Chapter 4: Probability and Probability Models

STAT 2601 – Business Statistics

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What is Probability?

Probability: A numerical measure of the likelihood or chance that an event will occur.

- Ranges from 0 (impossible) to 1 (certain)
- Can also be expressed as percentages (0% to 100%)

Two Main Approaches to Assigning Probabilities:

1. Classical/Theoretical Probability

$$P(A) = \frac{\text{Number of outcomes favorable to } A}{\text{Total number of equally likely outcomes in } S}$$

Assumptions:

- All outcomes are equally likely
- Sample space is finite

Examples:

- $P(\text{Head when flipping a fair coin}) = \frac{1}{2} = 0.5$
- $P(\text{Rolling a 5 on a fair die}) = \frac{1}{6} \approx 0.1667$
- $P(\text{Drawing an Ace from a standard deck}) = \frac{4}{52} = \frac{1}{13} \approx 0.0769$

What is Probability? (Continued)

2. Relative Frequency/Empirical Probability

$$P(A) = \lim_{n\to\infty} \frac{f}{n} \quad \text{or practically:} \quad P(A) = \frac{\text{Observed frequency of } A}{\text{Total number of trials}}$$

Based on:

- Historical data or experimental observations
- The **Law of Large Numbers**: As n increases, relative frequency stabilizes around true probability

Examples:

- **Quality Control:** 47 out of 500 widgets are defective

$$P(\text{Defective}) = \frac{47}{500} = 0.094 \quad (9.4\%)$$

- **Customer Behavior:** 320 out of 1000 customers clicked an email

$$P(\text{Email Click}) = \frac{320}{1000} = 0.32 \quad (32\%)$$

- **Loan Default:** 12 out of 1500 loans defaulted last year

$$P(\text{Default}) = \frac{12}{1500} = 0.008 \quad (0.8\%)$$

Basic Concepts

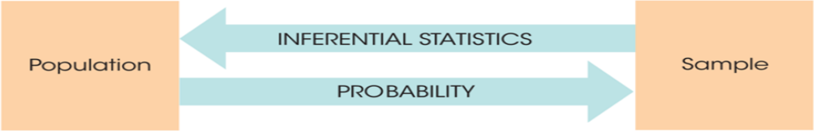


Figure: Relationships between samples and populations are defined in terms of probability

- **Experiment:** A process that yields a single outcome from all possible outcomes.
  Examples: Toss a coin, measure daily sales, test a product, record an opinion (yes, no), survey one customer about satisfaction.
- **Sample Space (S):** The complete set of all possible outcomes
  - Must be **exhaustive** (covers all possibilities)
  - Outcomes must be **mutually exclusive** (no overlap)

Sample Space Examples

- **Coin Flip:** $S = \{\text{Heads}, \text{Tails}\}$
- **Customer Satisfaction Survey:**
  $S = \{\text{Very Dissatisfied}, \text{Dissatisfied}, \text{Neutral}, \text{Satisfied}, \text{Very Satisfied}\}$
- **Product Quality Test:** $S = \{\text{Defective}, \text{Non-Defective}\}$
- **Two Dice Roll and Observe the Total Outcome:** $S = \{2, 3, 4, \ldots, 12\}$

Events: Simple and Compound

Simple Event

A **single** outcome from the sample space, denoted by E with a subscript.

- **Example 1:** Rolling a 4 on a die: $\{4\}$
- **Example 2:** Customer rating "Very Satisfied"
- **Example 3:** Stock price increases exactly 2%

Compound Event (Event)

Combination of **two or more** simple events.

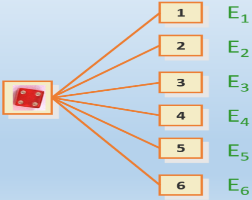
- **Example 1:** Rolling an even number: $\{2, 4, 6\}$
- **Example 2:** Customer rating "Satisfied or Very Satisfied"
- **Example 3:** Stock price increases by 1-3%: $\{1\%, 2\%, 3\%\}$

Important Distinction

- A **simple event** is an element of the sample space, which cannot be broken down further.
- An **Event** is any subset of the sample space S that may contain one or more elements and can be described using set operations such as union and intersection.

