Assignment 2 Solution (Week 3 - 4)

STAT 2601 - Business Statistics
School of Mathematics and Statistics, Carleton University

Total Marks: 40

Instructions: Please refer to the document entitled *Submission Instructions* posted at Brightspace. Follow the instructions carefully.

Q1: [12] Dish Detergent

(a) $P(A) = 0.65$[.5], $P(B) = 0.48$[.5], $P(A \cap B) = 0.25$[.5].

(b) 2×2 contingency table (2; .25 for each cell including marginal totals)

	B	$\bar{B}$	**Total**
A	0.25	0.40	**0.65**
$\bar{A}$	0.23	0.12	**0.35**
Total	**0.48**	**0.52**	1

(c) $P(A \cap \bar{B})(or A \cap B^c))$[.5] $= P(A) - P(A \cap B)$[.5] $= 0.40$[.5].

(d) $P(A \cup B)$[.5] $= P(A) + P(B) - P(A \cap B)$[.5] $= 0.65 + 0.48 - 0.25 = 0.88$[.5].

(e) $P(\overline{A \cup B})(or P(A \cup B)^c)$[.5] $= 1 - P(A \cup B)$[.5] $= 1 - 0.88 = 0.12$[.5].

(f) $P(A \cap \bar{B})$[.5] $+ P(B \cap \bar{A})$[.5] $= 0.40 + 0.23$[.5] $= 0.63$.

(g) The events A and B are NOT mutually exclusive [.5] because

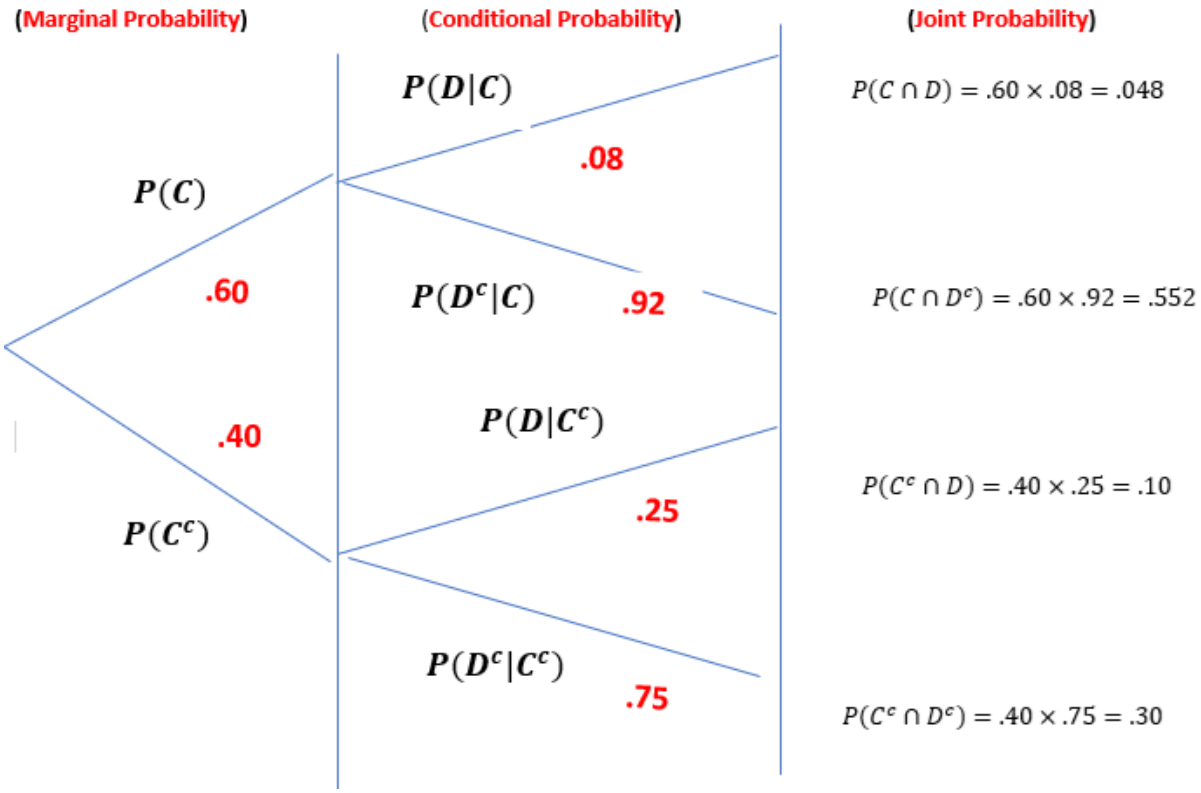
(i) $A \cap B \neq \varnothing$ OR

(ii) $P(A \cap B) = 0.25 \neq 0$ [.5].

(h) $P(\bar{B})(or P(B^c))$[.5] $= 1 - P(B)$[.5] $= 0.52$[.5].

Q2: [6] Credit Score and Default on Loans

(a) $P(C) = .60$[.5], $P(D|C) = .08$[.5], $P(D|C^c) = .25$[.5].



