and then assumed continuity in the intervening range—a frequently useful scientific approach to many problems. Only when we needed some function $f(E)$ such that $f(1) = 0$ did we resort to $\log E$ through empirical knowledge. Despite these elements of reasoning, equations 14.2 and 14.3 remain at this point primarily empirical. They are found to apply to some plurality elections featuring a variety of values of n: single-seat national assembly elections (n around 3), multi-seat blocs in the U.S. electoral college elections (n around 5), and union elections in very small constituencies (n around 1.5). It is unlikely that the seat–vote equations could fit all these cases by sheer accident. But as long as we have no theoretical justification for them it is always possible that the next plurality election system we happen to investigate might not fit.

The same applies to the further widening of the range of seat–vote equations by substituting equation 14.4 for equation 14.3. At least some PR elections are approximately accounted for, and of course the fit to plurality elections is not lost, since equation 14.4 includes equation 14.3 as the special case where $M' = 1$. The success of this further extension increases our confidence in the seat–vote equations, but we still have only a hypothesis and not a theoretical model, nor even a fully tested empirical rule.

Chapter 16 will give a rational basis for the seat–vote equations. However, a puzzling side issue also arises: if many values of n are possible, why have the Anglo-Saxon countries picked assembly sizes that approximately result in $n = 3$? The underlying "cube root law of assembly sizes" will be presented in the next chapter.

Seats & Votes

◆ 15 ◆

The Cube Ro[illegible] of Assemb[illegible]

TAAGEPERA + SHUGART

Larger nations tend to have larger national assemblies. Larger assemblies, like larger district magnitudes, tend to be more favorable to small-party representation. The difference is especially keen with plurality in single-member districts, because a larger number of such districts gives small parties a better chance. Thus the assembly size can have political consequence.

The way in which assembly size affects small-party representation is illustrated by a partly hypothetical example. Table 15.1 uses the British 1983 election results to show how the reduction in the number of assembly parties would proceed if one reduced the number of seats in Parliament by gradually fusing the existing districts.[1]

The number of seats in actual national assemblies tends to be close to the cube root of the populations of nations. The evidence for such a "cube root law of assembly sizes" is presented in this chapter. In contrast to many other relationships, this one is termed a "law," because the empirical connection is backed up by a rational model.

1. The result is exactly the same as for the bottom curve in Appendix C4, where plurality rule is applied at increasing district magnitudes. Applying plurality rule to multi-seat districts of increasing magnitude reduces the number of assembly parties. The percent seat shares of the parties are not changed if, instead of allocating all seats in the district to the plurality winner, one gives the plurality winner in the district only one seat so that the total assembly size is reduced.

EMPIRICAL EVIDENCE FOR THE CUBE ROOT LAW

National assemblies usually are either unicameral or bicameral. In the latter case we shall consider only the first chamber, the one that is meant somehow to represent the population as such. It usually has more seats than the second chamber, which is typically based on territorial units (the U.S. Senate), appointment (Canadian Senate), historical nobility (part of the British House of Lords), or indirect or limited-franchise elections. We shall first consider only twenty-one stable democracies with developed economies—the countries used in our discussion of issue dimensions, following Lijphart (1984b). Later we shall include all national assemblies in the world.

Figure 15.1 shows the number of seats (S) in the first or only chamber graphed against the population (P), for the twenty-one developed democracies. For twenty of them, only the situation around 1985 is shown. For the United States, the entire historical pattern since 1790 is shown, with both P and S gradually increasing. Both S and P scales are logarithmic. The straight line corresponding to the simple equation

Table 15.1
Reduction in the effective number of assembly parties (N_s) for British 1983 elections if fewer single-member districts had been used

Number of Seats	N_s	N_v	$r = \frac{N_v - N_s}{N_v}$
633 (actual)	1.98	2.93	.32
316	1.86	2.93	.37
158	1.80	2.93	.39
79	1.70	2.93	.42
. . .	. . .		
11	1.53	2.93	.48
1	1.00	2.93	.66

Source: Taagepera and Kaskla (1988).

