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THE ELUSIVE MEDIAN VOTER*

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While some political scientists have maintained that politics has little, if anything, to do with governmental output, economists have looked at expenditures as reflecting median voter outcomes. They have used the median voter framework extensively, both in theoretical work and in empirical analysis. This paper reviews the empirical work, concluding that the studies fail to indicate that actual expenditures correspond in general to those desired by the median voter.

The economic studies fail to identify whether expenditures are at the level desired by the median voter or at some multiple of this level. They also fail to identify whether the median voter is pivotal or a voter at some other fractile is pivotal. Moreover, the basic median voter model is rarely tested against competing theoretical or statistical models. In some studies parameter estimates are not consistent with the theoretical model designed on the basis of the median voter hypothesis.

The economic studies suggest that expenditures depend not only on the preferences of voters but also on the structure of political institutions. The presence of bureaucratic threats is offered as an institutional setting that can result in expenditures significantly in excess of those desired by the median voter.

1. Introduction

Undoubtedly the best known result of formal political theory is the median voter theorem attributed to Black (1958), Bowen (1943), and Hotelling (1929).[1] When the theorem applies, two candidates competing for office should converge to identical policy positions. In the case of a public expenditure, this position would be the median of the various expenditure levels most preferred by the individual voters.

While some writers have suggested that the median voter result could serve as a basic paradigm for political resource allocation, assuming the role that perfect competition has played for markets, political scientists have rarely, if ever, engaged in satisfactory empirical testing of the median voter model or its extensions. Indeed, in the political science literature on governmental expenditures, there are two prevailing positions that claim that

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[1]Numerous extensions of the median voter model have been carried out since the mid-1960s. For bibliography and discussion, see Mueller (1976).

politics has little, if anything, to do with governmental output. The first position, known as the incrementalist school, is that expenditures follow simple autoregressive rules like 'last year's budget plus five percent'.[2] The second position is that expenditures are largely determined by socioeconomic rather than political variables.[3]

Some economists would reply to the socioeconomic advocates that a median voter process does determine expenditures but that, in the crudest version of the economic theory, the median voter has median income. Thus, one would not be surprised to find that expenditures correlate with median income and other socioeconomic variables. The great advantage of the median voter paradigm is that it allows one to analyze *social* problems via the preferences of a single *individual*, the pivotal median voter. In theoretical work, many economists appear to accept the proposition that, quite generally, the median voter will play a pivotal role in the political process. Assumptions leading to median voter dominance have been employed to incorporate political processes in a wide range of economic contexts. These include pollution control;[4] income redistribution;[5] minimum wage legislation and union behavior;[6] and, especially, local governmental provision of goods and services.[7] In empirical work, economists have recently, beginning with a 1966 paper by Barr and Davis, adopted the median voter framework and applied it to the analysis of local government expenditures in Switzerland and the United States.[8]

The basic purpose of this paper is to evaluate these studies' empirical contribution toward establishing the median voter model as a paradigm for incorporating political factors into economic analysis.[9] Our conclusion is that the studies fail to indicate that actual expenditures correspond in general to those desired by the median voter.

Although the studies do show that operational measures of median income and median tax price can statistically account for expenditures, the studies in fact cannot identify whether observed spending is at the level actually desired by the median voter or is at, say, twice or one-third that level. We term this problem the 'multiple fallacy'.

[2]The seminal incrementalist model is found in Davis, Dempster and Wildavsky (1966).

[3]The case for socioeconomic primacy has recently been advanced by Lewis-Beck (1977). He references the previous literature.

[4]See Klevorick and Kramer (1973).

[5]See Atkinson (1973) and Romer (1975).

[6]See West (1974) and Farber (1978).

[7]See Barlow (1970), Bradford and Oates (1971), Buchanan and Flowers (1969), Denzau and Mackay (1976), Edelson (1974), Epple, Zelenitz and Visscher (1978), Eysenbach (1974), Lovell (1975), Sheshinski (1977), and Stiglitz (1977).

[8]We reference later studies as we discuss them below.

[9]Deacon (1977) usefully addresses a set of methodological issues, especially as they relate to measurement of variables and jurisdictional mobility, largely distinct from the issues raised in the present paper.

Moreover, the studies fail to show that empirical results based on the median voter model are superior to any reasonable alternative models. Most of the studies fail to test the median voter model not only against alternative formal models of political institutions but also against some simple alternative statistical models. For example, does the median of the income distribution predict better than the mean income, the income of other fractiles, etc.? The theory used to develop some empirical specifications in fact implies that, if one substitutes any fractile for the median, one should get the same empirical results. This problem is referred to as the 'fractile fallacy'.

After a brief review of the median voter model and related results in positive theory, the major portion of this paper addresses these and other methodological issues. We then contend that the economic studies, rather than validating the median voter model, suggest instead that political resource allocation can be understood only if there is an adequate modeling of the structure of political institutions. For example, expenditures determined by referendum may differ from those determined by representative legislative bodies. The actual institutional structure may result in expenditures far in excess of median desired expenditures, as perhaps evidenced by the overwhelming passage of Proposition 13 in California in June 1978. One possible source of such higher expenditure outcomes is the presence of bureaucratic threats that are discussed in the final section of the paper.

2. The median voter model

Consider a population of voters confronted with a choice between two alternatives. Each voter is always assumed to vote for his preferred alternative.[10] (There is no abstention.) Assume that alternatives can be represented in terms of a single dimension such as left–right or the amount of an expenditure. Also, assume that all voters have single peaked preferences in this dimension. (For example, each voter has a most preferred level of expenditures. As expenditures increase or decrease from this 'ideal' level, the voter's level of preference decreases.) A voter's ideal point is defined as his most preferred alternative among all the feasible alternatives on the dimension. The ideal points can obviously be ordered on the dimension. Define a median ideal point to be any alternative such that fewer than one-half the voters have ideal points below it in the ordering and no more than one-half the voters have ideal points above it. It follows that in a simple majority vote between a median ideal point and any other alternative, a median ideal point will never lose (although it may tie).

A referendum process leading to a median outcome can be devised in a straightforward manner. Each voter writes an expenditure level on the ballot.

[10]A number of arbitrary and unimportant assumptions can be made to handle the technicalities arising when voters are absolutely indifferent between two alternatives.

The process then selects the largest quantity such that this quantity and all larger quantities receive a majority of the votes. Such a system was used for school expenditures in Florida from 1939 to 1968 [Holcombe (1977, pp. 80–82)].

There are two additional institutional contexts where, in light of the theorem, we might expect the median ideal point to be the policy outcome. First, if the alternatives are proposed by two candidates competing for election, both candidates can be expected to propose a median ideal point in order to avoid losing the election. Secondly, if a committee votes by majority rule on a series of motions and any motion must be entertained, once a median ideal point has been passed no amending motion can defeat it.

As to the election institution, note that the result can break down if there are more than two candidates, if candidates are constrained in the alternatives they can offer, or if candidates derive utility from the policy outcome rather than just winning. With committees, the result would be vitiated by any form of agenda control that prevents a median from coming to the floor.

