A Model of Coalition Formation: Theory and Evidence

Itai Sened

Tel Aviv University

Coalition formation is modeled as a cooperative game. Each party enters the game endowed with a proportion (weight) of votes that it obtained in the election, and a preferred *policy* position. The payoffs to any party that joins a coalition are a function of the distance between the party's and the government's respective *policy* positions, and the *office* related payoff that the party receives as a member of the coalition. A new core solution concept, the IVCORE, is introduced. It allows the analysis of the trade-off between ideological, *policy* payoffs, and *office*-related sidepayments, in the bargaining process over future coalitions. It turns out that *policy* concerns of parties "induce" a core in a typical transferable payoffs game that would, otherwise, have a generically empty core. At the same time, the "budget constraint" on the *office*-related sidepayments, determines the composition of the coalition. The process of coalition formation in Israel, after the 1992 election, is used to illustrate the empirical relevance of the theoretical model.

1. INTRODUCTION

Formal models of coalition formation tend to assume that political parties coalesce to obtain either *office*-related payoffs (Riker 1962; Baron and Ferejohn 1989; Baron 1989, cf. Brown and Franklin 1973), or ideological, *policy* payoffs (Schofield 1986; Austen-Smith and Banks 1990; Laver and Shepsle 1990a; Baron 1993).

While most practitioners in the field agree that both types of payoffs are crucial in the process of coalition formation, most models in the literature assume that they are "orthogonal and hence can be analyzed independently of one another" (Austen-Smith and Banks 1990, 891).[1]

This article promotes a model that synthesizes the *office* and *policy* approaches to coalition formation, using a utility function that amalgamates the utility functions

[1] A notable exception to this general trend in the literature is Austen-Smith and Banks (1988). Even though my model is different from theirs in several aspects I obviously rely heavily on some of their earlier intuitions.

used by Riker and Schofield respectively: The payoffs each party gets from joining a coalition are a function of the distance between the government's position and the party's ideal point in the *policy space* and the share of the *office*-related payoffs the party receives as a member of the government in *office*.

The IVCORE is introduced as a new concept (cf. Sened 1995) that allows an analysis of the trade-off between *policy* and *office* considerations. It turns out that the ideological concerns of parties "induce" a core in a typical transferable payoffs game that would, otherwise, have a generically empty core. At the same time, the "budget constraint" on the sidepayments determines which parties would be included in the coalition.

The substantive meaning of the findings is that if parties bargained over side-payments, associated with being in *office*, alone, they would never reach stable agreements. Parties reach stable coalition agreements because they trade *office*-related sidepayments for *policy* compromises and vice versa.

Different parties are more or less prone to accept ideological compromises. Central, moderate parties are more likely to be included in a coalition because the ideological compromises they must make to join a coalition, are less costly. A coalition in *office* can only implement one *policy*. It is cheaper for the formateur of a coalition to "sell" this *policy* to moderate, central parties than to parties that insist on extreme ideological positions. The currency used in these trades are shares of the benefits of being *in office*, like portfolios and special budgets, distributed by the formateur to its partners in return for their willingness to endorse the government's *policy*.

Section 2 of this article introduces some basic concepts in contemporary coalition theory. Section 3 introduces the formal model. In section 4 through section 7 the formal results are derived and their implications for the study of proportional rule, parliamentary systems, are discussed. The coalition formation process in Israel, after the 1992 election, is used in section 8, as an illustration to the empirical relevance of the theoretical, formal model.

2. The Size Principle and the Structurally Stable Core

The most cited concept in the literature on coalition formation is Riker's "size principle." Riker (1962) shows that in *n*-person constant-sum-games with sidepayments, agents will form coalitions just as large as the minimum necessary to obtain control over the prize (32–3). The logic behind this principle is straightforward: if the prize is divisible, then those who gain control over it would like to share it with the smallest "minimum winning coalition" (MWC) possible.

The size principle seems to have a remarkable predictive power. In a data set compiled by Schofield (1990), 162 of 216 recorded government coalitions in European legislatures between 1945–1987 (75%), were either minimum winning or minority coalitions. In section 6, I show that minority governments are consistent

with Riker's size principle (Laver and Shepsle 1990b), in as much as they are usually formed by a dominant party when ideological differences prevent other parties from defeating the minority government.

Riker's size principle is criticized on the basis of three objections.

1. Many empirical tests have been less supportive than Schofield's data (for a detailed discussion see Schofield 1987, 1990; Laver and Schofield 1990).
2. Minimum winning coalitions (MWC) are inherently unstable: any MWC, say c_1, can be defeated by another MWC, say c_2, of parties not included in c_1, receiving 0 payoffs from c_1 and one member of c_1. Members of c_2 can always convince one member of c_1 to defect, promising more than he or she gets from c_1. This is often referred to as the "divide the dollar problem": three agents who have to divide a dollar, using majority rule (partitions must be approved by at least two players to be implemented) will never agree on how to divide the dollar for exactly the same reason (Ordeshook 1986, 349–55).
3. Riker's analysis ignored ideological *policy* payoffs. Parties do not bargain only over sidepayments, attached to holding *office*, but also over the *policy* that the government will implement (Laver and Schofield 1990, chap. 7). This aspect of the process is left out of Riker's analysis.

The field of formal models of coalition formation is currently dominated by a different analytical framework, called the "spatial theory of electoral competition" (STEC). Riker's "size principle" and related models assume that the only concern of parties in the coalition bargaining process, are *office*-related sidepayments. "STEC" is based on the assumption that the main issue in coalition bargaining is the compromise that must be reached over the *policy* position that the coalition will implement. Different parties have different ideological positions in the *policy space*. They must compromise and agree on a single *policy*, to be implemented by the future government.

"STEC" is based on the following theoretical primitives (Austen-Smith 1983): We have a set, X, of outcomes, which is a subset of the *policy space*, R^m. The dimensionality, m, of this space is determined by the number of political issues that dominate the political debate in the relevant country. Each issue is represented as one dimension of the policy space. Policy positions are represented as points in this space. The utility an agent gets from any policy position, $x \in R^m$, is assumed to be a function of the Euclidean distance between x and x_i: $u_i(x) = \phi_i(x - x_i)$, where x_i denotes the ideal policy point of agent i. Agents seek to realize policy points as close as possible to their ideal points. The central solution concept in this paradigm is the "core" or "simple majority equilibrium" (Austen-Smith 1983, 441):

> *Definition 1.* $x^* \in X \subseteq R^m$ is a simple majority equilibrium if, for any other feasible alternative $y \in X$, at least half of the agents prefer x^* to y.

When the core is nonempty, i.e., if there is at least one outcome, that satisfies definition 1, we expect the outcome of an interaction among rational agents using

majority rule, to be the core. Since, by definition, there is no position that a majority prefers to a core position, a core position will never be defeated by a majority of the agents.

McKelvey and Schofield show that in *policy spaces* with dimensionality $m \geq 2$, a core rarely exists, and if the core is empty, then agendas can lead to almost any policy outcome (McKelvey 1979; Schofield 1983; McKelvey and Schofield 1987). In other words, we do not expect any stable policy outcome to persist in such legislatures.

Shepsle (1979) was the first to emphasize, in this context, the role of institutions in inducing stable policy outcomes (Shepsle 1986, 51–2):

> ... [the] relationship between social choices and individual values is a mediated one. Standing between the individual ... and the social choices are institutions ... frameworks of rules ... [that] constrain the set of choosing agents, the manner in which their preferences may be revealed, the alternatives over which preferences may be expressed, the order in which such expressions occur ... the way in which business is conducted.

