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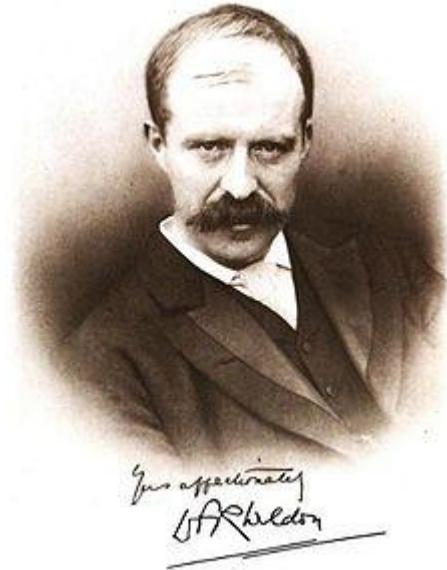
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Chi-Square test of GOF

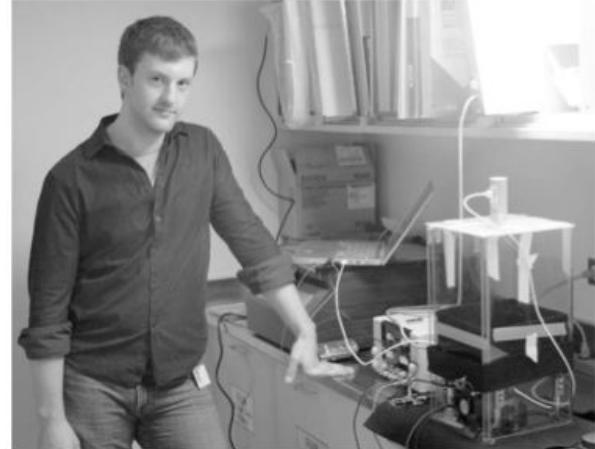
Weldon's dice

- Walter Frank Raphael Weldon (1860 - 1906), was an English evolutionary biologist and a founder of biometry. He was the joint founding editor of Biometrika, with Francis Galton and Karl Pearson.
- In 1894, he rolled 12 dice 26,306 times, and recorded the number of 5s or 6s (which he considered to be a success).
- It was observed that 5s or 6s occurred more often than expected, and Pearson hypothesized that this was probably due to the construction of the dice. Most inexpensive dice have hollowed-out pips, and since opposite sides add to 7, the face with 6 pips is lighter than its opposing face, which has only 1 pip.



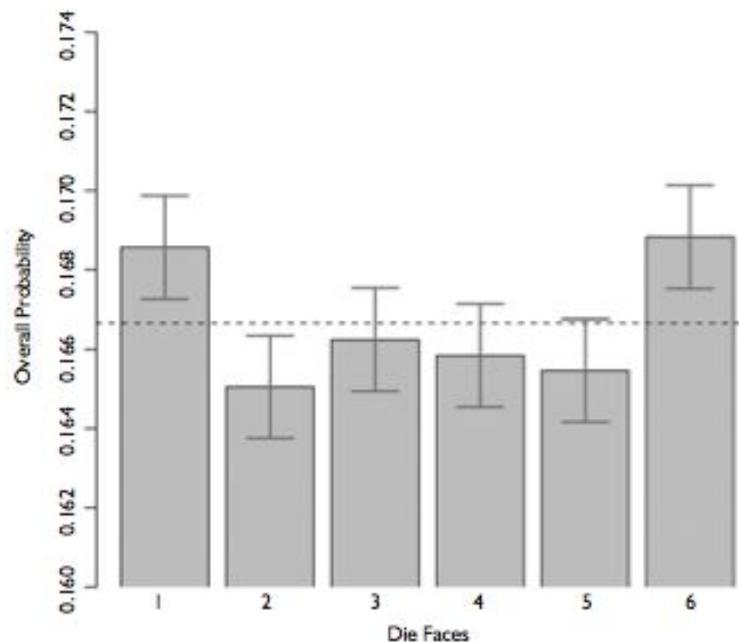
Labby's dice

- In 2009, Zacariah Labby (U of Chicago), repeated Weldon's experiment using a homemade dice-throwing, pip counting machine.
www.youtube.com/watch?v=95EErdouO2w
- The rolling-imaging process took about 20 seconds per roll.
- Each day there were ~150 images to process manually.
- At this rate Weldon's experiment was repeated in a little more than six full days.
- Recommended reading:
galton.uchicago.edu/about/docs/labby09dice.pdf



Labby's dice (cont.)

