Digit Recognition

Problem and Data

Digit Recognition

We'll consider a simple digit recognition problem

- Given an image representing a digit, we need to recognition the represented number
- This is an important building block for most OCR systems

The Dataset

We will use the classic [MNIST Digit Recognition Dataset] dataset

The dataset contains hand-written digits

- The original data was obtain from US Census Bureau and high-schools students
- Each digit is represented as a 28×28 greyscale image

In [22]:
```
from keras.datasets import mnist
# load the data, shuffled and split between train and test sets
(x_train, y_train), (x_test, y_test) = mnist.load_data()
```

The MNIST data is now stored in pairs of numpy arrays.

- The `x_train` and `x_test` arrays contain the greyscale value of each pixel
- The `y_train` and `y_test` arrays contain the class (digit) as an integer

Image Data

Let's inspect the output

In [23]:
```
print(f'Shape of y_train: {y_train.shape}')
print(f'Shape of y_test: {y_test.shape}')
n_tr = y_train.shape[0]
n_ts = y_test.shape[0]
```

```
Shape of y_train: (60000,)
Shape of y_test: (10000,)
```

- There are 60,000 training examples
- ...And 10,000 test examples

The target arrays are one-dimensional

Let's check a sample:

In [24]:
```
y_train
```

Image Data

Let's inspect the input

```
In [25]: print(f'Shape of x_train: {x_train.shape}')
         print(f'Shape of x_test: {x_test.shape}')
         x_h = x_train.shape[1]
         x_w = x_train.shape[2]
```

```
Shape of x_train: (60000, 28, 28)
Shape of x_test: (10000, 28, 28)
```

- The dataset input consists of 28x28 matrices

```
In [26]: print(f'Minimum: {x_train.min()} (train), {x_train.min()} (test)')
         print(f'Maximum: {x_train.max()} (train), {x_train.max()} (test)')
```

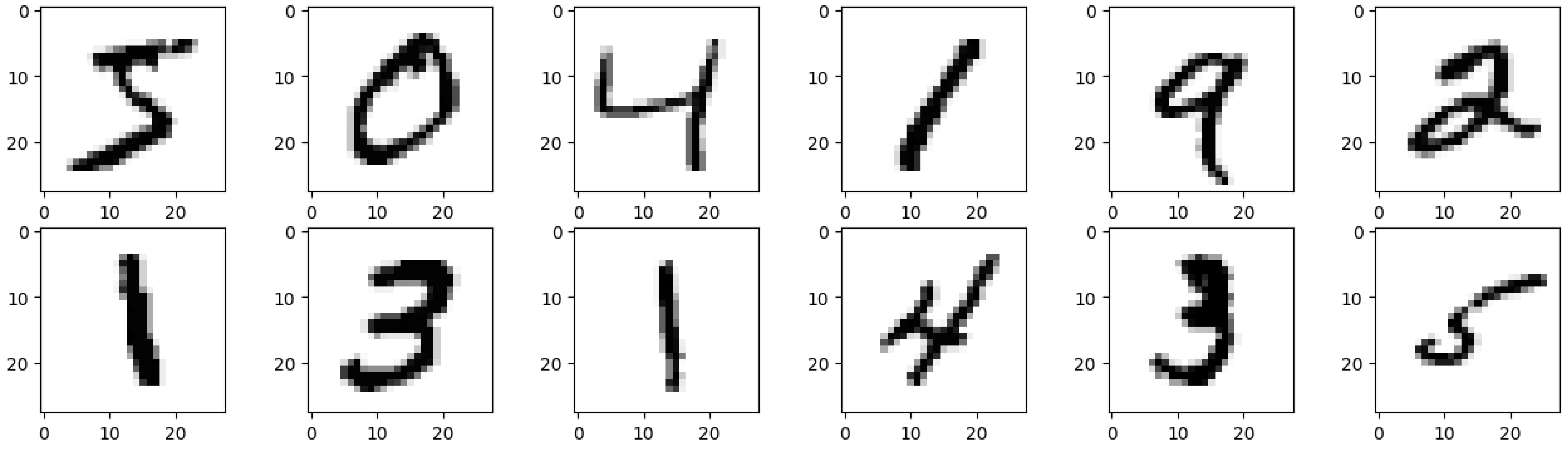
```
Minimum: 0 (train), 0 (test)
Maximum: 255 (train), 255 (test)
```