When preferences are not single-peaked and unidimensional, important recent work by McKelvey (1976; forthcoming) and by Cohen and Matthews (1977) has shown that, unless very restrictive and unrealistic conditions are satisfied, binary choice processes, including majority rule, will not lead to stable outcomes. Moreover, an individual who controls the agenda can generally manipulate matters so that he can obtain his preferred outcome regardless of the initial (*status quo*) point. Deviations from median voter outcomes are also discussed by Comanor (1976) and Hinich (1977).

3. The median voter approach to local expenditures

Despite the preceding theoretical caveats to the median voter model, expenditures of local governments (particularly in federal systems) might reasonably be expected to provide a fertile ground for empirical testing. First, expenditures are directly quantifiable so that 'objective' measurements of alternatives and policy outcomes are possible. Secondly, local populations tend to be more homogeneous than the population of the country as a whole. Thirdly, the largest local expenditure is for education. Since most American school boards are independent taxation and expenditure authorities, they deal with expenditure in only one area. This leads to an important element of unidimensionality. Whereas a national legislature must in some sense trade off defense, space, and antipoverty expenditures simultaneously, and city councils must juggle parks, potholes, and police, school boards and voters in school referenda must restrict their attention to a single area without any direct opportunity for logrolling or strategic voting in relation to other issues. True, school monies can be used either to raise salaries, hire more teachers, or resurface the football field. The presence of

private school substitutes may destroy single-peakedness.[11] Still, if researchers are ever to find an empirical context that is approximately unidimensional and single-peaked, school expenditures appear to be an appropriate choice.[12]

Assuming that local government expenditures reflect the desires of the 'median voter', economists have widely adopted the median voter hypothesis in empirical research. Several economists also assume that the median voter has median income in the community. Among them, Inman (1978) takes a forceful position. He claims that the median voter-median income assumptions produce 'an analytically powerful new "as if" proposition... which stands as political economy's counterpart to the market economy's supposition that firms are profit maximizers'. The median voter proposition, Inman further asserts, allows economists to 'bury politics' and analyze government expenditures by applying the individual utility maximizing model to the median income family.

Can politics be enshrined in the tomb of the median voter? Let us look at the empirical evidence on local expenditures contained in several papers inspired by the median voter hypothesis. (Appendix A provides a summary equation for each model.) These include the early study of Barr and Davis (1966). Bergstrom and Goodman (1973) estimated a constant elasticity demand function for municipal expenditures in ten American states. Their work has been replicated, with some extensions, for Switzerland, by Pommerehne and Frey (1976); Pommerehne (1978); and by Pommerehne and Schneider (1978). Inman's (1978) paper on school expenditures in Long Island is also based on the Bergstrom–Goodman study. He has refined the empirical application of their approach and attempted to correct for turnout factors. In contrast to Bergstrom and Goodman's direct specification of demand, Lovell's (1978) analysis of school expenditures in Connecticut, starts from a more basic level, a specification of the utility function for a voter. We also consider papers by Barkume (1976), Rubinfeld (1977), and Neufeld (1977). Although not directly inspired by the median voter model, they examine utility maximization in voting on school expenditures. Their work on voter behavior is directly relevant to the development of approaches to political resource allocation that are distinct from the median voter model.

All the studies cited above motivate voter behavior on the basis of the neoclassical microeconomic theory of utility maximization under a budget

[11]Stiglitz (1974) presents a detailed analysis of individual choice between public and private education.

[12]Robert C. Wood, an acute student and practitioner of American politics, has written, 'No real debate can take place about the comparative needs of schools and other functions for no one can seriously argue that the building of a new fire station should be made possible by cutting the school budget' [Wood (1958, p. 191)]. Furthermore, '[school officials] concentrate solely on the quantitative aspects... They make a "bricks and mortar" defense: more buildings, more teachers, and more money' [Wood (1958, p. 192)].

constraint. In effect, voters are constrained to allocate their disposable income between private consumption and some public expenditure. Private consumption can be arbitrarily measured so that its price is unity. The costs of the politically (publicly) produced good are shared by the voters. In the case of a property tax, a voter's tax share is simply the ratio of the assessed value of the voter's property to the value of all property. The voter's tax price, then, is found by multiplying his tax share by the unit cost of the politically produced good. A voter's preferences for various expenditure levels thus depend on (a) his tastes or utility function, (b) his income, (c) his tax price, and (d) a goods share function that relates total expenditure to the publicly financed goods received by the voter.[13]

3.1. Barr and Davis: The surrogate measure approach

In the earliest study, that of Barr and Davis (1966), voters in the various Pennsylvania counties are assumed to have identical tastes and, at first, identical incomes. However unrealistic, some assumption about uniformity of taste appears necessary to make headway with empirical work. The identical income assumption has been dropped from later studies. Given the Barr–Davis assumption, the median voter in Allegheny County will differ from his counterpart in Bucks County only in the effective tax price.

As surrogates for the tax price, Barr and Davis used assessed property value per capita (high values claimed to lead to a low tax price) and owner occupied residences per registered voter (high values lead to a high tax price). Empirically, the surrogates behaved as expected, suggesting that high tax prices lead to low expenditures.

Even forgetting the point, recognized by Barr and Davis, that their variables are questionable measures of tax price, we learn little about the median voter's power from these results. First, even if the surrogates measure the median voter's tax price, we know only that expenditures rise as this price falls. Expenditures everywhere could be 50% less or 100% greater than those desired by the median voter. Hence, the multiple fallacy. Similarly, the arguments Barr and Davis used to suggest that the median voter's tax price varies with the surrogates would also suggest that the price of voters in the 25th, 40th, or some other percentile also varies in the same manner. Hence, the fractile fallacy.

What, moreover, should be surprising to later advocates of the median voter hypothesis is the fact that, even allowing for the crude linear specification of Barr and Davis, median income failed to explain variance additional to that explained by the tax price surrogates. A similar result

[13]The technical development of the voter's decision calculus can be found in several sources, including Barr and Davis (1966) and Romer and Rosenthal (1978).

occurs in Denzau's (1975) regressions for Virginia school districts. All the other American studies show median income to be highly important, although it is not in Pommerehne's (1978) work on *representative* (nondirect) democracies in Swiss municipalities. While there is no such implication inherent in formal economic theory, economists would generally expect the demand for total government expenditure to increase with income *ceteris paribus*. (The absence of explanatory effect for income may be due to collinearity between income and wealth variables in some of these studies.)

There is little, then, in the Barr–Davis work that lends credence to the median voter theory. Their empirical results actually add little to previous studies by political scientists. For example, Wilson and Banfield (1964) demonstrated a negative relationship between percent of homes owner occupied and the percent voting yes on bond issues in Cleveland and Chicago. The value of the Barr–Davis contribution is not in the empirical results, but in the stimulus provided by their attempt to link empirical work with a formal model of the political process.