- Labby did not actually observe the same phenomenon that Weldon observed (higher frequency of 5s and 6s).
- Automation allowed Labby to collect more data than Weldon did in 1894, instead of recording "successes" and "failures", Labby recorded the individual number of pips on each die.



Expected counts

Labby rolled 12 dice 26,306 times. If each side is equally likely to come up, how many 1s, 2s, ..., 6s would he expect to have observed?

- (a) 1/6
- (b) 12/6
- (c) 26,306 / 6
- (d) $12 \times 26,306 / 6$

Expected counts

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- (a) $1/6$
- (b) $12 / 6$
- (c) $26,306 / 6$
- (d) $12 \times 26,306 / 6 = 52,612$

Summarizing Labby's results

The table below shows the observed and expected counts from Labby's experiment.

Outcome	Observed	Expected
1	53,222	52,612
2	52,118	52,612
3	52,465	52,612
4	52,338	52,612
5	52,244	52,612
6	53,285	52,612
Total	315,672	315,672

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Why are the expected counts the same for all outcomes but the observed counts are different? At a first glance, does there appear to be an inconsistency between the observed and expected counts?

Setting the hypotheses

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- Large deviations from what would be expected based on sampling variation (chance) alone provide strong evidence for the alternative hypothesis.
- This is called a *goodness of fit* test since we're evaluating how well the observed data fit the expected distribution.

Anatomy of a test statistic

The general form of a test statistic is

$$\frac{\text{point estimate} - \text{null value}}{\text{SE of point estimate}}$$

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These two ideas will help in the construction of an appropriate test statistic for count data.

Chi-square statistic

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χ^2 statistic

$$\chi^2 = \sum_{i=1}^k \frac{(O - E)^2}{E} \quad \text{where } k = \text{total number of cells}$$

Calculating the chi-square statistic

Outcome	Observed	Expected	$\frac{(O-E)^2}{E}$
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When have we seen this before?

The chi-square distribution

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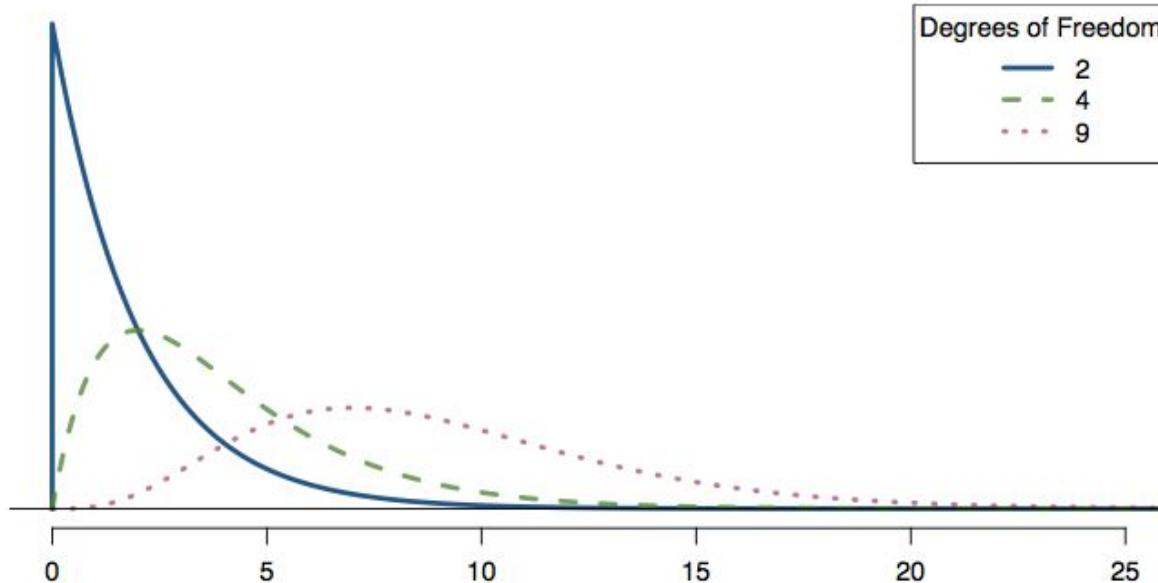
Remember

So far we've seen three other continuous distributions:

- normal distribution: unimodal and symmetric with two parameters: mean and standard deviation
- T distribution: unimodal and symmetric with one parameter: degrees of freedom
- F distribution: unimodal and right skewed with two parameters: degrees of freedom or numerator (between group variance) and denominator (within group variance)

Practice

Which of the following is false?

