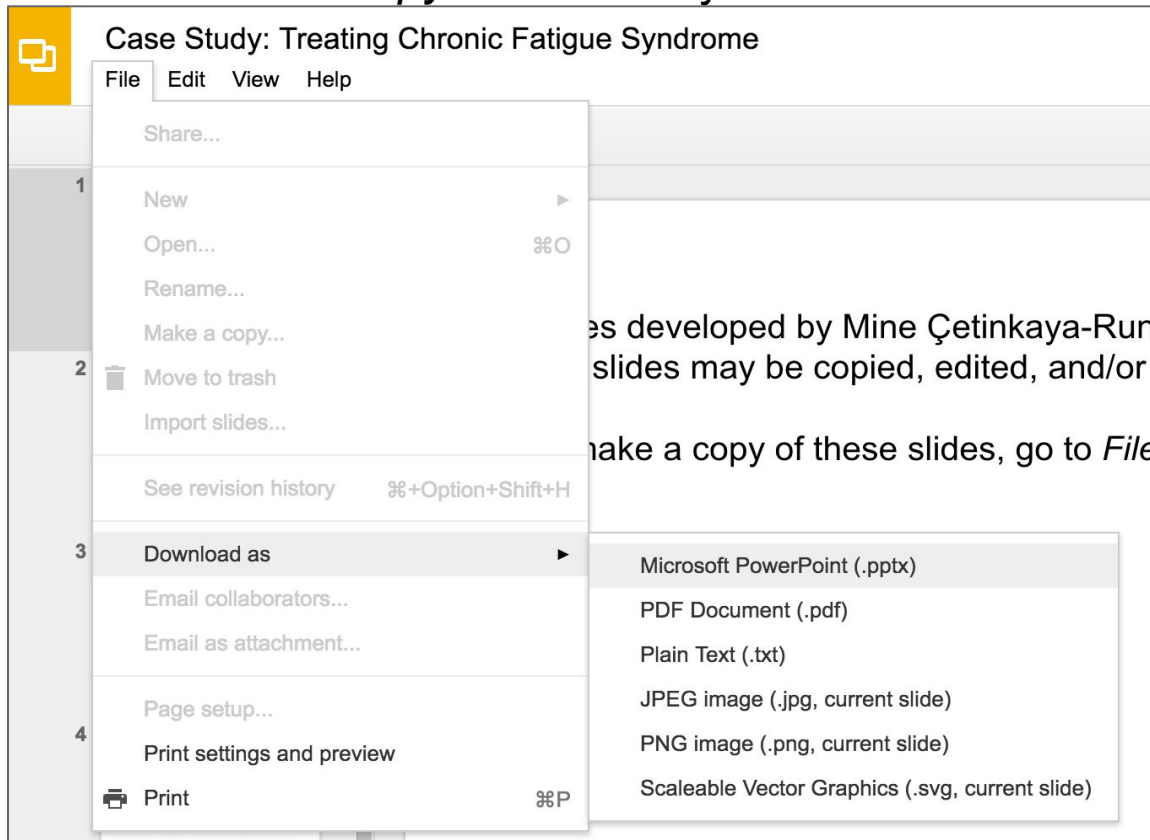


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One-sample mean with the t -distribution

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Friday the 13th

Between 1990 - 1992 researchers in the UK collected data on traffic flow, accidents, and hospital admissions on Friday 13th and the previous Friday, Friday 6th. Below is an excerpt from this data set on traffic flow. We can assume that traffic flow on given day at locations 1 and 2 are independent.

	type	date	6 th	13 th	diff	location
1	traffic	1990, July	139246	138548	698	loc 1
2	traffic	1990, July	134012	132908	1104	loc 2
3	traffic	1991, September	137055	136018	1037	loc 1
4	traffic	1991, September	133732	131843	1889	loc 2
5	traffic	1991, December	123552	121641	1911	loc 1
6	traffic	1991, December	121139	118723	2416	loc 2
7	traffic	1992, March	128293	125532	2761	loc 1
8	traffic	1992, March	124631	120249	4382	loc 2
9	traffic	1992, November	124609	122770	1839	loc 1
10	traffic	1992, November	117584	117263	321	loc 2

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No!

Hypotheses

What are the hypotheses for testing for a difference between the average traffic flow between Friday 6th and 13th?