- The content of the matrix cells ranges from 0 to 255

Image Data

Let's see some sample images

In [27]:
```
m, n = 2, 6
plt.figure(figsize=figsize)
for i in range(m):
    for j in range(n):
        plt.subplot(m, n, i*n + j + 1)
        plt.imshow(x_train[i*n + j], cmap='Greys')
plt.show()
```



Digit Recognition

A Solution Approach

Problem Model

This is a standard classification problem

...But it still best modeled in a probabilistic fashion

- We can view the image as a random (vector) variable X
- ...And the class a second random variable Y, with values in $\{c_1, c_2, \ldots c_n\}$

This the case for multiple reasons (e.g. labeling errors or ambiguous interpretation)

The two variables are correlated, which is captured via their joint distribution

$$X, Y \sim P(X, Y)$$

...But in practice, we are assuming X is observed, so we care about the conditional distribution:

$$P(Y \mid X)$$

Problem Model

We will approximate the $P(Y \mid X)$ via a parameterized function $\hat{f}(y, x; \theta)$

...Which we can train via Maximum Likelihood Estimation

- Given a training dataset $\{x_i, y_i\}_{i=1}^m$, we solve:

$$\operatorname*{argmax}_{\theta} \prod_{i=1}^{m} \sum_{j=1}^{n} [\![y_i = c_j]\!] \hat{f}(c_j, x; \theta)$$

- Where $[\![y_i = c_j]\!] = 1$ iff $y_i = c_j$
- ...And $\hat{f}(c_j, x; \theta)$ is the estimate probability of value c_j

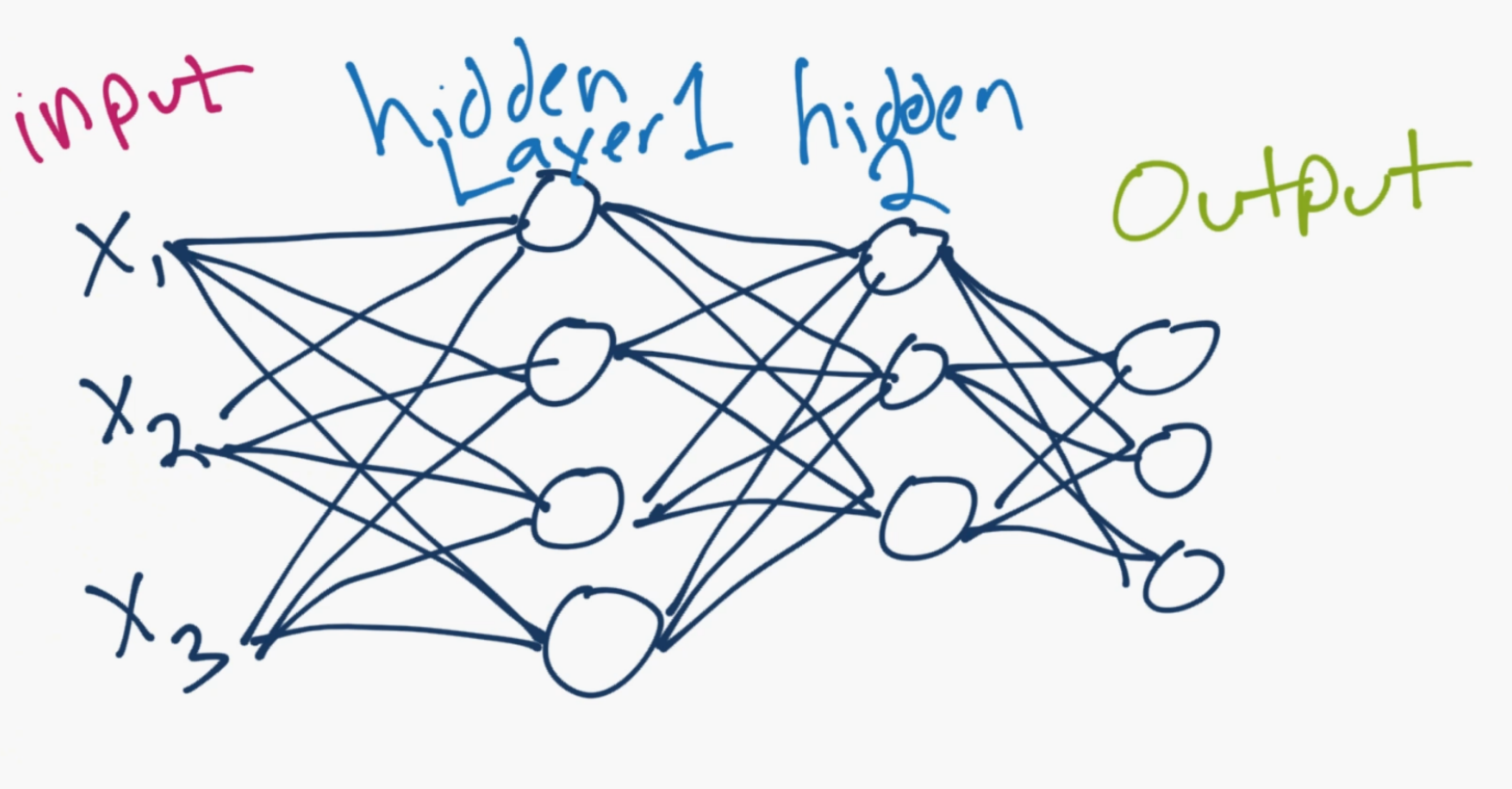
From the probability estimator, we can obtain a classifier

