Assignment 3: Due date: April 9 Tuesday

Let $Z_t \sim WN(0, \sigma^2)$ be white noise. Given a data set

$$Y = (1.33, -0.56, -1.31, -0.37, 0.05, 0.46, 2.00, -0.19, -0.25, 1.07, \\ -0.17, 1.14, 0.63, -0.75, 0.15, 0.71, 0.45, -0.14, 0.57, 1.43).$$

1) Draw a time series plot, ACF and PACF plot for the data.

2) Find the moment estimates of θ, σ^2 for fitting an MA(2) model $Y_t = Z_t + \theta Z_{t-2}$ to the data.

3) Find the least squares estimates of ϕ_1, ϕ_2, σ^2 for fitting an AR(2) model $Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} = Z_t$ to the data. Find a 95% confidence interval for each of ϕ_1 and ϕ_2.

4) Find the Yule-Walker estimates of ϕ_1, ϕ_2 for fitting an AR(2) model $Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} = Z_t$ to the data.

5) Find the conditional least squares estimates of ϕ, θ, σ^2 for fitting an ARMA(1,1) model $Y_t - \phi Y_{t-1} = Z_t + \theta Z_{t-1}$ to the data.

6) Find the maximum likelihood estimates of ϕ, θ, σ^2 for fitting an ARMA(1,1) model $Y_t - \phi Y_{t-1} = Z_t + \theta Z_{t-1}$ to the data. What is the maximized value of the log-likelihood?

7) Among AR(p) with $p = 1, 2, 3, 4, 5$, which model is the best in terms of FPE?

8) Among MA(q) with $q = 1, 2, 3, 4, 5$, which model is the best in terms of AICC?

9) Fit an MA(1) model to the data. Find the residuals. Find the portmanteau statistic $Q(10)$. Conduct a portmanteau test based on $Q(10)$ (State H_o and H_1, mention the test statistic, reference distribution, and the conclusion).

10) Which model will you select to describe the data?