A. $H_0: \mu_{6th} = \mu_{13th}$

$H_A: \mu_{6th} \neq \mu_{13th}$

B. $H_0: p_{6th} = p_{13th}$

$H_A: p_{6th} \neq p_{13th}$

C. $H_0: \mu_{diff} = 0$

$H_A: \mu_{diff} \neq 0$

D. $H_0: \bar{x}_{diff} = 0$

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Conditions

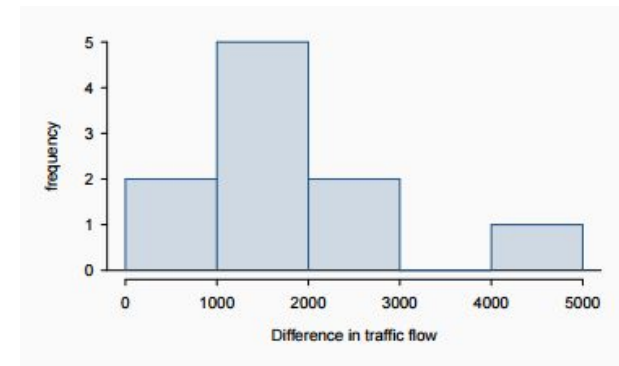
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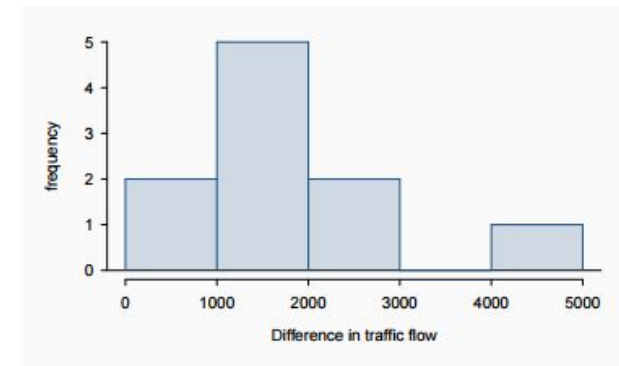
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 - The sample distribution does not appear to be extremely skewed, but it's very difficult to assess with such a small sample size. We might want to think about whether we would expect the population distribution to be skewed or not probably not, it should be equally likely to have days with lower than average traffic and higher than average traffic.
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So what do we do when the sample size is small?

Review: what purpose does a large sample serve?

As long as observations are independent, and the population distribution is not extremely skewed, a large sample would ensure that...

- the sampling distribution of the mean is nearly normal
- the estimate of the standard error, as $\frac{s}{\sqrt{n}}$, is reliable

The normality condition

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- The CLT, which states that sampling distributions will be nearly normal, hold true for *any* sample size as long as the population distribution is nearly normal
- While this is a helpful special case, it's inherently difficult to verify normality in small data sets
- We should exercise caution when verifying the normality condition for small samples. It is important to not only examine the data but also think about where the data come from
 - For example, ask: would I expect this distribution to be symmetric, and am I confident that outliers are rare?

The t distribution

- When the population standard deviation is unknown (almost always), the uncertainty of the standard error estimate is addressed by using a new distribution: the t distribution.

The t distribution

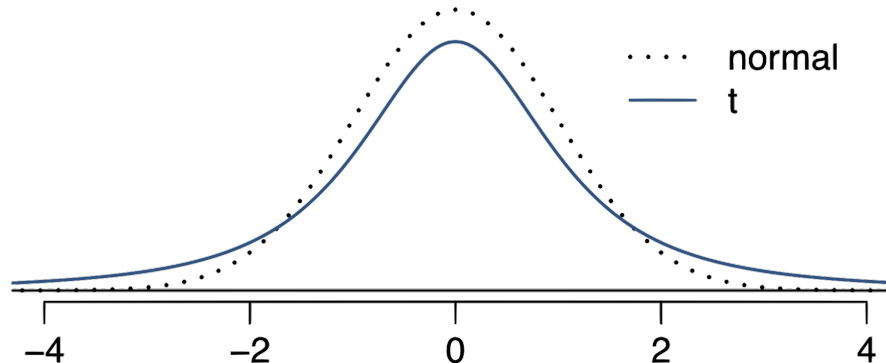
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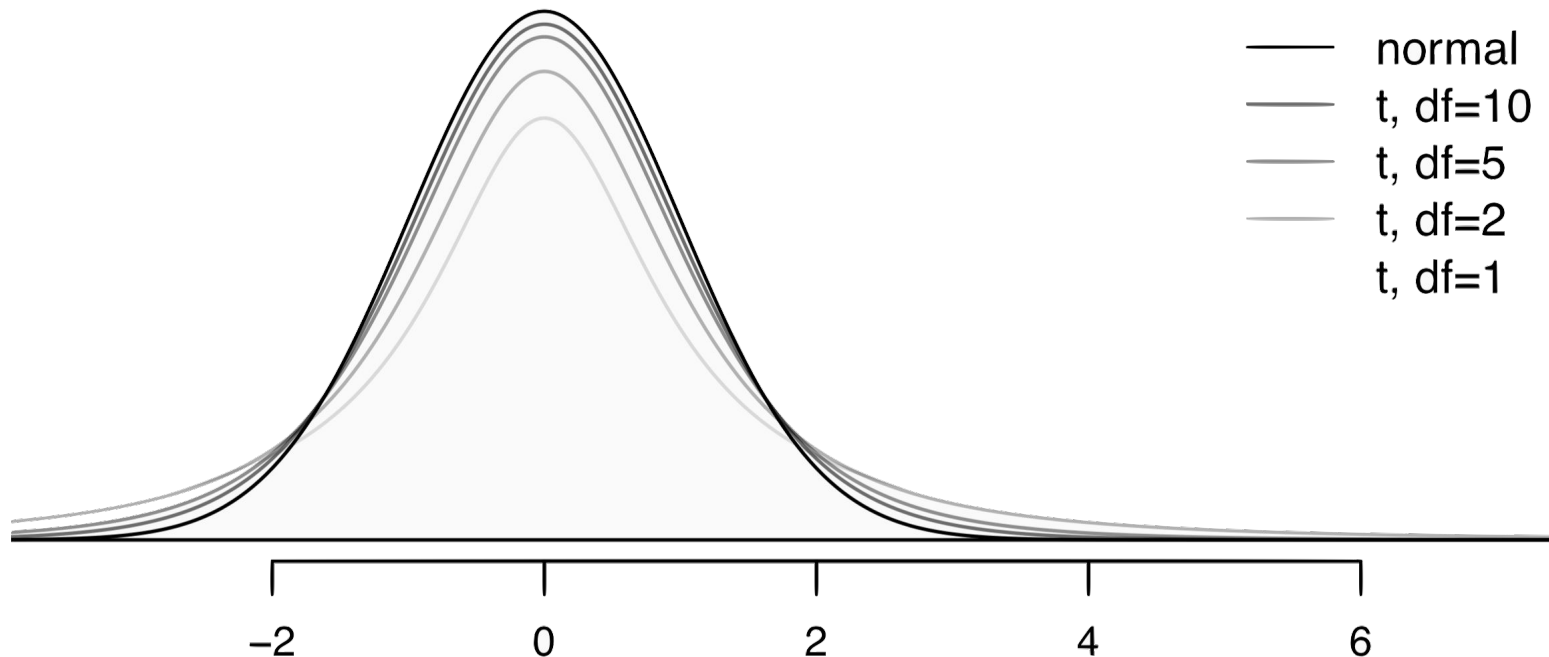
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- This distribution also has a bell shape, but its tails are **thicker** than the normal model's.
- Therefore observations are more likely to fall beyond two SDs from the mean than under the normal distribution
- These extra thick tails are helpful for resolving our problem with a less reliable estimate the standard error (since n is small)



