CHAPTER FIVE
THE OFFICER

The eighteenth century was a period of enlightenment throughout the Old and New World. France, the United States, and Poland granted themselves constitutions. Nations were in upheaval as their citizens started demanding equal justice for all, showing concern for human rights, and calling for a regulation of the social order. At the same time, demands for quality government arose and the question of how officials were to be elected to high positions became important again. In this atmosphere two eminent French thinkers appeared on the scene. One was a military officer with numerous distinctions in land and sea battles. His name was Chevalier Jean-Charles de Borda. The other was the nobleman Marquis de Condorcet. The two men, outstanding scientists in Paris during the time of the French Revolution, did something amazing: they reinvented the election methods that Llull and Cusanus had proposed a few hundred years earlier. Actually, they did more than that: they provided the appropriate mathematical underpinnings. At odds with each other on many subjects, they also engaged in a lively debate on the theory of voting and elections.

Born in 1733, Jean-Charles de Borda was the tenth of sixteen children. His parents, both of whose families belonged to the French nobility, were Jean-Antoine de Borda, Seigneur de Labatut, and Marie-Thérèse de la Croix. The boy exhibited great enthusiasm for mathematics and science at an early age and a cousin of his, Jacques-François de Borda, who was in touch with the leading mathematicians of his time, was to point Jean-Charles in the direction of his future career. Jacques-François tutored the boy until, at age seven, he was ready to enter the school of the Barnabite Fathers, whose curriculum was for the most part limited to the teaching of Greek and Latin. Four years later, it was Jacques-François again who convinced Jean-Charles's father to send his son to the Jesuit college of La Flèche, where the offspring of noblemen were educated. It was there, finally, Jean-Charles received a solid grounding in mathematics and the sciences. His achievements were far above average and upon graduation the

Jesuit teachers encouraged the fifteen-year-old boy to enter their order. But Jean-Charles had no interest in religion. He wanted to continue the family tradition and enter the military. The French army provided career opportunities not only for brave fighters but also for intellectuals. Jean-Charles's father allowed his son to follow his own wishes even though he had wanted him to become a magistrate. Thus began Borda's career as an army mathematician.

When Borda was twenty, his first mathematical paper, a piece on geometry, came to the attention of Jean le Rond d'Alembert, the renowned scientist in Paris. Three years later, while serving in the cavalry studying the flight path of artillery shells, Borda presented a theory of projectiles to the Académie des Sciences, whose members elected him to its ranks on the basis of this work.

But his calling still was the army, and the young officer climbed the rungs of the military's hierarchical ladder. As aide-de-camp to the Maréchal de Maillebois, Borda participated in the battle of Hastenbeck in July 1757, where the French army defeated the Duke of Cumberland. But by then he had had enough of horses and decided to exchange the cavalry for the sea. Completing the navy's two-year course in one year he devoted himself to naval construction and to the study of fluids. The navy was suspicious of this "terrestrian" who sought to gain entry into its close-knit officers' corps. Borda managed to prove himself through his academic achievements, however. Taking issue with Newton's theory of fluids for example, he proved that a spherical body offers only half the resistance to airflow than a cylindrical object of the same diameter, and that the resistance increases with the square of the velocity. By advocating spherical shapes, Borda became an early pioneer of submarine and airplane construction. Bodies of this shape would dominate travel under water and in the air—at least until it was discovered that for supersonic flight the most efficient aircraft body has a pointed shape.

The young officer also dealt with more prosaic gadgets like pumps and waterwheels. Throughout his life, Borda participated in many voyages, battles, adventures, and scientific endeavors. For now, we limit the narrative of his achievements to his preoccupation with elections, postponing other parts of his colorful life to the chapter's additional reading section.

The French Revolution, which cost so many of his contemporaries among the nobility, the officer class, and the scientific establishment their lives, left Borda unscathed. He did not participate in any political activity, sitting out the eleven months of the great Terror (September 1793 to July 1794) in his family estate in Dax, a town in the southwest of France. After a long illness, he died in 1799. Many internationally known scientists attended the funeral below Montmartre. His scientific achievements include important advances in experimental physics and engineering, in geodesy, cartography, and other areas.

Staying aloof of political matters during the revolution did not mean that Borda was uninterested in the political process. In fact, it was a sign of the times that one of the areas he dealt with was the theory of voting. In 1770 he had already delivered a lecture about his ideas on a fair election method before the Academy of Sciences. Too busy with military matters at the time, he neglected to publish anything, however. It was only eleven years later, in 1781, that Borda wrote an article titled "Mémoire sur les élections au scrutin" (Essay on ballot elections), which was published in the *Histoire de l'académie royale des sciences* three years later. A preface to Borda's paper, written by an unnamed discussant, lauded it profusely. The introductory essay ended with the sentence that Monsieur de Borda's observations about the inconveniences of election methods that had been nearly universally adopted are very interesting and absolutely new. (The discussant is nowadays believed to be the Marquis de Condorcet, hero of our next chapter.)

In the paper Borda analyzed the well-established method of electing a candidate to a post by majority decision. It seemed obvious to most that this was the correct and fair manner to elect officials. But was it really? Should majority decisions be accepted without question? Borda took issue with the basic, universally accepted axiom that underlies ballot elections, namely that the majority of votes expresses the wish of the electorate.

The axiom seemed reasonable enough and nobody ever made any objection to it. Everybody was convinced that the candidate who obtains the most votes is necessarily preferred to all competitors. It came as a great surprise, therefore, when Borda showed that very often this is not the case. In fact, he maintained that the method of majority elections is unquestionably correct only if no more than two contenders run for a position. If three or more people present their candidacies, Borda pointed out, majority decisions may lead to erroneous results. To make his point, he

presented an example in which a paradoxical situation arises. It is not at all contrived and can easily appear in everyday elections.

I will illustrate Borda's example with the election for class president at a high school. The class comprises twenty-four students. Peter, Paul, and Mary vie for the post; the twenty-one remaining students have to decide among them. Of course they use the age-old method of majority voting. Everyone puts the name of the preferred candidate on a piece of paper and drops the ballot into an urn. The count reveals that eight students voted for Peter, seven for Paul, and the remaining six for Mary. Peter, smiling broadly, thanks the voters for their confidence while Mary, disappointed at her poor showing, leaves the classroom in tears. But is the will of the twenty-one electors truly reflected in this result?

Let us poll the students more deeply about their preferences among all three candidates. The following becomes apparent. The eight students who put Peter first, would have put Mary second and Paul last. The seven who voted for Paul would also have put Mary second and relegated Peter to the end of the list. Finally, Mary's six supporters would have placed Peter behind Paul. The complete list of preferences can be summarized in the following table (where "preferred to" is indicated by " > "):

8 electors:	Peter > Mary > Paul
7 electors:	Paul > Mary > Peter
6 electors:	Mary > Paul > Peter

If we now scrutinize the preferences, we realize that in direct showdowns as advocated by Ramon Llull (see chapter 3), both Mary and Paul would have beaten Peter by thirteen votes against eight. (The seven electors in the second line of the table, and the six electors in the third line, place Peter behind both Paul and Mary.) So Peter, the undisputed winner of the majority vote, would already be out of the race. A comparison of the voters' preferences between Mary and Paul would then reveal that fourteen classmates (those in the first line and those in the third line) would have voted for her, and only seven for Paul. Now the shoe is definitely on the other foot: Mary wins, and Peter surreptitiously wipes away a tear. The results are the exact reverse of the ones obtained by majority election.

