STAT 6104 Financial Time Series
Final Exam
7:00-9:15. Monday, 7 Dec 2015

Name: ______________________ **Major:** ______________________

1. (15 marks)

   Let $Z_t \overset{i.i.d.}{\sim} N(0,1)$. Match each of the following time series with its corresponding acf plot.

   - Series A: $X_t = 0.9X_{t-1} + Z_t$
   - Series B: $X_t = 0.4X_{t-1} - 0.45X_{t-2} + Z_t$
   - Series C: $X_t = 3 - 2t + Z_t$
   - Series D: $X_t = 3\cos(\pi/4)t + Z_t$
   - Series E: $X_t = 3 - 2tZ_t$

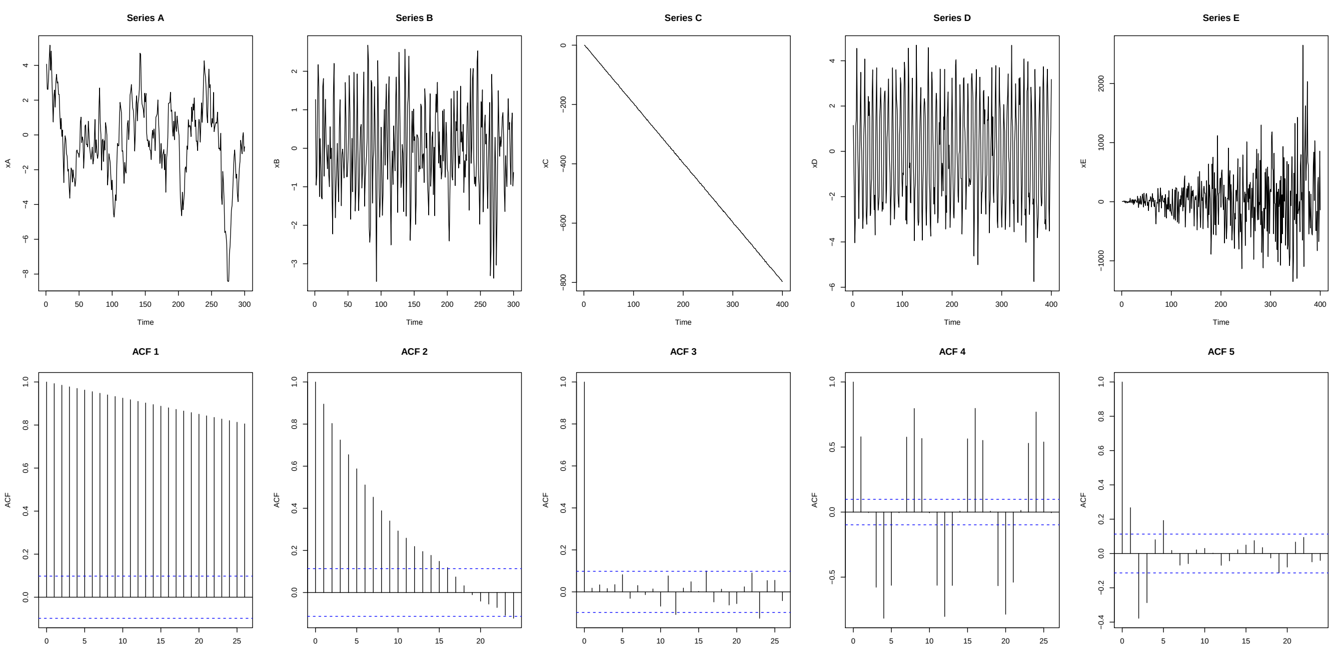

Figure 1: Match the Series plot and the ACF plot

2. (15 marks) Let $Z_t \overset{i.i.d.}{\sim} N(0,1)$ and $a_t \overset{i.i.d.}{\sim} N(0,1)$ denote two independent random variables, $X_t = \phi X_{t-1} + Z_t$ and $Y_t = a_t + \theta a_{t-2}$. Let $W_t = X_t + Y_t$.

   a) (5 marks) Find $E(W_t)$.

   b) (10 marks) Find the ACVF $\gamma(k) = Cov(W_t, W_{t+k})$ for all integers k.

3. (20 marks) A time series model is specified as follows

$$Y_t = \mu + 0.3Y_{t-1} + Z_t\,, \quad Z_t \overset{i.i.d.}{\sim} N(0,1)\,.$$

a) (6 marks) What is the name of this model? Is this model causal? Is this model invertible?

b) (4 marks) Find the expected value $E(Y_1)$ in terms of μ.

c) (10 marks) Suppose that we have three observations $Y_1 = 3$, $Y_2 = 4$ and $Y_3 = 6$. Find the least square estimate of μ.

4. (20 marks) Given a model

$$Y_t = \theta_1 Z_t + \theta_2 Z_{t-2}, \quad Z_t \overset{i.i.d.}{\sim} N(0,1)\,.$$

If the sample autocovariance of the data are given by $\hat{\gamma}(0) = 2.81$ and $\hat{\gamma}(2) = 1.4$, find the method of moment estimator for θ_1 and θ_2.

5. (20 marks) Consider the process $Y_t = 0.4Y_{t-1} - 0.04Y_{t-2} + a_t + 0.3a_{t-1}$ with $a_t \overset{i.i.d.}{\sim} N(0,9)$. Given that $a_{2013} = 0.5$, $Y_{2012} = 14$,Y_{2013}=15, $Y_{2014} = 18$ and $Y_{2015} = 12$.

a) (10 marks) Find the forecast Y_{2016}^{2015} and Y_{2017}^{2015}.

b) (10 marks) Find the 95% prediction interval of Y_{2016}.

6. (10 marks) For a process $\{Y_t\}$, the auto-covariance generating function is given by

$$g_Y(z) = \sum_{j=-\infty}^{\infty} \gamma(j) z^j\,,$$

where $\gamma(j)$ is the auto-covariance of lag j.

a) (10 marks) If $\{Y_t\}$ follows the MA(1) model $Y_t = a_t + \theta a_{t-1}$, $a_t \overset{i.i.d.}{\sim} N(0,\sigma^2)$, find $g_Y(z)$.

b) (10 marks) If $\{Y_t\}$ follows the AR(1) model $Y_t = \phi Y_{t-1} + a_t$, $a_t \overset{i.i.d.}{\sim} N(0,\sigma^2)$, find $g_Y(z)$.

End of paper