4

Government equilibrium

In the previous chapter we set out a stylized description of the government formation process in parliamentary democracies. This description is, in effect, a logical model of government formation. In this chapter, we begin to unfold some of the implications of our model, exploring ways in which it might increase our understanding of what it takes to make and break a government.

Since we are interested in what it takes for a government to form and stay formed, an important thread running through the rest of the argument of this book is the concept of an *equilibrium cabinet* – a notion that we hope is more or less intuitive. An equilibrium cabinet, once it is formed, stays formed because no political actor with the ability to act in such a way as to bring down the cabinet and replace it with some alternative has the incentive to do so. Conversely, no actor with the incentive to replace the cabinet with some alternative has the ability to do so. Thus we expect an equilibrium cabinet to be stable, remaining in place until a change in the external environment transforms either the incentives of some pivotal actor or the "pivotalness" of some actor already possessing the appropriate incentives.[1] (For example, an opposition party may, as a result of by-elections or defections from other parties, increase its weight and thereby the extent to which it is pivotal.) We do not expect a cabinet that is out of equilibrium to be stable, since at least one actor who has an incentive to bring down the government also has the ability to do so.

Political equilibriums can be attractive, retentive, or both. An *attractive* equilibrium cabinet is one that tends to come into being, even when it is not the original status quo. A *retentive* equilibrium cabinet remains in place if it is the original status quo, but may not come into being otherwise. For

[1]Thus, our notion of equilibrium must always be taken as relative to *anticipated* factors. Ex ante, actors choose a government in the expectation that it will remain in equilibrium. Ex post shocks to the system, unanticipated at the time of government formation, are potentially destabilizing factors.

example, a government may form in a particular strategic environment and this environment may change, perhaps because of an election result. The incumbent government may remain a retentive equilibrium, and therefore remain in office, even though, given the new election result, it would never have been able to *take* office if a different cabinet had been in power before the election.

The essential purpose of the discussion that follows, therefore, is to find ways of characterizing what it takes to form an equilibrium cabinet. This should enable us to identify those potential cabinets that are likely to be in equilibrium and those that are not, and thereby to enrich our understanding of the political factors that underpin cabinet stability. Such characterizations are based on a number of variable features of the political situation and the government formation process, and what our model does is allow us to explore each of these features in a systematic and rigorous manner. As it happens, the character of equilibrium cabinets, according to our model, is heavily conditioned by whether there exists some "strong" party that is in a particularly powerful bargaining position – in a sense to be made more precise – that allows it to dominate the making and breaking of governments.

In the remainder of this chapter we first recapitulate the basic building blocks of our model; we then characterize certain features of an equilibrium cabinet; next we discuss the role of strong parties; finally we look more generally at the distinctly centripetal tendencies implied by our model of government formation.

BUILDING BLOCKS FOR A MODEL OF GOVERNMENT FORMATION

The basic building blocks of our model are as follows:

1. There is a set of legislative parties, each with a weight and a policy position. A party's weight is measured in terms of its share of the total seats in the legislature. Each party's policy position is expressed in terms of the policy that the party is forecast to implement, if given the opportunity to do so, on each key policy dimension. This describes the political arena in which the making and breaking of governments takes place. Graphically this arena can be seen as a type of lattice, which we display in Figure 2.2 and exploit later in this chapter. The weights and policy positions of all parties are assumed in most of what follows to be common knowledge among all relevant actors.

2. There is a set of government departments, each responsible for the development and implementation of public policy in particular areas. Each department is responsible directly to a cabinet minister. While more than one department may be responsible to a single minister, no one

department may be responsible to two or more ministers at the same time.

3. There is a government formation process, set out in detail in Figure 3.1 and described in Chapter 3. Proposals for government involve the specification of which parties will hold cabinet portfolios with jurisdiction over the various key policy dimensions. A new cabinet replaces the status quo if it is proposed, receives the assent of every one of the parties participating in it, and is then supported by a majority of legislators.[2]

Winsets

The government formation process in a parliamentary democracy thus requires that, before a new government can replace the status quo, it must receive the support of each of its participants, on the one hand, plus the support of a legislative majority. The majority requirement means that one of the things of great concern to us is the set of potential cabinets preferred by a legislative majority to some cabinet that we have under consideration, which in turn depends on the policies that each potential alternative cabinet is forecast to implement if put into office. Thus, we must focus on two distinct, but related, preference relations – those over *policies* and those over *cabinets*.[3] For a policy x, the set of policies preferred by a majority to it is known as the *policy winset of x*, labeled $W^*(x)$. For a cabinet X, the set of cabinets preferred by a majority to it is known as the *lattice winset of* X, $W(X)$.[4] When we consider whether a particular cabinet might be in equilibrium, therefore, one of the things we do is look in its lattice winset, that is, at which cabinets are majority-preferred to it.

[2]To be precise, we are assuming a legislative decision rule based on majority voting in legislatures with an odd number of legislators, or in legislatures with an even number of legislators in which there are no "blocking coalitions" – coalitions such that the coalition and its complement both control exactly 50 percent of all seats. When there is a qualified majority decision rule, requiring the support of more than a bare majority of legislators, and when there are blocking coalitions, it may well be the case that more than one party is at the "median" position on some policy dimension – a situation that could of course be modeled, but which we do not consider here.

[3]The connection between these derives, as just noted, from the common-knowledge policy forecast associated with each cabinet. By convention, we will write cabinets in uppercase letters and associated policies in lowercase letters, when it is necessary to distinguish between them. Thus cabinet X is associated with policy forecast x. Because parties in our model are policy motivated, party i prefers cabinet X to cabinet Y if and only if it prefers policy x to policy y. In this context, x and y are *vectors* of policy positions.

[4]With a slight abuse of notation, we claim that $W(X) = W^*(x) \cap L$, where L is the lattice of portfolio allocations. By this we mean that the portfolio allocations preferred to cabinet X are those represented by points on L whose forecast policies are preferred to x (i.e., cabinets whose forecast policies are elements of $W^*(x)$).

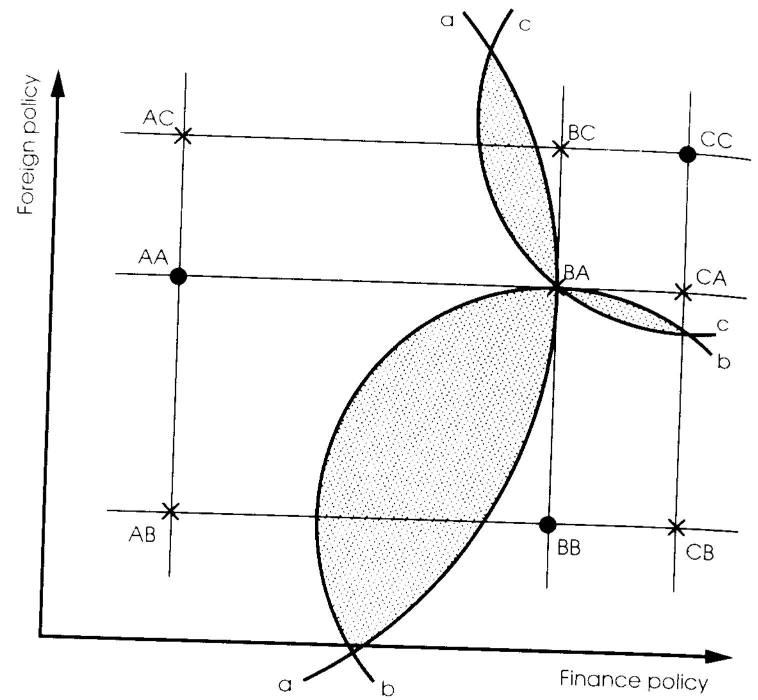

Figure 4.1. The winset of a BA cabinet

Returning to the lattice of party positions we first saw in Figure 2.2, we illustrate the policy winset of cabinet BA as the shaded area in Figure 4.1. Remember that cabinet BA gives the finance portfolio, on the horizontal axis, to Party B and the foreign affairs portfolio, on the vertical axis, to Party A. In order to arrive at this conclusion we first make the conventional political economist's assumption that the policy positions that each party prefers to the forecast policy of the BA cabinet are those that are closer than this to the party's ideal. If we also assume (once more conventionally) that closeness is denominated in the same Euclidean distances that we use in physical space, then the points that Party A prefers to BA, for example, are those inside the circle centered on Party A's policy position and passing through BA. A segment of this circle is shown in Figure 4.1, labeled aa. All points inside this circle are closer to Party A's ideal policy than is the policy forecast to be implemented by the BA government. Similarly, Party B prefers everything inside the circle centered on Party B's position and passing through BA – a segment of this is bb in Figure 4.1. The equivalent segment for Party C is cc. If the weights

of the three parties are such that any two can combine to form a legislative majority, then we know that policy positions are majority-preferred to those forecast for cabinet BA (and are therefore in the policy winset of BA) if they are in the intersection of any two of the three circles just described. This area is shaded in Figure 4.1.

Thus, this winset has three segments. First, there are policy positions preferred by both Party A and Party B to BA – these must be inside circle segments aa and bb. These positions are in the lower left-hand shaded petal-shaped area in Figure 4.1. Positions preferred by both Party A and Party C to BA must be inside circle segments aa and cc. They are in the upper left-hand shaded area. Similarly, positions preferred by both Party B and Party C to BA are in the middle right-hand shaded area. The union of the three shaded areas is the policy winset of BA, which we can abbreviate to W*(BA). Every policy in W*(BA) is preferred by some legislative majority to the policy of cabinet BA.[5]

Strategic equivalents and equilibrium

The government formation process described in Figure 3.1 has no beginning and no end, but we structure our analysis by imagining some incumbent cabinet, X, and investigating what would happen if the government formation process started from a status quo of X. Our rational foresight assumption implies that all political actors will be able to figure out what will happen as a result of starting the government formation process from X – let us call this forecast outcome Y. Thus starting at X implies, for the actors, ending at Y. As we saw in Chapter 2, we can think of Y as the *strategic equivalent of* X. Once we are at X, rational behavior in the strategic context of the process depicted in Figure 3.1 yields Y.

The notion of a strategic equivalent allows us to be more precise about what we mean by an equilibrium. A status quo X is in equilibrium if the strategic equivalent of X – which we write as SE(X) – is also X. An equilibrium is an X such that SE(X) = X. Conversely if the strategic equivalent of the status quo differs from the status quo, then the latter is not in equilibrium.

The next step in our argument is to note that, in the government formation process that we have described, the status quo can only be replaced by some alternative that is majority preferred to it. In other words, the strategic equivalent of the status quo, X, must either be X itself, or some element in the winset of X.[6] The reason for this is that

[5]Note that, while the policy winset W*(BA) is conspicuously nonempty, the lattice winset W(BA) is empty. That is, while there are *policies* preferred to those forecast for the BA government, there is no *cabinet* (point in L) that is forecast to implement such a policy. We will pursue this interesting fact.

[6]Formally, $SE(X) \in \{X\} \cup W(X)$.

replacing X involved investing some alternative cabinet. This investiture must be supported by a legislative majority. If the investiture would eventually result in some alternative Z that is not majority-preferred to X, then members of the legislative majority preferring X to Z, exercising their rational foresight, would vote against it.[7]

EQUILIBRIUM CABINETS AT THE GENERALIZED MEDIAN

Figure 4.1 portrays a *simple* jurisdictional arrangement in which horizontal policy is determined by the Finance Ministry and vertical policy by the Foreign Ministry. The potential cabinet that we are investigating here – BA – has one very important strategic feature; the portfolio with jurisdiction over each policy dimension is allocated to the party with the median legislator on that dimension. Since any two parties in this three-party arrangement, by assumption, compose a majority, it follows that the "middle" party on any dimension possesses the median legislator. The advantaged party is Party B on the horizontal dimension and Party A on the vertical dimension. The forecast policy output of this cabinet, then, is the dimension-by-dimension median (DDM) in the policy space. We can think of this cabinet as the DDM cabinet. As we saw in Chapter 1, Kadane (1972) has shown that any point in a continuous policy space that is an equilibrium (i.e., a point with an empty winset) must be the DDM.[8] As we shall shortly see, this particular strategic feature of the DDM cabinet is very important in the government formation process.

The shaded areas in Figure 4.1 show quite clearly that the BA policy position under investigation does not have an empty *policy* winset. Any policy in these shaded areas is preferred by a legislative majority to the policy forecast for BA. (And Kadane's result tells us that some other point is in turn preferred by some legislative majority to any point in these shaded areas.) If BA were simply a legislative policy proposal, therefore, it could be beaten in the legislature by a wide range of alternative policies and would not be in equilibrium.

[7]Note that we assume that actors behave strategically in the sense that their decisions take account of the effects of their choices and the forecast choices of others in future stages of any game they are playing. Thus, even if the first move away from the status quo, X, is to some interim point, Y (perhaps, but not necessarily, majority-preferred to the status quo), that makes some final alternative, Z, possible, members of the legislative majority who prefer X to Z will foresee the danger of allowing Y to form in the interim, since they know that SE(Y) = Z. They can prevent Z by refusing to vote for Y, keeping X in place even if some of them prefer Y to X.

[8]Kadane's result can be extended to the situation in which the set of alternatives to the DDM is the set of lattice points, rather than the full continuous policy space. A proof is provided in the appendix to this chapter.

In our model of the government formation process, however, BA is not simply a legislative policy proposal, but rather is the cabinet (and the forecast policy output associated with it) in which the finance portfolio is allocated to Party B and foreign affairs to Party A. As we have seen there is a finite set of cabinets that can be posed as alternatives to BA, and these are represented by the other points on the lattice in Figure 4.1. These are the three cabinets identified by black dots (AA, BB, and CC), which represent giving both key portfolios to a single party, and the five other cabinets identified by an "x" (AC, BC, CA, AB, CB), each of which represents a different way of allocating the portfolios to a coalition of two parties.

We can easily see by looking at Figure 4.1 that no lattice point is in the shaded area representing the winset of BA. This means that there is no *alternative cabinet* whose forecast policy outputs are preferred by a majority to those of BA. *BA, consequently, is preferred by some legislative majority to every alternative cabinet.* For this reason, BA is an equilibrium cabinet. If the status quo is BA, then any strategic move that will result in the investiture of an alternative to BA will be blocked by the legislative majority that prefers BA to the alternative in question.

This argument generalizes to support our first proposition characterizing equilibrium cabinets in parliamentary democracies:

> *Proposition 4.1*: The DDM cabinet is an equilibrium if there is no alternative government in its winset.

Informal proof: The strategic equivalent of any point is either the point itself or an element of its winset (see note 6). Thus, if the winset of the DDM cabinet is empty, then the strategic equivalent of the DDM must be the DDM itself. This is therefore in equilibrium.[9]

Remember that Kadane's result tells us that, when there is a simple jurisdictional structure, only the DDM cabinet can have an empty winset. No other cabinet can have an empty winset. Thus the DDM cabinet will remain in place if there is no lattice point in its winset. This was the case in Figure 4.1 and, as we shall see, it is in general quite common for there to be no alternative cabinet in the winset of the DDM. The finding that there is a potential equilibrium cabinet at the generalized median position holds for policy spaces of any dimensionality.[10]

It may be the case that some alternative is preferred by a majority to

[9]Formally, let the DDM cabinet be written as m^*. Since $SE(m^*) \in \{m^*\} \cup W(m^*)$ and $W(m^*) = \emptyset$, then $SE(m^*) = \{m^*\}$. When the DDM cabinet has an empty-lattice winset, we write it as m^{**}.

[10]However, we will demonstrate below that the probability that the generalized median cabinet is preferred to all others decreases as the dimensionality of the policy space and the number of parties increase.

