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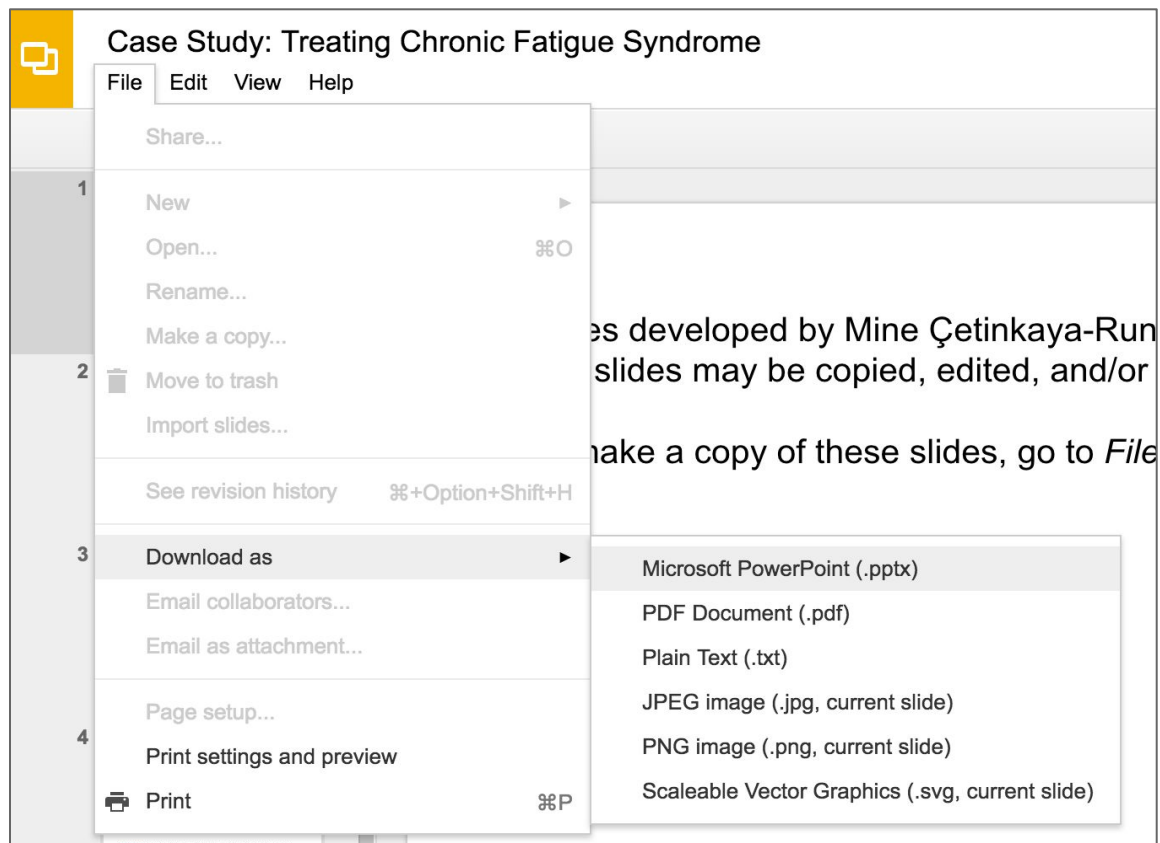
Chi-Square Test of Independence

Popular kids

In the dataset `popular`, students in grades 4-6 were asked whether good grades, athletic ability, or popularity was most important to them. A two-way table separating the students by grade and by choice of most important factor is shown below. Do these data provide evidence to suggest that goals vary by grade?

	Grades	Popular	Sports
4th	63	31	25
5th	88	55	33
6th	96	55	32

Chi-square test of independence

- The hypotheses are:

  H_0: Grade and goals are independent. Goals do not vary by grade.

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- The test statistic is calculated as

$$\chi^2_{df} = \sum_{i=1}^{k} \frac{(O-E)^2}{E} \quad \text{where} \quad df = (R-1) \times (C-1),$$

  where k is the number of cells, R is the number of rows, and C is the number of columns.

_____

Note: *we calculate df differently for one-way and two-way tables.*

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Note: *we calculate df differently for one-way and two-way tables.*

- The p-value is the area under the χ^2_{df} curve, above the calculated test statistic.

Expected counts in two-way tables

$$\text{Expected Count} = \frac{(\text{row total}) \times (\text{column total})}{\text{table total}}$$

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5th	88	55	33	176
6th	96	55	32	183
Total	247	141	90	478

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$$E_{row\ 1,col\ 1} = \frac{119 \times 247}{478} = 61 \qquad E_{row\ 1,col\ 2} = \frac{119 \times 141}{478} = 35$$

Expected counts in two-way tables

What is the expected count for the highlighted cell?

