5 Structural Equation Modeling 2: General Structural Models

References:

• Beaujean (2014). Chapter 3.

5.1. Latent Variable Analysis

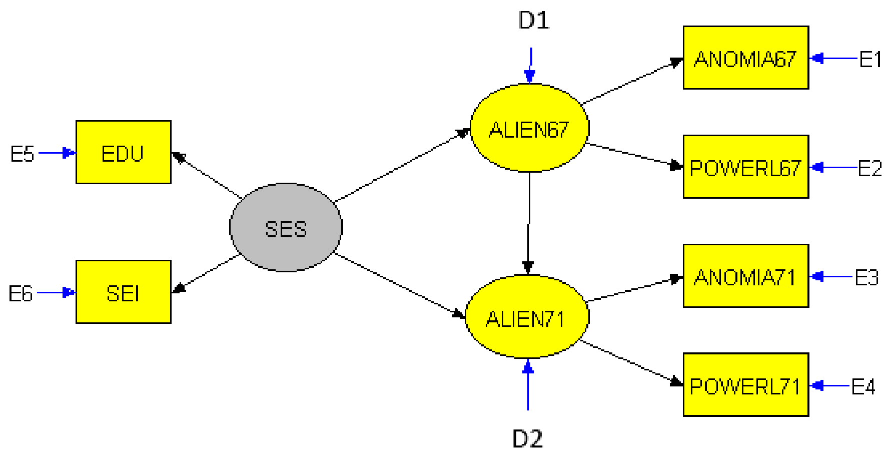
• Example 1. Stability of Alienation (Wheaton et al., 1977)

• Filename: *alien.cov* ($N = 932$)

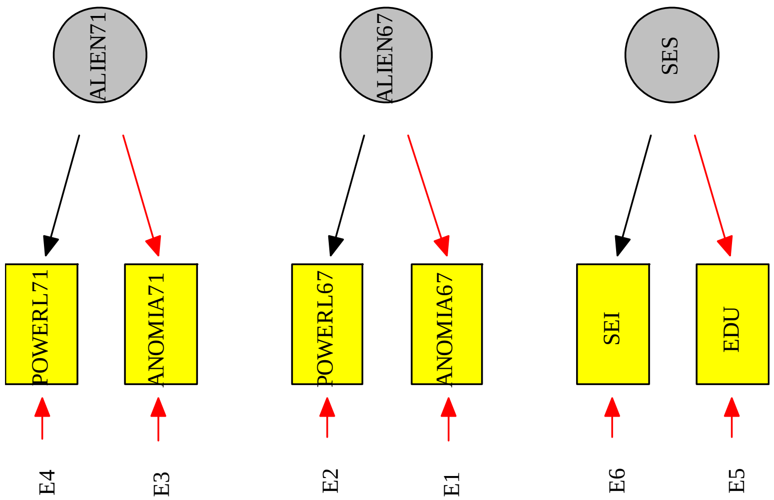
	ANOMIA67	POWERL67	ANOMIA71	POWERL71	EDUC	SEI
ANOMIA67	11.83					
POWERL67	6.95	9.36				
ANOMIA71	6.82	5.09	12.53			
POWERL71	4.78	5.03	7.50	9.99		
EDUC	-3.84	-3.89	-3.84	-3.63	9.61	
SEI	-2.19	-1.88	-2.72	-1.88	3.55	4.50

ANOMIA67, ANOMIA71=a scale measured feeling of isolation (in 1967 and 1971
POWERL67, POWERL71=a scale measured powerlessness in 1967 and 1971
EDUC=education level, SEI=socioeconomic index

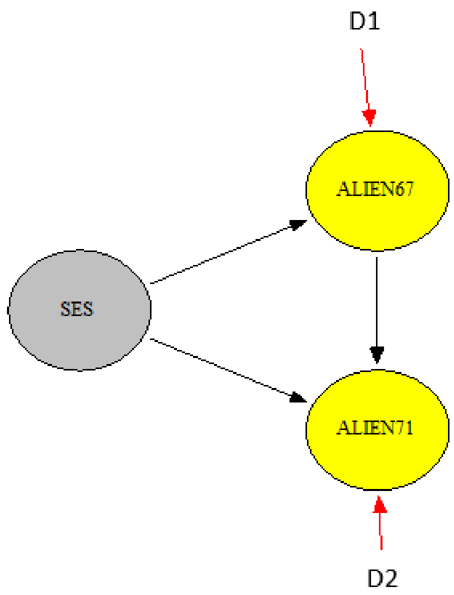
• In general, a latent variable model consists of (1) the measurement part, and (2) the structural part



• The measurement part (CFA) postulates the relationship between the factors and their indicators:



• The structural part postulates the structural relationship among the factors:



• Suppose there is a set of variables *v* that can be characterized by its population covariance matrix, Σ. The ***structural model*** represents our belief about how Σ is structured. That is, $\Sigma = \Sigma(\theta)$.

• Basic tasks involve

- parameter estimation

- model evaluation

- model modification

• We follow the same 5-step procedure as in CFA

5.2. Model Specification

• Measurement equation:

$$v = \mu + \Lambda f + e$$

v	is $p \times 1$ vector of observed variables
$f = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$	is $k \times 1$ vector of latent factors, $k = k_x + k_y$
e	is $p \times 1$ vector of measurement errors
μ	is $p \times 1$ vector of intercepts of v
Λ	is $p \times k$ factor loading matrix

- Structural equation:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \gamma & \beta \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} + \begin{bmatrix} f_x \\ d \end{bmatrix}$$

$$f \quad = \qquad Bf \qquad\quad + \quad z$$

f_x is $k_x \times 1$ vector of exogenous factors
f_y is $k_y \times 1$ vector of endogenous factors
d is $k_y \times 1$ vector of disturbances
γ is $k_y \times k_x$ matrix of structural coefficients from f_x to f_y
β is $k_y \times k_y$ matrix of structural coefficients among f_y

- Combining the measurement and structural equations,

$$v = \mu + \Lambda(I - B)^{-1} z + e$$

- Assuming $\mathrm{cov}(z, e) = 0$, the covariance matrix of v is

$$\Sigma = \Lambda(I - B)^{-1} \Psi (I - B)^{-1\prime} \Lambda' + \Theta = \Sigma(\theta)$$

where Λ is the $p \times k$ factor loading matrix, B is the $k \times k$ structural coefficient matrix, Ψ is the $k \times k$ variance covariance matrix of f_x and d, and Θ is the $p \times p$ variance covariance matrix of e.