- We simply look for the value (i.e. the class) in $\{c_1, \ldots c_n\}$
- ...That is associated to the largest probability

Neural Network Classifiers

We will rely on a Neural Network for our approximate model

NNs that are used for classification have a typical structure



- Their output layer has one neuron for each possible class
- ...And it uses a softmax activation function to ensure the output sum up to 1

By doing this, the output can be interpreted a discrete probability distribution

Preprocessing

Before we can start training we need to do some preprocessing

We will apply a min-max encoding to the input

- ...Since there is clear minimum and maximum for each pixel

```
In [28]: x_train_norm = x_train / 255.0
         x_test_norm = x_test / 255.0
```

We will adopt a one-hot encoding for the output

- ...Since we will need to build a network with one neuron per class

```
In [29]: y_train_cat = keras.utils.to_categorical(y_train)
         y_test_cat = keras.utils.to_categorical(y_test)
         y_train_cat[:2, :]

Out[29]: array([[0., 0., 0., 0., 0., 1., 0., 0., 0., 0.],
                [1., 0., 0., 0., 0., 0., 0., 0., 0., 0.]], dtype=float32)
```

Adding Channel Information

When working with image data, one extra step is needed

...Since images are not necessarily greyscale!

- Greyscale images can be represented as matrices
- ...But color images have a value of red, green, and blue for each pixel!

For this reason, an image is best described by a tensor not a matrix

Even if we have a single channel, it will be best to convert each input to a tensor

In [31]:
```
x_train_c = x_train_norm.reshape(-1, x_h, x_w, 1)
x_test_c = x_test_norm.reshape(-1, x_h, x_w, 1)
input_shape = (x_h, x_w, 1)
output_shape = (10,)
print(f'New shape of the training set: {x_train_c.shape}')
```

```
New shape of the training set: (60000, 28, 28, 1)
```

Training a Baseline Model

As a baseline, we will build an MLP model

We will have a look at the code, without going much into detail

```
def build_mlp(input_shape, output_shape, hidden, rate=0.05):
    mdl = keras.Sequential()
    mdl.add(keras.Input(shape=input_shape))
    mdl.add(keras.layers.Flatten())
    for k, h in enumerate(hidden):
        mdl.add(Dense(h, activation='relu'))
        mdl.add(keras.layers.Dropout(rate))
    mdl.add(Dense(output_shape[0], activation='softmax'))
    return mdl
```

