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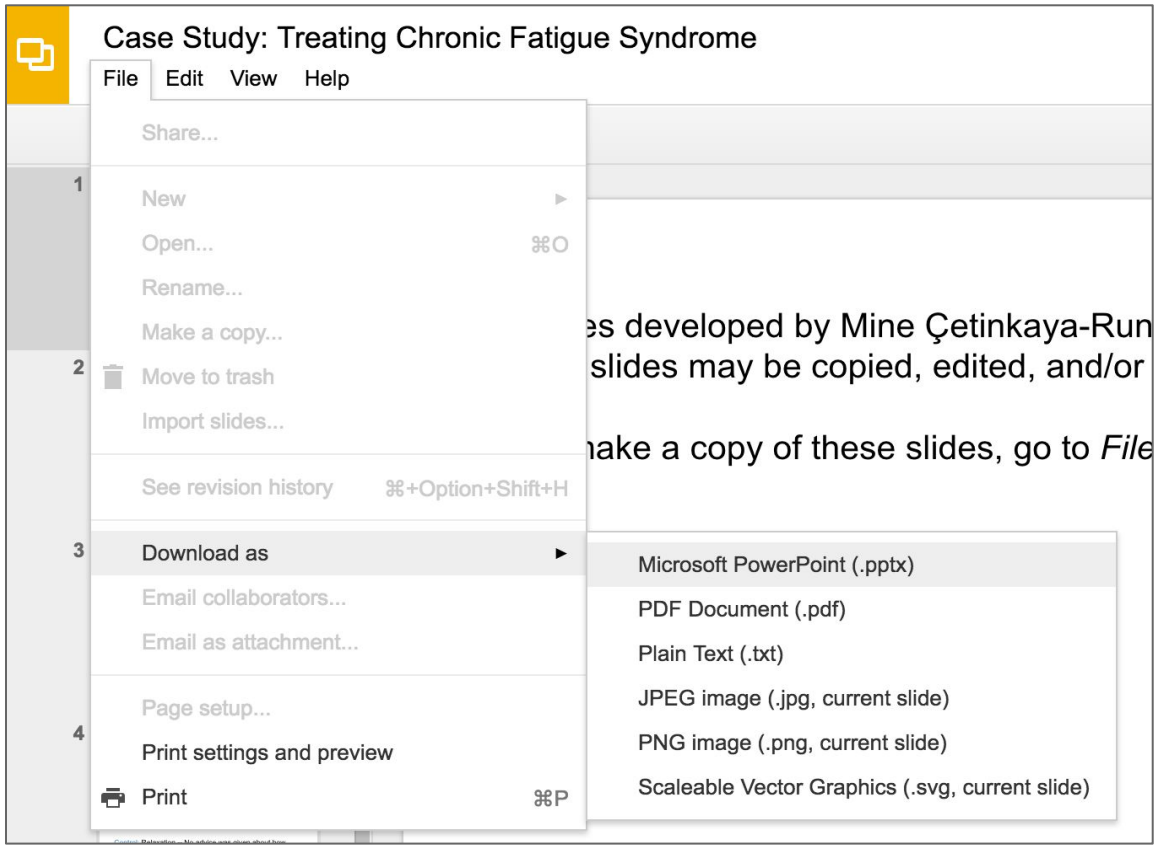

Difference of Two Proportions

Melting ice cap

Scientists predict that global warming may have big effects on the polar regions within the next 100 years. One of the possible effects is that the northern ice cap may completely melt. Would this bother you a great deal, some, a little, or not at all if it actually happened?

(a) A great deal
(b) Some
(c) A little
(d) Not at all

Results from the GSS

The GSS asks the same question, below are the distributions of responses from the 2010 GSS as well as from a group of introductory statistics students at Duke University:

	GSS	Duke
A great deal	454	69
Some	124	30
A little	52	4
Not at all	50	2
Total	680	105

Parameter and point estimate

- *Parameter of interest*: Difference between the proportions of *all* Duke students and *all* Americans who would be bothered a great deal by the northern ice cap completely melting.

$$p_{Duke} - p_{US}$$

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- *Point estimate*: Difference between the proportions of *sampled* Duke students and *sampled* Americans who would be bothered a great deal by the northern ice cap completely melting.

$$\hat{p}_{Duke} - \hat{p}_{US}$$

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Standard error of the difference between two sample proportions

$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

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2. *Independence between groups:*
   The sampled Duke students and the US residents are independent of each other.
3. *Success-failure:*
   At least 10 observed successes and 10 observed failures in the two groups.

Sample proportions are also nearly normally distributed

Construct a 95% confidence interval for the difference between the proportions of Duke students and Americans who would be bothered a great deal by the melting of the northern ice cap ($p_{Duke} - p_{US}$).

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$$= -0.011 \pm 0.097$$

$$= (-0.108, 0.086)$$

Practice

Which of the following is the correct set of hypotheses for testing if the proportion of all Duke students who would be bothered a great deal by the melting of the northern ice cap differs from the proportion of all Americans who do?

(a) $H_0 : p_{Duke} = p_{US}$
$H_A : p_{Duke} \neq p_{US}$

(b) $H_0 : \hat{p}_{Duke} = \hat{p}_{US}$
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(c) $H_0 : p_{Duke} - p_{US} = 0$
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Both (a) and (c) are correct.

Flashback to working with one proportion

- When constructing a confidence interval for a population proportion, we check if the *observed* number of successes and failures are at least 10.

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Pooled estimate of a proportion

- In the case of comparing two proportions where H_0: $p_1 = p_2$, there isn't a given null value we can use to calculated the *expected* number of successes and failures in each sample.

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- Therefore, we need to first find a common (*pooled*) proportion for the two groups, and use that in our analysis.
- This simply means finding the proportion of total successes among the total number of observations.

Pooled estimate of a proportion

$$\hat{p} = \frac{\#\ of\ successes_1 + \#\ of\ successes_2}{n_1 + n_2}$$

Practice

Calculate the estimated pooled proportion of Duke students and Americans who would be bothered a great deal by the melting of the northern ice cap. Which sample proportion ($\hat{p}_{Duke}$ *or* $\hat{p}_{US}$) the pooled estimate is closer to? Why?

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CI vs. HT for proportions

Do these data suggest that the proportion of all Duke students who would be bothered a great deal by the melting of the northern ice cap differs from the proportion of all Americans who do? Calculate the test statistic, the p-value, and interpret your conclusion in context of the data.

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$$p - value = 2 \times P(Z < -0.22) = 2 \times 0.41 = 0.82$$

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$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

- for CI: use $\hat{p}_1$ and $\hat{p}_2$
- for HT:
  - when H_0: $p_1 = p_2$: use $\hat{p}_{pool} = \frac{\#\ suc_1 + \#suc_2}{n_1 + n_2}$
  - when H_0: p_1 - p_2 = (some value other than 0): use $\hat{p}_1$ and $\hat{p}_2$
    - this is pretty rare

Reference - standard error calculations

	one sample	two samples
mean	$SE = \frac{s}{\sqrt{n}}$	$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
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- When working with means, it's very rare that σ is known, so we usually use *s*.
- When working with proportions,
  - if doing a hypothesis test, *p* comes from the null hypothesis
  - if constructing a confidence interval, use $\hat{p}$ instead

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