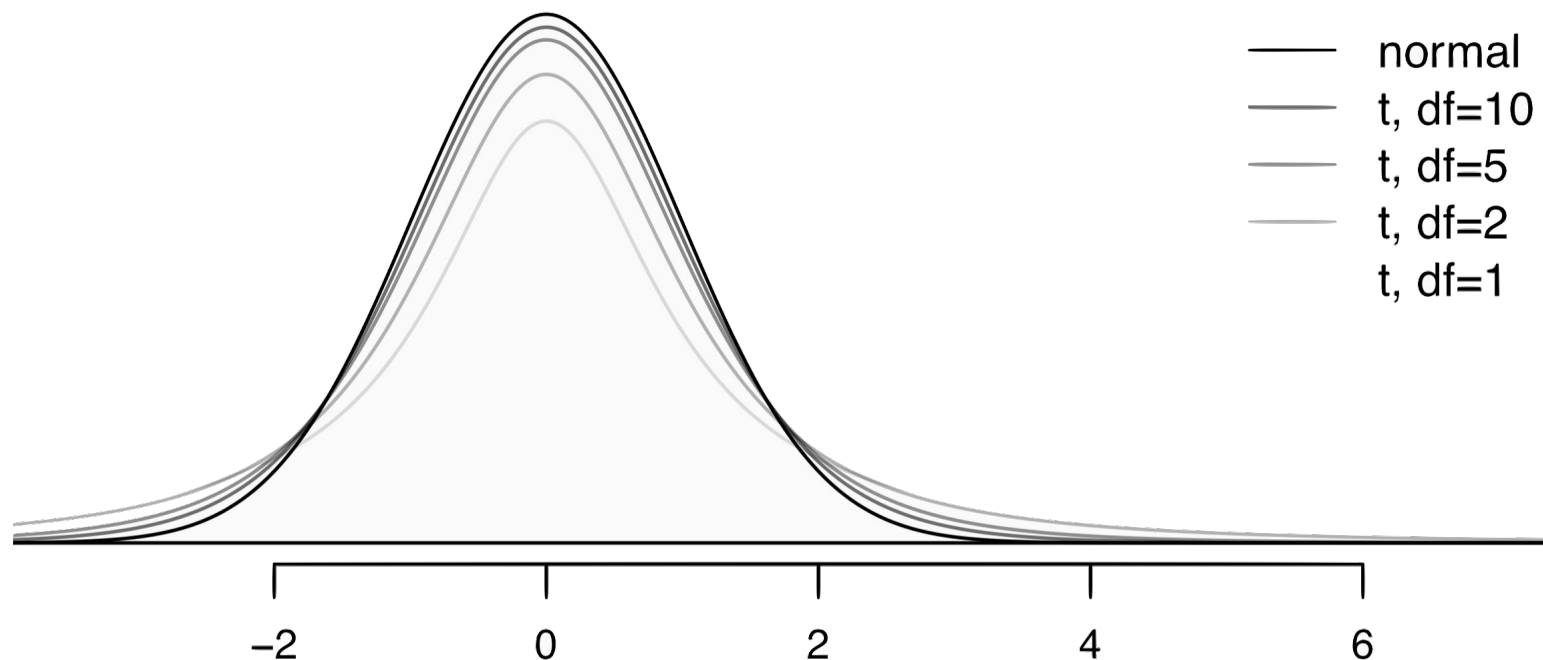
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- Has a single parameter: *degrees of freedom* (df).



