CHAPTER TEN
THE IVY LEAGUERS

Frustrated and dispirited, politicians looking for solutions—not always mathematical, as often as not political—to the seemingly intractable problems of apportioning seats in Congress finally turned to a professional, Walter F. Willcox. Willcox, professor of social science and statistics in the department of philosophy at Cornell University, had been active in the census of 1900 and later became the Census Bureau's chief statistician for population. He was instrumental in raising the debate about apportionment from the lowlands of politics to the realms of science.

The Census Bureau had been established very recently. At the outset, in 1790, the first census had been conducted by U.S. Marshals and for the next one hundred years counting the population was an ad hoc affair. Each time, after the data were published, the office would shut down until the next census. Toward the end of the nineteenth century the need for a permanent agency became apparent. Willcox explained how the establishment of the bureau was brought about: "Director William R. Merriam handled Congress very cleverly; got a stunning group of girls on his staff; nearly all of them, no doubt, wanted to remain in Washington and in the Census Office—at least until they got married. These girls, I was told, brought so much pressure on Congress that . . . the office was made permanent, not for any scientific reason, but to keep the staff from being disbanded." By an act of Congress in 1902 the Census Bureau became a permanent organization.

Willcox was the founder of sociology at Cornell. Born in 1861 in Reading, Massachusetts, he studied at Amherst as an undergraduate and then at Columbia University where he obtained LLD and PhD degrees. He also spent a year at the University of Berlin in Germany. Upon receiving his doctorate, he joined the faculty of Cornell University where his career would span forty years. (More on Willcox and on this chapter's other dramatis personae will be found in the additional reading section.)

The twentieth century's first census in the United States took place in 1910. It revealed a 20 percent population growth as compared to the pre-

vious decade. The United States' total population had grown from less than 75 million ten years earlier, to over 91 million. But higher birth rates, immigration, and the addition of Oklahoma do not provide a complete picture. Beyond the increase in the total number of citizens, the distribution of the population within the United States had changed. Poor farmers had begun to look for their fortunes in the cities, and migration from rural states toward urbanized centers resulted. The new realities needed to be reflected in the composition of the House. What method should be used to apportion the seats of Congress? The rural population was not going to take the erosion of their political power lying down. In an unlikely coalition, wealthy landowners and poor farmers were going to fight for their political rights, even if the rights weren't quite on their side. But first things first.

After studying the various apportionment methods, Willcox became convinced that Webster's technique of "major fractions" was the way to go. (Recall that this technique requires finding a divisor for the populations of the states, such that the result, when rounded *up* or *down* to the nearest integer number, gives the desired amount of seats.) It neither succumbs to any of the paradoxes, nor is it biased toward large or small states. Persuaded by the arguments, Congress began to lean toward the Webster-Willcox method. But Ohio, the 4th-largest state, and Mississippi, the 21st, did not like this one bit because they would each have received an additional seat under Hamilton's scheme. (Maine, the 34th state in terms of size, and Idaho, the 45th, would have each lost one.) So, to keep everybody happy, Congress adopted Webster's method but simultaneously increased the number of seats from 386 to 433. (One additional seat each was reserved for Arizona and New Mexico for whenever they would join the Union.) This occurred in 1912 and from then on, until today, the House comprises 435 seats. With this number of seats, Ohio and Mississippi kept the seats that had been allocated to them after the previous census, Maine and Idaho received the additional seat that Webster's method awarded them, and no other state would lose a seat. The fact that the inflation in the number of seats had eroded each representative's voice by about 11 percent apparently went unnoticed (386/433 – 1 = –11 percent).

But a sense of unease pervaded Congress. Whatever it was that had made Maine the odd man out, both in the Population Paradox and the New State Paradox, it gave everyone cold feet. What happens to Ohio and

Mississippi, Maine and Idaho, Alabama, New York, or Virginia one day could happen to any other state on a different day. A fresh idea was needed for the next apportionment, due in 1920. Eventually, one did come up. It was the brainchild of Willcox's colleague at the Bureau of the Census, the chief statistician of the Division of Revision and Results, Joseph A. Hill.

Looking for the fairest manner of apportionment, Hill considered the number of constituents that are required for one representative as the key variable. Evenhandedness requires, he felt, that the relative difference in this number between states be minimized. If one state requires 200,000 people to send a representative to Congress, and another state needs only 190,000, then the relative difference is 5 percent. Hill felt that seats should be apportioned in such a way that this relative difference be as small as possible. The following example shows how the apportionment is to be computed.

TABLE 10.1
The Hill method of apportionment

Total population: 4 million. 20 seats to be apportioned. Population per seat = 200,000

			Option 1			*Option 2*		
State	*Population*	*"Raw" seats*	*Allocated seats*	*Citizens/ seat*	*Diff*	*Allocated seats*	*Citizen/ seat*	*Diff*
A	3,300,000	16.5	16	206,250	17.8%	17	194,117	20.2%
B	700,000	3.5	4	175,000		3	233,333	
Total	4,000,000	20.0	20			20		

Since the relative difference in Option 1 (17.8%) is lower than in Option 2 (20.2%), the former is preferred.

How would Webster's method of rounding to the nearest integer value work in this example? Well, it wouldn't. In the rare case that the fractional seats of both states are precisely 0.5, Webster's method would give no indication as to which state's delegation to round up or down. The situation would be a tie.

Hill's idea appealed to Edward V. Huntington, a professor of mathematics and mechanics at Harvard University. Actually, Huntington's interest in apportionment was a sideline. The professor had known Hill from his undergraduate days at Harvard where they had been classmates. After studying all apportionment methods, Huntington became an avid advocate of Hill's proposal, which he called the "method of equal proportions," but which would henceforth be known as the Huntington-Hill (H-H) method.

Huntington formalized Hill's argument. An apportionment is good, he stated, if no reallocation of a seat from one state to another can reduce disparity. Of course, this left the notion of disparity to be defined. Huntington interpreted it—as did Hill—as the percentage difference in the number of people that is required for a congressional seat. The allocation of seats that minimizes the disparity is the best apportionment.

To recapitulate, the Webster-Willcox (W-W) method requires finding an appropriate divisor such that the seats—when rounded up or down—add up to 435. (For states that would be allocated less than one seat, the number would be rounded up in any case.) The H-H method requires finding a divisor and then rounding up or down, such that the relative difference in the numbers of constituents is decreased. Like many academic controversies, the dispute became fierce. As more scholars entered both sides of the fray, the proponents of the two methods became known, respectively, as the "Cornell school" (W-W) and the "Harvard school" (H-H).

Which is the preferable method was not only a question of mathematical superiority but also one of which method would be easier to explain to Congress. The Harvard school's method is not an easy one to implement. Not only must several divisors be tried, but it must also be verified for each one whether the reallocation of a seat between two states decreases the relative difference in the number of constituents required for a seat. Fortunately there is an easier way to implement the H-H method.

To explain it requires some preliminaries. Rounding up or down at the midpoint between two integers, that is, at a fraction of 0.5, is called "rounding at the arithmetic mean" because its computation requires just arithmetic. For example, the arithmetic mean is computed by adding one number to another and dividing the result by two (2 plus 18, divided by 2 equals 10). There is another mean, called the geometric mean. It is computed by multiplying the two numbers and then taking the square root. For example, multiplying 2 by 18 and then taking the square root gives 6. Thus the arithmetic mean of 2 and 18 is 10, while the geometric mean is 6.

Huntington claimed that finding an appropriate divisor and rounding at the *geometric* mean gives an apportionment that minimizes the relative difference in the number of people required for a congressional seat. Hence it produces exactly the result that the H-H method seeks.