The simple explanation for this paradoxical situation is that the support Peter receives from eight electors is more than offset by the utter rejection of his candidacy by thirteen others who place him dead last. The paradox had gone unnoticed for centuries because once an election was over, nobody ever bothered to compare the voters' preferences among the losers. There will be more to say about this paradox in the next chapter.

In one fell swoop Borda challenged an election method that had been used throughout the world for centuries. The example shows that different outcomes may occur as soon as voters become more farsighted and take into account the preferences beyond their first choice. Borda compared the situation to a sporting event in which three athletes vie for the title. After two competitors have worn themselves out in a first bout, both of them, by then tired and exhausted, may succumb to a weaker opponent.

But the navy officer did not simply criticize the age-old method, he also proposed a remedy. He called it "*Éléction par ordre de mérite*" (Election by ranking of merit). It would lead to a great debate between two outstanding French intellectuals of the eighteenth century.

In Borda's proposed voting method every elector jots down the names of the candidates in the order of merit he accords them. The ranking could be, for example, Peter on top, then Paul, then Mary. Borda proposed awarding one merit-unit—let's call it an *m-unit* for short—to each rank. Mary at the low end would get one, Paul two, and Peter three. If more candidates are present, the count would go higher. For eight candidates, the bottom-ranked candidate receives one m-unit, the top-ranked candidate eight.

This manner of awarding m-units rests on an assumption, however. Borda maintains that the degree of superiority the elector accords Peter over Paul is the same as the superiority of Paul over Mary. This assumption needs a justification and Borda provides it by employing some hand waving: since there is no reason to rank Paul (the middle candidate) closer to Peter than to Mary, the correct method would be to place him smack in the middle between them. So, given Borda's belief that the difference in merit between all ranks is identical, it is quite reasonable to award one additional m-unit to the next-higher rank. Of course, many people would dispute this assumption. The *intensities* with which electors prefer one candidate over another may differ.

Now on to the next stage. Peter in the above example was ranked first by eight, and last by thirteen electors. In Borda's manner of reckoning

Peter would therefore obtain 37 m-units ($[8 \times 3] + [13 \times 1]$). Paul would receive 41 ($[8 \times 1] + [7 \times 3] + [6 \times 2]$) and Mary a whopping 48 ($[8 \times 2] + [7 \times 2] + [6 \times 3]$). Now we understand why Mary should win. By the way, this manner of adding m-units also rests on an assumption: electors are considered to be equal. Then m-units awarded by different electors have the same value and can be summed. (Many people would dispute this assumption also. After all, my m-units may be different from yours.)

The astute reader may have recognized in Borda's count of m-units the method from the previous chapter, put forth by Cardinal Cusanus. The French navy officer was not aware of his predecessor. Indeed, the Cardinal's proposals for the election of popes and emperors were quite unknown during Borda's times and only rediscovered in the late twentieth century. But Borda would have provided an advance even if the earlier writings had been known to him. While Cusanus had implicitly assumed that one additional rank—be it from rank fifteen to fourteen, or from rank two to one—should always accord the candidate the same additional gain, Borda made the assumption explicit and gave it a justification. Granted, it was a hand-waving justification, but a justification it was nonetheless. The method suggested by the cardinal and by the navy officer, of choosing among candidates by assigning points, or m-units, according to their standing in the electors' rankings, is nowadays known as the Borda count.

Borda's and Cusanus's assumption that each additional rank is worth the same is crucial. Without it, several variations of the method can be thought of. The Eurovision song contest, mentioned in chapter 4, is a case in point. There, no m-units are awarded to the worst songs. Then one m-unit is given to the song ranked eleventh, and one additional m-unit is awarded for each rank up to the second-best song, which receives ten. Finally, the best song in a jury's opinion receives twelve m-units. Different variants of the method could be thought of, and they may result in different winners.

After presenting his method of voting "by order of merit," Borda went on to analyze under what circumstances the winner according to his scheme would coincide with the winner in a majority election. How many votes would a candidate need to receive in a conventional majority election so that he would also be guaranteed the top spot according to the rules of the Borda count? Let us say that Peter and Mary are ranked in first place by a and b electors, respectively. Borda investigates the worst-case scenario from Peter's viewpoint. Such a situation occurs if all of Peter's supporters put Mary second on their list, but Mary's supporters place Peter last.

a electors:	Peter > Mary > Paul
b electors:	Mary > Paul > Peter

In this case, Peter receives $3a$ m-units from his supporters and b m-units from Mary's fans, who placed him last. Mary receives $3b$ m-units from her supporters, and another $2a$ m-units from Peter's voters who placed her second. In order for Peter to get elected by the Borda count, the number of m-units he is awarded ($3a + b$) must be greater than Mary's m-units ($3b + 2a$). Note now that $a + b = n$, the number of voters. Simple arithmetic then shows that Peter must garner at least two-thirds of the votes in a conventional majority election in order to guarantee his win according to the Borda count.

More generally, if there are n candidates, the winning candidate must receive at least $1 - 1/n$ parts of the votes cast in a simple majority election to be certain that he would have won even by the Borda count. (I derive this simple result in the mathematical appendix to this chapter.) In the case of two candidates that means receiving at least half the votes, which is the same as saying that a simple majority suffices. This makes sense. But a line-up of, say, five candidates would require a candidate to receive four-fifths, or 80 percent, of the votes to make him the undisputed winner of both election methods. This may seem overly stringent and it is. Most often less support suffices because the worst-case scenario usually does not arise.

An interesting case appears when there are more candidates than there are electors. In order for a candidate to obtain the threshold of $1 - 1/n$ parts of the votes, there must be at least n electors. If there are less than n electors, unanimity among the electors is required. (If there are six candidates but only five electors, the winner would have to receive at least five-sixth of the votes. This means he needs to obtain all five votes.)

There are problems with the Borda count, some minor, some major. One of the minor ones is that draws may occur. Borda did not express himself on what should be done if two candidates receive the same number of m-units. It may have been obvious to him that a runoff election would decide between the two. What if three or more candidates receive

the same number of m-units? A second election by order of merit would be called for, and so on. And what about the case when an elector cannot rank two or more candidates because he is indifferent between them? Let us say there are five candidates and the elector ranked the first and second candidates, but is indifferent about the next three. Should they all receive three m-units, or one m-unit, or something in between?

A more serious problem is that, paradoxically, the winner of the Borda count may be nobody's favorite. It is easy to conjure up election results in which a candidate wins even though she is ranked no more than second best by all electors. For example:

11 electors:	Paul > Mary > John > Peter
10 electors:	Peter > Mary > John > Paul
9 electors:	John > Mary > Peter > Paul

Paul would get 63 m-units ($[11 \times 4] + [19 \times 1]$), Peter 69 ($[11 \times 1] + [10 \times 4] + [9 \times 2]$), John 78 ($[21 \times 2] + [9 \times 4]$) and Mary, who is neither liked nor hated by anybody, would get 90 and win (30×3). The ranking would be Mary > John > Peter > Paul. By the way, a simple majority election would have given the ranking Paul (11 votes) > Peter (10) > John (9) > Mary (zero), the exact opposite of the Borda count.