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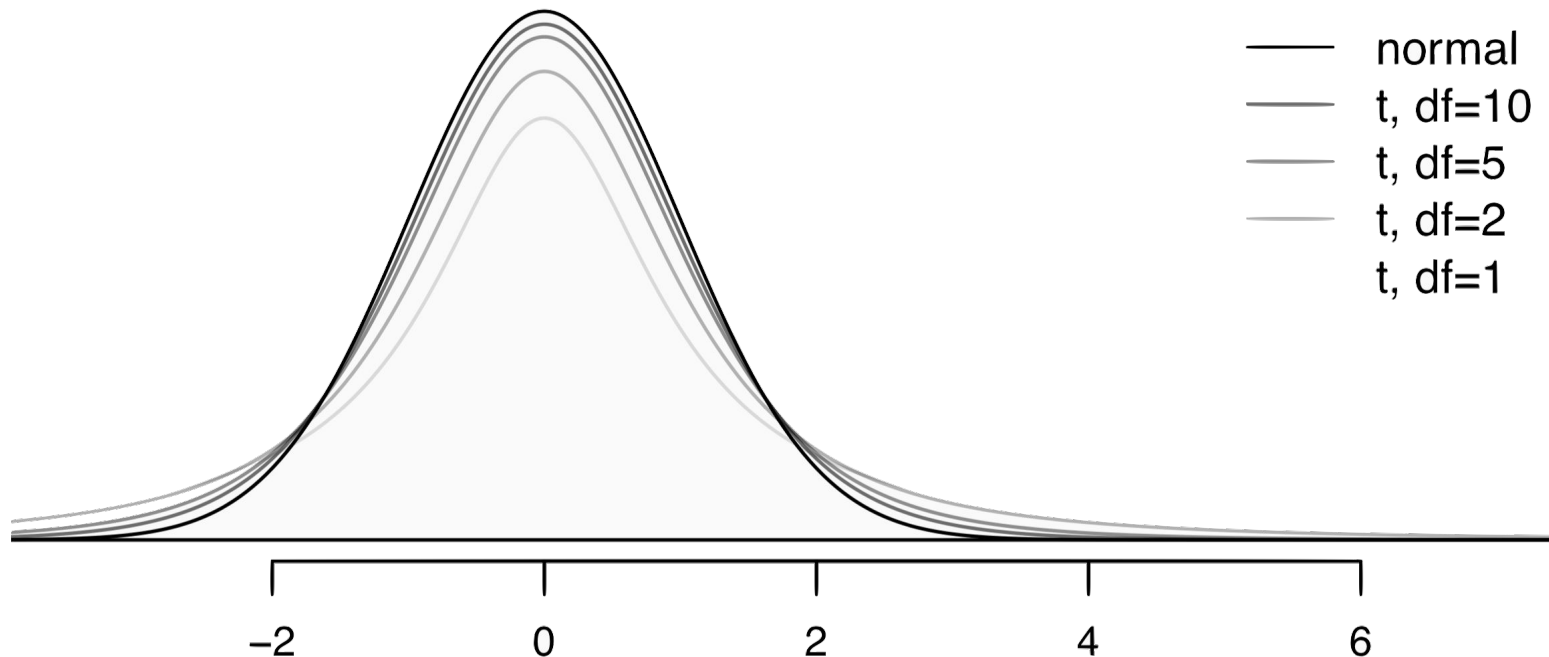
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What happens to the shape of the t distribution as df increases?

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Approaches normal

Back to Friday the 13th

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$$\bar{x}_{diff} = 1836$$

$$s_{diff} = 1176$$

Find the test statistic

Test statistic for inference on a small sample mean

The test statistic for inference on a small sample ($n < 50$) mean is the T statistic with $df = n - 1$

$$T_{df} = \frac{\text{point estimate} - \text{null value}}{SE}$$

Note: Null value is 0 because in the null hypothesis we set $\mu_{\text{diff}} = 0$

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$$df = 10 - 1 = 9$$

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- Or when these aren't available, we can use a t -table

Conclusion of the test

What is the conclusion of this hypothesis test?