Thus, rather than rounding exactly halfway between, say, 3 and 4 or 17 and 18, as the W-W method requires, the H-H method demands rounding at 3.4641 (square root of 3 times 4) or at 17.4928 (square root of 17 times 18). The difference between arithmetic and geometric means is quite small for consecutive integers, but for apportionment purposes it may mean the difference between getting an additional seat and not getting it.

Rounding at the geometric mean is a simple way of implementing the H-H method. But is it legitimate? Why should rounding at the geometric mean be related to minimizing relative differences in the number of people needed for a congressional seat? The astonishing statement that both procedures give identical results requires proof. Huntington provided it in an article in the *Transactions of the American Mathematical Society*. The paper, based on seven talks he had given to various audiences during the previous eight years, appeared in 1928. (It would lead us too far astray to reproduce the proof here, and we postpone this to the appendix.) Thus he not only formalized Hill's argument and found a simpler way of implementing it, he also gave it a mathematical underpinning.

Unfortunately, the discovery that rounding at the geometric mean is equivalent to the H-H method also brought to light a serious problem. Fair as the method seems at first blush—who would want to argue that relative differences should not be minimized?—it slightly favors smaller states. A small state requires a fractional seat of only 0.4142 to get rounded from one seat to two, whereas a large state would need 0.4959 fractional seats—nearly 20 percent more—to get rounded from 31 to 32 ($\sqrt{[1 \times 2]} = 1.4142$; $\sqrt{[30 \times 31]} = 30.4959$).

This is borne out by the facts. As compared to the Cornell (W-W) method, the Harvard (H-H) method would have allotted one additional seat each in 1920 to the relatively small states of Vermont, New Mexico, and Rhode Island and taken away one seat each from the relatively large states of New York, North Carolina, and Virginia. Obviously, the latter did not take kindly to H-H. And the less populated rural states did not take kindly to the W-W method; when compared to H-H it would have cost them eleven seats.

The squabbles between the supporters of Cornell and Harvard went on and on. In the end, it was all for naught. Faced with implacable opposition

to both proposals, Congress at first attempted to find a compromise. It tried to apportion the current 435 seats according to Webster's time-proven method. No agreement could be reached. Next, Congress considered an increase in the size of the House to 483 which would have, again, guaranteed all states at least as many seats as in the previous House—albeit with an erosion of again 11 percent due to the inflation in seats. This bill died in the Senate.

Other attempts fared no better. Congressmen and senators from the rural states blocked all reapportionment legislation and ensured that every attempt to change the status quo was doomed from the outset. Deadlock resulted and in the end Congress once again decided not to decide. In direct violation of the Constitution, no reapportionment was effected in 1921, and the composition of the House remained unchanged from its composition after the census of 1910. The gentlemen from the rural states welcomed this outcome. They were quite happy to postpone the inevitable for another ten years.

But looking the other way was no answer for the long run. With the end of the 1920s and the next census drawing nearer, a decision became more urgent. Congress could not snub the Constitution again; a solution had to be found. The debate reached acrimonious tones with the advocates even stooping to personal attacks. While Hill was a disimpassioned intellectual, Huntington was anything but. Pugnacious, with a taste for verbal sparring, he took up the fight for Hill's and his method with religious fervor.

The fact that the House Committee on the Census did not accept his method of equal proportions was "attributable entirely to one man, Professor W. F. Willcox, of Cornell," he charged in the journal *Science* in February 1928, claiming that this man's "entirely false description . . . supported by impressive charts and diagrams" had the effect of completely misleading the congressional committee. Calling Willcox's technique of major fractions "an obsolete method" and an "erroneous idea" he extolled the virtues of his own method of equal proportions, which for its "simplicity, directness, and intelligibility leaves nothing to be desired."

The harangue continued. While Willcox maintained that mathematicians and statisticians were in favor of his method he, in fact, quoted supporting statements only from constitutional lawyers and professors of political economy, Huntington claimed. Moreover, even their backing had been secured by misinformation. He contrasted this with his method of equal proportions that was "endorsed by a general consensus of scientific opinion," being "the only method that has the approval of any organized body of scholars." Omitting to specify who exactly the scholars were, he called upon Willcox to publish his views in some regular journal, not just in testimony before Congress, "so that they may be accessible to the scrutiny of all groups of scholars."

Half a year later, in December 1928, the pages of *Science* again served Huntington for a comparison between his "scientific method of equal proportions" and the "unscientific method of major fractions." Willcox's statements about the latter are "at variance with known mathematical facts" and serious errors in congressional hearings "will be the source of confusion to future students of the problem," he fumed. What was especially galling in his opinion was that "the appearance of such misstatements . . . in a permanent public document gives Congress a discouraging idea of the value of scientific methods."

Huntington further argued that "the method of equal proportions . . . has been mathematically shown to have no bias in favor of either the larger or the smaller states." This claim is very strange, if not downright dishonest. His paper in the *Transactions of the American Mathematical Society*, published only a few months previously, had shown unequivocally that the H-H method is equivalent to rounding at the geometric mean. And rounding at the geometric mean clearly favors small states. Did the professor feign ignorance or did he really not know any better?

One way of proceeding would have been to refer the question to a knowledgeable institution that would check out the matter and prepare an independent expert report. Indeed, Willcox had suggested exactly this in hearings before Congress. Unfortunately, the American Political Science Association, which would have been well placed to arbitrate between the competing methods, refused to get involved. In a glaring display of ivory tower mentality the organization's secretary wrote that his association "has the feeling that it ought not to undertake to decide a question of this sort." Apparently the association felt that studying and criticizing how public figures make tough decisions is all right, but taking a stand is not.

In February 1929, Willcox weighed in at *Science* for the first time. His manner was more gentlemanly than was Huntington's. Rather than attacking his opponent personally he dealt with the problem at hand. The real

danger loomed that the apportionment, first enacted in 1911 and unconstitutionally extended in 1921, would stay in effect for another ten years, until 1940, unless Congress decided to change it. Willcox wanted to deal with the big question instead of wasting time arguing about fractional seats. Hence, the objective was not to find the best or the fairest method but one that was workable and that would be acceptable to Congress. He expressed his goal succinctly: "What method is likely to give the bill the best chance of passing Congress?"

Convinced that his method of major fractions stood the better chance on this score, he could not but engage in a bit of demagoguery himself. Would the congressmen accept a "new, untried method" that is difficult to explain to laypeople, he wondered? Or would they prefer a method that had already been used in 1911 and involved no more than rounding fractions down if they are below 0.5, and up if they are above 0.5? Reminding the readers of a failed attempt to enact a previous apportionment law and eager to avoid a similar flop this time, he affirmed with selflessness that seems only slightly affected: "I would gladly abandon my preference . . . if I thought another method had a better chance of acceptance by Congress and the country." As for publishing in a recognized scientific journal, as Huntington demanded, Willcox did not care one bit. His work was not meant for the delectation of academics but as a service to Congress. "The judgment of the average representative or congressional committee is of far more importance than that of any group of scholars."

The riposte came after barely four weeks. Squeezed between "Geological Work in Tonga and Fiji" and "Importation of Cinematographic Films," a short note by Huntington appeared in *Science*, in which he outlines alleged errors in Willcox's paper. Huntington must have read the piece with a magnifying glass. Gleefully he jumped on a slightly ambiguous sentence in the paper. Willcox had written that the states' populations, divided by an appropriate divisor, give a series of quotients which, when rounded up or down, give each state's number of seats. "The whole series would add to 435." Deliberately misunderstanding "the whole series" to mean "the series of quotients," while Willcox had obviously meant "the series of representatives," Huntington termed the arguments in the paper crass misstatements of the mathematical facts, and the description of Willcox's method as grotesque. The note ends with a punch below the belt. "It appears to be only by evasive arguments like these that the method of major fractions can be defended." Ouch!