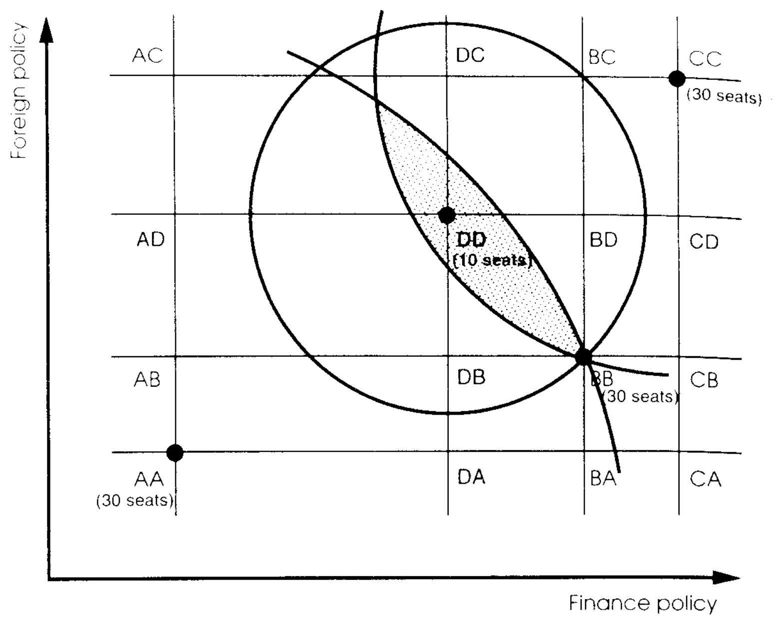

Figure 4.2. A DDM cabinet with a nonempty winset

the DDM cabinet, however, since Kadane's Theorem does not *guarantee* an empty winset for the DDM. An example of this is given in Figure 4.2. This shows a four-party system in which Parties A, B, and C control thirty seats each, and Party D controls ten seats. The DDM cabinet is BB, since Party B is at the median position on each policy dimension. The indifference curves through BB show how each of the other parties feels about BB.[11] Since Parties A and C are essential members of any winning legislative coalition that excludes Party B, while Party B obviously prefers its own ideal point to anything else, the winset of the BB cabinet is the intersection of the indifference curves centered on AA and CC and passing through BB – the shaded area in Figure 4.2. This area, however, contains an alternative cabinet, DD, preferred by both Party A and Party C (and obviously by Party D) to BB, despite the fact that BB is at the median position on both dimensions.

In this particular case, we know that both BD and DB are preferred by

[11]Recall that the points on the circle through a designated point (like BB) centered on a party's ideal are equidistant from that ideal and hence are equally preferable to the point in question (BB). Hence, the circle is called an *indifference curve.* Points inside the circle, being closer to the party ideal than is the point in question, are preferred to the latter.

a majority to DD since each of these is at the median position on just one more dimension.[12] By the same logic, BB is preferred by a majority to either BD or DB. In other words there is a cycle of potential cabinets, BB → DD → (DB or BD) → BB, with each cabinet in the cycle majority-preferred to the one preceding it. Nonetheless, as simulation results reported later in this volume confirm, most party configurations with a relatively small number of parties and salient policy dimensions do not generate cabinet cycles of this type. In these common cases, the DDM cabinet is an equilibrium, implementing policies at the generalized median of the policy space.

STRONG PARTIES AND EQUILIBRIUM CABINETS

What is a strong party?

Although the generalized median cabinet is often an equilibrium in the government formation process, it may not be the only one. The fact that any of the members of a proposed cabinet may veto it by refusing to participate means that there may be other equilibriums. To demonstrate that this can be so, Figure 4.3 shows the same government formation situation as Figure 4.1, but now displays how the various parties feel about a cabinet in which Party B takes both key portfolios. The intersection of indifference curves aa and cc – the shaded area – is the set of policies preferred by both Party A and Party C to the policy forecast for BB. We can see quite clearly that two alternative cabinets, BA and BC, are preferred by a legislative majority (comprising Party A and Party C) to BB. Note, however, that Party B is a participant in both BA and BC, and therefore can veto these proposals. There is no alternative to BB that is preferred by a legislative majority but which Party B cannot veto. This puts Party B in a very powerful bargaining position. Once a Party B minority government is in place, for example, it may prove difficult to dislodge, since to do so requires Party B's assent.[13] For this reason we call parties that are like Party B (in ways we will shortly make more specific) strong parties. Such parties are in a position to dominate the government formation process.

We can generalize the type of situation shown in Figure 4.3 by defining Party S as *strong* if it participates in every cabinet preferred by a majority

[12]We know this from Kadane's Improvement Algorithm, on which see footnote 8 of Chapter 1 and the appendix to the present chapter.

[13]For the time being we do not worry about how Party B came to be a single-party minority government in the first place. The possibilities are not so bizarre. For example, Party B might have been a majority government before an election in which it subsequently lost its majority. Government formation would then begin with a Party B minority government as the status quo.

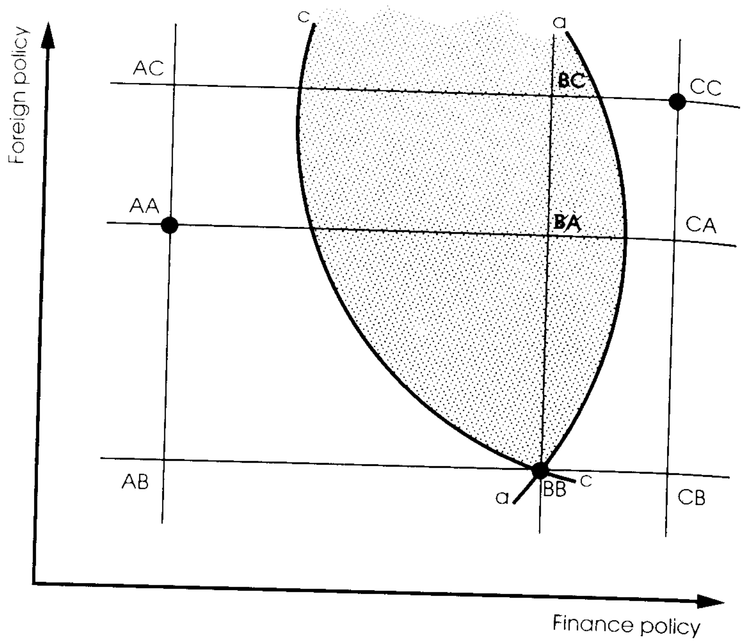

Figure 4.3. The winset of a BB cabinet

to the cabinet in which Party S takes all portfolios. In practice, there are two rather different types of situation in which this can happen. First, if no cabinet at all is preferred by any majority to some particular single-party government, then the ideal point of this particular party must have an empty winset and the party must obviously be strong. We know from Kadane's result that if a strong party has an ideal point that has an empty winset, then this is at the generalized median. In this case we designate the party as being *very strong*. Second, a party may have an ideal point with a nonempty winset, but be strong because the party participates in, and hence can veto, every cabinet in this winset. In this case, the strong party must use its power of veto in order to assert its strength. For reasons to which we will return, a particular strong party may not credibly be able to threaten such vetoes. We therefore designate a strong party whose ideal point has a nonempty winset, and whose strength derives from vetoes, as being *merely strong*.

Thus, the two different types of strong party derive their strength from different sources. The *very* strong party is strong quite simply because no majority prefers an alternative cabinet. The *merely* strong party does not find itself in this enviable position. Its strength derives from threats to

veto certain alternative cabinets. While the support of a parliamentary majority is the firm foundation on which the strength of a very strong party is based, the strength of a merely strong party derives from the credibility of threats that may be firm one moment and run like sand through its fingers the next.

Strong parties have a number of properties that help us to characterize equilibriums in the government formation process. The first of these is that *there can be at most one strong party*. If there were two strong parties, say S_1 and S_2, then the ideal point of one would have to be in the winset of the other.[14] But if, for example, S_1 were in the winset of S_2, then there is a point in S_2's winset in which S_2 does not participate. This contradicts the definition of a strong party.

The fact that there can be at most one strong party means that the strong party, if one exists, will be a focal actor in the government formation process. No other actor is in a position to insist on forming a government on its own. This is because, for every other party, not only is some alternative majority-preferred to the cabinet giving the party all key portfolios, but this is an alternative that it cannot veto.

We can show by example that there may be situations in which there is no strong party. Figure 4.4 shows such an example. This is a modification of the situation shown in Figure 4.2, resulting from splitting the original Party D into two parties, D and E. Party E has an ideal point at EE. We know from Figure 4.2 that DD is preferred by a majority to BB, so Party B cannot be strong. We can see from the cc and bb indifference curves in Figure 4.4 that Parties B and C, who have a majority between them, prefer EE to DD. So Party D cannot be strong. We know from the aa indifference curve and the fact that Party B prefers BB to anything else that BB is majority-preferred to EE. So party E cannot be strong. It is also easy to see that DD is majority-preferred to both AA and CC, so Parties A and C cannot be strong. Thus there is no strong party.

In general, however, as we shall show in Chapter 6, strong parties are quite common, both in simulated and real party systems. This is important, since strong parties have a significant impact on the government formation process, as we now see.

Strong parties in government formation

Since no alternative cabinet is majority-preferred to a cabinet in which a *very* strong party takes all key portfolios, we can easily see that this cabinet is in equilibrium. This is because a very strong party must have an ideal point (written s^{**}) at the DDM and, from Proposition 1, when the

[14]Assuming the majority preference relation is complete, then for any two cabinets, X and Y, either $X \in W(Y)$ or $Y \in W(X)$.

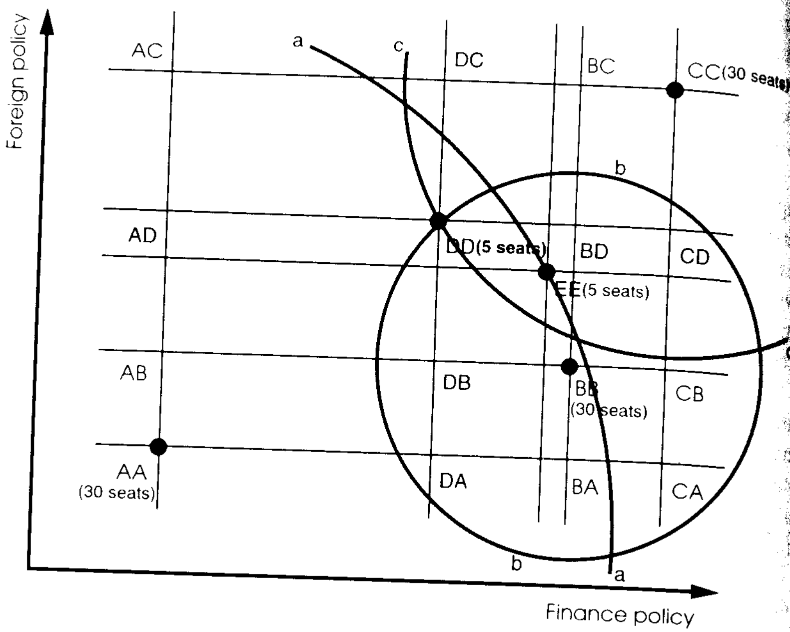

Figure 4.4. A system with no strong party

winset of the DDM is empty, the DDM cabinet is a retentive equilibrium.[15] That it is also an attractive equilibrium follows from a consideration of the government formation process described in Figure 3.1. In this, the very strong party simply waits until it is recognized and proposes its ideal point, a cabinet in which it gets all portfolios.[16] From the definition of a very strong party, this cabinet is majority-preferred to all others, including any other status quo. Since no other party participates in this cabinet, none can veto it. If some party proposes another cabinet, this proposal will lose since the strong party ideal is majority-preferred to it and no majority has any incentive to vote contrary to its preferences in this instance; this is because there is no alternative to a cabinet controlled by the very strong party that any majority could be aiming toward. So whatever the status quo might be, and however it may provisionally have been changed,[17] all parties will anticipate that the strong party ideal will

[15]Since $SE(s^{**}) \in \{s^{**}\} \cup W(s^{**})$ and $W(s^{**}) = \varnothing$, then $SE(s^{**}) = \{s^{**}\}$.

[16]Since every party has some probability of recognition, the strong party is bound to be recognized at some point.

[17]It is perfectly all right for the original status quo to be replaced by something else. Once s^{**} is proposed, it will nevertheless prevail. Consequently, with rational foresight at work, these interim changes in the status quo may never materialize (though, as noted, it won't affect our results if they do).

eventually be proposed, will not be vetoed, and will thus be invested in office. In short, a *very strong* party can form an equilibrium cabinet on its own, whether or not it controls a legislative majority.

The existence of any strong party, however, whether it is very strong or merely strong, has a major strategic impact on the government formation process. This is characterized by our second major proposition about government equilibrium in parliamentary democracies:

Proposition 4.2: When a strong party exists, it is a member of every equilibrium cabinet.

Another way of putting this proposition is that, if there is a strong party, then the equilibrium cabinet will either be the strong party ideal point, written s^*, or an element in the winset of this point, $W(s^*)$, in which the strong party, by definition, participates. A precise, and slightly more general, statement of this proposition and a full formal proof are provided in the appendix to this chapter.

The crucial feature of this result is that it is not possible to form an equilibrium cabinet without the participation of an existing strong party. This in turn means that the existence and identity of a strong party is a very important strategic feature of any government formation situation. We shall have much more to say in Part III of this book about the characteristics of a strong party, but it is clear at this stage that its identity is entirely determined by the configuration of party weights and policy positions. Thus changes in weights and policy positions may forge, revise, or obliterate the identity of the strong party. Quite small changes may have quite big effects, for instance, if they move into the winset of the strong party an alternative cabinet in which it does not participate.

What this means is that not only is the role of a strong party of considerable interest in itself, but also that particular party configurations might have striking strategic discontinuities. As a result of such discontinuities, a configuration of parties can be such that small perturbations of key parameters have big effects on the government formation process, by changing the existence and identity of a strong party, for example. If a party system is close to some strategic threshold, then small changes in party weights may change the decisive structure,[18] or small changes in party policy may change the location of key indifference contours. Both types of change will affect the contents of winsets. The addi-

[18]In the 1992 Irish election, for example, the entire system of possible coalitions depended on the very close outcome of the final seat in one constituency. The votes in this constituency were counted over and over again for a week, with each recount changing the result and being appealed by the loser, until a final result was declared. Serious coalition bargaining could not begin until this recounting had been completed. This case is developed in more detail in Chapter 6.

tion or subtraction of points from the winset of some party ideal may well determine whether that party is strong, and hence able to dominate the making and breaking of governments. Alternatively, a configuration of parties might be quite far from a strategic threshold, so that quite large changes in the parameters, while changing the shape of the winsets in the continuous policy space, may have no effect whatsoever on the list of alternative cabinets in key winsets. In such circumstances a wide range of different election results, or quite large shifts in party policy, would sustain the same cabinet equilibrium.

Strong parties and standoffs

A strong party, and in particular a *merely* strong party, need not necessarily get everything its own way, however. Although it is a participant in every equilibrium government, a merely strong party cannot necessarily impose its ideal point as the outcome of the government formation process. Remember that a very strong party has no alternative cabinet in the winset of its ideal point, s^{**}. A merely strong party does have alternative cabinets in the winset of its ideal, s^*. It is strong by virtue of its ability in principle to veto every one of these. However, it is possible to imagine situations in which a merely strong party may be forced to lift its veto on elements of $W(s^*)$ as the result of strategic interplay with other parties. We can gain some intuition about this by revisiting the example in Figure 4.3, in which Party B is the merely strong party and the generalized median cabinet is BA. (Hold onto your hats, however; we are about to explore some quite complex strategic thinking in this deceptively simple-looking case!)

Suppose that, for some reason that need not concern us here, the status quo cabinet in Figure 4.3 is CC. We know that the strategic equivalent of CC is either CC itself or a cabinet that is majority-preferred to CC.[19] The points that are majority-preferred to CC are {AB, BA, BB, BC, CA}. From a status quo of CC, therefore, the government formation process will thus end either with an equilibrium cabinet at CC or with any of these five alternatives to CC. We also know from Proposition 2 and Figure 4.3 that, since B is a strong party, the government formation process must end with an equilibrium cabinet that is either BB or an element in its winset, W(BB). Thus the equilibrium cabinet is in {BA, BB, BC}. Knowing that the status quo is CC and the strong party is B, therefore, allows us (and, far more important, the parties involved in government formation!) to concentrate on cabinets that are simultaneously in {AB, BA, BB, BC, CA} and {BA, BB, BC}, that is, on three cabinets – BA, BB, and BC.

Consider now what any of the parties can do if recognized to propose

[19] $SE(CC) \in \{CC\} \cup W(CC)$.

a new cabinet. Since it is the focal actor in the government formation process, consider first the strong party, Party B. Among the prospective replacements for CC, Party B clearly most likes its ideal point, BB. If recognized, Party B could propose this. However, BB is the point in {BA, BB, BC} least liked by Party A. Furthermore both Party A and Party C, jointly controlling a majority, prefer BA to BB. Party A could thus vote strategically against replacing CC with BB, even though it prefers BB to CC. It would do this in the anticipation of eventually being recognized and proposing BA, which is majority preferred to any other cabinet.