Shepsle (1979) showed that institutional arrangements often induce equilibria in cases where, without such arrangements, no equilibrium exists. Laver and Shepsle (1990a) and Austen-Smith and Banks (1990) use variations of Shepsle's argument to model coalition formation in European legislatures.

In a separate analytical effort Schofield (1986) shows that if instead of individual members, a legislature is made of political parties, "Structurally Stable Cores" (SSC) often exist in two-dimensional spaces. Schofield's work implies that the institution of "proportional rule systems" can induce a core where, without this voting mechanism, simple majority core would not exist.

To understand the SSC solution concept and prepare for the following analysis we need to define a set of formal primitives: Recall that $X \subseteq R^m$ is the set of possible outcomes. Let $P = \{1, \ldots, p\}$ be the set of parties and $W = \{w_1, \ldots, w_p\}$ be a set of weights determined by the number of seats each party gained in parliament. Without much loss of generality, assume that party ℓ is the biggest party in parliament: i.e., $w_\ell > w_i \ \forall \ i \in P \ i \neq \ell$. Let $\{x_1, \ldots, x_p\} \subseteq X$ be the set of ideal points of the relevant parties. Let $u = \{u_1, \ldots, u_p\}: X \Rightarrow R^p$ be a smooth utility function representing the preference profile of all parties over $X \subseteq R^m$. A voting rule, σ, is a social preference function which assigns to any utility function, u, a strict preference $\sigma(u)$.[2]

> *Definition 2* (Schofield 1984, 190). A coalition $c_r \subseteq N$ is decisive for σ if and only if, for any utility function, u, and any two alternatives $x, y \in X \ yP_i x \ \forall \ i \in c_r \Rightarrow y\sigma_{(u)}x$. Let D_σ be the set of decisive coalitions for σ.

> *Definition 3.* "$yP_i x \ \forall \ i \in c_i \Rightarrow y\sigma_{(u)}x$" denotes that y is preferred to x, given a utility space u, and a voting rule σ, if all members of a decisive coalition, $c_r \in D_\sigma$, prefer y to x.

[2]It also has to satisfy the *independence axiom* (see Schofield 1980, Def. 2.2).

Note that the rule σ defines the set of decisive coalitions: D_σ. In western democracies, any coalition of more than half of the legislators is decisive. This rule is known as majority rule.

Definition 4. The core $C(\sigma,u)$, in a weighted voting game σ, given u, is defined as: $CO(\sigma,u) = \{y \in X \mid x\sigma_{(u)}y,$ for no $x \in X\}$.

Definition 5. The core, $C(\sigma,u)$, is structurally stable, SSC (σ,u), if and only if $C(\sigma,u)$ is not empty and for any $x \in C(\sigma,u)$ there exists a neighborhood, $u_{(x)}$ of u and a neighborhood $v_{(x)}$ in X, s.t. $\forall u \in u'_{(x)}, \exists x' \in v_{(x)}$ s.t. $x' \in C(\sigma,u')$.

Definition 4 is a generalization of definition 1. Definition 1 stated that any outcome that is preferred by a majority over any other outcome is in the "simple majority core." Definition 4 states that any outcome, $y \in X$, that is preferred by a decisive coalition, defined by the voting rule σ, given the preference profile u, over any other outcome $x \in X$, is in the $C(\sigma,u)$ core. Definition 5 is a restriction on the $C(\sigma,u)$ core, requiring that if agents' preferences change slightly, the core does not "vanish," but remains in a "neighborhood," $v_{(x)}$, of the core $C(\sigma,u)$.

Schofield shows that in weighted majority voting games, if the SSC(σ,u) is not empty, it can only be at the ideal point of the largest party, x_ℓ:

Lemma 1. (Schofield (1995) SSC$(\sigma,u) \neq \phi \Leftrightarrow x_\ell \in$ SSC(σ,u).

Thus, if the SSC is nonempty, the policy position that will be implemented in equilibrium will be the policy position of the biggest party, ℓ.[3]

The models developed by Schofield and Shepsle show how institutional structures induce stable equilibria in multiparty European systems, but overlook the importance of transferable payoffs in coalition bargaining. Furthermore, these models do not yield clear predictions as to what coalitions may form by the end of the game. The model developed in the next section reintroduces sidepayment as a central factor in any coalition bargaining process. The new core solution concept, the IVCORE, allows the analysis of the trade-off between ideological, *policy* payoffs, and *office*-related sidepayments, in the bargaining process over future coalitions, and provides clear predictions about the coalitions that should emerge in such political environments.

3. THE MODEL

Considerable confusion exists in the literature concerning four stages that characterize proportional rule systems: In the first, preelection stage, parties position themselves to maximize their utility.[4] The second stage is the election game, in which voters choose whether and how to vote. The third stage is the coalition bar-

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[3]Throughout the article, I assume there is only one biggest party, ℓ.

[4]There is an open debate in the literature on whether parties maximize vote share or position themselves according to genuine policy preferences (Laver and Schofield 1990). This debate is of little consequence for this article.

gaining game. In the fourth legislative stage, a coalition that formed tries to implement its policy.

A comprehensive model must include all four stages. A good way to think about it is to use the notion of "backward induction." To play the coalition game, parties must have clear expectations about what will happen at the legislative stage. To vote, voters must have clear expectations about the coalition formation game. Finally, to position themselves so as to maximize their utility, in the preelection game, parties must have clear expectations about voting behavior (Austen-Smith and Banks 1988). Recently, attempts were made to incorporate all four stages into one model (Schofield 1995; Baron 1993; Austen-Smith and Banks 1988), but given the complexity of the problem, these attempts end up bypassing crucial aspects of the process (Baron 1993; Schofield 1995), or modeling very special cases (Austen-Smith and Banks 1988).

My modeling strategy is different. I endorse Schofield's description of the post-election game. I assume that parties assess the expected outcomes of the legislative game after obtaining the results of the election. Given this assessment, they can play the coalition game, as modeled in this article. Once this model is developed and tested we will be able to model the electoral and preelectoral stages. Meanwhile, for the sake of this article, the positions and weights of the parties are assumed to be exogenously fixed.

The game begins as "nature" assigns a vector of weights, $W = (w_1, \ldots, w_p)$, to a set of parties $P = (1, \ldots, p)$ with exogenously fixed ideal points $\{x_1, \ldots, x_p\}$. Research on legislative procedures of sophisticated legislators shows that the set of policy outcomes is likely to be contained in the *uncovered set* (Shepsle and Weingast 1984; McKelvey 1986; Ordeshook and Schwartz 1987). Unlike the *core*, the *uncovered set* always exists. It is centrally located and collapses to the core when the core exists (McKelvey 1986, 285). Thus, given a set of parties, $P = (1, \ldots, p)$, a vector of weights, $W = (w_1, \ldots, w_p)$ and a set of ideal points $\{x_1, \ldots, x_p\} \subseteq X$, the *uncovered set*, $\wp$, is the expected set of policy outcomes in the legislative game.[5] Coalition agreements about policy positions are usually not binding. Therefore, in this model, the results of the election, not the composition of any particular coalition, determine the policy outcome that will be implemented. This assumption distinguishes this model from models that make final policy outcomes depend on the composition of the coalition (cf. Laver and Shepsle 1990a; Austen-Smith and Banks 1988, 1990).[6]

[5]Some have suggested the *Pareto set* (Tullock 1967) or the *cycle set* (Laver and Schofield 1990, 122–26) as alternative solutions to voting games when the core is empty. These solutions have different justifications. All are "centrally located." *Cycle sets* and *uncovered sets* collapse to the core when a core exists, while the *Pareto set* does not. I chose the *uncovered set* since it was extensively studied and has a solid theoretical justification (Banks 1985). The results of this article would apply to any solution if the context suggests the use of alternative solution concepts.