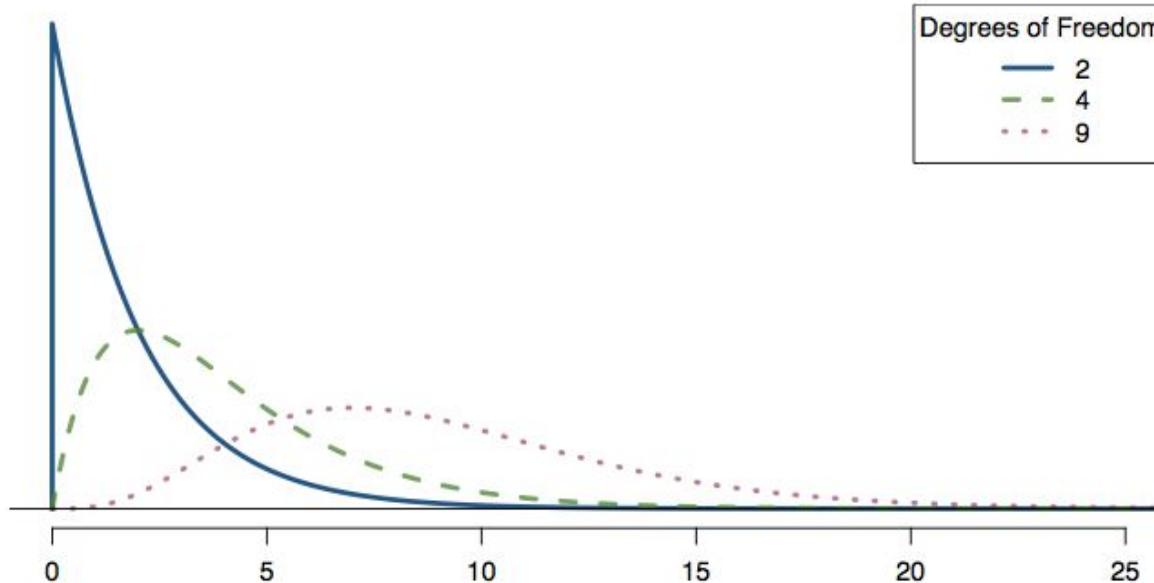


As the df increases,

- (a) the center of the χ^2 distribution increases as well
- (b) the variability of the χ^2 distribution increases as well
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Finding areas under the chi-square curve

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- For this we can use technology, or a *chi-square probability table*.

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Estimate the shaded area under the chi-square curve with $df = 6$.

Finding areas under the chi-square curve

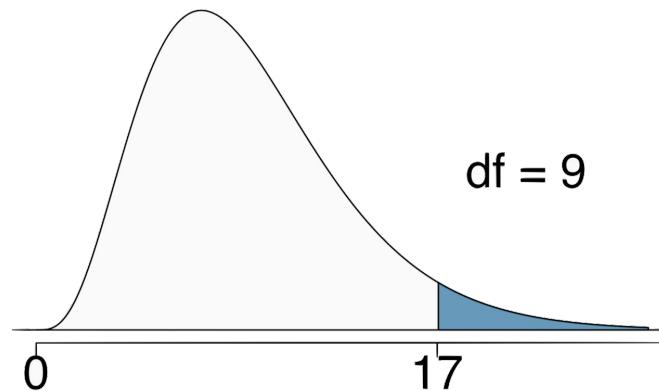
Estimate the shaded area under the chi-square curve with $df = 6$.

```
> pchisq(q = 10, df = 6, lower.tail = FALSE)  
[1] 0.124652
```

Finding areas under the chi-square curve

Estimate the shaded area above a cutoff value of 17 for the chi-square curve with $df = 9$.