Another paradoxical situation may arise through the sudden appearance of a clearly inferior candidate. Even though he would be ranked low on every voter's list, his addition to the roster may have a non-negligible influence on the election's outcome: the Borda counts of the front-ranked candidates could be changed, pushing a different winner forward. Let us assume that 51 electors prefer Ginger to Fred, and 49 prefer Fred to Ginger:

51 electors:	Ginger > Fred
49 electors:	Fred > Ginger

The Borda count declares Ginger the winner with 151 m-units ($[51 \times 2] + [49 \times 1]$), and Fred 149 ($[51 \times 1] + [49 \times 2]$). Now Bozo appears on the scene. Nobody really likes Bozo but his entry persuaded three of Fred's voters to rank Ginger even behind Bozo:

51 electors:	Ginger > Fred > Bozo
46 electors:	Fred > Ginger > Bozo
3 electors:	Fred > Bozo > Ginger

Now Ginger receives 248 m-units, Fred 249, and Bozo 102. Bozo's entry caused Fred to win.

Hence, by adding a dunce to the roster, the winner could be changed. The same may happen if a candidate drops out of the race or—may Heaven forbid—dies before the actual voting takes place. The most important challenge to the Borda count, however, is that it is open to manipulation through so-called strategic voting. This is the problem Pliny the Younger had been wrestling with (see chapter 2). We will have more to say about this practice in chapter 12.

Borda's suggestion was widely discussed in Paris. And the difficulties did not go unnoticed. Then another luminary appeared on the scene. His name was Marie-Jean-Antoine Nicolas de Caritat, Marquis de Condorcet.

BIOGRAPHICAL APPENDIX

Chevalier Jean-Charles de Borda

During several crossings of the Atlantic, Borda had the task of testing marine chronometers and studying methods to calculate the longitude of the ship's position. Toward the end of the eighteenth century, these were questions of paramount importance for maritime navigation. The latitude of a ship's position, that is, the distance north or south of the equator, can be ascertained relatively simply with the aid of a sextant or octant. Since these measurements are not affected by the earth's rotation, latitudinal positions can be determined by measuring angles of, say, the sun at noon over the horizon. Measurement of the longitude, though, is affected by the earth's rotation. Hence, a boat's east–west position cannot be established so easily. In order to ascertain the longitudinal position, a precise clock was needed that showed—wherever on the globe one may be—the local time at a reference point. Then, by comparing the reference time with the time at the current location, the ship's longitudinal position could be calculated. For example, if the sun at the current location is at its midday position and the clock, keeping the time of Le Havre, shows two o'clock in the afternoon, the navigator gathers that his ship is two hours, or 30 degrees, west of the port. (Twenty-four hours correspond to the full circle, that is, to 360 degrees. Hence, every hour's difference is equivalent to 15 degrees, which, along the equator is about 1,600 kilometers.) Together with the already determined latitude, the vessel's exact position on the globe is known. The clock's movement needed to be very precise, however. A deviation of just five minutes from the correct time at the reference point could translate into an error of up to 140 kilometers east or west. Many maritime disasters could have been avoided had exact timing devices been available to the ships' captains.

Pendulum clocks were of no use at sea, of course. They were meant to be hung on stable walls, not placed on vessels heaving and wallowing about in rough sea scapes. Watchmakers from various countries tried to invent timekeeping devices that would function sufficiently well even under extreme circumstances, but none were successful until the Swiss watchmaker Ferdinand Berthoud came to the rescue. He invented an isochronous balance wheel, driven by a spring that winds and unwinds at constant speed, which kept exact time even on boats rolling in foul weather. A first experiment showed that even after ten weeks of continuous operation the clock had accumulated an error of no more than one minute. In order to further test Berthoud's timepiece, King Louis XV ordered the mounting of an expedition. De Borda was appointed scientist in charge of the tests on board the *Flore*. The results exceeded the most optimistic expectations. After completion of the trip he and the ship's master composed a report titled "*Voyage fait par ordre du roi, en 1768 et 1769, dans différentes parties du monde, pour éprouver en mer les horloges de Monsieur Ferdinand Berthoud*" (Voyage undertaken by order of the king in 1768 and 1769 to different parts of the world in order to test the clocks of Mr. Ferdinand Berthoud at sea). The report was read to great acclaim at the Academy of Sciences. Berthoud was appointed the King's master watchmaker and awarded a yearly pension of 10,000 francs.

During the American war of independence Borda was promoted to captain and put in charge of a vessel. Cruising in the Caribbean and along the American coast on board the *Seine* he participated in many exploits. In the famed Battle of the Saints, in 1782, six ships were under his command. It was to be the end of his career at sea, however. The British enemy proved stronger and after several hours of battle—his vessel disabled and a large part of his crew killed—Borda was taken prisoner. He was lucky, though. His captivity was not very severe and did not last very long. Upon his liberation he returned to France and was named director of the Engineering School of the French navy.

Only then did Borda, who was already fifty years old, start his second career as a scientist. It was to immortalize his name to a far greater extent than would his military exploits. At that time, great confusion reigned in all parts of France. Merchants, traders, and shopkeepers in every province and in every town used different weights and measures—which sometimes carried the same name. The confusion made commerce extremely difficult. In 1790 King Louis XVI set up a commission to study how the units could be standardized. Half a year earlier, the tentative proposal had been made to base measurements of lengths on the pendulum. The unit of measurement was to equal the length of the pendulum whose swing back and forth lasts exactly one second. The method seemed acceptable to Britain and the United

States, and French scientists were quite enthusiastic about the fact that their proposal was about to gain international approval. A proposal was submitted to the National Assembly, which referred it to the Committee on Agriculture and Commerce, which recommended it to King Louis XVI, who passed it on to the Académie des Sciences, which established a committee to further study the matter. Now things got into high gear. The blue-ribbon committee consisted of Paris's most celebrated scientists: Jean-Louis Lagrange, Pierre-Simon Laplace, Gaspard Monge, the foremost mathematicians of their time; Antoine Lavoisier, the great chemist; and the Marquis de Condorcet, mathematician, politician, and economist of whom we will read much more in the next chapter. Jean-Charles de Borda was named the commission's president.

The commission saw a few problems with the pendulum; for one, they felt that basing one unit of measurement (length) on another (time) was not an appropriate approach. After all, the division of the day into 86,400 seconds was artificial and could be changed at any time. In fact, Borda advocated dividing the day into 10 hours of 100 minutes each, the latter being made up of 100 seconds. The second problem was even more serious; since the earth is flattened at the poles the gravitational constant, which is responsible for the swing times, is not identical everywhere. Hence, at different places on earth, different lengths of pendulums are needed to produce a one-second swing. This problem could have been rectified by determining a specific place on earth where the pendulum would be timed and measured, but such a decision would have challenged the national pride of all countries who—it was hoped—would adopt the new system. Another reason to reject the pendulum was that time appears as a squared term in the equation that determines the period of its swing. The scientists wanted to keep everything simple and linear.

So another solution was sought. In the committee's first report, of October 1790, the members decided to adopt a decimal division of money, weights, and measures. Actually, the subdivision of units had not really been the issue, but the scientists found it nonetheless important to address the question, presumably because it was so convenient to use the ten fingers to count off the digits. This report was a prequel to their second report, of March 1791, in which the committee announced its decision to define the unit of length as the 10 millionths part of the quarter meridian, that is 0.0000001 of the distance from the North Pole to the equator. All that now remained was to measure this distance . . .

