Chapter 2 Classification Tree

Classification Tree (C-Tree) is a method to generate a set of simple rules that can be applied to classify the observations. This is useful in **Data Mining** and has many potential applications. For example, it can be used to derive simple rules for credit approval, medical diagnosis, etc.

Classification Tree method was first developed by Breiman, Friedman, Olshen, and Stone in 1984. It has been an active research area since then. This method will build a classification tree based on the binary splitting of variables, one at a time. The tree is constructed such that each terminal node is as "pure" as possible. That is, in each terminal node, most of the observations are belongs to the same group. Once the classification tree is built, a set of simple classification can be easily obtained. Since this method will search for all possible binary splitting of the variables, it is computational intensive and may be time consuming.

2.1 The Iris flower data set

To illustrate the C-Tree, we first consider the famous Iris flower example. The data set iris.csv contains the information about Iris flower in 5 columns. The first four columns are the measurement of Sepal length, Sepal width, Petal length and Petal width respectively. The last column indicates three different species of the Iris flower (1= setosa, 2=versicolor 3= virginica). There are 150 observations and 50 observations from each species. Let us first read in this data set.

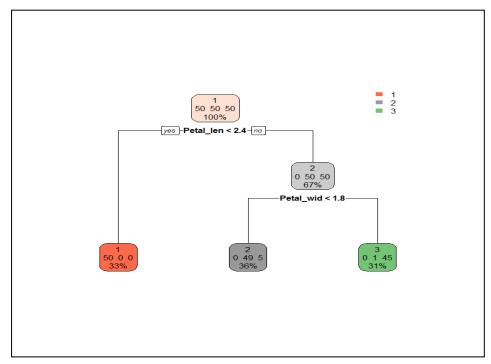
C-Tree has been implemented in R's built-in library *rpart* (stands for **Recursive Partitioning** and **Regression Trees**). Let us first load the *rpart* library and then built a C-Tree for this Iris data set.

```
library(rpart.plot) # load rpart.plot library
rpart.plot(ctree,extra=101) # use rpart.plot, extra=101
```

rpart() is a function in the rpart library to build C-Tree. We put the target variable to be classified on the left hand side of the model (specified by the symbol "~") and all the input variables on the right hand side. The result is then saved in the object *ctree*. (use the online

help in R *help(rpart)* for details). There is a function *rpart.plot()* for plotting CTREE output but you have to install rpart.plot library. First, mark sure your computer is connected to internet. Go to the top menu choose package->install package(s), choose a mirror site that near your location and choose rpart.plot to install. After installation, you should have the rpart.plot library in your local hard-disk.

We can plot *ctree* and display the information using *rpart.plot()*. The option *extra=101* will display information of the distribution in each node.



From the output, the classification rules is:

```
If Petal_{len} < 2.45 then species = 1 (setosa) (50/0/0)
else if Petal_wid < 1.75 then species = 2 (versicolor) (0/49/5)
else species = 3 (virginica). (0/1/45)
```

The numbers in the terminal nodes represent the number of cases. For example, in the node Petal_len<2.45, there are 50 setosa, 0 versicolor and 0 virginica; while in the node Petal_len>=2.45 and Petal_wid<1.75, there are 0 setosa, 49 versicolor and 5 virginica. Finally in the node *Petal_len>=2.45* and *Petal_wid>=1.75*, there are 0 setosa, 1 versicolor and 45 virginica.

We can also use print(ctree) to have numerical output instead:

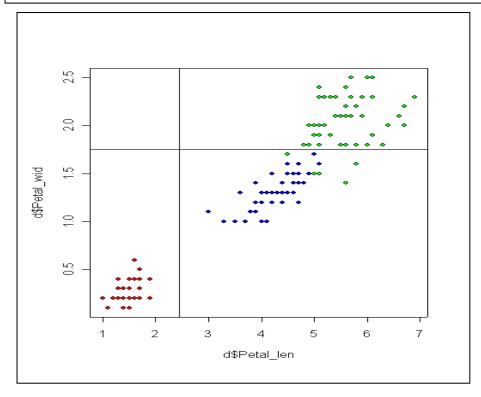
```
> print(ctree)
n = 150
```

This output shows the details at each node:

