Summary of Chapter 8

1 Concepts

- **Difference Between Two Means**:

1. σ_1 **and** σ_2 **are known (**Z **test)**: Test statistic $Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

Two-Tail Test: H_0: $\mu_1 - \mu_2 = 0$, H_1: $\mu_1 - \mu_2 \neq 0$

1) Use critical value: If $Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

3) Use C.I.: The C.I. for $\mu_1 - \mu_2$ is: $(\bar{X}_1 - \bar{X}_2) \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$.

If the C.I. does not include $\mu_1 - \mu_2$, reject H_0.

Upper-Tail Test: H_0: $\mu_1 - \mu_2 \leq 0$, H_1: $\mu_1 - \mu_2 > 0$

1) Use critical value: If $Z > Z_\alpha$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

Lower-Tail Test: H_0: $\mu_1 - \mu_2 \geq 0$, H_1: $\mu_1 - \mu_2 < 0$

1) Use critical value: If $Z < -Z_\alpha$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

2. σ_1 and σ_2 are unknown but assumed equal (pooled t test):

Test statistic $t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$, $\quad S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}}$

Two-Tail Test: H_0: $\mu_1 - \mu_2 = 0$, H_1: $\mu_1 - \mu_2 \neq 0$

1) Use critical value: If $t > t_{\alpha/2, n_1+n_2-2}$ or $t < -t_{\alpha/2, n_1+n_2-2}$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

3) Use C.I.: The C.I. for $\mu_1 - \mu_2$ is $(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2, n_1+n_2-2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

If the C.I. does not include $\mu_1 - \mu_2$, reject H_0.

Upper-Tail Test: H_0: $\mu_1 - \mu_2 \leq 0$, H_1: $\mu_1 - \mu_2 > 0$

1) Use critical value: If $t > t_{\alpha, n_1+n_2-2}$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

Lower-Tail Test: H_0: $\mu_1 - \mu_2 \geq 0$, H_1: $\mu_1 - \mu_2 < 0$

1) Use critical value: If $t < -t_{\alpha, n_1+n_2-2}$, reject H_0.

2) Use p-value: If p-value $< \alpha$, reject H_0.

- **Related Samples: Tests Means of Two Related Populations**

Two-Tail Test: $H_0 : \mu_{\bar{D}} = 0, \quad H_1 : \mu_{\bar{D}} \neq 0$

One-Tail Test: $H_0 : \mu_{\bar{D}} \leq 0 \; (\mu_{\bar{D}} \geq 0) \,, \quad H_1 : \mu_{\bar{D}} > 0 \; (\mu_{\bar{D}} < 0)$

1. σ_D **is known (**Z **test)**: Test statistic

$$Z = \frac{\bar{D} - \mu_D}{\frac{\sigma_D}{\sqrt{n}}}, \quad \text{where} \quad \bar{D} = \frac{\sum_{i=1}^{n} D_i}{n}$$

The C.I. for μ_D is $\bar{D} \pm Z_{\alpha/2} \frac{\sigma_D}{\sqrt{n}}$

2. σ_D **is unknown (**t **test)**: Test statistic

$$t = \frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}, \quad \text{where} \quad S_D = \sqrt{\frac{\sum_{i=1}^{n} (D_i - \bar{D})^2}{n-1}}$$

The C.I. for μ_D is $\bar{D} \pm t_{\alpha/2, n-1} \frac{S_D}{\sqrt{n}}$

Three approaches (critical value, p-value, and C.I.) can be used similarly as one sample test.

- **Two Population Proportion**:

  $(n_1p_1 \geq 5, n_1(1-p_1) \geq 5 \;\text{ and }\; n_2p_2 \geq 5, n_2(1-p_2) \geq 5)$

  Two-Tail Test: $H_0 : p_1 = p_2, \quad H_1 : p_1 \neq p_2$

  One-Tail Test: $H_0 : p_1 \leq p_2 \; (p_1 \geq p_2) \,, \quad H_1 : p_1 > p_2 \; (p_1 < p_2)$

  (Pooled) Z test: The test statistic is

$$Z = \frac{(p_{s_1} - p_{s_2}) - (p_1 - p_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \qquad \bar{p} = \frac{X_1 + X_2}{n_1 + n_2}, \;\; p_{s_1} = \frac{X_1}{n_1}, \;\; p_{s_2} = \frac{X_2}{n_2}$$

  The C.I. for $p_1 - p_2$ is

$$(p_{s_1} - p_{s_2}) \pm Z_{\alpha/2}\sqrt{\frac{p_{s_1}(1-p_{s_1})}{n_1} + \frac{p_{s_2}(1-p_{s_2})}{n_2}}$$

  Three approaches (critical value, p-value, and C.I.) can be used similarly as one sample test.

- **Hypothesis Tests for Variances**

F test: The test statistic is

$$F = \frac{S_1^2}{S_2^2} \qquad \text{where} \quad \text{df}_1 = n_1 - 1, \ \ \text{df}_2 = n_2 - 1$$

Two-Tail Test: H_0: $\sigma_1^2 = \sigma_2^2$, H_1: $\sigma_1^2 \neq \sigma_2^2$

If $F > F_U$ or $F < F_L$, reject H_0.