And this is where the real difficulties started. Measuring the earth in the late eighteenth century was a task comparable to building a space station in our days. The French scientists were not easily intimidated, however, and set themselves to the task. To facilitate the enormous undertaking, Borda invented a device that allowed the measurement of angles to a precision unheard of in his time. With this tool, measurements could be made by triangulating the landscape, and distances could be

computed using trigonometry. But then the committee became aware of another difficulty; nobody had ever set foot on the North Pole, let alone measured any distance emanating from it. The scientists bypassed this problem by deciding to make do with the distance between Dunkirk and Barcelona. Measuring the distance between these two towns, establishing their latitudinal positions, and taking into consideration the earth's flattening at the North Pole, the total length of the quarter meridian could be computed.

But the revolution interfered. France was at war, the Republic was established, Louis XIV was tried and put to death, the Terror took over, Lavoisier was executed, Condorcet committed suicide or was murdered, the Academy was abolished. In short, confusion reigned. In the midst of all this, the scientists went about carrying out their task. One team of surveyors made its way south from Dunkirk—with their poles and flags and measuring devices—while another team worked northward from Barcelona over the Pyrenees. The members of the teams were arrested numerous times. More than once did they escape death only narrowly by pointing out that they were working on a replacement for the hated royal measuring system. Undeterred by all the hardships, they continued with their task until they met at the town of Rodez, about 500 kilometers south of Paris.

The undertaking had lasted nearly eight years. On November 28, 1798 the committee announced that the ten millionth part of the distance between the North Pole and the equator corresponded to 0.513243 *toises*, or, as we are wont to say nowadays, to one meter. Along with the liter and the gram, the meter became the official unit of measurement through a law enacted on December 10, 1799. Recent measurements, performed with the help of satellites, show that the French surveyors' measurement of the distance between Dunkirk and Barcelona was off by only about the length of two football fields. Thus, the meter that they had established more than two centuries ago was correct to within one-fifth of a millimeter.

MATHEMATICAL APPENDIX

Borda Count and Majority Elections

Let us assume that there are n candidates and E electors. a electors rank Peter first. If a is greater than 50 percent, Peter would be elected by the majority. Under which circumstances would he also be guaranteed victory by Borda's method?

In Peter's worst-case scenario, Mary would be ranked second by the a electors who ranked him first, and first by everybody else, that is, by $E - a$ electors. Peter would be ranked last by $E - a$ electors:

a electors:	Peter > Mary > . . . . . .
$E - a$ electors:	Mary > . . . . . . > Peter

Peter would receive n m-units from each of the a electors who rank him first, and 1 m-unit from all others, for a total of $a \times n + (E - a) \times 1$. Mary would receive $n - 1$ m-units from the a fans of Peter, and n m-units from all other electors, for a total of $a \times (n - 1) + (E - a) \times n$.

For Peter's m-units to be greater than Mary's the following inequality must hold:

$$a \times n + (E - a) \times 1 > a \times (n - 1) + (E - a) \times n$$

Solving this, we obtain,

$$a/E > (n - 1)/n = 1 - 1/n$$

The term on the left-hand side, a/E, is the proportion of electors who place Peter first. If this proportion is greater than the right-hand side, $1 - 1/n$, Peter is guaranteed victory by the Borda count, even in the worst possible scenario.

CHAPTER SIX
THE MARQUIS

Scholarly debate in the French capital, with its newspapers, publishing houses, academies, and *salon* tradition, was always very lively. It was no different with Borda's voting scheme. As could be expected, his proposal of assigning points, or m-units, to preferences did not go unchallenged. The challenger came in the form of a nobleman, who was Borda's junior by ten years. His full name was Marie-Jean-Antoine Nicolas de Caritat, Marquis de Condorcet.

Born in 1743 in Ribemont, Condorcet was the only child of an ancient family of minor nobility. His father, a cavalry captain, was killed during a military exercise when Condorcet was only five weeks old. His mother, a fanatically religious woman, raised her son without any education. As a sign of devotion to the Virgin Mary and to the boy's childlike innocence, he was forced to wear white dresses until he was nine years old. This, his mother hoped, would guarantee her and her son's eternal salvation.

But then his uncle, a bishop, took over. Religion and devotion were all right, but even this man of the church thought this was going a bit too far. He hired a tutor so the boy could catch up with others his age, and then sent him to a Jesuit school in Reims, in the northern part of the country. Even though Jesuit schools were considered the best educational system Europe had to offer, they did not provide what one would nowadays consider a positive environment. Learning by rote and corporal punishment were the main instruments of instruction. Furthermore, rampant homosexuality among the monks and students left Condorcet with a hatred of the church that lasted throughout his lifetime. Nevertheless, he received a first-class education.

The boy's exceptional intellectual gifts soon became apparent and his uncle had him sent to the Collège de Navarre in Paris to continue his studies. In the first year, the college's program consisted of studies in philosophy, which Condorcet deeply disliked, and in the second of mathematics, at which he excelled. During his studies he had the good fortune of meeting the encyclopedist Jean le Rond d'Alembert. This celebrated mathematician and physicist had had a very unhappy childhood himself, born out of wedlock and abandoned by his mother upon his birth on the steps of a church. D'Alembert took the shy and awkward sixteen-year-old youth under his wings. Condorcet did not feel comfortable with the worldliness that reigned in the capital city. He was not good at speaking in company and would blush whenever spoken to. Nevertheless, he became a welcome guest at the salon of d'Alembert's companion, and possibly mistress, Julie de Lespinasse.

A first attempt to make a name for himself as a mathematician failed, because the results he had achieved were not new. But then, at age twenty-two, Condorcet published a work on integral calculus that was widely praised. Thus started his scientific career. Upon d'Alembert's recommendation he was elected to the Académie des Sciences four years later. Condorcet wrote more treatises, one of which was praised by his contemporary Joseph-Louis Lagrange, one of the leading mathematicians of the time, as a book "filled with sublime and fruitful ideas which could have furnished material for several volumes." After another four years he was elected the Académie's perpetual secretary. He had come to this position after taking to heart the advice given him by d'Alembert and a certain François-Marie Arouet, a.k.a. Voltaire. The two elder men had suggested to Condorcet that he gain experience in the most important skill the position required: writing obituaries for academy members who had passed away. In fact, the secretaries' *Éloges*, which covered all branches of the sciences, were not just simple summaries of scientists' lifetime achievements. They were more akin to learned chapters in the history of science than to the obituaries that we are accustomed to in today's newspapers. Actually Condorcet was no slouch in the literary realm, and in 1782 he was elected to the Académie Française, the highest literary honor to which a writer in France could aspire, again at the recommendation of his mentor d'Alembert.

While Condorcet was still preoccupied with his mathematical works he met Anne-Robert Jacques Turgot, a high official in the royal administration. Turgot, a brilliant economist, had a profound influence on Adam Smith, who lived in France at the time, and some of the ideas that Smith eventually incorporated into his *Wealth of Nations* came directly from Turgot. King Louis XVI named Turgot Minister of Finance in 1774. Looking

for people that could be trusted, he, in turn, appointed his friend Condorcet Inspecteur Général des Monnaies, inspector general of the mint. (It is interesting to note that, across the Channel, the great Isaac Newton had held a similar position.)

