Function approximation

wiki: https://en.wikipedia.org/wiki/Function_approximation

definition: **select a function among a well-defined class that closely matches ("approximates") a target function in a task-specific way**

Taylor series

wiki: https://en.wikipedia.org/wiki/Taylor_series

A special case of Pade approximation.

Pade approximation

wiki: https://en.wikipedia.org/wiki/Pad%C3%A9_approximant

target function: $f(x)$

function collection: $\frac{P_L(x)}{Q_M(x)} \approx f(x)$

1. $P_L(x) = p_0 + p_1 x + ... + p_L x^L$
2. $Q_M(x) = 1 + q_1 x + ... + q_M x^M$

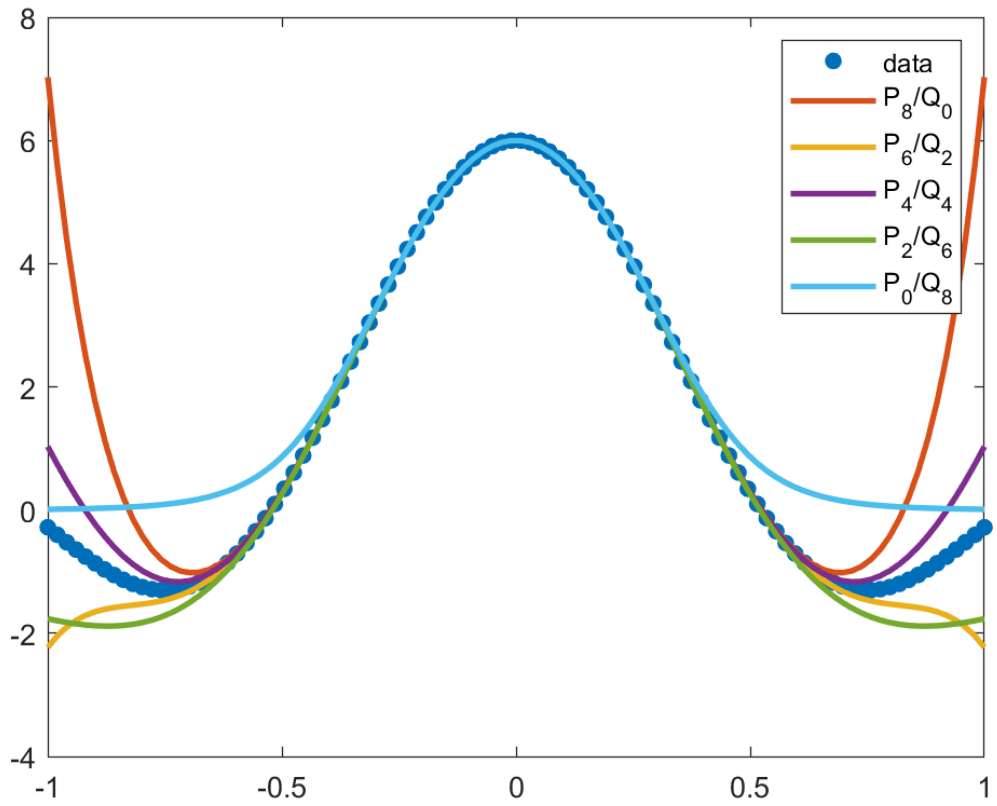
process

1. Taylor expansion: $f(x) \approx \sum_{n=0}^{n=L+M} A_n x^n$
2. then: $Q_M(x) \sum_{n=0}^{n=L+M} A_n x^n \approx P_L(x)$

$$
\begin{cases}
\begin{array}{rcl}
p_0 & = & A_0 \\
p_1 - A_0 q_1 & = & A_1 \\
p_2 - A_1 q_1 - A_0 q_2 & = & A_2 \\
 & ... & \\
p_L - A_{L-1} q_1 - ... - A_0 q_L & = & A_L \\
-A_L q_1 - ... - A_{L-M+1} q_M & = & A_{L+1} \\
 & ... & \\
-A_{L+M-1} q_1 - ... - A_L q_M & = & A_{L+M}
\end{array}
\end{cases}
$$

$$
\begin{pmatrix}
1 & & & & & 0 & & & & \\
& 1 & & & & -A_0 & 0 & & & \\
& & 1 & & & -A_1 & -A_0 & 0 & & \\
& & & \ddots & & \vdots & \vdots & \vdots & \ddots & \vdots \\
& & & & 1 & -A_{L-1} & -A_{L-2} & -A_{L-3} & \ddots & \vdots \\
& & & & & \vdots & \vdots & \vdots & \ddots & \vdots \\
& & & & & -A_{L+M-1} & -A_{L+M-2} & -A_{L+M-3} & \cdots & -A_L
\end{pmatrix}
\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_L \\ q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_M \end{pmatrix}
=
\begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ \vdots \\ A_L \\ \vdots \\ A_{L+M} \end{pmatrix}
$$

```
%symbolic toolbox needed, otherwise need to calculate high order taylor coeffcient by hand
syms x
expr1 = sin(6*x)/x;
taylor_coefficient = sym2poly(taylor(expr1,x,'order',9));

data_x = linspace(-1,1,100);
data_y = double(subs(expr1,x,data_x));

fig = figure();
ax = axes('Parent', fig, 'NextPlot', 'add', 'Box', 'on');

hline0 = plot(data_x, data_y, '.', 'MarkerSize',20);

[p,q] =  my_pade_approximation(taylor_coefficient, 8);
y = polyval(p,data_x)./polyval(q,data_x);
hline1 = plot(data_x, y,'LineWidth',2);

[p,q] =  my_pade_approximation(taylor_coefficient, 6);
y = polyval(p,data_x)./polyval(q,data_x);
hline2 = plot(data_x, y,'LineWidth',2);

[p,q] =  my_pade_approximation(taylor_coefficient, 4);
y = polyval(p,data_x)./polyval(q,data_x);
hline3 = plot(data_x, y,'LineWidth',2);

[p,q] =  my_pade_approximation(taylor_coefficient, 2);
y = polyval(p,data_x)./polyval(q,data_x);
hline4 = plot(data_x, y,'LineWidth',2);

[p,q] =  my_pade_approximation(taylor_coefficient, 0);
y = polyval(p,data_x)./polyval(q,data_x);
hline5 = plot(data_x, y,'LineWidth',2);

legend([hline0,hline1,hline2,hline3,hline4,hline5],{'data','P_8/Q_0','P_6/Q_2','P_4/Q_4','P_2/Q
```



function

```
function [p,q] = my_pade_approximation(taylor_coefficient, L)
% Pade approximation
% taylor_coeffient(int, (1,N1)): from high order to zero-order
% L(int, (1,1))
% p(float,(1,L+1)): from high order to zero-order
% q(float,(1,M+1)), from high order to zero-order, remember to add zero-order coefficient whicl
%
% reference:
%   wiki: https://en.wikipedia.org/wiki/Pad%C3%A9_approximant
%   http://mathworld.wolfram.com/PadeApproximant.html
%   http://mathfaculty.fullerton.edu/mathews/n2003/pade/PadeApproximationProof.pdf
M = size(taylor_coefficient,2) - 1 - L;
reverse_taylor = flip(taylor_coefficient, 2);
tmp1 = full(spdiags(repmat(reverse_taylor(1:(end-1)),M,1),-1:-1:(-L-M),L+M+1,M));
matA = [[eye(L+1); zeros(M,L+1)], -tmp1];
tmp1 = matA\reverse_taylor.';
p = flip(tmp1(1:L+1),1).';
q = [flip(tmp1(L+2:end),1).',1];
end
```