...Even if a classical MLP is not designed to handle images

- For this reason we'll start with a special `Flatten` layer

Training a Baseline Model

We can now train a 2-layer network as a baseline

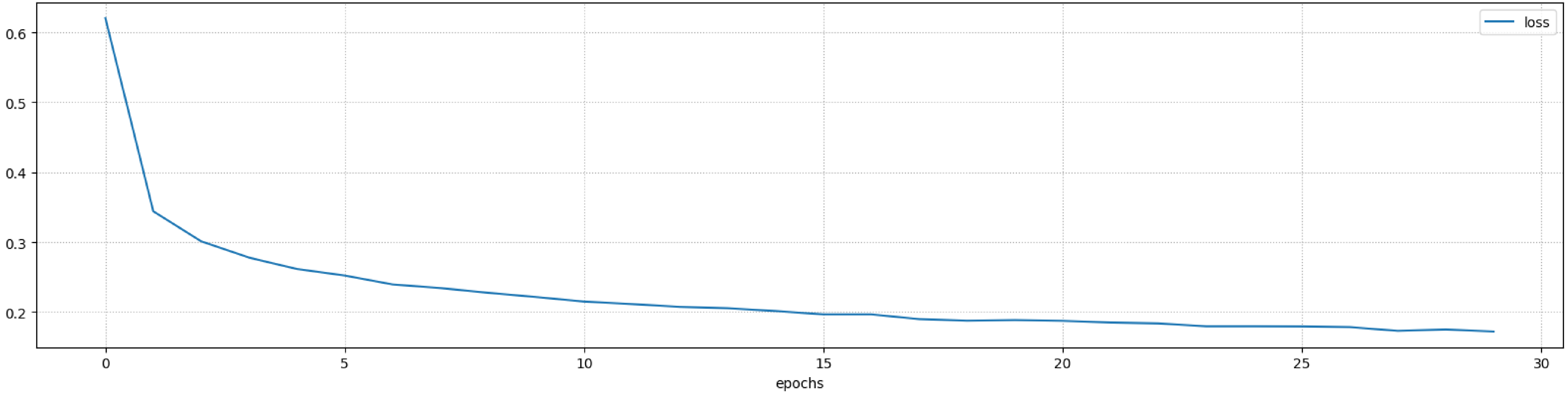
In [40]:
```python
nn1 = util.build_mlp(input_shape, output_shape, hidden=[16, 16])
history = util.train_nn(nn1, x_train_c, y_train_cat, batch_size=32, epochs=30, verbose
```

```
Epoch 1/30
1875/1875 [==============================] - 2s 862us/step - loss: 0.6205
Epoch 2/30
1875/1875 [==============================] - 2s 847us/step - loss: 0.3441
Epoch 3/30
1875/1875 [==============================] - 2s 864us/step - loss: 0.3011
Epoch 4/30
1875/1875 [==============================] - 2s 846us/step - loss: 0.2778
Epoch 5/30
1875/1875 [==============================] - 2s 857us/step - loss: 0.2615
Epoch 6/30
1875/1875 [==============================] - 2s 852us/step - loss: 0.2521
Epoch 7/30
1875/1875 [==============================] - 2s 848us/step - loss: 0.2393
Epoch 8/30
1875/1875 [==============================] - 2s 844us/step - loss: 0.2341
Epoch 9/30
1875/1875 [==============================] - 2s 860us/step - loss: 0.2274
```

Training a Baseline Model

Let's inspect the training curve

In [41]:
```
util.plot_training_history(history, figsize=figsize)
```




```
Final loss: 0.1721 (training)
```

There's still something to go before convergence, but we'll stop here

Evaluation

Now we can compute the model accuracy

In [45]:
```
nn1_p_tr = nn1.predict(x_train_c, verbose=0).argmax(axis=1)
nn1_p_ts = nn1.predict(x_test_c, verbose=0).argmax(axis=1)

nn1_acc_tr = accuracy_score(y_train, nn1_p_tr)
nn1_acc_ts = accuracy_score(y_test, nn1_p_ts)

print(f'2-layer network accuracy: {nn1_acc_tr:.3f} (train), {nn1_acc_ts:.3f} (test)')
```

```
2-layer network accuracy: 0.968 (train), 0.952 (test)
```

We are doing already pretty well!

What can we do to improve the results?

Beyond "stacking more layers" the answer is not clear

Digit Recognition

Convolutional Neural Networks

Exploiting Structural Information

DNs are very flexible learning models

- ...Since we can choose both how many layer to use
- ...And how big they should be

However, it's difficult to develop an intuition of which options work

- This is due to the poor interpretability of DNs
- ...To the point that a fully fledged research field focuses on automatic tuning

