STAT 6104 Financial Time Series
Exercise 7

1) Let X_t be an ARCH(1) process $X_t = \epsilon_t \sigma_t$ with

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 \, .$$

   a) Show that

$$E(\sigma_t^4) = \frac{\alpha_0^2}{1-\alpha_1} \frac{1+\alpha_1}{1-3\alpha_1^2} \, .$$

   b) Deduce that

$$E(X_t^4) = 3 \frac{\alpha_0^2}{1-\alpha_1} \frac{1+\alpha_1}{1-3\alpha_1^2} \, .$$

2) Show that for an IGARCH(1,1) model, for $j > 0$,

$$E(\sigma_{t+s}^2 | \mathcal{F}_{t-1}) = j\alpha_0 + \sigma_t^2 \, .$$

3) Let X_t follows a GARCH(2,3) model,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{2} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{3} \alpha_j X_{t-j}^2 \, .$$

Show that X_t^2 can be written as an ARMA(3,2) model in terms of the process $\nu_t = \sigma_t^2(\epsilon_t^2 - 1)$. Identify the parameters of the ARMA process in terms of the parameters of the given GARCH(2,3) model.

4) Perform a GARCH analysis on the U.S. Treasury bill data set `ustbill.dat` in `http://www.sta.cuhk.edu.hk/NHCHAN/TSBook2nd/dataset.html`.