[6]In the model developed by Austen-Smith and Banks (1988) the party that forms the coalition picks up a policy position that maximizes its utility. In the Laver and Shepsle model (1990a; cf. Austen-Smith and Banks 1990), parties who form the coalition, divide portfolios, assuming that each minister will impose his position in the dimension controlled by its department. These assumptions imply that the

An outcome of the coalition game is an ordered pair $(c_r, P_r \mid \wp)$, where $P_r \equiv (P_{\ell r}, \ldots P_{pr} \mid c_r, \wp)$ is a vector of shares, P_{ir}, of K, the total amount of *office*-related, available sidepayments.

The utility party i expects from joining a coalition c_r is a function: $u_{ir} = u_{ir}(c_r, P_{ir} \mid \wp)$. u_{ir} is assumed to be a linear function of P_{ir}, and a quadratic function of $d[x_i, E(x_r)]$, the Euclidean distance between the ideal policy position, x_i, of party i and the expected policy of coalition c_r, $E(x_r)$. Thus, $u_{ir} = \alpha_i \cdot P_{ir} - \beta_i \cdot d[x_i, E(x_r)]^2$ (Austen-Smith and Banks 1988).

The parameters α_i and β_i are crucial in this utility function. The debate over the extent to which parties are *office* or *policy* seeking (Budge and Laver 1985; Laver and Schofield 1990, chap. 7) can be reduced to the relationship between the two parameters. Of course, these parameters differ from party to party. If $\beta_i = 0$, then the party is purely *office seeking*. If $\alpha_i = 0$, then the party is purely *policy seeking*. The premise of this article is that such extremes are rare and that in most cases both parameters are strictly positive. Letting $\pi(x)$ be the probability that x becomes the policy of c_r, we get:

$$u_{ir}(c_r, P_r \mid \wp) = \alpha_i \cdot P_{ir} - \beta_i \cdot \int_{x \in \wp} d[x_i, x]^2 \cdot \pi(x) \, d(x) = \alpha_i \cdot P_{ir} - \beta_i(\wp) \quad (1)$$

$\beta_i(\wp)$ is a scalar, given the set $\wp$ of possible legislative outcomes implied by the election results, but it varies across parties depending on their location vis-à-vis $\wp$ and the magnitude of β_i. By Lemma 1, if the SSC is not empty then $\wp = x_\ell$, and:

$$u_{ir}(c_r, P_r \mid \wp) = \alpha_i \cdot P_{ir} - \beta_i \cdot \int_{x \in \wp} d[x_i, x]^2 \cdot \pi(x) \, d(x) =$$
$$\alpha_i \cdot P_{ir} - \beta_i \cdot d[x_i, x_\ell]^2 \quad (2)$$

The utility function specified in equations (1) and (2) relies on the premise that it is costly for a party to endorse a policy, other than its own. Parties endorse such policies to allow the formation of coalitions. The cost of such compromises is captured by $\beta_i(\wp)$ in equation (1), or $-\beta_i \cdot d[x_i, x_\ell]^2$ in equation (2). The benefits are captured by $\alpha_i \cdot P_{ir}$. Parties that stay in the opposition do not get these benefits, but avoid the cost of endorsing policies that deviate from their ideal points. Thus, parties that do not join the coalition are assumed to receive a utility of zero. Formally: $\forall \, i \in P, \forall \, c_r \subseteq N, u_{ir}(i \notin c_r) = 0$.

The game: Each party has to decide whether to accept a share, P_{ir}, to join c_r. If all the members of c_r accept what c_r allocates them, c_r forms. c_r can distribute sidepayments only if it is *invulnerable*, by definition 8, or *winning* by definition 7. Formally: $u_i(i \in c_r \mid c_r \text{ is } vulnerable) = 0, \forall \, i \in P, \forall \, c_r \subseteq N$.

Definition 7. A coalition c_r is *winning*, $c_r \in \{WC\}$, if it is *invulnerable*.

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coalition agreement can withstand pressures from parliament. My model assumes that coalition agreements cannot withstand such pressures. Therefore, the only policy position parties expect is the *core*, if one exists, or ever changing positions within $\wp$ if the *core* is empty.

Definition 8. A coalition, c_r is *vulnerable* if:

a. $\exists\, c_r'$ s.t. $\Sigma_{i\in c_r}'\, w_i > 50\%$), and
b. $\exists\, (c_r', P_r' \mid \wp)$ s.t. $u_i(c_r', P_r' \mid \wp) > u_i(c_r, P_r \mid \wp), \forall\, i \in c_r'$.

The literature usually treats the control, by a coalition c_r, of a majority of the seats in the legislature (formally: $\Sigma_{i\in c_r}\, w_i > 50\%$) as a necessary and sufficient condition for c_r to be *winning*. Numerous minority governments in European legislatures (Strøm 1990) prove that this condition is neither sufficient nor necessary. Definition 8 states that *only if* an alternative coalition, c_r', exists, its members control more than 50% of seats in parliament and they all prefer c_r' to c_r, c_r is *vulnerable*, and, therefore, *not winning* by definition 7.[7]

Let $u(x) = (u_{\ell r}(x), \ldots, u_{pr}(x))$ be the utility vector associated with outcome $x = (c_r, P_r \mid \wp) \in X$ and $U = \{U_\ell, \ldots, U_p\} \colon X \Rightarrow R^p$ be a utility function representing the preference profile of all parties over the set X of all possible outcomes.

Definition 9. A characteristic function of the coalition c_r, $V(c_r)$, is a collection of utility vectors, such that if c_r can guarantee to all its members $u_i(x)$ then $u(x)$ is in $V(c_r)$.

A game in characteristic function form is denoted by: $\Gamma = (V, P, U)$, where P is the set of all parties, V is the characteristic function that assigns to every coalition c_r, a collection of utility vectors and U is the set of all feasible utility vectors given the set of feasible outcomes X. $V(i)$ denotes the payoff a party i can guarantee itself acting as a "coalition of one."

Definition 10. $u(x)$ dominates $u(x')$ with respect to c_r if and only if $u(x)$ is in $V(c_r)$ (i.e., c_r can secure $u(x)$ to members) and $\forall\, i \in c_r, u_i(x) > u_i(x')$. $u(x)$ dominates $u(x')$ iff there exists a *winning (invulnerable)* coalition c_r such that $u(x)$ dominates $u(x')$ with respect to c_r.

Definition 11. The (IVCORE) is a set of ordered pairs such that $(c_r^*, P_r^* \mid \wp) \in$ (IVCORE) iff c_r^* is *invulnerable* and $u(c_r^*, P_r^* \mid \wp)$ is an undominated vector in U.

Definition 11 of the IVCORE is a standard definition of a core. It differs only in the nonstandard definition of $u(c_r^*, P_r^* \mid \wp)$ as a payoff vector in U, as defined by the utility function of the parties engaged in the coalition formation game, specified above.

Definition 12. The set of minimum winning coalitions {MWC} is defined as:

$$\text{MWC} = \{c_r \in \{\text{WC}\} \colon \forall\, i \in c_r,\ c_{r\backslash i} \notin \{\text{WC}\}\}.$$

[7]In most European legislatures, opposition parties can place motions of "no confidence" without proposing an alternative coalition. It is reasonable, however, to assume that motions of "no confidence" succeed only when some parties have an alternative in mind. Spain and Germany institutionalized a "constructive vote of no confidence." Parties are allowed to advance motions of "no confidence" only if they can name a feasible alternative.