Turgot, who saw the revolution approaching, realized the urgency of reform and the need to introduce competition and free markets into the French economy. Under the slogan "no bankruptcies, no new taxes, no debt" he attempted to make industry more efficient. He encouraged growth industries, abolished internal taxes on wheat, decreased government spending, curbed the extravagant outlays of the Royal Court, and ended the guild system, which had held a stranglehold over commerce and industry ever since the Middle Ages. All this did not sit well with the established orders, and after a while Turgot had made enemies of just about everyone in France. Nevertheless, for as long as the king supported him, he was safe. But then Turgot committed a grave error. Citing the need for economic belt-tightening he refused some favors to Queen Marie Antoinette's protégés. With that he broke one of the most important laws at Court: don't mess with the king's wife. Turgot was dismissed.

The Swiss banker Jacques Necker succeeded him at the ministry. He proceeded to reverse most of his predecessor's policies, which was one of the reasons for the eventual outbreak of the revolution. With his protector gone, Condorcet tendered his resignation. But the king refused and Condorcet stayed on at the mint for another fifteen years. Condorcet continued to write learned tracts on mathematics, economics, political science, and human rights.

At age forty-three Condorcet fell madly in love with Sophie de Grouchy, a lady more than twenty years his junior. The eldest daughter of the Marquis de Grouchy, a former page of Louis XV, she was said to be the most beautiful woman of her time in Paris. Condorcet and the young lady were of one mind in all their ideas and made an ideal couple. They were married in 1786. As was the custom for intellectual women in Paris, Sophie kept a salon at the couple's residence, the Hôtel des Monnaies. One of the guests who frequented these gatherings was an American by the name of Thomas Jefferson. Apart from organizing her salon, Sophie kept herself busy translating the works of Adam Smith—another of the guests at her salon—into French. She was also an accomplished portraitist, a skill that would become her sole means of support when hard times befell her. Four years after they got married the couple had a daughter, whom they named Eliza. Condorcet was a loving husband and a doting father.

As both president of the Académie des Sciences and member of the Académie Francaise Condorcet was by now one of France's foremost intellectuals. A true man of the enlightenment, he championed every liberal cause he could think of: economic freedom, tolerance toward Protestants and Jews, legal reform, public education, abolition of slavery, equality of all races. For example, he argued eloquently for women's rights. "Why should beings exposed to pregnancies and to passing indispositions not be able to exercise rights that no one ever imagined taking away from people who have gout every winter or who easily catch colds? . . . It is said that women have never been guided by what is called reason despite much intelligence, wisdom, and a faculty for reasoning developed to the same degree as in subtle dialecticians. This observation is false."

When turmoil broke out in 1789 Condorcet could not sit back. Leaving mathematics behind, he took a leading role in the revolution and was elected to the legislative assembly as a representative of Paris in 1791. Belonging neither to the more radical Montagnards nor to the more moderate Girondins, he tried to mediate among the various factions and temper the more extreme elements. As one of the more reasonable men in the legislative assembly, Condorcet was chosen to draft a constitution for the new nation. When the assembly was replaced by the convention a year later, Condorcet felt closer to the Girondins, who had in the meantime lost their power to the Montagnards. The latter, led by Robespierre, put down any opposing opinion with an iron fist, abolished royalty and put King Louis XVI on trial. Condorcet supported the trial but opposed the death penalty, a stand that did not make him a favorite of the Montagnards. It did not help the king either, who was executed on January 21, 1793.

Condorcet was not a speaker who could sweep away his audience. His rhetorical skills had not improved much since his youth; he was still shy and his voice did not carry. It was, therefore, not surprising that when he introduced his draft constitution to the assembly, he was unsuccessful. When his opponents presented their version of a constitution, an adulteration of his own initial draft, Condorcet protested against it with all his might and was promptly accused of being a traitor. I recount the tragic events that followed in the additional reading section.

The Marquis de Condorcet, one of the most remarkable men the French Revolution produced—politician, constitutional lawyer, mathematician, writer—left a great legacy. Important works in mathematics intermingle with texts on social issues. Some of his most intriguing texts, both as politician and as mathematician, were the contributions to the theory of voting and elections. His name is associated until today with one of the great social puzzles of all times: the Condorcet Paradox.

The paradox, to which we alluded in the previous chapter when talking about Jean-Charles de Borda, refers to a serious shortcoming of majority decisions. It is universally believed that decisions should be made, differences of opinion decided, judgments rendered, and officials elected, by taking votes and then counting which of the alternatives garnered most support. In fact, majority decisions represent one of the pinnacles of democracy. After all, the tenet of "one man one vote" rests on the assumption that the majority is always right. But to the surprise of many of Condorcet's, and our, contemporaries this assumption is fundamentally flawed. The Marquis showed that majority opinions are not always what they purport to be.

In 1785 Condorcet wrote a two-hundred-page pamphlet titled "*Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix*" (Essay on the application of probability analysis to majority decisions). He dedicated the work to Turgot, saying that it was he who had taught him that political science is amenable to the same degree of certainty as mathematics. As a case in point, Condorcet chose to demonstrate the power of mathematics by applying it to decisions made by majority votes. His analysis is not applicable only to citizens electing their leaders, he wrote, but also to judges in courts of law who must decide between guilt and innocence of an accused.

The essay was published fifteen years after Jean-Charles de Borda's address to the Académie des Sciences about ballot elections, and four years after the publication of his proposal to assign points, or *m-units*, to a candidate according to his ranking. Condorcet acknowledged Borda's contribution in a footnote saying that it had came to his attention only when his essay was already being printed.

As we shall see in the next chapter, Condorcet and Borda did not get along very well, but they did agree on one issue. Both had a somber view of majority decisions. Unlike Llull and Cusanus, who firmly believed that majorities reveal God's will and absolute truth, the two Frenchmen did not think that majority decisions were ipso facto correct decisions. Condorcet was convinced that societies had adopted majority rule for a much more pragmatic reason. Subordinating individuals to the will of the majority was meant to safeguard peace and quiet. Authority had to be placed where force is, and force is on the side on which most of the votes come down. Hence, for the good of the people, the will of the smaller number had to be sacrificed for the will of the larger in order to keep everybody quiet.

To buttress his claim, Condorcet cited instances from ancient times. The Romans and Greeks did not necessarily seek truth and try to avoid errors. What they strove to do was to balance the interests and passions of the various factions that made up their states. Whenever decisions were made, whether just or unjust, true or erroneous, reasonable or unreasonable, they had to be sustained by force. And since force is wielded by the majority, even incorrect decisions were adopted, if only they enjoyed the support of the majority. Subjecting decisions to tests of justice, truth, or reason would have put unnecessary restraints on the faction's authority. So might was right after all.

But eventually methods were sought that would permit decisions based on reason. The search for mechanisms less prone to error had started a long time before the age of enlightenment. During the centuries of deepest ignorance, a certain unease with majority decisions had already surfaced, especially when dispensing justice. Probably the most important problem a court of law faces is that judicial errors can result in people being convicted of crimes they did not commit. Therefore, attempts had been made in the Middle Ages to give the courts a form that would increase the probability that their decisions reflect the truth. Distrust for rulings in which one judge tipped the scales led the French to demand more than simple majorities to convict an accused. In England unanimity was required whenever juries rendered a decision. The Catholic Church's court of appeals demanded no less than three unanimous rulings in order for a judgment to be valid. (Of course, witch trials required neither a simple nor a qualified majority. Truth was elicited by the proven method of subjecting the poor women to various forms of torture.)