Note: To reduce the number of seats, contiguous districts were gradually fused. This quasi experiment preassumes that reduction of $E = S$ does not appreciably alter the voting pattern, that is, the effective number of electoral parties (N_v) remains the same.

$$S = P^{1/3} \qquad (15.1)$$

visibly expresses the relationship between S and P rather well. If population figures in million are used, the equation becomes

$$S = 100\, P^{1/3} \text{ (P in million).} \qquad (15.2)$$

However, figure 15.1 merely illustrates the plausibility of the cube root relationship; its validity will be discussed later.[2]

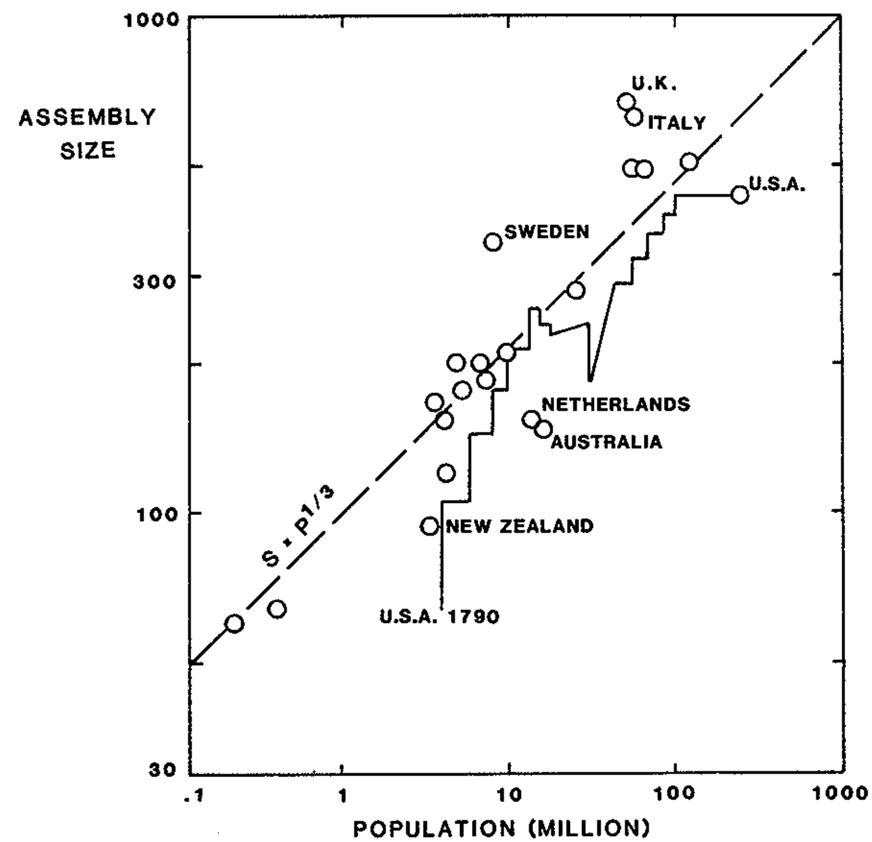

Figure 15.1. Population and national assembly size for the United States since 1790 and for 20 other stable, developed countries in the 1980s. Data from *NYT Family Almanac* (1972:22–36) and Banks (1985).

2. The log scales on the two axes are chosen so as to make the visual slope of the $S = P^{1/3}$ line equal to 1, because this approach subjects the relationship to the severest possible visual test. If the same scale were chosen on both axes, the slope would be 1/3,

Figure 15.2 extends the scope of study to all contemporary nations for which S and P are given in Banks (1985). The pattern becomes more scattered, and most points fall below the line $S = P^{1/3}$. Many fall even below the line $S = (1/2)P^{1/3}$, which is also shown. It is surprising that a common pattern exists at all, since the data now also include unstable regimes which have experimented with many different assembly sizes