However, if Party A proposes BA, then Party B can strategically veto this in favor of the CC status quo, even though B prefers BA to CC. It would do this in the anticipation of eventually being recognized and proposing BB and in the further anticipation that Party A will reluctantly accept that B's veto on BA will never be lifted. If Party A does accept this, then it will vote to replace CC with BB.

This generates a potential standoff between Party A and Party B. (Party C can do no better than to vote sincerely to retain CC.) Each party prefers both BA and BB to the status quo of CC. But Party A prefers BA to BB, while Party B prefers BB to BA. This standoff is quite close to a chicken game. Party B can threaten to keep vetoing BA, thereby keeping CC in place, until Party A gives in and votes for BB (which A does prefer to CC). Party A can threaten to continue voting strategically against BB, once more keeping CC in place, until Party B gives in and lifts its veto on BA (which B does prefer to CC).

If Party A always votes strategically against BB, and always proposes BA when recognized, and if Party B forms the opinion that Party A will continue to do this, then it is rational for Party B to lift its veto over BA, vote for this as a replacement for CC, and thus allow BA to form. If, on the other hand, Party B always vetoes BA, and always proposes BB when recognized, and if Party A forms the opinion that Party B will continue to do this, then it is rational for Party A to change its strategic vote against BB, vote for BB as a replacement for CC, and thus allow BB to form.

We cannot forecast how this standoff will be resolved without making new assumptions about the opinion formation process we have just described. For the purposes of Proposition 2, however, we do not need to know precisely which of the two outcomes involved in the standoff will actually prevail, since the outcome is either the strong-party ideal or some other cabinet that is majority-preferred to this. In either case, as claimed in Proposition 2, the strong party participates in the cabinet that takes office.

Readers who want to get a flavor of the strategic logic of this argument in the context of a real-world example will find this in Chapter 6. Among other things this describes a case in which a merely strong party – in this case Fianna Fáil in Ireland in 1993 – was forced to take a coalition partner despite holding all portfolios in a minority caretaker cabinet at the beginning of the government formation process. The reason for this was because Fianna Fáil was forecast to lose key standoffs with other parties, and thus could not impose its ideal point. The party did, however, participate in the cabinet that did actually form, which was at the dimension-by-dimension median position.

The possibility of standoffs thus explains why a merely strong party is not omnipotent, and may not be able to impose its ideal point even if this is the status quo. In the example in Figure 4.3, imagine the status quo is BB, in which Party B, the strong party, gets both key portfolios. Parties A and C both prefer BA to BB. If we assume that both A and C know that Party B does not have what it takes to win a standoff, however they might conceive of "what it takes," then they can force B to lift its veto of BA, by threatening to install CC as an interim cabinet. Party B cannot prevent CC from forming. Once CC has formed, Party B will then, by assumption, lose the standoff provoked by any attempt to reimpose BB by using its veto over BA. Given rational foresight, it will not actually be necessary to install CC, since B will recognize that its inability to win standoffs undermines its ability to maintain a status quo of BB. If a strong party cannot win standoffs, therefore, it may not be as strong as all that!

This last conclusion may be qualified, however, whenever the dimension-by-dimension median cabinet has an empty winset. In this circumstance we have

Proposition 4.3: When there is an empty-winset DDM, no cabinet in the winset of the strong-party ideal is in equilibrium if it is less preferred by the strong party to the DDM.[20]

A proof of this proposition is found in the appendix to this chapter. The proposition tells us that, even in those circumstances when a merely strong party does not have "what it takes" to win standoffs, the equilibrium will never be less desirable for it than a dimension-by-dimension median cabinet with an empty winset. Put another way, to the extent that a strong party can use its vetoes, it will be able to use these to impose outcomes that it likes at least as much as a DDM cabinet with an empty winset. In effect, it uses its strength to pull outcomes away from the center toward, even if not as far as, its ideal point. At the very least, it can use its vetoes to prevent the formation of cabinets it likes less than the DDM. In Figure 4.3, to continue our earlier example, BC is in the winset of the strong-party ideal and is thus in the equilibrium set identified in Proposition 4.2. BC can never be an equilibrium, however, because the strong party likes it less than BA, an empty-winset DDM. (There is no standoff between Party B and a legislative majority on the relative merits of BC and BA, since both Party B and a legislative majority prefer BA to BC.)

[20]Formally, no x in $W(s^*)$ for which $m^{**} >_s x$ is an equilibrium (where $>_s$ is the strong party's preference relation).

Before concluding this chapter, we note that the strong-party concept may be generalized. In the appendix to this chapter, we develop the notions of a *holdout party* and a *holdout point*, which may exist even when a strong party does not, and which specialize to the strong party and its ideal, respectively, when the latter do exist. The intuition here is that a party may not, like a strong party, be able to "hold out" for its ideal point, but nevertheless may be in a sufficiently distinguished strategic position to insist on some other portfolio allocation. Several propositions and an example on this idea are developed in the appendix.

STRONG PARTIES AND CENTRIPETAL TENDENCIES

Having specified a rather stylized generic model of the making and breaking of governments in parliamentary democracies, we have been able to identify a number of fascinating and important strategic aspects of this process.

First, one possible equilibrium cabinet is at the dimension-by-dimension median position, the DDM. Only for a cabinet at this position is it possible that no alternative cabinet is preferred by a legislative majority. If this is the case, then a cabinet at the DDM is a possible equilibrium (Proposition 4.1). This is a generalization of the strongly centripetal tendency to be found in the unidimensional spatial model of voting, with the pivotal role of the median voter. It suggests that, even when more than one dimension of policy is important, and the election does not manufacture a government by giving one party a legislative majority, the process of government formation nevertheless has strong centripetal tendencies.

Second, there may be many party configurations in which a single party is in a particularly powerful position in the government formation process. Some party may be *very* strong. Such a party has an ideal point at the dimension-by-dimension median, and no alternative cabinet is majority-preferred to it. In this case the conclusion is clear: It is very hard to see how a very strong party can be excluded from government (Proposition 4.1). In other cases a party may have an ideal point that is not at the DDM (so that there are indeed alternative cabinets that are majority-preferred to it), but may nonetheless be *merely* strong in the sense that it can veto any cabinet that is majority-preferred to its ideal point. In this event, the party clearly has an incentive to hold out for its ideal point in the government formation process although, if it is to achieve this, every pivotal actor will need to believe that the merely strong party can win strategic standoffs. Whether or not it can win standoffs, however, even a merely strong party will also be very difficult to exclude from government, either forming its ideal cabinet on its own, or participating in some cabinet that is majority-preferred to this (Proposition 4.2); in the latter case, the strong party can always ensure the formation of a cabinet that it prefers at least as much as a dimension-by-dimension median cabinet with an empty winset (Proposition 4.3).

What is striking about the role of these "stronger" parties is that they must have ideal points that are central in some sense. By definition, the very strong party is at the center of things – its ideal point *is* the DDM. In the case of a merely strong party, when the DDM cabinet is majority-preferred to all others, the merely strong party must be at the median position on at least one dimension of policy. This is because it has to be in a position to veto the DDM cabinet, which it otherwise could not prevent from forming. Thus each of these types of strong party has the intention of implementing some public policy position that is relatively central.

Finally, note how the existence of strong parties gives some structure to the government formation process, at the same time making government formation quite different from voting for bills or other items of legislative business. Strong parties are strong because of the way that their vetoes over possible cabinets can be used – and these vetoes arise precisely because parties are involved in forming a government rather than just voting on bills, and no party can be forced into government against its will. For this reason, forming a government may be more structured and less chaotic than generic policy voting in an assembly.[21]

We have nothing specific to say about government formation in those situations in which there is no strong party or when at least one alternative cabinet is majority-preferred to the dimension-by-dimension median cabinet (beyond the holdout party generalization in the appendix). In such situations, there may be cycles of cabinets – essentially of the same sort that generically plague multidimensional legislative models, that is, Cabinet B defeats Cabinet A; Cabinet C defeats Cabinet B; Cabinet A defeats Cabinet C; and so on, ad infinitum. Without making further assumptions about the government formation process, there is nothing in our model to imply anything other than a chaotic sequence of proposal and counterproposal.[22] Any cabinet that might take office would appear to be generically unstable, since some alternative must be majority-preferred to it, and this alternative cannot be prevented from forming by the vetoes of a strong party. Other things being equal, therefore (and in real life they may well not be), a party system that has no strong party and no empty-winset DDM cabinet seems likely to be more unstable than one that does.

[21]Furthermore, once the government has taken office, its control over the legislative agenda adds structure to other aspects of legislative decision making.

[22]Again, the generalization we offer in the appendix should be understood as a modest qualification to this assertion.

Appendix: Formal proofs and statements

TERMINOLOGY AND PRELIMINARY ARGUMENTS

Let L be the lattice of policy forecasts associated with each of the finite sets of possible portfolio allocations. For any point $x \in L$, if a point $x' \in L$ is preferred by a majority of legislators to x, we say that x′ *wins* against x. The set of points on L that win against x is the *lattice winset* of x, W(x) – we refer to this hereafter simply as the winset of x.[23]

We assume Euclidean preferences. Let y^i be the ideal point of party i; thus in Figure 4.1, $y^A = AA$, $y^B = BB$, $y^C = CC$. For any two arbitrary points, x and $z \in L$, party i prefers x to z, written $x\ R_i\ z$, if and only if $|y^i - x| \leq |y^i - z|$. The two sides of this inequality are, respectively, the Euclidean distance between x and party i's ideal and the distance between z and party i's ideal. Moreover, for any arbitrary point $z \in L$, we describe the set of lattice points party i prefers to it by $R_i(z) = \{w \in L: w\ R_i\ z\}$. $R_i(z)$ is i's *preferred-to-z set.*

From the assumption of Euclidean preferences, each $R_i(z)$ set will be the points on L contained in a circle in two-dimensional spaces (hypersphere in n-dimensional spaces) centered on y^i with radius equal to $|y^i - z|$. In Figure 4.1, $R_i(BA)$ for each party is indicated by the points on L inside the circle passing through BA and centered on their respective ideal points.[24] $R_i(z)$,

[23]In the appendix, lowercase letters will be used throughout to label both policies and governments. Unless otherwise specified, we shall always be referring to the *lattice* winset. If there is any possibility of confusion, we will be more explicit in distinguishing policies from governments.

[24]For preferences that weigh the dimensions according to their relative salience for i, party i's preferred-to set is, in the two-dimensional case, the set of points on L inside an ellipse (ellipsoid in the n-dimensional case). The lengths of the major and minor axes are proportional to the relative salience the party places on the various dimensions. The orientation of the ellipse relative to the dimensions of the space gives the degree to which preferences over the various dimensions are correlated. If the orientation of the ellipse yields axes parallel and perpendicular to the dimensions of the space, then preferences are said to be separable by dimension.

for each i and for any alternative $z \in L$, fully characterizes each party's preferences over alternative points on the lattice.

We may now use the preferred-to sets of individual parties to describe the preferences of majorities. To do so, recall that a party has a weight (w_i for party i) equal to its share of parliamentary seats. A collection of parties constitutes a majority if the sum of their weights exceeds one-half. Let M be the set of all majority coalitions, and let K be one such majority coalition in M.

Consider the preferences of the parties that comprise K. Specifically, for any arbitrary policy x, the $R_i(x)$ sets for i in K are the implementable policies preferred by each party in K to x. The intersection of all these sets gives the policies preferred to x by *every* party in K. Since K is in M, it follows that the intersection of the $R_i(x)$ sets, $i \in K$, contains points on the lattice that beat x in a majority rule contest with K decisive. The union of these intersections over all K in M fully describes the set of lattice points that are preferred by some majority to x. Specifically, the *winset of x*, W(x), is the set of points preferred to x by all the parties in some winning coalition.[25]

A party, S, may be such that it participates in every point in the winset of its ideal point, s^*. We call such a party a *strong party*. The median position on any policy dimension is defined in the normal way. The multidimensional median position is labeled m^*. If all portfolio jurisdictions are simple (unidimensional), then $m^* \in L$.[26]

If all jurisdictions are simple, then an extension of a result by Kadane (1972) shows that, if any point has an empty winset, it must be m^*. Kadane's proof uses an "improvement algorithm" whereby any point x that is not m^* is beaten in a majority vote by some other point x' that is identical to x in every respect save that there is some dimension for which x is not at the median position and x' is at the median position. Iterative application of the algorithm until $x' = m^*$ yields the proof: Either m^* has an empty winset or it is part of a majority cycle. (Note that, if at least one jurisdiction is complex and $m^* \notin L$, then it can be shown by example that it is possible for a point that is not m^* to have any empty winset.) Thus, when all jurisdictions are simple, at most one point can have an empty winset; if such a point exists, we designate it as m^{**}.

[25]For K a majority coalition, the set of points on L preferred by all its members to x may be written $\cap_{i \in K} R_i(x)$. We may now write the winset of x as:

$$W(x) = \cup_{K \in M}[\cap_{i \in K} R_i(x)].$$

[26]If at least one jurisdiction is complex (multidimensional), then m^* may not be an element of L, because the same party may not be at the median position on all dimensions within a given jurisdiction. In this appendix we restrict attention to simple jurisdictions; complex jurisdictions will be taken up in Chapter 11.

KADANE AND THE LATTICE

In Figure 4.1, notice that W(BA) is empty. The set of policies preferred by any majority coalition does not intersect L. There are clearly *policies* that majorities prefer to those that the BA government would implement. But there is no *cabinet* (i.e., lattice point) preferred to BA.[27]

Notice also that BA is the *multidimensional median*, m^*: Party B is median on the horizontal dimension, while party A is median on the vertical dimension. In fact, this is a special case of a more general proposition, an extension of Kadane's Theorem to the lattice:

> *Kadane's Theorem on the Lattice*: In a multidimensional, majority-rule, spatial model restricted to the lattice and simple jurisdictions, where parties have preferences that are single peaked and separable by dimension, either m^* has an empty winset or no point does.

Proof: To establish the result we must show that $W(x) \neq \varnothing$ for any lattice point $x \neq m^*$. Let $x = (x_1, x_2, \ldots, x_n)$ and $m^* = (m_1, m_2, \ldots, m_n)$, with $x_i \neq m_i$ for one or more index values by hypothesis. Specifically, assume $x_j \neq m_j$. Consider $z = (x_1, x_2, \ldots, x_{j-1}, m_j, x_{j+1}, \ldots, x_n)$ – z is simply x with the j^{th} component replaced with m_j. Since x and m^* are elements of L, then so is z. Next consider the hyperplane, H, through z and m^* (see Figure 4.5). From the definition of m^*, H partitions the space into two subspaces. For each subspace, the ideal points in it or on its boundary (H) have weights summing to one-half or more. Consider now the hyperplane H', the perpendicular bisector of the line from x to z. A fortiori, the set of ideals on H' or in the subspace containing z sum to one-half or more. Hence a majority prefers z to x, that is, $z \in W(x)$. Since x is any arbitrary lattice point other than m^*, the theorem is established.

STRONG PARTIES

In this section we establish Propositions 4.2 and 4.3. The statement below of Proposition 4.2 is more precise and slightly stronger than the version in the text. We write the strong party as S and its ideal point as s^*.

> *Proposition 4.2*: When S exists, the stationary subgame perfect equilibrium of the government formation game is in $\{s^*\} \cup W(s^*)$. Specifically, (i) if the status quo (SQ) is not an element of $W(s^*)$, then the equilibrium is s^* if S wins standoffs and an element of $W(s^*)$ otherwise; (ii) if SQ is an element of $W(s^*)$, then so is the equilibrium.

[27]If we write the conventional spatial modeling winset as $W^*(x)$ – this is the set of points in the full space (not restricted to L) preferred by a majority to x – in which case $W(x) = W^*(x) \cap L$, then W(BA) is empty, whereas W^*(BA) is not.

