THE CHINESE UNIVERSITY OF HONG KONG
Department of Statistics

STAT3007: Introduction to Stochastic Processes
Introduction and Some Basics - Exercises

1. The 17th century French nobleman, Chevalier de Méré, was a keen gambler who corresponded with some the time's most brilliant mathematicians. One problem he asked to Blaise Pascal (of Pascal's triangle, and Pascal's Wager) was the following: which is greater, the probability of getting at least one "6" in four rolls of a single die or the probability of getting at least one "double-6" in 24 throws of two dice? See if you can answer his question (like Pascal).

2. (Exercises 1.2.1, 1.2.2 and 1.2.5 in Pinsky and Karlin) Let A and B be arbitrary events. Use the addition law to verify the formula

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

then establish the general addition law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

and another addition law

$$\begin{aligned} P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ - P(B \cap C) + P(A \cap B \cap C). \end{aligned}$$

3. (Problem 1.2.4 in Pinsky and Karlin) A fair coin is tossed until the first time that the same side appears twice in succession. Let N be the number of tosses required.
   (a) Determine the p.m.f. for N.
   (b) Let A be the event that N is even and B be the event that $N \leq 6$. Evaluate $P(A)$, $P(B)$ and $P(A \cap B)$.

4. (Exercise 1.3.6 in Pinsky and Karlin) The discrete uniform distribution on $\{1, \ldots, n\}$ corresponds to the p.m.f.

$$p(k) = 1/n \text{ for } k = 1, \ldots, n \text{ and zero elsewhere.}$$

   (a) Determine the mean and variance of this distribution.
   (b) Suppose X and Y are independent random variables, each having the discrete uniform distribution on $\{0, \ldots, n\}$. Determine the p.m.f. for the sum $Z = X + Y$.
   (c) Under the assumptions of the question above, determine the p.m.f. for the minimum $U = \min\{X, Y\}$.

5. (Problem 1.3.11 in Pinsky and Karlin) Let X and Y be independent random variables sharing the geometric distribution whose mass function is

$$p(k) = (1-\pi)\pi^k \text{ for } k = 0, 1, \ldots,$$

where $0 < \pi < 1$. Let $U = \min\{X, Y\}$, $V = \max\{X, Y\}$, and $W = V - U$. Determine the joint p.m.f. for U and W and show that U and W are independent.

THE END