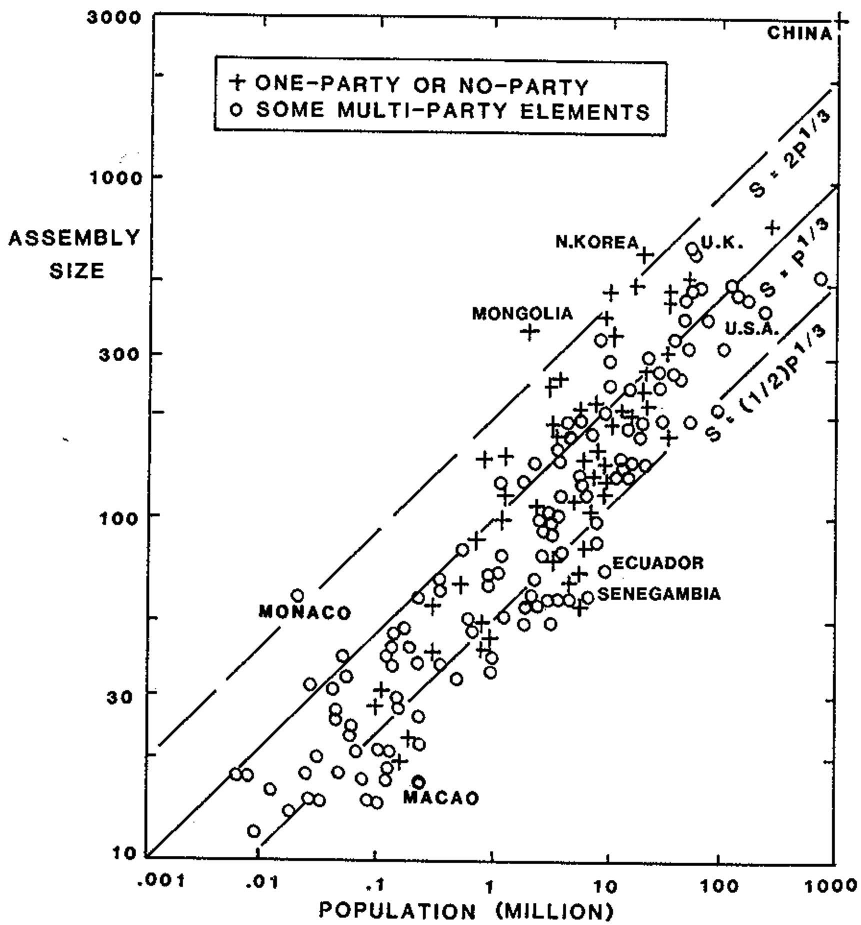

Figure 15.2. Assembly size vs. population for all countries with data in Banks (1985).

and with such a low slope the visual impression of the degree of fit is enhanced even for quite scattered data.

(or have done at times without any assembly) and nondemocratic regimes where the assembly is openly appointed by the rulers or covertly appointed through choiceless elections. Despite such variety in methods of choosing the assembly membership and the resulting roles of the assembly, the relationship between assembly size and population size tends to be the same. Countries with low literacy rates tend to have rather small assemblies (at given population size), and this leads us to the notion of "active population."

The number of assembly seats might be determined by literate adult population rather than by total population, since only adults vote and only literate voters are well placed to cast an informed vote. Many objections to such a simplistic argument can be raised, but rather than objecting let us try it out. If the argument has no merit, the literate adult population should not correlate with assembly size any more than the total population. On the other hand, if using adult literate population improves correlation with S, then we might be on the right track. We define "active population" (P_a) as

$$P_a = PLW, \tag{15.3}$$

where P is the total population, L is the literacy rate, and W is the working-age fraction of the population. As a rough rule of thumb, active population as defined here is about one-half of the total population (for example, for $L = .90$ and $W = .55$).[3]

Figure 15.3 shows the assembly sizes graphed against the active population. There are fewer data points than in the previous figure (S vs. P), because literacy figures for many countries could not be found.[4] The

3. Our definition of active population omits literate people past retirement age, which is unjustified intellectually but is forced upon us by data: working-age population figures are much more available than total adult population figures. Actually, W varies relatively little (.47 to .68) compared to L (0.05 to 1.00) so that reporting errors on literacy overshadow the slight systematic underestimation of adult population through neglect of retirees. Developed countries tend to have L close to 1.00 and W around .65, so that active population is close to two-thirds of total population. For the least developed countries, W is around 0.5, and low literacy ($L = .05$) can reduce P_a to as little as 2.5% of the total population.