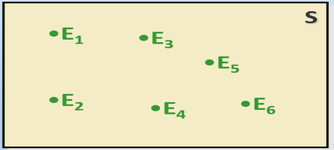
Example

- The die toss:
- Simple events:

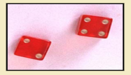


Sample space:

$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$



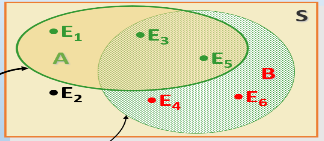
$S = \{1, 2, 3, 4, 5, 6\}$



- An **event** is a collection of one or more **simple events.**

- **The die toss:**
  - A: an odd number
  - B: a number > 2

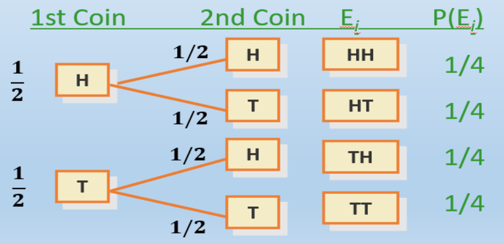
$A = \{E_1, E_3, E_5\} = \{1,3,5\}$



$B = \{E_3, E_4, E_5, E_6\} = \{3,4,5,6\}$

Probability and Events

Probability Tree: An effective and simpler method of applying the probability rules is the probability tree, wherein the events in an experiment are represented by lines.
Example: Toss a fair coin twice and draw the sample space.



(i) **Probability of <u>one H and one T</u>**

$$P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(ii) **Probability of <u>two T</u>**

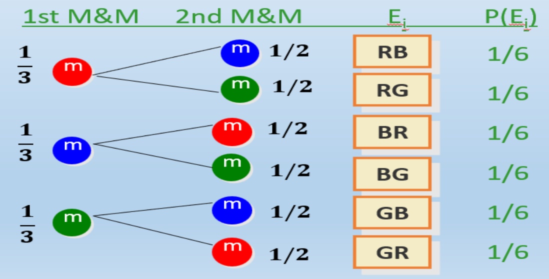
$$P(TT) = \frac{1}{4}$$

(iii) **Probability of <u>at least one H</u>**

$$P(HT) + P(TH) + P(HH) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Probability and Events

- Example: A bowl contains three M&Ms: one red, one blue and one green. A child selects (without replacement) two M&Ms at random.



(i) **Probability of <u>no</u> R**

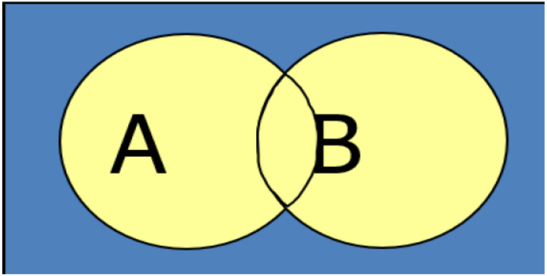
$$P(BG) + P(GB) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

(ii) **Probability of <u>at least one R</u>**

$$P(RB) + P(RG) + P(BR) + P(GR) = \frac{4}{6} = \frac{2}{3}$$

Venn Diagrams and Set Notation

Event Relationships: $A \cup B$ **(Union: Either A or B or Both)**



Example: Suppose we are throwing two fair dices.
Let event A comprise the tosses where the first outcome is 1:

$$A = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\}$$

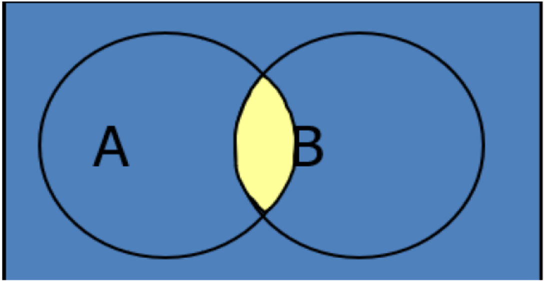
and let B tosses where the second outcome is 5:

$$B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

$$A \cup B = \{(1,1),(1,2),(1,3),(1,4),{\color{red}(1,5)},(1,6),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

