

# STAT 3007 Tutorial 2 suggested solutions

Example 1.  $\Pr(Y > y) = \Pr(\min\{X_1, \dots, X_n\} > y)$   
(答題不得寫在紅綫外)

$$= \Pr(X_1 > y, \dots, X_n > y)$$

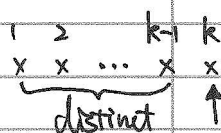
$$= \Pr(X_1 > y) \cdots \Pr(X_n > y) = (1 - F_X(y))^n$$

$$1 - F_X(y) = \Pr(X > y) = \int_y^{\infty} f_X(x) dx = \int_y^{\infty} 2e^{-2x} dx = e^{-2y}$$

$$F_Y(y) = 1 - e^{-2ny} \Rightarrow f_Y(y) = \frac{d}{dy} F_Y(y) = 2ne^{-2ny} \quad \square$$

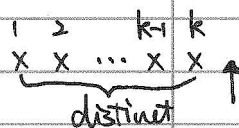
Example 2. For  $k = 2, 3, \dots, n+1$ ,

$$\Pr(N = k) = \frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-(k-2)}{n} \cdot \frac{k-1}{n}$$



Alternatively,

$$\Pr(N > k) = \frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-(k-1)}{n}$$



$$\Rightarrow \Pr(N = k) = \Pr(N > k-1) - \Pr(N > k)$$

$$= \frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-(k-2)}{n} \cdot \left(1 - \frac{n-(k-1)}{n}\right)$$

$$= \frac{n}{n} \cdot \frac{n-1}{n} \cdots \frac{n-(k-2)}{n} \cdot \frac{k-1}{n} \quad \square$$

Example 3. For  $k = 2, 3, \dots$

$$\Pr(N = k) = \Pr(X_2 > X_1, \dots, X_{k-1} > X_1, X_k \leq X_1)$$

$$= \int_0^{\infty} \int_0^{x_1} \int_{x_1}^{\infty} \cdots \int_{x_1}^{\infty} f(x_2) \cdots f(x_{k-1}) f(x_k) f(x_1) dx_2 \cdots dx_{k-1} dx_k dx_1$$

How?  $= \int_0^{\infty} (1 - F(x_1))^{k-2} F(x_1) f(x_1) dx_1$

$$= \int_0^{\infty} e^{-(k-2)x_1} (1 - e^{-x_1}) e^{-x_1} dx_1$$

$$= -\frac{1}{k-1} e^{-(k-1)x_1} \Big|_0^{\infty} - \frac{1}{k} e^{-kx_1} \Big|_0^{\infty} = \frac{1}{k-1} - \frac{1}{k}$$

Alternatively,

$$\Pr(N > k) = \Pr(X_2 > X_1, \dots, X_k > X_1)_{\text{題}} \\ \text{(答題不得寫在紅綫外)} \\ = \Pr(X_1 \text{ is the smallest among } X_1, X_2, \dots, X_k)$$

$$\text{Why?} = \frac{1}{k}$$

$$\Rightarrow \Pr(N = k) = \Pr(N > k-1) - \Pr(N > k) = \frac{1}{k-1} - \frac{1}{k} \quad \square$$

Example 4. (1)

$$P = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{pmatrix} 1-\alpha & \alpha \\ \alpha & 1-\alpha \end{pmatrix} \end{matrix}$$

$$(2) \Pr(X_0 = 0, X_1 = 0, X_2 = 0) = \Pr(X_0 = 0) \Pr(X_1 = 0 | X_0 = 0) \Pr(X_2 = 0 | X_1 = 0, X_0 = 0) \\ = (1-\alpha)^2$$

$$(3) \Pr(X_2 = 0) = \Pr(X_0 = 0, X_1 = 0, X_2 = 0) + \Pr(X_0 = 0, X_1 = 1, X_2 = 0) \\ = (1-\alpha)^2 + \Pr(X_0 = 0) \Pr(X_1 = 1 | X_0 = 0) \Pr(X_2 = 0 | X_1 = 1, X_0 = 0) \\ = (1-\alpha)^2 + \alpha^2 \quad \square$$

Example 5. Analogous to Example 4.

Example 6. Let  $X_n$  be the ordered pair of the results of  $(n-1)^{\text{th}}$  and  $n^{\text{th}}$  games for Michael.

(If the series has ended, interpret it as the results of the last two games.)

$$P = \begin{matrix} & \begin{matrix} (W,W) & (W,L) & (L,W) & (L,L) \end{matrix} \\ \begin{matrix} (W,W) \\ (W,L) \\ (L,W) \\ (L,L) \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 1-\alpha \\ \alpha & 1-\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad \square$$