- At node 1 (root), there is totally n=150 cases. Since each group has equal number (50) of cases, the program randomly assign a yval=1(setosa) to this node. Then there are 100 cases which are non-setosa, loss=100. The percentage of setosa, versicolor and virginica are given in the parenthesis.
- At node 2 (left of the split), the split condition is *Petal_len*<2.45. n=50 and all the cases 2. are setosa, therefore the group label is yval=1 and the loss=0.
- At node 3 (non-terminal nodel), n=100, therefore 50 versicolor and 50 virginica. Again 3. the program randomly assign a yval=2(versicolor) with loss=50.
- At node 6, n=54. Among them, there are 49 versicolor and 5 virginica. Therefore the 4. yval=2 and loss=5.
- At node 7, n=46. There are 1 versicolor and 45 virginica. Therefore the yval=3 and 5. loss=1.

Since this classification tree is relatively simple and the rule also depends on two variables, we can actually plot the observations for illustration.

```
# plot Petal wid versus Petal len with different color for each species
> plot(d$Petal_len,d$Petal_wid,pch=21,bg=c("red","blue","green")[d$Species])
                      # add a horizontal line
> abline(h=1.75)
> abline(v=2.45)
                      # add a vertical line
```



From this plot, we can clearly see how this classification rule works. The built-in function predict(ctree) will produce an 150x3 matrix. Each row represent the probability of that observation belongs to group 1, 2 or 3 respectively. Obviously, we should assign the group label where the prob. is largest. The function max.col() will return the column index corresponding to the column maximum which is the group label for the prediction. Finally we produce a classification table (confusion matrix) of this C-Tree.

From the table, there are five group 3 flowers misclassified into group 2 while there is one group 2 flower misclassified into group 3. The error rate is (1+5)/150=4%.

Although the *rpart()* function is quite easy to use, there are many options hidden in this function. See *help(rpart)* and *help(rpart.control)* for more details. There is one important option need to mention: the *method* option in *rpart* can be one of "anova", "poisson", "class" or "exp". If 'y' is a survival object, then *method="exp"* is assumed, if 'y' has 2 columns then *method="poisson"* is assumed, if 'y' is a factor then *method="class"* is assumed, otherwise *method="anova"* is assumed. It is wisest to specify the method directly, especially as more criteria are added to the function. In our example, our 'y' is a categorical variable; therefore *method="class"* is used.

2.2 Training, validation and testing data set

In data mining, the size of the dataset is huge and we usually partition the data into training, validation and testing data set. The training data is used to build the model. The validation set is used to tune the parameters in the model and the testing data set is used to assess the performance of the model. Let us consider the HMEQ example in chapter 1 again. Firstly we read in the cleaned dataset HMEQ1.CSV:

```
> d<-read.csv("hmeg1.csv")</pre>
                                  # get no. of row and display
> (n < -n row(d))
[1] 3067
> names(d)
 [1] "BAD"
                 "LOAN"
                            "MORTDUE"
                                       "VALUE"
                                                   "REASON"
                                                              "JOB"
                                                                          "YOJ"
 [8] "DEROG"
                 "DELINO"
                                        "NINO"
                                                   "CLNO"
                                                              "DEBTINC"
                            "CLAGE"
```

The total sample size is 3067. Now let us partition d into two parts: training dataset d1 (with sample size=2045, roughly r=2/3 of the total sample size) and testing dataset d2 (with sample size=1022).

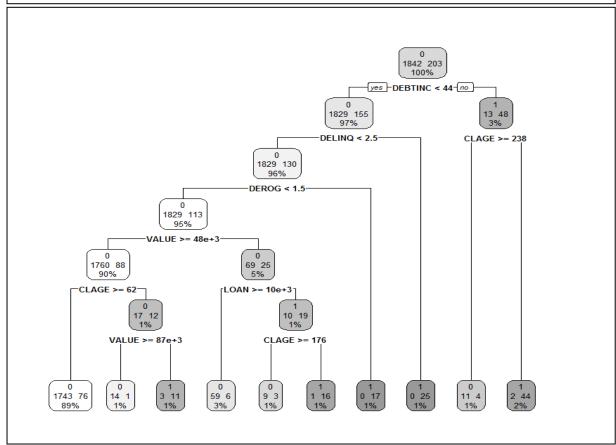
```
# display dimension of d1 and d2
> dim(d1)
[1] 2045
            13
> dim(d2)
[1] 1022
            13
> names(d)
 [1] "BAD"
                "LOAN"
                           "MORTDUE"
                                     "VALUE"
                                                 "REASON"
                                                           "JOB"
                                                                      "YOJ"
     "DEROG"
                           "CLAGE"
                                      "NINQ"
                                                            "DEBTINC"
                "DELINQ"
                                                 "CLNO"
 [8]
```

The random seed is set so that the training and testing dataset is the same every time when the same seed is used.