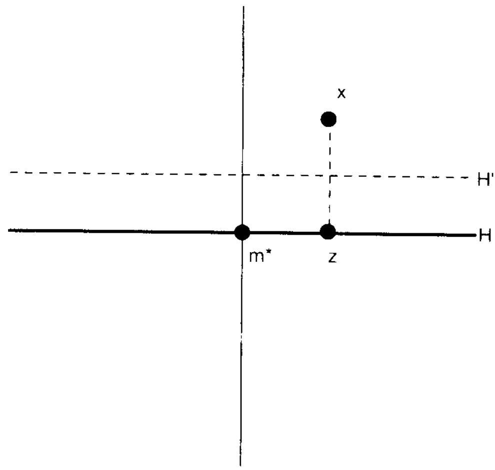

Figure 4.5. The Kadane Theorem and the lattice of credible points

We establish Proposition 4.2 by examining four mutually exclusive cases that between them exhaust all possibilities. These are defined according to whether the status quo is an element of $W(s^*)$ and on whether the resolution of the various standoffs favors S in the sense that it will prevail (written: "S favored"). Thus, we do not explicitly model beliefs about relative bargaining skill ("intestinal fortitude" or "resolve"); we take this as exogenously given and commonly known ex ante.[28] We derive four statements, set out schematically in Table 4.1, that jointly establish the proposition. The logic supporting each statement then follows.

Statement A: If S is favored and $SQ \notin W(s^*)$, then $SE(SQ) = s^*$.

Partition the actors into {S}, P_1, and P_2. P_1 is the set of actors (other than S) who prefer s^* to SQ, while actors in P_2 prefer SQ to s^*. Stationary subgame perfect strategies for actors in each group yield s^* as the equilibrium. Party S is in the driver's seat. Since it is common knowledge that S is favored, no other actor will propose a $y \in W(s^*)$, because he anticipates S's certain veto that cannot be undermined by strategic threats (this is what "S favored" means). Thus, the only conceivable proposals are s^* itself or some $y \notin W(s^*)$. For every such y from this latter set, there is a majority preferring s^*. Thus, for no y (other than s^*) is it true that $SE(y) = y$. Why? Because, if y becomes SQ, party S (or some member of P_1) will at some point be recognized, propose s^*, and it will pass.

[28]So long as the common-knowledge condition is met, the precise outcome of standoffs is of no consequence for the validity of the proposition.

Table 4.1. Statements establishing Proposition 4.2

	$SQ \notin W(s^*)$	$SQ \in W(s^*)$
S favored	Statement A: $SE(SQ) = s^*$	Statement B: $SE(SQ) \in W(s^*)$
S not favored	Statement C: $SE(SQ) \in W(s^*)$	Statement D: $SE(SQ) \in W(s^*)$

Statement B: If S is favored and $SQ \in W(s^*)$, then $SE(SQ) \in W(s^*)$.

This follows directly from Statement A. If the path from SQ were to pass through a $y \notin W(s^*)$, then $SE(SQ) = SE(y)$ and $SE(y) = \{s^*\}$ from Statement A. But because a majority prefers SQ to s^* by construction, and SE(y) is s^*, this majority would never permit a path through a $y \notin W(s^*)$. The equilibrium path from SQ, therefore, must remain in $W(s^*)$.

Statement C: If S is not favored and $SQ \notin W(s^*)$, then $SE(SQ) \in W(s^*)$.

In this situation, SQ involves a standoff between S and members of a majority who (i) prefer some $y \in W(s^*)$ to s^*, and (ii) prefer s^* to SQ. Part (i) follows since, if there were no such y for any majority, then S would be very strong with ideal point s^{**}, in which case Proposition 4.1 would apply. Part (ii) follows since the premise of the statement is that $SQ \notin W(s^*)$. This set of actors (a generic member of which is Alf) can vote strategically against s^*, if this is proposed, retaining SQ in the expectation of y being proposed. We know from the definitions of strategic equivalents and equilibriums that $SE(SQ) \in \{SQ\} \cup W(SQ)$. If S is forecast not to be favored in the ensuing standoff, then Alf must expect to realize a portfolio allocation y that is in both $W(s^*)$ and $W(SQ)$. Recall that Alf's preferences are $y >_\alpha s^* >_\alpha SQ$. If $SE(SQ) \notin W(s^*)$ and yet there were a standoff from SQ, then this would require Alf to be voting strategically for SQ over s^*, even though he preferred s^* to SQ and had no hope of getting anything better in $W(s^*)$, clearly contradictory behavior. This establishes that, when party S is not favored and $SQ \notin W(s^*)$, then $SE(SQ) \in W(s^*)$.

Statement D: If S is not favored and $SQ \in W(s^*)$, then $SE(SQ) \in W(s^*)$.

This follows directly from Statement C. If the equilibrium path from SQ to SE(SQ) passes through a y, then $SE(SQ) = SE(y)$. If $y \notin W(s^*)$ then

$SE(y) \in W(s^*)$, since, by assumption, S is not favored (Statement C). Thus $SE(SQ) \in W(s^*)$ as well. Otherwise, the equilibrium path from SQ remains within $W(s^*)$.

This concludes the proof of Proposition 4.2. It is worth noting that the *only* circumstance in which a strong party forms a single-party minority government is when $SQ \notin W(s^*)$ (SQ can be s^*, itself). And even here, S may be denied some of the portfolios if it cannot discourage standoffs. Consequently, the existence of a strong party is *not* equivalent to an assertion that it will uniquely form the government; the proposition only claims that S will always be a member of the government.

We assume that the bargaining skill of S is common knowledge, so that all actors know the row of Table 4.1 that applies. Of course, the common-knowledge assumption about preferences means they know the column as well. These common-knowledge conditions affect which government forms from the equilibrium set identified in Proposition 4.2. Party S nevertheless has some influence independent of bargaining skill. This effect is identified in the next proposition (although only for the case where an empty-winset DDM, m^{**}, exists).

Proposition 4.3: No $x \in W(s^*)$ for which $m^{**} >_S x$ is an equilibrium.

Assume, contrary to the proposition, that an x inferior to m^{**} for S is an equilibrium. Since S is strong, s^* is the unique holdout point (see text and next section of appendix); there are no other holdout points. Thus, no participant in m^{**}, other than S, can credibly veto a move to m^{**}. Reason: Any policy outcome they might seek by this move would then be their holdout point – contradiction. This means that, in accord with the government formation game of Figure 3.1, any player preferring m^{**} to x may move m^{**}, no non-S participant in m^{**} will credibly veto that motion, and a legislative majority prefers m^{**} to any other point. Consequently, the only ways in which m^{**} will not be the equilibrium are (1) if S vetoes m^{**} or (2) if some member(s) of the majority preferring m^{**} vote strategically. The first will occur only if S can obtain a $y \in \{s^*\} \cup W(s^*)$ that it prefers to m^{**}; in this case x is not an equilibrium and an outcome satisfying the proposition obtains. The second cannot occur: A majority, by construction, prefers m^{**} to any policy objective a potentially strategic voter might pursue and will block it; hence m^{**} will result, again consistent with the proposition. Thus, the assumed x, contrary to the proposition, cannot constitute an equilibrium.

HOLDOUT POINTS

We have just seen that a strong party can be in a position to have a considerable impact on the government formation process. Even when there is no strong party, however, one particular party may be in a stronger bargaining position than the others. It may be in a position to "hold out" for a cabinet that, while not its ideal point, is closer to its ideal cabinet than the generalized median.

This situation can arise for the following reasons. When there is no strong party there is, by definition, no party that participates in every cabinet that is majority-preferred to its ideal cabinet. However there may still be a party, H, that can find some cabinet, h (not necessarily Party H's ideal point, h^*), such that Party H both prefers h to every cabinet that is majority-preferred to h, and participates in every one of these cabinets. In this event, Party H, the *holdout party*, has an incentive to hold out in the government formation process for cabinet h, the *holdout point*, in just the same way as a strong party has an incentive to hold out for its ideal cabinet.

Formally, h is a *holdout point* and H is a *holdout party* iff:

1. H participates in every y in W(h);
2. H prefers h to every y in W(h);
3. other participants in h prefer it to every y in W(h); and
4. H prefers h to every other point satisfying conditions (1)–(3).

Throughout this development, we make use of an assumption that requires a modicum of preference diversity among party ideals:

Preference Diversity Assumption (PD): On each policy dimension a location is occupied by at most one party.

We are now able to establish a number of interesting properties for holdout points, properties we use subsequently to establish several theoretical propositions.

Property 1: The holdout point for party H is H's ideal point, h^*, if and only if H is strong.

Sufficiency follows by a direct application of the definition of a holdout point. Condition (1) of the definition is satisfied since H is strong and hence participates in every point in $W(h^*)$. Condition (2) is satisfied since h^* is H's ideal. Condition (3) is satisfied trivially. Condition (4) follows again because h^* is H's ideal. To establish necessity, suppose h^* were a holdout point, but H were not strong. But by condition (1), H must be strong. This contradiction establishes necessity.

Property 2: When there is no strong party, every holdout point must be at a median position on at least one dimension.

Consider a lattice point x at a median position on *no* dimension. Consider all the lattice points that result from projecting x onto median hyperplanes one dimension at a time. Call these projections $x^1, x^2, \ldots, x^n$ (where n is the dimensionality of the space). The point x^i is identical to x except that it is at the median position on the i^{th} dimension. From Kadane's Improvement Algorithm, we know that every such x^i is in W(x). For x to be the holdout point of some party, H, that party must participate in every point in W(x), and thus, by construction, in every one of the projections, x^i. This is possible only if x is H's ideal. But this means, from Property 1, that H is a strong party, contrary to hypothesis. Thus, when there is no strong party, every holdout point must be median on at least one dimension, and the statement is confirmed.

Property 3: There is no holdout point for any party not participating in m*.

Assume Party R, with ideal point r*, is a nonmedian party (it does not participate in m*), and its holdout point is h. Thus, h must be nonmedian on at least one dimension. Consider the projection of h onto the point x that differs from h only in that x is median on exactly one additional dimension (this is possible since h is not m*). There is, from the PD assumption, a unique party P that is median on this dimension. Thus, P participates in x. By the Pythagoras Theorem, $x >_P h$, and by the Kadane Improvement Algorithm, x is in the winset of h. Thus, party P is a participant in x, a point in the winset of the alleged holdout point, h, and prefers x to h. This contradicts part (3) of the definition of a holdout point. This implies that an h (distinct from m*) cannot be a holdout point for any party not participating in m*.

Property 4: There can be no more than one non-m* holdout point.

Since there is no holdout point for parties not participating in m* (Property 3), we may focus on median-participating parties, P and Q. Assume h_p and h_q are holdout points for P and Q, respectively, and that these points are distinct both from m* and from one another. From the completeness of the majority rule relation (putting ties to one side), without further loss of generality, let h_p be in the winset of h_q. This implies that Q participates in h_p from Condition (1) of the definition of holdout point applied to h_q.

The alleged holdout point, h_p, is nonmedian on at least one dimension (otherwise it would be indistinct from m*). On at least one of these dimensions, Q's ideal, q*, is median (since Q is a participant in m* and the PD assumption assures us that Q is the unique median participant on each of

these dimensions). Project h_p onto a point, x, so that x differs from h_p on exactly one of the dimensions on which Q is median. By the Kadane Improvement Algorithm, x is in the winset of h_p. By the Pythagoras Theorem, $x >_Q h_p$. Since Q participates in h_p, the latter is in violation of Condition (3) of the definition of a holdout point. Thus, h_p either cannot be distinct from m* or is identical to h_q.

Proposition 4.4: The holdout party is a participant in every equilibrium cabinet. The equilibrium cabinet is either the holdout point or an element of its winset. That is, the equilibrium cabinet is an element of $\{h^*\} \cup W(h^*)$.

Since there is at most one non-m* holdout point, h_q, the logic used to prove Proposition 4.2 (where party Q replaces party S and h_q is substituted for s*) can be used to show that $SE(SQ) \in \{h_q\} \cup W(h_q)$ for any SQ.

Paralleling the treatment of strong parties, a further generalization suggests that a holdout party influences the final equilibrium, whether it possesses the intestinal fortitude to win standoffs or not. We state this, without proof, as

Proposition 4.5: When there is an empty winset DDM, no cabinet in the winset of the holdout point is in equilibrium if it is less preferred by the holdout party to the DDM. That is, no $x \in W(h)$ for which $m^{**} >_H x$ is an equilibrium.

In sum, a holdout party is strong if there is a strong party, but a holdout party may exist even when there is no strong party. A nonstrong holdout party, like a strong party, has an advantage in government formation bargaining, and indeed should be a member of every equilibrium cabinet (Propositions 4.4 and 4.5).

The intuition concerning a holdout party works as follows. Suppose a party tried to hold out for its ideal point in the government formation process. That is, suppose its strategic purpose was to have itself installed as the sole party of government, taking all the key portfolios. If it were strong, it might be able to pull this off. Depending on the status quo and the beliefs of other parties about the intestinal fortitude of the party in question, the final government equilibrium will either be the strong party's ideal or a cabinet in its winset (Proposition 4.2). A nonstrong holdout party cannot hold out for its ideal (otherwise it would be strong). But it can hold out for some other cabinet. Specifically, on one of the dimensions on which it is median,[29] the holdout party can find a cabinet which it likes less than its ideal, but more than other feasible alternatives. An example will illustrate this.

[29]See Property 3 of holdout parties.

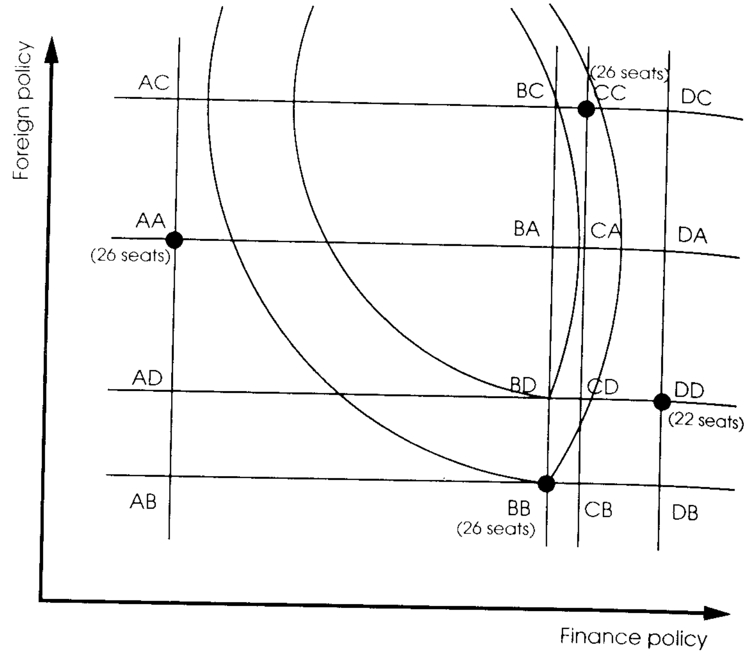

Figure 4.6. A system in which Party B can hold out for BD

Figure 4.6 shows a 100-seat parliament with two portfolios, finance and foreign affairs. Parties A, B, and C each have 26 seats, so that any pair of these have sufficient votes between them to install a government; Party D, with 22 seats, is a dummy. It is clear from the indifference curves through BB of Parties A and C that Party B is not strong, since there are lattice points in the winset of BB in which Party B does not participate. It may similarly be shown that none of the other parties is strong.

Since BA is the multidimensional median with Parties A and B its participants, these two parties are the only candidates for holdout party.[30] The only candidates for holdout point are lattice points lying on the median lines of the two median participants.[31] The best Party A can do is to hold out for BA itself; as shown in Figure 4.6, there is no non-DDM holdout point for A lying between AA and BA. So we turn our attention to Party B. The only holdout point candidates for B must lie on the dimension on which B is median,[32] and by inspection the only possibil-

[30]See Property 3 of holdout parties.
[31]See Property 2 of holdout parties.
[32]See Property 2 of holdout parties.

ity is BD. In fact, BD *is* a holdout point: (1) Party B participates in every lattice point in W(BD); (2) Party B prefers BD to each of these points; (3) Party D, the other participant, prefers BD to each of these points; and (4) Party B cannot hold out for anything it likes better.

This example suggests a general search procedure when looking for holdout points. First, look for a strong party. If no party is strong, then identify the median-participating parties as candidates for being a holdout party. Lattice points lying on the lines[33] containing both party ideals and the multidimensional median are holdout point candidates. At most, one of them can be a holdout point.[34] Our example establishes that a holdout party and holdout point can exist when a strong party does not. Thus, it does constitute a genuine generalization of the strong party concept.[35] However, we tend to find strong parties far more frequently than holdout parties in real data, so that the generalization may well turn out to be of greater theoretical than practical interest.