Definitions 7, 8, and 12 build to a nonstandard definition of MWC: by definitions 7 and 8, the set of {WC} can in fact contain minority, even single-party, coalitions. This terminology may be problematic in light of the standard use of the concept of *winning coalitions* (WC) in the literature, but it turns out to be an important novelty of the model discussed here. Minority governments can be *winning* if they are *invulnerable* to votes of no-confidence by the opposition. In section 6, I discuss this advantage of the model, in that it can explain the well-known phenomenon of the emergence and stability of minority governments. Until now, this phenomenon remained enigmatic from a theoretical point of view (Schofield 1990; Laver and Shepsle 1990b; Strøm 1990; Strøm and Leipart 1993).

Transferable payoffs games usually have generically empty cores (Ordeshook 1986, 349–55), so the core solution concept is of little help in analyzing them. The game defined above, however, will often have a nonempty IVCORE. What "induces" a core in this game is the cost, $\beta_i(\wp)$, that each party must pay if it joins a coalition, endorsing the policy of the government it joins, that usually deviate from its declared ideal positions in the *policy space*.

4. A GENERAL CHARACTERIZATION OF THE IVCORE: EXPLAINING THE STABILITY OF MWC

Lemma 2 states that, in equilibrium, all sidepayments will be allocated.[8] (The proofs to all the Lemmas and Theorems are found in the appendix at the end of the article.)

Lemma 2: If $(c_r, P_r) \mid \wp) \in$ (IVCORE) then $\Sigma_{i \in c_r} P_{ir} = K$.

Assume:

$$\forall j \in P, j \notin \ell \; \exists \; c_r \text{ s.t. } j \notin c_r, \; w(c_r) > 50\% \text{ and} \atop \Sigma_{i \in c_r}(\beta_i/\alpha_i) \cdot d[x_i, x_\ell]^2 < K \tag{3}$$

Condition (3) rules out the possibility that any party $j \in P$, except for the biggest party ℓ, takes all the transferable payoffs, K, because it is *invulnerable* to any coalition. If condition (3) does not hold, Lemma 3 does not hold and: $(c_r{}^* = (j), P_{rj}{}^* = K \mid \wp = x_\ell) \in$ (IVCORE) (cf. theorem 3). This type of degenerate outcomes is usually ruled out in similar models in the literature (cf. Austen-Smith and Banks 1988) by assuming that the biggest party is the first to try and form a coalition. Austen-Smith and Banks (1988) point out that this is a norm in many European legislatures. If the biggest party is the first to try and form a coalition, then Lemma 3 holds regardless of whether condition (3) holds or not. Lemma 3 proves that if the SSC is nonempty the biggest party will always be a member in the coalition.

Lemma 3: If SSC $\notin \varnothing$, and (3) holds then: $(c_r{}^*, P_r{}^* \mid \wp = x_\ell) \in$ (IVCORE) $\Rightarrow$ $\ell \in c_r{}^*$.

[8]I am assuming, of course, that $\Sigma_{i \in cr} P_{ir} > K$ is not feasible.

Lemma 4 shows that if $(c_r, P_r \mid \wp)$ is in the IVCORE, then the coalition c_r is a minimum winning coalition, as defined in definition 12.

Lemma 4: If $c_{r\setminus j} \in \{WC\}$, then $c_r \notin$ (IVCORE).

Let:

$$(SV_i \mid c_{\hat{r}}) = \text{Max}\left(\alpha_i \cdot \left(K - \left[\sum_{j\in c_r, j\notin c_{\hat{r}}, j\neq i}\beta_j(\wp) + \sum_{h\in c_r, h\notin c_{\hat{r}}, h\neq i}\alpha_h \cdot P_{h\hat{r}}\right]\right)\forall c_r \mid w(c_r) > 50\%\right) \quad (4)$$

$(SV_i \mid c_{\hat{r}})$ is the maximum value of sidepayments that party i can get, given a coalition, $c_{\hat{r}}$, from any other coalition, c_r, that can defeat $c_{\hat{r}}$, i.e., any alternative coalition that controls a majority in parliament (formally: $\forall\ c_r$ such that $w(c_r) > 50\%$). To compute it we subtract from the total value of sidepayments, K, the minimum that must be paid to the members of c_r, except i, to convince them to join c_r. If the relevant member, j, of c_r is not a member of $c_{\hat{r}}$, then, to join c_r, it must be paid enough to offset the cost of endorsing the expected policy position, $E(x_r)$ of c_r. This cost is captured by $\beta_j(\wp)$, summed up for all such members we get

$$\sum_{j\in c_r,\, j\notin c_{\hat{r}},\, j\neq i}\beta_j\,(\wp)$$

in equation (4). If member h, of c_r, is also a member of $c_{\hat{r}}$, it must be paid at least what it gets from $c_{\hat{r}}$, i.e., $\alpha_h \cdot P_{h\hat{r}}$, to join c_r. Summed up for all members of c_r we get

$$\sum_{h\in c_r,\, h\notin c_{\hat{r}},\, h\neq i}\alpha_h \cdot P_{h\hat{r}}$$

in equation (4). To convince party i to join c_r, to defeat $c_{\hat{r}}$, the formateur of c_r can offer party i, at most $(SV_i \mid c_{\hat{r}})$. Theorem 1 states necessary and sufficient conditions for the existence of a nonempty IVCORE.

Theorem 1: $(c_r^*, P_{ir}^* \mid \wp) \in$ (IVCORE) if and only if:

(i) c_r^* is minimum winning.
(ii) $\forall\ i \in P$, if $(SV_i \mid c_r^*) - \beta_i(\wp) \leq 0$, then $P_{ir}^* = 0$.

   Otherwise
$$(SV_i \mid c_r^*) \leq P_{ir}^* \leq \alpha_i \cdot \left(K - \left(\sum_{j\in c_r,\cdot, j\neq i}(SV_j \mid c_r^*)\right)\right).$$

(iii) $\sum_{i\in c_r^*}P_{ir*} = K$

In words: If the IVCORE is not empty it consists of a MWC, as defined in definition 12 (3.i). Each member of a coalition in the IVCORE gets at least its "reservation" price, $(SV_i \mid c_r^*)$—the maximum it can get from any coalition to which c_r^* may be *vulnerable*—and, at most, a share of K that allows c_r^* to pay all members big

enough shares to counter attempts to bribe members out of the coalition (condition 3.ii). Condition 3.iii is a budget constraint requiring that the total amount of side-payments be distributed by the coalition.

The IVCORE can be thought of as a remedy to a real theoretical problem with Riker's size principle: n-person constant-sum-games with sidepayments have a generically empty core (Ordeshook 1986, 349–551). In such environments only MWCs can emerge, but all MWCs are *vulnerable*, as defined in definition 8. In other words, in the Rikerian environment only minimum winning coalitions form but no coalition can last.

Theorem 1 explains why some of these coalitions do last: *policy* concerns of parties "induce" a core in this typical "divide the dollar" game. It is easy to show (Sened 1995) that there will always exist an upper bound $\bar{k}$ of K, such that if $K > \bar{k}$ the IVCORE is empty. So, ideological *policy* differences, if important enough, can induce equilibria in these games, but in the absence of such differences, coalitions will tend to collapse over disputes concerning the distribution of *office* related side-payments (cf. Mershon 1990).

5. A THEORETICAL EXPLANATION FOR DOMINANT PARTIES

Lemma 3 proved that if the SSC is nonempty, the biggest party will always be a member in the coalition; theorem 2 provides an elegant theoretical solution to the puzzle of single-party *dominance* in multiparty proportional rule systems. A *dominant* party (1) tends to be the biggest party over long periods of time; (2) it is likely to be a member of any governing coalition; (3) it has a visible advantage in the coalition bargaining process and (4) it sets the public policy agenda for the coalition, once it forms (Pempel 1990, 3–4).