- The variables are:

Name	Type	Cause/Effect	Dimension
v	observed	endogenous	$p \times 1$
f_y	latent	endogenous	$k_y \times 1$
f_x	latent	exogenous	$k_x \times 1$
e	latent	exogenous	$p \times 1$
d	latent	exogenous	$k_y \times 1$

- And the parameter matrices are:

Parameter matrix	Symbol	Name	Dimension
factor loading	Λ	lambda	$p \times 1$
variance-covariance matrix of errors, e	Θ	theta	$p \times p$
structural coefficients among the latent variables, f	B	beta	$k \times k$
variance-covariance matrix of f_x and d	Ψ	psi	$k \times k$

5.3. Identification

• The model $\Sigma = \Sigma(\theta)$ is ***identified*** if there are no vectors θ^* and θ such that $\Sigma(\theta^*)=\Sigma(\theta)$ unless $\theta^*=\theta$

• As in CFA, we have to fix the scale of the latent variables, which are, f_y, f_x, e, d

• Use ULI for f_y because the variances of f_y do not exist as free parameters in the system

• The t-$r$ule remains valid (necessary condition for identification)

5.3.1. The two-step rule

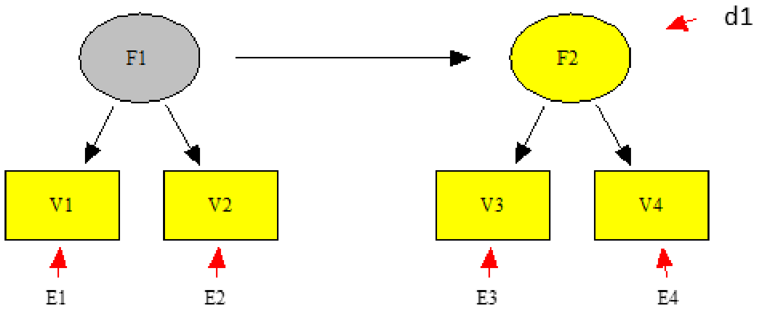
• Step 1: Reformulate the original model as a measurement (CFA) model and establish identification of measurement model

• Step 2: Establish identification of the latent variable model by treating it as structural equation with observed variables

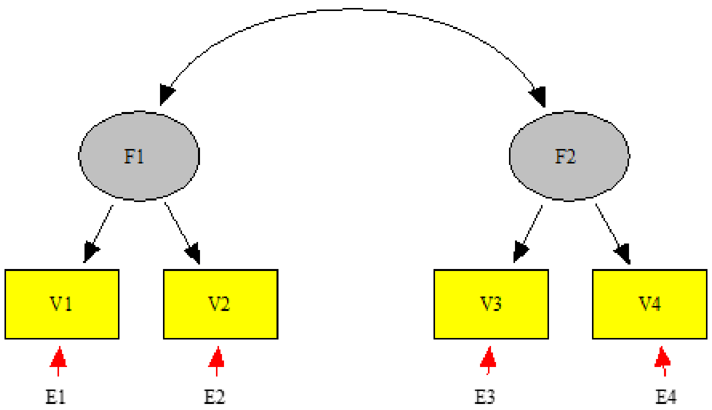
• A sufficient condition for identification

- Example 2

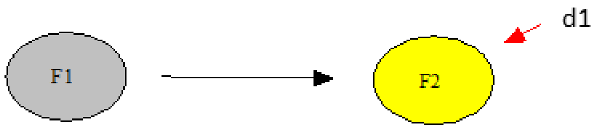
Model



Step 1

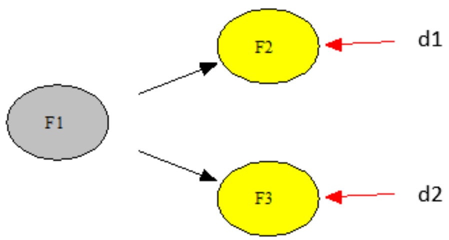


Step 2

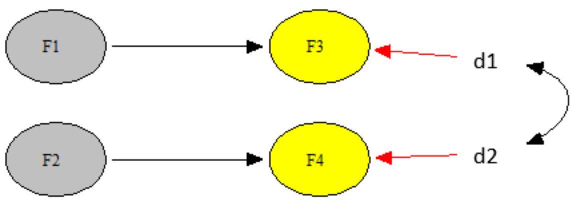


- Some valid Step 2 models (Long, 1983):

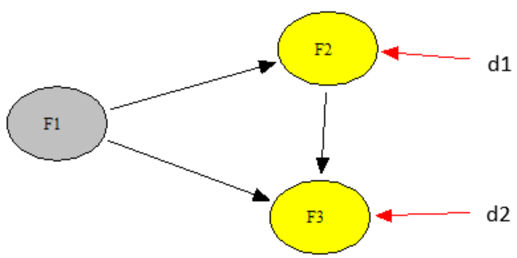
- Type I (regression models)