Condorcet gave a taste of how mathematical ideas, more specifically probability theory, could be applied to decisions rendered by courts of

law. He pointed out that the requirement of a plurality more stringent than a majority of one would render miscarriages of justice less probable. The larger the plurality required in a court, the smaller the probability that an innocent man would be convicted. This immediately begged the question, how far this certainty should be carried. And it raised a second problem: guilty defendants should not be declared innocent simply because the majority requirements had been carried too far.

So, after his rather pessimistic, if pragmatic, view of the advantages of majority decisions, Condorcet delved into the disadvantages. At the outset of his essay he apologized to mathematicians who would find the mathematical methods only of limited interest. Indeed, nothing more than basic arithmetic is needed to follow Condorcet's reasoning.

It is on page sixty-one of his essay that Condorcet presents the reader with the famous paradox. He illustrates it with an example of sixty voters who have to elect one of four candidates to a certain position. Here I give a simpler example with just three voters. Let us say Peter, Paul, and Mary must decide what to buy for their after-dinner drinks. Peter prefers Amaretto to Grappa, and Grappa to Limoncello. Paul prefers Grappa to Limoncello, and Limoncello to Amaretto. Finally, Mary prefers Limoncello to Amaretto, and Amaretto to Grappa.

Peter:	Amaretto > Grappa > Limoncello
Paul:	Grappa > Limoncello > Amaretto
Mary:	Limoncello > Amaretto > Grappa

Committed as they are to democratic values, the three diners decide to go by the majority opinion. They take votes and the preferences become quickly apparent. A majority prefers Amaretto to Grappa (Peter and Mary) and a majority prefers Grappa to Limoncello (Peter and Paul). Based on these two rounds they can make their decision: purchase a crate of Amaretto.

But surprise, surprise: Paul and Mary protest. What happened? The most reasonable selection method was used—one person one vote —and they still aren't happy? Do they want to change the rules in mid-game? Well, they have a legitimate grumble. Paul and Mary point out that they would prefer even Limoncello, the lowest ranked option, over Amaretto. How come? Here is the clincher: had the three campers had a third round of voting, between Limoncello and Amaretto, a majority would have preferred Limoncello (Paul and Mary). So let them buy Limoncello and get it over with. But wait a minute. Buy Limoncello, and Peter and Paul—yes, Paul, the guy who insisted on the third vote because of his dislike of Amaretto—will protest just as vigorously. They prefer Grappa to Limoncello. So here we have it, a paradox. One does not argue about tastes and Peter, Paul, and Mary have perfectly reasonable preferences. Try as you might, the final result is that Amaretto is preferred to Grappa, Grappa to Limoncello, Limoncello to Amaretto, Amaretto to Grappa, Grappa to Limoncello. . . . We could go on and on.

So what is the solution? The depressing answer is that there is none. There is no way out of Condorcet's Paradox. Whatever the choice, a majority always prefers a different option. Preferences cycle through all the alternatives and the paradox persists. The fact that the majority prefers Amaretto to Grappa, and Grappa to Limoncello, simply does not imply that Amaretto is preferred to Limoncello by the majority. In mathematical lingo, one says that "majority opinions are not transitive." What a letdown for democracy.

Condorcet's Paradox can be the source of much abuse. For example, a person setting the agenda at a board meeting can subtly influence the outcome of decisions by manipulating the order in which votes are taken. Let us say a company wants to reward its employees by installing a cafeteria, a health club, or a nursery on its premises. A decision must be made, and the company's CEO charges the personnel director with organizing a board meeting. The personnel director hates sweaty basements and has no predilection for screaming kids. She does, however, enjoy taking time off from her busy schedule for the occasional cup of coffee. At the meeting she takes the first two votes, and the cafeteria comes out on top. Since small talk and prevote discussions, stoked on by the personnel director herself, have taken up most of the morning, there is little time left for any more votes. Some board members need to use the bathroom facilities, others want to have a smoke, and anyway lunch is waiting in the executive dining room. "Let's just break up now," the sly operator may say. "Since the cafeteria is preferred to the health club and the health club to the nursery it is obvious that the majority wants a cafeteria." Nobody bothers to find out whether the nursery would have bested the cafeteria if a direct vote had been taken. And this is how the personnel director can have her way.

Hence with good reason the deeply troubled Condorcet feared that the paradox poses great dangers. Since ignorant masses could be manipulated by corrupt politicians and charlatans he decided that the people had to be informed about their rights and obligations as citizens. If a society's philosophers did not find the courage to enlighten the unsuspecting people, tyranny could set foot in the country and maintain itself. Condorcet, who devoted himself to seeking truth and to serving the fatherland, took that task upon himself.

As a vehicle for his educational efforts, Citizen Condorcet (it was no longer fashionable—in fact it was downright dangerous—to carry the title Marquis), together with Citizen Sieyes and Citizen Duhamel founded the *Journal d'Instruction Sociale* (Journal of Social Education), a weekly publication that would devote its pages to educating the public about their rights and duties. One should never forget, the editors reminded the readers in the prospectus for the new journal, that while liberty and equality were the most important assets of an enlightened people, they could also be the cause of the greatest harm if, due to ignorance, the people did not know how to safeguard these assets. The journal's aim was not to lecture its readers, the editors stressed. The objective was to enable them to form their own opinions.

The journal was to be launched in 1793 and be published every Saturday. Whatever profits there would be, the editors promised, would go to the National Institute of the Deaf and Dumb—the hearing impaired in modern parlance—at whose premises the journal was to be printed. The journal's first issue appeared on June 1, 1793, all of its three articles having been written by Condorcet. After a philosophical investigation into the meaning of the newly coined term *revolutionary* and an essay on progressive taxation, the booklet closes with the eight-page paper that is of primary interest to us. It is titled "*Sur les élections*" (On elections). In it Condorcet outlined his ideas on the electoral process.

The French were about to grant themselves a constitution that would decide the nation's fate. Would the people be governed by reason or by intrigue, by the will of all or by the will of a few? Would liberty be peaceful or agitated? The answer to these questions, the very survival of a well-functioning society, depended on the quality of the popular choices, Condorcet wrote. Constitutional shortcomings themselves pose no immediate dangers. As long as honest, publicly minded men rule the country—in spite of his avowed feminism Condorcet did not go so far as to include women—there would always be occasion to correct any threats to the nation that may arise. If corrupt men take over, however, even the best laws become only feeble ramparts against ambition and intrigue.

But honest people who base their elections, judgments, and decisions on a plurality of votes can be led into the feared cycles. So if majority decisions are not the solution, what is? Condorcet thought long and hard about the problem and finally came up with a suggestion. As in his essay of 1785, he recommends combinatorial methods and probability theory as the surest way to avoid the ills of conventional methods. His suggestion will sound strangely familiar to the readers of this book. It is very similar to the method that Ramon Llull had already proposed 500 years earlier.

When an elector chooses a candidate for a certain post, he performs a series of judgments. He does so by comparing all possible pairs of candidates, examining the reasons to vote for the one or the other, weighing them, and then expressing a choice. By doing this for all pairs of candidates, he obtains a ranking and the top-ranked candidate is the individual's favorite for the job. If not all electors express complete rankings—either because they are indifferent between some candidates or because some of them are unknown to them—the election outcome may not reflect the true preferences of the assembly. Nevertheless, Condorcet cautions, electors should not be forced to choose among candidates they do not know because this would simply result in a random ranking. Rather, he recommends—like Llull did half a millennium previously—that a list of acceptable candidates, well known to all electors, be drawn up before the voting starts. The electors must then make a complete listing (including indifferences) only among those deemed eligible.