4. Working-age data came from Taylor and Jodice (1983:95–97). Literacy figures were taken from *Information Please Almanac* 1985, which has much more recent information than Taylor and Jodice (1983) has. In case of missing L, the country was dropped. In case of missing W, the figure for a neighboring country with the same literacy was used; the error thus introduced is bound to be small, given the limited range of W.

theoretical model to be presented in the next section predicts that S equals the cube root of $(2P_a)$ rather than of P_a:

$$S = (2P_a)^{1/3}, \qquad (15.4)$$

and this corresponds to the central line shown in the graph. Also shown are lines at double and one-half of this expected value. Few countries have assemblies larger than twice the value predicted by the model, and few have assembly sizes smaller than one-half of the expected number. Equation 15.4 is close to the best-fit line, except that countries with very

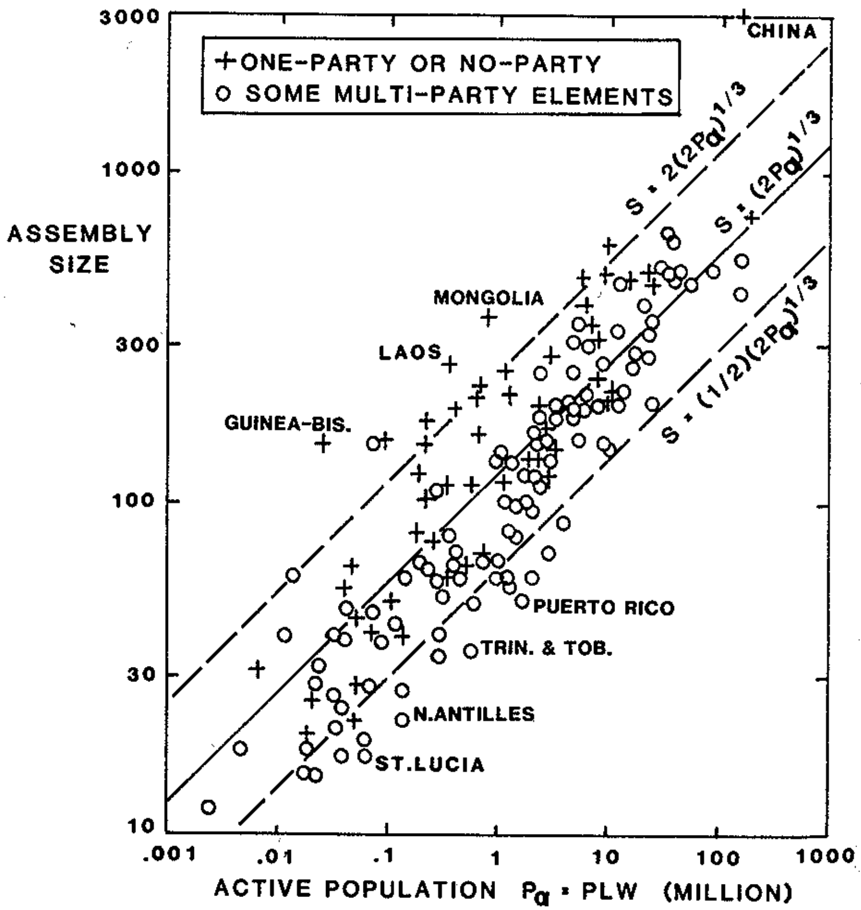

Figure 15.3. Assembly size vs. active (adult literate) population. Data from Banks (1985) and *Information Please Almanac* (1985) for literacy, and Taylor and Jodice (1983:95–97) for working-age population.

small adult literate populations tend to have assemblies smaller than expected. Correlation of S with P_a is better than with P, thus supporting the introduction of the notion of active population as defined here. An earlier study using data from around 1970 (Taagepera 1972) led to similar conclusions.