As Pempel states it (1990, 6): ". . . a central puzzle for single-party *dominance* within the industrialized democracies is how and why it occurs under situations of social dynamics and openness" (cf. Schofield 1995).

Traditional explanations to this phenomenon were given in historic terms. Such parties were said to be associated with formative historical events, national symbols or charismatic leaders (Duverger 1963, 308; Arian 1985, 108–15; Pempel 1990). Theorem 2 shows that *dominant* parties are just *core* parties. A combination of a central position in the *policy space* and size, gives such parties the advantages associated with *dominant* parties in industrialized democracies, allowing them to control the *policy agenda* and form consecutive coalitions.

Theorem 2: If SSC $\neq \varnothing$ and condition (3) of Lemma 3 holds, then:

$$(c_r^*, P_{ir}^* \mid \wp = x_\ell) \in (\text{IVCORE}) \Rightarrow c_r^* \in \{\text{MWC}\} \text{ and } \ell \in c_r^*.$$

Proof: Follows directly from Lemma 1, 3, and 4.

Theorem 2 shows that if the SSC is not empty and the IVCORE is not empty, the biggest party will form a MWC, as defined earlier, and implement its ideal policy position. Van Roozendaal (1990, table 2) provides empirical support for the

prediction of theorem 2: Using data from France, Germany, Ireland, Italy, and the Netherlands, he shows that in 75 of 80 coalitions that formed, the central *core* party was a member of the coalition.

6. MINORITY GOVERNMENTS

Just as intriguing as the puzzle of *dominant* parties, are *minority* governments (Strøm 1990). The puzzle is simple: how can governments who control only a minority in parliament implement their policy. For many years this was regarded as an important anomaly in coalition politics (Strøm and Leipart 1993; Van Roozendaal 1990). Theorem 3 explains this anomaly away. It specifies conditions under which coalitions may need less than a majority to be *winning*, providing a theoretical explanation to *minority* governments as a special case of MWC (Laver and Shepsle 1990b; Strøm and Leipart 1993).

Theorem 3: If the SSC $\neq \varnothing$ $(c_r{}^* = \ell,\ P_{\ell r}{}^* = K \mid \wp = x_\ell) \in$ (IVCORE) if and only if:

$$\forall\ c_r \subseteq N \text{ s.t. } \ell \notin c_r \text{ if } w(c_r) > 50\%, \text{ then } K \leq \Sigma_{i \in c_r} (\beta_i/\alpha_i) \cdot d[x_i x_\ell]^2 \qquad (5)$$

In words: If the ideological *policy* differences among parties are big enough, as captured by constraint (5), the biggest party will form a minority government and keep all the sidepayments to itself. How restrictive constraint (5) is and how big is "big enough" is a question for further empirical research. Theorem 3 states that if parties diverge on policy issues in a way that *office*-related sidepayments cannot offset the price of *policy* compromises among enough parties to form a majority in the legislature, the biggest party will form a minority government and keep all the sidepayments to itself.

Corollary 1: If SSC $\neq \varnothing$ and condition (3) and (5) above hold, then:

$$(c_r{}^* = \ell,\ P_{\ell r}{}^* = K \mid \wp = x_\ell) \text{ is the unique (IVCORE)}$$

Proof: Follows directly from Lemma 3 and theorem 3.

Theorem 3 and Corollary 1 have a straightforward interpretation: a *dominant* party can rely on extreme ideological, *policy* differences among the other parties in parliament to form a minority government. Van Roozendaal (1990, table 4) uses data from France, Germany, Ireland, Italy, and the Netherlands, to show that 26 of 32 minority coalitions that formed in these countries between 1945–1988 were formed by such central *core* parties.[9]

7. THE CASE OF AN EMPTY SSC

The biggest party loses some of its advantage when the SSC is empty. This is why the biggest party is not necessarily *dominant*. Contrary to earlier conjectures,

[9]The remaining cases are probably explained by the fact that condition 5 did not hold, or the fact that the SSC was empty (see section 7).

however, this does not necessarily imply that the coalition formation game will have an empty *core*. The IVCORE may well be nonempty even if the SSC is empty (Sened 1995).

The conclusion from this and the discussion earlier is that the difference between cases with empty and nonempty SSC's may not manifest itself in stability or instability of coalitions. Rather, the difference should be observed in the stability of the policies implemented. When the SSC is empty, policy positions should shift constantly. When the SSC is nonempty we should observe stable policies implemented. Whether coalitions will or will not be stable depends on whether the IVCORE is empty or not. The Israeli case study, discussed later, is a clear example of a case where the SSC is nonempty, the policy implemented is stable, but the coalitions that form tend to be unstable.

8. COALITION FORMATION IN ISRAEL AFTER THE 1992 ELECTION

This section looks at the process of coalition formation in Israel after the 1992 election, to illustrate the empirical relevance of the formal model promoted in the previous sections.

Method

To construct a precise empirical illustration of the model we[10] used the following method: Questions edited by authoritative survey data scholars in Israel— Arian and Shamir (AS)[11]—were used to build two different scales for the two most salient issues in Israeli politics: *religion* and *security*.[12] To establish the parties' ideal points in the *policy space* we used manifestoes and other official publications about the parties' positions on different issues. We looked for the answers that these publications implied for the different questions appearing in the AS survey. We used factor analysis to establish whether the answers to these questions, implied by the parties' manifestos and formal publications, had a common underlying factor motivating them. Using those items that performed well in the factor analysis, we constructed two Likert[13] scales, to estimate parties' ideal *policy* positions in two-

[10] The help of Dganit Olomoky is gratefully acknowledged.

[11] Starting from 1969, Asher Arian edits a series entitled "*The Elections in Israel.*" Michal Shamir joined him in this effort starting from 1982. This project includes a scientific survey run before every election and a book that analyses the results of this survey as well as other aspects of the election and appears immediately after the election (e.g., Arian and Shamir 1990, 1994).

[12] Note that the usual "economic" left-right dimension is missing from the analysis. Economic issues stir emotions in Israel as they do all over the world. Yet most students of Israeli politics agree that when it comes to voting behavior, economic issues play a secondary role.

[13] An excellent short text on the Likert scaling technique, its uses and its merits, compared to other scaling techniques, is Leege and Francis (1974: 202–4).

[14] We thank Asher Arian and Michal Shamir for allowing us to use their data for this project.

FIGURE 1

3RD KNESSET (NOV. 1955–DEC. 1959)

Party Distribution
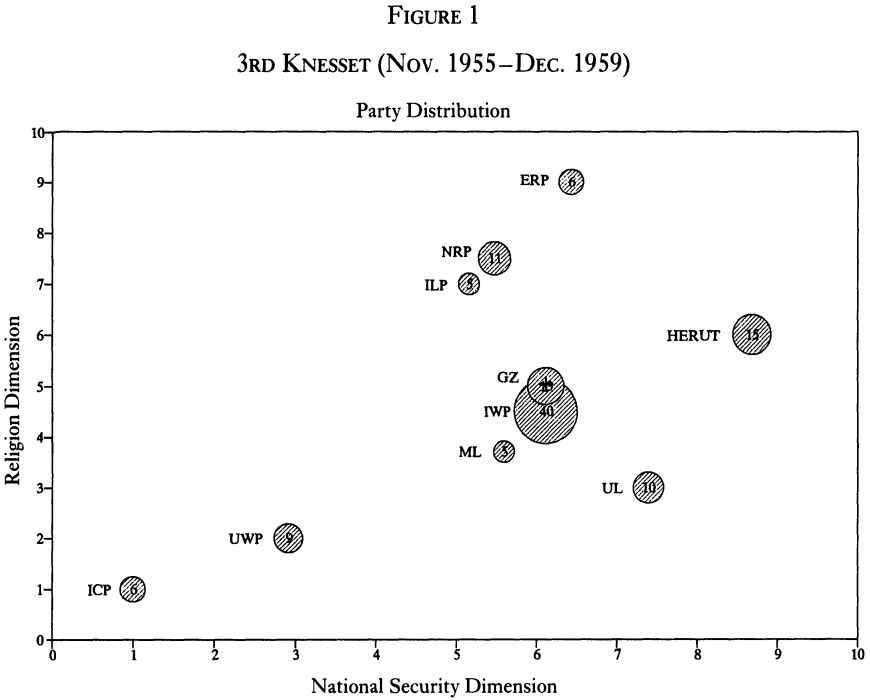
dimensional maps of the *policy space*.[14] Using the same technique we estimated the government position, using policy declarations that a prime minister gives when he presents his coalition for a vote of confidence in the Knesset.