3.2. Log-linear demand models

Following the Barr–Davis work, Bergstrom and Goodman (1973) sought to develop a more rigorously specified expenditure equation by estimating the standard log-linear demand function.[14] In this function (shown in Appendix A), demand depends, with constant elasticities, on income and tax price. The elasticities are assumed equal for all citizens. Now income can affect demand not only directly but also via the effect of income on tax price. If this relationship is assumed monotonic, the median voter has median income. Finally, this median voter's tax price is taken to be proportional to the ratio of the tax bill for the home with median value to total revenues for the municipality.

One unit of the politically produced good can be simultaneously consumed by all citizens only if it is a pure public good. Otherwise, more than one unit must be produced for an individual citizen to consume one unit. Bergstrom and Goodman attempted to evaluate the 'publicness' of collective goods by assuming that the unit price to the citizen was of the form qN^{γ}, where q is a constant, N is the municipality's population, and the 'crowding parameter' γ is a constant.[15] If γ is zero the expenditures represent pure public goods. At the other extreme, when $\gamma=1$, N units must be produced if the average citizen is to obtain one unit of the good, the good being purely private.

[14]The work of Borcherding and Deacon (1972) and of Perkins (1977) also extends Barr–Davis. Borcherding and Deacon, while assuming the median voter framework, do not attempt to identify the median voter, and use average income and tax share variables. Perkins builds on the Borcherding–Deacon analysis by including cross-price elasticities and allowing for state aid formulas in the expenditure regressions.

[15]$0 \leqq \gamma \leqq 1$. For the table in Appendix A, $\gamma = c_3/(1+c_2)$.

3.2.1. The multiple fallacy

Bergstrom and Goodman estimated their model separately for each of ten states. For total expenditures and for police expenditures estimated separately, all income elasticities were positive and all tax elasticities negative. This pattern was not quite as pervasive for parks and recreation expenditures, which showed one state with a negative income elasticity and three states with positive tax price elasticities. Pommerehne (1978) also obtained positive income elasticities and negative tax share elasticities. In his study of four types of Swiss communities and seven definitions of expenditures (administration, education, health, welfare, roads, environmental protection, and total), the only exceptions concerned three (out of 28) negative income elasticities. Pommerehne's work with Frey (1976) and with Schneider (1978) provides similar results.

Bergstrom and Goodman's results have been paralleled not only by Pommerehne and his colleagues but also, with somewhat different models, by Inman (1978), Lovell (1978), Rubinfeld (1977), and others. Expenditures or voting for expenditures will respond positively to income and negatively to price. These results are consistent with standard demand theory for normal goods. Do they also offer support for the median voter model? Note first that the multiple fallacy cannot be escaped. If expenditures everywhere were not those desired by the median voter but some multiple of this quantity, then the elasticity estimates would be unchanged but the multiple would be confounded in the intercept c_0 (see Appendix A). Since there are no prior constraints on c_0 that allow us to determine if the multiple is unity, we cannot know whether expenditures correspond to those desired by a voter with median income.

Nonetheless, this voter may be pivotal. While he may get some multiple of his druthers, his druthers are still critical to fixing the expenditure level. Unfortunately, all we know from the Bergstrom–Goodman and Inman studies is that the median statistics fit the data significantly better than the unconditional mean of the observed values. (We address Lovell and Rubinfeld later on.) More convincing evidence would be that the median fits the data substantially better than the mean, the 25th percentile, and other reasonable summary statistics of the income distribution. Similarly, is a median definition of tax price critical? Does it outperform the simple ratio of residential assessed value per household to all property value? Since renters tend to be poorer than homeowners in most communities, citizens with median incomes are likely to live in homes with less than median housing value for owner occupied homes. If the median income voter is indeed pivotal, then Bergstrom–Goodman (for obvious data reasons) probably used an incorrect housing value. A better measure (available from crosstabs on 1970 census tapes) would be a conditional average or median housing value for households close to the median of the income distribution. Would

changing the specification make a difference? These questions must be answered if one is to believe that a particular type of voter plays a pivotal role.

3.2.2. Mean vs. median?

Pommerehne and Frey (1976) attempted to test the median voter model of local expenditures against what they call a 'traditional model' in which mean income and tax share are used instead of medians. Although they interpret their results as supporting the median model, it is difficult to accept this claim on the basis of the evidence (from 74 Swiss communities) that they provide. Coefficients on income, tax share, and population are generally insignificant (at the 95% level) in the 'traditional model', while being significant in the 'median model'. Nonetheless, in three categories of spending (including total expenditure) the two models have virtually identical R^2; in three categories the 'median model' has higher R^2; and in two categories the 'traditional model' explains more of the variance. We suspect that there is substantial collinearity in the mean tax-share and population variables.[16] This may well lead to estimated coefficients with low t-values, even though the model with means fits the data about as well as the model with medians. While we agree with Pommerehne and Frey's well-placed attack on *ad hoc* (theory-free) regression models of public expenditures, their results cannot be interpreted as support for the median voter model.

Pommerehne (1978) did find substantially larger residual variances when he used mean rather than median income with 48 municipalities in his sample that employed direct democracy. In contrast, for 62 representative democracies, mean income actually led to slightly lower residual variances. Since Pommerehne's median statistic is taken from the income distribution of Swiss citizens while the mean is from the overall distribution, his results probably indicate only that the preferences of foreign residents (about 20% of the population) get little consideration in direct democracies. It is still open to question whether the median for Swiss citizens outperforms the mean, etc. for Swiss citizens. His results do, however, suggest a value to carrying out the comparisons we advocate.

3.2.3. Crowding

It is worthwhile to interrupt our discussion to mention the crowding parameter even though it does not directly enter the median voter story.

[16]The average of tax shares is, of course, just the inverse of population. We therefore interpret Pommerehne and Frey's 'average tax share' as the tax share of the voter with average income. Even in this case, if taxes are proportional in income and wealth (as they are in Pommerehne and Frey's sample), and the voter with mean income has mean wealth, we would still have the 'average tax share' equal to the inverse of population.

Although even for private goods this parameter should not exceed one, Bergstrom and Goodman (1973, pp. 288–289) obtained 20 out of 30 estimates greater than one. Similarly, computations made with Pommerehne's (1978, pp. 268–269) results show 18 of 28 greater than one. We believe these results can be explained by realizing that the estimated coefficient on population confounds economies of consumption (the crowding parameter) with scale economies or diseconomies in the production of publicly provided goods. If there are sharp diseconomies in production as city size increases, then the estimated 'crowding' parameter can exceed one, even for a pure public good.

3.2.4. *Attempts to model diversity of tastes*

Bergstrom and Goodman (1973, p. 286) realized that their specification depended heavily on the homogeneity of tastes assumed to exist across communities. It might be reasonable to assume that each community was made up of population subgroups with different tastes but that the tastes of these subgroups were constant across communities. But what if the communities differed sharply in their subgroup composition? Estimates based solely on median income and tax share might then be inappropriate.

