Assignment 1 (Due Date: Friday, Sept 21, 11:59pm)

1. Let

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

   (a) Find the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ and their corresponding normalized eigenvectors $(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3})$.

   (b) Let $\mathbf{P} = (x_1, x_2, x_3)$, a matrix formed by using the eigenvectors as columns. Find $\mathbf{PDP'}$ where $\mathbf{D}$ is a diagonal matrix with the eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ on its diagonal.

   (c) Find the singular value decomposition of $\mathbf{A}$.

2. Let the eigenvalue-eigenvector pairs of a 2×2 symmetric matrix $\mathbf{A}$ be

$$\left(3, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}\right) \quad \text{and} \quad \left(5, \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\right)$$

   (a) Find $\mathbf{A}$.

   (b) Find the determinant of $\mathbf{A}$.

   (c) Find the trace of $\mathbf{A}$.

3. Using $x_3 - x_9$ of the US Crime Data, compute

   (a) the sample mean $(\overline{\mathbf{x}})$,

   (b) the sample covariance matrix $(\mathbf{S})$, and

   (c) the sample correlation matrix $(\mathbf{R})$.