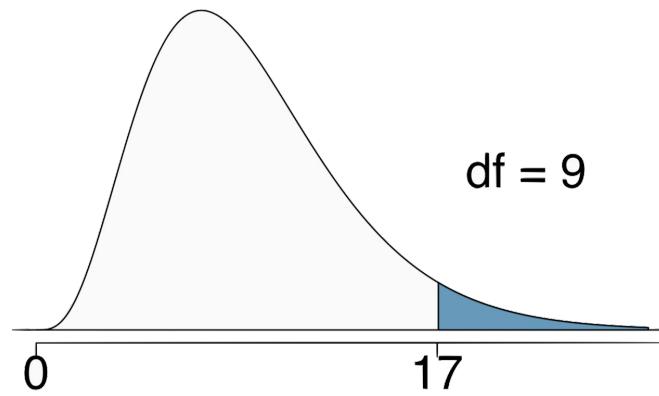
- (a) 0.05
- (b) 0.02
- (c) between 0.02 and 0.05
- (d) between 0.05 and 0.1
- (e) between 0.01 and 0.02



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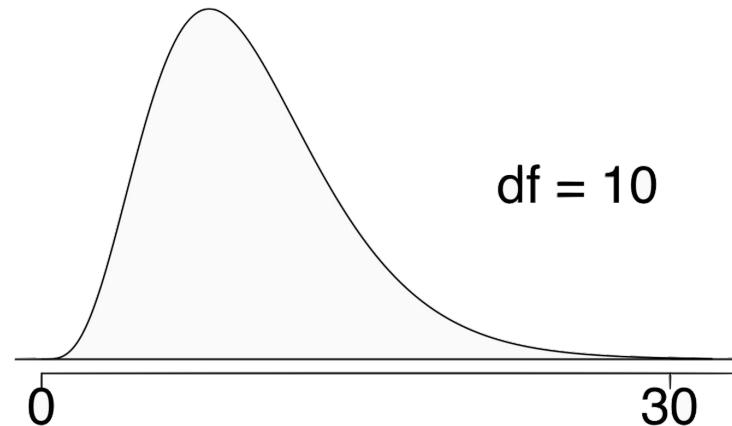


```
> pchisq(q = 17, df = 9, lower.tail = FALSE)  
[1] 0.04871598
```

Finding areas under the chi-square curve

Estimate the shaded area above a cutoff value of 30 for the chi-square curve with $df = 10$.

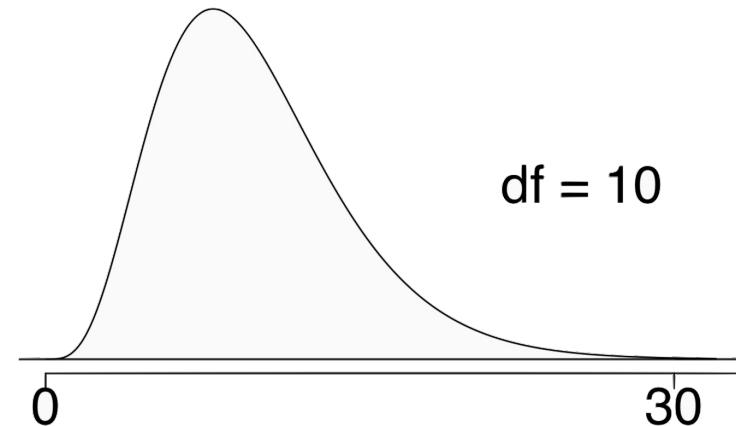
- (a) greater than 0.3
- (b) between 0.005 and 0.001
- (c) less than 0.001
- (d) greater than 0.001
- (e) cannot tell using this table



Finding areas under the chi-square curve

Estimate the shaded area above a cutoff value of 30 for the chi-square curve with $df = 10$.

- (a) greater than 0.3
- (b) between 0.005 and 0.001
- (c) less than 0.001**
- (d) greater than 0.001
- (e) cannot tell using this table



```
> pchisq(q = 30, df = 10, lower.tail = FALSE)  
[1] 0.0008566412
```

Back to Labby's dice

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- We had calculated a test statistic of $\chi^2 = 24.67$.
- All we need is the df and we can calculate the tail area (the p-value) and make a decision on the hypotheses.

Degrees of freedom for a goodness of fit test

- When conducting a goodness of fit test to evaluate how well the observed data follow an expected distribution, the degrees of freedom are calculated as the number of cells (k) minus 1.