Venn Diagrams and Set Notation

Event Relationships: $A \cap B$ **(Intersection: Joint Event)**



Example: Suppose we are throwing two fair dices.
Let event A comprise the tosses where the first outcome is 1:

$$A = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\}$$

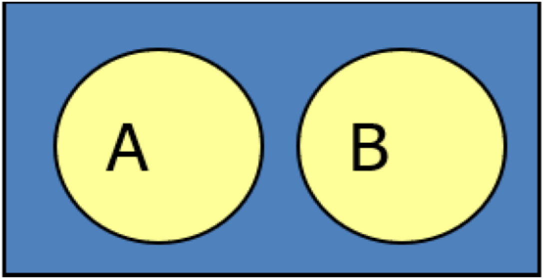
and let B tosses where the second outcome is 5:

$$B = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\}$$

$$A \cap B = \{(1,5)\} \Rightarrow A \cup B = A + B - A \cap B$$

Venn Diagrams and Set Notation

Event Relationships: $A \cup B$ **(Union: Either A or B) (Mutually Exclusive Event)**



Example: Suppose we are throwing two fair dices.
Let event A comprise the tosses where the outcomes are odd numbers:

$$A = \{1, 3, 5\}$$

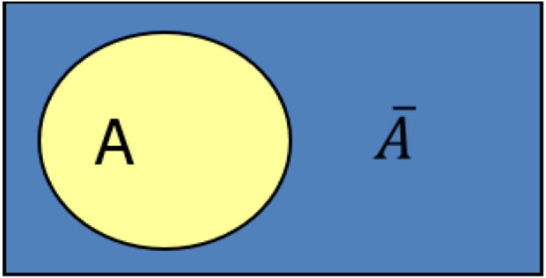
and B the tosses where the outcomes are event numbers:

$$B = \{2, 4, 6\}$$

$$A \cap B = \varnothing \Rightarrow A \cup B = A + B = \{1, 2, 3, 4, 5, 6\}$$

Venn Diagrams and Set Notation

Event Relationships: $\bar{A} = A^c$ **(Not A = A complement)**



Example: Suppose we are a single fair dice. Let event A comprise the tosses where the outcomes are odd numbers:

$$A = \{1, 3, 5\} \Rightarrow A^c = \{2, 4, 6\}$$

Probability Rules

Rule 1: Boundaries of Probability

For any event A,

$$0 \leq P(A) \leq 1$$

Explanation:

- Probability cannot be negative
- Probability cannot exceed 1 (a certain event)

Example:
Roll a fair six-sided die; let $A =$ event "roll a 4".

$$P(A) = \frac{1}{6} \quad \text{so} \quad 0 \leq \frac{1}{6} \leq 1$$

Another event $B =$ "roll a 10" (impossible)

$$P(B) = 0$$

Probability Rules

Rule 2: Probability of $\varnothing$ and S

$$P(\varnothing) = 0 \qquad \text{and} \qquad P(S) = 1$$

Explanation:

- $\varnothing$ is the empty event — cannot occur
- S is the sample space — one of the outcomes must occur

Example:
Experiment: Flip a fair coin.
$S = \{\text{Heads, Tails}\}$ so

$$P(S) = 1$$

An event "get a side that is both Heads and Tails simultaneously"

$$P(\varnothing) = P(\{\}) = 0$$

Probability Rules

Rule 3: Rule of the Complement

For any event A,

$$P(A^c) = 1 - P(A)$$

Explanation:

- A^c is the set of all outcomes in S that are not in A (i.e., event A does not occur)
- Either A occurs or A^c occurs — these are complementary

Example:

Roll a die. Let $A =$ "roll an even number"

$$A = \{2, 4, 6\} \quad \Rightarrow \quad P(A) = \frac{3}{6} = \frac{1}{2}$$

The complement $A^c =$ "roll an odd number"

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

Rule 4: Addition Rule (Union)

General Addition Rule

For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Explanation:

- $\cup$ (union): Either A or B or both occur
- Subtract $P(A \cap B)$ to avoid double-counting

Special Case: Mutually Exclusive Events

If A and B are **mutually exclusive (disjoint)**, then

$$P(A \cap B) = 0 \quad \Rightarrow \quad P(A \cup B) = P(A) + P(B)$$

Example 1: Addition Rule (Mutually Exclusive)

A company records the payment status of an invoice as Paid on time, Paid late, and Not paid. Define the events:

$$A = \text{invoice is paid on time,} \quad \text{where} \quad P(A) = 0.65$$
$$B = \text{invoice is paid late,} \quad \text{where} \quad P(B) = 0.2$$

An invoice cannot be paid both on time and late, so $A \cap B = \varnothing$.
Addition Rule (Mutually Exclusive):

$$P(A \cup B) = P(A) + P(B) = 0.65 + 0.20 = 0.85$$

There is an 85% probability that an invoice is paid either on time or late.

Example 2: Addition Rule (Not Mutually Exclusive)

Roll a die; let

$$A = \{2, 4, 6\}, \quad B = \geq 3 = \{3, 4, 5, 6\}$$
$$P(A) = \tfrac{1}{2}, \quad P(B) = \tfrac{4}{6} = \tfrac{2}{3}, \quad A \cap B = \{4, 6\}$$
$$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

Apply the rule:

$$P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$$

Contingency Table (Two-Way Table) and Probability

Contingency Table and Probability: Events may be placed in a $r \times c$ contingency table.

Example

Responses of 478 customers are placed in the following table:

		Prize Preferences		
Gender	**Skis**	**Camera**	**Bike**	**Total**
Man	117	50	60	227
Woman	130	91	30	251
Total	247	141	90	478

Example (Cont.)

Marginal Probability: Depends only on totals found in the margins of the table.

$$P(\text{woman}) = \frac{251}{478} = 0.525$$

Complement Probability:

$$P(\text{woman}^c) = 1 - 0.525 = 0.475$$

Joint Probability: Gives the probability of two events occurring together.

$$P(\text{woman} \cap \text{camera}) = \frac{91}{478} = 0.190$$

$$P(\text{woman} \cap \text{camera}^c) = \frac{130 + 30}{478} = 0.33$$

Joint Probability: Gives the probability of two events occurring together.

$$P(\text{woman}^c \cap \text{camera}) = \frac{50}{478} = 0.10$$

$$P(\text{woman}^c \cap \text{camera}^c) = \frac{117 + 60}{478} = 0.37$$

Addition Probability:

$$P(\text{woman} \cup \text{camera}) = P(\text{woman}) + P(\text{camera}) - P(\text{woman} \cap \text{camera})$$
$$= \frac{251}{478} + \frac{141}{478} - \frac{91}{478} = 0.63$$

Example: Contingency Table and Probability

National's cable has 38 million cable passings. Let us consider National cable's two services viz. **cable television service (A)** and **cable internet service (B)**. 10.9 million has only **cable television service** and 10.1 million has only **cable internet service**, while 8.2 million has both services. **Contingency Table:**

Create a 2×2 contingency table considering cable television service (A) in the row position and cable internet service in the column position (B).