The current worldwide population explosion has doubled the population of many countries over the last twenty years. Assemblies may fall below the predicted size because such nations are late in adjusting assembly size to increased population and literacy. If nations do adjust their assembly size to their active populations, there should be a noticeable trend toward larger assemblies. Increases in literacy should accentuate this trend even more. Combining the data used for figure 15.3 with that of the previous study of assembly sizes (Taagepera 1972), S and P_a can be determined for 105 countries during two time periods: around 1970 and around 1985. In 44 cases, S did not change (within plus or minus 10 percent). In 57 cases S went up by more than 10 percent, and in only four cases did S go down by more than 10 percent. Thus an upward trend in assembly sizes has indeed occurred.

THE THEORETICAL MODEL FOR ASSEMBLY SIZES

It is time to introduce the model. The reasoning presented here differs in minor ways from the one presented originally by Taagepera (1972). Most of the proof involves algebra only, but at one point calculus enters.

The model is easiest to visualize in the context of single-member districts. The most time-consuming activity of the members of a representative assembly is communication. We shall assume that two types of communication predominate: (1) communication with constituents, whose views are to be taken into account and to whom decisions have to be explained, and (2) communication with other representatives and monitoring communications among them, so as to have sufficient information about what is going on and to participate in decision-making. We shall consider the number of channels of communication of each type. Our simple model assumes that both types of channels are equally time-consuming on the average. The model neglects other types of communication and other time-consuming activities.

If S is the number of seats in the assembly and the total active population is P_a, then the average constituency of one assembly member consists of P_a/S people. (Strictly speaking, we should subtract the assembly member himself, but the error introduced by not doing so is negligible.) The average assembly member participates in the P_a/S channels that connect him to the constituents in a dual capacity: as a potential receiver and sender of information. Thus the total number of communications channels making demands on one member's time and effort in the constituency is

$$c_C = 2P_a/S. \tag{15.5}$$

Within the assembly itself, every assembly member communicates with the other $(S - 1)$ members in a dual capacity—as a potential listener and speaker. He also monitors the channels connecting the other $(S - 1)$ members to each other. The number of such channels is $(S - 1)(S - 2)/2$. Thus the number of communication channels making demands on one member's time and effort in the assembly is

$$c_S = 2(S - 1) + (S - 1)(S - 2)/2 = S^2/2 + S/2 - 1 \approx S^2/2, \tag{15.6}$$

provided that $S \gg 1$. The total number of channels making demands on an average assembly member is

$$c = c_S + c_C = S^2/2 + 2P_a/S. \tag{15.7}$$

We will assume that she/he is the least overworked and thus the most efficient when this total number of channels is minimized (for the given population size).

Figure 15.4 shows how the number of channels varies with S, for a country with $P_a = 125$ million, which is approximately the case for the United States. As the assembly size is increased, the number of constituency channels decreases but the number of assembly channels increases. Under optimal conditions (that is, the least total number of channels) according to the model, the assembly has 630 members, with each member having 397,000 constituency channels and 198,000 assembly channels, for a total of 595,000. For the actual U.S. House size (435 members), the number of assembly channels (as given by equation 15.6) is reduced to 95,000, but constituency channels (equation 15.5) increase to 575,000, so that the total becomes 670,000, in other words, 13 percent larger than the minimum possible. This is not a large extra load, and on the basis of the model one would not expect a heavy pressure to increase the House size.

The assembly size S_0 which minimizes C (at constant P_a) is obtained

by calculating the derivative dC/dS and making it zero. This is the only step involving calculus:

$$dc/dS = S - 2P_a/S^2 = 0. \tag{15.8}$$

Hence, for optimal assembly size S_0,

$$2P_a = S_0^3. \tag{15.9}$$

The outcome is the cube root law of assembly sizes:

$$S_0 = (2P_a)^{1/3}. \tag{15.10}$$

Strictly speaking, the model applies only to single-member districts in polities where assembly members have some independence from party discipline. There are various reasons why other types of assemblies might assume a similar relationship to population size, ranging from imitation to hidden processes analogous to the ones described in the model. In this sense the cube root law is not completely proved theoretically. On the other hand, it is not a purely empirical relationship, given the existence of a theoretical model covering at least part of the

