8. Structural Equation Modeling 5: Latent Growth Curve Models

References:

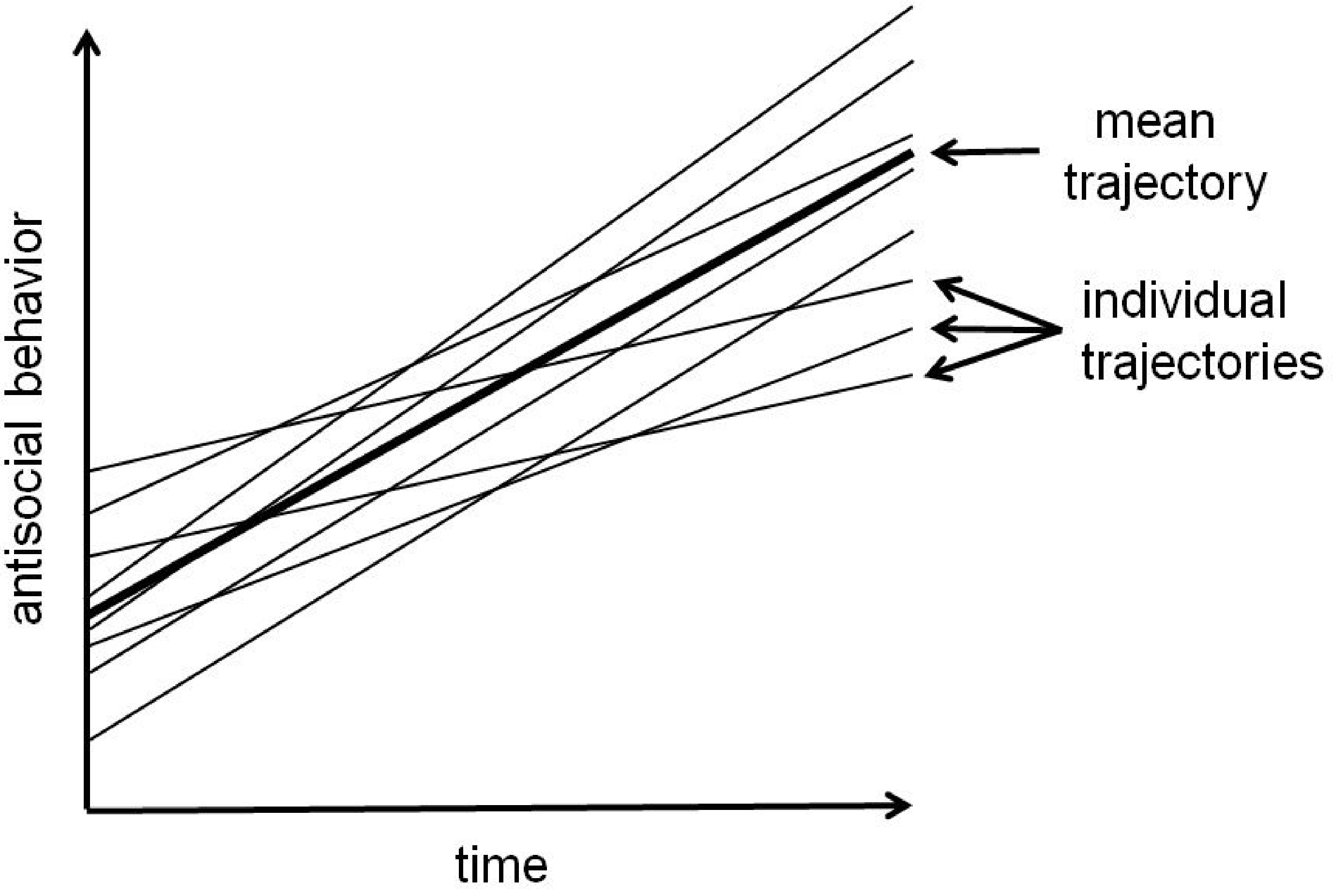
• Beaujean (2014). Chapter 5.

• Ducan, Ducan, & Strycker (2006). Chapter 2.

8.1. Introduction

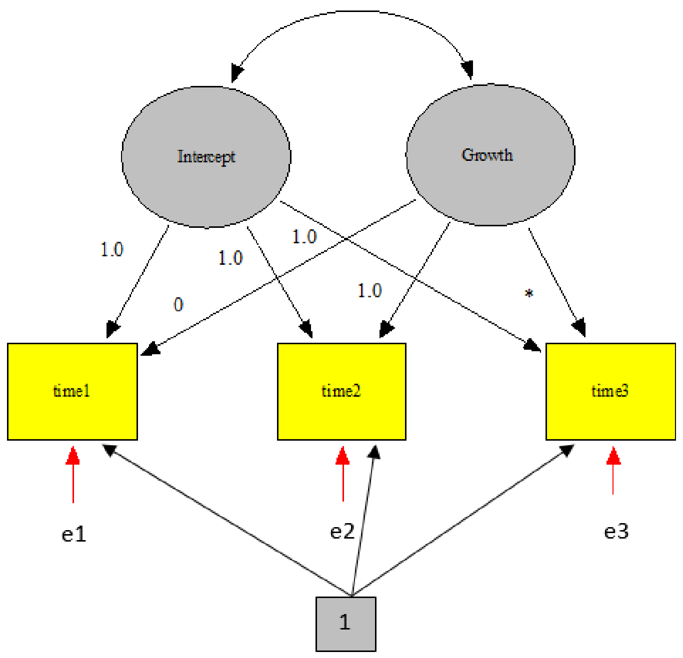
• The latent growth curve modeling (LGM) is a special modeling technique based on mean and covariance structure analysis. Its purpose is to examine individual developmental trajectory (change) over time in a longitudinal study.

• Also known as latent growth modeling, growth curve modeling, or latent curve modeling (LCM).


mean
trajectory
individual
trajectories
antisocial behavior
time

8.2. The Two-factor LGM

• Theoretical model with 3 time points:



• The intercept factor (f_1) describes individuals' initial status at the onset of the study.

• The growth or slope factor (f_2) describes individuals' changes from one time point to another.

- Methodologically, LGM is a CFA model with a mean structure.

- Model equations:

$$\begin{aligned}
\text{time1} &= V_1 = \mu_1 + 1.0 * f_1 + 0 * f_2 + e_1 \\
\text{time2} &= V_2 = \mu_2 + 1.0 * f_1 + 1.0 * f_2 + e_2 \\
\text{time3} &= V_3 = \mu_3 + 1.0 * f_1 + \lambda_{32} * f_2 + e_3
\end{aligned}$$

- In matrix form,

$$v = \mu + \Lambda f + e \qquad (1)$$

- Taking expectations of the observed variables,

$$E(v) = \mu + \Lambda \alpha$$

where $\alpha = E(f)$ is the mean vector of the latent factors

- With 3 time points,

$$E(f_1) = \alpha_1$$
$$E(f_2) = \alpha_2$$
$$E(V_1) = \mu_1 + 1.0{*}\alpha_1 + 0{*}\alpha_2 = \mu_1 + \alpha_1$$
$$E(V_2) = \mu_2 + 1.0{*}\alpha_1 + 1.0{*}\alpha_2 = \mu_2 + \alpha_1 + \alpha_2$$
$$E(V_3) = \mu_3 + 1.0{*}\alpha_1 + \lambda_{32}{*}\alpha_2 = \mu_3 + \alpha_1 + \lambda_{32}\alpha_2$$

• In a two-factor LGM, the variables are:

Name	Type	Cause/Effect	dimension
v	observed	DV	$p \times 1$
f	latent	IV	$k \times 1$
e	latent	IV	$p \times 1$

• And the parameter matrices are:

Parameter matrix	Symbol	Name	dimension
1) factor loading	Λ	lambda	$p \times k$
2) variance-covariance matrix of the factors	Ψ	psi	$k \times k$
3) variance-covariance matrix of errors	Θ	theta	$p \times p$
4)intercept of observed variables	μ	mu	$p \times 1$
5) latent means	α	alpha	$k \times 1$

8.2.1. Interpreting the model parameters

- μ_1, μ_2, μ_3 are the intercepts, which will be the means of the observed variables if the latent means are zeros

- α_1 is the mean of f_1, which measures the average initial status across individuals

- α_2 is the mean of f_2, which measures the average growth across individuals

- λ_{32} is the factor loading, which characterizes the growth pattern over time, e.g., whether the pattern is linear or nonlinear

- ψ_{11} is the variance of f_1, which measures how individuals are different in terms of their initial status