Events	**Internet Service**, B	**No Internet Service**, B^c	**Total**
Television Service, A	8.2	10.9	19.1
No Television Service, A^c	10.1	8.8	18.9
Total	18.3	19.7	38

Contingency Table and Probability

Addition Probability: What is the probability that a randomly selected cable passing has either cable television service or cable internet service?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{19.1}{38} + \frac{18.3}{38} - \frac{8.2}{38} \approx 0.77$$

Joint Probability: What is the probability that a randomly selected cable passing does not have National's cable television service and does not have National's cable internet service?

$$P(A^c \cap B^c) = \frac{8.8}{38} = 0.23$$

Conditional Probability

- Conditional Probability: The probability of an event given that another event has occurred is called a conditional probability.

$$P(\underbrace{A}_{\text{want}} | \underbrace{B}_{\text{already occurred}}) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) > 0$$

$$P(\underbrace{B}_{\text{want}} | \underbrace{A}_{\text{already occurred}}) = \frac{P(A \cap B)}{P(A)}, \text{ where } P(A) > 0$$

- Vertical line "|" between two events means given or provided.
- Event lying to the right of the vertical line "|" is the event which has "already occurred". Event lying to the left of the vertical line "|" is the "event of interest for which calculation is required".

Example

A company surveys its employees and records:

- Whether an employee works full-time
- Whether an employee works remotely at least one day per week

Define the events:

$$A = \text{employee works remotely} \quad \text{and} \quad B = \text{employee works full-time}$$

Suppose the following probabilities are known:

$$P(A) = 0.40, \quad P(B) = 0.70, \quad P(A \cap B) = 0.25$$

Question: What is the probability that an employee works remotely *given* that the employee works full-time?

Solution

We are asked to find $P(A \mid B)$. Using the conditional probability formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Substitute the given values:

$$P(A \mid B) = \frac{0.25}{0.70} \approx 0.36$$

Given that an employee works full-time, there is approximately a 36% chance that the employee also works remotely at least one day per week.

Example: MBA Graduate and Mutual Fund

Why are some mutual fund managers more successful than others? One possible factor is where the manager earned his or her MBA.

What is the probability that a fund will outperform the market given that the manager graduated from a top-20 MBA program? (Condition Explicit)

	B	B^c	**Total**
A	0.11	0.29	0.4
A^c	0.06	0.54	0.6
Total	0.17	0.83	1.0

Hers,

- $A =$ Fund manager graduated from a top-20 MBA program
- $A^c =$ Fund manager did not graduate from a top-20 MBA program
- $B =$ Fund outperforms the market
- $B^c =$ Fund does not outperform the market

Calculation:

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.11}{0.40} = 0.275$$

Conditional Probability and Independence

- **Independence:** One of the objectives of calculating conditional probability is to determine whether two events are related. In particular, we would like to know whether they are independent, that is, if the probability of one event is not affected by the occurrence of the other event.
- Two events A and B are said to be independent if one of the following is satisfied:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A) \times P(B)$

Example (MBA Graduate and Mutual Fund):

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.11}{0.40} = 0.275$$

$$P(B) = 0.17$$

Since $P(B|A) \neq P(B)$, the events A and B are NOT independent.

Rule 5: Multiplication Rule

The multiplication rule is used to compute the probability of the **joint occurrence** of A and B.

- $P(A \cap B) = P(A|B) \times P(B)$
- $P(A \cap B) = P(B|A) \times P(A)$
- $P(A \cap B) = P(A) \times P(B)$ (Only if A and B are independent events)

Additional Rules: Not explicitly mentioned in the textbook.

- $P(B) = P(A \cap B) + P(A \cap B^c)$
- $P(A) = P(A \cap B) + P(A^c \cap B)$
- $P(A \cap B) = 1 - P(A \cap B)^c = 1 - P(A^c \cup B^c)$
- $P(A \cup B) = 1 - P(A \cup B)^c = 1 - P(A^c \cap B^c)$
- $P(A^c|B) + P(A|B) = 1$
- $P(B^c|A) + P(B|A) = 1$

Example

In a company:

- 20% of employees work in the IT department
- 30% of employees hold a professional certification
- 50% of IT employees hold a professional certification

(Note: The condition is **implicit**; there is no explicit use of the word "given.")
Define the events:

$$I = \text{employee works in IT}$$
$$C = \text{employee holds a professional certification}$$

Questions:

1. Find the proportion of employees who work in IT and hold a certification.
2. Find the proportion of certified employees who work in IT.
3. Are the events "working in IT" and "holding a certification" independent?