$F_U = F_{\alpha/2, n_1-1, n_2-1}$, $\quad F_{U^*} = F_{\alpha/2, n_2-1, n_1-1}$, and $\quad F_L = \frac{1}{F_{U^*}}$

Upper-Tail Test: H_0: $\sigma_1^2 \leq \sigma_2^2$, H_1: $\sigma_1^2 > \sigma_2^2$

If $F > F_U$, reject H_0.

Lower-Tail Test: H_0: $\sigma_1^2 \geq \sigma_2^2$, H_1: $\sigma_1^2 < \sigma_2^2$

If $F < F_L$, reject H_0.

2 Examples

Example 1. Shipments of meat, meat by-products, and other ingredients are mixed together in several filling lines at a pet food canning factory. After the ingredients are thoroughly mixed, the pet food is placed in eight-ounce cans. Descriptive statistics concerning fill weights from two production lines, obtained from two independent samples are given in the following table.

	Line A	Line B
$\bar{X}$	8.005	7.997
S	0.012	0.005
n	11	16

a. At the 0.05 level of significance, is there evidence of a difference between the average weight of cans filled on the two lines?

b. Calculate the p-value in (a).

c. What assumptions do you have to make about the two populations in order to justify the use of the t test?

a. $H_0: \ \mu_1 = \mu_2 \quad \text{vs} \quad H_1: \mu_1 \neq \mu_2$.

Populations: 1 = Line A and 2 = Line B.

$\alpha = 0.05$, $df = n_1 + n_2 - 2 = 25$, and $t_{0.025,25} = 2.0595$, so we reject H_0 when $|t| > 2.0595$.

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)} = \frac{(11-1)\times 0.012^2 + (16-1)\times 0.005^2}{(11-1)+(16-1)} = 7.26 \times 10^{-5}.$$

The t statistic is:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(8.005 - 7.997) - 0}{\sqrt{7.26 \times 10^{-5} \times (\frac{1}{11} + \frac{1}{16})}} = 2.3972 > 2.0595.$$

So H_0 is rejected. There is enough evidence of a difference in the average weight of cans filled on the two lines.

b. p-value = $2P(t > 2.3972) = 0.0243 < 0.05$. According to the p-value, we also reject H_0.

c. We need to assume that the populations are normally distributed with equal variance.

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Example 2. Can student save money by buying their textbooks at *Amazon.com*? To investigate this possibility, a random sample of 15 textbooks used during the spring 2001 semester at Miami University was selected. The prices for these textbooks at both a local bookstore and through *Amazon.com* were recorded. The prices for the textbooks, including all relevant taxes and shipping are given below:

Textbook	Bookstore	*Amazon.com*
Access 2000 Guidebook	52.22	57.34
HTML 4.0 CD with Java Script	52.74	44.47
Designing the Physical Education Curriculum	39.01	41.48
Service Management: Operations, Strategy and IT	101.28	73.72
Fundamentals of Real Estate Appraisal	37.45	42.04
Investments	113.41	95.38
Intermediate Financial Management	109.72	119.80
Real Estate Principles	101.28	62.48
The Automobile Age	29.49	32.43
Geographic Information Systems in Ecology	70.07	74.43
Geosystems: An Introduction to Physical Geography	83.87	83.81
Understanding Contemporary Africa	23.21	26.48
Early Childhood Education Today	72.80	73.48
System of Transcendental Idealism (1800)	17.41	20.98
Principles and Lab for Fitness and Wellness	37.72	40.43

a. At the 0.01 level of significance, is there evidence of a difference between the mean price of textbooks at the local bookstore and *Amazon.com*?

b. What assumption is necessary to perform this test?

c. Find the p-value in (a) and interpret its meaning.

d. Set up a 99% confidence interval estimate of the mean difference in price. Interpret the interval.

e. Compare the results of (a) and (d).

a. $H_0 : \mu_{\bar{D}} = 0$ (There is no difference in the average price of textbooks between the local bookstore and *Amazon.com*) vs $H_1 : \mu_{\bar{D}} \neq 0$ (There is a difference in the average price of textbooks between the local bookstore and *Amazon.com*).

$\alpha = 0.01$, $\bar{D} = 3.5307$, $n = 15$, $df = n - 1 = 14$, $S_D = 13.8493$, and $t_{0.005,14} = 2.9768$, so H_0 is rejected when $|t| > 2.9768$. The t statistic is:

$$t = \frac{\bar{D} - \mu_{\bar{D}}}{S_D/\sqrt{n}} = \frac{3.5307 - 0}{13.8493/\sqrt{15}} = 0.9874 < 2.9768.$$

We cannot reject H_0, there is no enough evidence to conclude that there is a difference in the average price of textbooks between the local bookstore and *Amazon.com*.