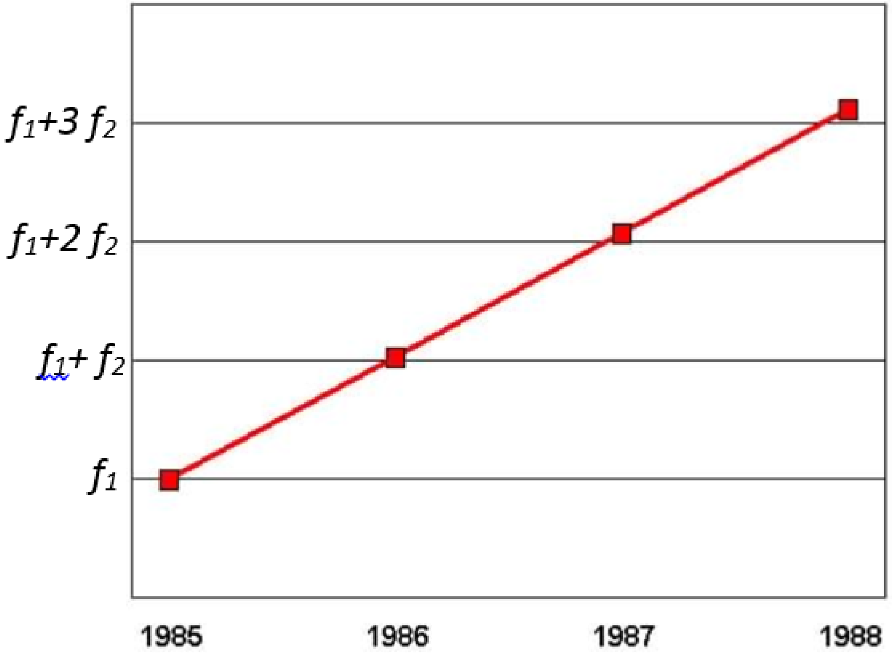
- ψ_{22} is the variance of f_2, which measures how individuals are different in terms of their growth

- ψ_{21} is the covariance of f_1 and f_2, which measures the relationship between initial status and growth

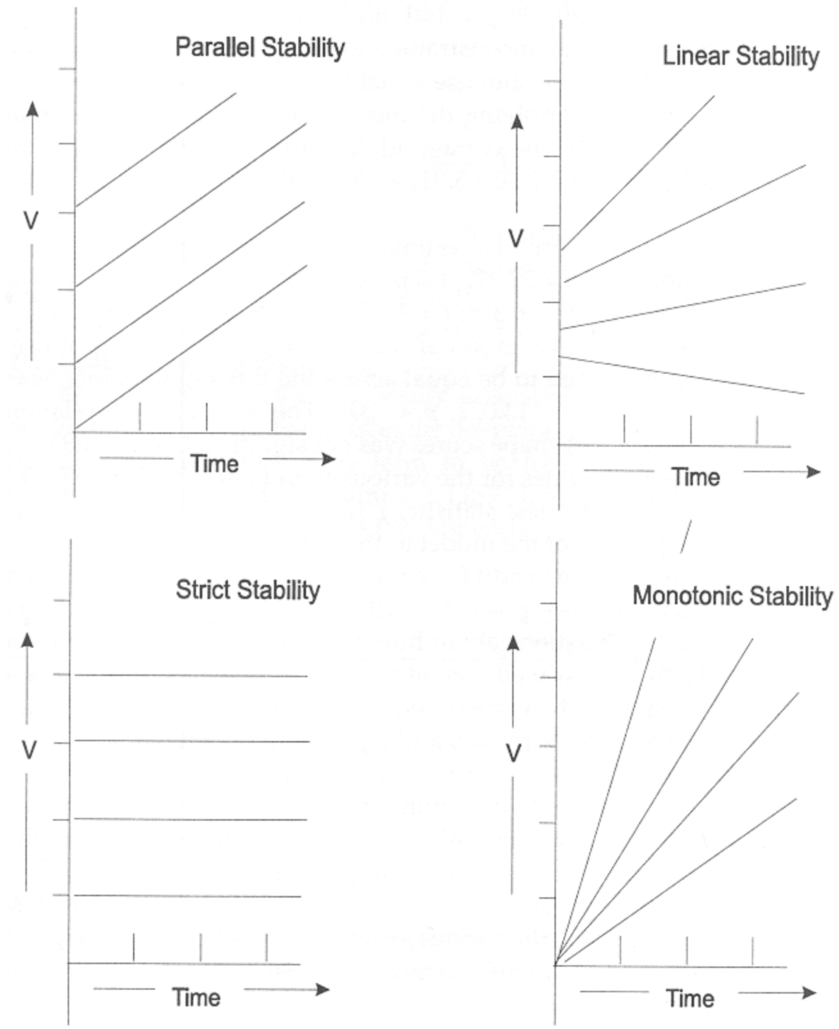
- θ_{11}, θ_{22}, θ_{33} are the error variances

- To fit a linear growth model, λ_{32} will become a fixed parameter with a known value.

- For example, $\lambda_{32} = 2$ and $\lambda_{42} = 3$ if four time points are selected <u>with equal intervals</u>.



- Different growth trajectories (Duncan et al., 2006; p.35)



8.3. Identification

• With 3 time points, the model cannot be identified because there are too many parameters for (1) mean structure and (2) covariance structure:

Structure	No. of information	No. of parameters	No. of constraints
Mean	3	$3+2=5$	3
Covariance	$(3 \times 4)/2 = 6$	$1+3+3=7$	2

• To overcome this, we typically (1) fix the intercepts of the observed variables at 0, that is, $\mu = (\mu_1, \mu_2, \mu_3)' = 0$, and (2) equate the error variances, that is, $\theta_{11} = \theta_{22} = \theta_{33}$

• For models with 4 or more time points, we may not need the equality of error variances constraints for identifying the covariance structure:

Structure	No. of information	No. of parameters	No. of constraints
Mean	4	$4+2=6$	4
Covariance	$(3 \times 4)/2 = 6$	$2+3+4=9$	0

8.4. Example 1: Alcohol Consumption

• In a longitudinal study of alcohol use, 343 participants were recruited. Each participant's level of alcohol consumption for the past 6 months was measured at three approximately equal time intervals over a 2-year period (Biglan et al., 1995):

12 months 12 months

Time 1 Time 2 Time 3

• Data: ($N = 343$, filename = *biglan.dat*)

time1	1.000	0.486	0.399
time2	0.486	1.000	0.533
time3	0.399	0.533	1.000
SD	7.390	7.990	8.080
MEAN	8.310	10.000	10.810

- Questions

1. What is the average initial alcohol consumption?

2. Are people different in their initial consumption?

3. What is the average growth in alcohol consumption?

4. Are people different in their growth?

5. Is there any relationship between initial consumption and growth?

6. What is the growth pattern? Is it linear?

- Summary of findings:

Question	Parameter of Interest	Results

• Example 1 (continued):

filename: *biglan1.R* (R script)

```
# Example 1: Alcohol Consumption

# set work directory and load the packages
setwd("c:/users/wchan/google drive/stat6108/data")
library(lavaan)
library(semPlot)

# data preparation
alc.corr <- matrix(
c(1.000,   0.486,   0.399,
  0.486,   1.000,   0.533,
  0.399,   0.533,   1.000),
nrow=3, ncol=3)
alc.sd <- c(7.390, 7.990, 8.080)
alc.mean <- c(8.310, 10.000, 10.810)
varname <- c("time1", "time2", "time3")
alc.cov <- cor2cov(alc.corr, alc.sd, names=varname)
names(alc.mean) <- varname

# specify Model 1 (Evaluating linear growth using Wald test)
model1 <- "
# measurement model
int =~ 1*time1 + 1*time2 + 1*time3
growth =~ 0*time1 + 1*time2 + la32*time3
# factor variance and covariance
int ~~ int + growth
growth ~~ growth
# error variance (constrained)
time1 ~~ c1*time1
time2 ~~ c1*time2
```

```
time3 ~~ c1*time3
# intercepts
time1 + time2 + time3 ~ 0*1
int + growth ~ 1
# evaluating linear growth
linear := la32-2"
# Fit Model 1 to data
fit1 <-lavaan(model1, sample.cov=alc.cov, sample.mean=alc.mean, sample.nobs=343)

# specify Model 2 (Evaluating linear growth using LR test)
model2 <- "
# measurement model
int =~ 1*time1 + 1*time2 + 1*time3
growth =~ 0*time1 + 1*time2 + 2*time3
# error variance (constrained)
time1 ~~ c1*time1
time2 ~~ c1*time2
time3 ~~ c1*time3"
# Fit Model 2 to data
fit2 <-lavaan(model2, sample.cov=alc.cov, sample.mean=alc.mean, sample.nobs=343, auto.var=TRUE,
              auto.cov.lv.x=TRUE, meanstructure=TRUE, int.ov.free=FALSE, int.lv.free=TRUE)

# save the output
sink("biglan1.out", split=TRUE)
writeLines("\n Example 1: Alcohol Consumption\n")
writeLines("\n Output for Model 1 (Evaluating linear growth using Wald test)\n")
inspect(fit1)
summary(fit1, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 2 (Evaluating linear growth using LR test)\n")
summary(fit2, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Model Comparisons\n")
lavTestLRT(fit1, fit2)
sink()

# create path diagram
semPaths(fit1,"path","est",nCharNodes=5)
```

filename: *biglan1.out* (output file)

Example 1: Alcohol Consumption

Output for Model 1 (Evaluating linear growth using Wald test)