Example

Given information:

$$P(I) = 0.20, \quad P(C) = 0.3, \quad P(C \mid I) = 0.5$$

(i) Proportion of employees who work in IT and hold a certification

$$P(I \cap C) = P(C \mid I)P(I) = (0.5)(0.2) = 0.1$$

Answer: 10% of employees work in IT and hold a certification.

(ii) Proportion of certified employees who work in IT

$$P(I \mid C) = \frac{P(I \cap C)}{P(C)} = \frac{0.10}{0.30} = 0.33$$

Answer: Approximately 33% of certified employees work in IT.

(iii) Independence of Events

Two events are independent if:

$$P(I \mid C) = P(I)$$

We found:

$$P(I \mid C) = 0.33 \quad \text{and} \quad P(I) = 0.20$$

Since:

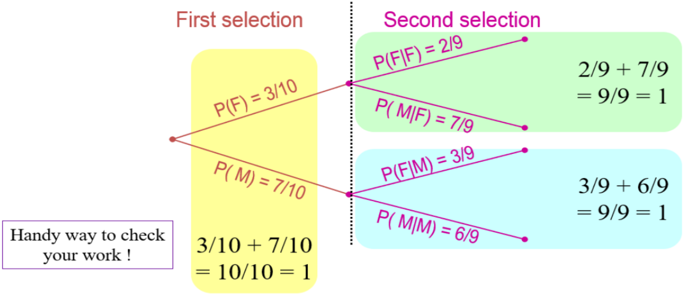
$$P(I \mid C) \neq P(I),$$

Conclusion: The events "working in IT" and "holding a professional certification" are **not independent**.

Probability Trees

- A probability tree provides a simple and effective way to apply probability rules by representing events as branching lines that form a tree-like diagram.
- **Structure:**
  - **First Branch:** Marginal Probability.
  - **Second Branch:** Conditional Probability.
  - **Last Branch:** Joint Probability (which is the product of first and second branches).
- **Rule:** The probabilities associated with any set of branches from one "node" must add up to 1.

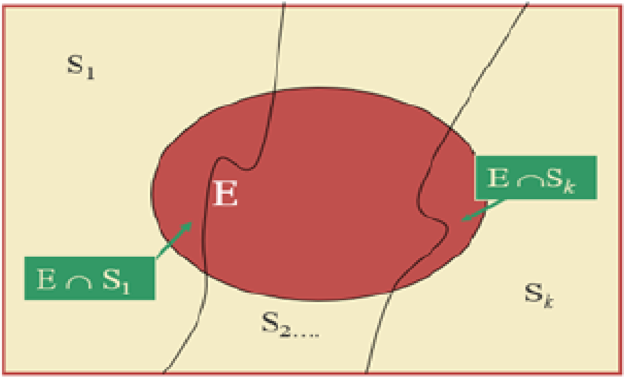
Probability Tree (Example): A graduate statistics course has seven male and three female students. The professor wants to select two students at random to help her conduct a research project. What is the probability that the two students chosen (selection without replacement) are female?