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Since the p-value is quite low, we conclude that the data provide strong evidence of a difference between traffic flow on Friday 6th and 13th.

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- We concluded that there is a difference in the traffic flow between Friday 6th and 13th
- But it would be more interesting to find out what exactly this difference is
- We can use a confidence interval to estimate this difference

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- ME is always calculated as the product of a critical value and SE
- Since small sample means follow a t distribution (and not a z distribution), the critical value is a t^* (as opposed to a z^* ?).

$$\text{point estimate} \pm t^* \times SE$$

Finding the critical value (t^*)

Using R:

```
> qt(p = 0.975, df = 9)
```

```
[1] 2.262157
```

Constructing a CI for a small sample mean

Which of the following is the correct calculation of a 95% confidence interval for the difference between the traffic flow between Friday 6th and 13th?

$$\bar{x}_{diff} = 1836 \quad s_{diff} = 1176 \quad n = 10 \quad SE = 372$$

- A. $1836 \pm 1.96 \times 372$
- B. $1836 \pm 2.26 \times 372$
- C. $1836 \pm -2.26 \times 372$
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Interpreting the CI

Which of the following is the *best* interpretation for the confidence interval we just calculated?

$$\mu_{diff:6th-13th} = (995, 2677)$$

We are 95% confident that...

- A. the difference between the average number of cars on the road on Friday 6th and 13th is between 995 and 2,677
- B. on Friday 6th there are 995 to 2,677 fewer cars on the road than on the Friday 13th, on average
- C. on Friday 6th there are 995 fewer to 2,677 more cars on the road than on the Friday 13th, on average
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Synthesis

Does the conclusion from the hypothesis test agree with the findings of the confidence interval?

Do you think the findings of this study suggests that people believe Friday 13th is a day of bad luck?

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Yes, the hypothesis test found a significant difference, and the CI does not contain the null value of 0.

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Does the conclusion from the hypothesis test agree with the findings of the confidence interval?

Yes, the hypothesis test found a significant difference, and the CI does not contain the null value of 0

Do you think the findings of this study suggests that people believe Friday 13th is a day of bad luck?

No, this is an observational study. We have just observed a significant difference between the number of cars on the road on these two days. We have not tested for people's beliefs

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Note: The example we used was for paired means (difference between dependent groups). We took the difference between the observations and used only these differences (one sample) in our analysis

Find more resources at openintro.org/os, including

- Slides
- Videos
- Statistical Software Labs
- Discussion Forums (free support for students and teachers)
- Learning Objectives

Teachers only content is also available for [Verified Teachers](#), including

- Exercise solutions
- Sample exams
- Ability to request a free desk copy for a course
- Statistics Teachers email group

Questions? [Contact us](#).

**Extra Slides from the
OS3 section on one-sample means**

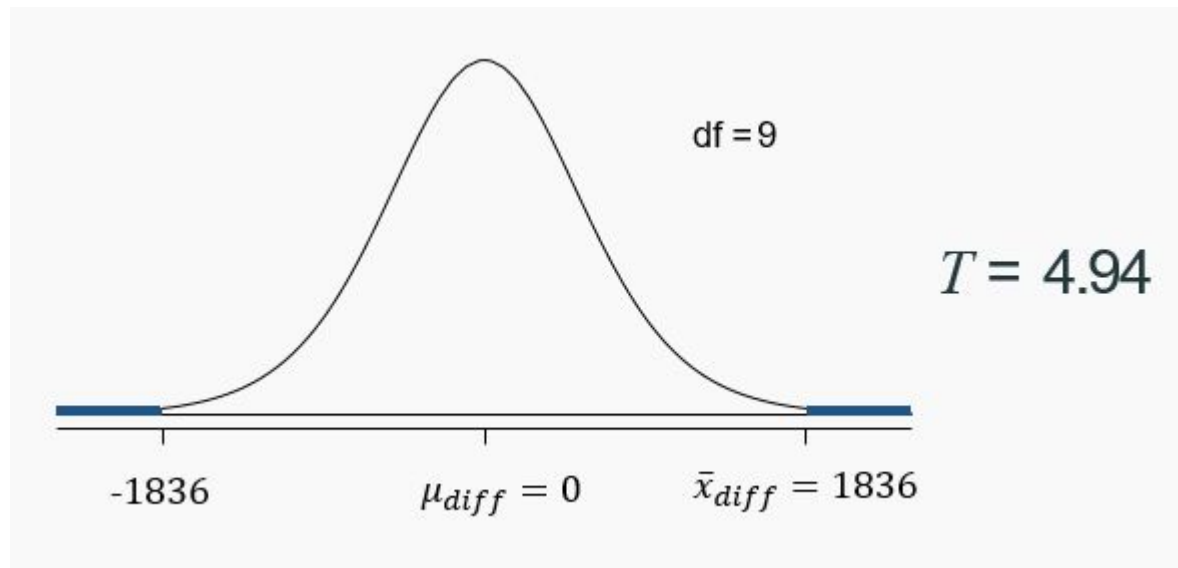
Finding the p-value

Locate the calculated T statistic on the appropriate df row, obtain the p-value from the corresponding column heading (one or two tail, depending on the alternative hypothesis).

