

STAT 6104  
ASSIGNMENT 2  
ANSWER

1. (a)  $E[\bar{X}] = \alpha$   
 (b)  $Var(\bar{X}) = \frac{1}{25}(5\gamma_0 + 8\gamma_1 + 6\gamma_2 + 4\gamma_3 + 2\gamma_4) = 0.7243$
2. (a) ARIMA(0, 0, 2)  
 (b)  $\{Z_t\}$  is stationary since it is MA(2).  
 (c) MA characteristic equation is  $1 + x + 0.25x^2 = 0$   
 $\Rightarrow |x| = |-2| = 2 > 1$   
 $\Rightarrow \{Z_t\}$  is invertible.  
 (d) 
$$\begin{cases} \gamma_0 = 41.25 \\ \gamma_1 = 25 \\ \gamma_2 = 5 \\ \gamma_k = 0, k \geq 3 \end{cases}$$
  

$$\begin{cases} \rho_0 = 1 \\ \rho_1 = \frac{20}{33} \\ \rho_2 = \frac{4}{33} \\ \rho_k = 0, k \geq 3 \end{cases}$$
  
 (e)  $Z_t = (1 + B + 0.25B^2)a_t = (1 + 0.5B)^2 a_t$   
 $\Rightarrow a_t = Z_t(\sum_{i=0}^{\infty} (-0.5)^i B^i)^2 = \sum_{k=0}^{\infty} (k+1)(-0.5)^k Z_{t-k}$   
 $\Rightarrow \pi_k = (k+1)(-0.5)^k, k = 0, 1, 2, \dots$
3. (a) AR characteristic equation is  $1 - 0.5x + 0.06x^2 = 0$   
 $\Rightarrow x_1 = 5, x_2 = \frac{10}{3}$   
 (b) Since  $|x_1| > 1, |x_2| > 1$   
 $\{Z_t\}$  is stationary and causal.  
 (c) 
$$\begin{cases} \gamma_0 = 0.5\gamma_1 - 0.06\gamma_2 + 1 \\ \gamma_1 = 0.5\gamma_0 - 0.06\gamma_1 \\ \gamma_2 = 0.5\gamma_1 - 0.06\gamma_0 \end{cases}$$
  
 $\Rightarrow \gamma_0 = 1.2908, \gamma_1 = 0.6089, \gamma_2 = 0.2270$
4. 
$$\begin{cases} \gamma_0 = 0.7\gamma_4 + \sigma^2 \\ \gamma_1 = 0.7\gamma_3 \\ \gamma_2 = 0.7\gamma_2 \\ \gamma_3 = 0.7\gamma_1 \\ \gamma_4 = 0.7\gamma_0 \\ \gamma_k = 0.7\gamma_{k-4}, k \geq 5 \end{cases}$$
  
 $\Rightarrow \begin{cases} \gamma_k = \frac{\sigma^2}{0.51} \cdot 0.7^n, \text{ if } k = 4n, n = 0, 1, 2, \dots \\ \gamma_k = 0, \text{ otherwise} \end{cases}$

$$\begin{aligned}
5. \quad & (1 - 0.6B)Z_t = (1 + 0.2B)a_t \\
& \Rightarrow Z_t = a_t(1 + 0.2B)(\sum_{i=0}^{\infty} (0.6)^i B^i) = a_t + \sum_{i=1}^{\infty} (\frac{4}{3})(0.6)^i a_{t-i} \\
& \Rightarrow a_t = Z_t(1 - 0.6B)(\sum_{i=0}^{\infty} (-0.2)^i B^i) = Z_t + \sum_{i=1}^{\infty} (4)(-0.2)^i Z_{t-i} \\
6. \quad & (a) \quad (1 - 0.5B)(1 - B)Z_t = (1 - 0.3B + 0.6B^2)a_t \\
& \Rightarrow \text{ARIMA}(1, 1, 2) \\
& (b) \quad (1 - B)^3 Z_t = (1 + 0.1B)a_t \\
& \Rightarrow \text{ARIMA}(0, 3, 1) \\
7. \quad & (a) \quad (1 - 0.3B)^2 Z_t = (1 - 0.2B)a_t \\
& \Rightarrow a_t = Z_t(1 - 0.3B)^2(\sum_{i=0}^{\infty} (0.2)^i B^i) = Z_t - 0.4Z_{t-1} + \sum_{i=2}^{\infty} (\frac{1}{4})(0.2)^i Z_{t-i} \\
& (b) \quad \begin{cases} E[a_t Z_t] &= E[a_t^2] = 1 \\ E[a_{t-1} Z_t] &= 0.6E[a_{t-1} Z_{t-1}] - 0.2E[a_{t-1}^2] = 0.4 \end{cases} \\
& \Rightarrow \begin{cases} \gamma_0 &= 0.6\gamma_1 - 0.09\gamma_2 + 0.92 \\ \gamma_1 &= 0.6\gamma_0 - 0.09\gamma_1 - 0.2 \\ \gamma_2 &= 0.6\gamma_1 - 0.09\gamma_0 \\ \gamma_k &= 0.6\gamma_{k-1} - 0.09\gamma_{k-2}, k \geq 3 \end{cases} \\
& \Rightarrow \begin{cases} \gamma_0 &= \frac{893600}{753571} = 1.1858 \\ \gamma_1 &= \frac{353620}{753571} = 0.4693 \\ \gamma_2 &= \frac{131748}{753571} = 0.1748 \end{cases} \\
& \Rightarrow \begin{cases} \rho_0 &= 1 \\ \rho_1 &= \frac{17681}{44680} = 0.3957 \\ \rho_2 &= \frac{131748}{893600} = 0.1474 \\ \rho_k &= \mathbf{0.6p_{k-1} - 0.09p_{k-2} \quad k = 3, 4, \dots} \end{cases}
\end{aligned}$$