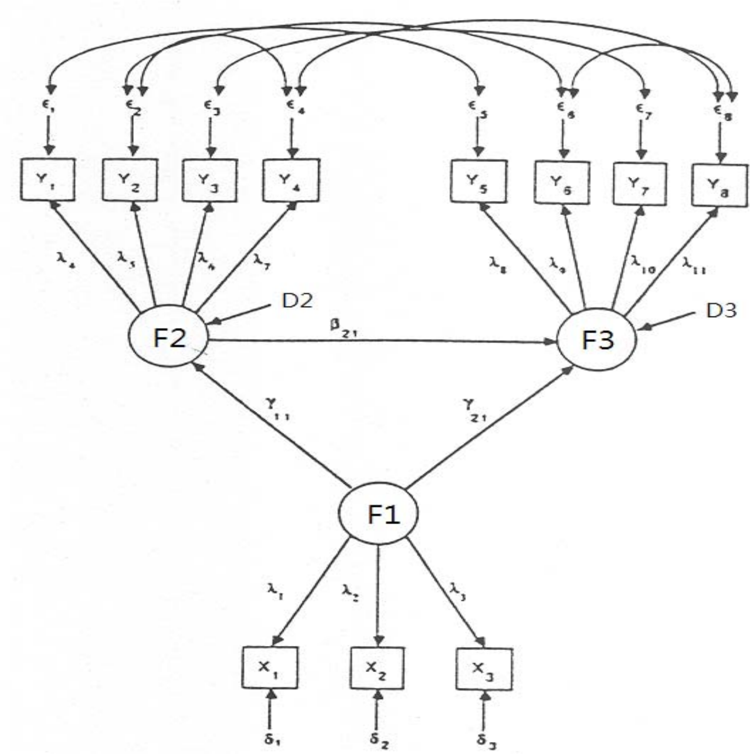
- Type II (seemingly unrelated regression)



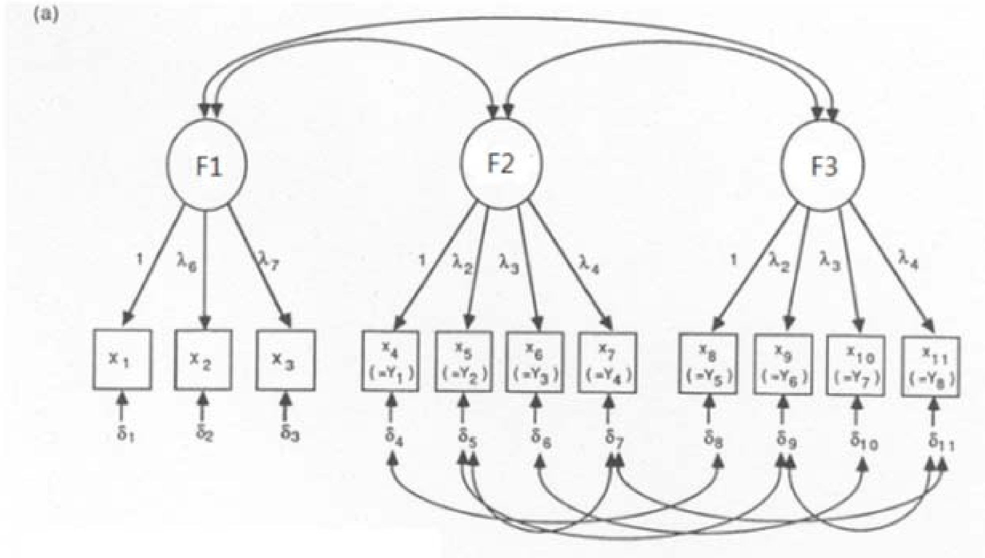
- Type III (recursive models)

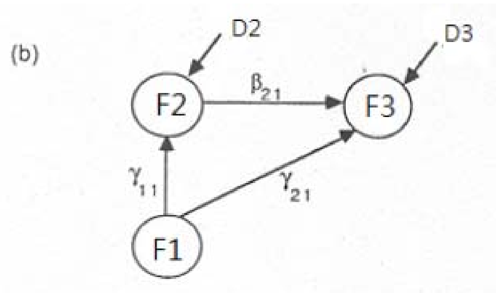


• Example 3: SEM Model of Political Democracy and Industrialization for Developing Countries, 1960 to 1965 (Bollen, 1989):