	Grades	Popular	Sports	Total
4th	63	31	25	119
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(a) 176 x 141 / 478

(b) 119 x 141 / 478

(c) 176 x 247 / 478

(d) 176 x 478 / 478

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(a) *176 x 141 / 478*   → 52

(b) 119 x 141 / 478   more than expected # of 5th graders

(c) 176 x 247 / 478   have a goal of being popular

(d) 176 x 478 / 478

Calculating the test statistic in two-way tables

Expected counts are shown in blue next to the observed counts.

	Grades	Popular	Sports	Total
4th	63 *61*	31 *35*	25 *23*	119
5th	88 *91*	55 *52*	33 *33*	176
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$$\chi^2 = \sum \frac{(63-61)^2}{61} + \frac{(31-35)^2}{35} + \cdots + \frac{(32-34)^2}{34} = 1.3121$$

Calculating the test statistic in two-way tables

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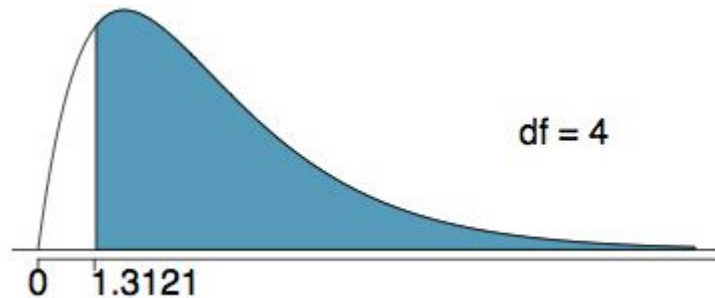$$\chi^2 = \sum \frac{(63-61)^2}{61} + \frac{(31-35)^2}{35} + \cdots + \frac{(32-34)^2}{34} = 1.3121$$

$$df = (R-1) \times (C-1) = (3-1) \times (3-1) = 2 \times 2 = 4$$

Calculating the p-value

Which of the following is the correct p-value for this hypothesis test?

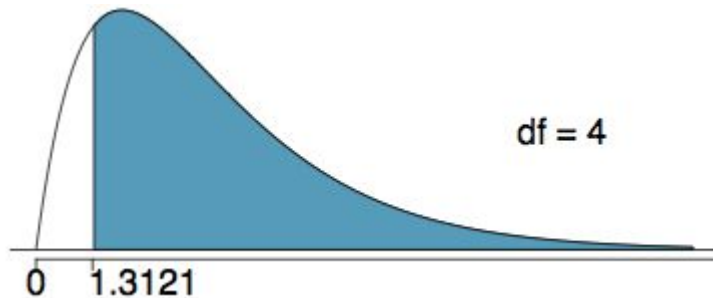$$\chi^2_{df} = 1.3121 \qquad df = 4$$

df = 4

0   1.3121

(a)   more than 0.3
(b)   between 0.3 and 0.2
(c)   between 0.2 and 0.1
(d)   between 0.1 and 0.05
(e)   less than 0.001

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Conclusion

Do these data provide evidence to suggest that goals vary by grade?

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Since the p-value is large, we fail to reject H_0. The data do not provide convincing evidence that grade and goals are dependent. It doesn't appear that goals vary by grade.

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- Sample exams
- Ability to request a free desk copy for a course
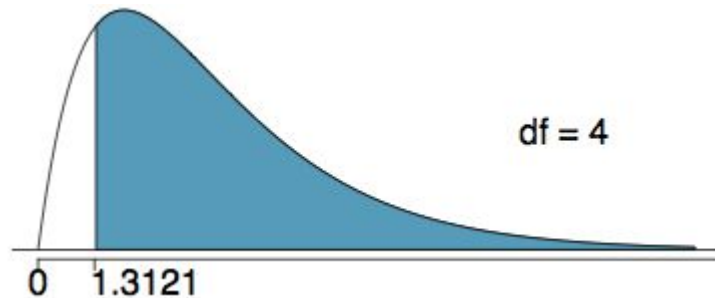- Statistics Teachers email group

Questions? Contact us.

Extra Slides from the
OS3 section on chi-square test of independence

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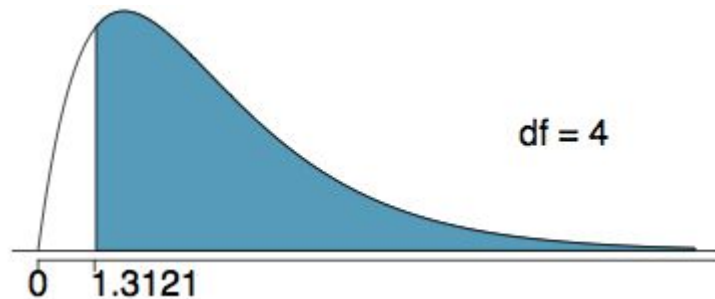
Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52

Calculating the p-value

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