**STAT 6104 Financial Time Series**
**Assignment 2**

Let $a_t \sim NID(0, \sigma^2)$ if not specified otherwise.

1. Let $X_t$ be a stationary time series with mean $\alpha$ and autocovariances $\gamma_k = 0.8^k$ and $\bar{X} = \sum_{t=6}^{10} X_t/5$
   (a) Find $E(\bar{X})$.
   (b) Find $\text{Var}(\bar{X})$.

2. Consider the process
$$Z_t = a_t + a_{t-1} + 0.25a_{t-2}\,, \quad \sigma_a^2 = 20.$$
   (a) Identify the order of the ARIMA model for the process.
   (b) Is $\{Z_t\}$ stationary?
   (c) Is $\{Z_t\}$ invertible?
   (d) Find the ACVF $\gamma(k)$ and ACF $\rho(k)$ of $\{Z_t\}$ for $k$ = 0, 1, 2, 3,....
   (e) Find the values of $\pi_k$, $k$ = 0, 1, 2, 3,... if the process is written as
$$a_t = \textstyle\sum_{k=0}^{\infty} \pi_k Z_{t-k}\,.$$

3. Consider the AR(2) process
$$Z_t = 0.5Z_{t-1} - 0.06Z_{t-2} + a_t\,,$$
   where $a_t$s are independently and identically distributed as $N(0, 1)$.
   (a) Find the roots of the AR characteristic equation.
   (b) Is the process $Z_t$ stationary and causal? Why?
   (c) Find the autocovariances $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$.

4. Find ACVF $\gamma(k)$, $k$=0,1,2,3,.... of the process
$$Z_t = 0.7Z_{t-4} + a_t\,.$$

5. Find the AR and MA representation of the process
$$Z_t = 0.6Z_{t-1} + a_t + 0.2a_{t-1}\,, \quad a_t \sim WN(0, 4)\,.$$

6. Identify the following as specific ARIMA models:
   a) $Z_t = 1.5Z_{t-1} - 0.5Z_{t-2} + a_t - 0.3a_{t-1} + 0.6a_{t-2}$.
   b) $Z_t = 3Z_{t-1} - 3Z_{t-2} + Z_{t-3} + a_t + 0.1a_{t-1}$.

7. Consider the ARMA(2,1) model
$$Z_t = 0.6Z_{t-1} - 0.09Z_{t-2} + a_t - 0.2a_{t-1}\,, \quad a_t \sim WN(0, 1)\,.$$
   a) Find the AR representation of $\{Z_t\}$.
   b) Find the ACF $\rho(k)$ of $\{Z_t\}$ for $k \in \mathbb{Z}$.