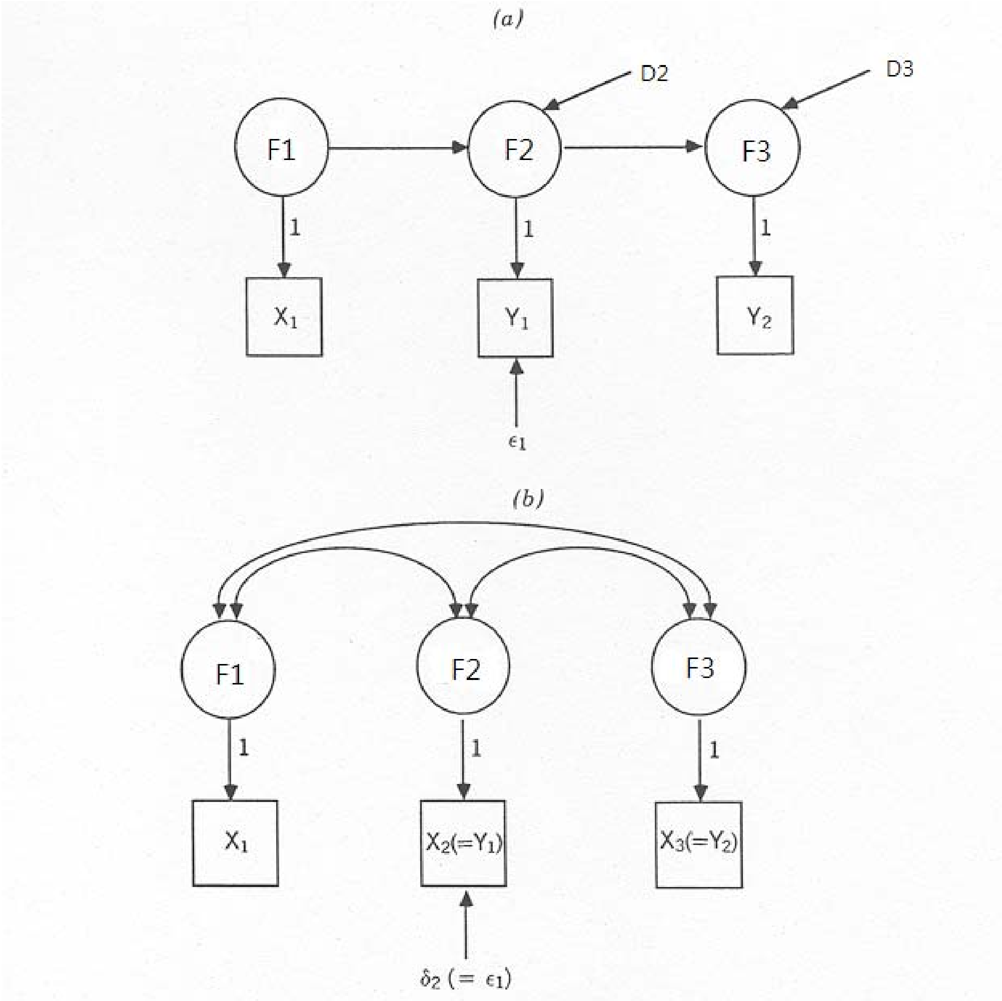


- Same model reformulated for two-step rule:





• Example 4. A model that fails the two-step rule but is identified (Bollen, 1989):



5.4. Estimation

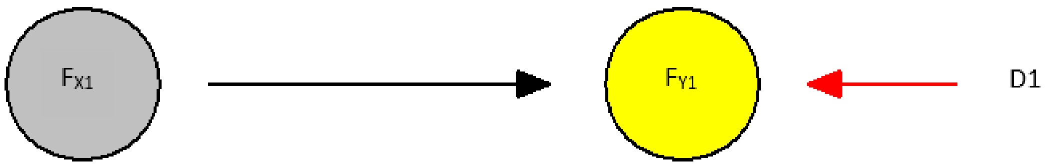
• Estimation methods are basically the same as that in §3.6

5.5. Goodness of Fit Assessment

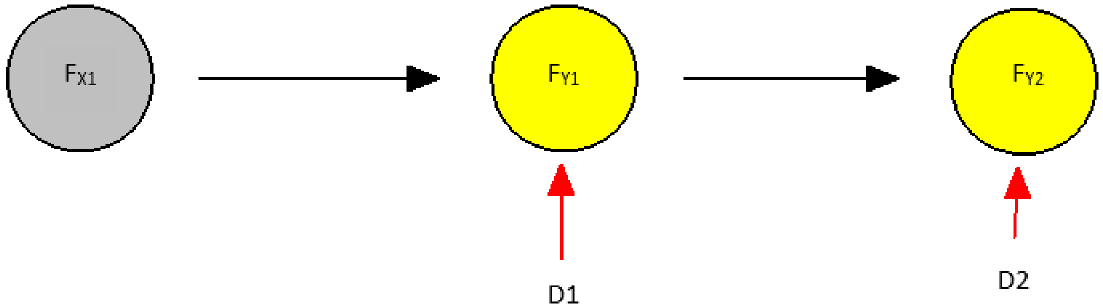
• Same as that in §3.7

5.6. Effect Decomposition

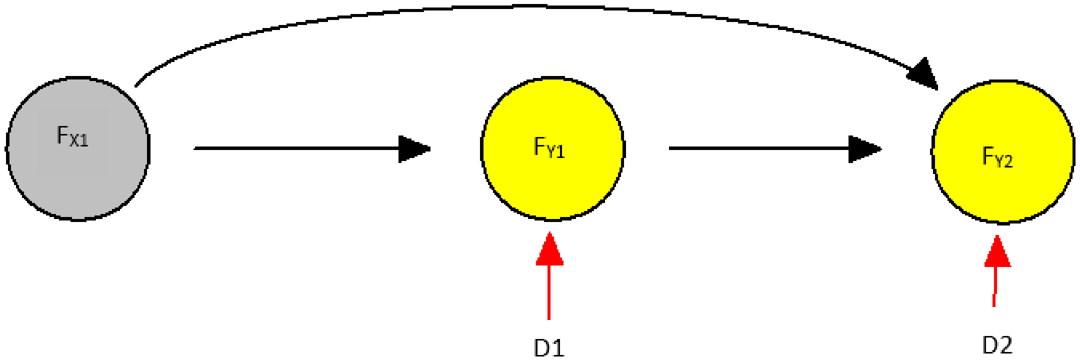
• ***Direct effect***: Influence of one variable on another that is unmediated by any other variables in a path model



• ***Indirect effect:*** Influence of one variable on another is mediated by at least one intervening variable (mediator)