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df						
1		3.08	6.31	12.71	31.82	63.66
2		1.89	2.92	4.30	6.96	9.92
3		1.64	2.35	3.18	4.54	5.84
.	
17		1.33	1.74	2.11	2.57	2.90
18		1.33	1.73	2.10	2.55	2.88
19		1.33	1.73	2.09	2.54	2.86
20		1.33	1.72	2.09	2.53	2.85
.	
.	
400		1.28	1.65	1.97	2.34	2.59
500		1.28	1.65	1.96	2.33	2.59
∞		1.28	1.64	1.96	2.33	2.58

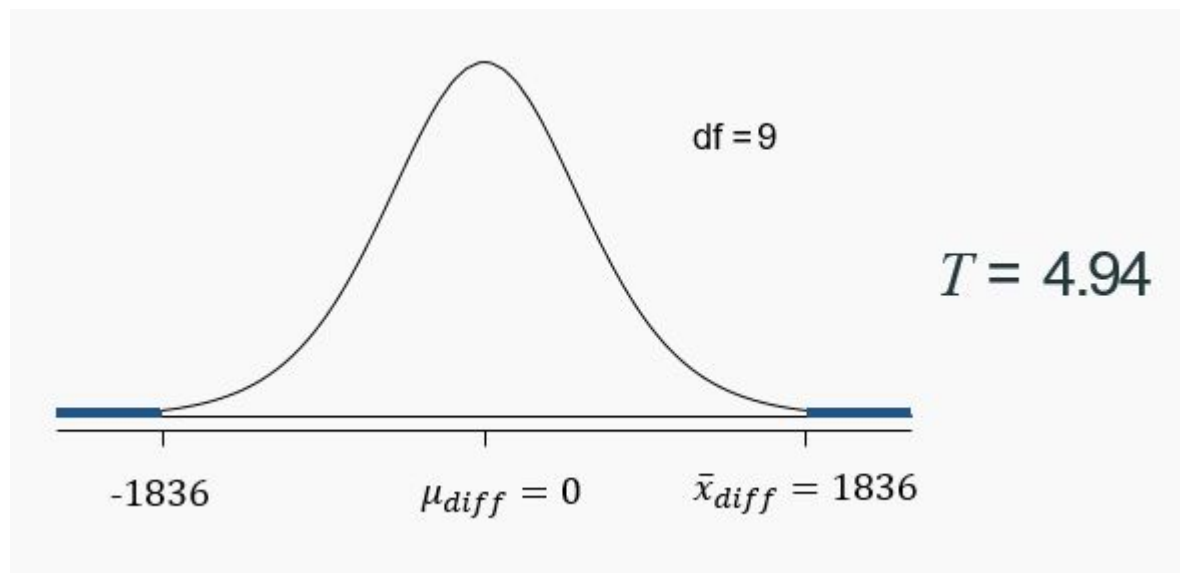
Finding the p-value (cont.)

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17



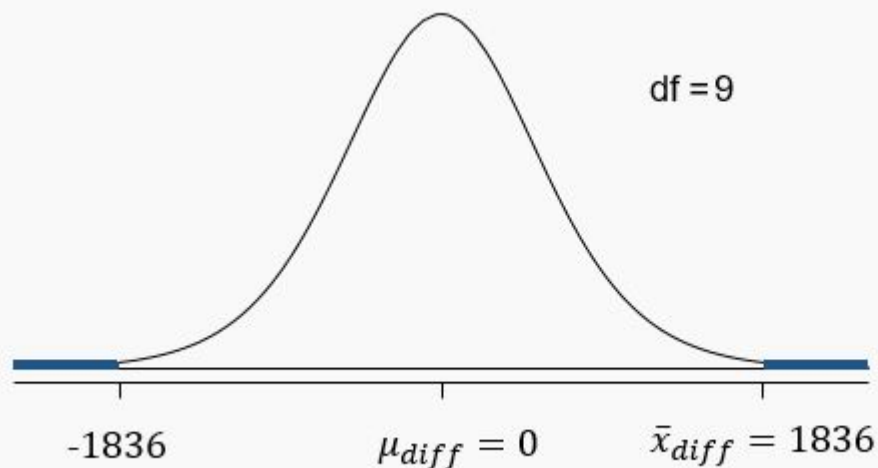
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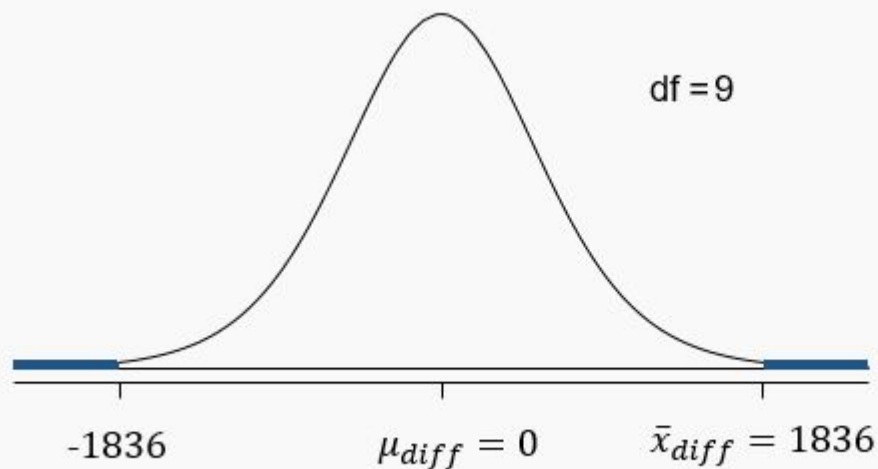
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Finding the p-value (cont.)