2.3 Controlling the Classification Tree

Now we try to build a C-Tree for the training dataset *d1* as follow:

```
> library(rpart)  # load rpart library
> y<-d1$BAD  # create target y
> ctree<-rpart(y~.,data=d1[,2:13],method="class") # alternate way to specify model
> rpart.plot(ctree,extra=101,box.palette=" Grays") # plot using grays level
```



The C-Tree built using the default option is far too complicate. Some terminal nodes contain small number of records which the rule may not be very useful. We may have a better control by using the *minsplit* and *maxdepth* options (see help(*rpart.control*) for details). *minsplit* set the minimum number of records in a node and *maxdepth* set the maximum depth of the C-Tree.

```
> ctree<-rpart(y~.,data=d1[,2:13],method="class",minsplit=40,maxdepth=4)</pre>
> rpart.plot(ctree,extra=101,box.palette="Grays") # use grays level instead of color
```

```
1842 203
                                              100%
                                     yes DEBTINC < 44-no
                       1829 155
                                                                   13 48
                         97%
                     DELINQ < 2.5-
                                                               CLAGE >= 238
         1829 130
           96%
       DEROG < 1.5
1829 113
```

```
print(ctree)
n = 2045
node), split, n, loss, yval, (yprob)
      * denotes terminal node
1) root 2045 203 0 (0.90073350 0.09926650)
  2) DEBTINC< 43.68142 1984 155 0 (0.92187500 0.07812500)
    4) DELINQ< 2.5 1959 130 0 (0.93363961 0.06636039)
      8) DEROG< 1.5 1942 113 0 (0.94181256 0.05818744) *
      9) DEROG>=1.5 17
                         0 1 (0.00000000 1.00000000) *
    5) DELINQ>=2.5 25
                        0 1 (0.00000000 1.00000000) *
  3) DEBTINC>=43.68142 61
                          13 1 (0.21311475 0.78688525)
    6) CLAGE>=238.0381 15
                            4 0 (0.73333333 0.26666667) *
    7) CLAGE< 238.0381 46
                            2 1 (0.04347826 0.95652174) *
```

By using the options minsplit=40 and maxdepth=4, we obtain a much simpler C-Tree. The classification rule is:

- 1. If (DEBTINC<43.68142) and (DELINQ<2.5) and (DEROG<1.5) then BAD=0. (1829/113)
- 2. If (DEBTINC<43.68142) and (DELINQ<2.5) and (DEROG>=1.5) then BAD=1. (0/17)
- 3. If (DEBTINC < 43.68142) and (DELINQ > = 2.5) then BAD = 1. (0/25)
- 4. If (DEBTINC > = 43.68142) and (CLAGE > = 238.0381) then BAD = 0. (11/4)
- 5. If (DEBTINC>=43.68142) and (CLAGE<238.0381) then BAD=1. (2/44)

2.4 Assessing the C-Tree

Continue with the example in section 2.3, we produce a misclassification table for this C-Tree as follow:

```
> pr<-predict(ctree)  # prediction on training dataset dl

> cl<-max.col(pr)  # find col. index of max in each row of pr

> table(cl,dl$BAD)  # classification table

cl 0 1

1 1840 117

2 2 86
```

The error rate is (2+117)/2045=0.0582. There are four classification rules. The quality of these classification rules are assessed by **support** (or coverage), confidence (or accuracy) and capture:

Support = total number of cases in that rule / total number of cases in the entire dataset, Confidence = proportion of correctly classified cases in that rule,

Capture = total number of cases of the majority group in that rule / total number of cases of that group in the entire dataset.