The Law of Total Probability

Let $S_1, S_2, S_3, \cdots, S_k$ be **mutually exclusive and exhaustive** events (that is, one and only one must happen and $P(S_i \cap S_j) = 0$ for all $i \neq j$).
Then the Total probability of a given event E can be written as:

$$\begin{aligned} P(E) &= P(E \cap S_1) + P(E \cap S_2) + \cdots + P(E \cap S_k) \\ &= P(S_1)P(E \mid S_1) + P(S_2)P(E \mid S_2) + \cdots + P(S_k)P(E \mid S_k) \end{aligned}$$



Bayes' Rule

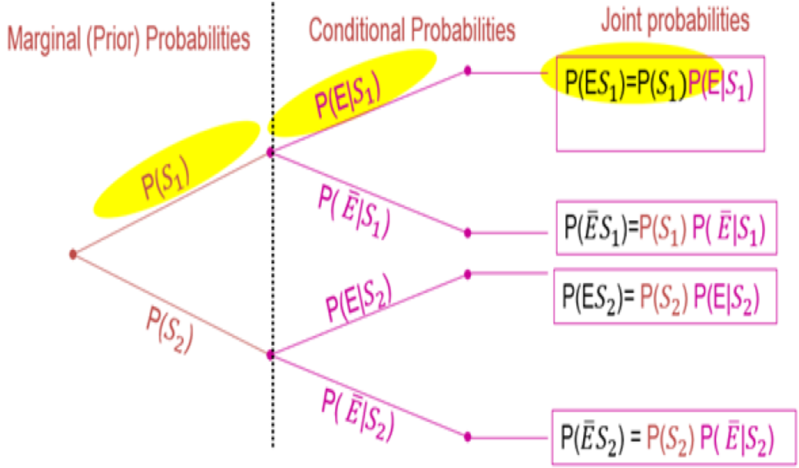
Let $S_1, S_2, S_3, \ldots, S_k$ be **mutually exclusive and exhaustive** events with **prior probabilities** $p(S_1), p(S_2), p(S_3), \cdots, p(S_k)$ ($P(S_i \cap S_j) = 0$ for all $i \neq j$). If an event E occurs, the **posterior probability** of S_i, given that E occurred, is

$$
\begin{aligned}
P(S_i|E) &= \frac{P(E|S_i)P(S_i)}{P(E)} \\
&= \frac{P(E|S_i)P(S_i)}{\sum_{j=1}^{k} P(E|S_j)P(S_j)}
\end{aligned}
$$

This is the classic **law of total probability combined with Bayes' Theorem**, where:

- $P(S_i)$: **prior probability**
- $P(E|S_i)$: **Likelihood**
- $P(S_i|E)$: **Posterior probability**

Tree diagram to conceptualize Bayes' Theorem



Bayes' Theorem Example: Heart Attack Risk by Gender

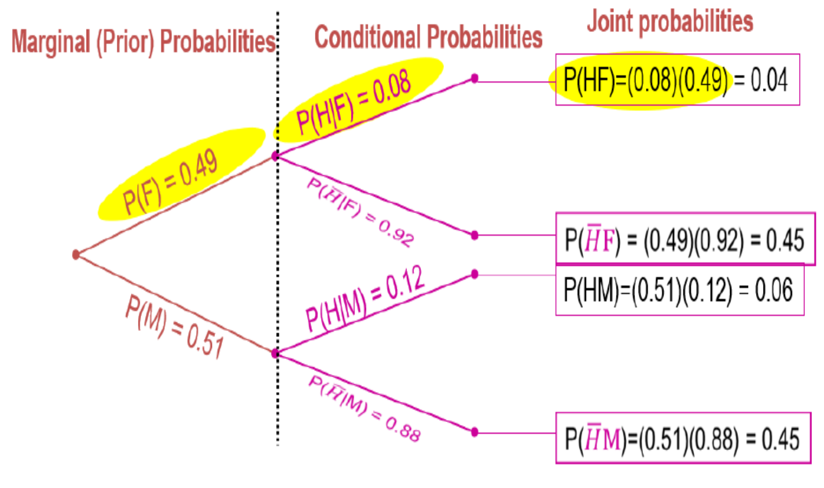
49% of the population are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. A single person is selected at random and found to be high risk. What is the probability that it is a male?