Note: model contains equality constraints:

```
  lhs op rhs
1   5 ==   6
2   5 ==   7

$lambda
      int growth
time1   0      0
time2   0      0
time3   0      1

$theta
      time1 time2 time3
time1 5
time2 0     6
time3 0     0     7

$psi
       int growth
int    2
growth 3   4

$nu
      intrcp
time1      0
time2      0
time3      0
```

```
$alpha
       intrcp
int         8
growth      9

lavaan 0.6-3 ended normally after 67 iterations

  Optimization method                           NLMINB
  Number of free parameters                          9
  Number of equality constraints                     2

  Number of observations                           343

  Estimator                                         ML
  Model Fit Test Statistic                       2.468
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.291

Model test baseline model:

  Minimum Function Test Statistic              219.563
  Degrees of freedom                                 3
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.998
  Tucker-Lewis Index (TLI)                       0.997

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3465.563
  Loglikelihood unrestricted model (H1)      -3464.329

  Number of free parameters                          7
  Akaike (AIC)                                6945.125
```

```
  Bayesian (BIC)                                6971.990
  Sample-size adjusted Bayesian (BIC)           6949.784

Root Mean Square Error of Approximation:

  RMSEA                                            0.026
  90 Percent Confidence Interval          0.000  0.114
  P-value RMSEA <= 0.05                            0.550

Standardized Root Mean Square Residual:

  SRMR                                             0.029

Parameter Estimates:

  Information                                   Expected
  Information saturated (h1) model            Structured
  Standard Errors                               Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                               5.262    0.702
    time2             1.000                               5.262    0.678
    time3             1.000                               5.262    0.644
  growth =~
    time1             0.000                               0.000    0.000
    time2             1.000                               2.505    0.323
    time3   (la32)    1.496    0.258    5.794    0.000    3.748    0.459

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth           -1.156    2.971   -0.389    0.697   -0.088   -0.088
```

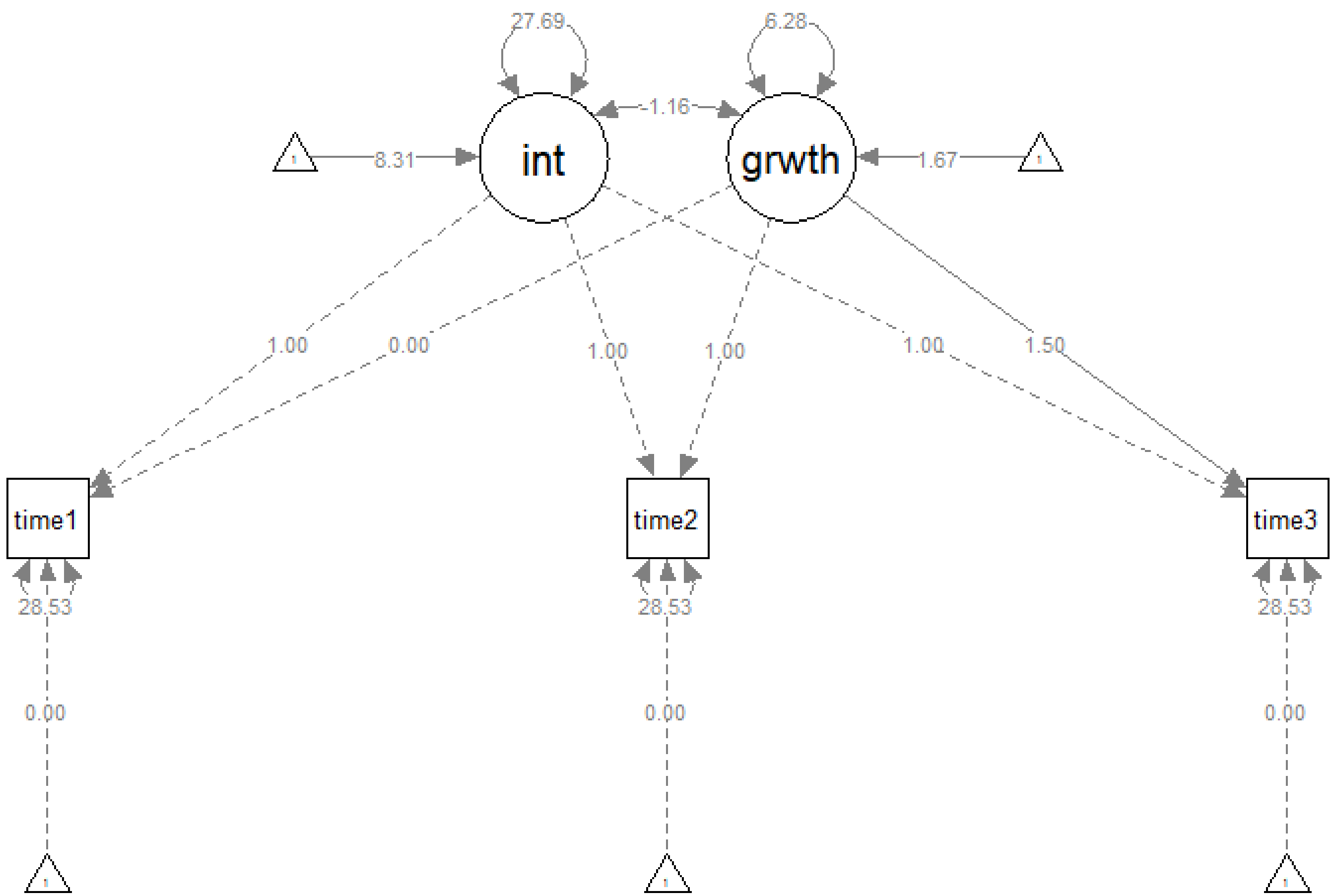
```
Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1             0.000                               0.000    0.000
   .time2             0.000                               0.000    0.000
   .time3             0.000                               0.000    0.000
    int               8.314    0.402   20.669    0.000    1.580    1.580
    growth            1.674    0.389    4.307    0.000    0.668    0.668

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    int              27.687    4.608    6.008    0.000    1.000    1.000
    growth            6.276    3.535    1.775    0.076    1.000    1.000
   .time1    (psi)   28.530    2.179   13.096    0.000   28.530    0.507
   .time2    (psi)   28.530    2.179   13.096    0.000   28.530    0.474
   .time3    (psi)   28.530    2.179   13.096    0.000   28.530    0.427

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    linear           -0.504    0.258   -1.950    0.051    1.748   -1.541
```

```
Output for Model 2 (Evaluating linear growth using LR test)

lavaan 0.6-3 ended normally after 55 iterations

  Optimization method                           NLMINB
  Number of free parameters                          8
  Number of equality constraints                     2

  Number of observations                           343

  Estimator                                         ML
  Model Fit Test Statistic                       4.614
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.202

Model test baseline model:

  Minimum Function Test Statistic              219.563
  Degrees of freedom                                 3
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.993
  Tucker-Lewis Index (TLI)                       0.993

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3466.636
  Loglikelihood unrestricted model (H1)      -3464.329

  Number of free parameters                          6
  Akaike (AIC)                                6945.271
  Bayesian (BIC)                              6968.298
  Sample-size adjusted Bayesian (BIC)         6949.264
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.040
  90 Percent Confidence Interval          0.000  0.107
  P-value RMSEA <= 0.05                          0.508

Standardized Root Mean Square Residual:

  SRMR                                           0.041

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                               5.335    0.707
    time2             1.000                               5.335    0.697
    time3             1.000                               5.335    0.648
  growth =~
    time1             0.000                               0.000    0.000
    time2             1.000                               1.944    0.254
    time3             2.000                               3.888    0.472

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth           -1.056    2.145   -0.492    0.622   -0.102   -0.102

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1             0.000                               0.000    0.000
   .time2             0.000                               0.000    0.000
```

```
   .time3             0.000                               0.000    0.000
    int               8.457    0.390   21.682    0.000    1.585    1.585
    growth            1.250    0.229    5.455    0.000    0.643    0.643

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1    (psi)   28.460    2.173   13.096    0.000   28.460    0.500
   .time2    (psi)   28.460    2.173   13.096    0.000   28.460    0.486
   .time3    (psi)   28.460    2.173   13.096    0.000   28.460    0.420
    int              28.464    4.377    6.503    0.000    1.000    1.000
    growth            3.780    1.753    2.156    0.031    1.000    1.000


Model Comparisons

Chi Square Difference Test

     Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit1  2 6945.1 6972.0 2.4680
fit2  3 6945.3 6968.3 4.6137     2.1456       1      0.143
```

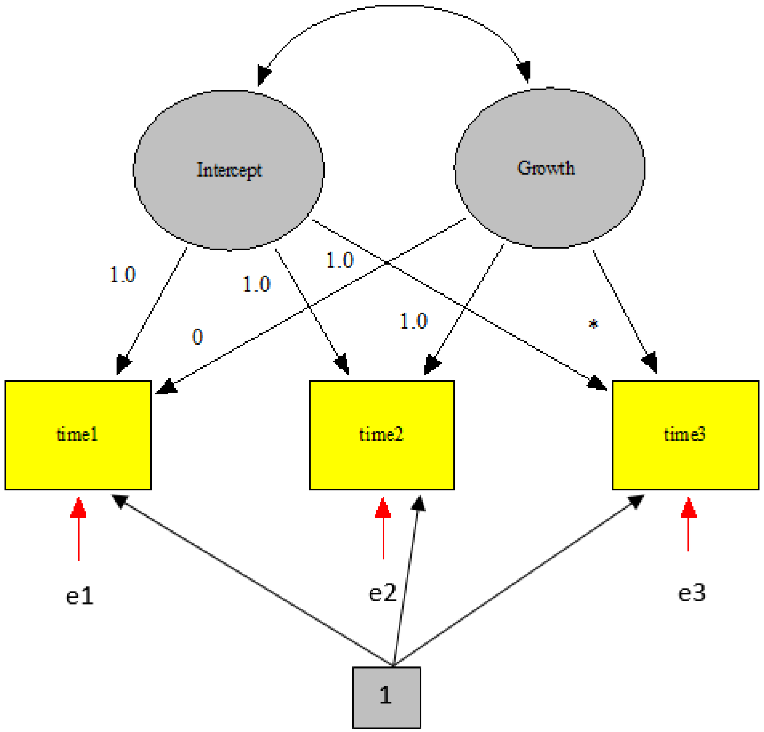
```
> library(semPlot)
> semPaths(fit1,"path","est",nCharNodes=5)
```