For example, there are totally 2045 records in d1. Among them, there are 1842 cases of BAD=0 and 203 cases of BAD=1. Therefore,

```
Rule 1: If (DEBTINC<43.68142) and (DELINQ<2.5) and (DEROG<1.5) then BAD=0. (1829/113) Support=1942/2045=0.9496, Confidence=1829/1942=0.9418, Capture=1829/1842=0.9929.
```

```
Rule 2: If (DEBTINC<43.68142) and (DELINQ<2.5) and (DEROG>=1.5) then BAD=1. (0/17) Support=17/2045=0.0083, Confidence=17/17=1, Capture=17/203=0.0837.
```

```
Rule 3: If (DEBTINC<43.68142) and (DELINQ>=2.5) then BAD=1. (0/25) Support=25/2045=0.0122, Confidence=25/25=1, Capture=25/203=0.1232.
```

```
Rule 4: If (DEBTINC>=43.68142) and (CLAGE>=238.0381) then BAD=0. (11/4) Support=15/2045=0.0073, Confidence=11/15=0.7333, Capture=11/1842=0.00597.
```

```
Rule 5: If (DEBTINC>=43.68142) and (CLAGE<238.0381) then BAD=1. (2/44) Support=46/2045=0.0022, Confidence=44/46=0.9565, Capture=44/203=0.2167.
```

In previous section, we have built the C-Tree based on the training dataset d1 and obtain classification rules for BAD and the error rate (training error) is about 5.82%. However, this error rate is computed using the training dataset d1 which built the rules. This is the **in-sample** assessment. We can apply these rules to the testing dataset d2 and produce a classification table for the testing dataset and computing the error rate. R has a built-in function predict() help us to apply the ctree to d2 easily as follow:

```
> pr<-predict(ctree,d2) # or using predict() function on d2

> cl<-max.col(pr)

> table(cl,d2$BAD) # classification table

cl 0 1

1 943 48

2 4 27
```

Therefore the error rate (generalization or prediction error) is (4+48)/1022=0.0509. Since this testing dataset is independent of the training dataset. This is called the out-of-sample assessment and is more accurately reflect the predictive power of these rules.

2.5 Algorithm for C-Tree building

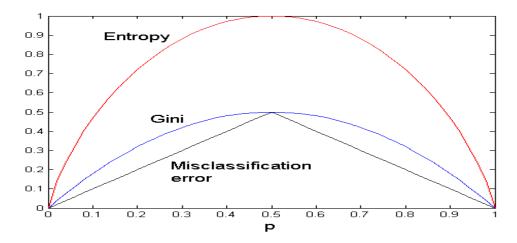
The objective of C-Tree is the find a set of rules such that the distribution of each terminal node is as "pure" as possible. To understand how to build a C-Tree, we need some measure of the impurity of the distribution. Suppose that there are c classes and p(i/t) is the proportion of records belonging to class i at node t, for i=0,1,..., c-1. There are several commonly used measures:

1.
$$Entropy(t) = -\sum_{i=0}^{c-1} p(i \mid t) \log_2 p(i \mid t)$$
, here we define $0 \log_2 0 = 0$.

2.
$$Gini(t) = 1 - \sum_{i=0}^{c-1} [p(i \mid t)]^2$$

3. Classification error(t) = $1 - \max_{i} [p(i \mid t)]$

The following is the plot of these functions for c=2:



Now we turn to the problem of how to choose the best variable for splitting. We need to compare the impurity of the parent node (before splitting) and the impurity of the child node (after splitting). We define the goodness of a splitting variable v by

$$\Delta(v) = I(parent) - \operatorname{Im}(v) = I(parent) - \sum_{i=1}^{k} N(v_i)I(v_i) / N$$

where I(.) is the impurity measure of a given node, N is the total number of records in the parent node, k is the number of attribute value of v, $N(v_i)$ is the number of records associated with the child node v_i . We choose the variable v such that $\Delta(v)$ is largest, or equivalently the impurity measure Im(v) is smallest.

Let us first consider a very simple example to illustrate the calculations. Assume that we need to choose between two binary variables A and B such that

```
root (6,6)
 node 1:A=0 (4,3) *
                                                     node\ 1:B=0\ (1,4)\ *
 node\ 2:A=1\ (2,3)\ *
                                                     node\ 2:B=1\ (5,2)\ *
Using variable A,
                                                  Using variable B,
at node 1: Gini=1-(4/7)^2-(3/7)^2=0.4898
                                                  at node 1: Gini=1-(1/5)^2-(4/5)^2=0.32
at node 2: Gini=1-(2/5)^2-(3/5)^2=0.48
                                                  at node 2: Gini=1-(5/7)^2-(2/7)^2=0.408
\Delta = 0.5 - (7/12)(0.4898) - (5/12)(0.48) = 0.5 - 0.486 \Delta = 0.5 - (5/12)(0.32) - (7/12)(0.408) = 0.5 - 0.3715
```