- **Prior Events:** F (Female), M (Male).
- **Prior Probabilities:** $P(F) = 0.49$, $P(M) = 0.51$ (first branch: given).
- **New Event:** H (Heart Attack)
- **Conditional Probabilities:** $P(H|F) = 0.08$, $P(H|M) = 0.12$. (second branch: given)
- **Joint Probabilities:** $P(HF)$, $P(H^cF)$, $P(HM)$, $P(H^cM)$ (third branch)
- **Revised Probabilities:**

$$P(M|H) = \frac{P(H \cap M)}{P(H)} = \frac{P(H \cap M)}{P(H \cap M) + P(H \cap F)} = \frac{0.06}{0.06 + 0.04} = 0.6$$

Example: Heart Attack Risk by Gender (Cont.)

49% of the population are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. A single person is selected at random and found to be high risk. What is the probability that it is a male?



Bayes' Theorem Example: Quality Control in Manufacturing

A company has three production lines (S1, S2, S3) with different defect rates:

- Line S1 produces 70% of output, defect rate: 4%
- Line S2 produces 20% of output, defect rate: 2%
- Line S3 produces 10% of output, defect rate: 96%

If a randomly selected product is defective, what's the probability it came from Line S1?

Solution: Let the events: S_1, S_2, S_3 = product from each line and H = defective rate.

Given:

$$\begin{aligned} P(S_1) &= 0.7, & P(H|S_1) &= 0.04 \\ P(S_2) &= 0.2, & P(H|S_2) &= 0.02 \\ P(S_3) &= 0.1, & P(H|S_3) &= 0.96 \end{aligned}$$

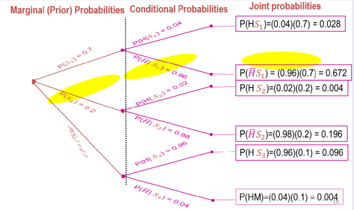
Joint probabilities:

$$P(H \cap S_1) = (0.04)(0.7) = 0.028$$
$$P(H \cap S_2) = (0.02)(0.2) = 0.004$$
$$P(H \cap S_3) = (0.96)(0.1) = 0.096$$

Total probability:

$$P(H) = 0.028 + 0.004 + 0.096 = 0.128$$

Bayes' Theorem



$$P(S_1|H) = \frac{P(H \cap S_1)}{P(H)} = \frac{0.028}{0.128} \approx 0.219$$

A defective product has about 21.9% chance of coming from S1 (down from 70% prior probability). Line S3, despite producing only 10% of output, causes most defects due to its high 96% defect rate.

Probability Formulas Summary

Basic Rules

- $0 \leq P(A) \leq 1$
- $P(S) = 1, P(\varnothing) = 0$
- $P(A^c) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Mutually Exclusive:** $P(A \cup B) = P(A) + P(B)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$

Conditional & Independence

- **Conditional:** $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- **Multiplication:** $P(A \cap B) = P(A|B)P(B)$
- **Independent Events:** $P(A \cap B) = P(A)P(B)$
- **Tests for Independence:**
  - $P(A|B) = P(A)$
  - $P(B|A) = P(B)$
  - $P(A \cap B) = P(A)P(B)$

Bayes' Theorem

$$P(S_i|E) = \frac{P(E|S_i)P(S_i)}{P(E)}$$

$$= \frac{P(E|S_i)P(S_i)}{\sum_{j=1}^{k} P(E|S_j)P(S_j)}$$

- $P(S_i)$: Prior probability
- $P(E|S_i)$: Likelihood
- $P(S_i|E)$: Posterior probability

Set Relations

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$
- $P(A) = P(A \cap B) + P(A \cap B^c)$

Law of Total Probability

For $S_1, S_2, \ldots, S_k$ mutually exclusive and exhaustive:

$$P(E) = \sum_{i=1}^{k} P(E|S_i)P(S_i)$$

Key Notes

- Independence $\neq$ mutually exclusive
- $\sum P(S_i) = 1$ for any partition
- Always check: $0 \leq P(A) \leq 1$