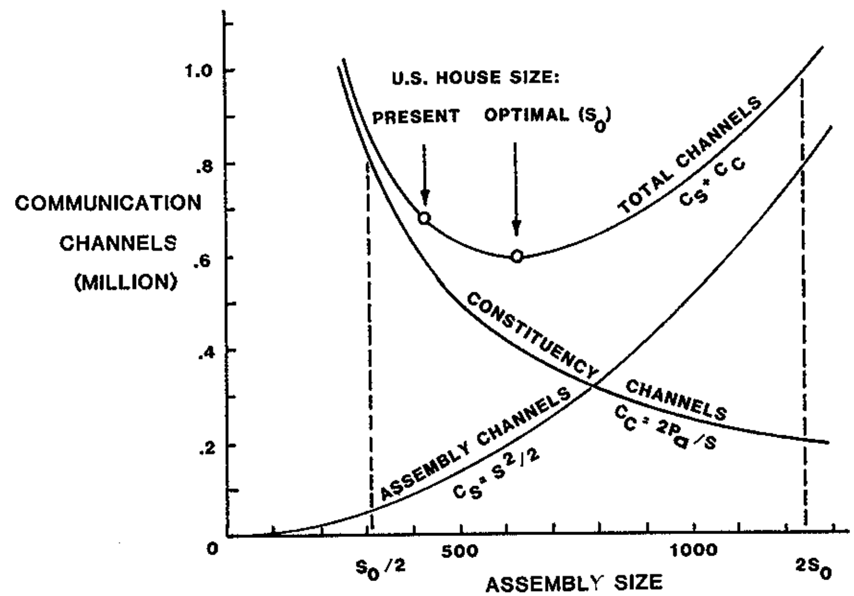

Figure 15.4.Number of communication channels for one assembly member as a function of assembly size, for an active population of 125 million.

assemblies. It is thus definitely more lawlike than the cube rule of elections, which traditionally has been called the "cube law," and it compares favorably with Duverger's law. Given the paucity of genuine theory-based quantitative laws in political science, the cube root law of assembly sizes is among the strongest.

According to the model, what is the ratio of an assembly member's channels to the assembly (c_S) and to the constituents (c_C)? Combining equations 15.5 and 15.9 yields $c_C = S^2$. Thus under optimal conditions the number of assembly channels are one-half of the constituency channels, and the total number of channels is

$$c = 1.5\, S^2 = 1.5\, (2P_a)^{2/3} \qquad (15.11)$$

at the minimum. The number of channels at other, nonoptimal, assembly sizes can also be calculated, as we did in connection with figure 15.4. An assembly twice the optimal size increases the number of channels to 67 percent above the optimal, and an assembly half the optimal size has 42 percent more channels than optimal. As we saw in figure 15.3, most actual assemblies fall into this range.

EXPLANATION OF THE CUBE RULE OF ANGLO-SAXON ELECTIONS

It was observed in the previous chapter that the cube rule of Anglo-Saxon parliamentary elections emerged from a wider seat–vote equation for plurality elections when the ratio $n = (\log V)/(\log E)$ was close to 3, with E standing for the number of districts within which the plurality rule was applied. At that stage, it was puzzling why $n \approx 3$ for Anglo-Saxon national assemblies. The puzzle now receives an explanation. Taking logarithms on both sides of equation 15.10 leads to

$$(\log P_a + \log 2)/(\log S) = 3, \qquad (15.12)$$

and log 2 is small compared to log P_a. The logarithms of active population, adult population, and the number of voters are not very different (see table 14.1) as long as literacy or electoral participation rates are not extremely low. Thus $(\log V)/(\log S)$ has a value close to 3 in nearly all parliamentary elections. In single-seat plurality elections the number of districts equals the number of seats ($E = S$), and thus the cube rule automatically results.

The cube rule no longer remains an isolated empirical rule but

results from combining the cube root law of assembly sizes with the seat–vote equations. The theoretical justification for the cube root law has been presented. The next chapter establishes a rational model for the seat–vote equations, and then the cube rule also receives a theoretical basis. Various other attempts to explain the cube rule are reviewed by Taagepera (1986).