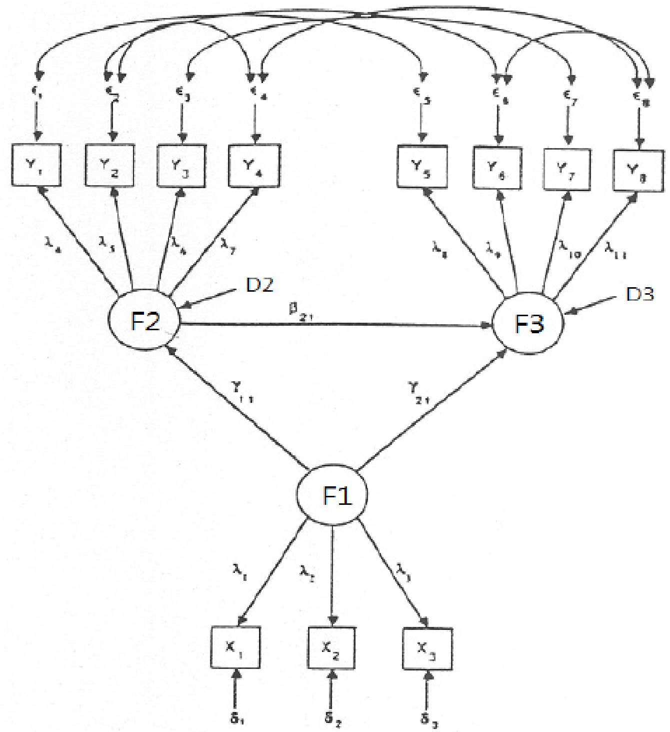


• ***Total effect:*** The total of the direct effect and all indirect effects



F_{X1} to F_{Y2}: direct effect $= \gamma_{21}$
indirect effect $= \gamma_{11}\beta_{21}$
total effect $= \gamma_{21} + \gamma_{11}\beta_{21}$

• Example 3 (cont.):



	Direct Effect	Indirect Effect	Total Effect
F1 → F3			
F1 → Y_8			
F1 → X_1			
F2 → F3			
F2 → Y_8			
F2 → X_1			

5.7. Example 1 (cont.)

filenames: alien.R (R script)

```
# Example 1: Stability of Alienation (Wheaton et al., 1977)

# set work directory
setwd("c:/users/wchan/google drive/stat6108/data")

# load the lavaan package
library(lavaan)

# write the input data into a full covariance matrix
alien <- scan("alien.cov")
alien.cov <- getCov(alien, names=c("anomia67", "powerl67", "anomia71", "powerl71", "educ", "sei"))

# specify Model 1

model1 <- "
# measurement model
 SES =~ educ + sei
 ALIEN67 =~ anomia67 + powerl67
 ALIEN71 =~ anomia71 + powerl71

# structural model
 ALIEN67 ~ a*SES
 ALIEN71 ~ b*ALIEN67 + c*SES

# effect decomposition
 direct := c
 indirect := a*b
 total := direct + indirect
"
```

```
# Fit Model 1 to data
fit1 <- lavaan(model1, sample.cov=alien.cov, sample.cov.rescale=FALSE, sample.nobs=932, auto.var=TRUE,
auto.fix.first=TRUE)
mi1 <- modificationIndices(fit1, sort.=TRUE)
```

```
# specify Model 2 (with error covariances added)

model2 <- "
# measurement model
SES =~ educ + sei
ALIEN67 =~ anomia67 + powerl67
ALIEN71 =~ anomia71 + powerl71

# structural model
ALIEN67 ~ a*SES
ALIEN71 ~ b*ALIEN67 + c*SES

# error covariances
anomia67 ~~ anomia71
powerl67 ~~ powerl71

# effect decomposition
direct := c
indirect := a*b
total := direct + indirect

# comparing direct vs indirect effect
effect_d := indirect - direct
"

# fit Model 2 to data
fit2 <- lavaan(model2, sample.cov=alien.cov, sample.cov.rescale=FALSE, sample.nobs=932, auto.var=TRUE,
auto.fix.first=TRUE)
```

```
# save the output
sink("alien.out", split=TRUE)
writeLines("\n Example 1: Stability of Alienation\n")
writeLines("\n Output for Model 1\n")
inspect(fit1)
summary(fit1, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
writeLines("\n Modification indices\n")
list(mi1)
writeLines("\n Output for Model 2\n")
inspect(fit2)
summary(fit2, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
writeLines("\n Comparing Model 1 and Model 2\n")
lavTestLRT(fit1, fit2)
sink()
```

filename: alien.out (output file)

```
 Example 1: Stability of Alienation

 Output for Model 1

$lambda
         SES ALIEN6 ALIEN7
educ       0      0      0
sei        1      0      0
anomia67   0      0      0
powerl67   0      2      0
anomia71   0      0      0
powerl71   0      0      3

$theta
         educ sei anom67 pwrl67 anom71 pwrl71
educ      7
sei       0    8
anomia67  0    0   9
powerl67  0    0   0     10
anomia71  0    0   0      0     11
powerl71  0    0   0      0      0     12

$psi
        SES ALIEN6 ALIEN7
SES     13
ALIEN67  0  14
ALIEN71  0   0     15

$beta
        SES ALIEN6 ALIEN7
SES       0      0      0
ALIEN67   4      0      0
ALIEN71   6      5      0

lavaan 0.6-3 ended normally after 58 iterations
```