Hence variable B is prefer to A since Im(A)=0.486 and Im(B)=0.3715. The calculation of Im(v) can be easily extended to find the best splitting condition for continuous variable where we divide the value of v into intervals and obtain the distribution within the interval. For example:

	Cheat	No		No)	No		Yes		Yes		Υe	es N		o N		lo N		10		No	
			Taxable Income																				
Sorted Values Split Positions	\rightarrow	60			70		75		85		90		95		100		120		125		220		
	\rightarrow	55		65		72		80		8	87		92		97 1		10	12	22 17		72 230		
		\u00cm	^	<=	>	<=	>	=	^	<=	^	<=	>	=	^	=	^	<=	^	<=	^	<=	>
	Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
	No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
	Gini	0.420		0.400		0.375		0.343		0.417		0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

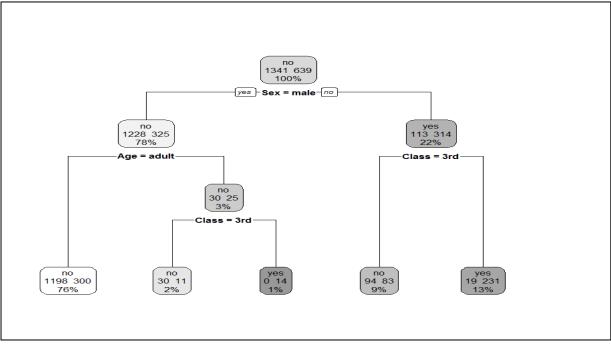
The value of income is sorted and the mid-point between two successive values is used as the split position. The distribution of the target variable is obtained and the impurity measure is computed accordingly. In the above example, the splitting condition *Income*<=97 provides the best (smallest) impurity measure.

```
Child node:
               <=97,
                         (Y,N)=(3,3),
                                         Gini=1-(3/6)^2-(3/6)^2=1/2
               >97,
                         (Y,N)=(0,4)
                                         Gini=1-(0/4)^2-(4/4)^2=0
Impurity measure Im = (6/10)(1/2) + (4/10)(0) = 3/10 = 0.3.
```

2.6 The Titanic dataset

Let use illustrate the C-Tree again by an interesting example. The file "Titanic.csv" contains the information about the 2201 people on board during the first and also the last voyage of Titanic. There are 4 columns: Class (1st, 2nd, 3rd, crew), Age(adult, child), Sex(Female, Male) and Survive(Yes, No). We want to build a classification tree to "predict" who is going to survive in the crash. Instead of using the whole dataset, we randomly sampled 1980 cases (about 90%) as our training data and the remaining 221 cases as testing data.

```
> d<-read.csv("titanic.csv")</pre>
                                                  # read in data
                                                  # display variables
> names(d)
[1] "Class"
               "Age"
                         "Sex"
                                    "Survive"
> set.seed(12345)
                                                  # set random seed
> (n < -nrow(d))
                                                  # get and display sample size
[1] 2201
> r < -0.9
                                                  # set sampling ratio
> id<-sample(1:n,round(r*n))</pre>
                                                  # create random sample
> d1<-d[id,]
                                                  # select training data
> d2 < -d[-id,]
                                                  # select testing data
> dim(d1)
[1] 1981
> dim(d2)
[1] 220
> ctree<-rpart(Survive~Class+Age+Sex,data=d1,method="class")
> rpart.plot(ctree,extra=101,box.palette="Grays") # use grays level instead of color
```



```
> pr<-predict(ctree)  # prediction on training data d1
> cl<-max.col(pr)
> table(cl,dl$Survive)  # classification table for d1
cl no yes  # in-sample error rate is (19+395)/1981=20.9%
1 1322 395
2 19 245
```

The classification rule from the training data is as follow:

- 1. If (Sex=male) and (Age=adult) then **NO** (1198/300)
- 2. If (Sex=male) and (Age=child) and $(Class=3^{rd})$ then **NO** (30/11)
- 3. If (Sex=male) and (Age=child) and $(Class <> 3^{rd})$ then **YES** (0/14)
- 4. If (Sex=female) and $(Class=3^{rd})$ then **NO** (94/83)
- 5. If (Sex=female) and $(Class <> 3^{rd})$ then **YES** (19/231)

The above rules can be simplified into:

- 1. If $(Class=3^{rd})$ then **NO**.
- 2. If $(Class <> 3^{rd})$ and (Sex=female) then **YES**.
- 3. If $(Class <> 3^{rd})$ and (Sex=male) and (Age=child) then **YES**.