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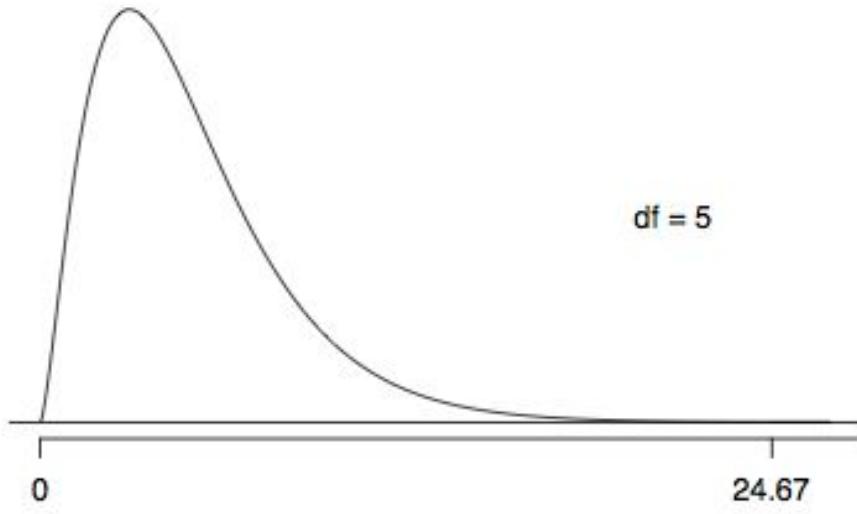
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- For dice outcomes, $k = 6$, therefore

$$df = 6 - 1 = 5$$

Finding a p-value for a chi-square test

The *p-value* for a chi-square test is defined as the *tail area above the calculated test statistic*.



$$\text{p-value} = P(\chi^2_{df=5} > 24.67)$$

is less than 0.001

Conclusion of the hypothesis test

We calculated a p-value less than 0.001. At 5% significance level, what is the conclusion of the hypothesis test?

- (a) Reject H_0 , the data provide convincing evidence that the dice are fair.
- (b) Reject H_0 , the data provide convincing evidence that the dice are biased.
- (c) Fail to reject H_0 , the data provide convincing evidence that the dice are fair.
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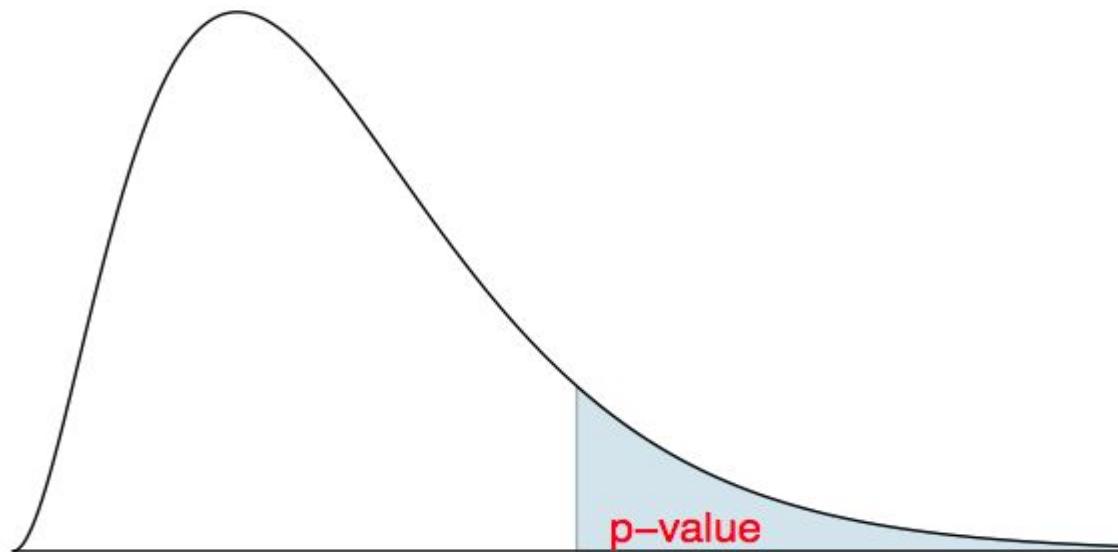
Turns out...

- The 1-6 axis is consistently shorter than the other two (2-5 and 3-4), thereby supporting the hypothesis that the faces with one and six pips are larger than the other faces.
- Pearson's claim that 5s and 6s appear more often due to the carved-out pips is not supported by these data.
- Dice used in casinos have flush faces, where the pips are filled in with a plastic of the same density as the surrounding material and are precisely balanced.



Recap: p-value for a chi-square test

- The p-value for a chi-square test is defined as the tail area *above* the calculated test statistic.
- This is because the test statistic is always positive, and a higher test statistic means a stronger deviation from the null hypothesis.



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Failing to check conditions may unintentionally affect the test's error rates.

2009 Iran Election

There was lots of talk of election fraud in the 2009 Iran election. We'll compare the data from a poll conducted before the election (observed data) to the reported votes in the election to see if the two follow the same distribution.