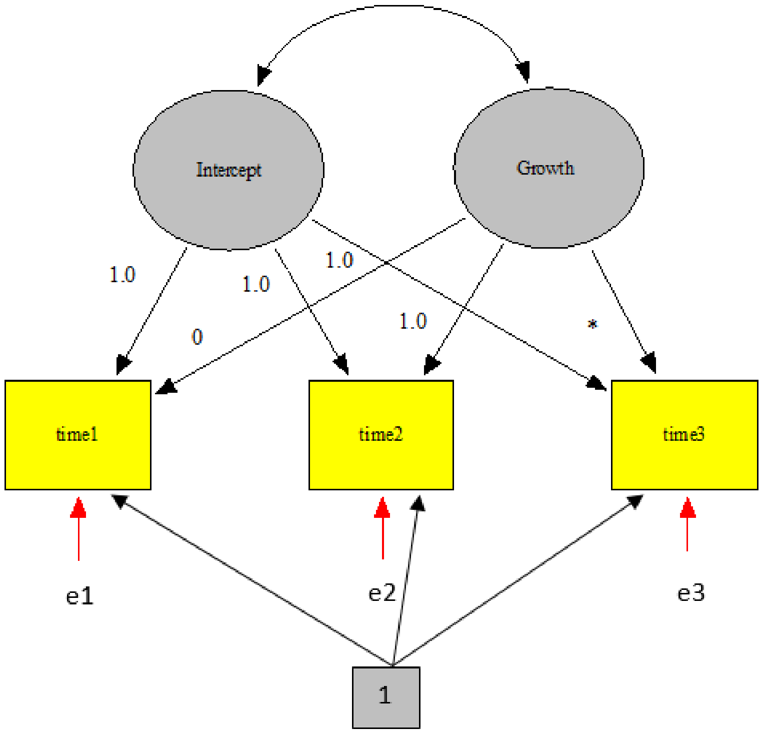
8.5. Multisample LGM

• To determine whether a common developmental model exists, or whether there are different growth patterns across groups (Duncan et al., 2006; Ch. 5)

Group 1:



Group 2:



8.5.1. Identification

• For the covariance structure, since the factor loading matrix has a specific pattern with some fixed values, we can estimate the variances of the latent factors

• For the mean structure, we are interested in estimating α, the latent means of the intercept and growth factors. This is achieved by setting the intercept $\mu = 0$ in each group

8.6. Example 2: Comparing Alcohol Use Between Females and Males

• In a longitudinal study of adolescent alcohol use, 196 females and 95 males were recruited. Each participant's level of alcohol consumption for the past 6 months was measured at three approximately equal time intervals over a 2-year period (Biglan et al., 1995).

• Data:

Female ($N_1 = 196$)
(filename = *female.dat*)

time1	1.0000	0.4641	0.4200
time2	0.4641	1.0000	0.5614
time3	0.4200	0.5614	1.0000
SD	1.3282	1.5136	1.5346
MEAN	1.4430	1.7230	1.8310

Male ($N_2 = 95$)
(filename = *male.dat*)

1.0000	0.4708	0.3915
0.4708	1.0000	0.6679
0.3915	0.6679	1.0000
1.3932	1.4910	1.6520
1.5540	1.8640	2.2800

- Questions:

1. Do they have equal growth pattern?

2. Are females (males) different in their initial alcohol use and growth?

3. Are such differences identical between female and male drinkers?

4. Do the two groups have equal average initial status?

5. Do they have equal average growth?

- Summary of findings:

	Parameter of Interest		
Question	Females	Males	Results

• Example 2 (continued):

filename: *biglan2.R* (R script)

```
# Example 2: Comparing Alcohol Use Between Females and Males

# set work directory and load lavaan packages
setwd("c:/users/wchan/google drive/stat6108/data")
library(lavaan)

# data preparation

# group 1: Females
female.corr <- matrix(
c(1.0000,  0.4641,    0.4200,
  0.4641,  1.0000,    0.5614,
  0.4200,  0.5614,    1.0000),
nrow=3, ncol=3)
female.sd <- c(1.3282, 1.5136, 1.5346)
female.mean <- c(1.4430, 1.7230, 1.8310)

# group 2: Males
male.corr <- matrix(
c(1.0000,  0.4708,    0.3915,
  0.4708,  1.0000,    0.6679,
  0.3915,  0.6679,    1.0000),
nrow=3, ncol=3)
male.sd <- c(1.3932, 1.4910, 1.6520)
male.mean <- c(1.5540, 1.8640, 2.2800)

varname <- c("time1", "time2", "time3")
female.cov <- cor2cov(female.corr, female.sd, names=varname)
male.cov <- cor2cov(male.corr, male.sd, names=varname)
names(female.mean) <- names(male.mean) <- varname
```

```
# specify Model 1 (Using Wald test to compare the groups)
model1 <- "
# measurement model
int =~ 1*time1 + 1*time2 + 1*time3
growth =~ 0*time1 + 1*time2 + c(la1,la2)*time3
# factor variance and covariance
int ~~ c(ps11,ps12)*int
growth ~~ c(ps21,ps22)*growth
int ~~ c(ps31,ps32)*growth
# error variance (constrained)
time1 ~~ c(theta1,theta2)*time1
time2 ~~ c(theta1,theta2)*time2
time3 ~~ c(theta1,theta2)*time3
# intercepts and factor means
time1 + time2 + time3 ~ 0*1
int ~ c(al11,al12)*1
growth ~ c(al21,al22)*1
# comparing females and males
la_d := la1-la2
ps1_d := ps11-ps12
ps2_d := ps21-ps22
ps3_d := ps31-ps32
al1_d := al11-al12
al2_d := al21-al22
"
# Fit Model 1 to data
fit1 <-lavaan(model1, sample.cov=list(Females=female.cov, Males=male.cov),
sample.mean=list(Females=female.mean, Males=male.mean),
              sample.nobs=c(196, 95))
```

```
# specify Model 2 (Using LRT test to compare the groups)
model2 <- "
# measurement model
int =~ 1*time1 + 1*time2 + 1*time3
growth =~ 0*time1 + 1*time2 + time3
# error variance (constrained)
time1 ~~ c(theta1,theta2)*time1
time2 ~~ c(theta1,theta2)*time2
time3 ~~ c(theta1,theta2)*time3
"
# Fit Model 2 to data
fit2 <-lavaan(model2, sample.cov=list(Females=female.cov, Males=male.cov),
sample.mean=list(Females=female.mean, Males=male.mean),
              sample.nobs=c(196, 95), auto.var=TRUE, auto.cov.lv.x=TRUE, meanstructure=TRUE,
int.ov.free=FALSE, int.lv.free=TRUE,
              group.equal=c("loadings","lv.variances","lv.covariances","means"))

# save the output
sink("biglan2.out", split=TRUE)
writeLines("\n Example 2: Comparing Alcohol Use Between Females and Males\n")
writeLines("\n Output for Model 1 (Using Wald test to compare the groups)\n")
summary(fit1, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 2 (Using LRT test to compare the groups)\n")
summary(fit2, fit.measures=TRUE, standardized=TRUE)
lavTestLRT(fit1,fit2)
sink()
```