If the elector suddenly realizes that he prefers Alexander to Bertram, Bertram to Charles, and at the same time Charles to Alexander, then, Condorcet asserts, at least one of the choices must have been based on an erroneous assessment of the relative strengths of the candidates. (Condorcet tacitly assumes that choices must accord with common sense and therefore be transitive.) In this case the elector will reexamine his set of choices and clean it of the ones that led to inconsistencies. Reevaluating all judgments, he identifies those that may lead to the absurd situation,

and drops, or reverses, the one that he deems most improbable. For example, if he strongly prefers Alexander to Bertram and strongly prefers Bertram to Charles, while his preference of Charles over Alexander is only slight, he will drop the latter preference. Thus the elector will end up with a complete ranking of the candidates.

Now comes the actual election. An election by an electoral college is the aggregation of the individual rankings. Condorcet suggests that after the electors have made up their minds about the candidates' relative positions, they get together to judge the candidates. Each candidate is paired against each of the other candidates in a series of showdowns. The electors express their preferences, and the candidate who receives more votes is considered to be superior to the other. After all pairings have been performed, the candidates are ranked. In the final list, a candidate who won the showdown against a particular competitor will be ranked above him and the person who comes out on top of the list will be declared the winner.

In the ideal situation, where the best candidate wins all contests against the other candidates, an unambiguous winner exists; he will be declared the "Condorcet winner." (A candidate who loses all showdowns against the other candidates is called a Condorcet loser.) But things are not usually as simple since, in general, no unambiguous list can be drawn up. A Condorcet winner, the ideal candidate, superior to every other contestant, exists only rarely. Usually no candidate wins every single showdown. Even an exceptionally strong contestant is bound to lose some of them. This, of course, produces cycles, and when that happens, no Condorcet winner exists. What is to be done?

When cycles appeared in an individual's preferences, Condorcet argued that the situation does not accord with common sense and one of the preferences should be reversed. But there is an important difference between a single man's preferences and an election. As was shown in the Grappa-Amaretto-Limoncello example, when aggregating the preferences of three or more electors, cycles may occur even if the individual electors' judgments are perfectly consistent. So Condorcet suggested that an assembly use the same method to resolve a cycle as an individual does when examining his choices: at least one of the showdown results must be deleted. But which one? It cannot be argued that preferences expressed by the electors were unreasonable. After all, they were based on a majority of votes. Condorcet saw a way out: the preference with the feeblest majority is to be dropped.

So Condorcet's proposal entails the two-by-two showdowns with which we are already familiar due to Ramon Llull's work in the thirteenth century. But there is an important difference. Llull had advocated the election without further ado of the candidate who won most of the contests. Condorcet, in contrast, proposed to check the whole ranking, all the way to the bottom, for inconsistencies. If the top-ranked candidate turns out to be inferior to another candidate, the result of the showdowns that produce the cycle should be dropped. In the end, the Condorcet winner may not be identical to the winner according to Llull.

Let us analyze an election for a monastery's prior. Eleven candidates vie for the post. The best result was achieved by Brother Angelo who won the showdowns against all competitors, except for the one against Brother Giulio. He thus gets nine points. Brother Giulio who won all showdowns except the ones against Brother Innocenzo and one other competitor came next with eight points. Brother Innocenzo, who lost against Brother Angelo, won against Brother Giulio, and won another six showdowns, came in third with seven points. The following table summarizes the results.

TABLE 6.1

	Showdown against:				
	Angelo	*Giulio*	*Innocenzo*	. . . .	*Total*
Angelo	-	loses	wins	8 additional wins	9 points
Giulio	wins	-	loses	7 additional wins	8 points
Innocenzo	loses	wins	-	6 additional wins	7 points

Based on the total score, Ramon Llull would have had Angelo elected prior. But Angelo was beaten by Giulio. And Giulio was beaten by Innocenzo, who in turn was beaten by Angelo. We have a cycle. Who should become prior?

Condorcet suggests scrutinizing the election results more closely. Let us say that Angelo lost the crucial showdown against Giulio with a whopping 1 vote to 8, while Giulio lost his showdown against Innocenzo very narrowly with 4 votes to 5. Finally, Innocenzo lost his duel against Angelo with 2 votes to 7:

TABLE 6.2

	Showdown against:		
	Angelo	*Giulio*	*Innocenzo*
Angelo	-	loses 1:8	wins
Giulio	wins	-	loses 4:5
Innocenzo	loses 2:7	wins	-

According to Condorcet's proposal, Brother Giulio's loss, being the narrowest, would be deleted from the tally. Hence, the cycle would be broken and Brother Giulio would become the new prior.

What happens when several cycles appear, as often happens when more than three candidates present themselves? In this case inconsistencies are even more probable. But Condorcet sees no special problem. If dropping one preference does not fix the cycles, the majority must have erred more than once. The solution consists in dropping as many judgments as needed, until an unambiguous winner can be determined. All judgments that lead to inconsistencies are reexamined and dropped one by one, starting with the ones that have the narrowest majorities.

The cycle-breaking mechanism Condorcet proposed seems like a good idea. So why not use it? The problem is that it is not easy to implement. With, say, ten candidates there are forty-five judgments. (The first candidate meets nine competitors, the next eight, and so forth. More generally: with n candidates there will be $n(n-1)/2$ showdowns.) It is no simple task to single out those judgments—among the forty-five—that lead to the inconsistencies. But an even more serious problem can arise if two or more inconsistent judgments obtain equal majorities. Let us look again at the simplest example, the after-dinner drinks. Each choice has the same two-to-one majority. So where should the cycle be broken? Which showdown should be dropped? The one between Amaretto and Grappa, the one between Grappa and Limoncello, or the one between Limoncello and Amaretto?

Condorcet's proposal seems a very reasonable method, but in its purest form it is fairly useless. Of course a Condorcet winner, the candidate who beats all others, would be the preferred winner. If there are only two contestants, the outcome is obvious: the one who beats the other is the Condorcet winner. But even one additional candidate may lead to a messy situation, as witnessed by the Amaretto-Grappa-Limoncello example. The more candidates there are, the more unlikely it is that a Condorcet winner exists. And then, there is the large number of showdown contests that would have to be performed, a near impossible task even for a moderate number of candidates.

* * *

After centuries of making do with majority elections, all of a sudden the method was shown to be defective. And now there were not one, but two new proposals, both of which had their advantages and disadvantages. In Condorcet's two-by-two showdowns an inferior candidate would never be elected, but there was no guarantee of a winner. Borda's m-units-for-rank scheme takes the electors' true preferences into account, but the eventual winner could very possibly turn out to have been nobody's favorite. And if there are Borda and Condorcet winners, the two may not be identical. Paradoxes abounded everywhere. Neither of the methods was undisputedly superior to the other. This did not keep the two savants from flaunting the advantages of their own election method while putting down the other.

Nevertheless, both men deserved great honor and great honor they received: in Paris's 9th arrondissement a street was named after Condorcet, and in the 3rd we find a street named after Borda. The honors did not end with the rue Condorcet and rue Borda either. To underscore their international, and even outer-spatial reputations two lunar craters on the moon have been named after Borda and Condorcet. For good measure there is also a Cusanus Crater, but nothing on the moon has so far been named for Ramon Llull.

