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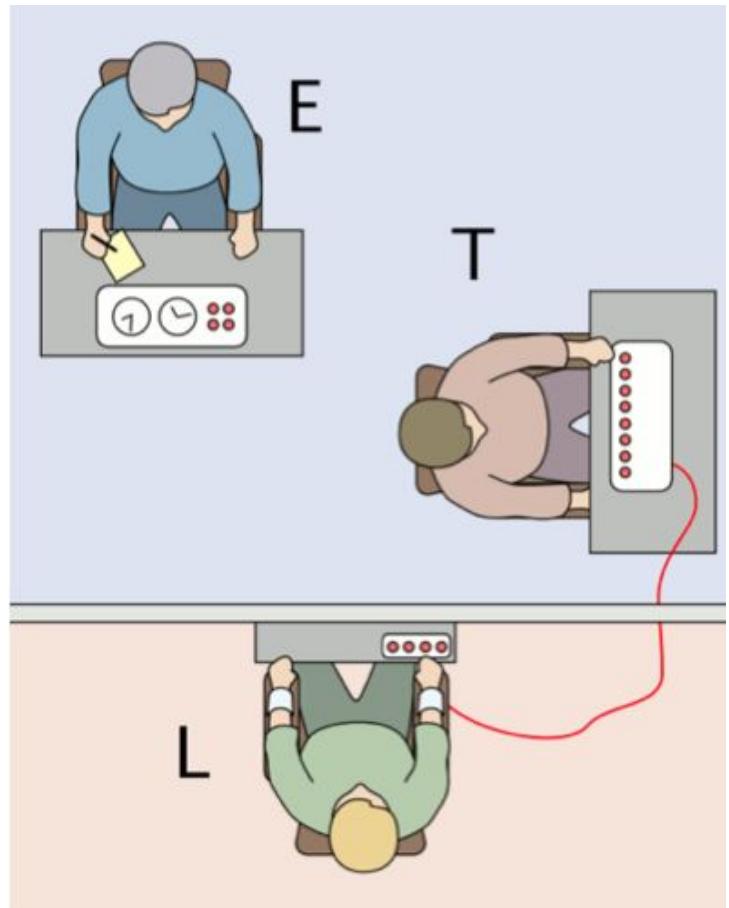


# Geometric Distribution

# Milgram experiment

Stanley Milgram, a Yale University psychologist, conducted a series of experiments on obedience to authority starting in 1963.

- Experimenter (E) orders the teacher (T), the subject of the experiment, to give severe electric shocks to a learner (L) each time the learner answers a question incorrectly.
- The learner is actually an actor, and the electric shocks are not real, but a pre-recorded sound is played each time the teacher administers an electric shock.



# Milgram experiment (cont.)

- These experiments measured the willingness of study participants to obey an authority figure who instructed them to perform acts that conflicted with their personal conscience.
- Milgram found that about 65% of people would obey authority and give such shocks.
- Over the years, additional research suggested this number is approximately consistent across communities and time.

# Bernoulli random variables

- Each person in Milgram's experiment can be thought of as a *trial*.
- A person is labeled a *success* if she refuses to administer a severe shock, and *failure* if she administers such shock.
- Since only 35% of people refused to administer a shock, *probability of success* is  $p = 0.35$ .
- When an individual trial has only two possible outcomes, it is called a *Bernoulli random variable*.

# Geometric distribution

Dr. Smith wants to repeat Milgram's experiments but she only wants to sample people until she finds someone who will not inflict a severe shock. What is the probability that she stops after the first person?

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... the third person?

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$$P(9 \text{ shock, } 10^{\text{th}} \text{ refuses}) = \underbrace{\frac{S}{0.65} \times \cdots \times \frac{S}{0.65}}_{9 \text{ of these}} \times \frac{R}{0.35} = 0.65^9 \times 0.35 \approx 0.0072$$

# Geometric distribution (cont.)

The *geometric distribution* describes the waiting time until a success for *independent and identically distributed (iid)* Bernoulli random variables.

- independence: outcomes of trials don't affect each other
- identical: the probability of success is the same for each trial

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## Geometric probabilities

If  $p$  represents probability of success,  $(1 - p)$  represents probability of failure, and  $n$  represents number of independent trials

$$P(\text{success on the } n^{\text{th}} \text{ trial}) = (1 - p)^{n-1} p$$

# Practice

Can we calculate the probability of rolling a 6 for the first time on the 6<sup>th</sup> roll of a die using the geometric distribution? Note that what was a success (rolling a 6) and what was a failure (not rolling a 6) are clearly defined and one or the other must happen for each trial.

1. no, on the roll of a die there are more than 2 possible outcomes
2. yes, why not

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- A. no, on the roll of a die there are more than 2 possible outcomes
- B. yes, why not

$$P(6 \text{ on the } 6^{\text{th}} \text{ roll}) = \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) \approx 0.067$$

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But how can she test a non-whole number of people?

# Expected value and its variability

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- Dr. Smith is expected to test 2.86 people before finding the first one that refuses to administer the shock, give or take 2.3 people.
- These values only make sense in the context of repeating the experiment many many times.

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