Young speak of a "disturbing discovery." It is disturbing all right. But when one digs for the reason, it does not really surprise. The quota requirement is, after all, a very stringent condition and is easily violated. Let us see why. Rounding the raw seats of a small state entails a much greater adjustment than when the same is done for a large state. The quota of a state with 1.5 raw seats spans 66 percent (33 percent when rounded up from 1.5 to 2, and another 33 percent when rounded down). A state whose raw allocation is 41.5 has a quota that spans less than 2.5 percent. Since the requirement to stay within the quota is much more stringent for a large state than for a small one, it is not compatible with the idea that the number of seats be exactly proportional to the populations. Recall that the divisor methods entail changing the divisor whenever this is necessary to bring the allocated seats in line with the available number of seats. This has precisely the effect of moving some states' delegations beyond the rounded numbers.

So we can't have it all ways, something has to give. As the following table shows, either the Population Paradox or the quota requirement must

be abandoned. Balinski and Young opt for the latter. "Achieving apportionments that accurately reflect relative changes in populations seems more important than always staying within the quota," they declare. Actually, one does not give up very much by allowing quota violations since they do not occur very frequently in practice. Comparing theoretical estimates for the five common methods, the authors concluded that the Adams and Jefferson methods violate the quota requirement nearly always, whereas the Dean method does so only in 1.5 percent of the times, and Hill's H-H method in a bit less than 0.3 percent. But the hands-down winner is, once again, Webster's method: quota violations crop up in only 0.06 percent of the times. With reapportionments occurring every ten years, Webster's W-W method will produce quota violations only once every 16,000 years on average. (Actually, the H-H method would also not be doing too badly, with a quota violation occurring on average only once every 3,500 years.)

TABLE 12.4

Method	*Hamilton*	*Adams*	*Dean*	*Hill*	*Webster*	*Jefferson*
Quota Violation	No	Yes	Yes	Yes	Yes	Yes
Paradox						
Alabama	Yes	No	No	No	No	No
Population	Yes	No	No	No	No	No
New State	Yes	No	No	No	No	No

If an apportionment method fulfills the quota requirements, it produces paradoxes; if it is immune to paradoxes it violates the quota requirement. The sad conclusion of all this is that not all items on Balinski and Young's wish list can be satisfied simultaneously. In spite of this gloomy state of affairs, the news is not all bad. There is one method that does come close to the ideal: the W-W method. "The simplest and most intuitively appealing method of all is the best one . . . it avoids the paradoxes, it is unbiased, and in practice it stays within the quota."

So why is it not in use? Balinski and Young's book was published in 1982, but in spite of all criticism, Huntington-Hill has remained the method of choice. As pointed out at the beginning of the chapter, the state of Montana challenged it after the census of 1990 but had to resort to the largely disregarded method of James Dean in its lawsuit since both H-H and W-W took away one seat. And after the 2000 census nobody had a bone to pick.

The persistence of a method that is known to be deficient is a puzzling phenomenon. We wait breathlessly to see what will happen in 2011, in 2021 . . .

BIOGRAPHICAL APPENDIX

Michel L. Balinski

Balinski was born in Switzerland into a Polish family active in international affairs. His grandfather Ludwik Rajchman, a medical doctor by profession, was a prominent socialist intellectual who devoted his life and career to the service of humanity. After World War II he would be the founder of UNICEF and the spiritual father of the World Health Organization. From Switzerland the family moved to France, but as a Jew and a prominent opponent of the Nazis, Rajchman had to flee to the United States, taking young Michel with him. There the family took on U.S. citizenship and Michel received a thoroughly American education: BA in mathematics from Williams College in 1954, MA in economics from MIT two years later, and doctorate, in mathematics again, from Princeton University in 1959.

His studies were followed by a rich career as a consultant and professor of mathematics, economics, statistics, management, decision sciences, and operations research at various universities in the United States—for a while he also served on the advisory board to the mayor of the city of New York—before he returned to France in the 1980s. He became director of the Laboratoire d'Econométrie of the École Polytechnique in Paris. Balinski was the founding editor of the journal *Mathematical Programming*, is a noted authority on mathematical optimization and operations research, and served as president of the Mathematical Programming Society from 1986 to 1989.

H. Peyton Young

Young completed his BA in general studies at Harvard University in 1966 and then went on to obtain a PhD in mathematics at the University of Michigan. After he finished his PhD he was quite fed up with the ivory tower and its lack of connection with the "real" world. Thus he chose, in 1971, to work for a study commission in Washington rather than go into academia. But after a year in DC he was pretty much fed up with the real world too (it was the Watergate era) and decided to take another look at academia. This is when he hooked up with Balinski.

Subsequently, Young taught economics, public policy, decision sciences, and business at Johns Hopkins University, the University of Maryland, and the University of Chicago. He also held positions in Europe, as visiting professor at the University of Siena in Italy, professorial fellow at Nuffield College in Oxford, and deputy chairman of system and decision sciences at the International Institute for Applied Systems Analysis in Austria. Young is a senior fellow at the Brookings Institution in Washington DC, was elected president of the Game Theory Society in 2006, and appointed professor at Oxford University a year later. Young is truly interdisciplinary. His many dozens of publications deal with various subjects in applied mathematics, economics, game theory, and political theory. In his latest research, he is concerned with the evolution of norms, conventions, and other forms of social institutions.