filename: *biglan2.out* (output file)

```
 Example 2: Comparing Alcohol Use Between Females and Males

 Output for Model 1 (Using Wald test to compare the groups)

lavaan 0.6-3 ended normally after 50 iterations

  Optimization method                           NLMINB
  Number of free parameters                         18
  Number of equality constraints                     4

  Number of observations per group
  Females                                          196
  Males                                             95

  Estimator                                         ML
  Model Fit Test Statistic                       2.553
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.635

Chi-square for each group:

  Females                                        0.712
  Males                                          1.841

Model test baseline model:

  Minimum Function Test Statistic              212.491
  Degrees of freedom                                 6
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
```

```
  Tucker-Lewis Index (TLI)                       1.011

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1468.702
  Loglikelihood unrestricted model (H1)      -1467.425

  Number of free parameters                         14
  Akaike (AIC)                                2965.404
  Bayesian (BIC)                              3016.830
  Sample-size adjusted Bayesian (BIC)         2972.433

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.102
  P-value RMSEA <= 0.05                          0.770

Standardized Root Mean Square Residual:

  SRMR                                           0.023

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard


Group 1 [Females]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                                0.885    0.663
    time2             1.000                                0.885    0.596
```

```
    time3             1.000                               0.885    0.572
  growth =~
    time1             0.000                               0.000    0.000
    time2             1.000                               0.482    0.324
    time3    (la1)    1.270    0.286    4.437    0.000    0.611    0.395

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth  (ps31)    0.095    0.149    0.640    0.522    0.224    0.224

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1             0.000                               0.000    0.000
   .time2             0.000                               0.000    0.000
   .time3             0.000                               0.000    0.000
    int     (al11)    1.440    0.095   15.163    0.000    1.627    1.627
    growth  (al21)    0.299    0.096    3.103    0.002    0.620    0.620

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    int     (ps11)    0.783    0.202    3.870    0.000    1.000    1.000
    growth  (ps21)    0.232    0.192    1.207    0.227    1.000    1.000
   .time1   (tht1)    0.996    0.101    9.899    0.000    0.996    0.560
   .time2   (tht1)    0.996    0.101    9.899    0.000    0.996    0.452
   .time3   (tht1)    0.996    0.101    9.899    0.000    0.996    0.416


Group 2 [Males]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                               1.057    0.766
    time2             1.000                               1.057    0.727
    time3             1.000                               1.057    0.631
```

```
  growth =~
    time1               0.000                               0.000    0.000
    time2               1.000                               0.663    0.456
    time3    (la2)      1.717    0.308    5.567    0.000    1.139    0.680

Covariances:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth  (ps32)     -0.115    0.170   -0.680    0.497   -0.165   -0.165

Intercepts:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1               0.000                               0.000    0.000
   .time2               0.000                               0.000    0.000
   .time3               0.000                               0.000    0.000
    int     (al12)      1.523    0.140   10.897    0.000    1.440    1.440
    growth  (al22)      0.416    0.123    3.382    0.001    0.626    0.626

Variances:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    int     (ps12)      1.118    0.282    3.960    0.000    1.000    1.000
    growth  (ps22)      0.440    0.218    2.016    0.044    1.000    1.000
   .time1   (tht2)      0.788    0.114    6.892    0.000    0.788    0.413
   .time2   (tht2)      0.788    0.114    6.892    0.000    0.788    0.373
   .time3   (tht2)      0.788    0.114    6.892    0.000    0.788    0.281

Defined Parameters:
                     Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    la_d               -0.447    0.421   -1.063    0.288   -0.528   -0.285
    ps1_d              -0.335    0.347   -0.964    0.335    0.000    0.000
    ps2_d              -0.208    0.291   -0.716    0.474    0.000    0.000
    ps3_d               0.211    0.226    0.933    0.351    0.388    0.388
    al1_d              -0.083    0.169   -0.492    0.623    0.187    0.187
    al2_d              -0.117    0.156   -0.750    0.453   -0.006   -0.006
```

Output for Model 2 (Using LRT test to compare the groups)

```
lavaan 0.6-3 ended normally after 32 iterations

  Optimization method                           NLMINB
  Number of free parameters                         18
  Number of equality constraints                    10

  Number of observations per group
  Females                                          196
  Males                                             95

  Estimator                                         ML
  Model Fit Test Statistic                      10.518
  Degrees of freedom                                10
  P-value (Chi-square)                           0.396

Chi-square for each group:

  Females                                        3.811
  Males                                          6.707

Model test baseline model:

  Minimum Function Test Statistic              212.491
  Degrees of freedom                                 6
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.997
  Tucker-Lewis Index (TLI)                       0.998

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1472.684
```

```
  Loglikelihood unrestricted model (H1)      -1467.425

  Number of free parameters                          8
  Akaike (AIC)                                2961.368
  Bayesian (BIC)                              2990.755
  Sample-size adjusted Bayesian (BIC)         2965.385

Root Mean Square Error of Approximation:

  RMSEA                                          0.019
  90 Percent Confidence Interval          0.000  0.093
  P-value RMSEA <= 0.05                          0.667

Standardized Root Mean Square Residual:

  SRMR                                           0.065

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard


Group 1 [Females]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                               0.959    0.704
    time2             1.000                               0.959    0.651
    time3             1.000                               0.959    0.596
  growth =~
    time1             0.000                               0.000    0.000
    time2             1.000                               0.563    0.382
    time3   (.p6.)    1.522    0.226    6.734    0.000    0.857    0.532
```

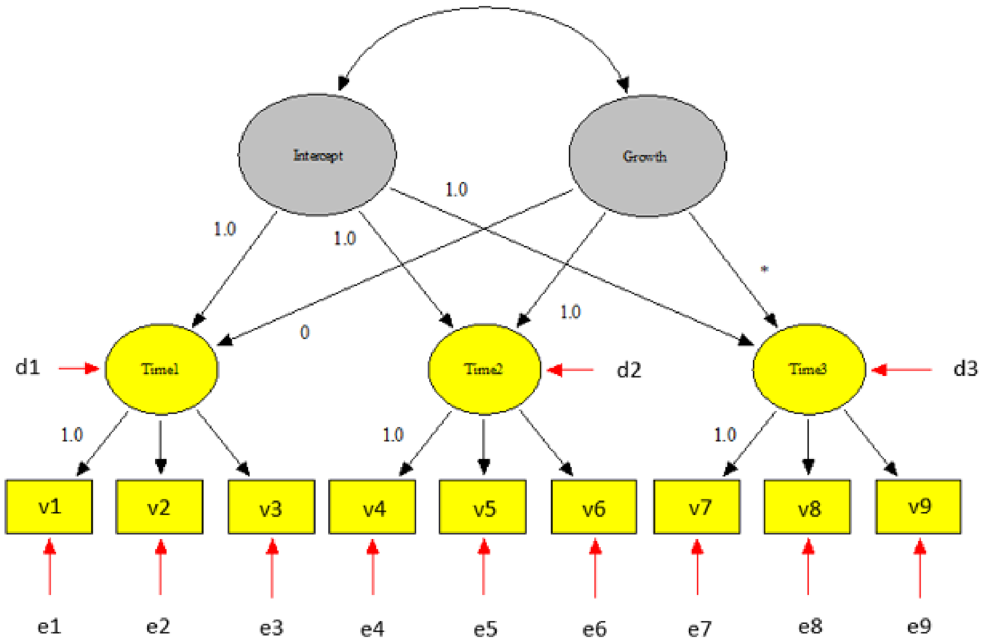
```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth  (.12.)   -0.000    0.105   -0.002    0.998   -0.000   -0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1             0.000                               0.000    0.000
   .time2             0.000                               0.000    0.000
   .time3             0.000                               0.000    0.000
    int     (.16.)    1.470    0.079   18.668    0.000    1.533    1.533
    growth  (.17.)    0.324    0.074    4.374    0.000    0.575    0.575

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1   (tht1)    0.937    0.088   10.623    0.000    0.937    0.505
   .time2   (tht1)    0.937    0.088   10.623    0.000    0.937    0.431
   .time3   (tht1)    0.937    0.088   10.623    0.000    0.937    0.362
    int     (.10.)    0.920    0.163    5.642    0.000    1.000    1.000
    growth  (.11.)    0.317    0.137    2.307    0.021    1.000    1.000


Group 2 [Males]:

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int =~
    time1             1.000                               0.959    0.712
    time2             1.000                               0.959    0.657
    time3             1.000                               0.959    0.601
  growth =~
    time1             0.000                               0.000    0.000
    time2             1.000                               0.563    0.386
    time3   (.p6.)    1.522    0.226    6.734    0.000    0.857    0.537
```

```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  int ~~
    growth  (.12.)   -0.000    0.105   -0.002    0.998   -0.000   -0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1             0.000                               0.000    0.000
   .time2             0.000                               0.000    0.000
   .time3             0.000                               0.000    0.000
    int     (.16.)    1.470    0.079   18.668    0.000    1.533    1.533
    growth  (.17.)    0.324    0.074    4.374    0.000    0.575    0.575

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1   (tht2)    0.894    0.113    7.942    0.000    0.894    0.493
   .time2   (tht2)    0.894    0.113    7.942    0.000    0.894    0.420
   .time3   (tht2)    0.894    0.113    7.942    0.000    0.894    0.351
    int     (.10.)    0.920    0.163    5.642    0.000    1.000    1.000
    growth  (.11.)    0.317    0.137    2.307    0.021    1.000    1.000

Chi Square Difference Test

     Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
fit1  4 2965.4 3016.8  2.5531
fit2 10 2961.4 2990.8 10.5177     7.9646       6     0.2407
```

8.7. LGM with Multiple Indicators

• To determine a growth model of a set of related measures (e.g., language ability, logical reasoning, memory) simultaneously (Duncan et al., 2006; Ch. 4)

• Curve-of-factors LGM (McArdle, 1988)



V1, V2, V3 = Time 1 indicators (e.g., language, logic, memory test score) measuring Time 1 Factor (e.g., intelligence)
V4, V5, V6 = Time 2 indicators (e.g., language, logic, memory test score) measuring Time 2 Factor (e.g., intelligence)
V7, V8, V9 = Time 3 indicators (e.g., language, logic, memory test score) measuring Time 3 Factor (e.g., intelligence)