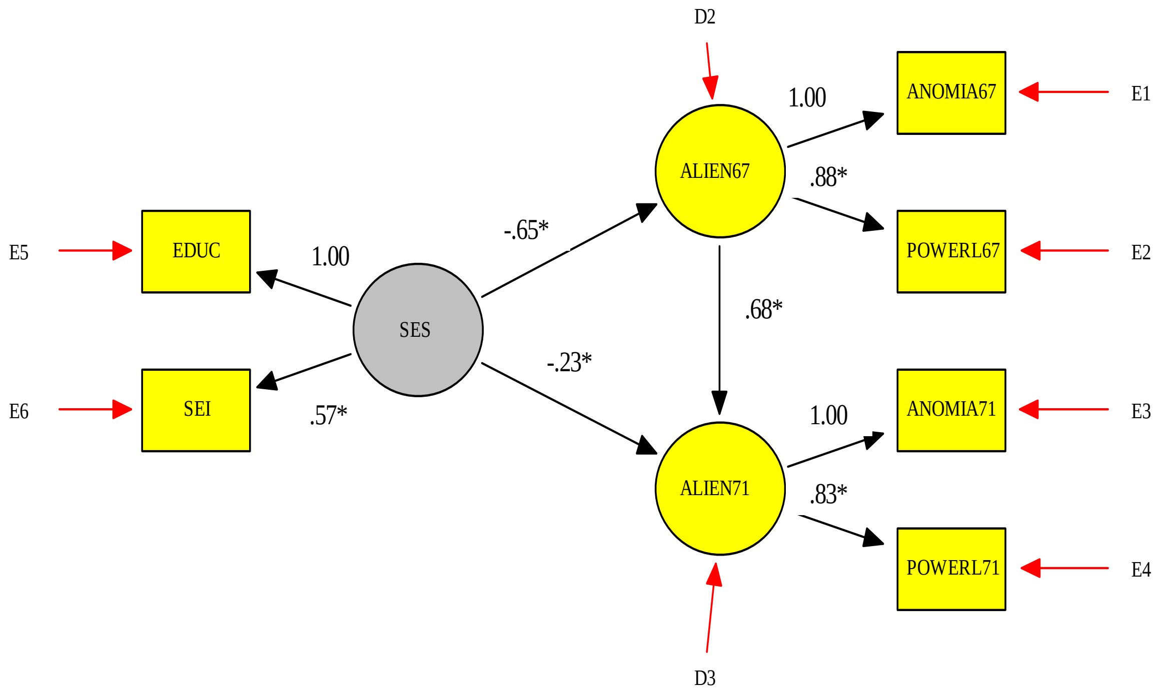
```
  Optimization method                           NLMINB
  Number of free parameters                         15

  Number of observations                           932

  Estimator                                         ML
  Model Fit Test Statistic                      88.293
  Degrees of freedom                                 6
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic             2159.906
  Degrees of freedom                                15
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.962
  Tucker-Lewis Index (TLI)                       0.904

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -13098.966
  Loglikelihood unrestricted model (H1)     -13054.819

  Number of free parameters                         15
  Akaike (AIC)                               26227.931
  Bayesian (BIC)                             26300.491
  Sample-size adjusted Bayesian (BIC)        26252.852

Root Mean Square Error of Approximation:

  RMSEA                                          0.121
  90 Percent Confidence Interval          0.100  0.144
  P-value RMSEA <= 0.05                          0.000
```

```
Standardized Root Mean Square Residual:

  SRMR                                           0.025

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  SES =~
    educ              1.000                               2.492    0.804
    sei               0.572    0.044   13.010    0.000    1.425    0.672
  ALIEN67 =~
    anomia67          1.000                               2.809    0.816
    powerl67          0.881    0.041   21.307    0.000    2.473    0.808
  ALIEN71 =~
    anomia71          1.000                               2.998    0.847
    powerl71          0.834    0.039   21.385    0.000    2.500    0.791

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  ALIEN67 ~
    SES        (a)   -0.646    0.057  -11.250    0.000   -0.574   -0.574
  ALIEN71 ~
    ALIEN67    (b)    0.682    0.053   12.771    0.000    0.639    0.639
    SES        (c)   -0.229    0.058   -3.976    0.000   -0.190   -0.190

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .educ              3.398    0.455    7.462    0.000    3.398    0.354
   .sei               2.472    0.181   13.687    0.000    2.472    0.549
   .anomia67          3.946    0.346   11.420    0.000    3.946    0.333
   .powerl67          3.246    0.273   11.906    0.000    3.246    0.347
   .anomia71          3.546    0.372    9.545    0.000    3.546    0.283
```

```
   .powerl71          3.735    0.289   12.910    0.000    3.735    0.374
    SES               6.212    0.597   10.413    0.000    1.000    1.000
   .ALIEN67           5.292    0.474   11.176    0.000    0.671    0.671
   .ALIEN71           3.738    0.389    9.606    0.000    0.416    0.416

R-Square:
                   Estimate
    educc             0.646
    sei               0.451
    anomia67          0.667
    powerl67          0.653
    anomia71          0.717
    powerl71          0.626
    ALIEN67           0.329
    ALIEN71           0.584

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    direct           -0.229    0.058   -3.976    0.000   -0.190   -0.190
    indirect         -0.441    0.048   -9.215    0.000   -0.367   -0.367
    total            -0.670    0.060  -11.195    0.000   -0.557   -0.557


 Modification indices

[[1]]
        lhs op      rhs     mi    epc sepc.lv sepc.all sepc.nox
44 anomia67 ~~ anomia71 57.693  1.863   1.863    0.498    0.498
46 powerl67 ~~ anomia71 54.108 -1.597  -1.597   -0.471   -0.471
45 anomia67 ~~ powerl71 45.229 -1.429  -1.429   -0.372   -0.372
47 powerl67 ~~ powerl71 41.895  1.219   1.219    0.350    0.350
41      sei ~~ anomia71 18.443 -0.647  -0.647   -0.219   -0.219
37     educ ~~ anomia71 16.531  0.917   0.917    0.264    0.264
36     educ ~~ powerl67 12.065 -0.685  -0.685   -0.206   -0.206
42      sei ~~ powerl71  6.804  0.353   0.353    0.116    0.116
38     educ ~~ powerl71  6.744 -0.516  -0.516   -0.145   -0.145
27  ALIEN67 =~      sei  5.182  0.222   0.624    0.294    0.294
```

```
26  ALIEN67 =~     educ  5.182 -0.389  -1.092   -0.352   -0.352
30  ALIEN71 =~     educ  5.182  0.296   0.888    0.287    0.287
31  ALIEN71 =~      sei  5.182 -0.169  -0.508   -0.239   -0.239
40      sei ~~ powerl67  4.840  0.293   0.293    0.103    0.103
32  ALIEN71 =~ anomia67  3.497  0.310   0.931    0.271    0.271
33  ALIEN71 =~ powerl67  3.497 -0.273  -0.820   -0.268   -0.268
22      SES =~ anomia67  3.497  0.137   0.341    0.099    0.099
23      SES =~ powerl67  3.497 -0.120  -0.300   -0.098   -0.098
35     educ ~~ anomia67  3.276  0.402   0.402    0.110    0.110
28  ALIEN67 =~ anomia71  0.215  0.104   0.293    0.083    0.083
29  ALIEN67 =~ powerl71  0.215 -0.087  -0.245   -0.077   -0.077
25      SES =~ powerl71  0.215 -0.029  -0.073   -0.023   -0.023
24      SES =~ anomia71  0.215  0.035   0.087    0.025    0.025
39      sei ~~ anomia67  0.064 -0.038  -0.038   -0.012   -0.012
```