Candidate	Observed # of voters in poll	Reported % of votes in election
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Hypotheses

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H_0 : The observed counts from the poll follow the same distribution as the reported votes.

H_A : The observed counts from the poll do not follow the same distribution as the reported votes.

Calculation of the test statistic

Candidate	Observed # of voters in poll	Reported % of votes in election	Expected # of votes in poll
(1) Ahmedinajad	338	63.29%	$504 \times 0.6329 = 319$
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$$\chi^2_{df=3-1=2} = 30.89$$

Conclusion

Based on these calculations what is the conclusion of the hypothesis test?

- (a) p-value is low, H_0 is rejected. The observed counts from the poll do not follow the same distribution as the reported votes.
- (b) p-value is high, H_0 is not rejected. The observed counts from the poll follow the same distribution as the reported votes.
- (c) p-value is low, H_0 is rejected. The observed counts from the poll follow the same distribution as the reported votes
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- Learning Objectives

Teachers only content is also available for [Verified Teachers](#), including

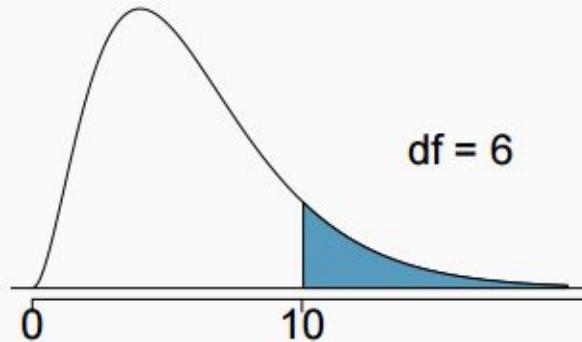
- Exercise solutions
- Sample exams
- Ability to request a free desk copy for a course
- Statistics Teachers email group

Questions? [Contact us.](#)

**Extra Slides from the
OS3 section on testing for goodness of fit**

Finding areas under the chi-square curve

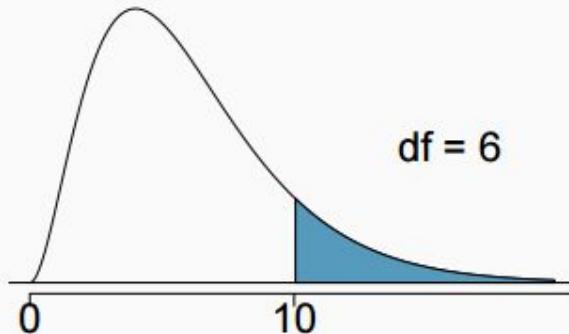
Estimate the shaded area under the chi-square curve with $df = 6$.



Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
	6	7.23	8.56	10.64	12.59	15.03	16.81	18.55	22.46
	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32

Finding areas under the chi-square curve (cont.)

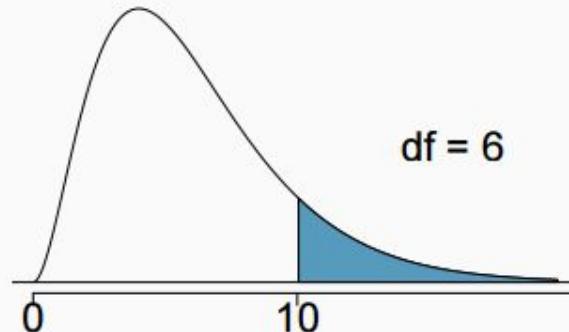
Estimate the shaded area under the chi-square curve with $df = 6$.



Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	1.07	1.64	2.71	3.84	5.41	6.63	7.88	10.83
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	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52
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Finding areas under the chi-square curve (cont.)

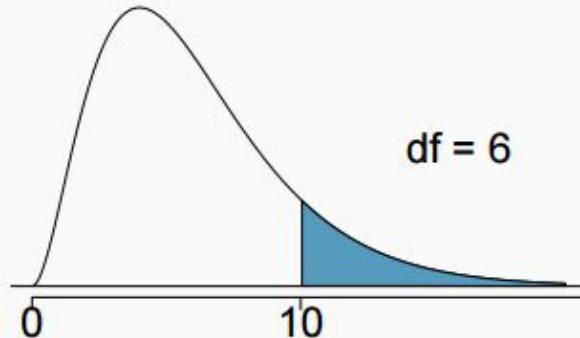
Estimate the shaded area under the chi-square curve with $df = 6$.



Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	2	3	4	5	6	7	8
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Finding areas under the chi-square curve (cont.)

Estimate the shaded area under the chi-square curve with $df = 6$.

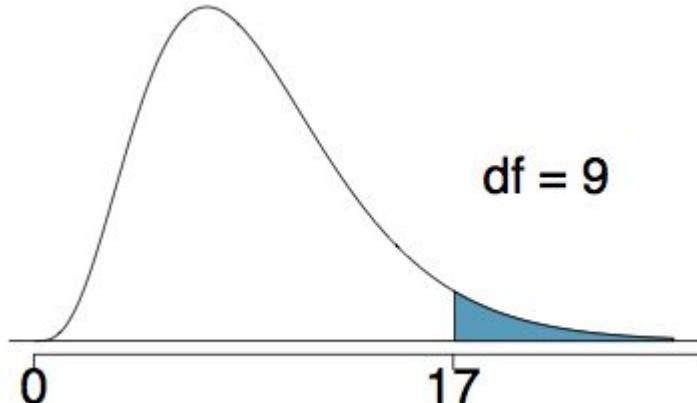


$P(\chi^2_{df=6} > 10)$
is between 0.1 and 0.2

Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	1	2	3	4	5	6	7	8
	1.07	2.41	3.66	4.88	6.06	7.23	8.38	9.55
	1.64	3.22	4.64	5.99	7.29	8.56	9.80	10.64
	2.71	4.61	6.25	7.78	9.24	10.64	12.02	12.83
	3.84	5.99	7.81	9.49	11.07	12.59	14.07	14.77
	5.41	7.82	9.84	11.67	13.39	15.03	16.62	17.34
	6.63	9.21	11.34	13.28	15.09	16.81	18.48	19.19
	7.88	10.60	12.84	14.86	16.75	18.55	20.28	21.00
	10.83	13.82	16.27	18.47	20.52	22.46	24.32	25.08

Finding areas under the chi-square curve (cont.)

Estimate the shaded area (above 17) under the χ^2 curve with $df = 9$.

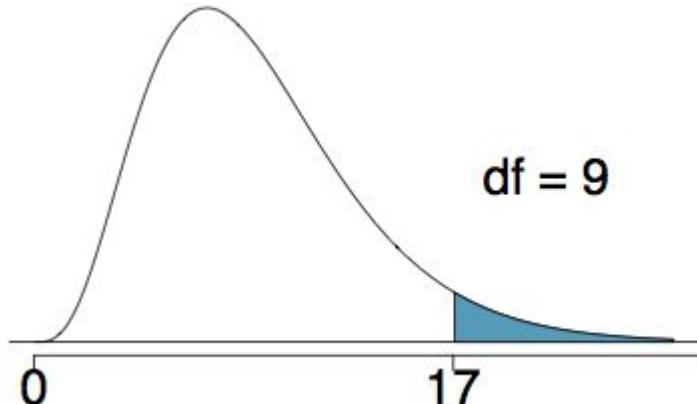


- (a) between 0.01 and 0.02
- (b) 0.02
- (c) between 0.02 and 0.05
- (d) 0.05
- (e) between 0.05 and 0.10

Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	7	8	9	10	11	12	13	14
	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26

Finding areas under the chi-square curve (cont.)

Estimate the shaded area (above 17) under the χ^2 curve with $df = 9$.

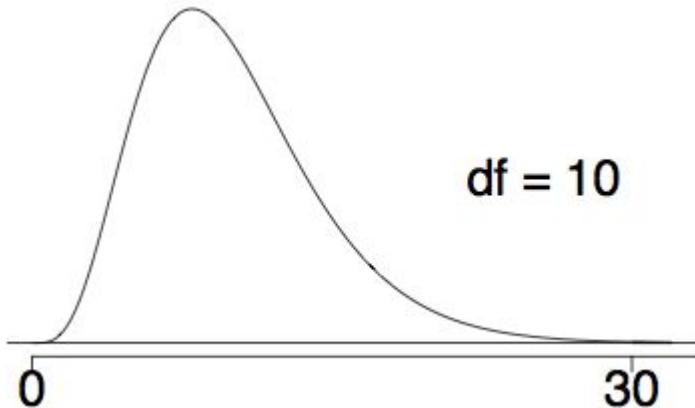


- (a) between 0.01 and 0.02
- (b) 0.02
- (c) *between 0.02 and 0.05*
- (d) 0.05
- (e) between 0.05 and 0.10

Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001	
df	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26

Finding areas under the chi-square curve (one more)

Estimate the shaded area (above 30) under the χ^2 curve with $df = 10$.