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b. We have to assume the the average price of textbook from the local bookstore and *Amazon.com* are normally distributed.

c. p-value = $2P(t > 0.9874) = 0.3402 > 0.05$.

Under null hypothesis, the probability of obtaining a mean difference that gives rise to a test statistic that deviates from 0 by 0.9874 or more is 0.3402.

d.

$$\bar{D} \pm t_{\alpha/2,n-1}\frac{S_D}{\sqrt{n}} = 3.5307 \pm 2.9768\frac{13.8493}{\sqrt{15}}$$

$$\text{or} \qquad -7.1141 < \mu_D < 14.1755$$

e. The results in (a) and (d) are the same. The hypothesized value of 0 for the difference in the average price for textbooks between the local bookstore and *Amzon.com* is inside the 99% confidence interval.

Example 3. The results of a study conducted as part of a yield-improvement effort at a semiconductor manufacturing facility provided defect data for a sample of 450 wafers. The following contingency table presents a summary of the responses to two questions: “Was a particle found on the die that produced the wafer?” and “Is the wafer good or bad?”

	Quality of Wafer		
Particles of Wafer	Good	Bad	Totals
Yes	14	36	50
No	320	80	400
Total	334	116	450

a. At the 0.05 level of significance, is there a significant difference between the proportion of good and bad wafers that have particles?

b. Calculate the p-value in (a) and interpret its meaning.

c. Set up a 95% confidence interval estimate of the difference between the population proportion of good and bad wafers that contains particles.

d. What conclusions can you draw from this analysis?

a. $H_0: \; p_1 = p_2$ vs $H_1: \; p_1 \neq p_2$.

Populations: 1 = good, 2 = bad.

$\alpha = 0.05$, and $Z_{0.025} = 1.96$, so we reject H_0 when $|Z| > 1.96$.

$$n_1 = 334, \quad n_2 = 116, \quad p_{s_1} = \frac{14}{334} = 0.0419 \quad p_{s_2} = \frac{36}{116} = 0.3103,$$

$$\bar{p} = \frac{14+36}{334+116} = \frac{50}{450} = 0.111.$$

The test statistic is:

$$Z = \frac{(p_{s_1} - p_{s_2}) - (p_1 - p_2)}{\sqrt{\bar{p}(1-\bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(0.0419 - 0.3103) - 0}{\sqrt{0.111(1-0.111)(\frac{1}{334} + \frac{1}{116})}} = -7.9254.$$

Since $|Z| > 1.96$, we reject H_0.

There is enough evidence that a significant difference exists in the proportion of good and bad wafers that have particles.

b. p-value = $2P(Z < -7.9254) = 2.27 \times 10^{-15} \simeq 0 < 0.05.$

Under null hypothesis, the probability of obtaining a test statistic as small as -7.9254 or smaller is 0.

c.

$$(p_{s_1} - p_{s_2}) \pm Z_{\alpha/2}\sqrt{\frac{p_{s_1}(1-p_{s_1})}{n_1} + \frac{p_{s_2}(1-p_{s_2})}{n_2}}$$

$$= -0.2684 \pm 1.96\sqrt{\frac{0.0419(0.9581)}{334} + \frac{0.3103(0.6897)}{116}}$$

$$\text{or} \qquad -0.3553 < p_{s_1} - p_{s_2} < -0.1815$$

Note: The hypothesized value of 0 is outside the 95% confidence interval.

d. There is significant difference in the proportion of good and bad wafers that have particles.

Example 4. The computer Anxiety Rating Scale (CARS) measures an individual's level of computer anxiety on a scale from 20 (no anxiety) to 100 (highest level of anxiety). Researchers at Miami University administered CARS to 172 business students. One of the objectives of the study was to determine if there is a difference between the level of computer anxiety experienced by female students and male students.

	Males	Females
$\bar{X}$	40.26	36.85
S	13.35	9.42
n	100	72

a. At the 0.05 level of significance, is there evidence of a difference in the variability of the computer anxiety experience by females and males?

b. Calculate the p-value.

c. What assumptions do you have to make about the two populations in order to justify the use of the F test?

a. $H_0: \ \sigma_1^2 = \sigma_2^2$ vs $H_1: \ \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05, \ S_1 = 13.35, \ S_2 = 9.42, \ df_1 = n_1 - 1 = 99, \ df_2 = n_2 - 1 = 71.$

$F_{0.975,99,71} = 1.556, \ \ F_{0.025,99,71} = 1/F_{0.975,71,99} = 0.653$

R command: qf(c(0.975, 0.025), 99, 71)

So we reject H_0 when $F > 1.556$ or $F < 0.653$. The test statistic is:

$$F = \frac{S_1^2}{S_2^2} = \frac{13.35^2}{9.42^2} = 2.008 > 1.559.$$

We reject H_0, there is enough evidence to conclude that the two population variances are different.

b. p-value = $2P(F > 2.008) = 0.0022.$

R command: 2*[1-pf(2.008,99,71)]

c. We need to assume that each population is normally distributed.

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