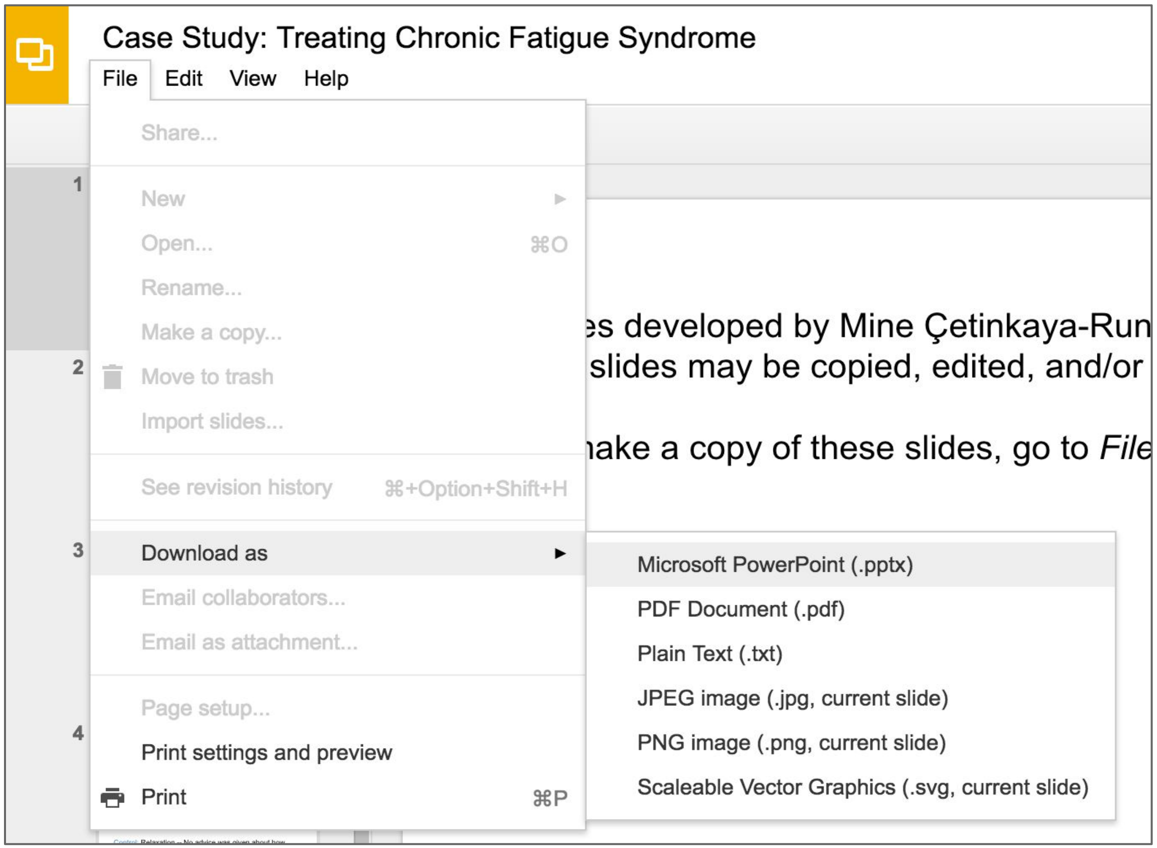
Slides developed by Mine Çetinkaya-Rundel of OpenIntro
Translated from LaTeX to Google Slides by Curry W. Hilton of OpenIntro.

To make a copy of these slides, go to *File > Download as > [option]*, as shown below. Or if you are logged into a Google account, you can choose *Make a copy...* to create your own version in Google Drive.



Poisson Distribution

Poisson distribution

- The Poisson distribution is often useful for estimating the number of rare events in a large population over a short unit of time for a fixed population if the individuals within the population are independent.
- The **rate** for a Poisson distribution is the average number of occurrences in a mostly-fixed population per unit of time, and is typically denoted by λ.
- Using the rate, we can describe the probability of observing exactly *k* rare events in a single unit of time.

$$\text{P(observe } k \text{ rare events)} = \frac{\lambda^k e^{-\lambda}}{k!}$$

where k may take a value 0, 1, 2, and so on, and *k!* represents *k*-factorial. The letter $e \approx 2.718$ is the base of the natural logarithm. 57 The mean and standard deviation of this distribution are λ and $\sqrt{\lambda}$, respectively

Example #1

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that in a given week the electricity fails only once.

