Differentiation

wiki: https://en.wikipedia.org/wiki/Numerical_differentiation

$$\frac{d^n}{dx^n} f(x)\Big|_{x=a}$$

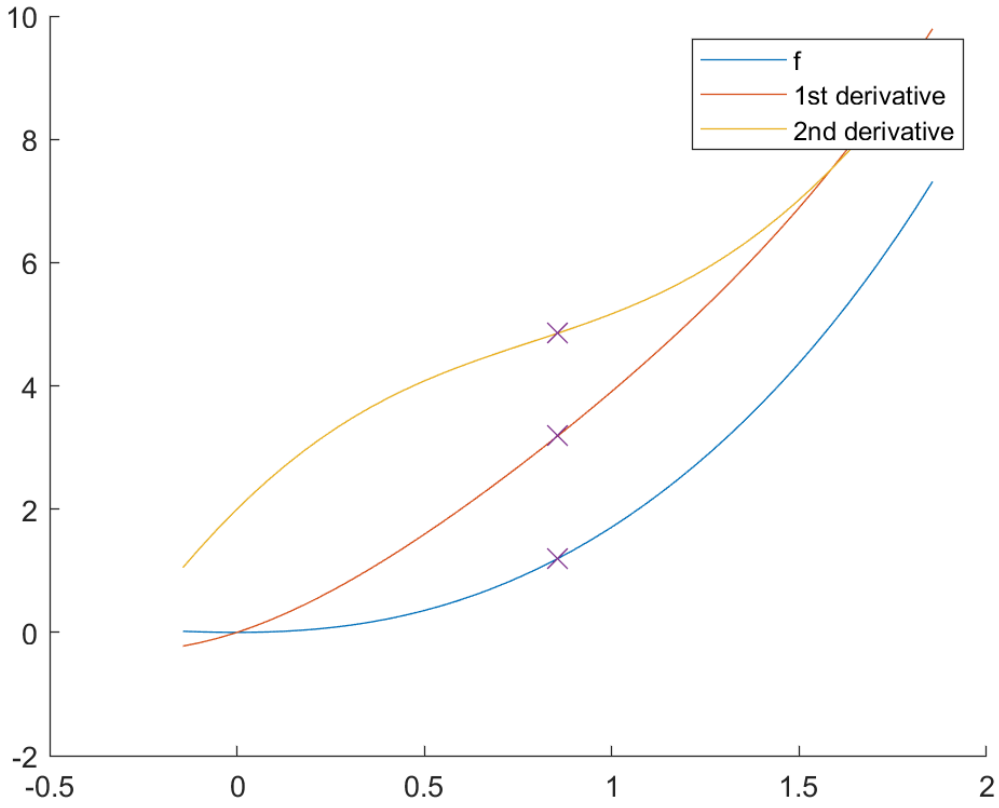
1. function: f
2. x value: a
3. order: n

Parameter

```
hf1 = @(x) x.^3 + sin(x).^2;
hf1_1 = @(x) 3*x.^2 + 2*sin(x).*cos(x);
hf1_2 = @(x) 6*x + 4*cos(x).^2 - 2;
a = rand(1);
```

Graph

```
fig = figure();
ax = axes(fig);
ax.NextPlot = 'add';
fp1 = fplot(hf1, [a-1,a+1]);
fp2 = fplot(hf1_1, [a-1,a+1]);
fp3 = fplot(hf1_2, [a-1,a+1]);
plot([a,a,a],[hf1(a),hf1_1(a),hf1_2(a)], 'x', 'MarkerSize', 10);
legend([fp1,fp2,fp3], {'f', '1st derivative', '2nd derivative'})
```



Analytical results

```
h = 1e-3;
standard_fval1 = hf1(a);
standard_fval1_1 = hf1_1(a);
standard_fval1_2 = hf1_2(a);
disp(['first derivative result: ',num2str(standard_fval1_1,15)])
```

```
first derivative result: 3.18787490507905
```

```
disp(['second derivative result: ',num2str(standard_fval1_2,15)])
```

```
second derivative result: 4.85437033819734
```

```
hf_error_1 = @(x) abs(x-standard_fval1_1)/abs(standard_fval1_1);
hf_error_2 = @(x) abs(x-standard_fval1_2)/abs(standard_fval1_2);
```

First Derivative-1

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$$

```
ret1 = (hf1(a+h)-hf1(a-h))/2/h;
```

```
disp(['First Derivative-1: ',num2str(ret1,15)])
```

```
First Derivative-1: 3.18787524503306
```

```
disp(['relative error: ',num2str(hf_error_1(ret1))])
```

```
relative error: 1.0664e-07
```

First Derivative-2

$$f'(a) = \frac{f(a-2h) - 8f(a-h) + 8f(a+h) - f(a+2h)}{12h}$$

```
ret1 = (hf1(a-2*h) - 8*hf1(a-h) + 8*hf1(a+h) - hf1(a+2*h))/12/h;
disp(['First Derivative-2: ',num2str(ret1,15)])
```

```
First Derivative-2: 3.18787490507869
```

```
disp(['relative error: ',num2str(hf_error_1(ret1))])
```

```
relative error: 1.1019e-13
```

Second Derivative-1

$$f''(a) = \frac{f(a+h) + f(a-h) - 2f(a)}{h^2}$$

```
ret1 = (hf1(a+h) + hf1(a-h) - 2*hf1(a))/h^2;
disp(['Second Derivative-1: ',num2str(ret1,15)])
```

```
Second Derivative-1: 4.85437043185044
```

```
disp(['relative error: ',num2str(hf_error_2(ret1))])
```

```
relative error: 1.9293e-08
```

Second Derivative-2

$$f''(a) = \frac{-f(a-2h) + 16f(a-h) - 30f(a) + 16f(a+h) - f(a+2h)}{12h^2}$$

```
ret1 = (-hf1(a-2*h) + 16*hf1(a-h) -30*hf1(a) +16*hf1(a+h) - hf1(a+2*h))/12/h^2;
disp(['Second Derivative-2: ',num2str(ret1,15)])
```

```
Second Derivative-2: 4.85437033833265
```

```
disp(['relative error: ',num2str(hf_error_2(ret1))])
```

```
relative error: 2.7874e-11
```