Newton Method

Bin Fu
fubin1991@outlook.com
Rm 418, CYM Physics BLG

Course website: http://www.physics.hku.hk/~phys4150/

Newton Method

Use Newton Raphson Method to find the root of function:

$$f(x) = (x-1)^3 - 1$$

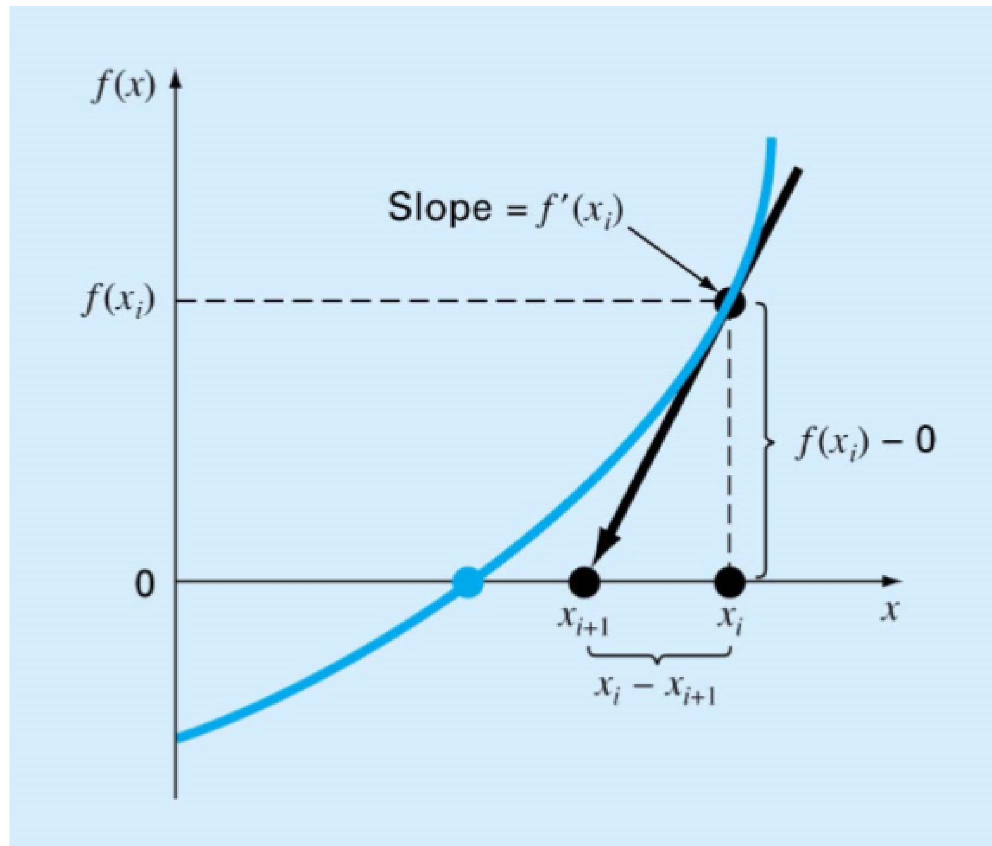
Newton Method

- Widely used
- Requires one single starting value of x
- Possibility of diverging (depend on the staring x and how close is it to the root)

If the initial guess at the root is xi, a tangent can be extended from the point [xi, f(xi)].

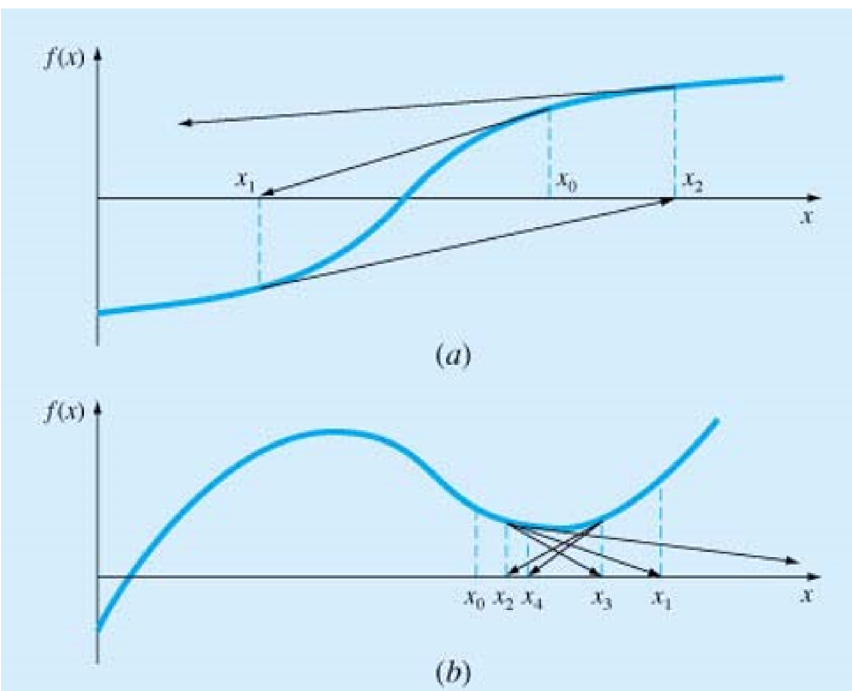
The point where this tangent crosses the x axis usually represents an improved estimate of the root.

