Eigenvalue of Complex Matrix

Reference: see lecture05.pdf page79-81

Definition: given a hermitian matrix C, find its eigenvalue λ_i and eigenvectors $\vec{v}_i$.

Denote $A = real(C), B = \mathrm{Im}age(C)$ which have property $A = A^T, B = -B^T$

$$(A + iB)\left(\vec{x} + i\vec{y}\right) = \lambda\left(\vec{x} + i\vec{y}\right)$$

seperate real part and image part of this equation:

$$\begin{cases} A\vec{x} - B\vec{y} = \lambda\vec{x} \\ B\vec{x} + A\vec{y} = \lambda\vec{y} \end{cases}$$

write in matrix form (eigenvalue problem of a real symmetry matrix)

$$\begin{bmatrix} A & -B \\ B & A \end{bmatrix} \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \lambda \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix}$$

Unsolved problems: hermitian matrix C only has N_0 eigenvalue, while this real symmetry matrix has $2N_0$ eigenvalues

Generate Hermitian Matrix

```
tmp1 = randn(3) + randn(3)*1i;
matA = tmp1 + tmp1';
```

built-in

```
disp(sort(eig(matA).'))
```

```
   -1.4889    4.0441    6.4511
```

first convert hermitian matrix into double size reall symmetry matrix

```
[~,EVL] = eig_complex(matA);
disp(sort(EVL.'))
```

```
   -1.4889   -1.4889    4.0441    4.0441    6.4511    6.4511
```

```
function [EVC,EVL] = eig_complex(matA)
% solve eigenvalue problem of complex hermitian matrix
```

```
% matA(N0,N0)
% (ret1)EVC(N0,2*N0)
% (ret2)EVL(2*N0,1): remove duplication: TODO
% TODO: add reference
assert(ishermitian(matA), 'eig_complex requires hermitian matrix');
N0 = size(matA,1);
tmp1 = real(matA);
tmp2 = imag(matA);
[tmp1,tmp2] = eig([tmp1, -tmp2; tmp2, tmp1]);
EVC = tmp1(1:N0,:) + 1i*tmp1(N0+1:end,:);
EVL = diag(tmp2);
end
```