• For metric invariance, factor loadings of the same variable are constrained to be equal across time (i.e., λ_{21}=λ_{52}=λ_{83}; λ_{31}=λ_{62}=λ_{93})

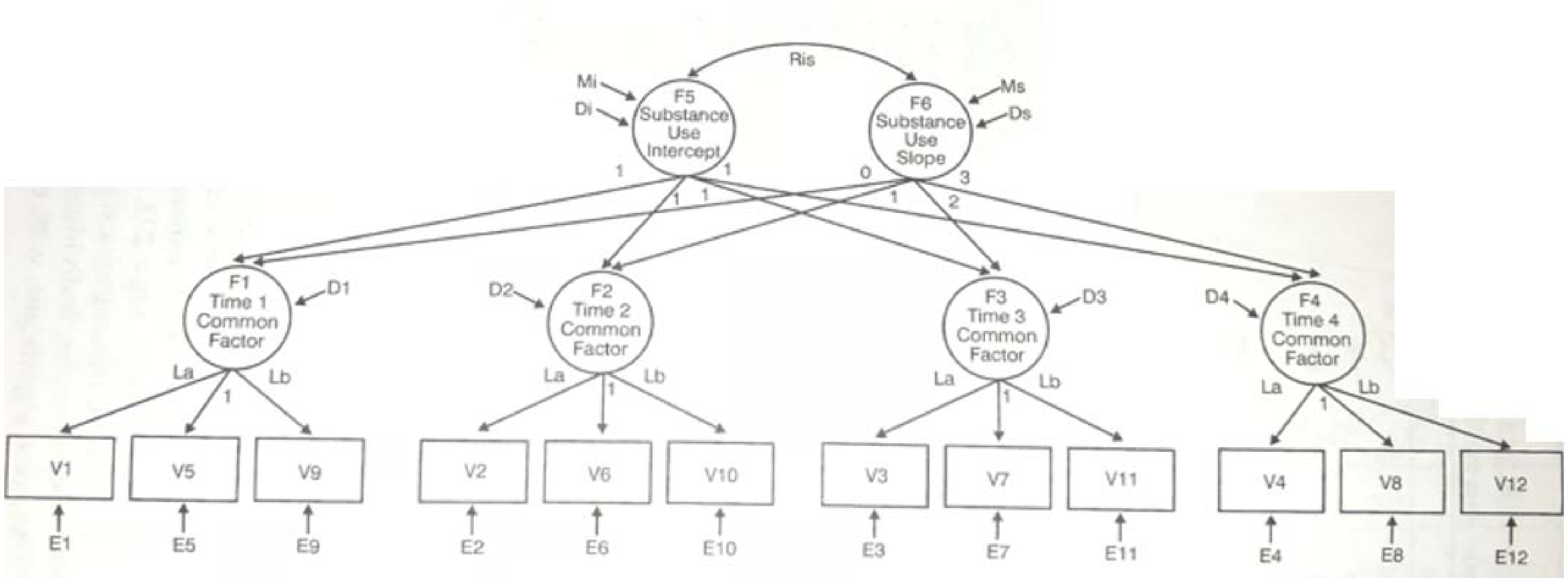
• In order to estimate the mean of the intercept and growth factor, α_1 and α_2, we need to fix (1) the intercept of the time factors at zeros (i.e., α_3=α_4=α_5=0), and (2) the intercepts of the reference variables (*V*1, *V*4, *V*7) at zeros (i.e., μ_1=μ_4=μ_7=0)

• Errors of the same variable are allowed to covary to improve model fit

$$\Psi_e = \begin{bmatrix} \psi_{11} & & & & & & & & \\ & \psi_{22} & & & & & & & \\ & & \psi_{33} & & & & & & \\ \psi_{41} & & & \psi_{44} & & & & & \\ & \psi_{52} & & & \psi_{55} & & & & \\ & & \psi_{63} & & & \psi_{66} & & & \\ \psi_{71} & & & \psi_{74} & & & \psi_{77} & & \\ & \psi_{82} & & & \psi_{85} & & & \psi_{88} & \\ & & \psi_{93} & & & \psi_{96} & & & \psi_{99} \end{bmatrix}$$

8.8. Example 3: Drug Use

• In a longitudinal study of drug use, 3 indicators were used to measure the factor: alcohol use, tobacco use, and marijuana use. 357 participants were recruited and each participant's level of substances consumption for the past 6 months was measured at four approximately equal time intervals (Duncan et al., 2006, Ch. 4)



- Data ($N = 357$, filename=*drug.dat*)

Table 4.1

Descriptive Statistics for Adolescent Alcohol, Tobacco, and Marijuana Use

	Alcohol Use				*Tobacco Use*				*Marijuana Use*			
	T1	*T2*	*T3*	*T4*	*T1*	*T2*	*T3*	*T4*	*T1*	*T2*	T3	*T4*
	V1	*V2*	*V3*	*V4*	*V5*	*V6*	*V7*	*V8*	*V9*	*V10*	*V11*	*V12*
V1	1.000											
V2	.725	1.000										
V3	.595	.705	1.000									
V4	.566	.624	.706	1.000								
V5	.419	.281	.303	.283	1.000							
V6	.344	.362	.350	.367	.671	1.000						
V7	.224	.281	.353	.360	.548	.783	1.000					
V8	.183	.234	.300	.384.	.458	.696	.823	1.000				
V9	.579	.482	.410	.303	.455	.333	.244	.179	1.000			
V10	.532	.571	.501	.440	.347	.444	.352	.272	.663	1 .000		
V11	.439	.507	.648	.496	.378	.419	.430	.345	.551	.709	1.000	
V12	.431	.469	.527	.571	.345	.424	.427	.412	.499	.682	.736	1.000
M	1.338	1.591	2.019	2.364	.862	1.218	1.445	1.756	.554	.890	1.033	1.123
SD	1.260	1.334	1.440	1.376	1.709	1.948	2.117	2.265	1.199	1.432	1.496	1.503

Note. Correlation matrix is in the triangle; means and standard deviations are presented in the bottom rows of the matrix.

- Summary of findings:

1. Average initial status

2. Average growth

3. Variability of people's initial status

4. Variability of people's growth

5. Relationship between initial status and growth

6. Pattern of growth

filename: *drug.R* (R script file)

```
# Example 3: Drug Use

# set work directory and load lavaan package
setwd("c:/users/wchan/google drive/stat6108/data")
library(lavaan)

# data preparation
data <- read.table("drug.dat")
data <- as.matrix(data)
corr <- data[1:12,]
sd <- data[13,]
mean <- data[14,]
cov <- cor2cov(corr, sd)
rownames(cov) <- colnames(cov)

# specify Model 1 (Curve-of-factors LGM)
model1 <- "
# measurement model
Time1 =~ la1*V1 + 1*V5 + la2*V9
Time2 =~ la1*V2 + 1*V6 + la2*V10
Time3 =~ la1*V3 + 1*V7 + la2*V11
Time4 =~ la1*V4 + 1*V8 + la2*V12
Int =~ 1*Time1 +1*Time2 + 1*Time3 + 1*Time4
Growth =~ 0*Time1 +1*Time2 + la3*Time3 + la4*Time4

# error variance
V1 ~~ V1 + V2 + V3 + V4
V2 ~~ V2 + V3 + V4
V3 ~~ V3 + V4
V4 ~~ V4
V5 ~~ V5 + V6 + V7 + V8
V6 ~~ V6 + V7 + V8
```

```
V7 ~~ V7 + V8
V8 ~~ V8
V9 ~~ V9 + V10 + V11 + V12
V10 ~~ V10 + V11 + V12
V11 ~~ V11 + V12
V12 ~~ V12

# intercepts
Time1 + Time2 + Time3 + Time4 ~ 0*1
Int + Growth ~ 1
V1 + V2 + V3 + V4 ~ 0*1
V5 + V6 + V7 + V8 + V9 + V10 + V11 + V12 ~ 1

# evaluating linear growth
linear1 := la3-2
linear2 := la4-3
"

# Fit Model 1 to data
fit1 <-lavaan(model1, sample.cov=cov, sample.mean=mean, sample.nobs=357, auto.var=TRUE,
auto.cov.lv.x=TRUE)

# specify Model 2 (Testing Linear Growth using LR test)
model2 <- "
# measurement model
Time1 =~ la*V1 + 1*V5 + lb*V9
Time2 =~ la*V2 + 1*V6 + lb*V10
Time3 =~ la*V3 + 1*V7 + lb*V11
Time4 =~ la*V4 + 1*V8 + lb*V12
Int =~ 1*Time1 +1*Time2 + 1*Time3 + 1*Time4
Growth =~ 0*Time1 +1*Time2 + 2*Time3 + 3*Time4

# error variance
V1 ~~ V1 + V2 + V3 + V4
V2 ~~ V2 + V3 + V4
V3 ~~ V3 + V4
```

```
V4 ~~ V4
V5 ~~ V5 + V6 + V7 + V8
V6 ~~ V6 + V7 + V8
V7 ~~ V7 + V8
V8 ~~ V8
V9 ~~ V9 + V10 + V11 + V12
V10 ~~ V10 + V11 + V12
V11 ~~ V11 + V12
V12 ~~ V12

# intercepts
Time1 + Time2 + Time3 + Time4 ~ 0*1
Int + Growth ~ 1
V1 + V2 + V3 + V4 ~ 0*1
V5 + V6 + V7 + V8 + V9 + V10 + V11 + V12 ~ 1
"

# Fit Model 2 to data
fit2 <-lavaan(model2, sample.cov=cov, sample.mean=mean, sample.nobs=357, auto.var=TRUE,
auto.cov.lv.x=TRUE)

# save the output
sink("drug.out", split=TRUE)
writeLines("\n Example 3: Drug Use\n")
writeLines("\n Output for Model 1 (Curve-of-factors LGM)\n")
summary(fit1, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 2 (Testing Linear Growth using LRT)\n")
summary(fit2, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Model Comparisons\n")
lavTestLRT(fit1, fit2)
sink()
```