To respond to this concern, Bergstrom and Goodman proved a theorem that gives sufficient conditions for the continued use of median statistics in expenditure regressions. Use a partition into subpopulations to classify the voters of each local governmental area. Across governments, each subpopulation's income distribution is assumed identical up to a *proportional* constant that is government-specific. For example, if the partition is Catholics and non-Catholics, the Catholic and non-Catholic income distributions should have the same shape in Alphaville and Betatown, but every Catholic fractile and non-Catholic fractile in Alphaville could represent an income k times the corresponding income in Betatown. Moreover, the subpopulations are assumed to vary in demand only via the constant c_0 in Appendix A and via the tax share which must take the form: $\tau = \tau_i \tau_j Y^{\varepsilon}$. That is to say, the tax share has constant elasticity with respect to income, Y, and its logarithm has additive components τ_i for a town factor, τ_j, a subpopulation factor, and an income factor. The subpopulations are still restricted to having identical tax and income elasticities. With the foregoing assumptions and other technical assumptions, the theorem is developed. Consider a voter from any subpopulation who has median income for the *entire* community. The median quantity demanded in the community will be the quantity demanded by this voter multiplied by a shift factor. The shift factor is solely a function of the subpopulation proportions. (In our example, the percentage Catholic and percentage non-Catholic.)

Bergstrom and Goodman make only limited use of the theorem in their empirical work. The tax share is computed on the basis of median housing values for the entire community rather than for a specific reference subpopulation. Rather than explicitly modeling a shift factor, Bergstrom and Goodman simply introduce demographic variables as linear regressors. Detailed empirical use of the theorem awaited Inman's work.

In studying school expenditures for 58 Long Island communities, Inman (1978) chose voting, homeowning non-Catholics under 65 as a reference subpopulation. Although his tax share (certainly for data reasons) continues to be based on overall median housing values rather than a tax share model for his reference group, Inman explicitly estimates a shift function. Inman claims that if the estimated shift function is approximately unity then expenditures behave 'as if' the median income voter were decisive. He also claims that the shift function estimated with his data is approximately unity.

In refining the Bergstrom–Goodman approach, Inman (1) unfortunately falls into the fractile fallacy trap. (The multiple fallacy obviously continues to be present.) We also contend that he (2) fails to present a convincing argument that his shift function is indeed unity. Moreover, (3) the Bergstrom–Goodman approach makes demanding empirical requirements. Investigators would need to know the correct shift function for the correct group. Not all shift factors will be unity. If anything, Inman has shown that a voting, homeowning, young non-Catholic with median income is 'as if' decisive and not that any median income voter is decisive. We elaborate on the first two points below.

3.2.5. *The fractile fallacy*

Regarding the fractile fallacy, the important consideration is that, to justify use of the Bergstrom–Goodman theorem, Inman assumes that all income distributions are lognormal and that each subgroup distribution is proportional not only across cities (as required by Bergstrom–Goodman) but also to the overall distribution for the city. Under these assumptions, it is shown in Appendix B that the regression of any fractile of the income distribution will give the same estimates as regression of the median, except for the intercept. Empirically, then, Inman's method is caught in a crossfire. If a median regression proves superior to the use of other fractiles, the assumptions used to justify the model are contradicted. On the other hand, if all fractiles give similar results, then any voter appears 'as if' decisive.[17]

3.2.6. *The shift function*

Our second point relates to testing whether the shift function is unity.

[17]Another assumption made by the Bergstrom–Goodman theorem is the constant elasticity relation between income and housing value. Inman measured tax share as $\pi b/B$, where b is median housing value, B is total assessed value, and π is the net burden of a dollar of local tax as corrected for matching aid to the schools and personal tax deductions. If the constant elasticity assumption is correct, the regression 'fit' to the data when π/B is used as a regressor in place of tax share should be roughly equal to the 'fit' when tax share is used. This test could be used in future research.

Inman takes the parameter estimates of a shift function regression and the observed subpopulation proportions for the community to compute an estimated shift for each of the 58 communities in his sample. He uses these estimates to conduct 58 separate tests of the null hypothesis that the shift equals one. Since he rarely rejects the null, he concludes that the 'as if' median is supported. However, another hypothesis test may be proposed. If the shift is indeed one, it follows that the regression estimates will not be improved by including the independent variables representing the shift. In the case of Inman's OLS runs, the appropriate test of no improvement would be the ordinary F-test comparing the sums of squared errors from a regression with the shift variables and one without them. Performing this test led us to reject the null at the 5% level and conclude that the shift is not one.[18]

In any event, a better specified shift may be even more significantly different from unity. For example, Rubinfeld's (1977) work suggests that family size heavily affects demand. A shift based on census data for family size could well lead to different results. More generally, given that the proportions entering the shift must, by the definition of a partition, satisfy the identity that they all sum to one, we conjecture that the only shift that can both satisfy the identity and be one is the identity itself. If this is the case, the median income median voter will always elude Inman's approach.

Our critique of the Bergstrom–Goodman line of studies should not mask the fact that they represent an important advance over the early Barr–Davis regressions. The later studies represent an explicit modelling of demand. Inman, in particular, has refined the calculation of tax price by correcting for the effect of matching aid to the school districts and for federal tax deductions for local taxes paid. He has also estimated the income effect on demand of lump sum state aid. (He finds that state aid increases expenditure by an amount only slightly smaller than the increase in state aid. This 'flypaper effect' is in itself inconsistent with the median voter model.) Finally, since voters represent a characteristic of the subpopulation partition, Inman has introduced a turnout factor into the regression via the shift function.

3.3 *Lovell's model of the median voter utility function*

Lovell's (1978) work on school expenditures in Connecticut is based on an explicit specification of the household utility function. This function is taken to be

$$U=(C^{-\beta}+\alpha n^{-\delta\theta}S^{-\theta})^{-1/\beta}, \tag{1}$$

[18]Using the values reported in Inman's table 2 for the 4OLS model, we compute $F(3, 51)=3.62$, which is significant at the 0.05 level. Allowing for round-off error in reporting in table 2 and making the assumptions that lead to the lowest possible F-value, we have $F=3.07$, which is also significant at 0.05.

where C is household consumption of private goods; S public educational expenditures *per pupil*; n the number of school age children in the household; and α, β, δ, θ parameters common to all voters.[19]

Lovell's approach presents several advantages in comparison to the Inman study. First, Lovell avoids assumptions that lead directly to a fractile fallacy. Secondly, he attempts to model the effect of family size on demand. Thirdly, in using as a regressor the ratio of median to mean income (SKEW), rather than just the median, Lovell potentially builds a case for the decisiveness of the median voter. Were the mean voter decisive, Lovell's model would require using the ratio of the mean to the mean or one. Thus, there would be no income variable in the model. But Lovell's results show that SKEW is a significant regressor.

Lovell claims that his paper 'provides strong empirical support for Hotelling's proposition that the median voter dominates the ballot box' [Lovell (1978, p. 493)]. We do not believe that this claim is justified, for the following reasons. (1) At a given level of per student expenditure the cost to a household of sending an additional child to public school is zero. In Lovell's model, this standard 'free rider' situation becomes a 'free bearer' problem, as households can indefinitely increase their utility simply by having more children. The model is thus misspecified since it ignores the private cost of children. (2) Even given Lovell's assumptions, desired expenditures may not be monotonic in income; consequently, the pivotal voter may not have median income. (3) Even with monotonicity of desired expenditures, the coefficient on SKEW may be confounding tax price and family size effects. (4) The treatment of private school alternatives is inadequate. (5) The empirical results yield an implausible value for a key parameter of the household utility function. We discuss each of these points in some technical detail below.