(b) **Law of Total Probability:**

$$P(D)[.5] = P(C \cap D) + P(C^c \cap D)[.5] = .(.60 \times .08) + (.40 \times .25)[.5] = .148[.5]$$

(If the tree diagram is provided, 1 mark of the intermediate steps will be based on tree diagram)

(c) **Revised Probability:**

$$P(C^c|D)[.5] = \frac{P(C^c \cap D)}{P(D)}[.5] = \frac{P(C^c \cap D)}{P(C^c \cap D) + P(C \cap D)}[.5] = \frac{.10}{.10 + .048}[.5] \approx .68[.5]$$

Q3: [11] Starbucks Coffee

(a) Binomial [0.5] probability distribution, with parameters $n = 10$ [0.5] and $p = 0.15$, [0.5].

(b) (i) Mean: np[0.5] $= 10 \times 0.15 = 1.5$ [0.5]

(ii) Standard Deviation: $\sqrt{npq}$[0.5] $= \sqrt{10 \times 0.15 \times 0.85} = 1.13$. [0.5]

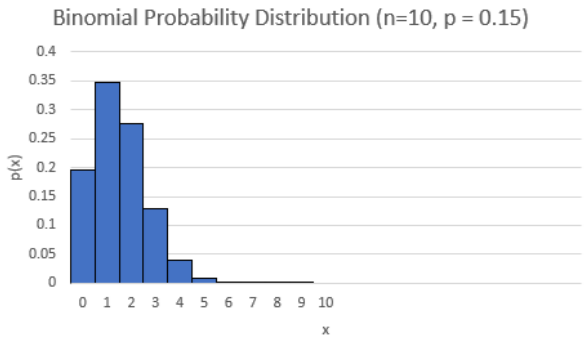
(c) Here, success = prefer Starbucks coffee, so $p = 0.15$.

$$P(X = 2)[0.5] = \binom{10}{2}(0.15)^2(0.85)^{10-2}[0.5]$$
$$\approx 0.28[0.5]$$

(d) **Binomial Probability Distribution: [1]**

x	p(x)
0	0.196874
1	0.347425
2	0.275897
3	0.129834
4	0.040096
5	0.008491
6	0.001249
7	0.000126
8	8.33E-06
9	3.27E-07
10	5.77E-09

Histogram: [0.5]



Shape: [0.5] Right or positively skewed distribution.

(e) Here, success = prefer Starbucks coffee, so $p = 0.15$.

$$\begin{aligned} P(1 \leq X < 3)[0.5] &= P(X = 1) + P(X = 2)[0.5] \\ &= \binom{10}{1}(0.15)^1(0.85)^{10-1} + \binom{10}{2}(0.15)^2(0.85)^{10-2}[0.5] \\ &\approx 0.62[0.5] \end{aligned}$$

(Students may use the EXCEL output to answer this question, so they should be awarded full marks in this case)

(f) Here, success = not preferring Starbucks coffee, so $p = 0.85$.

$$\begin{aligned} P(X > 7)[0.5] &= P(X = 8) + P(X = 9) + P(X = 10)[0.5] \\ &= \binom{10}{8}(0.85)^8(0.15)^{10-8} + \binom{10}{9}(0.85)^9(0.15)^{10-9} \\ &+ \binom{10}{10}(0.85)^{10}(0.15)^{10-10}[0.5] \\ &= 0.28 + 0.35 + 0.20 \\ &\approx 0.83[0.5] \end{aligned}$$

(Students may use the EXCEL output to answer this question, so they should be awarded full marks in this case)

Q4: [11] Pizza Hut

(a) Poisson Distribution [0.5] with parameter, average number of pizza orders Pizza Hut receives during 30-minute period in Friday night, $\mu = 8/30\text{-minute}$. [0.5]

(b) $\mu = 8/30\text{-minute}$.

$P(X = 5)$[0.5] $= \dfrac{e^{-8}8^5}{5!}$[0.5] $= 0.09$. [0.5]

(c) Duration $= 7$ minutes, Adjusted $\mu = \dfrac{8 \times 7}{30} = 1.87/7\text{-minute}$[0.5]

$$\begin{aligned} P(X \geq 4)[0.5] &= 1 - P(X < 4)[0.5] \\ &= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)][0.5] \\ &= 1 - 0.88 \\ &\approx 0.12.[0.5]. \end{aligned}$$

(d) Duration $= 15$ minutes, Adjusted $\mu = \dfrac{8 \times 15}{30}$[0.5] $= 4/15\text{-minute}$[0.5]

$$\begin{aligned} P(X < 3)[0.5] &= P(X \leq 2)[0.5] \\ &= P(X = 0) + P(X = 1) + P(X = 2)[0.5] \\ &\approx 0.24.[0.5] \end{aligned}$$

[Note: If the original μ is used instead of adjusted μ, only 1.5 mark is awarded (out of 3) for the first two probability statements if they are correct.]

(e) EXCEL POISSON FUNCTIONS and PROBABILITIES:

(b) $P(X = 5)$ = POISSON.DIST(5,8,FALSE)[0.5] $= 0.0916$. [0.5]

(c) $P(X \geq 4) = 1 - P(X < 4) = 1 -$ POISSON.DIST(3,1.87,TRUE)[0.5] $=$ 0.12021. [0.5]

(d) $P(X < 3) = P(X \leq 2) =$ POISSON.DIST(2,4,TRUE)[0.5] $= 0.2381$. [0.5]