To test a hypothesis, we take a random sample from the population under study, compute an appropriate **test statistic,** and then either reject or fail to reject the null hypothesis H_0. The set of values of the test statistic leading to rejection of H_0 is called the **critical region** or **rejection region** for the test.

Two kinds of errors may be committed when testing hypotheses. If the null hypothesis is rejected when it is true, then a type I error has occurred. If the null hypothesis is not rejected when it is false, then a type II error has been made. The probabilities of these two types of errors are denoted as

$$\alpha = P\{\text{type I error}\} = P\{\text{reject } H_0 | H_0 \text{ is true}\}$$ → producer's risk

$$\beta = P\{\text{type II error}\} = P\{\text{fail to reject } H_0 | H_0 \text{ is false}\}$$ → consumer's risk

Sometimes it is more convenient to work with the **power** of the test, where

$$\text{Power} = 1 - \beta = P\{\text{reject } H_0 | H_0 \text{ is false}\}$$ → probability of correctly reject H_0

Thus, the power is the probability of *correctly* rejecting H_0.

β — reflect the sensitivity of the hypothesis testing

Power — reflect the ability to detect a false hypothesis

3-3.6 The Probability of Type II Error and Sample Size Decisions

$$\beta = \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) \tag{3-46}$$

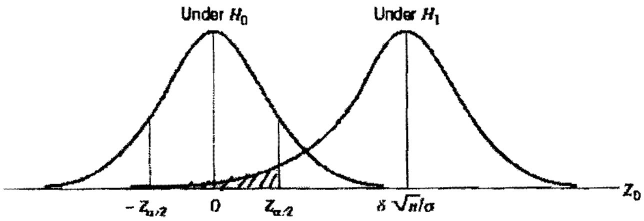

Figure 3-6 The distribution of Z_0 under H_0 and H_1.

$H_0: \mu = \mu_0$ vs. $H_1: \mu = \mu_1 = \mu_0 + \delta$

$$\beta = P\left(\left| \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right| < Z_{\alpha/2} \;\middle|\; H_0 \text{ is false} \right)$$

$$= P\left(-Z_{\alpha/2} < \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} < Z_{\alpha/2} \;\middle|\; \mu = \mu_0 + \delta \right)$$

$$= P\left(-Z_{\alpha/2} < \frac{\bar{X} - (\mu_0 + \delta) + \delta}{\frac{\sigma}{\sqrt{n}}} < Z_{\alpha/2} \;\middle|\; \mu = \mu_0 + \delta \right)$$

$$= P\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} < \frac{\bar{X} - (\mu_0 + \delta)}{\frac{\sigma}{\sqrt{n}}} < Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} \;\middle|\; \mu = \mu_0 + \delta \right)$$

$$= \Phi\left(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} \right) - \Phi\left(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma} \right).$$