• Model 1



$T_{ML} = 88.29$, $df = 6$, $p < .001$
$NNFI = .904$, $CFI = .962$, $RMSEA = .121$, $SRMR = .025$

• From *SES* to *ALIEN71*,

- direct effect is $c =$ -0.23, $p <$ .01

- indirect effect is $ab =$ (-0.65)(0.68) $=$ -0.44, $p <$ .01 (this is known as the Sobel test)

- total effect is $c + ab =$ -0.23 $-$ 0.44 $=$ -0.67, $p <$ .01

• Full mediation occurs when the direct effect is no longer significant given the mediator

• Partial mediation occurs when the magnitude of the direct effect is reduced (but still significant) after controlling for the mediator

• Problems of Sobel test: the asymptotic standard error of the indirect effect may be incorrect for small sample sizes (Bollen & Stine, 1990), because the distribution of the product of the two coefficients (*a* and *b*) is skewed

• Use bootstrap standard error to improve the accuracy of statistical inference for a mediation effect

```
# fit Model 1 to data
fit1 <- lavaan(model1, sample.cov=alien.cov, sample.cov.rescale=FALSE,
sample.nobs=932, auto.var=TRUE, auto.fix.first=TRUE, se="bootstrap",
bootstrap=1000)
```

• Bootstrapping is a resampling technique and it requires (1) raw data and (2) a sufficiently large number of bootstrap replications to work (e.g., brep=1000)

• Model fit can be much improved if error covariances are allowed

• This is easy to justify as the same scale was administered twice at 2 time points
$\Rightarrow$ Model 2

• How do we know which effect, direct or indirect, is stronger?

Output for Model 2

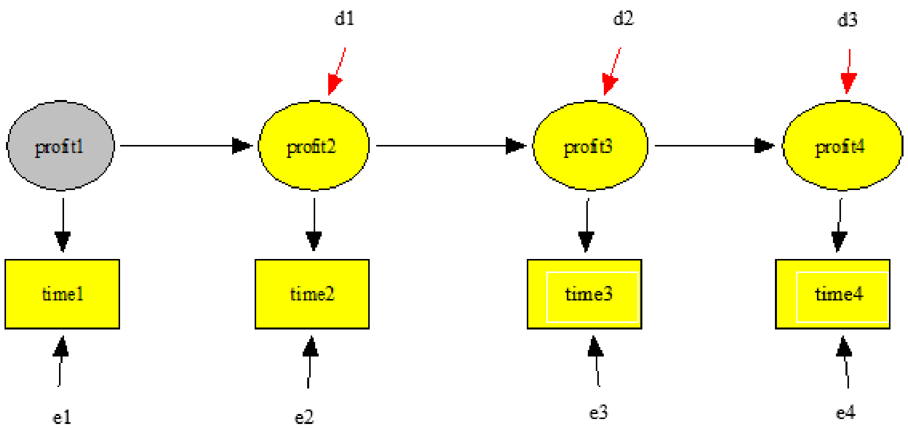
```
$lambda
         SES ALIEN6 ALIEN7
educ       0      0      0
sei        1      0      0
anomia67   0      0      0
powerl67   0      2      0
anomia71   0      0      0
powerl71   0      0      3

$theta
         educ sei anom67 pwrl67 anom71 pwrl71
educ      9
sei       0   10
anomia67  0    0  11
powerl67  0    0   0     12
anomia71  0    0   7      0     13
powerl71  0    0   0      8      0     14

$psi
        SES ALIEN6 ALIEN7
SES     15
ALIEN67  0  16
ALIEN71  0   0     17

$beta
        SES ALIEN6 ALIEN7
SES       0      0      0
ALIEN67   4      0      0
ALIEN71   6      5      0

lavaan 0.6-3 ended normally after 70 iterations

  Optimization method                           NLMINB
  Number of free parameters                         17
```

```
  Number of observations                           932

  Estimator                                         ML
  Model Fit Test Statistic                      23.742
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.000

Model test baseline model:

  Minimum Function Test Statistic             2159.906
  Degrees of freedom                                15
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.991
  Tucker-Lewis Index (TLI)                       0.965

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)             -13066.690
  Loglikelihood unrestricted model (H1)     -13054.819

  Number of free parameters                         17
  Akaike (AIC)                               26167.380
  Bayesian (BIC)                             26249.615
  Sample-size adjusted Bayesian (BIC)        26195.624

Root Mean Square Error of Approximation:

  RMSEA                                          0.073
  90 Percent Confidence Interval          0.046  0.102
  P-value RMSEA <= 0.05                          0.076

Standardized Root Mean Square Residual:

  SRMR                                           0.016
```

```
Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  SES =~
    educ              1.000                               2.522    0.814
    sei               0.559    0.043   12.948    0.000    1.408    0.664
  ALIEN67 =~
    anomia67          1.000                               2.704    0.786
    powerl67          0.951    0.059   15.981    0.000    2.571    0.840
  ALIEN71 =~
    anomia71          1.000                               2.925    0.827
    powerl71          0.875    0.055   15.926    0.000    2.559    0.810

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  ALIEN67 ~
    SES        (a)   -0.614    0.058  -10.630    0.000   -0.572   -0.572
  ALIEN71 ~
    ALIEN67    (b)    0.586    0.052   11.330    0.000    0.542    0.542
    SES        (c)   -0.286    0.057   -5.015    0.000   -0.246   -0.246

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
 .anomia67 ~~
   .anomia71          1.412    0.312    4.524    0.000    1.412    0.333
 .powerl67 ~~
   .powerl71          0.535    0.251    2.132    0.033    0.535    0.174

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .educ              3.250    0.463    7.013    0.000    3.250    0.338
   .sei               2.519    0.180   14.014    0.000    2.519    0.559
```

```
   .anomia67          4.533    0.460    9.856    0.000    4.533    0.383
   .powerl67          2.750    0.394    6.983    0.000    2.750    0.294
   .anomia71          3.961    0.516    7.677    0.000    3.961    0.316
   .powerl71          3.442    0.408    8.444    0.000    3.442    0.344
    SES               6.360    0.606   10.489    0.000    1.000    1.000
   .ALIEN67           4.916    0.479   10.259    0.000    0.672    0.672
   .ALIEN71           4.221    0.419   10.086    0.000    0.493    0.493

R-Square:
                   Estimate
    educ              0.662
    sei               0.441
    anomia67          0.617
    powerl67          0.706
    anomia71          0.684
    powerl71          0.656
    ALIEN67           0.328
    ALIEN71           0.507

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    direct           -0.286    0.057   -5.015    0.000   -0.246   -0.246
    indirect         -0.360    0.042   -8.587    0.000   -0.310   -0.310
    total            -0.645    0.060  -10.753    0.000   -0.556   -0.556
    effect_d         -0.074    0.080   -0.927    0.354   -0.064   -0.064


 Comparing Model 1 and Model 2

Chi Square Difference Test

     Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit2  4 26167 26250 23.742
fit1  6 26228 26301 88.293     64.551       2  9.615e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5.8. Profit Growth: Model 3 (see Example 3 in Chapter 4)




