Exercises for Chapter 5

1. Let $Z_t = a_t + 0.4a_{t-1}$, $a_t \sim N(0, 4)$. Given $Z_1 = 0.5$, $Z_2 = 2.1$, $Z_3 = 1.2$, $Z_4 = -0.8$.

   (a) Using all the information up to day 4, predict Z_5. Find the prediction error and 95% prediction interval.

   (b) Using all the information up to day 4, predict Z_6. Find the prediction error and 95% prediction interval.

   (c) Using all the information up to day 4, predict Z_{100}. Find the prediction error and 95% prediction interval.

2. Let $Z_t = 0.1Z_{t-1} + 0.2Z_{t-2} + a_t$ with $\sigma_a^2 = 25$. Let $Z_{199} = 120$, $Z_{200} = 100$.

   (a) Find the Z_{201}^{200}, Z_{202}^{200} and Z_{205}^{200}.

   (b) Find the 95% prediction interval of Z_{201} and the 95% prediction interval of Z_{203}.

   (c) Update your forecast for Z_{205} given $Z_{201} = 11$.

   (d) Find $\text{Cov}[e_t(m), e_t(k)]$, where m and k are both positive integers.