Willcox did not take the affront lightly. Handing copies of the ambiguous passage to a class of thirty undergraduates he asked them their opinion, without letting on the reason for the inquiry. Three-quarters of the class thought "the series" meant "series of representatives," only one-quarter interpreted the phrase to mean "series of quotients." Willcox reported the results of his inquiry in *Science* in March 1929 as if it had been a scientific experiment of great importance. "It is hard to understand how a scholar of the position of Professor Huntington could have given my words the meaning he did," he comments and closes his paper with the remark "Hitherto I have not answered Professor Huntington's personal attacks but this case is so clear and typical that I have made an exception."

The debate raged throughout 1928 and the first half of 1929. If nothing else, it served to show that politics in science could be every bit as bitter as politics in Congress. Huntington stepped in again in May 1929, but by that time the question had become moot.

It had become obvious by early 1929 that feuding politicians would be unable to come to an agreement and that academics, left to their own devices, did not behave any better. So Congress turned to the one institution that could save it from the impending imbroglio. It comprised experts, pundits in their fields, who could be counted on to settle any scientific question in an unbiased manner, uninfluenced by partisan or nationalistic considerations: the National Academy of Sciences.

So when Congressman Ernest Gibson of Vermont remarked that "the apportionment of representatives ... is a mathematical problem," and posed the question "why not use a method that will stand the test ... under a correct mathematical formula?" the Speaker of the House, Nicholas Longworth of Ohio, decided to put an end to the discussions by requesting the National Academy of Sciences (NAS) to decide on the appropriate method of apportionment.

Founded in 1863 by Abraham Lincoln, the academy's task is to give advice to the federal government and to the public about the impact of scientific and technological issues on policy decisions. As mandated in its Act of Incorporation, the NAS must "investigate, examine, experiment, and report upon any subject of science or art," whenever any department of the government asks it to do so. The members of the NAS—professors

at universities, scholars at research laboratories, scientists in private companies—work outside the framework of government to ensure their independence. So essential has the academy's service to government become that over the years Congress and the White House have repeatedly issued legislation and executive orders that reaffirm its unique role.

In contrast to the American Political Science Association the NAS did take up the gauntlet. A commission was created that was to decide on the best method to use for apportionment. The blue-ribbon study group was comprised of the three mathematicians Gilbert A. Bliss from the University of Chicago, Ernest W. Brown from Yale University, Luther P. Eisenhart from Princeton University, and the group's chairman, the biologist and geneticist Raymond Pearl from Johns Hopkins University in Baltimore. Among the mathematicians, Pearl was the odd man out. However, he was well versed in statistics, having spent a year in England with Karl Pearson, the founder of the world's first university statistics department at University College in London. It was a comfort to the members of the House, indeed to the citizens of the United States, to know that such high-powered scientists from Chicago, Yale, Princeton, and Johns Hopkins would be dealing with this problem even though it involved nothing more than basic arithmetic.

The report started out innocently enough. If apportionment were computed by simply dividing each state's population by the total population, it stated, the number of representatives would in nearly all cases consist of a whole number and a fraction "as, for example 7.3." (The latter remark was apparently added so that the report would be understandable even by the most obtuse reader.) Now if fractional voting were permitted in the House there would be no problem, the committee members remarked; each state would receive the exact number of representatives corresponding to the whole votes, plus—this was an innovative idea—an additional representative with the fractional vote. The latter would be an incomplete representative, so to say. But the Constitution did not provide for fractional voting in Congress. Hence this was a nonstarter.

Sharp as razors, they deduced that it was necessary to reach a solution to the apportionment problem in whole numbers. But by setting this condition, the mathematical nature of the problem was altered fundamentally. "It should be understood that frequently a problem in applied mathematics may have no unique solution, for the reason that the data initially given do not completely characterize the solution mathematically. In such cases a solution must be chosen for other than mathematical reasons among those which are mathematically possible."

The committee members went hunting for reasons other than mathematical ones. They considered all apportionment methods then known that avoided the Alabama Paradox and did lead to workable solutions. There were five: those proposed by Webster, Adams, Jefferson, and Hill, as well as another method suggested by James Dean, a mathematics and physics professor at the University of Vermont during the 1820s. I have not spoken about Dean's method because it was never used, except in a legal challenge by the state of Montana in 1991. (It consists of adjusting the value of "population per seat" for the United States as a whole in such a manner that "population per seat" for each state is as close as possible to that value.)

The four men liked the idea of minimizing relative differences; thus from the outset the odds were stacked in favor of H-H, which does exactly that. Sure enough, the report culminated in the assessment that the method of equal proportions is preferable to the others because it minimizes relative differences. "After full consideration of these various methods your committee is of the opinion that, on mathematical grounds, the method of equal proportions is the method to be preferred."

The way the professors arrived at their conclusion is somewhat reminiscent of circular reasoning. First they stated their preference for minimizing relative differences, next they chose the method that was designed to do just that. Not much left to decide then, really. They did concede that if the *absolute* difference in the number of constituents required for a representative should be minimized, a different method would be optimal. And if the absolute difference of the inverse of this number, that is, representatives per constituents, were considered, yet a different method would be optimal. "Each of the other four methods listed is, however, consistent with itself and unambiguous," they concluded.

But the professors gave a further justification for their choice. They preferred H-H to the other methods because "it occupies mathematically a neutral position with respect to emphasis on larger and smaller states." Signed: G. A. Bliss, E. W. Brown, L. P. Eisenhart, Raymond Pearl, Chairman.

By "neutral" the four committee members did not mean that H-H is unbiased—we know that the geometric mean penalizes larger states—but rather that in terms of bias H-H occupied the middle place among the five methods. Modern scholars remarked that it was fortunate that the committee had an odd number of methods to start with; otherwise it would have been difficult to identify the best method as the one in the middle.

Huntington could breathe a sigh of relief; his method of equal proportions had been vindicated by the committee. In a recapitulation of the NAS's findings he gives a gleeful account of the committee's report in *Science*. "All controversy surrounding the mathematical aspects of the problem of reapportionment of Congress should be regarded as closed by the recent authoritative report of the National Academy of Sciences," he states at the outset, and then he proceeds to take another swipe at Willcox's "complicated and artificial" method. "The hold which this now obsolete method still maintains on the imagination of many congressmen is due mainly, it appears, to a misconception."

In another punch below the belt, Huntington did not forego the occasion to repeat the allegation that Willcox had meant to add a series of quotients instead of congressmen. But all this may now be forgotten because "the National Academy of Sciences confirms that . . . the method of equal proportions . . . is logically superior to the method of major fractions." The method of equal proportions received the unanimous stamp of approval by the NAS because it allocates seats in Congress in a way that minimizes disparity. An apportionment made according to it cannot be improved, in the sense that the shift of a seat from one state to another will increase disparity. Huntington writes, Willcox's "useless complications . . . are completely done away with in the modern theory which provides a simple and direct test for the settlement of any dispute between two states." The diatribe ends with a blow at the regrettable failures of recent history. "The purely political attempts which have been made to retain the obsolete method of major fractions in current legislation have proved to be a serious menace to the whole reapportionment movement."

Did we say that the NAS committee had issued its report unanimously? We did, and formally this is not incorrect. But a few years after the report was published, Willcox dropped a dark hint. He suggested that the report had not been issued unanimously. How could that be? The four members of the committee, Bliss, Brown, Eisenhart, and Pearl had all signed the report. Did Willcox mean to insinuate that one of the professors had been coerced? This was preposterous and any such suggestion should immediately be dismissed. Huntington was appalled by the suggestion. "Any attempt to show . . . that these signatures do not mean what they say, is a gross insult to these distinguished scholars," he wrote with deep indignation. So was the non-unanimity a figment of Willcox's imagination?