3.3.1. *Free bearers*

If we neglect private schooling (a point to which we return below), a household faces the budget constraint

$$Y=C+p(Y)\bar{n}S, \tag{2}$$

where Y is household income; $\bar{n}$ the community's average number of school children per household; and $p(Y)\bar{n}$ the tax price paid by a household with income Y, for a dollar of per pupil expenditure. For a given tax structure the cost to a household of having an additional child is zero.

Since C is total *household*, not per household member, consumption, then

[19]In Appendix A, $c_0=(\theta+1)^{-1}\ln(\alpha\theta/\beta)$, $c_1=(\beta+1)/(\theta+1)$, $c_2=-(\theta+1)^{-1}$, $c_3=-(\delta\theta+1)/(\theta+1)$.

as long as the marginal utility of a child is positive, an additional child increases household utility without imposing additional resource costs on the family.[20] Lovell's formulation thus leads to a situation where households are unlimited 'free bearers' – it is in the private interest of each household to have indefinitely many children. To correct for this, it would be necessary to incorporate a private cost to having children, either in the utility function or in the budget constraint.[21]

3.3.2. *Is the ideal point monotonic in income?*

Even if we abstract from the free bearer problem and treat family size as an exogenous variable, as Lovell does, the model still confronts the hunter of the median voter with serious difficulties. For a given value of n and a given tax structure, a household's 'ideal' expenditure per pupil, S^*, will depend on household income *and* the number of school children in the household. Even if this ideal expenditure is monotonic in income (for fixed n) and in family size (for fixed Y), the total dependence of S^* on income need not be monotonic. To take a simple example, suppose that for households of a given size, S^* increases with income and, holding income fixed, S^* increases with the number of school age children in the household. Suppose also that, on average, household size declines as income increases. Then both high and low income households may be relatively low demanders of public education, with highest S^* belonging to middle income households. We could not then assert that the median S^* is demanded by the household with median income. Lovell ignores this problem and takes the pivotal voter as coming from a household with median income for the community. He presents no evidence that his (implicit) assumption of monotonicity is justified.

3.3.3. *The effect of family size*

No doubt for data reasons, Lovell assumes that the number of children attending public school from the pivotal (median income) household is equal to the *mean* number of public school children per household in the community. If the pivotal household does not have the mean number of public school children, then Lovell's regressions omit a term that relates the pivotal household's number of children to the mean. If this omitted term is

[20]Of course, each additional child in the school system marginally changes $\bar{n}$ and S. But the household can be presumed to ignore these effects, as it ignores the effects of its own purchasing decisions on price in competitive private markets.

[21]More children in the household will typically lead to greater expenditure on housing. If tax price depends on housing, then p will depend on n, and there will be a private cost to having children. A budget constraint more realistic than (2) would include a component for housing and allow p to be a function of housing expenditure. Such a budget constraint is used by Rubinfeld (1977).

correlated with SKEW, then the coefficient on SKEW may be confounding the tax price effect with the influence of family size. Thus, it may be incorrect to interpret a negative coefficient on SKEW as supporting the median voter hypothesis (details in Appendix C.1).

3.3.4. *Private school alternatives*

Our discussion has so far assumed that private schools are not a significant alternative to public education. In regressions based on his theoretical model, Lovell also does not allow for private schools. We have already pointed out that private alternatives to public provision may lead to multipeaked household preferences. Let us, however, ignore this unhappy possibility. Even so, if the pivotal household has children in public school, then its tax price is reduced when other households send their children to private school. The increased use of the private alternative acts as a subsidy for the first household, as long as that household remains pivotal. This effect should be taken into account in the calculation of tax price. A second, more complex effect of private schools is caused by users of private schools preferring a lower (or zero) level of expenditures than they prefer when there are only public schools. This can affect the identity of the pivotal household. The first effect (on tax price) will be to increase the pivotal household's desired per-student expenditure, while the second effect may work in the opposite direction. Since private schools are not negligible in Connecticut (in large towns, private enrollments ranged from 7.3 to 29.4% of total school enrollments, according to the 1970 census), the specification should seek to adjust for these effects.[22]

3.3.5. *Interpreting the estimates*

Finally, the estimates reported by Lovell in his tables 4 and 5 allow us to calculate the structural parameters of the underlying household utility function. For all six regressions in these two tables the value of δ implied by the estimates is *negative*. Together with estimates of the other parameters, this means that, for the utility function given by (1), the marginal utility of children is negative (see Appendix C.2) regardless of the number of children in the household. This result is clearly inconsistent with the model, since it implies that no household would have any children. (Of course, it also solves the free bearer problem!) This analysis casts doubt on the validity of the parameter estimates and further undermines our willingness to accept the claim that the results support the median voter model.

[22]The subsidy effect should prove far more amenable to empirical investigation than the median shift effect. We expect the subsidy effect to far outweigh any variation across communities in the number of children in the pivotal household.

In spite of these reservations, we feel that Lovell's efforts at utility function modeling indicate a promising line of attack. Our critique should not deter researchers from an appropriate revision of this model.

4. Institutions and voter choice

4.1. Models of voter choice

Why the median voter may not get his druthers is suggested by two studies in which the dependent variable is voter behavior rather than expenditures. Both concern voting in referenda on school millage rates. Using aggregate data for census tracts, Barkume (1976) studied Santa Barbara, California; using a panel survey, Rubinfeld (1977) examined Troy, Michigan. Both models view the voter as making a choice between the school board's proposal and some *status quo* or reversion level of taxes were the proposal to fail. Note that the voter does not compare the proposal to his ideal point but to the *status quo*. This means, as we have pointed out elsewhere,[23] that the board can threaten the voters with a low level of expenditure or indeed, in some cases, no schools at all (*vide* the recent closings in Ohio, Oregon, Illinois, etc.). The threat possibility does not preclude the median voter from being the pivotal voter. But rather than offering him his ideal point, the board will make the median voter indifferent between the proposal and the reversion. These potential threats explain the stress we have placed on the multiple fallacy. At least where referenda are used for school taxes/expenditures, decisiveness does not necessarily imply obtaining the ideal point. This is the case only if political institutions guarantee that the median ideal point is placed on the ballot.

Of these two studies, Rubinfeld's is particularly interesting. As with the aggregate data studies of expenditure, Rubinfeld found that income and price behaved as expected. Other significant variables accorded with a narrow self-interest view of voting. School teachers voted more heavily 'yes', *ceteris paribus*, than other citizens. And, in line with the 'free rider' aspect of public education, each additional child in the public school system increased the probability a voter would vote yes.

Rubinfeld uses the concept of an ideal point in developing his model. Unfortunately, for technical reasons inherent in the logit model used by Rubinfeld, ideal points cannot be estimated from the data. While Rubinfeld's research informs us as to the *relative* importance of various factors for voting decisions, the research does not (nor was it intended to) test the median voter hypothesis. Moreover, our wariness of using surveys to conduct such a test is fuelled by the potentially quite severe problems of response bias and sample self-selection bias involved in eliciting voter preferences directly.

