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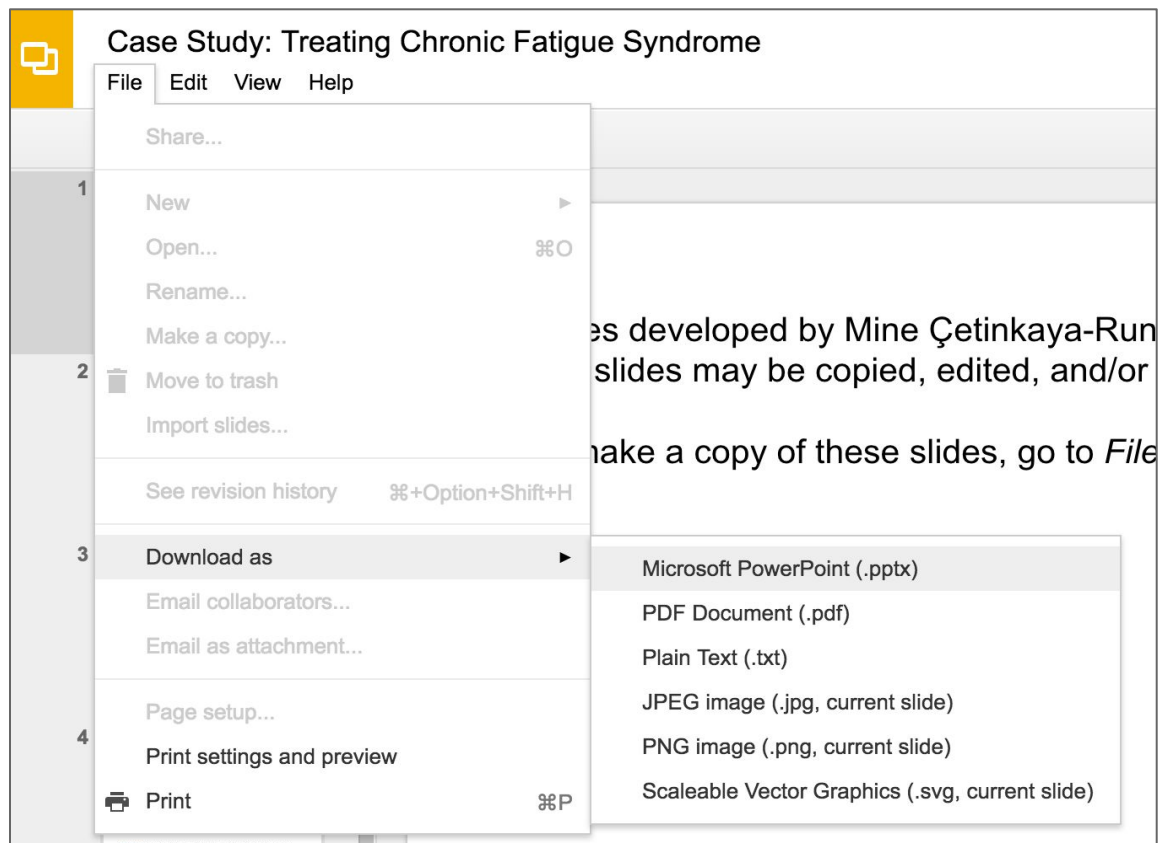
Negative binomial distribution

Negative binomial distribution

- The negative binomial distribution describes the probability of observing the kth success on the nth trial.
- The following four conditions are useful for identifying a negative binomial case:
  1. The trials are independent.
  2. Each trial outcome can be classified as a success or failure.
  3. The probability of success (p) is the same for each trial.
  4. The last trial must be a success.

  Note that the first three conditions are common to the binomial distribution.

Negative binomial distribution

$$P(k^{th} \text{ success on the } n^{th} \text{ trial}) = \binom{n-1}{k-1} p^k (1-p)^{n-k},$$

where p is the probability that an individual trial is a success. All trials are assumed to be independent.

Practice

A college student working at a psychology lab is asked to recruit 10 couples to participate in a study. She decides to stand outside the student center and ask every 5th person leaving the building whether they are in a relationship and, if so, whether they would like to participate in the study with their significant other. Suppose the probability of finding such a person is 10%. What is the probability that she will need to ask 30 people before she hits her goal?

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Given: p = 0.10, k = 10, n = 30. We are asked to find the probability of 10th success on the 30th trial, therefore we use the negative binomial distribution.

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$$= 0.00012$$

Binomial vs negative binomial

How is the negative binomial distribution different from the binomial distribution?

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How is the negative binomial distribution different from the binomial distribution?

- In the binomial case, we typically have a fixed number of trials and instead consider the number of successes.

- In the negative binomial case, we examine how many trials it takes to observe a fixed number of successes and require that the last observation be a success.

Practice

Which of the following describes a case where we would use the negative binomial distribution to calculate the desired probability?

(a) Probability that a 5 year old boy is taller than 42 inches.

(b) Probability that 3 out of 10 softball throws are successful.

(c) Probability of being dealt a straight flush hand in poker.

(d) Probability of missing 8 shots before the first hit.

(e) Probability of hitting the ball for the 3rd time on the 8th try.

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