There is one type of choice that is intuitive and has a big impact

...This concerns the idea of exploiting structural information

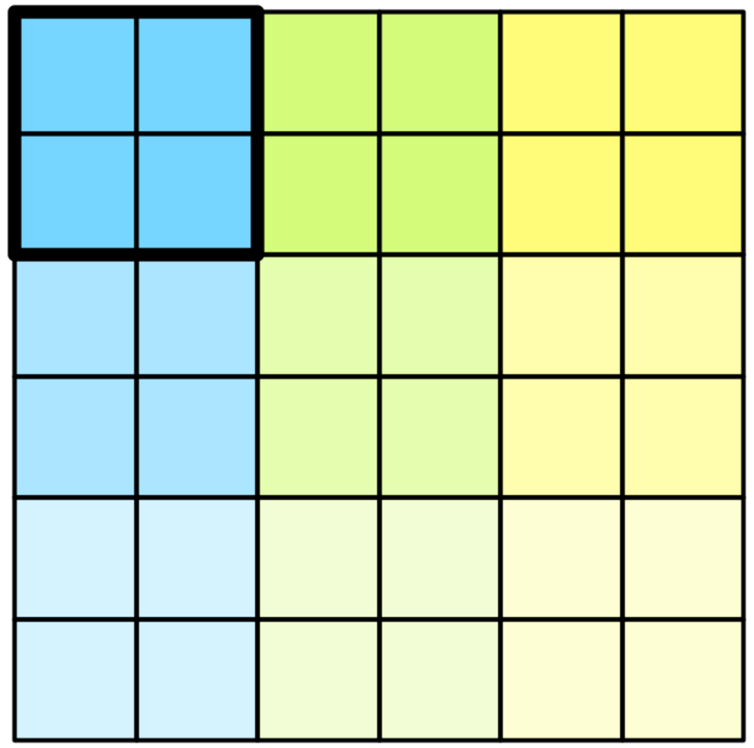
- For example, nearby pixels in an image may be semantically linked
- ...And the same goes for nearby points in time
- ...Or nearby words in a sentence

Convolutional Layers

This idea is at the basis of convolutional layers

A 2D convolution layer...

- Starts from an input tensor with shape (m, n, c)
- ...And slides a linear n_f, m_f filter (or kernel) on top of the image, with a certain step size (stride)



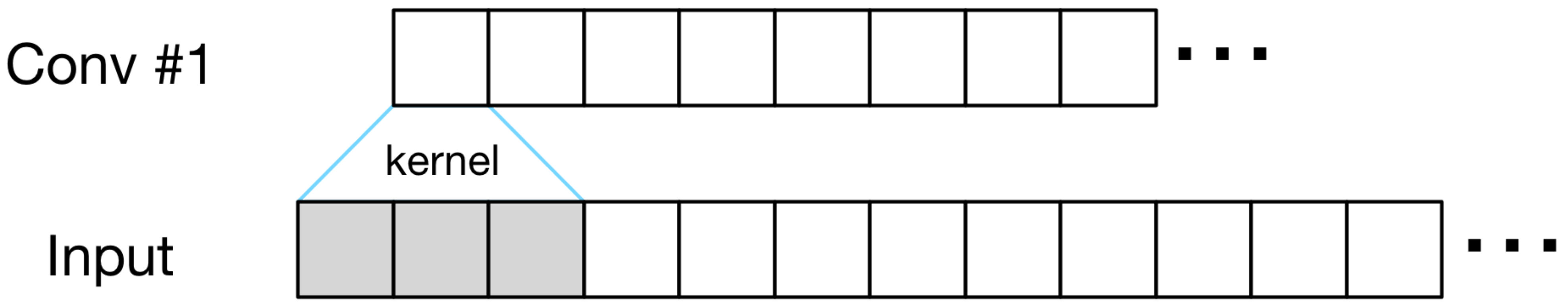
- You can think of that as moving an n_f, m_f mask across an image
- The figure shows a 2x2 convolution with stride 2

Convolutional Layers

Each application of the kernel...

- Computes a dot product (involving all channels) to obtain a scalar
- ...The optionally applies an activation function

Here we see the effect along 1 dimension:



Therefore, by applying a 2D convolution to an input tensor

...We get a slightly smaller output tensor (like smaller image)

- Every kernel we apply builds a new "channel" in the output

Convolutional Layers

Convolutional layers have some interesting properties

Their weights are associated only to the filter

- So, all applications of the filter/kernel use the same weights
- ...And the number weights does not depend on the input size

This allows a huge reduction in terms of number of weights

Of course the model will be less expressive

...But still capable of laerning useful relations!

