## Exercises for Chapter 2

1. Show that the process $Z_t = a_t a_{t-2}$ is weakly stationary, where $a_t$ is a white noise sequence with zero mean and variance $\sigma_a^2$

2. Let $X_t = (-1)^t Z$, where $X$ is a random variable
   Give necessary condition(s) on Z so that $X_t$ is weakly stationary.

3. Is it possible to have series $\{Z_t\}$ with a constant mean and $\mathrm{Corr}(Z_t, Z_{t+k})$ free of $t$, but $\{Z_t\}$ is not stationary? If the answer is yes, give an example. If the answer is no, explain why.

4. Let $W_t = (-1)^t Y_t$ where $Y_t$ are independently and identically distributed with $P(Y_t = k) = 0.2, k = -2, -1, 0, 1, 2$ for all integers $t$.

   (a) Is $W_t$ strictly stationary?

   (b) Is $W_t$ weakly stationary?

5. Suppose that $\{a_t\}_{t=1,\dots}$ are independent and identically distributed random variables with mean 0 and finite variance $\sigma_a^2$, and $\theta_i$ is a constant for $i = 0, 1, 2, 3, 4$. If

$$X_t = \theta_0 a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \theta_3 a_{t-3} + \theta_4 a_{t-4},$$

   then compute the autocorrelation function $\rho(k)$ of $X_t$ for $k = 0, 1, 2, \dots..$

6. Consider the process $Z_t = a_t - 1.7a_{t-1} + 0.72a_{t-2}$ where $\{a_t\}$ are i.i.d. with finite variance $\sigma_a^2 = 4$.

   (1) Find $\mathrm{Var}(Z_t)$.

   (2) Find the autocovariance function of the process.

   (3) Find $\mathrm{Var}(\frac{1}{15}\sum_{t=1}^{15} Z_t)$.

7. Consider a time series $(-1.08, -0.33, 0.18, 0.42, 0.18, -0.29, -0.03,$
   $-1.09, 0.18, -0.62, -2.18, -2.87, -3.61, -3.46, -3.92)$.

   (1) Find the sample ACVF $C(h)$ for $h = 0, 1, \dots, 5$.

   (2) Find the sample ACF $r(h)$ for $h = 0, 1, \dots, 5$.

   (3) Is this time series a white noise? Why?