Figure 1 and figure 2 are maps of seat distributions and *policy* positions of parties in the Knesset. Figure 1 is an example of the period between 1948–1973, when the Labor party captured a *core* position. The black cross marks the government's position. Its proximity to the position of the *dominant* labor party is typical of that period.

Figure 2 is an example of the period between 1977–1992. All the coalitions of this period included the Likud party, but unlike the Labor party, the Likud never captured a SSC position. This period is characterized by an empty SSC, resulting, among other things, in unstable government positions that were relatively far from the position of the Likud party.

Table 1 provides statistical evidence supporting this prediction: coalition positions during the period 1948–1973, when the SSC was nonempty, were significantly closer to the declared position of the biggest party than policy positions during the period between 1977–1988, when the SSC was empty. Note that the standard deviation for the first period is substantially smaller than the standard deviation for the second period.

FIGURE 2

10TH KNESSET (AUG. 1981–SEPT. 1984)

Party Distribution
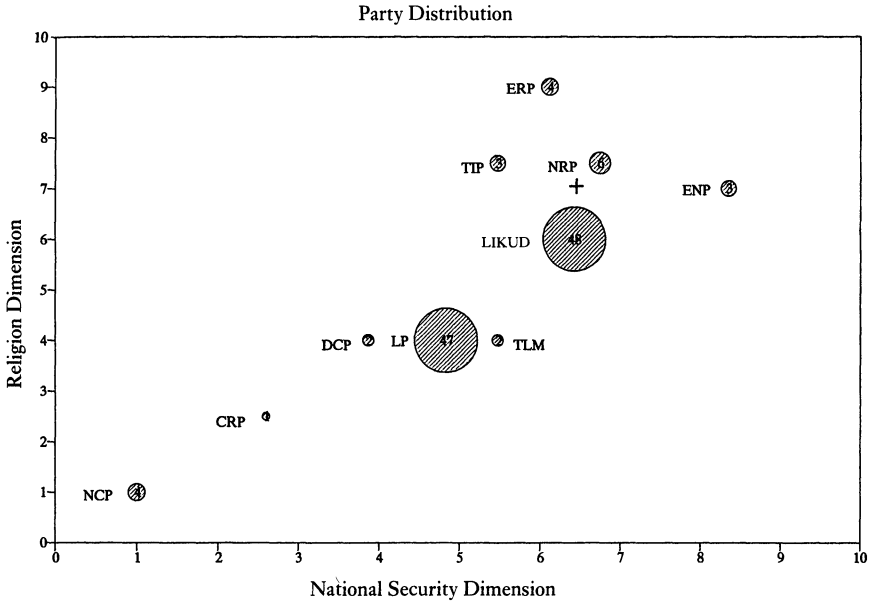
THE 1992 ELECTION MINORITY MWC GOVERNMENTS
AND THE PEACE NEGOTIATION

The four coalitions, formed after the 1992 election in Israel, illustrate the empirical relevance of the central claims derived from the formal model in the previous sections: After the results of the election became known, on June 23, 1992, it was apparent to observers and politicians alike, that the Labor party recaptured the SSC position (Sened 1992).

Figure 3 indicates parties' policy positions and seat distribution in the Israeli Knesset after the 1992 election. The cross marks the declared position of the gov-

TABLE 1

DISTANCE OF THE DECLARED POSITION OF THE COALITION
FROM THE DECLARED POSITION OF THE COALITION FORMER WHEN THE
CORE IS EMPTY AND WHEN THE CORE IS NONEMPTY

Core	Mean	Standard Deviation	Minimum	Maximum	F	F Prob
Nonempty	1.685	0.496	0.620	2.500	5.810	0.000
Empty	1.959	1.195	0.620	5.200	$N = 32$	

FIGURE 3

13TH KNESSET (JUNE 1992–)

Party Distribution
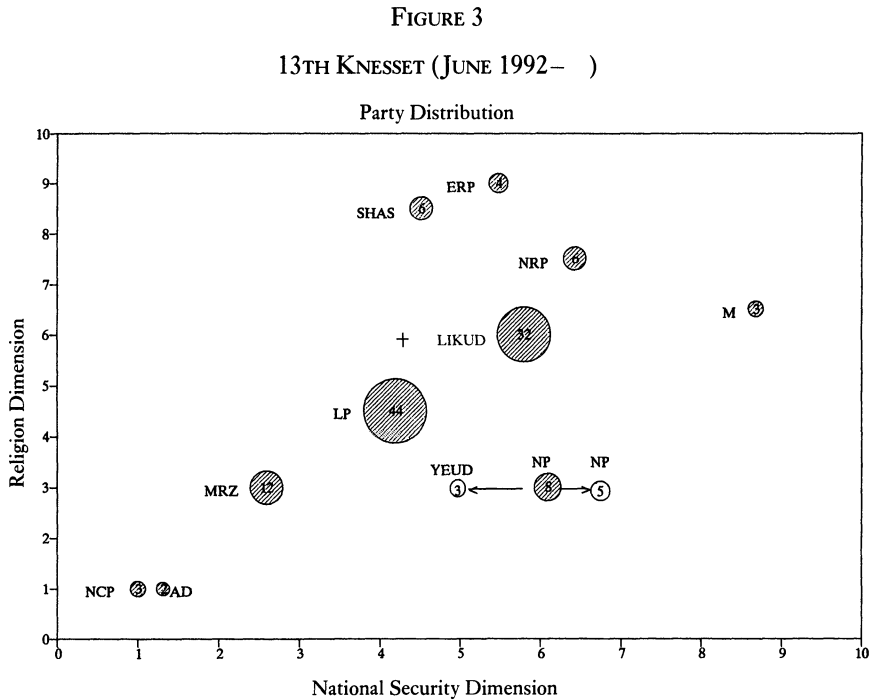
ernment Rabin formed after the election. It is easy to show that the Labor party's *policy* position is the SSC in this configuration.

After learning the results of the election, Rabin declared that he would implement the policy of the Labor party, regardless of the composition of the coalition. Many took it to be an arrogant statement by an arrogant leader, but Rabin was relying on the advice of his advisers, derived from models similar to the one developed here (Doron, forthcoming).

