Summary of Chapter 5

1 Concepts

- **Types of Sampling Methods**

1. Nonprobability Sampling Methods

   Convenience sampling
   Judgement sampling
   Quota sampling
   Chunk sampling

2. Probability Sampling Methods

   Simple random sampling
   Systematic sampling
   Stratified sampling
   Cluster sampling

• Sampling Distribution

1. Sampling distribution of the mean

   1) If $X \sim N[\mu, \sigma^2]$, then $\bar{X}$ is also normally distributed with

$$\mu_{\bar{X}} = \mu \qquad \text{and} \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

   2) If the distribution of X is not normal, we can apply the **Central Limit Theorem** when n is large enough (> 30). In this case, the sampling distribution of $\bar{X}$ is approximate normal with

$$\mu_{\bar{X}} = \mu \qquad \text{and} \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

2. Sampling distribution of the proportion

   If $X \sim \text{Binomial}(n, \pi)$, then $p = \frac{X}{n}$ is called the sample proportion.

   When n is large enough ($n\pi \geq 5$ and $n(1-\pi) \geq 5$), the sampling distribution of p is approximate normal with

$$\mu_p = \pi \qquad \text{and} \qquad \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

2 Examples

Example 1. An electronics company manufactures resistors that have a mean resistance of 100 ohms and a standard deviation of 10 ohms. The distribution of resistance is normal. Find the probability that a random sample of $n = 25$ resistors will have an average resistance less than 95 ohms.

The sampling distribution of $\bar{X}$ is normal, with

$$\mu_{\bar{X}} = 100 \text{ ohms} \qquad \text{and} \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2 \text{ ohms}$$

Then,

$$P(\bar{X} < 95) = P(Z < \frac{95 - 100}{2}) = P(Z < -2.5) = 0.0062$$

R command: pnorm(95, 100, 2) or pnorm(-2.5).

Example 2. Researchers at Student Monitor estimate that 13% of U.S. college students purchased items from an online book-seller during the spring semester of 2000. If random samples of 400 U.S. college students are selected,

a. what proportion of samples are likely to have between 10% and 15% who purchased items from an online bookseller?

b. If a current sample of 400 U.S. college students has more than 18% who purchased items from an online bookseller, what can you infer about the population estimate of 13%. Explain.

c. If a current sample of 100 U.S. college students has more than 18% who purchased items from an online bookseller, what can you infer about the population estimate of 13%. Explain.

The sampling distribution of p is normal, with

$$\mu_p = \pi = 0.13, \qquad \text{and}$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.13 \times (1-0.13)}{400}} = 0.016815$$

a.

$$P(0.10 < p < 0.15) = P(\frac{0.10 - 0.13}{0.016815} < p < \frac{0.15 - 0.13}{0.016815})$$
$$= P(-1.7841 < Z < 1.1894) = 0.8457$$

b.

$$P(p > 0.18) = P(Z > 2.9735) = 0.0015$$

If the proportion of U.S. college students in the population who purchased items from an online bookseller is indeed 13%, the proportion of samples of 400 U.S. college students that will have more than 18% of students who purchased items from an online bookseller will be 0.0015, which is very low proportion. Hence, the current sample of 400 U.S. college students revealing more than 18% who purchased items from an online bookseller is rather strong evidence that the population estimate of 13% is very likely to be an under estimation.

c.

$$P(p > 0.18) = P(Z > 1.4868) = 0.0685$$

If the proportion of U.S. college students in the population who purchased items from an online bookseller is indeed 13%, the proportion of samples of 100 U.S. college students that will have more than 18% of students who purchased items from an online bookseller will be 0.0685, which is rather low proportion but not at all unlikely. Hence, the current sample of 100 U.S. college students revealing more than 18% who purchased items from an online bookseller suggests that the population estimate of 13% is probably an under estimation.

R command:

a. pnorm(1.1894) − pnorm(-1.7841)

b. 1 − pnorm(2.9735)

c. 1 − pnorm(1.4868)