THE CHINESE UNIVERSITY OF HONG KONG
Department of Statistics

STAT3007: Introduction to Stochastic Processes
Markov Chain - First Step Analysis - Exercises

1. (Slide 7 of the "Markov Chains - First Step Analysis" notes) Consider the Markov chain described by the transition probability matrix on Slide 2 of the "Markov Chains - First Step Analysis" notes. Let T be the time to absorption for this chain. Show that $Pr(T > k|X_0 = 1) = \beta^k$ for $k = 0, 1, \ldots$.

2. (Exercise 3.4.2 in Pinsky and Karlin) Consider the Markov chain whose transition probability matrix is given by

$$\mathbb{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{pmatrix}.$$

   (a) Starting in state 1, determine the probability that the Markov chain ends in state 0.

   (b) Determine the mean time to absorption.

3. (Exercise 3.4.4 in Pinsky and Karlin) A coin is tossed repeatedly until two successive heads appear. Find the mean number of tosses required. *Hint: let X_n be the cumulative number of successive heads. What is the state space? What are the transition probabilities?*

4. (Exercise 3.4.7 in Pinsky and Karlin) Consider the Markov chain whose transition probability matrix for states 0, 1, 2, 3 is given by

$$\mathbb{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

   Starting in state 1, determine the mean time that the process spends in state 1 prior to absorption and the the mean time that process spends in state 2 prior to absorption. Verify that the sum of these is the mean time to absorption.

5. (Problem 3.4.12 in Pinsky and Karlin) A Markov chain $\{X_n\}$ on states 0, 1, 2 has transition probability matrix

$$\mathbb{P} = \begin{pmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0 & 0 & 1 \end{pmatrix}.$$

Eventually, the process will end up in state 2. What is the probability that, given it a start in state 0, when the process moves into state 2, it does so from state 1?

THE END