[23] See Romer and Rosenthal (1978 and forthcoming).

Rather than confirming simple economic models of the median voter, Rubinfeld's findings cast doubt on all formal models of the political process that ignore how voters are informed and mobilized. In May 1973, Rubinfeld's voters rejected, by 65% to 35%, a proposal that they passed one month later with 51% of the vote. He shows a sharp increase in turnout, especially for voters who were not long-term residents of Troy. Furthermore, separate estimates of the model for May and June indicate a relative increase in 'yes' voting by females, presumably because females place relatively high value on the custodial function of the schools. These results pose a new direction and a challenge to formal theorists.

4.2. Institutional considerations

Rather than deal with the rich but difficult problems of political communications, we will spend the balance of this paper emphasizing institutional considerations that can cause public expenditures to deviate from the median voter model. Two of the studies previously discussed suggest the importance of institutional considerations.

First, Bergstrom and Goodman (1973) estimate their model separately for ten states. These results show striking differences. Statistically, Bergstrom and Goodman report that an F-test would not allow pooling of the data. The income elasticities on general expenditures range from 1.73 in Illinois to 0.16 in the neighboring state of Wisconsin. Tax share elasticities range from -0.50 in New York to -0.01 in Wisconsin. For both Pennsylvania and Wisconsin, neither elasticity is significant at the 0.05 level, despite a sample size of 124 in Pennsylvania. True, the differences could reflect problems with aggregation or omitted variables. Ethnicity, a key variable for Wilson and Banfield (1964), for example, is not included. Yet Bergstrom and Goodman did include seven census demographics (nonwhite, aged, population density, owner occupied, population change, employment–residential ratio, residential mobility). Thus, the results could also readily reflect institutional differences in determining expenditures.

Rather than being made by the three median voter institutions we initially discussed, most of the municipal expenditure decisions analyzed by Bergstrom and Goodman are in fact made by representative legislatures. Lovell acknowledges that institutions could matter, in writing, 'The heteroscedasticity correction is excessive if the electoral process is less noisy than would otherwise be expected in smaller districts employing town meetings rather than representative governments' [Lovell (1978, p. 491)]. But Lovell does not see representative institutions as distorting the median voter's preference on average. For Lovell, politics only increases the error term.

Only in the work of Pommerehne (1978) and Pommerehne and Schneider (1978) can we see explicitly that institutions matter. Pommerehne replicated

the Bergstrom–Goodman median voter model for four types of Swiss municipalities: (1) 32 direct democracies that had obligatory as well as optional (citizen-initiated) referenda on expenditures; (2) 16 direct democracies with optional referenda only; (3) 35 representative democracies with some form of expenditure referenda; and (4) 27 representative democracies with no referendum process. For the simple model that includes only median (of Swiss citizens') income and tax share, the residual variance, adjusted for degrees of freedom, increases uniformly as one goes from the 'pure' direct democracies of type 1 to the 'pure' representative democracies of type 4. The type 4 variance is more than triple the type 1 variance [Pommerehne (1978, p. 266)]. Representative institutions may well be 'noisy', as Lovell conjectured.

Are there also systematic differences in spending between the direct and representative democracies? The work of Pommerehne and Schneider contains some suggestive evidence. When the mean income of *all* households is substituted for the median income of *Swiss* households in the regression, the type 1 and 2 residual variances increase by 50% and 100%, respectively. This result suggests that the direct democracies in some way respond to the preferences of their voters. The foreign residents may get little consideration. (However, for testing the median voter model it would be more appropriate to compare median income of Swiss citizens with mean and other statistics of the Swiss citizen income distribution.) In contrast, the type 3 and 4 residual variances both decrease by 10% when mean is substituted for median. This may suggest that expenditures in representative democracies reflect the 'exploitable' tax base more than citizen preferences. The mean–median difference is slight, however. The high residual variance of both models suggests that simple income and tax price variables cannot provide an adequate specification of expenditures in representative democracies.

Pommerehne and Schneider indeed sought to extend the specification of the simple Bergstrom–Goodman model. For direct democracies the simple median voter model with a crowding parameter of unity remains the best fitting model. No reduction in adjusted residual variance is gained from demographics, ethnicity (French–German–Italian), political composition or fiscal illusion variables.[24] In contrast, the best fitting equation for the representative democracies augments the median voter model by two terms. First, the complexity of the revenue system (CRS) is measured by the Herfindahl concentration index.[25] This index is one when all the revenue comes from one source and it approaches zero if there are many revenue sources with equal shares. This variable is highly significant, dramatically reducing explained variance. Complex revenue systems lead to large expenditures, perhaps reflecting the American adage 'you can nickel and dime 'em to death'. The second variable, TBE^{-1}, is the inverse of the time before the next election. The more time before an election, the greater the expenditure, although this variable is significant only at the 0.10 level.[26] Both variables reflect information cost problems evoked in our discussion of Rubinfeld. It is striking that neither variable improves the direct democracy equation. In direct democracies, both the town meetings and the frequent referenda should serve to keep citizens better informed and to allow for the free amendment process that we indicated was an essential ingredient of median voter 'committees'.

[24]Oral communication, Werner W. Pommerehne.

[25]A variable conceptually similar to CRS gives a slightly better fit in type 3.

If the results on the time pattern of expenditures and the complexity of the revenue system suggest that representative democracies are indeed distinct from direct democracies, we ought to pose several caveats.

(1) Tax payments of foreign residents have been prorated to the income of Swiss citizens rather than entered in the calculation of the tax price. (In American studies, nonresidential property values lower the tax price of voters.)

(2) Wide fluctuations of estimated coefficient values as new variables are entered suggest the presence of severe problems of multicollinearity or small sample outliers.

(3) Even with the TBE and CRS variables, the estimated variance for representative democracies is about 50% greater than that of the simple median voter model for direct democracies.

These caveats have prevented us from assessing whether, in a *ceteris paribus* sense, expenditures are higher in representative democracies than in direct democracies. One would like to make such an evaluation since the possibilities of both monopoly power acquired by elected officials and coalition of minorities politics suggest that expenditures will be greater in representative democracies.

Pommerehne (1978, p. 277) reports that the mean level (across municipalities) of expenditures is roughly the same in all four types. Since the predicted values in regressions with poor R^2 values lie near the mean, we would likely find that representative democracies are 'predicted' to spend more for some *ceteris paribus* assumptions, less for others. The specification of the representative democracy model must be improved before a more definitive evaluation can be made.

5. Summary of empirical review

The various studies we have reviewed have not provided strong, broadly based support for the median voter hypothesis. We found methodological problems that made tests of the hypothesis inherently difficult; and we found

[26]Separate regressions for salaries indicate that wage increases represent much of the fluctuation contained in the TBE term.

that median voter models were inadequately tested against competing models. An implication of our findings is that, while theorists may find the median voter assumption convenient in the analysis of local public goods and elsewhere, there currently is hardly any empirical basis for making the assumption.