[33]Hyperplanes in more dimensions.
[34]See Property 4 of holdout parties.
[35]It is somewhat deflating to rely on a made-up example to illustrate holdout points. In fact, as the empirical analysis in Part III demonstrates, real instances of holdout points do arise in the data (but any of these would take some time to elaborate, so we relied for the purposes of exposition on a fictional case).

5

Strong parties

One of the key conclusions of the previous chapter – and indeed of our entire analysis – had to do with the role of strong parties in the making and breaking of governments. In a nutshell, if a party is *very* strong, then the cabinet giving that party all portfolios is preferred by some legislative majority to any alternative cabinet. A party may also be *merely* strong, however, if it participates in every cabinet that is preferred by a legislative majority to the cabinet in which it controls all portfolios. In the previous chapter we show that there can be at most one strong party. Our central result, stated in Proposition 4.2, is that a strong party can dominate the business of government formation and guarantee itself a place at the cabinet table. This result has fundamental implications, both for the analysis of particular cases and for our general understanding of the making and breaking of governments in parliamentary democracies.

If we want to analyze a particular government formation situation using our model, one of the first things we need to know is which party (if any) is the strong party. If we want to understand the process of government formation more generally, then we need to know about the conditions under which a configuration of legislative parties generates a strong party. Thus in this chapter we take a closer look at strong parties and the party configurations that sustain them.

We do this in several ways. First, we briefly discuss how to squeeze as much juice as we can out of our model by focusing on necessary and sufficient conditions for the existence of a strong party. Second, since this formal analysis does not give us much intuition about the existence of strong parties in actual party systems, we switch to an alternative mode of analysis, using simulation experiments to provide us with a feel for the circumstances in which a strong party might exist. We conclude the chapter by comparing our notion of a strong party to distinguished parties identified by other authors; in particular, we look at van Roosendaal's *central party* and Schofield's *core party*.

EXISTENCE OF STRONG PARTIES

Our model bases its strategic account of government formation on three types of information. The first is the jurisdictional structure of government decision making. This is seen in terms of the allocation of key policy dimensions to the jurisdiction of particular cabinet portfolios. The second is the decisive structure of the legislature. This is generated by the set of legislative parties, their weights, and a decision rule. The decisive structure describes the combinations of parties whose joint weight exceeds a threshold defined in the decision rule. Such combinations of parties are said to be decisive and, in this context, determine whether a government constitutionally retains the support of the legislature. In this book we take the decision rule to be majority voting, but there is absolutely no reason why our model should not be modified to work with other decision rules. The third type of information we use concerns the policy positions of legislative parties on key policy dimensions. With information on each of these three matters, we can implement our model to perform a strategic analysis of a particular case to determine the existence and identity of empty-winset DDMs and strong parties, and thereby identify potential equilibrium cabinets. In almost all of what follows, we assume a simple jurisdictional structure in which each cabinet portfolio has jurisdiction over a single key policy dimension (or at least over a very highly correlated set of key policy dimensions – we return to explore such matters in Chapters 11 and 12). What we will be concerned with in this chapter, therefore, are the ways in which particular configurations of party weights and policy positions sustain the existence of strong parties.

We have elsewhere conducted a formal analysis of the conditions for the existence of a strong party (Laver and Shepsle, 1993). Part of this analysis does offer us some intuition in the very simple special case in which there are two policy dimensions and three parties, any two of which are needed to support a government in a vote of confidence. Formal conditions for the existence of a strong party in the three-party case with two simple jurisdictions are derived in the appendix to this chapter. We can summarize these briefly in the following terms. Generically, the policy positions of three parties in two dimensions will form a triangle.[1] The longest side of this triangle will be the distance between the two parties who are farthest apart. Neither of these parties can be the strong party.[2] Thus only the third – "more central" – party

[1]If the three parties are precisely arranged on some line in the two-dimensional space, the analysis in this section collapses to one dimension in which the "central" party will always be strong

[2]By construction, each of these parties prefers the ideal point of the third party to the ideal point of the other. The third party prefers its ideal point to anything. Thus the

can be strong.[3] The formal results in the appendix to this chapter (to which the technical reader may wish to repair at this point) show that it usually is strong, but that it is not always strong.[4] (The simulation results to be reported provide more precise estimates of frequencies.) Thus the findings in this simple case reinforce the conclusion of the previous chapter that strong parties tend to be central, because they typically need to be participants in (so as to be able to veto) the dimension-by-dimension median cabinet. This implies that, while the power of strong parties may be used to pull cabinet policy away from the dead center of the party system, it will not be used to impose extreme policy positions, since parties in extreme positions cannot be strong in our sense.

We have also generalized our conditions for the existence of a strong party in more complex cases (Laver and Shepsle, 1993: 441–443). Unfortunately, while these conditions are not difficult to write down, they are not easy to relate intuitively to actual party configurations. They are thus not very useful – although they do teach us the general lesson that, even though strong parties are quite common in practice, the conditions for the existence of a strong party are subtle and complex. In practice this has the important implication that, while we can simply look at a particular party configuration and make guesses as to the existence and identity of a strong party, these guesses will not always be right, and a comprehensive strategic analysis can yield surprising results.

Paradoxically, therefore, the complexity of the formal conditions identifying a strong party is actually rather encouraging – the strong-party concept is not simply restating something obvious. Nonetheless the "ugliness" of these expressions indicates diminishing marginal returns from formal analysis and suggests that there is probably little to be learned from further mathematical manipulation. In order to deepen our intuitions, therefore, we offer a somewhat different perspective on strong parties – one gleaned from large-scale simulation experiments.

third-party ideal will be in the winset of the ideals of the two parties who are farthest apart, a point neither can veto. Thus, they cannot be strong.

[3]This party is more central in the sense that its ideal is closer to the ideals of the other two parties than the other two party ideals are to each other – a generalization of the notion of the central party of three parties arranged on a line.

[4]The "central" party will not be strong if the cabinet giving one portfolio to each of the other parties is in the winset of the central party's ideal – an unlikely but possible circumstance. To construct such an example, place the three parties in a triangle that is almost (but not quite) equilateral, identify the central party (not the two who are furthest apart) and rotate the triangle in relation to the lattice until the central party is median on neither policy dimension.

SIMULATED PARLIAMENTS

We have just seen that the conditions specifying the existence and identity of the strong party in a given party system depend on complex interactions between key variables. These interactions can be written down as mathematical expressions, but these expressions do not give us much intuition about the practical circumstances in which we are likely to encounter strong parties. We can of course begin to answer this question by looking at government formation in the real world, and we do this in Part III, where we provide an empirical evaluation of our model. However, strange as it may seem at first sight, the real world may not be the best laboratory for a full exploration of the possibilities for equilibrium in the government formation process we model. The reason is simple. When we study government formations we are studying important, but rather rare, events. In each of the countries we study, few governments actually form, while many variables change both between countries and between the formation of two consecutive governments in the same country. The real world simply does not provide us with the range of variation in key parameters that would allow us to conduct a full exploration of the equilibrium characteristics of our model.

Our attempts to elaborate the full implications of our model are thus constrained in two directions. Formal theoretical analysis (such as is found in Laver and Shepsle, 1993) yields expressions that, while rigorously derived, do not give us much intuition about what is likely to happen in particular real-world cases. Yet there are too few real-world cases to generate a rich empirical universe in which we can systematically explore our approach. Furthermore, one of the most valuable benefits of any model, including ours, is to allow us to address counterfactual – "what if?" – questions about situations that have not yet arisen in the real world. One solution to such problems is to use our model to conduct experiments in a simulated world. Although it is still rather rare in political science, the simulation approach has a number of advantages. We can generate as many cases as we like, and we can control as many variables as we like, carefully exploring the effect of manipulating our variables in a very systematic manner. We can thereby derive generalizations that are in many ways analogous to those derived from a formal deductive analysis, but which are much more closely related to things that we can observe in the real-world processes we are studying. Simulations are not a substitute for the real world that we analyze in Part III, and they certainly do not "test" our model in the way that we can with real-world data. Rather they are a theoretical development tool. In a complex theoretical environment, they allow us to develop generalizations that are useful surrogates for the type of abstract theoretical proposition we have just found so unhelpful when trying to determine conditions for the existence of a strong party.

For our present purposes, the use of simulation experiments allows us to generate a large number of hypothetical government formation settings in order to study their equilibrium properties. This allows us to explore systematically the ways in which particular characteristics of the government formation setting affect the existence and identity of a strong party.

For example, we can investigate systems with five equally weighted parties embedded in a policy space in which two dimensions of policy are important. We can use a computer to generate one, two, ten thousand, or ten million cases of such systems, in each of which the parties have different policy positions while everything else remains exactly the same. (This is the kind of dataset that it would take longer than the expected life of the universe for the real world to generate!) For each of these simulated parliaments, we can use our model to identify the strong party, if one exists. Taking the dataset as a whole, we can then explore the configurations of party policy positions that tend to sustain a strong party.

We can then repeat the analysis for different five-party decisive structures, for example, one with five parties in which one party can form a winning coalition with any of the other four, while all of the other four must combine to exclude the "dominant" party. We can explore three-, four-, six-, seven-, and eight-party systems, with various decisive structures. We can look at three-, four-, and five-dimensional policy spaces, and so on. In short, only the practical limits of our endurance and our computer firepower constrain our ability to create simulated worlds that allow us to explore relationships between variables that are too complex to characterize analytically, at least in an intuitively useful manner.

An analogy might give a better feel than a sermon for the role of simulation experiments such as these. Imagine that we want to find out some interesting things about a dart board – say, the area of the bull's eye. Calculating this area is a straightforward application of elementary geometry. Suppose, however, that you replaced the bull's eye with a picture of the face of your worst enemy. It is impossible, analytically, to calculate the precise area of this irregular and unpleasant object, but all is not lost. You can conduct an experiment to estimate very easily the ratio of this area to the area of the board as a whole. And, since you can easily calculate the area of the dart board, you can then estimate the area of your enemy's face. Pin your enemy's effigy to the bull's eye. Put on a blindfold and throw a thousand darts at random in the direction of the board, as hard as you like. If you have not been cheating and if the darts have really been thrown randomly, then the ratio of the number of

darts hitting your enemy's face to the number hitting the board gives the area of that face relative to the area of the whole board. The dart throwing experiment enables you to estimate something that you could not calculate analytically. Improving the precision of the estimate is simple, and might even be fun in this case. Just throw more darts, but don't peek till you're done! Note that you derive an estimate of the size of your enemy's face, not a precise calculation of its area; if the experiment was repeated, a different, but not very different, number of darts would hit the face.

The situation in which a strong party can exist, like the face of your worst enemy, is one of those ugly objects that defies analytical manipulation. We are able to give a precise characterization in the simplest of cases (as we do in the appendix to this chapter), but the analysis fails to provide insight as we complicate things, considering party systems with more than three parties, governments consisting of more than two portfolios, jurisdictions consisting of more than one dimension, and so on. Indeed, as we mention in the preceding few paragraphs (and demonstrate explicitly in Laver and Shepsle, 1993), the analytical conditions for the existence of a strong party in these more complex circumstances imply irregularly shaped regions of the policy space within which a strong party position might be located (generalizations of Figure 5.4 in the appendix to this chapter). Rather like throwing random darts at a board, however, we can generate random configurations of party positions for particular controlled sets of parameters and estimate the proportion of these in which a strong party exists. We can thereby get an idea of the relative size of the area in the policy space within which a party will be strong, relative to the area defined by the range of possible party positions. Repeating these experiments for different parameter settings allows us to identify the way in which changing particular parameters changes the likelihood of a strong party. We can do this even if we cannot provide a closed-form solution to the ugly set of conditions defining this area analytically.

Obviously, our simulated world will differ from the real world in many important respects, the most significant of which is that real party positions are not random. Rather, they are the result of complicated strategic interactions in larger political struggles of which government formation is but a part. An important consequence of this fact is that, even if a particular party configuration is extremely unlikely in random data, rational and strategic parties might home in on it with great frequency, making this configuration not at all atypical of the real world. Two Downsian parties in a unidimensional spatial world, for example, will home in on the ideal point of the median voter, even though randomly generated locations for these parties would render such a configuration very improbable.

This highlights the very important point that we do not present these simulation results as a test of our model in any sense whatsoever.[5] Rather, we emphasize that we are using simulations to get a feel for the way in which particular conditions, such as the decisive structure, the number of parties, or the dimensionality of the policy space, have an impact on the existence and identity of a strong party.

In subsequent chapters, notably Chapters 6 and 10, we go beyond this basic use of simulation technology and apply it to an enterprise to which it is ideally and uniquely suited. We use simulations to conduct "sensitivity" analyses on equilibriums in particular cases. Two quite different types of sensitivity analysis can be used to add considerably to our account of the making and breaking of governments. The first, which we will encounter in Chapter 6, allows us to estimate the potential impact of unreliable empirical data. The second, which we will encounter in Chapter 10, relates to cabinet stability. We briefly preview each of these analyses in the next two paragraphs.

In our empirical work in Part III of this book we rely on an expert survey of party policy positions in particular countries (described further in Chapter 7). We treat the mean expert opinion of a party's policy position as an estimate of where a particular party is *really* located. For particular countries, the number of experts can be small and the variance in their opinions can be large. As in any empirical research, there is inevitably some error in our data, therefore. Using our simulation technology, however, we can estimate the sensitivity of our findings on particular equilibrium cabinets to measurement error by simulating the error process. We do this by taking a particular "base case" in which we are interested, and generating a large number of simulated cases in each of which party policy positions are scattered randomly around our point estimates of the "real" party position. Each simulated party position is our base-case estimate of this plus an error term, the error term varying according to a variance estimated from observed variations in actual expert opinions in this case. We have less statistical confidence in the equilibrium forecasts of our analysis when our sensitivity analysis reveals that these are not robust to the variance in the experts' collective wisdom.

Perhaps the most important application of our simulation technology, however, is developed in Chapter 10 and has to do with the stability of cabinet equilibriums. Our model of Chapters 3 and 4 describes an environment in which there are major strategic discontinuities. In certain

[5]We do not after all want the suspicious reader to form the impression that we are setting out to test our model on data we have made up ourselves!

circumstances, for example, big changes in party positions may make no difference at all to the existence and identity of a strong party. Other circumstances may be much more delicately balanced, with very small changes in party positions making big differences as to whether some party is strong.[6] The more delicately balanced equilibrium is obviously far more susceptible to shocks to the system that could not have been anticipated at the time of government formation. In that sense, it is less stable. In Chapter 10, therefore, we use our simulation technology to conduct a different type of sensitivity analysis, estimating the sensitivity of particular cabinet equilibriums to random shocks, and thereby deriving estimates of the relative stability of cabinets.

In the next section, however, we use simulation as an exploratory tool to get a general sense of which factors affect the existence and identity of a strong party. While we can use pencil-and-paper techniques to apply our model to the strategic analysis of simple two-dimensional cases of government formation, if we add more dimensions, complex jurisdictions, and many parties, we quickly find that manual analysis is tedious and error-prone at best, impossibly time-consuming for even a single example at worst. However, the arithmetic of calculating the set of cabinets that are closer to some party ideal point than the particular cabinet under investigation – the arithmetic underlying the calculation of winsets – is essentially straightforward, if incredibly boring. In other words it is just the job for a computer, and we have designed a computer program, WINSET, that does the job for us.[7] This program can in principle take any number of differently weighted parties, each with positions in any number of dimensions, which may be allocated to a set of jurisdictions, and calculate the lattice winsets of any points in which we might be interested. The program can also identify empty-winset DDMs, search for the strong party if one exists, identify holdout points (the generalization developed in the appendix to Chapter 4), generate thousands of random party systems in a very carefully programmed manner to allow us to conduct controlled simulation experiments, and bake bread.

[6]Some party ideal may be very close to a crucial indifference curve defining the winset of the strong party ideal, for example. In this case, a small movement in party positions may easily undermine the strong party. In another case, no party ideal may be anywhere near a crucial indifference curve, so that huge changes in party positions are necessary before the status of the strong party changes.

[7]The WINSET program, which runs under DOS, is freely available for personal research and teaching use via Internet. The program itself, program manuals, and sample data files can be downloaded by connecting to FTP.TCD.IE, logging on as user "anonymous," and supplying a complete e-mail address as a password. The latest release versions of all files are located in the directory /PUB/POLITICS, which contains a README file describing what is available.

