STAT 6104 Time Series
Midterm
7:00-9:30. Monday, 23 Oct 2017

Name: ________________________ **Major:** ________________________

1. (45 marks) The data of quarterly property price index of a city is given by

$$\mathbf{x} = (129, 138, 132, 144, 169, 188, 170, 206, 220, 248, 226, 248)\,.$$

a) (5 marks) Sketch a time series plot of $\mathbf{x}$.

b) (10 marks) Suggest the period of seasonal effect. Suggest a suitable filter (of the form $(a_{-k}, \ldots, a_0, \ldots, a_k)$) to estimate the trend of the data. Find $\hat{T}_4$ and $\hat{T}_9$ (estimated trends at time 4 and 9) using your suggested filter.

c) (10 marks) Consider an estimated trend

$$\hat{T} = (120, 132, 144, 155, 167, 179, 191, 203, 215, 226, 238, 250)\,.$$

Estimate the seasonal effects.

d) (10 marks) Using c), find the residuals $\hat{\mathbf{N}}$ of the data after accounting for the trend and seasonal effects. Sketch a time series plot of $\hat{\mathbf{N}}$.

e) (10 marks) Find the lag one sample ACF of $\hat{\mathbf{N}}$. Based of the lag one sample ACF, test whether the residual $\hat{\mathbf{N}}$ is white noise.

2. (25 marks) Given $a_t \overset{i.i.d.}{\sim} N(0,1)$. Identify the following ARIMA models (state the order of the model):

a) $Z_t = 0.2Z_{t-1} + 0.24Z_{t-2} + a_t - 3a_{t-2}$.

b) $Z_t = 0.49Z_{t-2} + a_t - 0.5a_{t-1} - 0.14a_{t-2}$.

c) $Z_t = 1.8Z_{t-1} - 0.8Z_{t-2} + a_t - 4a_{t-1} + 6a_{t-2} - 4a_{t-3} + a_{t-4}$.

d) $Z_t = 3Z_{t-1} - 3Z_{t-2} + Z_{t-3} + a_t - 1.2a_{t-1} + 0.2a_{t-2}$.

e) $Z_t = -8Z_{t-1} + 0.81Z_{t-2} + a_t + 7.6a_{t-1} - 4.05a_{t-2}$.

State whether the models have stationary solutions. State whether the models are causal and invertible.

3. (20 marks) Consider the ARMA(1,1) model

$$X_t - 0.5X_{t-1} = Z_t + 1.5Z_{t-1}\,, \quad Z_t \overset{i.i.d.}{\sim} N(0,9)\,,$$

where $N(\mu, \sigma^2)$ denotes normal distribution with mean μ and variance σ^2.

a) (10 marks) Find the values of ψ_k, k =1,2,3... if the model is represented as

$$X_t = Z_t - \sum_{s=1}^{\infty} \psi_k Z_{t-k}\,.$$

b) (8 marks) Find the auto-covariances (ACVF) γ_k, k = 0,1,2,3,... of the model.

c) (2 marks) Is this model useful in practice? Why?

4. (5 marks) Consider the time series $Y_t = Z_t + 0.5Z_{t-1} \times Z_{t-2}$, where $Z_t \overset{iid}{\sim} N(0,1)$. Find the ACVF function of Y_t.

5. (5 marks) Suppose that a time series $\{Y_t\}$ has covariance structure $(\gamma(0), \gamma(1), \gamma(2)) = (1.2, 1, 0.2)$.

a) (3 marks) By consider some property of the quantity $Y_t - 2Y_{t-1} + Y_{t-2}$, argue that the covariance structure is not reasonable.

b) (2 marks) Which set(s) are more reasonable covariance structure?

i) $(\gamma(0), \gamma(1), \gamma(2)) = (2, 1, 0.2)$
ii) $(\gamma(0), \gamma(1), \gamma(2)) = (1, 1, 0.2)$
iii) $(\gamma(0), \gamma(1), \gamma(2)) = (1.2, 1, 0)$

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