filename: *drug.out* (output file)

```
 Example 3: Drug Use


 Output for Model 1 (Curve-of-factors LGM)

lavaan 0.6-3 ended normally after 87 iterations

  Optimization method                           NLMINB
  Number of free parameters                         57
  Number of equality constraints                     6

  Number of observations                           357

  Estimator                                         ML
  Model Fit Test Statistic                      76.481
  Degrees of freedom                                39
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic             3173.220
  Degrees of freedom                                66
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.988
  Tucker-Lewis Index (TLI)                       0.980

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -6425.102
  Loglikelihood unrestricted model (H1)      -6386.861
```

```
  Number of free parameters                         51
  Akaike (AIC)                               12952.204
  Bayesian (BIC)                             13149.969
  Sample-size adjusted Bayesian (BIC)        12988.173

Root Mean Square Error of Approximation:

  RMSEA                                          0.052
  90 Percent Confidence Interval          0.034  0.069
  P-value RMSEA <= 0.05                          0.406

Standardized Root Mean Square Residual:

  SRMR                                           0.051

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Time1 =~
    V1      (la1)     0.977    0.080   12.258    0.000    0.928    0.729
    V5                1.000                               0.950    0.557
    V9      (la2)     1.108    0.095   11.670    0.000    1.053    0.844
  Time2 =~
    V2      (la1)     0.977    0.080   12.258    0.000    0.915    0.697
    V6                1.000                               0.937    0.494
    V10     (la2)     1.108    0.095   11.670    0.000    1.038    0.758
  Time3 =~
    V3      (la1)     0.977    0.080   12.258    0.000    1.014    0.726
    V7                1.000                               1.038    0.495
    V11     (la2)     1.108    0.095   11.670    0.000    1.150    0.807
```

```
  Time4 =~
    V4       (la1)    0.977    0.080   12.258    0.000    1.033    0.735
    V8                1.000                               1.058    0.474
    V12      (la2)    1.108    0.095   11.670    0.000    1.172    0.789
  Int =~
    Time1             1.000                               0.940    0.940
    Time2             1.000                               0.954    0.954
    Time3             1.000                               0.860    0.860
    Time4             1.000                               0.844    0.844
  Growth =~
    Time1             0.000                               0.000    0.000
    Time2             1.000                               0.230    0.230
    Time3    (la3)    2.430    0.328    7.399    0.000    0.505    0.505
    Time4    (la4)    3.568    0.530    6.739    0.000    0.728    0.728

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .V1 ~~
   .V2                0.475    0.077    6.135    0.000    0.475    0.579
   .V3                0.400    0.075    5.327    0.000    0.400    0.478
   .V4                0.426    0.075    5.708    0.000    0.426    0.512
 .V2 ~~
   .V3                0.514    0.085    6.063    0.000    0.514    0.570
   .V4                0.412    0.084    4.922    0.000    0.412    0.459
 .V3 ~~
   .V4                0.492    0.094    5.213    0.000    0.492    0.537
 .V5 ~~
   .V6                1.452    0.161    9.011    0.000    1.452    0.621
   .V7                1.263    0.167    7.572    0.000    1.263    0.489
   .V8                1.154    0.174    6.629    0.000    1.154    0.415
 .V6 ~~
   .V7                2.247    0.215   10.474    0.000    2.247    0.747
   .V8                2.133    0.222    9.621    0.000    2.133    0.658
 .V7 ~~
   .V8                2.844    0.260   10.934    0.000    2.844    0.795
 .V9 ~~
```

```
   .V10              0.191    0.083    2.316    0.021    0.191    0.319
   .V11              0.118    0.078    1.526    0.127    0.118    0.210
   .V12              0.117    0.078    1.507    0.132    0.117    0.192
 .V10 ~~
   .V11              0.371    0.095    3.928    0.000    0.371    0.494
   .V12              0.391    0.098    3.978    0.000    0.391    0.479
 .V11 ~~
   .V12              0.358    0.108    3.306    0.001    0.358    0.467
  Int ~~
    Growth          -0.041    0.022   -1.840    0.066   -0.213   -0.213

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    Time1            0.000                                0.000    0.000
    Time2            0.000                                0.000    0.000
    Time3            0.000                                0.000    0.000
    Time4            0.000                                0.000    0.000
    Int              1.358    0.130   10.424    0.000    1.520    1.520
    Growth           0.296    0.055    5.389    0.000    1.370    1.370
   .V1               0.000                                0.000    0.000
   .V2               0.000                                0.000    0.000
   .V3               0.000                                0.000    0.000
   .V4               0.000                                0.000    0.000
   .V5              -0.499    0.142   -3.520    0.000   -0.499   -0.292
   .V6              -0.430    0.168   -2.555    0.011   -0.430   -0.227
   .V7              -0.629    0.202   -3.123    0.002   -0.629   -0.300
   .V8              -0.656    0.228   -2.874    0.004   -0.656   -0.294
   .V9              -0.954    0.135   -7.075    0.000   -0.954   -0.764
   .V10             -0.936    0.163   -5.749    0.000   -0.936   -0.683
   .V11             -1.265    0.196   -6.452    0.000   -1.265   -0.888
   .V12             -1.550    0.225   -6.897    0.000   -1.550   -1.044

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .V1               0.761    0.089    8.581    0.000    0.761    0.469
   .V2               0.884    0.099    8.893    0.000    0.884    0.514
```

.V3	0.921	0.111	8.314	0.000	0.921	0.472
.V4	0.911	0.115	7.923	0.000	0.911	0.460
.V5	2.008	0.169	11.904	0.000	2.008	0.690
.V6	2.723	0.223	12.200	0.000	2.723	0.756
.V7	3.320	0.270	12.299	0.000	3.320	0.755
.V8	3.857	0.311	12.388	0.000	3.857	0.775
.V9	0.449	0.093	4.819	0.000	0.449	0.288
.V10	0.800	0.114	6.988	0.000	0.800	0.426
.V11	0.707	0.123	5.732	0.000	0.707	0.348
.V12	0.832	0.135	6.143	0.000	0.832	0.377
Time1	0.105	0.048	2.208	0.027	0.116	0.116
Time2	0.115	0.034	3.421	0.001	0.131	0.131
Time3	0.205	0.044	4.608	0.000	0.190	0.190
Time4	0.022	0.058	0.377	0.706	0.019	0.019
Int	0.798	0.135	5.906	0.000	1.000	1.000
Growth	0.047	0.018	2.583	0.010	1.000	1.000

Defined Parameters:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
linear1	0.430	0.328	1.310	0.190	-1.495	-1.495
linear2	0.568	0.530	1.074	0.283	-2.272	-2.272