EXISTENCE OF STRONG PARTIES IN SIMULATED PARLIAMENTS

In what follows we use WINSET to create large numbers of datasets, each comprising 1,000 randomly generated parliaments. In each dataset we explore a particular type of party system, holding constant the dimensionality of the policy space, the number of parties in the system, and the decisive structure of the legislature – what we allow to vary randomly are the positions of each party on each policy dimension. We calculate the proportion of all such cases for which strong and very strong parties exist. We then repeat this experiment for different configurations of party system. We vary the number of parties, the decisive structure of the legislature, and the dimensionality of the policy space. Each time we generate a 1,000-case random dataset for a particular configuration of party system, with each of the 1,000 cases having all parameters constant except for randomly varying party positions. As a result of exploring the impact of the variation of various parameters on the identity and existence of strong parties, we gain some insight into the types of party configuration that can sustain a strong party.

Table 5.1 reports the results of a number of such simulation experiments. Each row represents a run of 1,000 experiments under specified conditions. Columns 1 and 2 describe those conditions. Columns 3 and 4 report the frequencies of all strong parties, then just of very strong parties, out of 1,000 different spatial party configurations. Each party policy position on each dimension in each configuration was an independent random draw from the uniform probability distribution on the [0,1] interval. The number of dimensions varies between two and four.

In column 1 we identify party systems ranging in size from three to seven for which we examine specific decisive structures. In all cases we restrict attention to nondictatorial structures – those in which no party's weight exceeds one-half. It is worth noting that there are far fewer different decisive structures for a given number of parties than there are different allocations of seats between parties. In the three-party case, for example, it does not matter what weights we assign parties, so long as none exceeds one-half. In the four-party case we examine the two "generic" decisive structures. The first involves a large party that can win with any coalition partner and three small parties that can only win without the large party if all join together; hereafter we refer to this as a *dominated decisive structure*. The second consists of three strategically equal parties plus a dummy (the latter a party that contributes essential weight to no winning coalition).[8] Of the many five-party decisive structures, we focus

[8]There are two other four-party structures, both involving blocking coalitions. One gives some party exactly half of the weight. The other makes any set of three parties

Table 5.1. Simulation experiments: frequency of strong and very strong parties under various dimensionalities and decisive structures.

Decisive structure	Number of dims	Frequency of: Strong party	Frequency of: Very strong party
Three party			
(.33, .33, .33)	2	907	327
	3	734	102
	4	536	36
Four party			
(.40, .20, .20, .20)	2	791	331
	3	554	131
	4	370	43
(.30, .30, .30, .10)	2	803	339
	3	578	103
	4	383	31
Five party			
(.40, .15, .15, .15, .15)	2	766	411
	3	516	201
	4	326	106
(.20, .20, .20, .20, .20)	2	622	201
	3	311	35
	4	150	1
Six party			
(.40, .12, .12, .12, .12, .12)	2	730	428
	3	477	240
	4	316	117
(.18, .18, .18, .18, .18, .10)	2	551	175
	3	236	27
	4	92	4
Seven party			
(.46, .09, .09, .09, .09, .09, .09)	2	740	471
	3	504	261
	4	333	133
(.14, .14, .14, .14, .14, .14, .14)	2	455	131
	3	130	9
	4	37	1

on two – one in which a large party can win with any partner and another in which any three of the five parties is winning. Similarly, in the six-party case, we focus on two decisive structures – one in which a large party can win with any partner and another in which any three of five specific parties constitute a winning coalition while the sixth party is a dummy.[9] Finally, we examine two seven-party structures. The first involves a dominant party that wins with any partner. The second is such that any four of the seven parties are winning.

In each of the party systems in Table 5.1, the two generic decisive structures we consider represent extremes among decisive structures. One is the most egalitarian, making all parties (or nearly all) equal. The other concentrates the most bargaining power possible on one party, without making it a dictator. Other experiments, not reported here, suggest that results for other decisive structures lie between these extremes.

Column 3 gives the number of times per 1,000 runs that a strong party emerges in different random party systems. We may summarize this set of results quite straightforwardly. There is almost always a strong party in low-dimensional three-party systems; there is almost never one in high-dimensional, strategically equal, seven-party systems. The frequency of strong parties decreases monotonically as either the size of the party system or the dimensionality of the policy space increases, and is affected by the decisive structure. *All other things equal, strong parties are more likely in party systems with fewer parties, fewer policy dimensions, and dominated decisive structures* (and these variables interact with one another).

The impact of a dominated decisive structure on the frequency of strong parties is particularly striking in large party systems of high dimensionality. A strong party is nine times as likely in the seven-party, four-dimensional case with a dominant party (333 of 1,000) than in the same case with equal-sized parties (37 of 1,000). In contrast, the decisive structure matters much less in four- and five-party systems. In all but the most extreme circumstances (six or seven roughly equal-sized parties), strong parties are common. The equilibrium concept on which our propositions of Chapter 4 are based is not a rare and esoteric idea that depends rigidly on very specific parameter settings.

Very strong parties – parties that are strong because there is no

winning. The first can only arise in an even-seat parliament in which one party wins precisely half the seats. The other only arises in a parliament in which the seat total is divisible by four with each party winning precisely the same number of seats. These decisive structures are considered pathological rather than generic and, for the sake of parsimonious presentation, are not investigated further. Of course, if such a decisive structure proved of interest, our simulation technology would be perfectly applicable.

[9]To enable all six parties to be strategically identical, it is necessary for the parliament to consist of a seat total divisible by six with each party possessing an identical number of seats. This is regarded as nongeneric.

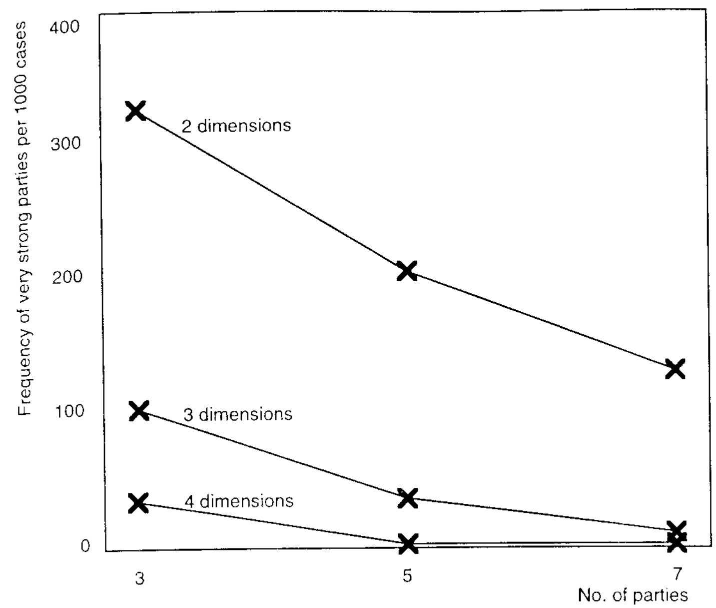

Figure 5.1. Frequency of very strong parties in "egalitarian" party systems of different sizes and dimensionalities

lattice point in the winset of their ideal – arise much less frequently than strong parties, as shown in column 4 of Table 5.1. Their frequency varies with both the size of the party system and the dimensionality of the policy space. This is exhibited in Figure 5.1, where the frequency per 1,000 of very strong parties in the most egalitarian decisive structure is displayed. Moreover, their frequency is quite sensitive to the decisive structure. In five-, six-, and seven-party systems with four policy dimensions, for instance, very strong parties are roughly 100 times more frequent in systems with a dominant party than in those without one.

Overall, the lessons from Table 5.1 and Figure 5.1 are quite clear. The existence of very strong parties is highly sensitive to the configuration of the party system. They become much less likely as the dimensionality of the party system increases and the decisive structure becomes more egalitarian. Strong parties in general are more likely, other things being equal, in small low-dimensional party systems with unequal distributions of bargaining weight, and much less likely in large high-dimensional systems with equal distributions of bargaining weight.

Table 5.2. Frequency of strong parties in random five-party, two-dimensional party systems with different decisive structures

Decisive structure	Frequency of strong party (per 1,000)	Identity of strong party				
		1	2	3	4	5
Weight of party (1, 2, 3, 4, 5)						
A (40, 15, 15, 15, 15)	766	559	51	58	44	54
B (40, 17, 17, 17, 9)	712	407	87	104	102	12
C (44, 24, 24, 4, 4)	671	297	131	149	49	45
D (26, 26, 26, 11, 11)	683	223	226	211	9	14
E (26, 26, 16, 16, 16)	644	188	207	71	81	97
F (20, 20, 20, 20, 20)	622	133	142	121	105	121

Table 5.3. Frequency of very strong parties in random five-party, two-dimensional party systems with different decisive structures

Decisive structure	Frequency of very strong party (per 1,000)	Identity of very strong party				
		1	2	3	4	5
Weight of party (1, 2, 3, 4, 5)						
A (40, 15, 15, 15, 15)	411	373	6	6	14	12
B (40, 17, 17, 17, 9)	306	241	15	23	27	0
C (44, 24, 24, 4, 4)	268	160	43	58	4	3
D (26, 26, 26, 11, 11)	284	94	95	95	0	0
E (26, 26, 16, 16, 16)	191	77	64	10	15	25
F (20, 20, 20, 20, 20)	201	38	41	40	36	46

SIZE AND STRENGTH

Table 5.1 gives us a general picture of the relationship between the existence of a strong party and some overall features of the party system, but it still does not answer the question of what it takes for a party to be strong. Specifically, given the parameters that determine the out-

Table 5.4. Frequency of merely strong parties in random five-party, two-dimensional party systems with different decisive structures

Decisive structure	Frequency of merely strong party (per 1,000)	Identity of merely strong party				
		1	2	3	4	5
Weight of party (1, 2, 3, 4, 5)						
A (40, 15, 15, 15, 15)	355	186	45	52	30	42
B (40, 17, 17, 17, 9)	406	166	72	81	75	12
C (44, 24, 24, 4, 4)	403	137	88	91	45	42
D (26, 26, 26, 11, 11)	399	129	131	116	9	14
E (26, 26, 16, 16, 16)	453	111	143	61	66	72
F (20, 20, 20, 20, 20)	421	95	101	81	69	75

put of our model, how is strength affected by a party's position in the decisive structure? And how is it affected by a party's policy position? We begin to answer these questions by building on the simulation experiments of Table 5.1 to explore the relationship between a party's position in the decisive structure and its likelihood of being strong. To do so we intensively analyze all nondictatorial decisive structures of a five-party parliament with two policy dimensions, each constituting a simple jurisdiction. We report a five-party legislature for this analysis because it is rich enough to allow a wide range of variation in strategic parameters, while still remaining tractable. Unreported experiments in other types of party systems suggest similar results. Again we use our computer program, WINSET, to generate simulated parliaments. For each decisive structure a dataset consists of 1,000 cases in which five parties are assigned policy positions in a two-dimensional space, their positions on each dimension drawn independently from a uniform distribution over the [0,1] interval.

An analysis of these simulations is presented in Tables 5.2, 5.3, and 5.4. The rows present data on each of the six generic decisive structures for the five-party case.[10] Column 2 of Table 5.2 gives for each decisive structure the frequency of strong parties per 1,000 runs of our simulated parliament, and the final panel of columns identifies which of the five parties is

[10]For details on these decisive structures, see Laver and Shepsle (1992).

strong.[11] Tables 5.3 and 5.4 partition these data according to whether we are dealing with *very* strong parties or *merely* strong parties.[12]

Table 5.2 presents a fairly clear picture of the impact of a party's position in the decisive structure on whether it is strong. In decisive structures A and B, Party 1 is dominant in the intuitive sense that it may win in coalition with any other party in decisive structure A and with any other nondummy party in decisive structure B.[13] In each of these cases a strong party is very likely to exist. If a strong party does exist in such circumstances, it is typically the dominant party. The remaining non-dummy parties in decisive structures A and B are about equally likely to be strong and, perhaps surprisingly, even a dummy party may be strong by virtue of its position in the policy configuration of parties. Party 5 in decisive structure B is a dummy in the sense that it is an essential member of no winning coalition. Any coalition that is winning with it is also winning without it. Yet it is a strong party in 12 trials out of 1,000.[14]

Among the remaining four, more egalitarian, decisive structures, a strong party also exists quite frequently. Nonetheless, the final panel of columns shows that a party's position in any decisive structure has a great bearing on its likelihood of being strong. When a party is in a more powerful position in the decisive structure, it is much more likely to be strong. This can be seen very clearly in decisive structure C, where Party 1 is in a more powerful position (in the sense of being a pivotal member of more winning coalitions) than Parties 2 and 3, which are in turn more powerful than Parties 4 and 5. The likelihood of each party being strong is strongly related to its position in the decisive structure.

Tables 5.3 and 5.4 disaggregate Table 5.2 and contrast what it takes to be very strong with what it takes to be merely strong. Between them they show that decisive structures with a dominant party (such as decisive structures A and B) are much more likely to generate *very* strong parties. Table 5.3 shows that the likelihood of a very strong party is crucially affected by the decisive structure, declining rapidly as the decisive structure becomes less dominated. As might be expected, the very strong party is almost invariably the dominant party. This is because a dominant player cannot be excluded from a winning parliamentary coalition unless (nearly) all the other parties gang up on it. In the simulation experiments

[11]Thus, for each row the entries in panel (3) sum to the entry in column (2).

[12]Thus common cell entries in Table 5.3 and 5.4 sum to the corresponding cell entry in Table 5.2.

[13]This is nearly identical to the formal definition of *dominant player* given in Peleg (1981).

[14]Parties 4 and 5 in the decisive structure D are also dummies; jointly, they are strong in about 4 percent of the instances in which there is a strong party. A strong dummy party is illustrated in Figure 4.2 for a four-party, two-dimensional setting; in that figure, Party D is strong.

with random party systems, it is quite unusual for those other parties to have spatial locations that provide incentives for all of them simultaneously to want to gang up in this way.

Put differently, a very strong party, by definition, has an ideal point with an empty winset. We know from our adaptation of the Kadane Theorem in Chapter 4 that, in the simple jurisdictional structure we investigate here, this ideal must be at the median on each policy dimension. This is much more likely to happen for a dominant party than for any other party.[15] It may be noted, moreover, that a dummy party can *never* be at the median on any dimension and thus can never satisfy this necessary condition for being very strong. In confirmation, Table 5.3 reveals that Party 5 in the decisive structure B and Parties 4 and 5 in decisive structure D – all dummy parties – are never very strong.

In contrast to the results for a very strong party, Table 5.4 shows that the existence of a merely strong party is much less a product of the decisive structure. Merely strong parties are more or less equally likely in each of the five-party decisive structures. The probability of a party being merely strong is still distinctly related to its position in the decisive structure but much less strikingly so than the likelihood of a party being very strong. Parties with a weaker position in the decisive structure may nonetheless be strong. But they are far more likely to be merely strong, and hence to have to rely on vetoes, than to be very strong – having only to rely on majority support for their ideal point in the legislature.

Thus a powerful position in the decisive structure enhances a party's control over the making and breaking of governments in two ways. First, a more dominant position in the decisive structure is far more likely to make a party strong, and hence an essential member of any government. Second, while even parties with weaker positions in the decisive structure can be strong if they occupy the right position in the configuration of party positions, dominant parties are far more likely than these to be very strong, and thus not to have to rely on their ability to win standoffs. Parties less well placed in the decisive structure are far more likely, if they

[15]The likelihood that a party located randomly will be at the multidimensional median is a probability that may be calculated a priori. In decisive structure F of Tables 5.2–5.4, the probability that a specific party is at the median on *one* dimension is .20. The probability that this party is at the median on *both* dimensions, given independent random draws, is $.20 \times .20 = .04$. In contrast, in decisive structure A, Party 1 has a probability of .60 of being median on one dimension – this is the chance that it is neither the left-most nor right-most party on this dimension – and thus a probability of $.6 \times .6 = .36$ of being located at the multidimensional median, a probability *nine* times larger than in the equal-size structure. This is borne out in the simulation results of Table 5.3, where the strong party in the equal-size structure is very strong, on average, 40 times per 1,000, whereas the dominant party in the first structure is very strong 373 times per 1,000.

are strong, to be merely strong, and thus to have to be able to win standoffs with other parties if they are to exploit their strength to the full.