- Intuitively, filters will learn to recognize local features
- Earlier convolutions will focus on fine-grain details
- ...While later convolution will aggregate them

This property allows CNN to work very well on image data

CNNs in Keras

We'll glance again at the code to build a Convolutional NN (in Tensorflow/Keras)

```python
def build_cnn(input_shape, output_shape, hidden, convs, rate=0.05):
    mdl = keras.Sequential()
    mdl.add(keras.Input(shape=input_shape))
    for nf in convs:
        mdl.add(Conv2D(nf, kernel_size=(3,3), activation='relu'))
    mdl.add(keras.layers.Flatten())
    for h in hidden:
        mdl.add(Dense(h, activation='relu'))
        mdl.add(keras.layers.Dropout(rate))
    mdl.add(Dense(output_shape[0], activation='softmax'))
    return mdl
```

- We start by building the convolutions layer
- ...Then we add some fully connected layers
- ...And we finish with an output layer using a softmax activation

Training a CNN

CNNs can be trained as usual, but the process is much slower

...Since even with few weights, we still need to do a lot of computations

- Using GPUs can considerably accelerate this step

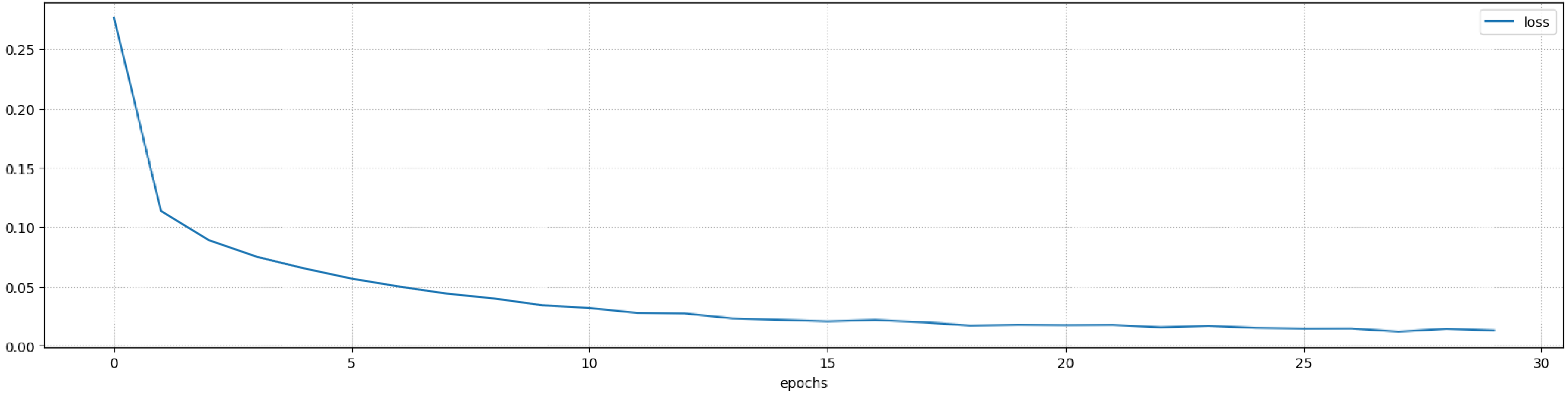
```
In [46]: cnn1 = util.build_cnn(input_shape, output_shape, hidden=[16], convs=[16])
         history2 = util.train_nn(cnn1, x_train_c, y_train_cat, batch_size=32, epochs=30, verbo
```

```
Epoch 1/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.2761
Epoch 2/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.1133
Epoch 3/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.0889
Epoch 4/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.0750
Epoch 5/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.0653
Epoch 6/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.0566
Epoch 7/30
1875/1875 [==============================] - 5s 3ms/step - loss: 0.0500
```

Training a CNN

Let's check the training curve

In [48]:
```
util.plot_training_history(history2, figsize=figsize)
```




```
Final loss: 0.0130 (training)
```

Again, there is still some way to go, but we'll stop here for a fair comparison

Quality Evaluation

In [49]:
```
cnn1_p_tr = cnn1.predict(x_train_c, verbose=0).argmax(axis=1)
cnn1_p_ts = cnn1.predict(x_test_c, verbose=0).argmax(axis=1)

cnn1_acc_tr = accuracy_score(y_train, cnn1_p_tr)
cnn1_acc_ts = accuracy_score(y_test, cnn1_p_ts)

print(f'Shallow network accuracy: {nn1_acc_tr:.3f} (train), {nn1_acc_ts:.3f} (test)')
print(f'Convolutional network accuracy: {cnn1_acc_tr:.3f} (train), {cnn1_acc_ts:.3f} (
```

```
Shallow network accuracy: 0.968 (train), 0.952 (test)
Convolutional network accuracy: 0.999 (train), 0.980 (test)
```

The results are much better!

- Even if the CNN has much fewer weights than the fully connected one
- ...And the same number of hidden layers

Exploiting structural information is a powerful idea in DL