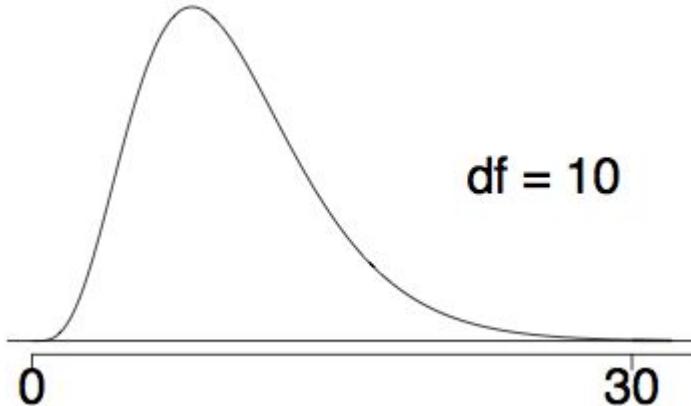


- (a) between 0.005 and 0.001
- (b) less than 0.001
- (c) greater than 0.001
- (d) greater than 0.3
- (e) cannot tell using this table

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26

Finding areas under the chi-square curve (one more)

Estimate the shaded area (above 30) under the χ^2 curve with $df = 10$.



- (a) greater than 0.3
- (b) between 0.005 and 0.001
- (c) *less than 0.001*
- (d) greater than 0.001
- (e) cannot tell using this table

Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001	
df	7	8.38	9.80	12.02	14.07	16.62	18.48	20.28	24.32
	8	9.52	11.03	13.36	15.51	18.17	20.09	21.95	26.12
	9	10.66	12.24	14.68	16.92	19.68	21.67	23.59	27.88
	10	11.78	13.44	15.99	18.31	21.16	23.21	25.19	29.59
	11	12.90	14.63	17.28	19.68	22.62	24.72	26.76	31.26



Finding the tail areas using computation

- While probability tables are very helpful in understanding how probability distributions work, and provide quick reference when computational resources are not available, they are somewhat archaic.

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- While probability tables are very helpful in understanding how probability distributions work, and provide quick reference when computational resources are not available, they are somewhat archaic.
- Using R:

```
pchisq(q = 30, df = 10, lower.tail = FALSE)  
# 0.0008566412
```

Finding the tail areas using computation

- While probability tables are very helpful in understanding how probability distributions work, and provide quick reference when computational resources are not available, they are somewhat archaic.

- Using R:

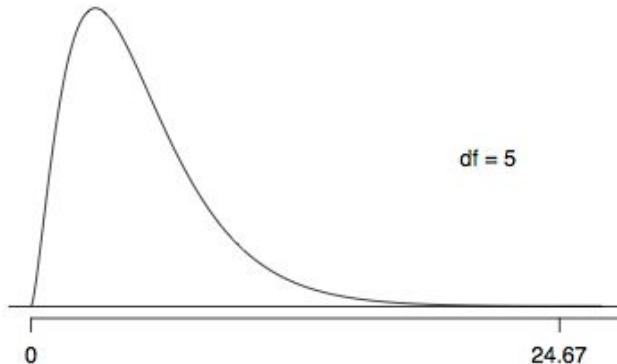
```
pchisq(q = 30, df = 10, lower.tail = FALSE)  
# 0.0008566412
```

- Using a web applet:

http://bitly.com/dist_calc

Finding a p-value for a chi-square test

The *p-value* for a chi-square test is defined as the *tail area above the calculated test statistic*.



$$\text{p-value} = P(\chi^2_{df=5} > 24.67)$$

is less than 0.001

Upper tail	0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001	→
df	1	2	3	4	5	6	7	8	
	1.07	2.41	3.66	4.88	6.06	7.82	9.84	11.67	
	1.64	3.22	4.64	5.99	7.29	9.21	11.34	13.28	
	2.71	4.61	6.25	7.78	9.24	10.60	12.84	14.86	
	3.84	5.99	7.81	9.49	11.07	13.82	16.27	18.47	
	5.41	7.84	9.84	11.67	13.39	15.09	16.75	20.52	→
	6.63	9.21	11.34	13.28	15.09	16.75	20.52		
	7.88	10.60	12.84	14.86					
	10.83	13.82	16.27	18.47					