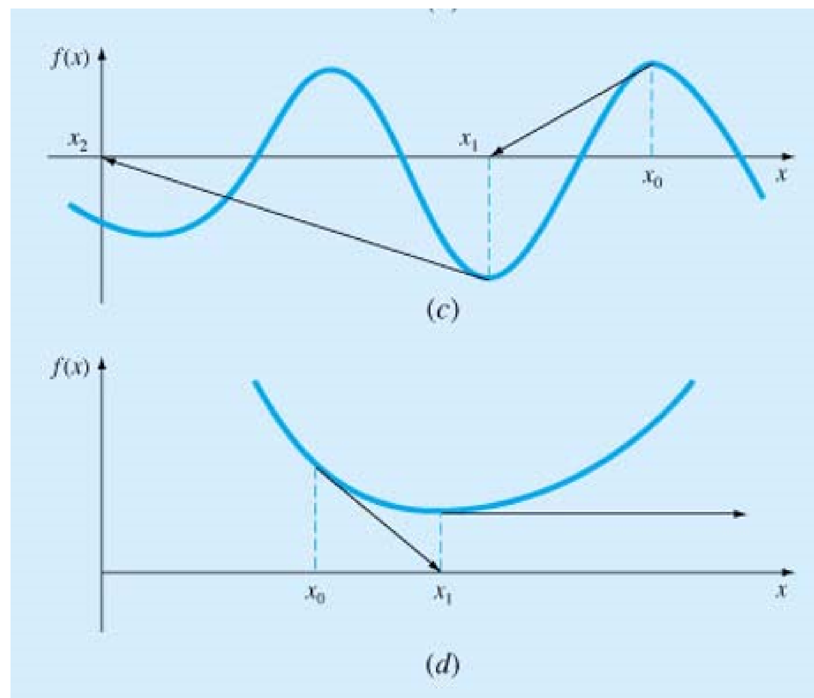
$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$



Newton Method

- **Failure of Newton-Raphson method**
  - Inflection point in vicinity of root
  - Oscillate around local maximum or minimum
  - Jump away for several roots
  - Disaster from zero slope





Newton Method

Algorithm: The steps of the Newton-Raphson method to find the root of an equation $f(x)=0$ are

1. Evaluate $f'(x)$

2. Use an initial guess of the root, x_i, to estimate the new value of the root, x_{i+1}, as

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

3. Find the absolute relative approximate error $|\in_a|$ as

$$|\in_a| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100$$

4. Compare the absolute relative approximate error with the pre-specified relative error tolerance, $\in_s$. If $|\in_a| > \in_s$ then go to Step 2, else stop the algorithm. Also, check if the number of iterations has exceeded the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user.

Newton Method

```
x0=3;
crit=1;ii=0;tol=0.00001;err=1;
while err>tol
    ii=ii+1;
%     fx=x0.^(0.5)-1;
    fx=(x0-1).^(3)-1;
%     fx=cos(x0);
    [dfx]=diff_1ord(x0);
    x1=x0-(fx/dfx);
    x_list(ii,1)=x1;
    err=abs((x1-x0)/x1);
    x0=x1;
end
x1
EE=abs(x1-x_list);
```

0.4167
0.1105
0.0106
1.1156e-04
1.2443e-08
0

$$\epsilon_{i+1} \sim c\epsilon_i^2$$

THANKS FOR ATTENTION !