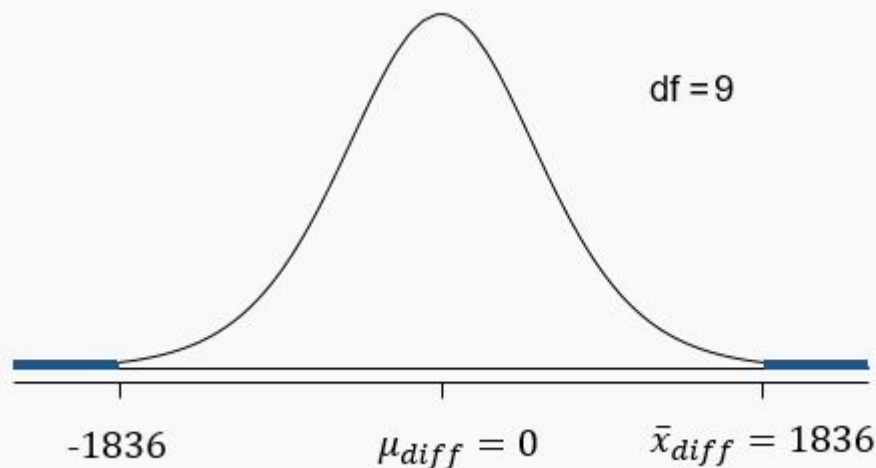
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9	1.38	1.83	2.26	2.82	3.25 →
10	1.37	1.81	2.23	2.76	3.17



What is the conclusion of the hypothesis test?

Finding the p-value (cont.)

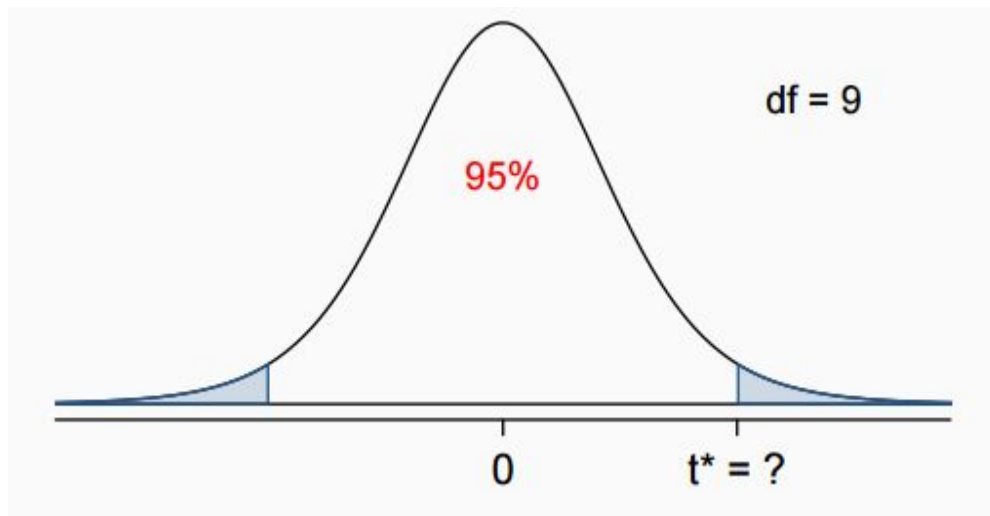
one tail	0.100	0.050	0.025	0.010	0.005
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9	1.38	1.83	2.26	2.82	3.25 →
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What is the conclusion of the hypothesis test?

