STAT5102
Regression in Practice

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Preamble: Matrix Operations

In this chapter, we shall cover

- Basic notations
- Matrix multiplication
- Inverse of a matrix
- Solving system of simultaneous linear equations

Notation

Definitions:

- A *matrix* is a rectangular array of numbers.

- A number in a particular row-column position is called an *element* of the matrix.

- A matrix containing r rows and c columns is aid to be an $r \times c$ *matrix* where r and c are the dimensions of the matrix.

- If $r = c$, a matrix is said to be a *square* matrix.

- If $\mathbf{A}$ is any matrix, then a matrix I is defined to be an *identity* matrix if $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$.

Matrix Multiplications

Requirement for matrix multiplication

<u>Example:</u>

Given the matrices A, B, and C:

$$\mathbf{A} = \begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 3 \\ -2 & 1 & 2 \end{bmatrix}$$

Find AB, AC and BA.

Inverse

Definition:

The square matrix $\mathbf{A}^{-1}$ is said to be the inverse of the square matrix $\mathbf{A}$ if

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$

<u>Examples:</u>

Find the inverse of the following matrices:

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & -1 \\ 2 & 3 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 4 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Solving Systems of Simultaneous Linear Equations

A matrix solution to a set of simultaneous linear equations, $\mathbf{AV} = \mathbf{G}$:

$$\mathbf{V} = \mathbf{A}^{-1}\mathbf{G},$$

if $\mathbf{A}^{-1}$ exists.

<u>Example:</u>

$$\begin{aligned} 3v_1 &+ v_2 = 5 \\ v_1 &- v_2 = 3 \end{aligned}$$

Obtain a general form that involves matrices, i.e. What are **V**, **A** and **G** respectively?