It was not, and the files at the National Academy of Sciences contain a dark secret. Nobody was coerced but what is missing from the historical record is that the committee had originally included a fifth member, the Harvard mathematician William Fogg Osgood. What had happened?

Osgood had been recruited to the committee with Bliss, Brown, Eisenhart, and Pearl and had joined in the drafting of the report's first version. It was already clear that the committee would endorse the method of Huntington, his colleague at Harvard, but as work progressed, Osgood became disillusioned. He felt that the Harvard method was not given strong enough support. Finally, he had a change of heart about his collaboration. He announced his resignation in a telegram of January 30, 1929 to the NAS home secretary. It read:

> Report Committee has been so far weakened through introduction of irrelevant material that I find myself unable to subscribe to the revised form STOP in order not to obstruct the proceedings I beg to be relieved from further service on the committee.

Two days later, in a telegram to committee chairman Pearl, he provided more detail:

> My resignation from the Committee was intended to expedite a unanimous report from the other four members STOP As you have sent me third draft, I am glad to comment as follows STOP I should to add [*sic*] to paragraph five QUOTE but the present problem does not belong to this class UNQUOTE or omit the whole paragraph STOP in nine I should wish to omit the last sentence STOP You must realize that I have consistently adhered to first part of report as drafted and unanimously accepted in Baltimore and circulated STOP My objections are to new material introduced since then at the instigation of President of the Academy. This new material in my opinion robs the report of its essential meaning STOP My resignation therefore stands.

Three days after that telegram, Raymond Pearl gave an account of the situation in a letter to the president of the National Academy of Sciences:

> This report has been signed by four members of the Committee, namely Bliss, Brown, Eisenhart, and Pearl. The fifth member of the Committee, Prof. William F. Osgood, resigned from the Committee during the period of its work, for reasons which he has stated to you. I am informed by him that he wishes his resignation to stand, and desires to take no part in the further work of the Committee from the time his resignation was dated. I see no alternative, therefore, except for you to accept his resignation and consider the report to have been drawn by the four members of the Committee who signed it. I am very sorry that Professor Osgood felt obliged to resign. I did everything in my power to induce him not to do so, as did every other member of the Committee. Furthermore, every member of the Committee went just as far as he consienciously [*sic*] could to meet Professor Osgood's views at every point.

Willcox was not so naive as to believe that Osgood's resignation came about because the Harvard professor agreed with the Cornell method. To the contrary, it was obvious to him that Osgood had resigned because the committee's support for the Harvard method was not strong enough. He had accused the president of the NAS of having watered down the committee's conclusions to such a point that he could no longer agree with them. So actually Osgood's continued collaboration with the committee would not have done anything to further Willcox's cause.

For a long time, nothing further was heard on the subject from Willcox. Only in 1941 did he reveal that he had received a letter from Raymond Pearl, the NAS committee's chairman. In it Pearl belatedly vindicated Willcox's method. "Your efforts I heartily and unreservedly endorse. . . . In my opinion you are now doing a real service in bringing [the method of smallest divisors] to the fore."

If—in the wake of the NAS report—Congress would at long last decide on the method to be used, then finally, after twenty years, a new apportionment could be effected. Not everybody was happy with that prospect, however. Whichever apportionment method would be used, it would certainly be to the detriment of the rural states whose populations had diminished significantly. They were determined to fight tooth and nail against the looming danger to their interests. If they once again could torpedo attempts to reapportion the House, there was a chance that the present apportionment would be kept at least for the next ten years. Senators and congressmen from the rural states railed and fulminated against the "unholy, unrighteous, and unjust" method that "would run us down under the wheels, crush us by the system of major fractions . . . ruthlessly and without mercy." Their main aim was not just to derail W-W, but also to prevent any kind of agreement.

Notwithstanding their resistance, in the summer of 1929 Congress passed a bill that stipulated that the president would send the census data to Congress, together with two apportionment suggestions, one computed according to the W-W method, and another computed according to the H-H method. If Congress could not decide between the two methods the W-W method used in 1911 would automatically be employed again.

But then a remarkable if not wholly surprising coincidence occurred. After the census data were collected and counted, and the apportionments were computed and compared, it turned out that the W-W and the H-H methods gave identical results. Congress did not have to decide which method to use, nobody was crushed under the wheels, and apportionment could be effected in 1931 in idyllic harmony. Everyone breathed a sigh of relief and Congress could rest for the next ten years—at least concerning the vexatious apportionment issue.

The next time around, Congress was not so lucky. Following the 1940 census, everything came tumbling down again. When President Franklin D. Roosevelt presented the apportionments according to the two methods, forty-six states did indeed get the identical number of seats. But Arkansas and Michigan did not. Under the H-H method Arkansas would receive seven seats and Michigan seventeen, under the W-W method Arkansas would receive six and Michigan eighteen seats.

That was enough to get the controversy going again. And not only the two states joined the fight. Since Arkansas was a solidly Democratic state and Michigan usually voted Republican, the question became a partisan issue for the House. All of a sudden, the correct answer to a mathematical question boiled down to whether you were a Democrat or a Republican. The former preferred H-H, the latter W-W. (For what follows it is important to know that in 1941 Democrats held the majority in the House.)

TABLE 10.2A AND B
Comparison between H-H and W-W (1940 census)

(A) Huntington-Hill method (equal proportions)

Total population: 7,205,493. 24 seats to be apportioned. Population per seat = 300,229

			Option 1			Option 2		
State	*Population*	*"Raw" seats*	*Allocated seats*	*Citizens/seat*	*Diff*	*Allocated seats*	*Citizens/seat*	*Diff*
AR	1,949,387	6.493	6	324,898	11.26%	7	278,484	11.02%
MI	5,256,106	17.507	18	292,006		17	309,183	
Tot.	7,205,493	24.000	24			24		

Since the relative difference in Option 1 (11.26%) is higher than in Option 2 (11.02%), the latter is preferred.

(B) Webster-Willcox method (major fractions)

Find a divisor such that the rounded seats add up to 24: 300,000. (Actually, any divisor between 299,906 and 300,349 would work.)

	Population	*"Raw" seats*	*Rounded seats*
Arkansas	1,949,387	6.498	6 (rounded down)
Michigan	5,256,106	17.520	18 (rounded up)
Total	7,205,493	24.018	24

On February 17, 1941, the House started debating the issue. As stipulated by a resolution that the House had passed a year previously, the clerk of the House had presented the proposed apportionments among the states, based on both the Cornell and the Harvard method, on January 8. Unless the House took some action on the matter within sixty days, that is, by March 9, the apportionment would automatically be effected according to the old W-W (Cornell) system of major fractions. Thus it was high time for the Democrats to become active; otherwise, Arkansas could wave its seventh seat good-bye at least for the next ten years.

The discussion was kicked off by J. Bayard Clark from North Carolina. He pointed out that Michigan would gain a seat to the detriment of Arkansas if the method of major fractions were employed—even though the population of Arkansas had increased faster than that of Michigan. "The House would not want to let some mathematical formula result in an inequity or an injustice of this sort," he exclaimed, capitalizing on the congressmen's subliminal distrust of mathematics. (Little did it matter that his statement was quite untrue anyway: between 1930 and 1940, the population of Arkansas increased by 5.1 percent while the population of Michigan grew by 8.5 percent.) Joseph W. Martin from Massachusetts interjected: "We are trying to upset what we agreed upon last year. . . . Why should we change the understanding just because of a particular advantage to any one State?" Clark would have none of that. "The House ought not to permit any mere mathematical formula to defeat equity," he thundered from the floor, again driving home his point about the alleged vagaries of mathematics.

