Exercises for Chapter 1

1. Suppose that $X_t = 10 + a_t + 0.3a_{t-1}$, $a_t \sim N(0,1)$.

   (a) Find $\mathrm{Var}(X_t)$, $\mathrm{Cov}(X_t, X_{t+1})$, $\mathrm{Corr}(X_t, X_{t-1})$ and $\mathrm{Cov}(X_t, X_{t+k})$ for $|k| \geq 2$.

   (b) Let $\bar{X} = \sum_{t=1}^{100} X_t/100$. Find $\mathrm{Var}(\bar{X})$.

   (c) Give an example (a value of $\bar{X}$) such that for the hypothesis H_o : $E(X_t) = 10$ verses $H_1 : E(X_t) \neq 10$, the tests
   1) accounting for time dependence, and
   2) without accounting for time dependence,
   give different conclusions.

2. Design a filter such that a quadratic trend would not be distorted.

3. Let $\{a_{-2}, a_{-1}, a_0, a_1, a_2\}$ be a filter and $\{x_1, x_2, \ldots, x_{10}\}$ be the observations.

   a) If the filter is applied to the observations, what is the length of the filtered sequence?

   b) Write, in terms of a_k and x_ks, the filtered value for the 6-th observation (x_6).

   c) Suppose that $a_0 = 0.5$, $a_1 = 0.2$, $a_2 = 0.05$ and the filter is symmetric. Does the filter pass through a linear trend without distortion? Does the filter passes through a quadratic trend without distortion?

   d) If $a_0 = 0.5$, $a_1 = a_{-1} = a_2 = a_{-2} = 0.1$, do you think this filter is useful? Explain.

4. Use the following R-code to generate a series with trend, quarterly seasonal component and noise:

```
set.seed(6104)
season.effect=c(-20,-10,60,-30)
sea.com=rep(season.effect,20)
t=1:80
trend=-2+0.3*t+0.025*(t^2)
noise=rnorm(80,0,10)
data=trend+sea.com+noise
ts.plot(data)
```

   (a) Decompose the series `data` into three components. Compare the true and the estimated components.

   (b) How many times of differencing should one take to obtain a stationary sequence?