The first coalition that Rabin formed was a minimum winning coalition[15] with Shas (6 seats), Meretez (12 seats) and Labor (44 seats), controlling 62 of 120 seats in parliament. Rabin seems to have combined the Austen-Smith and Banks (1988) logic, together with my logic. He seems to have "bought" Meretz with a disproportionate number of government positions, like my model predicts. The initial policy position of the Rabin government is quite far from Meretz's ideal policy position. Initially Meretz, with 12 seats in parliament, received three portfolios. The Labor party received only 9 portfolios for 44 seats. Shas received only one portfolio—the Ministry of the Interior—for six seats. Rabin seems to have paid them

[15]Looking at figure 3 one is tempted to say that Rabin had "closer" potential partners, but this implies the fallacy of interpersonal comparison of utility. We cannot infer from these maps any absolute value of these distances for potential coalition partners.

by promising to implement a policy close enough to them in the *policy space*, as the logic of the Austen-Smith and Banks (1988) model suggests.

In the first year of his government, Rabin seemed to implement a policy position even more to the right than the cross in figure 3 indicates. Meretz was particularly angry with his decision to deport numerous Hamas activists to southern Lebanon in the winter of 1992. Meretz threatened to leave the coalition. The crisis was resolved when Sarid, a major figure in Meretz, received a fourth portfolio—the Ministry of Ecology.

Since Sarid received the Ecology portfolio, Meretz never threatened to leave the coalition until the redistribution of portfolios in February 1995. By then, the policy of the government shifted back to the left with the signature of the peace agreement with Jordan and the Oslo agreement with the P.L.O. In January 1995 it became clear that Shas would not return to the coalition. As a result two portfolios remained vacant: the Interior Ministry, previously held by Shas and the Ministry of Religion that was never assigned. Rabin gave two portfolios to members of the Labor party and nothing to Meretz. Meretz threatened to leave the coalition. Rabin called Sarid and talked about the importance of the government's stability for the peace process. Meretz withdrew its threat and the crisis was over.

In the summer of 1993 Shas resigned from the government. The main reason for Shas' resignation was that Rabin was unable to maintain the policy he promised Shas to implement (marked by the black cross in figure 3) on the religious dimension. The government's policy position shifted closer to the Labor party's ideal policy position, i.e., the *core* position in parliament, and farther away from Shas, as my model predicts. This shift became apparent with the approval, by the end of 1992, of the *Basic Law: Freedom and Human Dignity* and the related *Basic Law: Freedom of Occupation*. These laws limit the ability of the religious authorities in Israel to intervene in the private life of Israeli citizens.[16]

Shas' resignation left Rabin at the head of his second coalition that controlled only 56 seats in parliament, i.e., a *minority* government. Numerous motions of no-confidence were advanced in the Knesset, but to no avail. Knowing that no coalition could form to replace Rabin's coalition, parties not in the coalition often bargained into supporting the coalition, or abstaining in return for small favors from the coalition in office.

Shas' resignation worried Israeli and international observers because it came just as the peace process started to gain momentum. But Rabin's minority government continued to function. Numerous motions of no-confidence failed. The opposition parties were frustrated to learn that a minority government could function so well (cf. Strøm 1990). Among other things, this frustration brought about a split in the

[16]Israel does not have a constitution. It uses an English type "common law" system instead. In recent years, some laws were passed in the Israeli Knesset with the special status of "basic laws" that require special majorities to be changed. The *Basic Law: Freedom and Human Dignity* and the *Basic Law: Freedom of Occupation*, are two such laws. They worry the religious parties because these laws included several defenses against the intrusion of religious law into the private sphere of the individual citizen.

second largest opposition party, Tzomet, into two parties—Yeud with three seats and Tzomet with five, in January 1994.

For about two years a *minority* government, made of the Labor party and Meretz, remained in power controlling only 56 out of 120 seats. This *minority* government was strong enough to pursue the historic and controversial peace initiative with the P.L.O. and the less controversial, but just as important, peace agreement with Jordan.

A third coalition was about to form when Shas tried to rejoin the government. This time it seemed to be willing to accept the status quo on the religious dimension in return for a bigger share of portfolios: in addition to the Interior Ministry, Shas was to get the Ministry of Religious Affairs. Yet, by now, the spiritual leader of Shas, Rabi Ovadia Yosef, required assurances to prevent Rabin from moving farther away, on the religious dimension, from Shas' ideal position: The agreement included a statement (#3) promising to change any law that may be used by the Supreme Court to rule in any way that may change the status quo on the religious dimension. This agreement between Rabin and Shas, that was to serve as the basis for the third coalition in Israel since the 1992 election was challenged in the Supreme Court in Israel. The ruling of the Supreme Court was vague,[17] but the agreement was never implemented. Shas stayed in the opposition and the third coalition died a "pre-natal" death.

After the deal with Shas fell apart, the opposition began to embarrass the coalition and managed to win several votes in the Knesset. In response to this embarrassing situation, Rabin formed, in December 1994, a new coalition, the fourth since 1992, by joining Yeud, the half of the Tzomet split, that was closer to the Labor ideal policy position (see figure 3). To join, Yeud required and obtained the Ministry of Energy.[18] This fourth coalition, since 1992, is still a minority government controlling only 59 out of 120 seats in parliament.

The problem in subjecting a theoretical model like the one advanced here to an empirical test is that even a relatively simple model, like the one promoted here, raises critical problems of estimating its parameters. This is not a simple technicality, it has to do with the problem of interpersonal comparison of utilities: When measuring utility we can not use common yard sticks across different parties or individuals. If politics was more static, we could use observations over years, for each party, to estimate the relationship between *office*-related sidepayments and *policy* compromises for each party. But parties come and go and their leaderships constantly change, so comparison over time is practically meaningless.

For the time being, all we can do is use "stylized" case studies, such as the Israeli case discussed earlier, to illustrate, rather than actually "test," the usefulness of

[17]The Court ruled that the agreement was "problematic" but not enough so, to justify an intervention by the Supreme Court.

[18]The energy resources in Israel are centralized. Electricity is supplied by a government enterprise, and the Ministry of Energy controls many aspects of the supply of other sources of energy like natural gas and gasoline.

mathematical models of politics like the one promoted here. What we learn from such case studies is the dynamics of coalition formation. What we want to test by such case studies is whether reality is consistent with the model's predictions and whether the model provides useful descriptions of the main features of reality, given the best account we can get of the events.

9. Concluding Remarks

McKelvey's (1979) work on "Intransitivities in Formal Voting Models" led many to believe that instability in policy outcomes should be the rule, and not the exception, in politics (Riker 1980; Austen-Smith 1983). Instability of policy outcomes has, since, been shown to be more common than previously thought, but stability is by no means an exception. This observation motivated researchers to try to explain this *excess* stability.

One research program initiated by Shepsle (1979) has proved to be useful in this respect. It claims that institutional constraints explain excess stability. Schofield's (1986) work, for example, implies that the multiparty, proportional rule, institutional setup, can induce stability in government policies. Relying on Schofield's work, but introducing *office*-related payoffs as an element in the bargaining process, I showed that when the SSC is nonempty one can predict the policy position and the composition of the coalition that will form. When the SSC is empty one cannot predict the final policy outcome, but stable coalition agreements can still be predicted. Some parties are better placed—because they need less partners or because of their central position—to cope with unstable policy outcomes.

The IVCORE can be thought of as a remedy to a decades old problem with Riker's size principle. The problem with the original size principle was that, contrary to common use, it should never have served as a predictive tool. It is well known that *n*-person constant-sum-games with sidepayments have a generically empty core (Ordeshook 1986, 349–55). In the Rikerian environment only minimum winning coalitions form, but no coalition should ever last. The central result of this article, theorem 1, explains why some of these coalitions do last: Ideological *policy* differences, if important enough, induce equilibria in these games. In the absence of such differences, coalitions will tend to collapse over disputes concerning the distribution of *office*-related sidepayments to members (cf. Mershon 1990).