Example #1

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that in a given week the electricity fails only once.

Given $\lambda = 2$.

Example #1

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that in a given week the electricity fails only once.

Given $\lambda = 2$.

$$P(\text{only 1 failure in a week}) = \frac{2^1 \times e^{-2}}{1!}$$

Example #1

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that in a given week the electricity fails only once.

Given $\lambda = 2$.

$$
\begin{aligned}
P(\text{only 1 failure in a week}) &= \frac{2^1 \times e^{-2}}{1!} \\
&= \frac{2 \times e^{-2}}{1}
\end{aligned}
$$

Example #1

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that in a given week the electricity fails only once.

Given $\lambda = 2$.

$$
\begin{aligned}
P(\text{only 1 failure in a week}) &= \frac{2^1 \times e^{-2}}{1!} \\
&= \frac{2 \times e^{-2}}{1} \\
&= 0.27
\end{aligned}
$$

Example #2

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that on a given day the electricity fails three times.

Example #2

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that on a given day the electricity fails three times.

We are given the weekly failure rate, but to answer this question we need to first calculate the average rate of failure on a given day: $\lambda_{day} = \frac{2}{7} = 0.2857$. Note that we are assuming that the probability of power failure is the same on any day of the week, i.e. we assume independence.

Example #2

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that on a given day the electricity fails three times.

We are given the weekly failure rate, but to answer this question we need to first calculate the average rate of failure on a given day: λ_{day} = 2 = 0.2857. Note that we are assuming that the probability 7 of power failure is the same on any day of the week, i.e. we assume independence.

$$P(\text{3 failures on a given day}) = \frac{0.2857^1 \times e^{-0.2857}}{3!}$$

Example #2

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that on a given day the electricity fails three times.

We are given the weekly failure rate, but to answer this question we need to first calculate the average rate of failure on a given day: λ_{day} = 2 = 0.2857. Note that we are assuming that the probability 7 of power failure is the same on any day of the week, i.e. we assume independence.

$$\begin{aligned} P(\text{3 failures on a given day}) &= \frac{0.2857^1 \times e^{-0.2857}}{3!} \\ &= \frac{0.2857 \times e^{-0.2857}}{6} \end{aligned}$$

Example #2

Suppose that in a rural region of a developing country electricity power failures occur following a Poisson distribution with an average of 2 failures every week. Calculate the probability that on a given day the electricity fails three times.

We are given the weekly failure rate, but to answer this question we need to first calculate the average rate of failure on a given day: λ_{day} = 2 = 0.2857. Note that we are assuming that the probability 7 of power failure is the same on any day of the week, i.e. we assume independence.

$$
\begin{aligned}
P(\text{3 failures on a given day}) &= \frac{0.2857^1 \times e^{-0.2857}}{3!} \\
&= \frac{0.2857 \times e^{-0.2857}}{6} \\
&= 0.0358
\end{aligned}
$$

Is it Poisson?

- A random variable may follow a Poisson distribution if the event being considered is rare, the population is large, and the events occur independently of each other
- However we can think of situations where the events are not really independent. For example, if we are interested in the probability of a certain number of weddings over one summer, we should take into consideration that weekends are more popular for weddings.
- In this case, a Poisson model may sometimes still be reasonable if we allow it to have a different rate for different times; we could model the rate as higher on weekends than on weekdays.
- The idea of modeling rates for a Poisson distribution against a second variable (day of the week) forms the foundation of some more advanced methods called *generalized linear models*. There are beyond the scope of this course, but we will discuss a foundation of linear models in Chapters 7 and 8.

Practice

A random variable that follows which of the following distributions can take on values other than positive integers?

(a) Poisson

(b) Negative binomial

(c) Binomial

(d) Normal

(e) Geometric

Practice

A random variable that follows which of the following distributions can take on values other than positive integers?

(a) Poisson

(b) Negative binomial

(c) Binomial

(d) *Normal*

(e) Geometric