Output for Model 2 (Testing Linear Growth using LRT)

```
lavaan 0.6-3 ended normally after 77 iterations

  Optimization method                           NLMINB
  Number of free parameters                         55
  Number of equality constraints                     6

  Number of observations                           357

  Estimator                                         ML
  Model Fit Test Statistic                      79.062
  Degrees of freedom                                41
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic             3173.220
  Degrees of freedom                                66
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.988
  Tucker-Lewis Index (TLI)                       0.980

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -6426.393
  Loglikelihood unrestricted model (H1)      -6386.861

  Number of free parameters                         49
  Akaike (AIC)                               12950.785
  Bayesian (BIC)                             13140.794
  Sample-size adjusted Bayesian (BIC)        12985.343
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.051
  90 Percent Confidence Interval          0.034  0.068
  P-value RMSEA <= 0.05                          0.438

Standardized Root Mean Square Residual:

  SRMR                                           0.050

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Time1 =~
    V1        (la)    0.973    0.080   12.229    0.000    0.916    0.723
    V5                1.000                               0.942    0.553
    V9        (lb)    1.110    0.095   11.630    0.000    1.045    0.842
  Time2 =~
    V2        (la)    0.973    0.080   12.229    0.000    0.920    0.699
    V6                1.000                               0.946    0.498
    V10       (lb)    1.110    0.095   11.630    0.000    1.049    0.762
  Time3 =~
    V3        (la)    0.973    0.080   12.229    0.000    1.010    0.724
    V7                1.000                               1.038    0.495
    V11       (lb)    1.110    0.095   11.630    0.000    1.152    0.809
  Time4 =~
    V4        (la)    0.973    0.080   12.229    0.000    1.031    0.735
    V8                1.000                               1.060    0.475
    V12       (lb)    1.110    0.095   11.630    0.000    1.177    0.791
  Int =~
    Time1             1.000                               0.954    0.954
```

```
    Time2             1.000                                   0.951    0.951
    Time3             1.000                                   0.866    0.866
    Time4             1.000                                   0.848    0.848
  Growth =~
    Time1             0.000                                   0.000    0.000
    Time2             1.000                                   0.283    0.283
    Time3             2.000                                   0.515    0.515
    Time4             3.000                                   0.757    0.757

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .V1 ~~
   .V2                0.475    0.077    6.139    0.000    0.475    0.576
   .V3                0.401    0.075    5.344    0.000    0.401    0.476
   .V4                0.426    0.074    5.744    0.000    0.426    0.512
 .V2 ~~
   .V3                0.513    0.085    6.031    0.000    0.513    0.568
   .V4                0.406    0.084    4.827    0.000    0.406    0.453
 .V3 ~~
   .V4                0.492    0.094    5.227    0.000    0.492    0.538
 .V5 ~~
   .V6                1.453    0.161    9.020    0.000    1.453    0.622
   .V7                1.265    0.167    7.577    0.000    1.265    0.490
   .V8                1.159    0.174    6.655    0.000    1.159    0.416
 .V6 ~~
   .V7                2.243    0.214   10.467    0.000    2.243    0.747
   .V8                2.126    0.222    9.599    0.000    2.126    0.657
 .V7 ~~
   .V8                2.842    0.260   10.931    0.000    2.842    0.794
 .V9 ~~
   .V10               0.191    0.083    2.308    0.021    0.191    0.321
   .V11               0.115    0.078    1.483    0.138    0.115    0.206
   .V12               0.120    0.078    1.546    0.122    0.120    0.197
 .V10 ~~
   .V11               0.366    0.095    3.841    0.000    0.366    0.491
   .V12               0.383    0.099    3.853    0.000    0.383    0.472
```

```
  .V11 ~~
    .V12              0.355    0.109    3.262    0.001    0.355    0.466
   Int ~~
     Growth          -0.057    0.024   -2.321    0.020   -0.236   -0.236

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
     Time1            0.000                               0.000    0.000
     Time2            0.000                               0.000    0.000
     Time3            0.000                               0.000    0.000
     Time4            0.000                               0.000    0.000
     Int              1.351    0.129   10.445    0.000    1.504    1.504
     Growth           0.355    0.037    9.713    0.000    1.327    1.327
    .V1               0.000                               0.000    0.000
    .V2               0.000                               0.000    0.000
    .V3               0.000                               0.000    0.000
    .V4               0.000                               0.000    0.000
    .V5              -0.495    0.141   -3.503    0.000   -0.495   -0.291
    .V6              -0.472    0.170   -2.775    0.006   -0.472   -0.248
    .V7              -0.624    0.199   -3.130    0.002   -0.624   -0.297
    .V8              -0.660    0.229   -2.888    0.004   -0.660   -0.296
    .V9              -0.952    0.135   -7.063    0.000   -0.952   -0.767
    .V10             -0.986    0.165   -5.962    0.000   -0.986   -0.716
    .V11             -1.263    0.194   -6.504    0.000   -1.263   -0.887
    .V12             -1.558    0.226   -6.900    0.000   -1.558   -1.047

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    .V1               0.766    0.088    8.677    0.000    0.766    0.477
    .V2               0.886    0.100    8.865    0.000    0.886    0.511
    .V3               0.923    0.111    8.347    0.000    0.923    0.475
    .V4               0.907    0.115    7.895    0.000    0.907    0.460
    .V5               2.011    0.169   11.913    0.000    2.011    0.694
    .V6               2.715    0.223   12.182    0.000    2.715    0.752
    .V7               3.321    0.270   12.298    0.000    3.321    0.755
    .V8               3.853    0.311   12.383    0.000    3.853    0.774
```

```
   .V9                 0.447    0.093    4.809    0.000    0.447    0.290
   .V10                0.793    0.115    6.868    0.000    0.793    0.419
   .V11                0.702    0.124    5.663    0.000    0.702    0.346
   .V12                0.830    0.136    6.084    0.000    0.830    0.375
    Time1              0.079    0.045    1.749    0.080    0.089    0.089
    Time2              0.128    0.034    3.814    0.000    0.143    0.143
    Time3              0.211    0.043    4.848    0.000    0.196    0.196
    Time4              0.014    0.049    0.276    0.783    0.012    0.012
    Int                0.808    0.137    5.918    0.000    1.000    1.000
    Growth             0.072    0.015    4.688    0.000    1.000    1.000
```

```
 Model Comparisons

Chi Square Difference Test

     Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit1 39 12952 13150 76.481
fit2 41 12951 13141 79.062     2.5812       2     0.2751
```

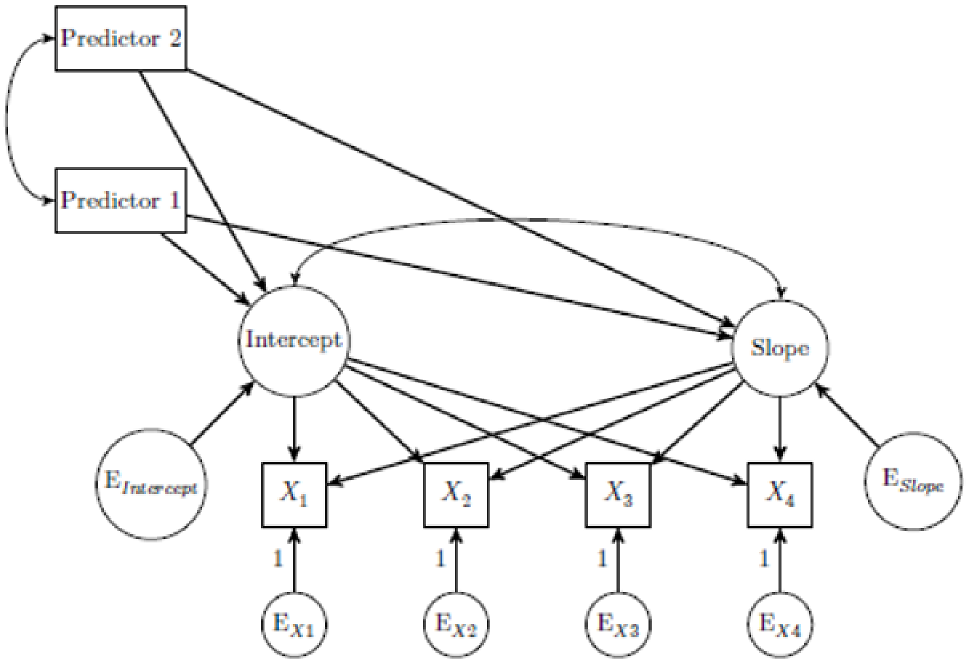
8.9. LGM with Covariates

8.9.1. Models with Time-Invariant Covariates

• Time-invariant covariates are stable measures over all the data collection time periods.

• Examples are number of siblings, socioeconomic status.

• The following is a 4-point LGM model with 2 time-invariant covariates, Predictor 1 and Predictor 2 (Beaujean, 2014; Figure 5.3a):

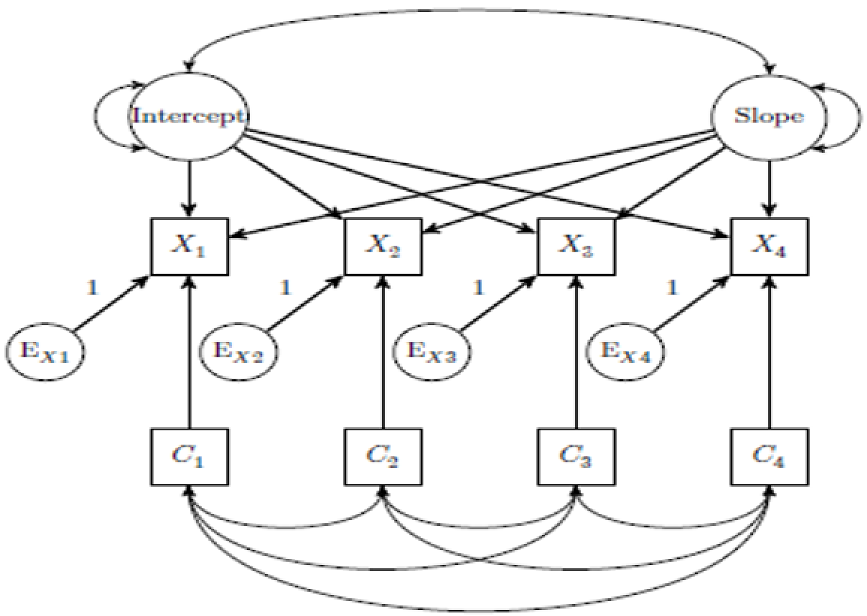


8.9.2 Models with Time-Dependent Covariates

• Time-dependent covariates vary with time.

• Examples are investment return, mood, blood pressure.

• The following is a 4-point LGM model one time-dependent covariate, *C* (Beaujean, 2014; Figure 5.3b):



8.10. Limitations

- Balanced-on-time design (Ware, 1985)

    - equal number of observations for all individuals

    - equal spacing of assessments for all individuals