BIOGRAPHICAL APPENDIX

Marquis de Condorcet

Branded a traitor and fearing for his life, the Marquis de Condorcet took refuge in the house of a devoted woman, Madame Rose Vernet. This lady, a widow who supported herself by renting out rooms in her house in the rue des Fossoyeurs, was a person of exceptional character. Her name would be unknown to us today, had it not been for the exceptional courage she displayed during the great Terror by sheltering the wanted fugitive.

Only two of Mme Vernet's tenants knew of Condorcet's identity. One of them was a Montagnard by the name of Marcoz, who was told of the secret

but kept it and, in fact, provided Condorcet with newspapers and information about the developments in the outside world. The other tenant who was in on the secret was Mme Vernet's cousin, the mathematician Sarret. The only other person who knew of Condorcet's identity was Mme Vernet's loyal servant, Mademoiselle Manon. Shared secrets and the cramped quarters made for a romantic atmosphere, and according to some accounts Mme Vernet and Monsieur Sarret eventually got married.

While Madame and Monsieur may have had a budding romance, Condorcet was very lonely. The only contacts he had in his hideout were with Sarret, Marcoz, and Mme Vernet. Occasionally Sophie came to see her beloved husband, but visits were dangerous and therefore rare. He never got to see his four-year-old daughter again, whom he so dearly loved. A letter that he sent Eliza touches the readers' hearts even two hundred years later: "Whatever the circumstances in which you read these lines, which I am writing far away from you, indifferent as to my own fate but preoccupied by yours and your mother's, remember that nothing can guarantee that those circumstances will last. Get into the habit of working, so that you are self-sufficient and need no external help. Work will provide for your needs; and though you may become poor, you will never become dependent on others. . . . My child, one of the best ways to ensure your happiness is to preserve your self-respect, so that you can look back on your whole life without shame or remorse, without seeing a dishonorable act, nor a time when you have wronged someone without having made amends. . . . If you want society to give you more pleasure and comfort than sorrow or bitterness, be indulgent and guard yourself against egoism as a poison which ruins all its pleasures. . . ."

In order not to arouse suspicion concerning the whereabouts of her husband, Sophie made one of the hardest decisions of her life; with his consent she divorced the Marquis. By then Sophie was nearly penniless and had to support herself and her daughter by drawing portraits. Actually portrait drawing was quite a good business in these uncertain times. Many Parisians, not knowing what the future may bring, wanted to leave their likeness to their next of kin.

Condorcet spent a cold and lonely winter working in solitude on his last text, *Esquisse d'un tableau historique des progrès de l'esprit humain* (Sketch for a historical picture of the progress of the human mind). The partly imaginary description of the human race's progress from savagery to a future state in which equality among classes and nations would reign, and human nature would be perfected, was to become Condorcet's legacy.

After staying at Mme Vernet's boarding house for five months, the Marquis had reason to fear that his hideout in the rue des Fossoyeurs (renamed today rue Servandoni) was being staked out. An unknown man had shown up at Mme Vernet's doorstep, on the pretext of wanting to rent a room. He had asked strange questions and then left again. Condorcet felt that it was no longer safe to stay in this hideout. Had he been discovered, it would have meant the guillotine not only for him but also for Mme Vernet, and his wife's life would not have been spared either. Against the express wishes of his devoted landlady, Condorcet left the boarding house.

Dressed as a commoner, he set out for the house of Amélie and Jean-Baptiste Suard, close friends of his from better times. In the countryside outside Paris, he hoped to receive temporary shelter. It was a perilous journey. Condorcet managed to cross the city lines where only six days earlier a former member of the convention by the name of Masuyer had been recognized, tried, and immediately executed.

After a long and strenuous walk—Condorcet had not been able to exercise his legs for nearly half a year—he finally arrived at the Suards' home. A maid opened the door only to tell him that her master and mistress had left for Paris that same morning. The lonely fugitive spent the next two days without food, wandering around and sleeping under the open sky. When his friends finally returned from Paris they were too afraid to take him in. One could hardly have blamed them. Giving refuge to a wanted man was punishable by death, and the less than trustworthy maid had taken a close look at the unshaven stranger. Suard promised that he would try to obtain a passport for him, and Condorcet left the house.

He took refuge at a country inn trying to blend in with the locals. But his noble demeanor soon betrayed him and when he ordered an omelet with an "aristocratic amount of eggs"—believed to have been 12—his cover was definitely blown. Asked to identify himself, Condorcet, who carried no papers, tried to pass himself off as a chamber valet by the name of Pierre Simon. Pending verification of his identity, the unknown man was put in a prison cell. Two days later he was found dead. The cause of his demise was never determined. Did he die of natural causes, did he commit suicide, or was he murdered because he was too popular in Paris to be executed? One version has it that a friend of his, a medical doctor, had given him a vial of poison a long time ago that Condorcet kept hidden in a ring on his finger. It was to spare him the guillotine if and when things should go terribly wrong. Had he used it? We will never know.

Sophie was also arrested but soon set free. She survived her husband by twenty-eight years. When Eliza was seventeen, she married an Irish general, twenty-seven years her senior, by the name of Arthur O'Connor. O'Connor had been an indefatigable fighter for Irish independence. Arrested by the English and kept in prison for five years, he finally agreed to exile in France and became a Général de Division under Napoleon. He and Eliza bought an estate south of Paris where they raised three sons who tragically died. (Other sources indicate that Eliza and O'Connor's offspring served as officers in the French army.) After his retirement from the army O'Connor became a prolific writer on social and political subjects, even helping to edit the twelve volumes of Condorcet's works.

CHAPTER SEVEN
THE MATHEMATICIAN

When writing his essay in 1785, Condorcet was apparently already aware of Borda's contribution of 1781. He admitted as much in a caustic footnote in which he acknowledged that the existence of Borda's paper had been pointed out to him by friends, at a time when his own paper was already being printed. Somewhat patronizingly, he claimed that he would not have known anything about the paper save for the fact that some people had mentioned it to him. As is now believed, however, Condorcet was being less than truthful. In 1781, he was the perpetual secretary of the Académie des Sciences, and as such was responsible for editing the academy's *Mémoires*. He could not have been unaware of what was being printed. More likely, he himself decided to publish Borda's paper.

Condorcet did not think highly of Borda. In fact, he did not even consider him a very capable mathematician. While undeniably talented, Condorcet said, Borda had to take recourse to the lesser science of engineering, building ships and fortifications, after failing at mathematics. According to Condorcet, Borda was not even qualified to be a member of the academy of sciences and had gained entry to this hallowed temple of scholarship not by erudition or scholarship but by wish of the king. In a letter to a friend, Condorcet wrote that Borda likes to talk a lot and wastes his time tinkering with childish experiments.

Why would Condorcet, the self-styled gatekeeper of French science, publish a supposedly inferior paper? Actually he did even more than have it published. Borda's paper was prefaced by a highly complimentary review, and according to custom it was the editor who wrote the prefaces. So Condorcet, who considered Borda unworthy of even being a member of the academy, not only published his paper but also sang its praises. Why did he do that? Had he not recognized Borda's method as a challenge to his own? Whatever the motives, publishing the paper gave him occasion to put forth his own—allegedly superior—election method.

Condorcet attacks the Borda count by means of an example. Without even mentioning Borda, except for a caustic reference to "a famous math-