POLICY AND STRENGTH

Having examined the connection between party size and the likelihood it is strong, we now turn to the relationship between a party's position in the policy space and that likelihood. We continue with our intensive investigation of five-party, two-dimensional party systems, considering all nondictatorial decisive structures. Characterizing a party's position in two-dimensional policy space relative to the positions of other parties, is not a straightforward matter, however, especially when the number of parties moves beyond three.[16] As the number of dimensions moves beyond two, the problem becomes even more complex. One important feature of a party's policy position, however, and one that is of great significance to our approach, concerns whether a party is at the median position on one or more policy dimensions. If a party is at the median on any key policy dimension, it is a participant in the dimension-by-dimension median cabinet – a cabinet that is often one potential equilibrium. In addition, as we saw in Chapter 4, the strong party is typically a participant in the DDM cabinet – thus the strong party is typically at the median on at least one policy dimension.[17] A very strong party, as we have seen, must have an ideal at the median position on *all* policy dimensions.[18] In what follows, therefore, we use a party's occupation of median positions on key policy dimensions as an indicator of its policy position relative to other parties.

In Tables 5.5, 5.6, and 5.7, we investigate all five-party, two-dimensional, decisive structures. For each decisive structure, we compute the frequency per 1,000 trials that a party is strong, given that it is median on no, one, or two policy dimension(s). The results for when a party is at the median on no policy dimension are reported first, in Table 5.5. Each cell reports two pieces of information. The first number is the proportion of trials in which Party X is strong given that it is at no median. The second number gives the number of times per 1,000 trials that Party X is indeed median on no policy dimension.[19]

[16]Note our characterization of a generalized "center" party and two "extreme" parties in the three-party, two-dimensional case.

[17]If the DDM cabinet has a nonempty winset, however, the strong party can in theory be nonmedian on every dimension.

[18]Assuming, as we have here, a simple jurisdictional structure.

[19]This proportion can be computed a priori. In the case of the largest party in the most dominated structure, it is nonmedian on a dimension if and only if it is first or fifth in the ordering of the parties. This occurs with probability .40. Thus, the probability that it is median on neither dimension is .40 × .40 = .16, so we should expect it to be nonmedian on both dimensions approximately 160 times per 1,000 simulations.

Table 5.5. *Relative frequency of strong parties among parties occupying no median position*

	Party				
Decisive structure	*1*	*2*	*3*	*4*	*5*
Weight of party (1, 2, 3, 4, 5)					
A (40, 15, 15, 15, 15)	0.0 (161)	0.0 (809)	0.0 (809)	0.0 (834)	0.0 (811)
B (40, 17, 17, 17, 9)	0.00 (242)	0.01 (707)	0.01 (698)	0.00 (671)	0.01 (1000)
C (44, 24, 24, 4, 4)	0.00 (345)	0.00 (585)	0.01 (578)	0.01 (881)	0.02 (892)
D (26, 26, 26, 11, 11)	0.00 (425)	0.00 (418)	0.00 (463)	0.01 (1000)	0.01 (1000)
E (26, 26, 16, 16, 16)	0.01 (497)	0.01 (492)	0.01 (738)	0.01 (751)	0.02 (730)
F (20, 20, 20, 20, 20)	0.02 (649)	0.01 (622)	0.01 (638)	0.02 (644)	0.01 (661)

Note: Cell entries give the proportion of trials for which a party is strong if it is at no median. In parenthesis is the number of trials (per 1,000) for which the party was in fact at no median.

The results of Table 5.5 provide startling evidence of the strategic importance for the making and breaking of governments for a party to be at the center of things. If a party is at no median position, it has almost no chance of being strong. This is clearly true for dummy parties (Party 5 of decisive structure B and Parties 4 and 5 of decisive structure D), who by definition are never median. But it is even true for the largest party in the most dominated decisive structure. Party 1 in decisive structure A is at no median in 161 of the 1,000 simulated cases; in no instance is it strong.

*Table 5.6. Relative frequency of strong parties among parties at m**

Decisive structure	Party 1	2	3	4	5
Weight of party (1, 2, 3, 4, 5)					
A (40, 15, 15, 15, 15)	0.96 (386)	1.00 (6)	1.00 (5)	1.00 (14)	0.92 (13)
B (40, 17, 17, 17, 9)	0.97 (246)	0.94 (16)	0.85 (27)	0.93 (29)	– (0)
C (44, 24, 24, 4, 4)	0.95 (167)	0.90 (48)	0.97 (59)	1.00 (4)	1.00 (3)
D (26, 26, 26, 11, 11)	0.90 (101)	0.89 (102)	0.92 (103)	– (0)	– (0)
E (26, 26, 16, 16, 16)	0.91 (82)	0.91 (69)	0.77 (13)	0.79 (19)	1.00 (25)
F (20, 20, 20, 20, 20)	0.95 (40)	0.93 (43)	0.91 (44)	0.85 (41)	1.00 (46)

Note: Cell entries give the proportion of trials for which a party is strong if it is at m*. In parenthesis is the number of trials (per 1,000) for which the party was in fact at m*.

The results of Table 5.5 suggest unequivocally that, *almost without regard to the position of a party in the decisive structure, if a party is at the median on no policy dimension, then it will almost never be strong.* We have of course shown analytically in Chapter 4 that, in the simple jurisdictional structure, a strong party must be at the median on at least one policy dimension if the DDM cabinet has an empty winset (so as to be able to veto this). In theory, a strong party can be median on no dimension when the DDM has a nonempty winset. Table 5.5 adds considerably to our understanding of this, however, by showing that a strong

Table 5.7. Relative frequency of strong parties among parties at median on only one dimension

Decisive structure	Party 1	2	3	4	5
Weight of party (1, 2, 3, 4, 5)					
A (40, 15, 15, 15, 15)	0.41 (453)	0.23 (185)	0.24 (186)	0.18 (152)	0.22 (176)
B (40, 17, 17, 17, 9)	0.33 (512)	0.25 (277)	0.28 (275)	0.25 (300)	– (0)
C (44, 24, 24, 4, 4)	0.28 (488)	0.23 (367)	0.25 (363)	0.30 (115)	0.25 (105)
D (26, 26, 26, 11, 11)	0.28 (474)	0.28 (480)	0.27 (434)	– (0)	– (0)
E (26, 26, 16, 16, 16)	0.26 (421)	0.32 (439)	0.20 (249)	0.24 (230)	0.24 (245)
F (20, 20, 20, 20, 20)	0.27 (311)	0.28 (335)	0.23 (318)	0.18 (315)	0.23 (293)

Note: Cell entries give the proportion of trials for which a party is strong if it is at the median on one dimension. In parenthesis the number of trials (per 1,000) for which the party was in fact at the median on one dimension.

party can be nonmedian, regardless of the winset of the DDM, only in very particular circumstances.[20]

By way of dramatic contrast, consider now the frequency with which a party is strong if it is median on both dimensions. These results are given

[20]This is an appropriate point to repeat our caveat about the relationship between simulations and the real world. When party positions and weights are strategic rather than random, and information is perfect and complete, it is possible that actual outcomes may home in relentlessly on those very particular party configurations.

in Table 5.6. Each cell gives the proportion of times that Party X is strong if it is located at the dimension-by-dimension median (the DDM) and the frequency per 1,000 that Party X is at the DDM. As Table 5.6 reveals, parties at the DDM are nearly always strong, quite regardless of their position in the decisive structure (though the latter very significantly affects the frequency of being median on both dimensions).

Our simulation experiments once more add considerably to our understanding by suggesting that, *whatever its weight, and whatever the decisive structure, a party at the dimension-by-dimension median is nearly certain of being strong, and therefore of being a party of government.*

Tables 5.5 and 5.6 thus present refreshingly clear-cut findings. If a party is at no median it has almost no chance of being strong. If it is at both medians it is almost certain to be strong. In each case the finding holds almost regardless of the weights and positions of other parties. What, however, if a party is at the median position on only one of the two policy dimensions? The results of our simulations for such are displayed in Table 5.7.

The results are again quite intriguing. As in the previous tables, the two bits of information provided for each party in each decisive structure are the proportion of times that Party X is strong given that it occupies the median position on precisely one dimension, and the frequency per 1,000 that it occupies precisely one median position. Not surprisingly, the dominant party in a decisive structure is much more likely than other parties in that structure to occupy one median (compare the second bit of information for Party 1 in decisive structures A and B to that for all the other parties in those structures). More surprising is that the largest party(ies) occupies one median with approximately the same frequency in all but the equal-size decisive structure. Most surprising of all is that, with the exception of the dominant party in the most dominated decisive structures, the proportion of times a party is strong if it occupies precisely one median is about 0.25, and is independent of party size or decisive structure. Even the very tiny parties in the third decisive structure are strong about 25 percent of the time when they are at the median position on one key policy dimension.

The conclusions we can draw from the simulation experiments reported in Tables 5.5, 5.6, and 5.7 can be summarized quite succinctly. If a party is at no median, then it is almost certain not to be strong. If a party is at both medians, a situation that is much more likely if the party is the largest in a dominated decisive structure, then it is almost certain to be strong (thereby guaranteeing itself a place in government). If a party is at one of the two medians, then it has about a 25 percent chance of being strong. With very few qualifications, then, the likelihood that a party is strong depends on whether it is at two, one, or no median positions. Once

this is determined – and of course larger parties are more likely to be at the median on some dimension, other things being equal, than smaller parties – then the prospect of a party being strong does not vary much either with party size or decisive structure. These results clearly suggest that being in median policy positions – being at the center of things in this sense – gives a party considerable hold over the making and breaking of governments.

CONCLUSION: STRONG PARTIES AND OTHER "DISTINGUISHED" PARTIES

We have covered a great deal of ground in this chapter. We displayed the logic undergirding the existence of a strong party in the simplest of settings – that of two unidimensional jurisdictions and three parties. Formal conditions for the general case are "uglier" and, in our judgment, not particularly pregnant with further insights.

We thus turned to simulation experiments in order to explore conditions affecting the existence and identity of strong parties. The conclusions we drew from these are quite compelling. Strong parties exist with great regularity, especially in lower-dimensional policy spaces, in small(ish) party systems, and in decisive structures with a dominant player. Moreover, and quite consistent with casual intuition, both a party's position in the decisive structure and its position relative to others on key policy dimensions have a profound influence on whether it is strong. In Part III of this book, we examine the making and breaking of real cabinets in a range of postwar European parliamentary democracies. As we shall then see, the empirical case for the importance of strong parties is quite compelling. Strong parties exist with great frequency in real party systems (and very strong parties exist in a surprisingly large proportion of these cases). Such parties are indeed much more likely to get into government.

The final task for this theoretical chapter, therefore, is to locate our concept of a strong party in the now well-established literature on coalition theory. This is harder than one might imagine, both because the search for distinguished policies, parties, and coalitions has been an odyssey with a long history in game theory and coalition modeling, and because our own theory is a rather radical departure from this theoretical tradition.

In Chapter 4 we argued that the making and breaking of governments, a process involving several parties in a multidimensional issue space, nevertheless may possess some Archimedean points. We say "nevertheless" because spatial modelers and coalition theorists have for some time been frustrated by the apparent (theoretical) chaos associated with majority rule processes (including those by which governments are formed). We provided the prospect of ducking between the horns of this dilemma by formulating a specific sequential model of government formation and wedding it to a novel interpretation of the "space" of alternatives (namely, the lattice of feasible governments). Governments that form in our model, moreover, are subjected to an especially stern test. Since we endow our actors with rational foresight, the government they form must not only prevail over the existing status quo, but also must be expected to survive subsequent confidence motions.

Thus, the salient features of our model of government formation consist of:

- policy-motivated parties;
- a lattice of feasible governments;
- a status quo government;
- a sequential process by which the status quo government may be replaced;
- common knowledge permitting each actor to exercise rational foresight; and
- no exogenous enforcement of deals between parties.

Nearly every one of these features departs from conventional models of coalition formation in general and government formation in particular.

First, early coalition models, of which Riker (1962) is the exemplar and pioneer, assumed that parties were *office* seeking, not policy motivated; over the 30 years of coalition studies, this is probably the modeling convention that has been relaxed the most.

Second, all coalition theories based on an assumption of *policy*-seeking politicians were imbedded in a spatial model that assumed that any point in the policy space was a feasible basis around which a winning coalition could assemble. Our lattice is a major departure, emphasizing as it does the fact that in practice it is not policy agreements that come into being, but governments, each government comprising a set of cabinet ministers. Every actual or potential government is a discrete entity with a particular forecast policy output. Our argument is that the only really effective way for legislators to control a government is to (threaten credibly to) replace it with another government, which is another discrete entity, also with a particular forecast policy output.

Third, with some exceptions (Brams and Riker, 1973, and Austen-Smith and Banks, 1988, 1990), formal models of government formation processes have been timeless and ahistorical. Ours, by contrast, commences from a historically determined status quo, on the one hand, and unfolds in a constitutionally established sequential form, on the other (the latter inspired by the work of Baron and Ferejohn, 1989).

Fourth, and closely tied to the sequential nature of our model, our

political actors are not myopic. Put differently, they are strategic in their thinking and thus capable of seeing beyond their noses. In proposing portfolio allocations, in deciding whether to consent to cabinets in which they participate, and in determining whether to support particular investiture and/or confidence motions, they think ahead. This constitutes a stiffer test for equilibrium concepts than those based on the myopic behavior explicitly or implicitly assumed by many earlier theories.

Fifth, nearly all of the early game-theoretic approaches to the subject of government formation took a cooperative approach. The major departures from this convention, and our own model departs in the same way, were displayed in the work of Baron and Ferejohn, as well as of Austen-Smith and Banks. These noncooperative approaches of course accept that parties engage in "cooperative" ventures such as coalitions. They claim, however, that any cooperation that does transpire – for example, in forming coalitions, making policy compromises, and allocating portfolios – does so because it is in all cooperators' interests to stick to the deal they have negotiated. Their cooperation, that is, depends only on *self-enforcement;* it does not require or assume some unmodeled exogenous enforcement agent.[21]

Given the variety of ways in which our model departs from so many of its predecessors, it exceeds our own capabilities to compare and contrast these alternative approaches in a systematic manner. Instead, we focus on a few recent approaches that may contain a familial resemblance to our own. Specifically, we briefly examine van Roozendaal's central party and Schofield's core party, though even here we can hardly be thorough.

In a number of recent papers, van Roozendaal (1990, 1992, 1993) has emphasized the centripetal tendencies of government formation. He defines a *central party* as the weighted-voting analogue to the median voter in simple voting models.[22] This concept is tied expressly to a unidimensional ordering of parties, so it is spatially much less complex than our own approach. Through any of a number of different empirical means – expert surveys of leaders or voters, factor analyses of party manifestos or other campaign statements, parliamentary voting analyses – the parties of a po-

[21]Cooperative game theory *does* assume exogenous enforcement, so that once parties to an agreement sign on the dotted line, the agreement is implemented without a hitch. There is no reneging, no ex post departures from ex ante commitments. The early inspiration for this modeling convention was economic exchange in which it was not implausible on its face to assume an exogenous umpire or court system that costlessly and perfectly enforced contracts between traders. Even in economics, however, with its more modern attention to incomplete contracts and enforcement problems, cooperative game theory conventions have been called into question. See, for example, Williamson (1985).

[22]In related work, van Deemen (1989) defines a *dominant party* that is "centripetal" in terms of weight, not spatial policy location (van Roozendaal works with this concept as well). Because it is a nonspatial concept, we do not pursue it further here.

litical system are ordered on a single underlying dimension of policy, usually the conventional left–right continuum. Let R_P be the sum of weights of all parties at or to the right of Party P. Define L_P in a symmetrical fashion for parties to the left of P. Then, for a given set of parliamentary weights, the central party is that party for which $R_P \geq ½$ and $L_P \geq ½$. It is the median of the weighted party ordering on the underlying dimension. In one dimension a central party always exists and, except for rare configurations of weights, is unique. The central party is pivotal to everything that happens in the parliament that requires majority support – it "controls the political game," as van Roozendaal (1993: 40) puts it. Consequently, "it follows that the central party will not be left out of the cabinet"; van Roozendaal predicts that, "when a central party is present, it will be included in all cabinets that are formed" (van Roozendaal, 1993: 41). He does not provide a theoretical rationale for this prediction in the form of an explicitly modeled government formation process, though in the unidimensional case it is compatible with numerous theoretical formulations.