At best, the work of Lovell and of Pommerehne suggests that a median model dominates a mean model in *some* jurisdictions. Rather than confirming Inman's notion that the political process could be encapsulated in the median voter outcome, the comparative regressions of Bergstrom–Goodman and Pommerehne–Schneider suggest that institutions matter. Further indication that institutions matter is available from an overview of school expenditures in the United States.

6. An alternative to the median voter model: threats in school referenda

Were the aim of political process to enact the preferences of the median voter, the simplest method would be to adopt the Florida system mentioned earlier. To our knowledge, Florida is the only state to have used such a system. Florida itself abandoned this system in 1968, replacing it with state financing and a limit on additional local spending. In 1973, Florida school districts were given the power to levy up to the local limit without any need for voter approval [Holcombe (1977, p. 82)].

In 28 states, school referenda of various types permit some direct voter control of expenditures.[27] Do these referenda, as Holcombe (1977, pp. 78–79) has argued, represent the 'voting on alternative motions' institution that yields the median outcome?[28] We answer no, in that the school boards exercise significant agenda control. Arkansas has a statutory limit of only one election per school year.[29] In Ohio this limit is three.[30] In Oregon it has been six to eight.[31] Given that only school boards can make proposals and that the number of proposals is limited, proposals are not freely amendable. Thus, both in the referendum states and in the nonreferendum states, in order to obtain a median voter outcome we must rely on candidates for school board or other elected representatives (or, in some cases, appointed officials) promising and enacting the expenditures desired by the median voter.

[27]See Tron (1976).

[28]Deacon (1977) refers to unpublished work by Holcombe (1976), saying that it conducts a direct test, yielding 'evidence that strongly supports the median voter outcome' [Deacon (1977, p. 386)]. Our reading of Holcombe's paper leads us to regard Deacon's characterization as unduly optimistic.

[29]Hamilton and Cohen (1974, p. 37).

[30]Hamilton and Cohen (1974, p. 28).

[31]Cf. *Oregon Laws 1973*, ch. 796, p. 2043 and *Oregon Laws Special Session 1974*, ch. 45, p. 133.

While political competition within districts or competition across districts in a mobile society may drive spending toward the median voter level, two pieces of evidence argue that elected representatives do not represent the median voter. First, school closings occasioned by voters refusing to approve the board's proposals are direct evidence of tension between the electorate and the board. Secondly, outcomes of the referendum process appear to reflect directly the aforementioned threat possibilities in reversion level.

Neufeld (1977), in his analysis of Michigan school expenditures, recognizes that the reversion is an important determinant of a voter's response to a proposed tax rate change. Neufeld nevertheless believes that over time expenditures would approach the median desired expenditure. His empirical work is not well tied to his theoretical model. His results do indicate that the percentage of voters approving a proposal depends both on the proposal and the reversion. The greater the average tax bill decrease if a proposal is defeated, the greater the percentage voting yes. At the same time, an increase in the proposed tax increment will reduce the yes vote percentage. This suggests (though Neufeld does not say so) that voters do perceive the referendum as a take-it-or-leave-it offer, and are disinclined to 'hold out' for a lower offer from the school board. If voters expected the board to keep cutting the budget proposal until the median ideal point were reached, then we would not expect the reversion to play so significant a role.

Elsewhere, we have shown that when the board can threaten the voters via an all-or-none choice, a budget-maximizing board will not hold an election when the reversion is greater than the median ideal point.[32] The board will, however, assess all of the reversion budget. Now Colorado and Oregon have similar referendum systems except for the reversion. In Oregon, nearly all school districts are unable to operate without voter approval of the current budget. Most reversions are zero and many others are only a small fraction of total operating expenses. In contrast, Colorado has uniformly high reversions. By statute, Colorado districts are guaranteed 106% of the previous year's expenditure, adjusted for projected enrollment changes. Cohen and Hamilton (1974, p. 17) report, 'From contextual evidence, one may infer that a few, but only a few, budget elections occur.' Oregon is just the reverse. In recent years, fewer than ten of the more than 300 districts in Oregon have been able to forgo budget elections.[33]

Clearly, institutions, through the reversion statutes, affect whether voting actually occurs. Do they also affect expenditures? In Oregon, those few districts that do not hold elections assess, in the aggregate, over 99% of their reversion levels. The Portland school district has, at least since 1970, always

[32]See Romer and Rosenthal (forthcoming). Deacon and Shapiro (1975) and Barkume (1976) recognize the possible all-or-nothing structure of referenda, but they are not concerned with exploring the implication of this for possible agenda control.

[33]Data for Oregon are drawn from Romer and Rosenthal (forthcoming).

assessed all of its reversion. Presumably, Colorado districts have an appetite as large as those in Oregon. Defenders of the median ideal point model would now have to explain why locally financed expenditures increase at 6% annually in nominal dollars in Colorado with enrollment adjustments, while in Portland and a few other Oregon districts the increase is 6% without these adjustments.[34]

We have discussed districts not holding budget elections. Our threat model conversely holds that elections will be held when the reversion is below the ideal point of the median voter. In this case the amount voted, rather than being the median ideal point, will exceed the median, increasing as the reversion level decreases. Spending will thus depend not only on voter preferences but also on the reversion level and, perhaps, other institutional factors. Although we are a long way from modeling the full complexity of political institutions, even a simple threat model would endow politics with a more complex role than that attributed by the median voter hypothesis.[35]

[34]Portland enrollments declined from 72,161 in 1970–71 to 55,389 in 1976–77. (Oregon Department of Education, 'Oregon school districts: Type, location, size', issues of 1 July 1971 and 1 July 1977.)

[35]Empirical tests of these propositions are presented in Romer and Rosenthal (1979).

Appendix A: Summary of expenditure models

Authors	Model	Remarks
Barr and Davis (1966)	$\frac{E}{N}=c_0+c_1\frac{V}{N}+c_2H+u$	Median income and demographic variables did not significantly reduce error variance
Bergstrom and Goodman (1973); Pommerehne and Frey (1976); Pommerehne (1978); Pommerehne and Schneider (1978)	$\ln E=c_0+c_1\ln\hat{Y}+c_2\ln\hat{\tau}+c_3\ln N+\sum\beta X+u$	$\sum\beta X$ contains significant demographics for Bergstrom and Goodman $\sum\beta X$ absent for Pommerehne and Frey Key $\sum\beta X$ variables for Pommerehne and Pommerehne and Schneider are time before election and complexity of revenue system
Inman (1978)	$\ln\left(\frac{E}{ADM}\right)=c_0+c_1\ln\hat{Y}+c_2\ln\hat{\tau}+\ln[(X)]+u$	Shift function (X) has coefficients linear in demographic variables or products of demographic variables
Lovell (1978)	$\ln\left(\frac{E}{ADM}\right)=c_0+c_1\ln\left(\hat{Y}-\hat{\tau}\frac{E}{ADM}\right)+c_2\ln\hat{\tau}+c_3\ln ADM^*+u$	$\ln\hat{\tau}=\ln(\hat{Y}/Y)+\beta_1\ln G+\beta_2\ln H$
Rubinfeld (1977)	$\frac{E^*}{ADM}-\frac{E'}{ADM}c_0+c_1\ln Y+c_2\ln\tau+\sum\beta X+u$	E^*, E' cannot be identified. Significant X variables include dummies for children enrolled in school, and for school teachers

Appendix A (*cont.*)

Authors	Model	Remarks
Barkume (1976)	$U(E_p)-U(E_q)=c_0+c_1\ln\hat{Y}+c_2\ln\hat{\tau}+\sum\beta X+u$	$\hat{Y}$ and $\hat{\tau}$ are medians for each census tract in the sample
Neufeld (1977)	$\ln\left[\dfrac{\%\text{ yes vote}}{100-\%\text{ yes vote}}\right]=c_0+c_1\bar{I}+c_2\bar{D}+c_3\bar{\tau}E+c_4\dfrac{\bar{W}}{\bar{Y}}+\sum\beta X+u$	

Notation:

E = total expenditure
Y = household income
ADM = school enrollment
N = population
V = total assessed property value
H = proportion of homes owner occupied
τ = tax price
ADM^* = average number of public school children per household
X = auxilliary (demographic) variables
G = (total assessed housing value)/V
E_p = expenditure if referendum passes
E_q = expenditure if referendum fails
$E_q<E'<E_p$
E^* = individual's 'ideal' expenditure
$U(\cdot)$ = individual's utility function
$\bar{I}$ = average tax bill increase if referendum passes
$\bar{D}$ = average tax bill decrease if referendum fails
$\bar{W}$ = average home value
u = error term; logistic distribution for Rubinfeld and Barkume, normal for others
c,β = coefficients to be estimated
bars ($^-$) denote average values
carets ($\hat{}$) denote median values.

Appendix B

Let Y be household income. If the distribution of $\ln Y$ is normal with mean μ and variance σ^2, then Y has lognormal distribution with

$$\text{mean } Y=\exp(\mu+\tfrac{1}{2}\sigma^2),$$

$$\text{var } Y=\exp(2\mu+\sigma^2)(e^{\sigma^2}-1),$$

$$\text{coefficient of variation}=\frac{(\text{variance})^{1/2}}{\text{mean}}=(e^{\sigma^2}-1)^{1/2}, \qquad \text{(B1)}$$

and

$$\text{median } Y=\hat{Y}=e^{\mu}.$$

The fractile Y_f is given by

$$Y_f=\exp(\mu+k_f\sigma)=\hat{Y}\exp(k_f\sigma)=m_f\hat{Y},$$

where k_f is the corresponding fractile of the normal distribution with zero mean and unit variance [Aitchison and Brown (1957, pp. 8–9)].

Consider two income distributions, Y^1 and Y^2, both lognormal, with parameters (μ_1,σ_1) and (μ_2,σ_2), respectively, and having the same coefficient of variation. From (B1), $\sigma_1=\sigma_2$. Consequently,

$$Y_f^1=m_f\hat{Y}^1 \quad\text{and}\quad Y_f^2=m_f\hat{Y}^2.$$

Thus, replacing $\ln\hat{Y}$ by $\ln Y_f$ in Inman's regressions would leave all estimates except the constant term unchanged. As long as we have lognormal distributions satisfying the proportionality requirement, any fractile of the income distribution would have the same explanatory power as the median.

Appendix C

1. For simplicity, assume that $p(Y)=Y/\bar{Y}$, as with a proportional income tax. Let $\hat{Y}$ be the median household income. Suppose this household is pivotal, and let $\hat{n}$ be the number of children in it. For this household, maximization of (1) subject to (2) gives

$$\ln S-\gamma\ln C=\text{const.}-\sigma\ln(\hat{Y}/\bar{Y})-[\delta(1-\sigma)+\sigma]\ln\bar{n}$$
$$-\delta(1-\sigma)\ln(\hat{n}/\bar{n}), \qquad \text{(C1)}$$

where $\sigma=1/(\theta+1)$ and $\gamma=\sigma(\beta+1)$.

Lovell's 'generalized CES' regression is

$$\ln S - \gamma \ln C = \text{const.} - \sigma \ln (\hat{Y}/\bar{Y}) - [\delta(1-\sigma)+\sigma] \ln \bar{n}, \quad \text{(C2)}$$

which is missing the last term in (C1) as long as $\hat{n} \neq \bar{n}$. To see one possible effect of this omission, suppose that family size is related to income approximately as $n = aY^b$, so that (C1) can be approximated by

$$\ln S - \gamma \ln C = \text{const.} - [\sigma + \delta(1-\sigma)b] \ln (\hat{Y}/\bar{Y}) - [\delta(1-\sigma)+\sigma] \ln \bar{n}. \quad \text{(C3)}$$

If equation (C3) is 'correct', then Lovell's estimated coefficient of $\ln(\hat{Y}/\bar{Y})$ captures both tax price and family size effects. If $b>0$ and $\delta>0$, the true σ could even be *zero*, yet the estimated coefficient of $\ln(\hat{Y}/\bar{Y})$ would be negative.

2. For Lovell's utility function,

$$\frac{\partial U}{\partial S} = \frac{\alpha\theta}{\beta} n^{-\theta\delta} U^{1+\beta} S^{-(1+\theta)}$$

and

$$\frac{\partial U}{\partial n} = \frac{\alpha\theta\delta}{\beta} U^{1+\beta} n^{-(1+\theta\delta)} S^{-\theta}.$$

Positive $\partial U/\partial S$ requires that $\alpha\theta/\beta > 0$; therefore, $\partial U/\partial n > 0$ requires $\delta > 0$.

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A RECONCILIATION OF RECENT RESULTS IN OPTIMAL TAXATION THEORY

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The recent papers by Guesnerie and Diewert on tax reforms are interpreted as contributions to the characterization of second-best optima. This paper demonstrates that when it is possible to achieve any feasible direction of change in supplies by a differential change in producer prices, there are unique producer support prices. Under these circumstances, the apparent differences between Guesnerie and Diewert are reconciled. Optimality conditions with nonunique support prices are also considered.

1. Introduction

One of the major concerns of optimal taxation theory is the presentation of conditions which characterize the Pareto optimal states of an economy which must operate with a limited set of tax instruments. Recently Diewert (1978) and Guesnerie (1977) have addressed themselves to the slightly different problem of analyzing tax reform proposals. Weymark (1978b) provides further results on tax reform. Diewert and Guesnerie consider whether it is possible to propose a set of marginal changes in the tax instruments so as to achieve a Pareto improvement. Necessary conditions for the nonexistence of Pareto-improving tax changes are necessary conditions for the initial allocation to be Pareto optimal. Consequently, it is possible to consider this aspect of Diewert's and Guesnerie's contributions in terms of the more traditional approach of characterizing optima.

Unfortunately, it is difficult to see the relationship between the conditions developed by Diewert and Guesnerie. To correspond with Guesnerie's model, Diewert's problem must be reformulated by replacing his ad valorem commodity taxes by specific taxes, by fixing profit tax rates at 100%, and by replacing compensated demand functions with ordinary demand functions.

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