STAT 6104 Financial Time Series
Final Exam
7:00-9:15. Monday, 5 Dec 2016

Name: ______________________ **Major:** ______________________

1. (20 marks) Consider the model

$$X_t = 0.2X_{t-1} - 0.01X_{t-2} + Z_t - 1.2Z_{t-1}\,, \qquad Z_t \sim N(0,1)$$

   (a) (10 marks) If X_t can be represented as $X_t = Z_t - \sum_{s=1}^{\infty}\psi_s Z_{t-s}$, then find the values of ψ_k, k =1,2.

   (b) (4 marks) Is X_t causal? Is X_t invertible?

   (c) (6 marks) Write down the Yule-Walker equations for solving the ACF $\gamma(k)$ of the process $\{X_t\}$.

2. (20 marks) Let $Z_i \overset{i.i.d.}{\sim} N(0,1)$, $i = 1,2$, be two independent random variables. Also, let $X_t = Z_1\cos(\lambda t) + Z_2\sin(\lambda t)$ and $Y_t = 2Z_1\cos(\lambda t)$.

   a) (2 marks) Find $E(X_t)$ and $E(Y_t)$.

   b) (6 marks) Find the ACVF $\gamma_X(t,k) = Cov(X_t, X_{t+k})$ for all integers t,k. Is $\{X_t\}$ stationary?

   c) (6 marks) Find the ACVF $\gamma_Y(t,k) = Cov(Y_t, Y_{t+k})$ for all integers t,k. Is $\{Y_t\}$ stationary?

   d) (6 marks) Find the $\eta(t,k) = Cov(X_t, Y_{t+k})$ for all integers t,k.

3. (15 marks) Consider the model $Z_t = 0.5Z_{t-1} + 0.2Z_{t-2} + a_t + 0.2a_{t-1} - 0.4a_{t-2}$. Suppose that $Z_{200} = 4, Z_{199} = 5, a_{200} = 1, a_{199} = 0.5$ and $a_{198} = 0.7$.

   a) (5 marks) Let $\hat{Z}_n(h)$ be the h-step forecast given information up to time n. Find $\hat{Z}_{200}(1)$, $\hat{Z}_{200}(2)$, $\hat{Z}_{200}(3)$, $\hat{Z}_{200}(4)$ and $\hat{Z}_{200}(5)$.

   b) (6 marks) Suppose that width of the 95% prediction interval for Z_{201} is 4. Find the 95% prediction intervals of Z_{202} and Z_{203}, respectively. Note that if $X \sim N(0,1)$, then $P(X \leq 1.96) = 0.975$.

   c) (4 marks) Update forecasts Z_{203} and Z_{204} given $Z_{201} = 18$

4. (15 marks) Consider the stationary GARCH(2,1) model

$$\begin{aligned} X_t &= \epsilon_t\sigma_t\,, \qquad \epsilon_t \overset{i.i.d.}{\sim} N(0,1) \\ \sigma_t^2 &= 0.1 + 0.2X_{t-1}^2 + 0.3X_{t-2}^2 + 0.35\sigma_{t-1}^2\,. \end{aligned}$$

   a) (5 marks) Express X_t^2 as an ARMA model.

   b) (2 marks) Find $E(X_t^2)$ and $E(\sigma_t^2)$.

   c) (5 marks) Find $\mathrm{Cov}(X_t^2, \sigma_{t-1}^2)$.

   d) (3 marks) Given a data set $\{X_1, X_2, X_3, X_4\} = \{1.3, 0.1, -2.2, 1.4\}$, find the likelihood for the GARCH model.

5. (10 marks) Suppose that $\{X_t\}$ is the noninvertible MA(1) process

$$X_t = Z_t + \theta Z_{t-1}, \quad Z_t \sim WN(0, \sigma^2)$$

where $|\theta| > 1$. Define a new process $\{W_t\}$ as

$$W_t = \sum_{j=0}^{\infty} (-\theta)^{-j} X_{t-j}.$$

Show that $W_t \sim WN(0, \sigma_W^2)$, and express σ_W^2 in terms of σ^2 and θ. Show that $\{X_t\}$ has the invertible representation

$$X_t = W_t + \frac{1}{\theta} W_{t-1}.$$

End of paper