We apply this rule to the testing dataset (d2) to assess this rule.

```
> table(d2$Survive,d2$Class,d2$Sex,d2$Age)
                                                                   # display multi-way table
      = female, = adult
1st 2nd 3rd crew
0 1 10 0
s 11 8 5 2
                                          \# error = 1 + 5 = 6
   no
  yes
      = male, = adult
        1st 2nd 3rd crew
14 13 46 58
8 2 5 22
                                          # error=8+2+5+22=37
   no
  yes
      = female, = child
1st 2nd 3rd crew
0 0 2 0
s 0 2 2 0
                                          # error=2
  no
   yes
      = male, = child
        1st 2nd 3rd crew
                                          # error=2
             0
   no
   yes
```

There are totally 6+37+2+2=47 error cases out of 220 testing cases, so the error rate is 47/220=2136%. Alternatively, we can simply using *predict()* function on d2,

```
> pr<-predict(ctree,d2)  # classification for d2
> cl<-max.col(pr)
> table(cl,d2$Survive)  # out-of-sample error rate = (1+46)/220

cl no yes
    1 148 46
    2 1 25
```

2.7 Random Forest

Random Forest is an ensemble (also known as classifier combination) method. The idea is simple but computationally intensive. We repeatedly draw many (e.g, ns=100) random samples from the original training data and using the model built from each sample to predict the testing data. Each of these ns prediction result may not be the same. We use the majority

vote for these *ns* predictions to be our final prediction. In fact, this ensemble method can be applied to any classification methods and not just restricted to classification tree. However, since we built many c-trees using different random samples, we have this fancy name as **Random Forest**. When the random samples drawn from the training data is **bootstrap** sample (i.e., sampling with replacement with same size as the training dataset), we call this as **bootstrap aggregation** method, or simply **Bagging**. Let us formalize the Bagging algorithm of random forest as follow:

- 1. Suppose we have d1 as training dataset with n1 observations and d2 as testing dataset with n2 observations. Let ns (e.g. 100) as the number of trees built in the random forest.
- 2. Draw a random sample from d1 with replacement with sample size n1 and save it to ds.
- 3. Built a c-tree using ds and make prediction on the testing dataset d2 using this c-tree.
- 4. Repeat step 2 to 3 ns times. Save the prediction in step 3 each time.
- 5. Finally use the majority classification result in these ns predictions as the final prediction.

We will illustrate this random forest using the HMEQ data as follow:

```
# random forest using HMEQ data
d<-read.csv("hmeq1.csv")
                                           # read in dataset
set.seed(5104)
                                           # set the random seed
n < -nrow(d)
                                           # get sample size
                                           # set sampling ratio
r < -2/3
id<-sample(1:n,size=round(r*n),replace=F) # sample r*n random integers from 1 to n
d1<-d[id,]
                                           # training dataset
d2 < -d[-id,]
                                           # testing dataset
n1 < -nrow(d1)
                                      # get sample size of d1
                                      # get sample size of d2
n2 < -nrow(d2)
ns<-100
                                      # set no. of tree in random forest
                                      # initialize the prediction results to zero
pred<-matrix(0,nrow=n2,ncol=ns)</pre>
for (i in 1:ns) {
                                      # loop for ns
  id<-sample(1:n1,replace=T)
                                     # draw n1 random integers with replacement
  ds<-d1[id,]
                                      # draw random sample
  ctree<-rpart(ds[,1]~.,data=ds[,2:13],method="class",minsplit=40,maxdepth=4)
 pr<-predict(ctree,d2)
                                     # prediction on d2
                                      # change predition to 0 or 1
  cl < -max.col(pr)-1
  pred[, i]<-cl</pre>
                                      # save prediction to pred
pr<-apply(pred,1,mean)
                                      # compute row mean of pred
pr1<-(pr>0.5)+0
                                      # make final prediction
                                      # classfication table for random forest
table(pr1,d2$BAD)
    0 1
pr1
  0 946 47
         28
  1
```

Note that the error rate of this random forest is (47+1)/1022=4.7% and has slight improvement over the ordinary C-TREE on the HMEQ1 dataset!