```
# specify Model 3 (first order model with latent variables)
model3 <- "
# measurement equations (Lambda)
Profit1 =~ 1*time1
Profit2 =~ 1*time2
Profit3 =~ 1*time3
Profit4 =~ 1*time4

# structural equations (Beta)
Profit2 ~ Profit1
Profit3 ~ Profit2
Profit4 ~ Profit3

# constraining error variances (Theta)
time1 ~~ vare*time1
time2 ~~ vare*time2
time3 ~~ vare*time3
time4 ~~ vare*time4"

fit3 <- lavaan(model3, sample.cov=ability.cov, sample.cov.rescale=FALSE, sample.nobs=200, auto.var=TRUE)
```

Output for Model 3

```
Note: model contains equality constraints:

  lhs op rhs
1   4 ==   5
2   4 ==   6
3   4 ==   7

$lambda
      Proft1 Proft2 Proft3 Proft4
time1      0      0      0      0
time2      0      0      0      0
time3      0      0      0      0
time4      0      0      0      0

$theta
      time1 time2 time3 time4
time1 4
time2 0     5
time3 0     0     6
time4 0     0     0     7

$psi
        Proft1 Proft2 Proft3 Proft4
Profit1  8
Profit2  0      9
Profit3  0      0     10
Profit4  0      0      0     11

$beta
        Proft1 Proft2 Proft3 Proft4
Profit1      0      0      0      0
Profit2      1      0      0      0
Profit3      0      2      0      0
Profit4      0      0      3      0
```

```
lavaan 0.6-5 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         11
  Number of equality constraints                     3
  Row rank of the constraints matrix                 3

  Number of observations                           200

Model Test User Model:

  Test statistic                                 4.397
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.111

Model Test Baseline Model:

  Test statistic                               822.319
  Degrees of freedom                                 6
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.997
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2368.824
  Loglikelihood unrestricted model (H1)      -2366.626

  Akaike (AIC)                                4753.648
  Bayesian (BIC)                              4780.034
  Sample-size adjusted Bayesian (BIC)         4754.689

Root Mean Square Error of Approximation:
```

```
  RMSEA                                          0.077
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.178
  P-value RMSEA <= 0.05                          0.234

Standardized Root Mean Square Residual:

  SRMR                                           0.008

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard errors                             Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Profit1 =~
    time1             1.000                               5.767    0.912
  Profit2 =~
    time2             1.000                               6.782    0.934
  Profit3 =~
    time3             1.000                               7.317    0.942
  Profit4 =~
    time4             1.000                              10.013    0.968

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Profit2 ~
    Profit1           1.120    0.065   17.247    0.000    0.952    0.952
  Profit3 ~
    Profit2           1.050    0.046   22.710    0.000    0.973    0.973
  Profit4 ~
    Profit3           1.303    0.054   24.205    0.000    0.952    0.952

Variances:
```

		Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.time1	(vare)	6.737	0.881	7.644	0.000	6.737	0.168
.time2	(vare)	6.737	0.881	7.644	0.000	6.737	0.128
.time3	(vare)	6.737	0.881	7.644	0.000	6.737	0.112
.time4	(vare)	6.737	0.881	7.644	0.000	6.737	0.063
Profit1		33.263	4.096	8.121	0.000	1.000	1.000
.Profit2		4.294	2.048	2.097	0.036	0.093	0.093
.Profit3		2.863	1.433	1.998	0.046	0.053	0.053
.Profit4		9.377	2.625	3.572	0.000	0.094	0.094

R-Square:

	Estimate
time1	0.832
time2	0.872
time3	0.888
time4	0.937
Profit2	0.907
Profit3	0.947
Profit4	0.906