The data provides convincing evidence of a difference between traffic flow on Friday 6th and 13th

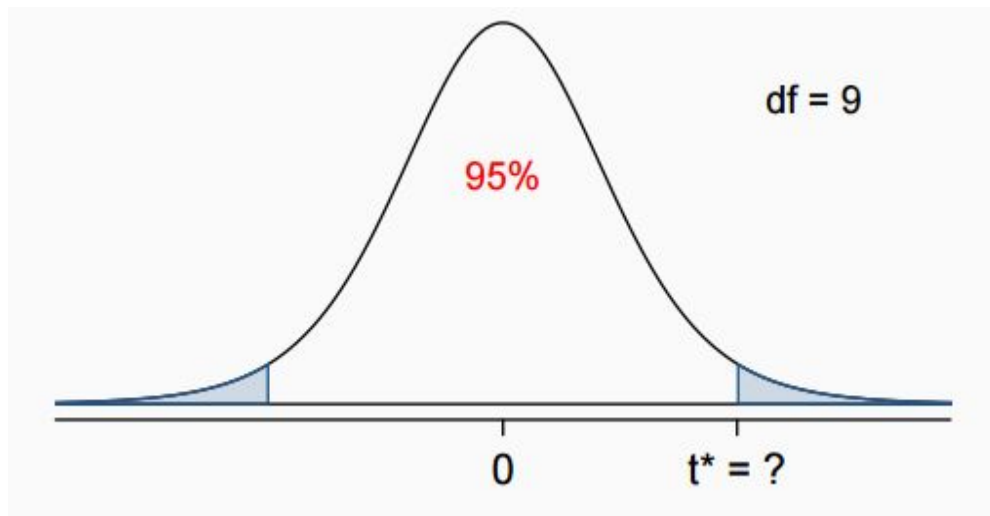
Finding the critical t (t^*)



$n = 10$, $df = 10 - 1 = 9$, t^* is at the intersection of rows $df = 9$ and two tail probability 0.05

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17

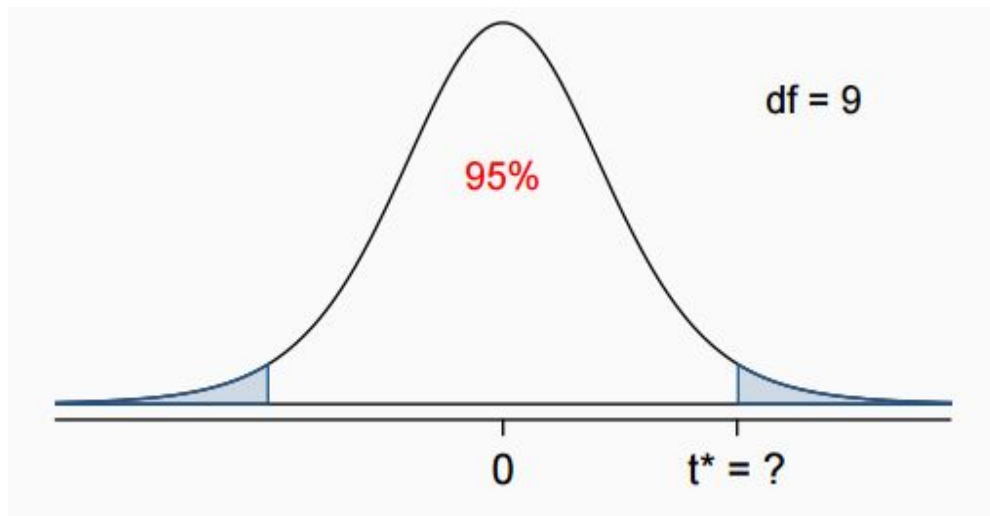
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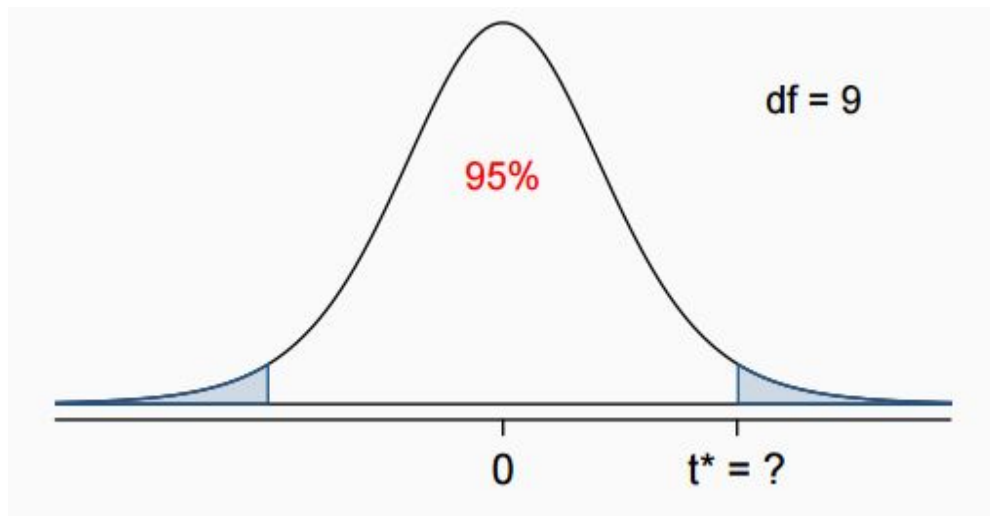
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