Exercises for Chapter 4

1. From a series of 100 observations, we calculate r_1= -0.5, r_2 = 0.3, r_3 = -0.2, r_4 = 0.1, $|r_k| < 0.09$ for $k > 4$. On the basis of this information alone, what ARIMA model would we tentatively specify for the series?

2. Consider an AR(1) series of length 200 with $r_1 = 0.5$. Perform a statistical test at α= 0.05 that H_0: ρ_1= 0.9 versus H_1: $\rho_1 \neq 0.9$.

3. For a series Z_t with length 100, we have computed r_1= 0.7,r_2= 0.45, r_3 = 0.3, $\bar{Z}$= 2 and S^2= 5, where $\bar{Z}$ and S^2 are the sample mean and variance of the data set respectively. Consider an AR(2) process with a constant term θ_0,

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t, \text{ where } a_t \sim WN(0, \sigma_a^2).$$

   Find the estimates of ϕ_1, ϕ_2, θ_0, and σ_a^2 by the method of moment.

4. Find the method of moment estimates of ϕ and θ for the stationary and invertible ARMA(1, 1) process

$$Z_t = \phi Z_{t-1} + a_t - \theta a_{t-1}, \text{ where } a_t \sim WN(0, \sigma_a^2).$$

   based on the first two sample autocorrelations r_1= 0.2 and r_2 = -0.1.

5. Suppose we have a AR(1) model $Z_t = \theta_0 + \phi Z_{t-1} + a_t$ with $a_t \sim WN(0, \sigma_a^2)$. We regress the series $\{Z_t\}$ against the lag-1 counterpart $\{Z_{t-1}\}$, and have the following computer output:

   Coefficients:

	Value	Std.Error	t value	$Pr(>\|t\|)$
Intercept	5.6421	0.8921	6.3245	0.0000
Zt.1	0.6250	0.1019	6.1335	0.0000

   Residual standard error: 2.862 on 78 degrees of freedom
   Multiple R-Squared: 0.3090
   Find the estimates of μ, ϕ, θ_0, and σ_a^2 based on conditional least square method.

6. Consider an MA(1) process, $Z_t = a_t - \theta a_{t-1}$. Based on a series of length 4, we observed Z_1= 0, Z_2= 0, Z_3= 2 and Z_4=1.

   a) Find the conditional least-square estimate of θ.
   b) Find an estimate of σ_a^2.

7. For a MA(1) model

$$Y_t = \alpha Z_t + \beta Z_{t-1}, \quad \text{for } Z_t \overset{i.i.d.}{\sim} N(0, 1)\,.$$

   Use the method of moment to estimate parameters α and β.

8. For a ARMA(1,1) model

$$Y_t = \phi Y_{t-1} + Z_t + \theta Z_{t-1}, \quad \text{for} \;\; Z_t \overset{i.i.d.}{\sim} N(0, \sigma^2),$$

write down $\sum_{t=1}^{3} Z_t^2$ in terms of ϕ and θ and $\{Y_1, Y_2, Y_3\}$.

9. Given $\sum_{t=2}^{n} Y_{t-1}Y_t = 328$ and $\sum_{t=1}^{n-1} Y_t^2 = 413$ and $Y_1 = -0.2$, $Y_n = 2$ with $n = 200$, for a AR(1) model

$$Y_t = \phi Y_{t-1} + Z_t, \quad \text{for} \;\; Z_t \overset{i.i.d.}{\sim} N(0, \sigma^2),$$

a) Estimate the parameters ϕ and σ^2.

b) Find a 95% confidence interval for ϕ.

10. Given the sample autocorrelation functions $\gamma_0 = 1, \gamma_1 = 0.416, \gamma_2 = .37$, use Yule-Walker equation to estimate the parameters in the AR(2) model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + Z_t, \quad \text{for} \;\; Z_t \overset{i.i.d.}{\sim} N(0, \sigma^2).$$

11. For a MA(1) model

$$Y_t = Z_t + \theta Z_{t-1}, \quad \text{for} \;\; Z_t \overset{i.i.d.}{\sim} N(0, \sigma^2).$$

write down the likelihood function for the observations $\{Y_1, Y_2, Y_3\}$.

12. For a MA(2) model

$$Y_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}, \quad \text{for} \;\; Z_t \overset{i.i.d.}{\sim} N(0, 1)$$

find the lag-2 PACF for Y_t.

13. Figure 1 and Figure 2 display the ACF and PACF of the process $\{X_t\}$, respectively. What model would you suggest for $\{X_t\}$?

Figure 1: ACF
plot1.pdf

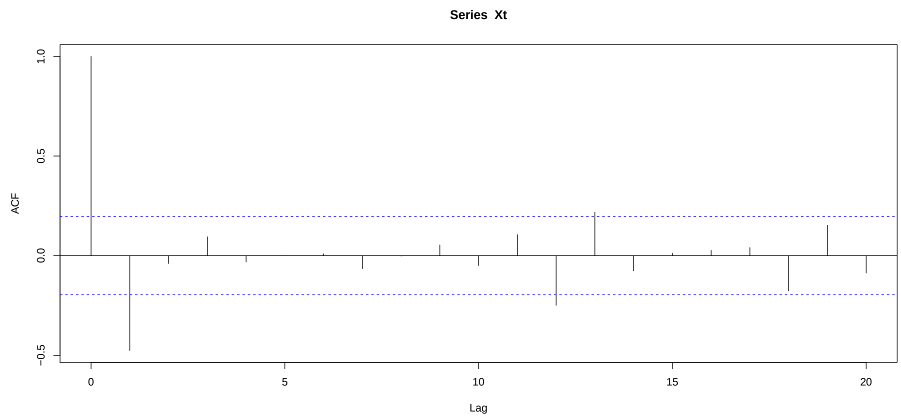
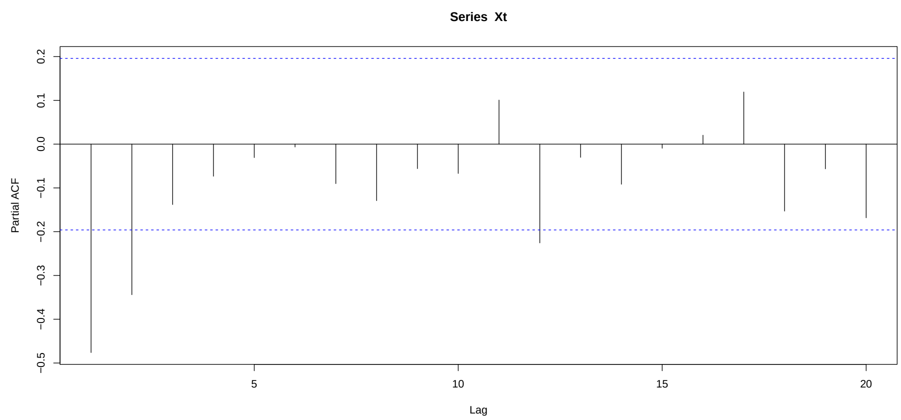


Figure 2: PACF
plot2.pdf



14. A number of models are fitted to the data set with $n = 400$ and the following results are obtained:

   a) ARMA(1,2) log-likelihood=-634

   b) ARMA(2,3) log-likelihood=-636

   c) ARMA(4,3) log-likelihood=-641

   d) ARMA(1,1) log-likelihood=-630

   Which model should you choose in terms of AICC?