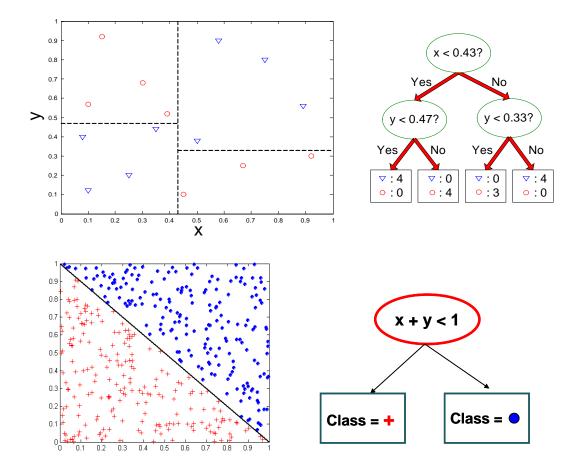
2.8 C-Tree using EXCEL

There is a good implementation of Classification tree using EXCEL found in the internet developed by Dr. Saha. The file ch2-ctree-iris.xls contains the iris data. Inside the EXCEL file, there are seven separate sheets.

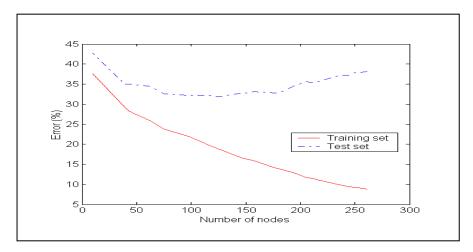
- 1. ReadMe sheet contains the basic information for using this file. Basically, this file can handle up to 10,000 observations and 50 predictor variables.
- 2. Data are stored in L24 of the Data sheet. We can specify which variable is class, continuous, categorical or omit. We can copy and paste your own data here to build our own classification tree.
- 3. In the UserInput sheet, we can specify the parameters to build the tree: minimum node size, maximum purity at each node, and maximum depth of the tree. The data set is divided into two subsets: training data set and testing data set. We can specify the percentage of data points used as testing data. Once the parameters are specified, we can click the Build Tree button to build the tree.
- 4. Tree sheet contains the classification tree and the Node View contains information in each node. There is a button View node link to the Node view sheet. Information about the **splitting condition, label, confidence** and **support** of each terminal node is given. In node 1, there are 50 records out of total 150 records, therefore the support is 50/150=33%. The majority class is *setosa* (node *ID*=1). There is no misclassification; therefore the confidence is 100%. In node 3, the node *ID*=2. There are 54 records, therefore the support is 54/150=36%. The misclassification rate is 5/54=9.26%; therefore the confidence is 100-9.26=90.74%. Finally, in node 4, the node *ID*=3. There are 46 records and the support is 46/150=30.67%. The misclassification rate is 1/46=2.17% and hence the confidence is 100%-2.33%=97.83%.
- 5. The result sheet contains the overall information about this tree as well as the classification (confusion) table of the training data and testing data.
- 6. Finally a set of classification rules is induced from the tree. The support and confidence of each rule is given as in the Tree sheet. There is an additional **Capture** is also given. The capture is the percentage of correctly classified records captured by the rule. For example, the capture of rule 1 is 100% since all 50 setosa species is captured by rule 1. For rule 2, there are 49 out of 50 (=98%) virginica species are captured. For rule 3, there are 45 out of 50 (=90%) versicolor species are captured.

2.9 Final remarks

- 1. C-Tree is a nonparametric method. It does not require any distributional assumptions.
- 2. Relatively easy to interpret, especially for small tree.
- 3. Computationally intensive.
- 4. The variables are based on binary splitting. It is possible to extend it to multi-way split.
- 5. The binary splitting is suitable for problem with rectangular decision boundaries. For other type of decision boundaries, the C-Tree may be very complicate.



6. Generally speaking, the training error will decrease when the size of the C-Tree increase. However, the generalization error may increase again after certain point. That is, large C-Tree will easily lead to model over-fitting.



Reference:

Chapter 4 of Introduction to Data Mining by Tan, Steinbach and Kumar, Addison Wesley.