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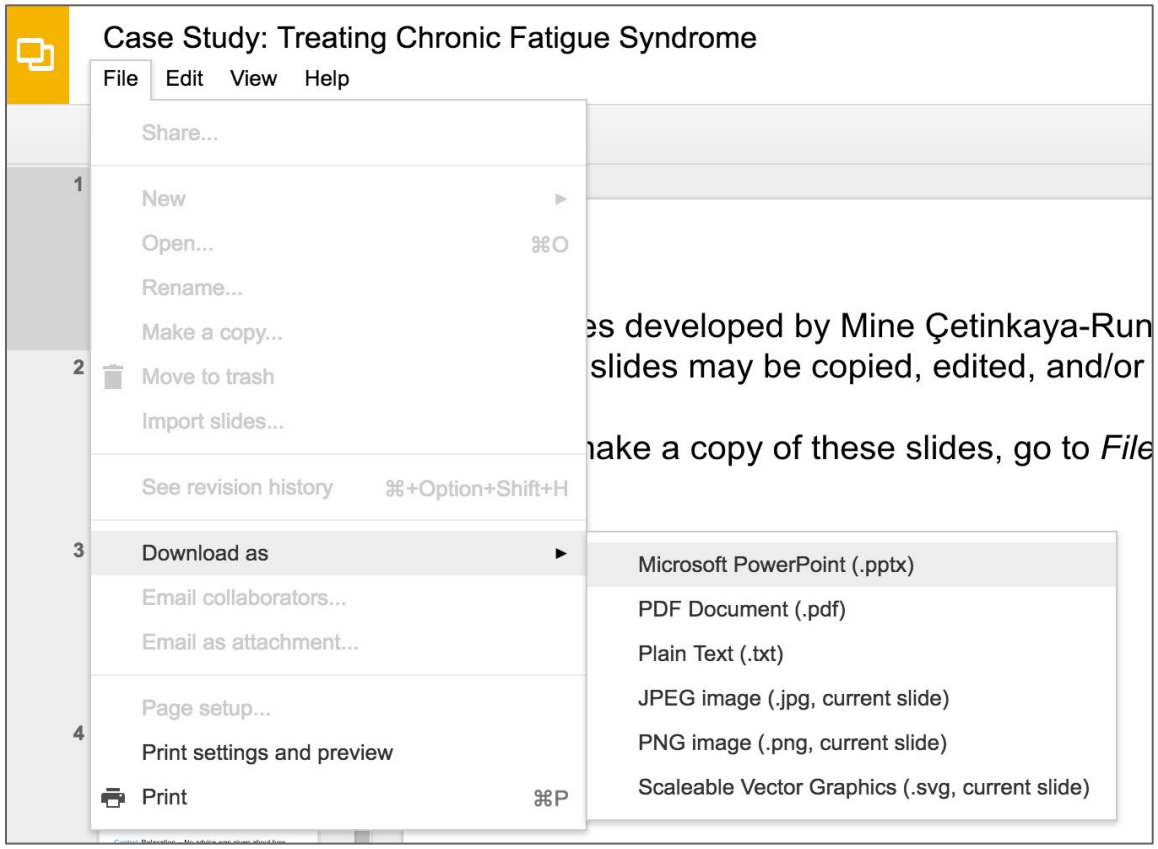

Sampling from a Small Population

Sampling with replacement

When sampling with replacement, you put back what you just drew.

- Imagine you have a bag with 5 red, 3 blue and 2 orange chips in it. What is the probability that the first chip you draw is blue?

  5 O, 3 O, 2 O

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  1st Draw - 5 O, 3 O, 2 O

  2nd Draw - 5 O, 3 O, 2 O

  P(2nd chip B | 1st chip B) = 3 / 10 = 0.3

Sampling with replacement (cont.)

- Suppose you actually pulled an orange chip in the first draw. If drawing with replacement, what is the probability of drawing a blue chip in the second draw?

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- If drawing with replacement, what is the probability of drawing two blue chips in a row?

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  P(1st chip B) x P(2nd chip B | 1st chip B) = 0.3 x 0.3 = 0.09

Sampling with replacement (cont.)

When drawing with replacement, probability of the second chip being blue does not depend on the color of the first chip since whatever we draw in the first draw gets put back in the bag.

$$Prob(B \mid B) = Prob(B \mid O)$$

In addition, this probability is equal to the probability of drawing a blue chip in the first draw, since the composition of the bag never changes when sampling with replacement.

$$Prob(B \mid B) = Prob(B)$$

When drawing with replacement, draws are independent.

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  1st Draw - 5 **O**, 3 **O**, 2 **O**

  2nd Draw - 5 **O**, 2 **O**, 2 **O**

  P(2nd chip B | 1st chip B) = 2 / 9 = 0.22

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  P(1st chip B) x P(2nd chip B | 1st chip B) = 0.3 x 0.22 = 0.066

Sampling without replacement (cont.)

When drawing without replacement, the probability of the second chip being blue given the first was blue is not equal to the probability of drawing a blue chip in the first draw since the composition of the bag changes with the outcome of the first draw.

$$Prob(B \mid B) \neq Prob(B)$$

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This is especially important to take note of when the sample sizes are small. If we were dealing with, say, 10,000 chips in a (giant) bag, taking out one chip of any color would not have as big an impact on the probabilities in the second draw.

Practice

In most card games cards are dealt without replacement. What is the probability of being dealt an ace and then a 3? Choose the closest answer.

(a) 0.0045

(b) 0.0059

(c) 0.0060

(d) 0.1553

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P(ace, then 3) = (4 / 52) x (4 / 51) ~ 0.0060

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