

# Models

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## 1 Phenomenological Growth Models

### 1.1 Models

- Exponential growth model

$$\frac{dC}{dt} = rC_t$$

- Generalized growth model

$$\frac{dC}{dt} = rC_t^p$$

- Logistic growth model

$$\frac{dC}{dt} = rC_t \left(1 - \frac{C_t}{K}\right)$$

- Generalized logistic model

$$\frac{dC}{dt} = rC_t^p \left(1 - \frac{C_t}{K}\right)$$

- Richards growth model

$$\frac{dC}{dt} = rC_t \left(1 - \left(\frac{C_t}{K}\right)^a\right)$$

- Generalized Richards growth model

$$\frac{dC}{dt} = rC_t^p \left(1 - \left(\frac{C_t}{K}\right)^a\right)$$

where  $C_t$  is cumulative incidence at time  $t$ ,  $p$  is scaling factor of growth,  $K$  is final epidemic size.

## 1.2 Pseudo-code

1. `SimpleGrowth` function: differential equation
2. `solvedSimpleGrowth` function: solve differential equation to get predicted incidence
3. `ResidFun` function: calculate residual = predicted incidence - observed incidence
4. `fittingSimpleGrowth` function: fitting data to the model by minimizing sum of squared residuals (L2-norm) and obtain parameter estimates `Ptrue`
5. `confint` function: obtain uncertainty around parameter estimates, by running `nsim` simulations. Each simulation includes:
  - (a) Obtain predicted incidence `bestfit` by executing `solvedSimpleGrowth` with parameter inputs being `Ptrue`
  - (b) Generate simulated dataset around predicted incidence, by assuming a certain error structure (i.e., Poisson distribution, Negative Binomial distribution)
  - (c) Fitting simulated dataset to the model to obtain a new set of parameters