Out of breath by now, Clark yielded ten minutes of his speaking time to Congressman Ed Gossett from Texas. Gossett put his colleagues at ease. "We should not go into the intricate mathematical and geometrical formulas necessary to understand these various methods," he suggested, alleviating their innate fear of mathematics. Instead, he proposed to take the National Academy of Sciences's word for what the best method is. "Scientific opinion has concluded . . . that equal proportions tend to equalize the size of congressional districts." The Huntington-Hill method thus vindicated, Arkansas would keep its seventh seat.

Michigan would not take this lying down. Making the case for his state, Congressman Earl C. Michener started out in a conciliatory tone. "None of the methods is perfect," he remarked soothingly, "and it is the rule

rather than the exception that experts do not agree." Conceding that "everybody is sincere and thoroughly convinced that his philosophy should be adopted by the Congress in the name of justice and equity," he went on to admit that there is no absolute answer. In this case, rather than making unwarranted changes, it was preferable to keep the method that had been decided upon previously. "No one knew at that time where the shoe was going to pinch." And then he went for the jugular. "The professors, statisticians, and mathematicians presented their arguments and after most careful consideration by the Census Committee and the Congress itself, the method of major fractions was adopted as preferable under all circumstances," he exclaimed, conveniently ignoring the National Academy of Sciences' contrary conclusions.

Then it was Arkansas' turn again. "There is nothing sacrosanct about the various methods used," the state's representative David Terry conceded. The House must simply seek a method of apportionment that would "settle this vexing decennial problem in a way that would be most equitable and fair to all the States of the Union, large, medium, and small." Then he got to the point. No longer content with hiding behind niceties, Terry cut to the chase. "The Republican side of the House is making a desperate effort to control this body," he exclaimed to the applause of his fellow Democrats. Ezekiel Gathings from Arkansas feigned surprise at the surge in activity of the lawmakers from Michigan. Why this protest, all of a sudden? After all, when the Committee on the Census debated the issue, no representative from Michigan had ever bothered to attend in order to argue his state's case. This was too much for Jesse P. Wolcott. He and his colleagues from Michigan had been under the impression that everything had already been settled, that is why they had not shown up. But Gathings did not let this go by. Everybody had known for weeks that the matter was being discussed. Apparently the gentlemen from Michigan did not really want the seventeen congressional districts in their state disturbed, he concluded. "Oh just a minute," an enraged Wolcott shot back. "What does the gentleman think we are fighting for? We are not sitting here with our mouths open just for the fun of it. . . . Michigan is entitled to the seat and we are putting up a fight for it."

The debate became even more incensed. Fred C. Gilchrist from Iowa introduced a totally new aspect into the discussion. He did not care which method would be used, because Iowa was going to lose a seat whatever method was going to be used. He had a different cat to skin: non-naturalized immigrants, at that time referred to as aliens. Using a chart prepared by the Immigration and Naturalization Service, he pointed out that of the nearly 5 million noncitizens living in the United States, one-quarter lived in New York State and another 11 percent in California. Thus four extra congressmen were allocated to New York and two to California, on the basis of aliens living in these states. "No reason can be given why any man who is born in a different country and who does not think enough of America to become a citizen . . . should be counted in determining the apportionment of representation." Worse still, Gilchrist reminded his listeners, "some of them hid behind foreign flags and refused to fight for America, but stayed at home and received $10 or $12 or $15 a day as wages while our own boys gave up their jobs and got from $1 to $1.10 per day, and many thousands of them never came home at all, but were killed in the shambles of European battlefields." Then he turned to his real gripe. "A very large majority of these aliens are located in the cities, and this fact tends to take away from the rural sections and farm sections a fair and equitable share in the control of legislation." Thus, the rural states were doubly punished by the aliens, Gilchrist lamented. Not only did they take the jobs of American boys, but also by settling in towns and cities they prevented the rural states from being correctly represented in Congress.

As the discussion progressed, others joined the fray. Leland M. Ford from California had his say and so did August H. Andresen from Minnesota; John R. Kinzer from Pennsylvania made his point and A. Leonard Allen from Louisiana made his. Carl T. Curtis from Nebraska gave his opinion and so did many others. And so it went, on and on. Of course, the whole exercise was ultimately futile. No argument could justify one method over the other since the question boiled down to whether one wanted Arkansas or Michigan to get the additional seat. Appeals to fairness and equity carried no weight. Neither did the invocation of mathematical authorities or institutions, be it the NAS, the Brookings Institution, or the Census Bureau, since for every argument a counterargument could be found that would justify—always for the most lofty reasons—whichever method one wished.

While more or less polite banter took place in the House, tempers were rising among the academics again. This time, much of their learned debate was vented on the pages of *Sociometry*, a journal founded in 1937

and devoted to matters of social psychology or, if you will, psychological sociology. (*Sociometry* appeared until 1977 when it changed its name to *Social Psychology*. Today it is called *Social Psychology Quarterly*.) Huntington started the debate with an article titled "The Role of Mathematics in Congressional Apportionment." Prefaced by an editor's note that "Huntington's suggested method of apportionment . . . is before Congress as we go to press," the paper purports to give a mathematical justification to the Harvard method. A mathematical theorem is either true or false, Huntington asserts, and then produces a "theorem of equal proportions." Actually this seems to be the first time that anybody ever spoke of such a theorem, but Huntington claims that "the truth of the theorem is vouched for by the unanimous Report of the Census Advisory Committee . . . and the unanimous report of a committee of the National Academy of Sciences." Such weighty support certainly should sway Congress, but further inspection reveals that Huntington was being disingenuous by appealing to the readers' respect for mathematical theorems and their proofs. (Note how Huntington tried to curry favor with the readers of *Sociometry* by appealing to their respect for mathematics, while Congressman J. Bayard Clark had tried to convince his colleagues by appealing to their disrespect for mathematics.)

What Huntington did was to posit his favorite question—does the reallocation of a seat from one state to another minimize the percentage inequality of the congressional districts in them?—as an appropriate test of a good apportionment. Then he stated, as a theorem, that the Harvard method satisfies this test. This is quite sneaky; since everything depends on the assumption, the argument is circular. If one accepts the test, the Harvard method is acceptable. If one does not, everything is open. Hence, the "theorem" says nothing about which method is the better one.

To hide the obvious shortcoming in his argument, Huntington takes refuge in demagoguery. He deplores that some "professional politicians . . . largely influenced by Professor Willcox of Cornell, sharply resent the intrusion of mathematical theories in a field which they regard as purely political, not at all mathematical. Professor Willcox flatly rejects (or, more accurately, deliberately ignores) the mathematical theorems cited above." He further accuses Willcox, who had maintained that the Harvard method was difficult to understand while the theory underlying the Cornell method is persuasive to the nonmathematical mind, of insulting Congress. "This is perhaps the first time in history that advocates of any measure have openly accused the Congress of the United States of being unable to multiply and divide," he proclaims.

Willcox would not let that pass. "So long as [Huntington's articles] were safely immersed in the catacombs of Public Documents I ignored them. But now that [they] are accessible to uninformed persons whose opinions I value . . . the antediluvian octogenarian must turn away from more congenial tasks to answer him," the by then eighty-year-old professor wrote in a reply in the same journal.

Choosing the proper method of apportionment is a political one, Willcox maintained, since the choice is made by a political body for political motives. "The force behind the bill is not a tardy conversion of Congress to the method of equal proportions but a discovery by the leaders of the majority that [a switch of method] will transfer a seat from Michigan to Arkansas." Hence the problem at hand was not "a choice between two methods. In reality it turns out to be a choice between two parties." Allowing a conversion to the Harvard method now, Willcox fulminated, would open a Pandora's box of future trouble, with the party in power switching apportionment method to suit its needs.

The basic problem is "to reach a result that is as near as may be to an exact apportionment," Willcox remarks and then correctly points out that this begs the question "how is 'nearness' to an exact apportionment to be measured?" As the NAS report had pointed out twelve years earlier, much depends on whether "nearness" is defined in absolute terms or as a percentage. The proponent of the Cornell school then stoops to some demagoguery of his own, albeit in a subtler and more poetic fashion than the advocate for the Harvard school. Huntington's articles "reveal the writer as a modern Don Quixote roaming in an unreal world where he tilts against a Congressional windmill the structure of which he fails to understand and the forces governing which he has been unable to influence." Concluding his reply, Willcox regrets that Huntington did not let sleeping dogs lie. "In reviving the struggle of 1929 he has now innocently but nonetheless efficiently dragged the problem of apportionment back into the quagmire of politics from which his academic opponents had long struggled to extricate it and has left it for his successors and mine in a shape far more complicated and menacing than it would have been had he never touched it."

To make a long story short, Willcox was not convincing enough and President Roosevelt did not abandon the Democratic Party. On November 15, 1941, without much consideration as to the merit of the method, he signed into law "An Act to Provide for Apportioning Representatives in Congress among the Several States by the Equal Proportions Method," thus decreeing that Huntington-Hill would henceforth be used to apportion Congress. The Harvard method had carried the day and the gentlemen from Arkansas could breathe a sigh of relief. They got their seventh seat. And the Democratic majority in the House increased its majority by one.

But misgivings about the crude manner in which H-H had been adopted abounded in the House, and in 1948 the question was put to a scientific test once again. The confused Congress turned to the National Academy of Sciences for help, asking it, once again, to investigate which apportionment method the Congress should use. The NAS formed another committee. This time around, the committee was purely a Princeton affair and the committee was, if anything, even more blue ribbon than the first one. John von Neumann from the Institute of Advanced Study (IAS) was its chairman. An émigré from Hungary, he is today considered one of the most important mathematicians of the twentieth century. (The IAS provided him—as well as his colleagues Albert Einstein, Kurt Gödel, and other refugees from Nazi-Germany—a quiet place to further human knowledge, undisturbed by banal duties such as teaching students or supervising doctoral candidates.) Another member of the committee was Marston Morse, von Neumann's colleague at the Institute for Advanced Study at Princeton. The committee's third member was Luther Eisenhart from Princeton University who had already been a member of the NAS's first committee two decades earlier. He was chosen to be the chairman.

The three mathematicians went to work. They had been asked by the president of the NAS—who, in turn, had been asked by the Speaker of the House—to report on any new developments in the mathematical aspects of the apportionment problem since the previous report of 1929. Actually, there was not much new to report. The only paper written since then, and worthy of consideration by the committee, was one prepared by Walter E. Willcox for a meeting of the International Statistical Institute. In it he proposed a new apportionment technique: the modern House method. But even that was no innovation. Upon inspection, it turned out that the modern House method was no more than a method that had already been considered in the NAS's previous report, albeit under the name method of smallest divisors. So the committee could limit itself to covering substantially the same ground as its predecessor.

After again lamenting the fact that "fractional representation has not been so far introduced," Morse, von Neumann, and Eisenhart subjected the various methods of apportionment once more to intense scrutiny. They compared the H-H method of equal proportions that had already been considered superior in the previous NAS report with the competing methods. This they did by subjecting the methods to the test whether the differences between the numbers of citizens required for a seat decrease if any of the other methods are used. One member of the committee—the report remains silent on his identity—was designated to work out the comparisons algebraically. He did, and it comes as no surprise that the mathematical paperwork confirmed the conclusion of the report that had been submitted to Congress nineteen years previously. "In the above four comparisons EP [the H-H method of equal proportions] scored decisively in each case," the report concluded. This was not altogether mind blowing since the new committee was not about to disavow its forerunner, especially since one-third of the members—the committee's chairman Eisenhart—provided continuity.

But there remained a loose end. After all, Willcox had put forth the modern House method a.k.a. the method of smallest divisors and "this report would not be a reply to the request of the National Academy if it did not analyze the recent paper of Professor Willcox." Indeed, apart from repackaging and changing the name, Willcox believed he had found a further justification for his modern House method. He suggested comparing the different apportionment methods not just between pairs of states but also between *all* states. What is the difference, he asked, between the largest population and the smallest population required for a seat? Willcox called this new variable, computed over all forty-eight states, the "range." The apportionment method that results in the smallest range would be the preferred one.

Inspecting the ranges for the 1940 census according to the different apportionment schemes, Willcox found that it was smallest under the modern House method. It should therefore be the preferred method, he

argued. But his analysis had a snag. True to his suggestion that the comparisons encompass all states, Willcox had included Nevada in his calculations. This state, with a population of only 110,247, was too small to receive representation in the House by force of its size; it had come by its seat only because of the law that each state was to have at least one representative. Hence, the number of citizens required for Nevada's single seat (110,247) was exceptionally low, especially when compared to South Carolina, which received six seats for a population of 1,899,804 or 316,634 citizens for a seat.

Morse, von Neumann, and Eisenhart argued that Nevada should be excluded from any comparison since the allocation of this state's seat was not based on any of the apportionment methods. And—here comes the knockout punch—once Nevada is excluded from the computations, the range is slightly lower under the H-H method. It would have been the other way around after the 1930 census. Thus, including or excluding this or that state could completely change the result. Exasperated, the committee members threw their hands collectively up into the air. "To use the language of gun-fire," they wrote in their report, "the test of minimum range makes the evaluation of a method dependent upon a few eccentric shots, and in this sense is a random determination of value." Unwilling to succumb to such randomness, the committee stuck to its preference of the Huntington-Hill method.

Such was the state of affairs toward the middle of the twentieth century. Whatever its advantages or disadvantages, Huntington-Hill a.k.a the Harvard method has been the method of choice for Congress ever since. Of all methods considered, it represented the middle way and therefore was the most acceptable. Nevertheless, the whole affair left a bad taste because on the theoretical level the question had not been settled at all. One could not get rid of the arbitrariness to which the mathematical imprecision of the rounding process gives rise. It was only partly tongue-in-cheek that some wags suggested explicitly introducing randomness into the apportionment method. They proposed a roulette-based method to distribute fractional seats. The width of each cell on the roulette wheel would correspond to the size of each state's fractional seat and—*les jeux sont faits*—the state into whose cell the marble falls, gets the leftover seat. This is less absurd than it sounds because, on average, the method is absolutely unbiased; in the long run, every state gets its fair share of left-over seats. But since apportionments take place only once every ten years, it would take a very long run indeed for the odds to average out. And as the eminent English economist John Maynard Keynes once remarked, "in the long run, we are all dead."

BIOGRAPHICAL APPENDIX

Walter F. Willcox

Many social scientists consider Willcox the "father of American demography." He started the teaching of statistics at Cornell. Under the heading "applied ethics" he offered "an elementary course in statistical methods with special treatment of vital and moral statistics" in the department of philosophy in 1892. It was one of the earliest courses in social statistics in the United States. Willcox's main achievement was the application of the still young science of statistics to the area of demography. By today's standards he was not a sophisticated statistician. According to Frank Notestein, a former student who would later become director of the Office of Population Research at Princeton University, he barely knew what mean, median, and mode were and used only simple methods instead of high-powered techniques. Then again, statistics was a new subject at the time, so this was not very surprising. Most importantly, he had a very healthy respect for data and everybody agreed that he was a great teacher.

Many scholars today accuse him of having been one of the proponents of scientific racism. Apparently Willcox believed in the racial inferiority of Afro-Americans—he still called them Negroes, in the same manner that he spoke of "girls" when we would today respectfully say "women"—and tried to explain the plight of black farmers with this alleged inferiority. Willcox was active well into his nineties and died at age 103. At various times, he served as president of the American Economic Association, president of the American Statistical Association, and president of the American Sociological Association. He was a prominent member of the highly selective and narrowly restricted International Statistical Institute, attending most of its meetings, be they in Tokyo, Warsaw, or Rio. He was even present at a meeting in Prague in 1938 that had to be cut short when Hitler invaded Czechoslovakia.

Joseph A. Hill

One does not expect the life of a statistician to be as adventurous as that of, say, an archaeologist like Indiana Jones, but Hill's vita seems to have

been gray even by the modest standard that one would apply to a member of his profession. Hill studied at Harvard and then became a statistician for the government. "The nature of his work was such as to afford little opportunity for publicity," an obituary in the *Journal of the American Statistical Association* read and then went on to describe the "arduous and relatively thankless job of production and processing of statistics" that Hill performed throughout his life. "Relative anonymity was inherent in the nature of his job," the biographer continues, and then can do no more than to extol Hill's "necessary care and extravagant meticulousness." Wistfully, he adds that "this sort of service produces no renown." At a loss to say anything extraordinarily positive, he presents Hill's compilation of a "well-constructed statistical volume with clear and precise headings and titles" as a highlight of his "patient, imaginative, but unspectacular work."

His output, though wide-ranging—his statistical tractates cover crime, fecundity, insanity, migration patterns, child labor, marriage, and divorce—was quite unspectacular. "In his work relatively little of higher mathematical and statistical technique was required," the biographer recounts. Since one does not end an obituary without mentioning so much as a single outstanding aptitude, Hill is described as a master at "the art of extracting the last drop of legitimate meaning out of a body of statistics." But, commendably, he never let his quest for more information get the better of him. Hill squeezed information from numbers while strictly adhering to an "unflinching recognition of what the figures do not and cannot prove." Especially praiseworthy in the eyes of the biographer was the self-discipline that kept Hill from "stretching the material to prove some pet theory." Not only that, but his publications were always provided with an "honest and precise statement of the margin of error." Truly a virtuous man.

Then there was, of course, the census. In fact, for a whole generation of budding statisticians Hill was identified with the work of the Census Bureau. He gave lectures on the census to the American Historical Association, the American Sociological Society, the American Statistical Association, the Federal Council of Churches, and he wrote about it for *Youth's Companion*, the *National Republic*, the *New York Times*, and the *Monthly Labor Review*. In addition, he authored numerous unpublished internal documents about this or that aspect of the census. As chairman of the Quota Board, he was instrumental in allotting immigration quotas to countries in the same proportion that the American people traced their origins to those countries.

Although far from the limelight, the behind-the-scenes work of this dedicated professional must not be belittled. His whole life was devoted to improving the quality of the material on which public policy is based. It was in this spirit of service to the public that he thought deeply about apportionment methods. His ideas on that subject were published in the *Congressional Digest*.

Edward V. Huntington

Born in 1874, Huntington was educated at Harvard, became an instructor at Williams College, and went to Europe to obtain his doctorate in mathematics in Strasbourg, then in Germany. Upon his return to the United States he began a career at Harvard, starting as an instructor and gradually making his way up the academic ladder to assistant professor, associate professor, and full professor. In contrast to most colleagues at math departments throughout the world, Huntington especially enjoyed teaching mathematics to engineering students, which earned him the additional title of professor of mechanics. World War I found him in Washington where he dealt with statistical problems for the military. Huntington's primary research interests were the foundations of mathematics, and he did important work on axiomatic systems in algebra, geometry, and number theory.

MEMBERS OF THE NAS COMMITTEES

Gilbert A. Bliss

Bliss, professor of mathematics at the University of Chicago, was born in 1876 into a wealthy family. His father was the president of the Chicago Edison Company, which supplied most of Chicago's electricity. But the family fell on hard times during the Depression and the young man had to earn his way through university as a professional mandolin player. After obtaining his PhD at the University of Chicago, Bliss went to the then stronghold of modern mathematics, the University of Göttingen in Germany, for a year. There he met the towering giants, Felix Klein and David Hilbert. World War I found him designing firing tables for the artillery. Bliss was best known for his work on the calculus of variations.

Ernest W. Brown

Brown was born in England to a farmer and lumber merchant. Educated at Cambridge, he moved to the United States when he was twenty-five years old. Brown first taught mathematics at Haverford College in Pennsylvania, and was appointed professor at Yale in 1907. His primary interest was astronomy, and he published important work on lunar theory and the movement of the moon. For example, he correctly ascribed hitherto unexplained wobbles in the moon's orbit to irregular changes in the Earth's period of rotation.

Luther P. Eisenhart

Son of a sometime-dentist, Eisenhart had been a precocious child. At Gettysburg College he excelled in his studies as well as in baseball. After obtaining his doctorate in mathematics from Johns Hopkins University, Eisenhart spent his entire academic career at Princeton University, beginning as an instructor at age twenty-four in 1900 and continuing through the ranks until his retirement, as Head of Mathematics, forty-five years later. His chosen fields of specialization were differential geometry and, after his retirement, Einstein's theory of general relativity. Apart from his research and teaching duties, Eisenhart was also very active in administrative matters. At different times he served as president of the American Mathematical Society, officer of the American Philosophical Society, president of the American Association of Colleges, and editor of the *Annals of Mathematics* and the *Transactions of the American Mathematical Society*. He was awarded seven honorary degrees for services rendered to mathematics and to higher education in general, and King Leopold III of Belgium made him an Officer of the Order of the Crown.

Raymond Pearl

Pearl, from Johns Hopkins University in Baltimore, was among the first scientists to apply statistical methods and procedures to biological problems. The author of "Breeding Better Men," he at first supported eugenics and was a dues-paying member of the American Eugenics Society. Later he turned against this pseudoscience and his criticism of eugenics, published in the influential magazine *The American Mercury*, made it into the national headlines. It also earned him the enmity of many biologists. But progressive views on a pseudoscientific aberration did not prevent him from holding racist and anti-Semitic views. He was proud about how Johns Hopkins University dealt with the problem of the Jews. "For a number of years past, very quietly and skillfully, means have been taken and are being planned for the future to keep down our Jewish percentage," he wrote to a friend at Harvard. The secret was not to use crude quotas, as Harvard tried, but to practice discrimination. After all, "whose world is this to be, ours, or the Jews?"

In 1927, Pearl was the center of a high-profile case of academic infighting. Offered the position of head of a research institute at Harvard University, Pearl immediately resigned his professorship at Johns Hopkins in order to accept the prestigious post. The step was somewhat premature because a faculty member at the same institute, who had felt slighted by Pearl's earlier attack on eugenics, appealed the job offer to Harvard's Board of Overseers. In a rare step, the offer was rescinded and Pearl had to ask Johns Hopkins meekly to reinstate him—which the university did.

William Fogg Osgood

Osgood, son of a medical doctor, was born in 1864, and as a young man first wanted to study the classics. But after two years at Harvard he was persuaded by his teachers to switch to mathematics. He finished his undergraduate studies brilliantly, coming in second among 286 students. After an additional year at Harvard to earn his master's degree, Osgood obtained a three-year fellowship that enabled him to study in Germany. He learned to speak German and spent his time first in Göttingen, where, like Bliss, he was taken under the wings of the towering figure of Felix Klein, and later at the University of Erlangen, where he received his doctorate. In Göttingen he married Theresa Anna Amalie Elise Ruprecht, the daughter of the owners of a local publishing house. The couple would have three children before their marriage broke up and ended in divorce.

At Harvard, Osgood became instructor, then assistant professor, and then full professor. During his three years abroad he had absorbed much European mathematics, and he was instrumental in bringing methods and techniques to the United States. Osgood had taken a liking to all things German, supported Germany during the First World War, and even took to emulating the mannerisms of a German professor. From 1905 to 1906, he served as president of the American Mathematical Society. Somewhat late in life, Osgood decided to tie the knot again. The already sixty-eight-year-old mathematician married a forty-year-old woman, Celeste Phelpes Morse, the divorced wife of his eminent Harvard colleague, the mathematician Marston Morse, about whom I will have more to say below. Upon learning of this liaison, Morse got a shock, and in the ensuing scandal Osgood retired from Harvard. For two years he taught in Beijing before moving back to Massachusetts. Today he is best remembered for his work on differential equations and the calculus of variations. He died in 1943.

John von Neumann

Jancsi, as he was called then, was born in 1903 and became a child prodigy. At his father's request, a well-to-do banker in Budapest who wanted a practical profession for his son, Jancsi enrolled at the Swiss Federal Institute of Technology in Zurich to study chemical engineering. Simultaneously, and secretly, he studied mathematics at the University of Budapest—in spite of a quota against Jews—albeit without attending any classes. He put in a presence only for exams, which he passed brilliantly. In 1926 he received a diploma in chemical engineering from the Federal Institute of Technology (ETH) in Zurich and a doctorate from the University of Budapest. There followed a study year with David Hilbert at the world-

famous math department of the University in Göttingen. Von Neumann was already considered a genius, and everyone who met him recognized his superior intellect. "By his mid-twenties, von Neumann's fame had spread worldwide in the mathematical community. At academic conferences, he would find himself pointed out as a young genius," the biographer William Poundstone wrote. During the years 1930 to 1933 he held positions both in Germany and at Princeton University. After IAS was founded, he became one of its six original professors of mathematics. In 1937 von Neumann, now Johnnie, became an American citizen. He died of cancer at the age of fifty-four.

Von Neumann is considered the father of modern computers. Actually he fathered many disciplines and subdisciplines. His groundbreaking contributions to mathematics, quantum theory, economics, decision theory, computer science, neurology, and other fields are too vast to be listed here. We just mention two areas. As a consultant to the Manhattan Project he was instrumental in the development of the atom bomb in Los Alamos. He worked out the theory of "implosion" that proved to be the key to the success of Little Boy, dropped over Hiroshima, and Fat Man, dropped over Nagasaki. The idea was that explosives, shaped in a certain way, should surround a subcritical mass of plutonium. Just after detonation, the shock wave would turn inward, crushing the plutonium into a supercritical mass. It is for his association with the Manhattan Project (and for the fact that the cancer-stricken professor was confined to a wheelchair during the last months of his life) that Stanley Kubrick reportedly had von Neumann in mind when he created the character Dr. Strangelove in his 1963 film. Among the many honors von Neumann received are two presidential awards: the Medal for Merit in 1947 and the Medal for Freedom in 1956.

Marston Morse

Morse served as a soldier in World War I in France after earning his PhD in mathematics at Harvard and was awarded a Croix de Guerre for his outstanding service in the ambulance corps. After the war, he taught at Cornell, Brown, and Harvard, before joining the Institute for Advanced Study at Princeton. He is most famous for developing "Morse theory," an area in topology, that is, the study of shapes. Morse was awarded twenty honorary degrees and named a Chevalier in France's Légion d'Honneur. A member of the second NAS committee, Morse provided some continuity from the first, one might say, since he was the first husband of the second wife of William Osgood who had served on, and then resigned from, the first NAS committee.

MATHEMATICAL APPENDIX

Rounding at the Geometric Mean

In this chapter the question was asked, why rounding at the geometric mean should be related to minimizing relative differences in the number of people needed for a congressional seat. Here we present a proof of the astonishing statement that both procedures give identical results.

Let $p_1, p_2, \ldots p_n$ be the populations of the states, let d be the divisor, and let $a_1, a_2, \ldots a_n$ be the apportionment that results from rounding the ratios p_i/d at the geometric mean.

Then for every state i we have

$$\sqrt{(a_i[a_i - 1])} \leq p_i/d \leq \sqrt{(a_i[a_i + 1])}.$$

This is equivalent to

$$a_i(a_i - 1)/p_i^2 \leq 1/d^2 \leq a_i(a_i + 1)/p_i^2.$$

Since this holds for every i, it follows that for every i and j,

$$a_i(a_i - 1)/p_i^2 \leq a_j(a_j + 1)/p_j^2.$$

Now assume, by way of contradiction, that the apportionment $a_1, a_2, \ldots a_n$ does not minimize relative differences in the number of people needed for a congressional seat. In that case there exists a pair of states i, j such that state i is better represented than state j, and a transfer of one seat from i to j would lessen the relative inequality between them.

This means that

$$([a_j + 1]/p_j)/([a_i - 1]/p_i) < (a_i/p_i)/(a_j/p_j).$$

But this implies that

$$a_i(a_i - 1)/p_i^2 > a_j(a_j + 1)/p_j^2,$$

which contradicts the earlier inequality. Hence, the apportionment that results from rounding the ratios p_i/d at the geometric mean must minimize relative differences in the number of people needed for a congressional seat.

After: Peyton H. Young, *Equity in Theory and Practice*, Princeton University Press, 1995.

I would like to thank Daniel Barbiero, Manager of Archives and Records of the National Academy of Sciences, for making available to me the content of the documents quoted here. The documents are contained in the folder: NAS-NRC Archives, Central File: ADM: ORG: NAS: Committee on Mathematical Aspects of Reapportionment: 1928–29.

CHAPTER ELEVEN

THE PESSIMISTS

We now leave the matter of apportionment for a while and return to the troublesome problem of electing a leader. Remember Condorcet and his paradox? And how Lewis Carroll wrestled with it? Well the problem did not go away. Nor did it mellow with age. If anything it became more vexing. Enter Kenneth Arrow, Nobel Prize winner of economics in 1972 and one of the most important economists of the twentieth century.

An outstanding graduate student at Columbia University in the late 1940s, Arrow was thinking about his doctoral thesis. It was an exciting time for budding economists, observing and shaping subjects in the making. Arrow was caught up in these "heady days of emerging game theory and mathematical programming," as he would put it later. In the meantime he neglected his PhD thesis. He had high aspirations and his teachers and colleagues also expected a lot from him. But it was as if he was spellbound. No topic that he considered seemed sufficiently challenging. Even though his coursework had been completed at Columbia back in 1942, he was still short a thesis six years later. While everybody knew he was brilliant, the years passed without his putting pen to paper.

There was hope, however. Just a few years earlier, at the Institute for Advanced Study in Princeton, John von Neumann, together with Oskar Morgenstern, a refugee from the Nazis in Austria, had finished a thick primer that would become one of the most influential scientific works of the twentieth century. *Theory of Games and Economic Behavior*, published in 1944, was to have a profound influence on the further development of economics and political science. Based on only a handful of axioms, the theory contained in their book, henceforth called "game theory," ushered in the age of mathematical economics. What Euclid did for geometry, von Neumann and Morgenstern did for economic behavior.

One of the fundamental assumptions of their new theory was that each participant in a game has a so-called utility function. As we shall see presently, utility functions are a fundamental concept not only for the understanding of economic behavior but also of the Condorcet Paradox.