The central party is subsumed by our model of government formation. In the one-dimensional special case of our model, the weighted median party is very strong. In other words, in this special case, van Roozendaal's central party is also a very strong party. Consequently, the strategic outcome of the government formation process as we model it explicitly is for this party to become the government. Hence, our one-dimensional forecast subsumes van Roozendaal's. In more than one dimension (something on which van Roozendaal is silent) the natural generalization is the dimension-by-dimension weighted median (DDM). If the DDM is a party ideal point, then, in the spirit of van Roozendaal, that party is *very central*. But it need not be very strong (it is so only if its winset is empty).

The most comprehensive theoretical alternative to our own notion of a strong party, however, is that of the core party, invented by Norman Schofield, a grizzled veteran of the coalition theory wars. In a series of papers authored singly (an important subset of which is Schofield, 1978, 1986, 1987, 1992, 1993) and collaboratively (McKelvey and Schofield, 1986, 1987), he provides theoretical conditions for the existence of spatial equilibrium points in a weighted majority-rule setting analogous to voting in legislatures. This generalizes well-known results for a majority rule equilibrium in a one-person–one-vote committee (Plott, 1967).

In the pure, unweighted, majority-rule setting – a legislature with no parties in which every legislator voted on his or her own initiative, for example – a majority-rule equilibrium, or *majority core*, is a very rare bird indeed. In an odd-numbered committee, such a point exists if n−1 of the voters may be paired up in a manner so that the respective contract loci of each pair (in two dimensions the lines joining their ideal points) intersect at the ideal point of the remaining voter. That voter's ideal is the

majority core. Small perturbations in the location of one or more of these voters destroys the radial symmetry that defines the majority core – hence this is, in Schofield's terms, *structurally unstable*.

However, since one of Schofield's main interests, like ours, is in government formation in legislatures with political parties of various sizes, he is interested in a majority core in circumstances in which each "voter" (party) may not have the same number of votes. Moreover, he is decidedly more interested in majority cores that are robust to small perturbations – in his terms, these are *structurally stable majority cores*.

Schofield establishes a necessary and sufficient condition for the existence of a core point. He begins with a multidimensional policy space in which party ideal points are located. For any minimum winning coalition of parties[23] there is a Pareto set – the set of policy points from which departure to any other point harms at least one of the coalition parties. For policy spaces based on Euclidean distance, this is simply the convex hull of coalition party ideals – the points contained in the geometric body created by the lines connecting party ideal points. Schofield also refers to this as the coalition's *compromise set*. A policy point is a core point if and only if it lies in the compromise set of every minimum winning coalition. If the intersection of these compromise sets is empty, then there is no core. Moreover, if, for a particular core point, small changes in party ideals still leave it in the compromise set of every minimum winning coalition, then that core point is structurally stable.

A *core party* is a party whose ideal policy is a core point. Schofield suggests that when such a party exists, the government formation game is likely to conclude in a minority government in which the core-party ideal becomes government policy (Schofield, 1993: 8).

When a core party does not exist, as is often the case, Schofield shifts attention to the region of the space within which cycles among coalitions and policies are likely to be located. He calls the latter the *cycle set*. The union of the cycle set and the core is known as the *heart*. Schofield establishes that the heart always exists – either there will be a core point or a cycle set (not both and not neither).

The cycle set can be shown in a partially made-up example. We reproduce figure 3 of Schofield (1993) as Figure 5.2. Five Dutch parties, holding 90 seats of the 100 seats of the Tweede Kamer, are the principals of the government formation game after the June 1952 election. The threshold is 51, and the remaining 10 seats are held by Communists and independents who are assumed to sit on the sidelines during this process. The Labor Party (PvdA) and the Catholic People's Party (KVP)

[23]This is defined conventionally, namely that the weight of parties in the coalition exceeds one-half and that the removal of any party reduces the coalition's weight to less than one-half.

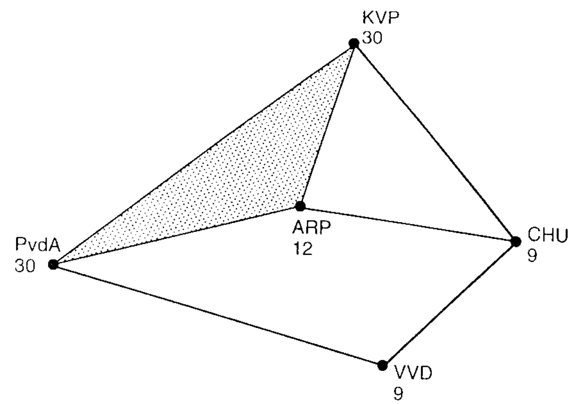

Figure 5.2. An empty core and nonempty cycle set (taken from Schofield, 1993)

each hold 30 seats, the Anti-Revolutionary Party (ARP) 12, and the Christian Historical Union (CHU) and Liberals (VVD) hold 9 seats each. The parties are given spatial locations hypothetically. The three lines, respectively connecting [PvdA,KVP], {KVP,ARP], and [PvdA,ARP] are known as *median lines*. Each line has the property that either on or to one side of the line can be found parties that between them compose a legislative majority. Thus, to take the [PvdA,ARP] line, parties holding 72 seats lie on or above the line, while parties holding 60 seats lie on or below the line. The same holds for each of the other two median lines. If there were a core point, all median lines would intersect at it, clearly not the case here.

The shaded area bounded by the median lines is identified by Schofield as the cycle set. For any point outside this region, majority voting will pull policy toward it. Thus if, for example, a policy to the east of the shaded region were proposed, say by CHU, the majority on or to the west of the [KVP,ARP] median line – KVP, ARP, and PvdA – would pull policy in a westerly direction. On the other hand, if particular policies are proposed that are inside the cycle set, then there will be pulls and tugs, but these will never take the process outside the boundaries of the cycle set.

The implications of Schofield's theory can be divided according to whether a core party exists or not. We should emphasize, however, that he does not explicitly model the government formation process or the actual allocation of portfolios, but rather concentrates on the equilibrium policy that emerges given the constellation of forces in the parliament.[24]

[24]This focus on *policy* rather than on *portfolio allocation* is what makes comparisons between the two theories so speculative.

First, suppose a core party exists. Schofield's theory implies that policy will reflect the core party's ideal point. It may form a single-party minority government or form a coalition with other parties. The assignment of portfolios does not matter to Schofield's parties, since they care only about policy (like parties in our model) and policy deals can be struck without the policing mechanism of portfolio allocation (unlike our model). Schofield's is a cooperative game-theoretic account, so that he assumes any deals are exogenously enforced – deals that have been struck, stay stuck!

It may be shown that the winset of the ideal of a core party is empty – no policy is preferred by a parliamentary majority to it. Schofield's core party is, by our lights, *very strong*, though our very strong party need not be a core party.[25] Thus the core party is a special case of a very strong party. According to Proposition 4.2, the equilibrium will be the ideal point of a very strong party, if one exists. So, it would appear that the predictions of the two theories are compatible in this event.

If no core party exists, Schofield's theory predicts an outcome in the cycle set. In the partially made-up example of Figure 5.2, he speculates (Schofield, 1993: 11) that either the "surplus" cycle set coalition {PvdA,ARP,KVP], the minimum winning coalition {PvdA,KVP}, or one of the two minority coalitions {PvdA,ARP} or {KVP,ARP} will form. A specific claim that he makes is that CHU and VVD are weak players – "we might expect them not to be formal members of government" (Schofield, 1993: 11).[26]

Our theory comes to a potentially different conclusion. Just for the fun of it, we took seat distributions and party positions exactly as Schofield gave them in Figure 5.2,[27] superimposed our lattice on his spatial representation (Figure 5.3), and input these data into WINSET.

We have maintained Schofield's convention of treating Communists and small religious parties as voting against all proposed cabinets; thus the decision rule requires 51 of the 90 votes controlled by the "coali-

[25]Why? Because our very strong party has the property that its ideal is preferred by a majority to every other *lattice* point, not to every point in the space.

[26]This claim by Schofield, though intuitively plausible, is entirely speculative since his theory has *nothing* to say about portfolio allocations.

[27]This yielded the following configuration, rescaling positions to lie on the [0, 1] interval. The threshold is 51 seats. This in effect makes the conservative assumption that Communists and fundamentalist religious parties are liable to vote against any proposed government.

Party	*Seats*	*Policy position*
PvdA	30	0.000, 0.313
ARP	12	0.589, 0.500
VVD	9	0.714, 0.000
KVP	30	0.700, 1.000
CHU	9	1.000, 0.438

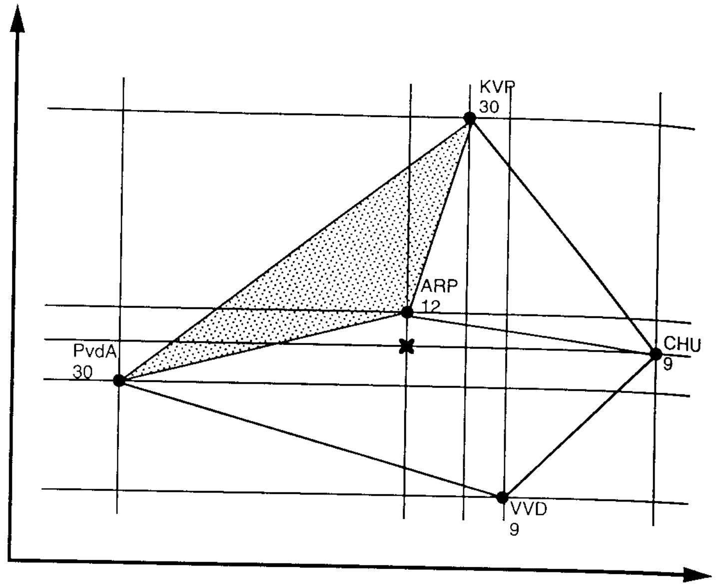

Figure 5.3. An empty core, a very strong party, and an empty-winset DDM

tionable" parties. In Figure 5.3, both the ARP ideal and the (ARP, CHU) coalition cabinet – marked with an "x" – are equilibriums, according to Propositions 4.1 and 4.2.[28] Indeed, the ARP is a very strong party. This leads to three implications. First, the ARP is very strong on our account, without being a core party, according to Schofield. Second, our account produces a set of potential equilibriums (the ARP ideal and the point marked with an "x" in Figure 5.3) that only partially overlaps the equilibrium set of Schofield's account (shaded in Figure 5.3). Finally, we note that CHU is not a "weak" player – as it is according to Schofield.

Which theory is right? It's hard to say, since these data are made up. But it does suggest that the two theories can be compared in principle. tverall, however, the comparison between the two approaches is quite striking in one important regard. Schofield's approach, which does not consider the government formation process at all and concentrates instead on voting in legislatures, identifies a core party as an important equilibrium concept. Our approach subsumes this in the notion of a very strong party. But note that a very strong party has an empty lattice

[28]Because there is a restricted set of coalitionable parties, the effective decision rule requires more than simple majorities (51 votes out of 90 available) among the parties in Figures 5.2 and 5.3. We note as a consequence that there can be – and, in this instance, are – multiple empty-winset points.

winset. In an important sense, its position also depends only on voting in legislatures, since the very strong party does not need to exercise its vetoes. Our notion of a merely strong party (and holdout party, for that matter) has no analogue in Schofield's work or indeed in any other approach to government formation of which we are aware. The reason is simple. A merely strong party is strong because it is in a position to veto cabinets that are majority-preferred to its own ideal cabinet (in which it gets all portfolios). These vetoes arise because parties cannot be forced into cabinets against their will, and it is only by modeling cabinet formation, as opposed to voting in legislatures, that the logic of this process, and the rationale for strong parties, becomes apparent.

Appendix: Formal conditions for the existence of a strong party with three parties and two jurisdictions[29]

Consider three parties, I, J, and K. Each has weight, w_I, w_J, and w_K, respectively, and a spatial position, i, j, and k, respectively.[30] If some party controls more than half of the weight, then it is very strong by definition. We exclude that case in what follows and assume no party's weight exceeds one-half; that is, we assume a *nondictatorial* decisive structure. This means that any pair of parties can form a parliamentary majority. Finally, let $d_t(uv)$ be the Euclidean distance between party T's ideal point, t, and the government in which the horizontal portfolio is given to party U and the vertical portfolio to party V (where T, U, and V are each elements, not necessarily different, of {I,J,K}). This government implements the policy uv – a horizontal-dimension policy given by U's ideal, u, and a vertical-dimension policy given by V's ideal, v. We now state the conditions under which party I is strong.

Strong Party Characterization Proposition: Party I is a strong party in a three-party, two-jurisdiction setting if and only if

$$d_j(jk) < d_j(i) \rightarrow d_k(jk) > d_k(i). \quad (1)$$
$$d_j(kj) < d_j(i) \rightarrow d_k(kj) > d_k(i). \quad (2)$$
$$d_k(i) < d_k(j). \quad (3)$$
$$d_j(i) < d_j(k). \quad (4)$$

Conditions 1–4 lay out the circumstances in which no policy of a government (no lattice point) that excludes party I is majority-preferred to party I's ideal. Condition 1 states that if a JK government is preferred by party J to a government in which party I controls all portfolios (because the JK government policy, jk, is closer in Euclidean distance to party J's ideal, j, than i, the policy that an exclusively party I government would pursue), then this JK government is not preferred by party K. (The converse is

[29]This development is drawn from Laver and Shepsle (1993).
[30]Spatial positions are vectors in the two-dimensional policy space.

implied by this statement as well, namely, if party K prefers the JK government, then party J does not.) Condition 2 requires the same for a KJ government. Finally, conditions 3 and 4 require that no other single-party government is preferred to a government in which party I gets all portfolios. The only remaining governments are those in which party I is a participant, so we thus may validly claim that conditions 1–4 are *sufficient* for party I to be strong. Conversely, if any of these conditions is violated, then there exists a lattice point in the winset of the putative strong party in which the latter does not participate. This establishes the *necessity* of conditions 1–4.

Figure 4.1 in the previous chapter shows these conditions at work. Party B is strong because:

1. Although party A prefers an AC government to one comprising only party B, party C does not.
2. Although party C prefers a CA government to one comprising only party B, party A does not.
3. Party A prefers a government comprising only party B to one comprising only party C.
4. Party C prefers a government comprising only party B to one comprising only party A.

Having established the conditions for the existence of a strong party in the three-party, two-jurisdiction case, we may now determine when, in fact, these conditions will be realized. We arbitrarily locate parties K and J in the two-dimensional policy space displayed in Figure 5.4. This determines the lattice points of the JK and KJ governments, as well as of the governments in which either party J or party K gets all portfolios. Now we seek to determine the set of locations for party I that simultaneously satisfies conditions 1–4 of the Strong Party Characterization Proposition. Condition 3 requires that I be located inside a circle centered on K through J. Condition 4 requires that I be located inside a circle centered on J through K. Arcs of these circles running through J and K are displayed in Figure 5.4. Condition 1 requires that I either be inside the circle centered on J through JK or inside the circle centered on K through JK. Condition 2 requires that I either be inside the circle centered on J through KJ or inside the circle centered on K through KJ. The shaded area in the figure comprises the locus of points simultaneously satisfying these requirements. Given the fixed locations for parties J and K, if party I's position is in the shaded area, then I is strong.

To this region we may add, though we do not display it in the figure, the locus of policy positions for party I that render either party J or party K strong. The area of the union of these three regions, relative to the area of the two-dimensional space, gives a measure of how "difficult" it is to

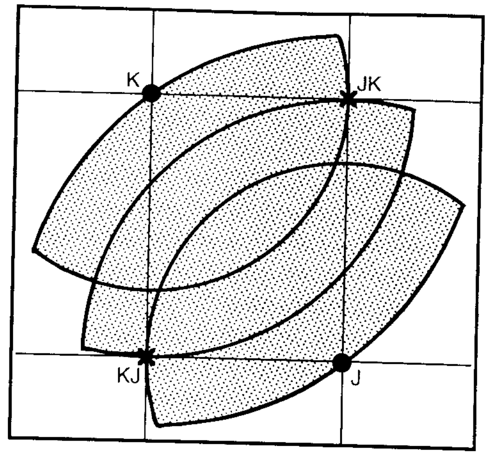

Figure 5.4. Locating a strong party in a three-party two-dimensional system

satisfy the conditions of the Strong Party Characterization Proposition. As our simulation experiments reported in the body of the this chapter reveal, this irregularly shaped region comprises approximately 90 percent of the area of the policy space in which parties are able to locate. That is, in the three-party, two-jurisdiction case, a very high proportion of all possible spatial party configurations supports the existence of a strong party.

PART III

Empirical investigations