At the same time, I have shown that: (1) concerns about *office*-related sidepayments may induce an IVCORE even when the majority rule core in the *policy space*, is empty. (2) The "budget constraint," induced by the limited amount of *office*-related sidepayments, determines the composition of the coalition that will form after any election. Central, moderate parties are more likely to be included in a coalition because the ideological, *policy* compromises they make when they join a government are less costly in terms of what the formateur of the coalition must pay them in return for such compromises. It is cheaper for the formateur of a coalition to "sell" the government's policy to moderate, central parties. The currency used

in these trades are *office*-related sidepayments such as governmental portfolios and special budgets, that the formateur distributes to its partners in return for their willingness to endorse the government's *policy*.

Manuscript submitted 6 June 1994
Final manuscript received 12 July 1995

Appendix

Lemma 2: If $(c_r, P_r) \mid \wp) \in$ (IVCORE) then $\Sigma_{i \in c_r} P_{ir} = K$

Proof: if $\Sigma_{i \in c_r} P_{ir} < K$, there must be a payoff vector that dominates $u(c_r, P_r) \mid \wp)$ giving a share of $(\Sigma_{i \in c_r} P_{ir})$ to each party member in c_r. $\square$.

Lemma 3: If SSC $\neq \varnothing$, and (3) below holds then if $(c_r, P_r \mid \wp\} \in$ *(IVCORE)* $\Rightarrow$ $\ell \in c_r$.

$$\forall j \in P, j \neq \ell \exists c_r \text{ s.t. } j \notin c_r, \text{ and } w(c_r) > \\ 50\%, \Sigma_{i \in c_r} (\beta_i / \alpha_i) \cdot d[x_j, x_\ell]^2 < K \tag{3}$$

Proof: I show that for any coalition c_r, if $\ell \notin c_r$, $\exists$ a *winning* coalition c_r' with $j \in c_r, j \notin c_r', \ell \in c_r'$, and $\forall i \in c_r, \ell \neq i \neq j i \in c_r' u_{ir}' > u_{ir}$. Lemma 1 and the hypothesis of Lemma 3 (SSC $\neq \phi$) imply $x_r = x_r' = x_\ell x_j \neq x_\ell \Rightarrow d[x_j, x_\ell = x_r]^2 > 0$, which implies that $\alpha_j \cdot P_{jr} \geq \beta_j \cdot d[x_j, x_r]^2 > 0$, or else, $u_{jr} < 0$. $x_r' = x_\ell \Rightarrow$ $d[x_r', x_\ell]^2 = 0$, therefore, $\exists$ an infinitesimally small number ϵ, s.t. $P_{ir}' = P_{ir} +$ $\epsilon \forall i \in c_r'$ and $P_{\ell r}' = P_{jr} - (r-1) \cdot \epsilon$ where r is the number of parties in c_r'. $\forall i \in$ $c_r' u_{ir}' > u_{ir}$, since $x_r = x_r' = x_\ell$, if c_r is *winning*, so is c_r'.[19] If $c_r = (j)$, by condition (3), c_r is *vulnerable*. $\square$.

Lemma 4: If $c_r \backslash j \in$ {WC}, then $c_r \notin$ (IVCORE).

Proof: Unless SSC $\neq \phi$ and $j = \ell, j \in c_r \Rightarrow P_{jr} \geq \beta_j(\wp) > 0$, or else $u_{jr} < 0$. Since $P_{jr \backslash j} = 0$, $\exists \epsilon$ s.t. $\forall i \in c_{r \backslash j}, P_{ir \backslash j} = P_{ir} + \epsilon \Rightarrow u_{ir \backslash j} > u_{ir}$. By hypothesis $c_r / j \in$ {WC}. By Lemma 3, if SSC $\neq \phi$ and $j = \ell, c_{r \backslash \ell} \notin$ {WC} $\square$.

Theorem 1: $(c_r^*, Pi_r^*/\wp) \in$ (IVCORE) if and only if:

(i) c_r^* is minimum winning.
(ii) $\forall i \in P$, if $(SV_i \mid c_r^*) - \beta_i(\wp) \leq 0 \Rightarrow P_{ir}^* = 0$.

[19]Note that c_r is not necessarily *vulnerable* to c_r' because, as I show later, c_r' could be a minority government. Yet if SSC $\neq \phi$ if $\ell \notin c_r$, as the proof above demonstrates c_r cannot be in the core, since members of any coalition can be made better off by including ℓ, unless the coalition is $c_r = (j)$, which is vulnerable by condition (3) of Lemma 3.

Otherwise

$$(SV_i \mid c_r^*) \le P_{ir}^* \le \alpha_i \cdot \left(K - \left(\sum_{j \in c_r^*, j \notin i} (SV_j \mid c_r^*) \right) \right).$$

(iii) $\Sigma_{i \in c_r} {}^* P_{ir}^* = K$

Proof: Necessity: Condition 3.i is necessary by Lemma 3 and 4. Condition 3.ii guarantees that c_r^* is *invulnerable* to any outside threat. It also guarantees that any member of the coalition is offered its "reservation" price, defined as the price it can get from any coalition to which c_r^* may be *vulnerable*. It is necessary because otherwise c_r^* would be *vulnerable*, or would pay some members more than it has to. Condition 3.iii is necessary by Lemma 2.

Sufficiency: Condition 3.ii guarantees that c_r^* is *invulnerable*, and that $u(c_r^*, P_r^* \mid \wp)$ is undominated. Condition 3.iii sets the budget constraint and guarantees that $\Sigma_{i \in c_r} {}^* P_{ir^*} < K$ or $\Sigma_{i \in c_r} {}^* P_{ir^*} > K$ never occurs. $\square$.

Theorem 3: If the SSC $\ne \phi$, $(c_r^* = (\ell), P^*_{\ell r} = K \mid \wp = x_\ell) \in$ (IVCORE) iff

$$\forall \ c_r \in C \text{ s.t. } \ell \notin c_r, \text{ if } w(c_r) > 50\%, \text{ then } K \le \Sigma_{i \in c_r} (\beta_i / \alpha_i) \cdot d[x_i, x_\ell]^2 \quad (5)$$

Proof: Sufficiency: If condition (5) holds, $u(c_r^* = (\ell), P_{\ell r}^* = K \mid \wp = x_\ell)$ is undominated because $u_\ell(c_r^* = (\ell), P_{\ell r}^* = K \mid \wp = x_\ell) = K$ and $\forall \ i \in P, \ i \ne \ell$, $u_i(c_r^* = (\ell), P_{\ell r}^* = K \mid \wp = x_\ell) = 0$. Condition (5) implies that any coalition c_r that can make (ℓ) *vulnerable* either pays a negative payoff to one of its members, if $\ell \notin c_r$, or pays $u_{\ell r} < K$ to ℓ, if $\ell \in c_r$. Thus $u(c_r = (\ell), P^*_{\ell r} = K \mid \wp = x_\ell)$ dominates any $u(\cdot)$ with respect to $c_r = (\ell)$ because if condition (5) holds $u(c_r^* = (\ell)$, $P^*_{\ell r} = K \mid \wp = x_\ell)$ is in $V(c_r = (\ell))$ and $u_\ell(c_r^* = (\ell), P_{\ell r}^* = K \mid \wp = x_\ell) = K$ is the maximum ℓ can get in this game.

Necessity: If condition (5) does not hold, ℓ forming a minority government is not an equilibrium since it is *vulnerable*, making $u(c_r = (\ell), P_{\ell r} = K \mid \wp = x_\ell)$ unfeasible for coalition (ℓ), i.e. $u(c_r = (\ell), P_{\ell r} = K \mid \wp = x_\ell)$ is not in $V(c_r = (\ell))$. $\square$.

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Itai Sened is assistant professor of political science, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel.