4 Structural Equation Modeling 1: Path Analysis

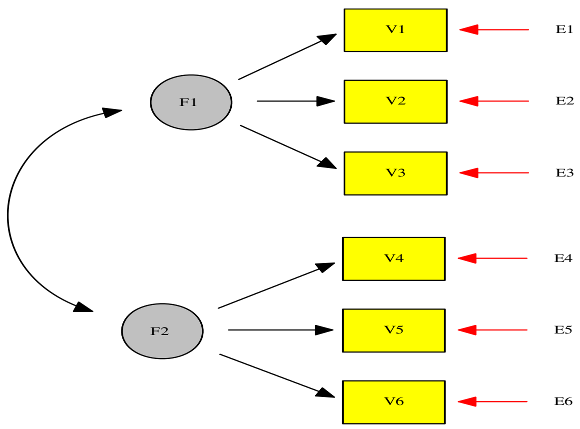
References:

• Beaujean (2014). Chapter 2.

4.1. Why SEM?

1. Confirmatory factor analysis

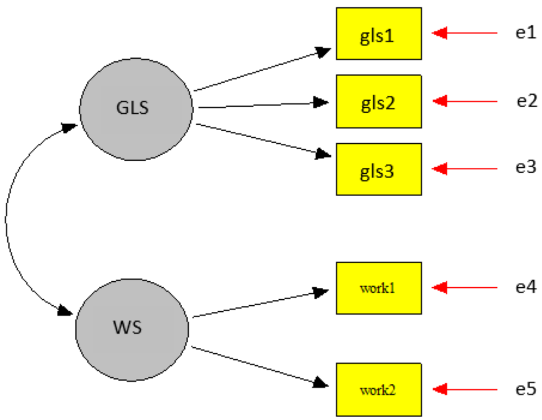
• SEM examines how well the observed variables measure the underlying constructs



- Example 1. The Subjective Well Being Model

- Filename: *swb.cov* ($N = 500$)

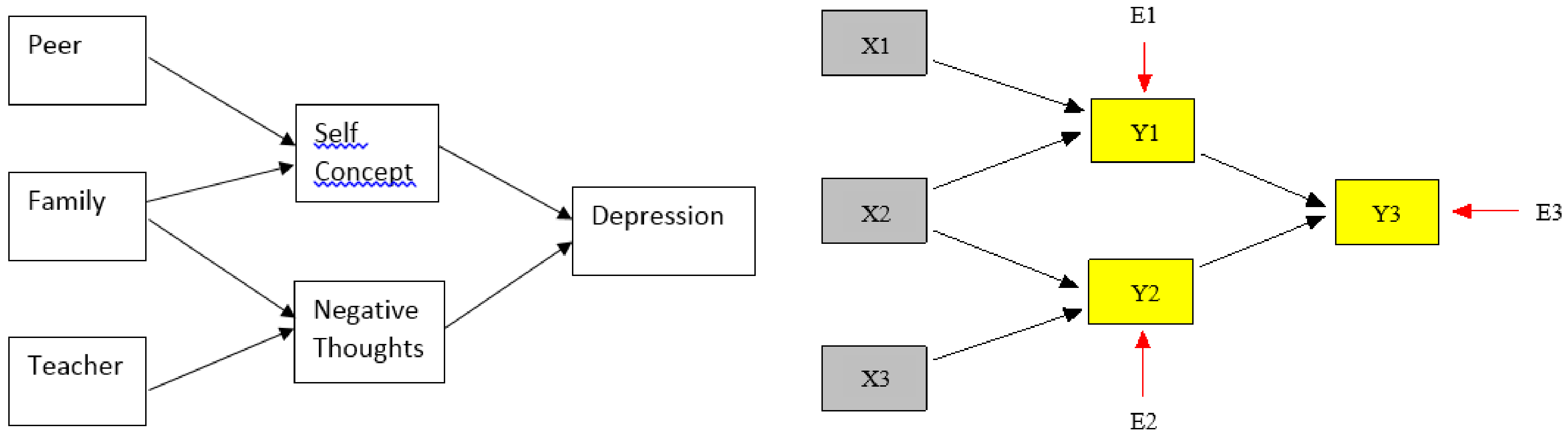
	V1	V2	V3	V4	V5
V1 (gls1)	198				
V2 (gls2)	82	86			
V3 (gls3)	54	28	24		
V4 (work1)	52	30	18	151	
V5 (work2)	16	10	7	44	28



- See lecture notes in Chapter 3 for details

2. Path analysis

- SEM examines the relationship among a set of observed variables



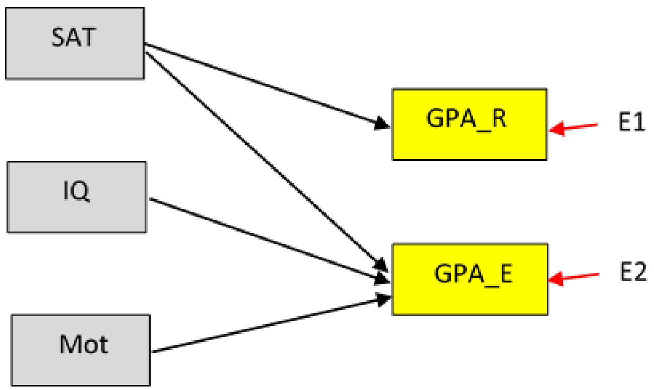
- Extension of multiple regression analysis

- Example 2. College Academic Performance (Raykov, 2006)

- Filename: *college.cov* ($N = 150$)

	GPA_R	GPA_E	SAT	IQ	Motiv
GPA_R	.594				
GPA_E	.483	.754			
SAT	3.993	3.626	47.457		
IQ	.426	1.757	4.100	10.267	
Motiv	.500	.722	6.394	.525	2.675

GPA_R=GPA in required courses, GPA_E=GPA in elective courses, SAT=Scholastic aptitude test, IQ=intelligence score, Motiv=motivation



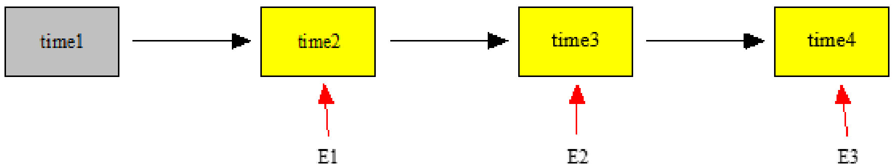
- Example 3. Profit Growth

- Filename: *profit.cov* ($N = 200$ companies)

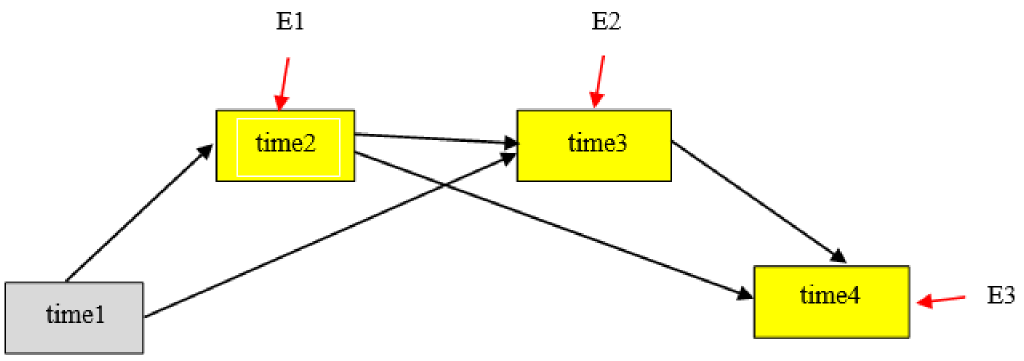
	time1	time2	time3	time4
time1	40			
time2	37	53		
time3	40	48	60	
time4	50	63	70	107

time1=profit at time 1, time2=profit at time 2, time3=profit at time 3, time4=profit at time 4

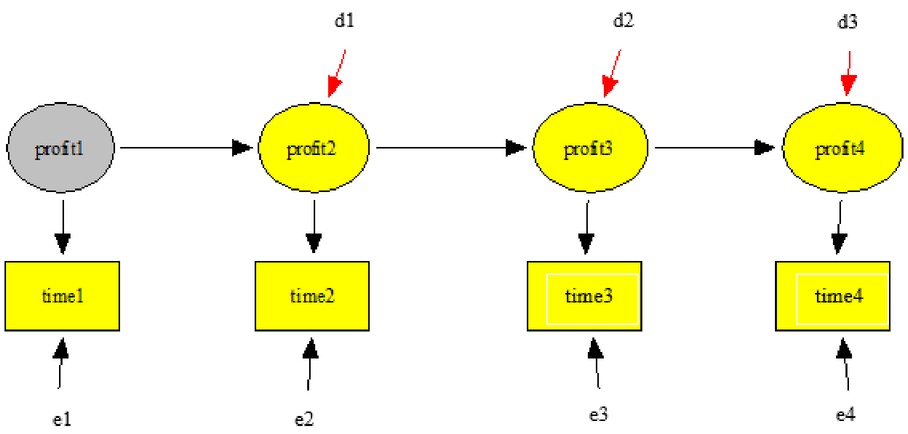
- Model 1: First-order model



- Model 2: Second-order model

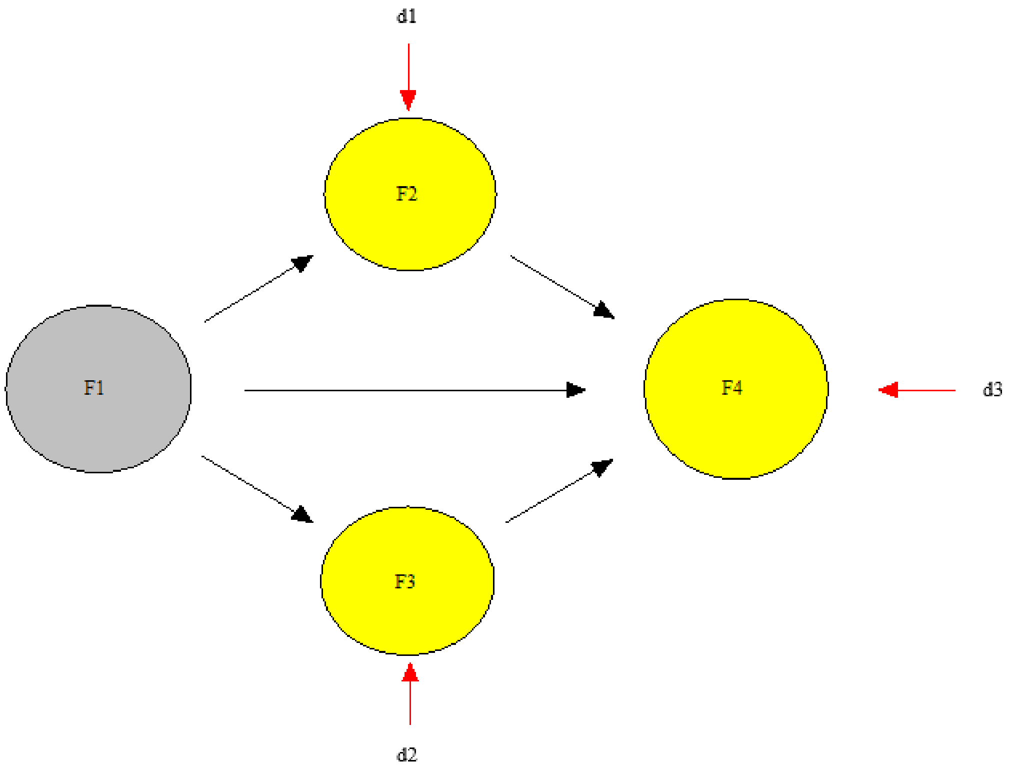


- Model 3: Latent variable model



3. Latent variable analysis

• SEM examines the relationship among a set of latent constructs



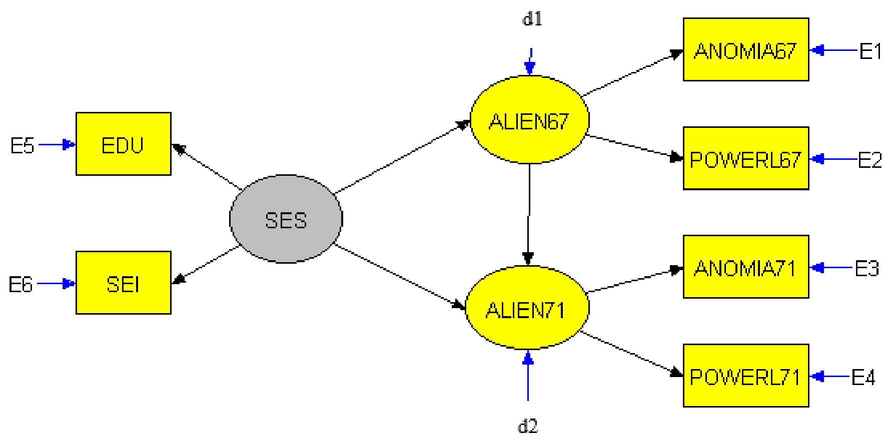
• Combining CFA (measurement model) and path analysis (with latent variables)

- Example 4. Stability of Alienation (Wheaton et al., 1977)

- Filename: *alien.cov* ($N = 932$)

	ANOMIA67	POWERL67	ANOMIA71	POWERL71	EDUC	SEI
ANOMIA67	11.83					
POWERL67	6.95	9.36				
ANOMIA71	6.82	5.09	12.53			
POWERL71	4.78	5.03	7.50	9.99		
EDUC	-3.84	-3.89	-3.84	-3.63	9.61	
SEI	-2.19	-1.88	-2.72	-1.88	3.55	4.50

ANOMIA67, ANOMIA71=a scale measured anomia in 1967 and 1971
POWERL67, POWERL71=a scale measured powerlessness in 1967 and 1971
EDUC=education level, SEI=socioeconomic index



4.2. Path Analysis

• Suppose there is a set of p variables (y, x) with a covariance matrix Σ. The ***structural model*** represents our belief about how Σ is structured. That is, $\Sigma = \Sigma(\theta)$.

• Basic tasks involve

- parameter estimation

- model evaluation

- model modification

• We follow the same 5-step procedure as in CFA

4.3. Model Specification

- Model equation:

$$y = \mu_y + \beta y + \gamma x + e$$

$$x = \mu_x + (x - \mu_x)$$

y is $p_y \times 1$ vector of observed dependent variables
x is $p_x \times 1$ vector of observed independent variables
μ_y is $p_y \times 1$ vector of intercepts of y
β is $p_y \times p_y$ matrix of path coefficients among the y variables
γ is $p_y \times p_x$ matrix of path coefficients from x to y
e is $p_y \times 1$ vector of measurement errors
μ_x=$E(x)$ is $p_x \times 1$ vector of means of x

- Combining the observed variables *y* and *x*, we have

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix} + \begin{bmatrix} \beta & \gamma \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} + \begin{bmatrix} e \\ x - \mu_x \end{bmatrix}$$

$$v \quad = \quad \mu \quad + \quad Bv \quad + \quad z$$

- Covariance matrix of *v* is

$$\begin{aligned} \Sigma &= \mathrm{cov}\{(I - B)^{-1}(\mu + z)\} \\ &= (I - B)^{-1}\Psi(I - B)^{-1\prime} \\ &= \Sigma(\theta) \end{aligned}$$

where B is the $p \times p$ path coefficient matrix, and Ψ is the $p \times p$ variance-covariance matrix of e and x.

• In path analysis, the variables are:

Name	Type	Cause/Effect	Dimension
y	observed	DV	$p_y \times 1$
x	observed	IV	$p_x \times 1$
e	latent	IV	$p_y \times 1$

• And the parameter matrices are:

Parameter matrix	Symbol	Name	Dimension
path coefficients among the observed variables	B	beta	$p \times p$
variance-covariance matrix of e and x	Ψ	psi	$p \times p$

4.4. Identification

• The model $\Sigma = \Sigma(\theta)$ is ***identified*** if there are no vectors θ^* and θ such that $\Sigma(\theta^*) = \Sigma(\theta)$ unless $\theta^* = \theta$

• ULI and UVI are irrelevant

• The t-rule remains valid (necessary condition for identification)

4.5. Estimation

• Estimation methods are basically the same as that in §3.6

4.6. Goodness of Fit Assessment

• Same as that in §3.7

filename: college.R

```
# Example 2: College Academic Performance (Raykov, 2006)

# set work directory
setwd("c:/users/wchan/google drive/stat6108/data")

# load the lavaan package
library(lavaan)
data <- scan("college.cov")

# write the input data into a full covariance matrix
college.cov <- getCov(data, names=c("GPA_R", "GPA_E", "SAT", "IQ", "Motiv"))

# Specify Model 1
college1.model <- "
# regression equation (B)
GPA_R ~ SAT
GPA_E ~ SAT + IQ + Motiv

# error variance of e (psi)
GPA_R ~~ GPA_R
GPA_E ~~ GPA_E

# variance-covariance of x (psi)
SAT ~~ SAT
IQ ~~ IQ
Motiv ~~ Motiv
SAT ~~ IQ + Motiv
IQ ~~ Motiv
"

# Fit Model 1 to the data
fit1 <- lavaan(college1.model, sample.cov=college.cov, sample.nobs=150, fixed.x=FALSE)
mi1 <- modindices(fit1, sort.=TRUE)
```

```
# Specify Model 2 (based on modification indices from Model 1)
college2.model <- "
# Regression Equation (B)
GPA_R ~ SAT
GPA_E ~ SAT + IQ + Motiv

# variance-covariance of x (psi)
SAT ~~ SAT
IQ ~~ IQ
Motiv ~~ Motiv
SAT ~~ IQ + Motiv
IQ ~~ Motiv

# error Variance and Covariance (psi)
GPA_R ~~ GPA_R
GPA_E ~~ GPA_E
GPA_R ~~ GPA_E
"

# Fit Model 2 to the data
fit2 <- lavaan(college2.model, sample.cov=college.cov, sample.nobs=150, fixed.x=FALSE)
mi2 <- modindices(fit2, sort.=TRUE)


sink("college.out", split=TRUE)
writeLines("\n Example 2: College Academic Performance (Raykov, 2006) \n")
writeLines("\n Output for Model 1 \n")
writeLines("\n Sample covariance matrix \n")
list(college.cov)
writeLines("\n residual matrix \n")
residuals(fit1, type="raw")
writeLines("\n standardized residual matrix \n")
residuals(fit1, type="cor")
writeLines("\n free parameters in Model 1 \n")
inspect(fit1)
```

```
summary(fit1, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
writeLines("\n modification indices \n")
list(mi1)
writeLines("\n Output for Model 2 \n")
writeLines("\n residual matrix \n")
residuals(fit2, type="raw")
writeLines("\n standardized residual matrix \n")
residuals(fit2, type="cor")
writeLines("\n free parameters in Model 2 \n")
inspect(fit2)
summary(fit2, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
writeLines("\n modification indices \n")
list(mi2)
writeLines("\n Comparing Model 1 and Model 2 \n")
lavTestLRT(fit1, fit2)
sink()
```

```
filename: college.out

 Example 2: College Academic Performance (Raykov, 2006)

 Output for Model 1

 Sample covariance matrix

[[1]]
      GPA_R GPA_E    SAT     IQ Motiv
GPA_R 0.594 0.483  3.993  0.426 0.500
GPA_E 0.483 0.754  3.626  1.757 0.722
SAT   3.993 3.626 47.457  4.100 6.394
IQ    0.426 1.757  4.100 10.267 0.525
Motiv 0.500 0.722  6.394  0.525 2.675

 residual matrix

$type
[1] "raw"

$cov
      GPA_R  GPA_E  SAT    IQ     Motiv
GPA_R  0.000
GPA_E  0.177  0.000
SAT    0.000  0.000  0.000
IQ     0.080  0.000  0.000  0.000
Motiv -0.038  0.000  0.000  0.000  0.000

 standardized residual matrix

$type
[1] "cor.bollen"

$cov
```

```
      GPA_R  GPA_E  SAT    IQ     Motiv
GPA_R  0.000
GPA_E  0.266  0.000
SAT    0.000  0.000  0.000
IQ     0.033  0.000  0.000  0.000
Motiv -0.030  0.000  0.000  0.000  0.000

 free parameters in Model 1

$lambda
      GPA_R GPA_E SAT IQ Motiv
GPA_R     0     0   0  0     0
GPA_E     0     0   0  0     0
SAT       0     0   0  0     0
IQ        0     0   0  0     0
Motiv     0     0   0  0     0

$theta
      GPA_R GPA_E SAT IQ Motiv
GPA_R 0
GPA_E 0     0
SAT   0     0     0
IQ    0     0     0   0
Motiv 0     0     0   0  0

$psi
      GPA_R GPA_E SAT IQ Motiv
GPA_R  5
GPA_E  0     6
SAT    0     0     7
IQ     0     0    10   8
Motiv  0     0    11  12  9

$beta
```

```
      GPA_R GPA_E SAT IQ Motiv
GPA_R     0     0   1  0     0
GPA_E     0     0   2  3     4
SAT       0     0   0  0     0
IQ        0     0   0  0     0
Motiv     0     0   0  0     0

lavaan 0.6-5 ended normally after 52 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         12

  Number of observations                           150

Model Test User Model:

  Test statistic                               100.104
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.000

Model Test Baseline Model:

  Test statistic                               463.244
  Degrees of freedom                                10
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.786
  Tucker-Lewis Index (TLI)                       0.286

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1357.836
```

```
  Loglikelihood unrestricted model (H1)      -1307.784

  Akaike (AIC)                                2739.672
  Bayesian (BIC)                              2775.799
  Sample-size adjusted Bayesian (BIC)         2737.822

Root Mean Square Error of Approximation:

  RMSEA                                          0.465
  90 Percent confidence interval - lower         0.389
  90 Percent confidence interval - upper         0.545
  P-value RMSEA <= 0.05                          0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.070

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  GPA_R ~
    SAT               0.084    0.006   13.975    0.000    0.084    0.752
  GPA_E ~
    SAT               0.046    0.007    6.523    0.000    0.046    0.366
    IQ                0.146    0.013   11.599    0.000    0.146    0.539
    Motiv             0.131    0.029    4.449    0.000    0.131    0.247

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  SAT ~~
```

```
    IQ                 4.073    1.821    2.237    0.025    4.073    0.186
    Motiv              6.351    1.051    6.045    0.000    6.351    0.567
  IQ ~~
    Motiv              0.521    0.427    1.221    0.222    0.521    0.100

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .GPA_R              0.256    0.030    8.660    0.000    0.256    0.434
   .GPA_E              0.234    0.027    8.660    0.000    0.234    0.312
    SAT               47.141    5.443    8.660    0.000   47.141    1.000
    IQ                10.199    1.178    8.660    0.000   10.199    1.000
    Motiv              2.657    0.307    8.660    0.000    2.657    1.000

R-Square:
                   Estimate
    GPA_R              0.566
    GPA_E              0.688

 modification indices

[[1]]
     lhs op   rhs     mi    epc sepc.lv sepc.all sepc.nox
23 GPA_E  ~ GPA_R 72.199  0.663   0.663    0.588    0.588
13 GPA_R ~~ GPA_E 72.199  0.170   0.170    0.694    0.694
20 GPA_R  ~ GPA_E 38.579  0.373   0.373    0.420    0.420
22 GPA_R  ~ Motiv  0.463 -0.021  -0.021   -0.044   -0.044
32 Motiv  ~ GPA_R  0.457 -0.146  -0.146   -0.069   -0.069
16 GPA_R ~~ Motiv  0.457 -0.038  -0.038   -0.045   -0.045
21 GPA_R  ~    IQ  0.385  0.008   0.008    0.034    0.034
15 GPA_R ~~    IQ  0.380  0.080   0.080    0.049    0.049
28    IQ  ~ GPA_R  0.380  0.312   0.312    0.075    0.075
24   SAT  ~ GPA_R  0.238  0.755   0.755    0.084    0.084
14 GPA_R ~~   SAT  0.238  0.194   0.194    0.056    0.056
```

```
 Output for Model 2

 residual matrix

$type
[1] "raw"

$cov
      GPA_R  GPA_E  SAT    IQ     Motiv
GPA_R  0.000
GPA_E  0.006  0.008
SAT    0.000  0.000  0.000
IQ     0.080  0.054  0.000  0.000
Motiv -0.038 -0.025  0.000  0.000  0.000


 standardized residual matrix

$type
[1] "cor.bollen"

$cov
      GPA_R  GPA_E  SAT    IQ     Motiv
GPA_R  0.000
GPA_E  0.005  0.000
SAT    0.000 -0.003  0.000
IQ     0.033  0.016  0.000  0.000
Motiv -0.030 -0.021  0.000  0.000  0.000


 free parameters in Model 2

$lambda
      GPA_R GPA_E SAT IQ Motiv
GPA_R     0     0   0  0     0
```

```
GPA_E      0      0   0  0      0
SAT        0      0   0  0      0
IQ         0      0   0  0      0
Motiv      0      0   0  0      0

$theta
      GPA_R GPA_E SAT IQ Motiv
GPA_R 0
GPA_E 0     0
SAT   0     0     0
IQ    0     0     0   0
Motiv 0     0     0   0  0

$psi
      GPA_R GPA_E SAT IQ Motiv
GPA_R 11
GPA_E 13    12
SAT    0     0     5
IQ     0     0     8   6
Motiv  0     0     9  10   7

$beta
      GPA_R GPA_E SAT IQ Motiv
GPA_R      0      0   1  0      0
GPA_E      0      0   2  3      4
SAT        0      0   0  0      0
IQ         0      0   0  0      0
Motiv      0      0   0  0      0

lavaan 0.6-5 ended normally after 62 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         13
```

```
  Number of observations                           150

Model Test User Model:

  Test statistic                                 0.845
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.656

Model Test Baseline Model:

  Test statistic                               463.244
  Degrees of freedom                                10
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.013

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1308.206
  Loglikelihood unrestricted model (H1)      -1307.784

  Akaike (AIC)                                2642.412
  Bayesian (BIC)                              2681.550
  Sample-size adjusted Bayesian (BIC)         2640.408

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.126
  P-value RMSEA <= 0.05                          0.745
```

```
Standardized Root Mean Square Residual:

  SRMR                                           0.014

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  GPA_R ~
    SAT               0.084    0.006   13.975    0.000    0.084    0.752
  GPA_E ~
    SAT               0.045    0.006    6.910    0.000    0.045    0.357
    IQ                0.141    0.009   15.550    0.000    0.141    0.522
    Motiv             0.145    0.021    6.851    0.000    0.145    0.274

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  SAT ~~
    IQ                4.073    1.821    2.237    0.025    4.073    0.186
    Motiv             6.351    1.051    6.045    0.000    6.351    0.567
  IQ ~~
    Motiv             0.522    0.427    1.221    0.222    0.522    0.100
 .GPA_R ~~
   .GPA_E             0.171    0.024    7.001    0.000    0.171    0.697

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    SAT              47.141    5.443    8.660    0.000   47.141    1.000
    IQ               10.199    1.178    8.660    0.000   10.199    1.000
    Motiv             2.657    0.307    8.660    0.000    2.657    1.000
   .GPA_R             0.256    0.030    8.660    0.000    0.256    0.434
```

```
   .GPA_E             0.235    0.027    8.660    0.000    0.235    0.317

R-Square:
                   Estimate
    GPA_R             0.566
    GPA_E             0.683

 modification indices

[[1]]
     lhs op   rhs    mi    epc sepc.lv sepc.all sepc.nox
22 GPA_R  ~ Motiv 0.463 -0.021  -0.021   -0.044   -0.044
16 GPA_R ~~ Motiv 0.457 -0.038  -0.038   -0.045   -0.045
33 Motiv  ~ GPA_E 0.457 -0.220  -0.220   -0.116   -0.116
32 Motiv  ~ GPA_R 0.457 -0.146  -0.146   -0.069   -0.069
21 GPA_R  ~    IQ 0.385  0.008   0.008    0.034    0.034
15 GPA_R ~~    IQ 0.380  0.080   0.080    0.049    0.049
28    IQ  ~ GPA_R 0.380  0.312   0.312    0.075    0.075
29    IQ  ~ GPA_E 0.380  0.468   0.468    0.126    0.126
25   SAT  ~ GPA_E 0.238  1.133   1.133    0.142    0.142
14 GPA_R ~~   SAT 0.238  0.194   0.194    0.056    0.056
24   SAT  ~ GPA_R 0.238  0.755   0.755    0.084    0.084
20 GPA_R  ~ GPA_E 0.087  0.025   0.025    0.028    0.028


 Comparing Model 1 and Model 2

Chi-Squared Difference Test

     Df    AIC    BIC    Chisq Chisq diff Df diff Pr(>Chisq)
fit2  2 2642.4 2681.6   0.8445
fit1  3 2739.7 2775.8 100.1044      99.26       1  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Example 2. College Academic Performance

Model	T	df	p	$NNFI$	CFI	$RMSEA$	$SRMR$	$\Delta\chi^2(\Delta df)$	p

filename: profit.R

```
# Example 3: Profit Growth

# set work directory
setwd("c:/users/wchan/google drive/stat6108/data")

# load the lavaan package
library(lavaan)
ability <- scan("profit.cov")

# write the input data into a full covariance matrix
ability.cov <- getCov(ability, names=c("time1","time2", "time3", "time4"))

# specify Model 1 (first order model)
model1 <- "
# regression Equation (Beta)
time2 ~ time1
time3 ~ time2
time4 ~ time3

# Variance of x (Psi)
time1 ~~ time1

# error variance (Psi)
time2 ~~ time2
time3 ~~ time3
time4 ~~ time4
"

# specify Model 2 (second order model)
model2 <- "
# regression Equation (Beta)
time2 ~ time1
time3 ~ time1 + time2
```

```
time4 ~ time2 + time3

# Variance of x (Psi)
time1 ~~ time1

# error variance (Psi)
time2 ~~ time2
time3 ~~ time3
time4 ~~ time4
"

# specify Model 3 (first order model with latent variables)
model3 <- "
# measurement equations (Lambda)
Profit1 =~ 1*time1
Profit2 =~ 1*time2
Profit3 =~ 1*time3
Profit4 =~ 1*time4

# structural equations (Beta)
Profit2 ~ Profit1
Profit3 ~ Profit2
Profit4 ~ Profit3

# constraining error variances (Theta)
time1 ~~ vare*time1
time2 ~~ vare*time2
time3 ~~ vare*time3
time4 ~~ vare*time4
"

# Fit Models 1-3 to the data
fit1 <- lavaan(model1, sample.cov=ability.cov, sample.cov.rescale=FALSE, sample.nobs=200, fixed.x=FALSE)
fit2 <- lavaan(model2, sample.cov=ability.cov, sample.cov.rescale=FALSE, sample.nobs=200, fixed.x=FALSE)
fit3 <- lavaan(model3, sample.cov=ability.cov, sample.cov.rescale=FALSE, sample.nobs=200, auto.var=TRUE)
```

```
# save the output
sink("profit.out", split=TRUE)
writeLines("\n Example 3: Profit Growth\n")
writeLines("\n Output for Model 1\n")
inspect(fit1)
summary(fit1, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
writeLines("\n Output for Model 2\n")
inspect(fit2)
summary(fit2, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
lavTestLRT(fit1, fit2)
writeLines("\n Output for Model 3\n")
inspect(fit3)
summary(fit3, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
sink()
```

filename: profit.out

```
 Example 3: Profit Growth

 Output for Model 1

$lambda
      time2 time3 time4 time1
time2     0     0     0     0
time3     0     0     0     0
time4     0     0     0     0
time1     0     0     0     0

$theta
      time2 time3 time4 time1
time2 0
time3 0     0
time4 0     0     0
time1 0     0     0     0

$psi
      time2 time3 time4 time1
time2 5
time3 0     6
time4 0     0     7
time1 0     0     0     4

$beta
      time2 time3 time4 time1
time2     0     0     0     1
time3     2     0     0     0
time4     0     3     0     0
time1     0     0     0     0

lavaan 0.6-5 ended normally after 16 iterations
```

```
  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                          7

  Number of observations                           200

Model Test User Model:

  Test statistic                                68.771
  Degrees of freedom                                 3
  P-value (Chi-square)                           0.000

Model Test Baseline Model:

  Test statistic                               822.319
  Degrees of freedom                                 6
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.919
  Tucker-Lewis Index (TLI)                       0.839

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2401.011
  Loglikelihood unrestricted model (H1)      -2366.626

  Akaike (AIC)                                4816.022
  Bayesian (BIC)                              4839.110
  Sample-size adjusted Bayesian (BIC)         4816.933

Root Mean Square Error of Approximation:

  RMSEA                                          0.331
```

```
  90 Percent confidence interval - lower         0.266
  90 Percent confidence interval - upper         0.401
  P-value RMSEA <= 0.05                          0.000

Standardized Root Mean Square Residual:

  SRMR                                           0.073

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  time2 ~
    time1             0.925    0.048   19.094    0.000    0.925    0.804
  time3 ~
    time2             0.906    0.039   22.935    0.000    0.906    0.851
  time4 ~
    time3             1.167    0.046   25.392    0.000    1.167    0.874

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    time1            40.000    4.000   10.000    0.000   40.000    1.000
   .time2            18.775    1.877   10.000    0.000   18.775    0.354
   .time3            16.528    1.653   10.000    0.000   16.528    0.275
   .time4            25.333    2.533   10.000    0.000   25.333    0.237

R-Square:
                   Estimate
    time2             0.646
    time3             0.725
    time4             0.763
```

Output for Model 2

```
$lambda
      time2 time3 time4 time1
time2     0     0     0     0
time3     0     0     0     0
time4     0     0     0     0
time1     0     0     0     0

$theta
      time2 time3 time4 time1
time2 0
time3 0     0
time4 0     0     0
time1 0     0     0     0

$psi
      time2 time3 time4 time1
time2 7
time3 0     8
time4 0     0     9
time1 0     0     0     6

$beta
      time2 time3 time4 time1
time2     0     0     0     1
time3     3     0     0     2
time4     4     5     0     0
time1     0     0     0     0

lavaan 0.6-5 ended normally after 20 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                          9
```

```
  Number of observations                           200

Model Test User Model:

  Test statistic                                 0.687
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.407

Model Test Baseline Model:

  Test statistic                               822.319
  Degrees of freedom                                 6
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.002

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2366.969
  Loglikelihood unrestricted model (H1)      -2366.626

  Akaike (AIC)                                4751.939
  Bayesian (BIC)                              4781.623
  Sample-size adjusted Bayesian (BIC)         4753.110

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.175
  P-value RMSEA <= 0.05                          0.514
```

```
Standardized Root Mean Square Residual:

  SRMR                                           0.005

Parameter Estimates:

  Information                                 Expected
  Information saturated (h1) model          Structured
  Standard errors                             Standard

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  time2 ~
    time1             0.925    0.048   19.094    0.000    0.925    0.804
  time3 ~
    time1             0.458    0.069    6.623    0.000    0.458    0.374
    time2             0.586    0.060    9.751    0.000    0.586    0.551
  time4 ~
    time2             0.479    0.087    5.527    0.000    0.479    0.337
    time3             0.783    0.082    9.604    0.000    0.783    0.586

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    time1            40.000    4.000   10.000    0.000   40.000    1.000
   .time2            18.775    1.877   10.000    0.000   18.775    0.354
   .time3            13.555    1.356   10.000    0.000   13.555    0.226
   .time4            21.977    2.198   10.000    0.000   21.977    0.205

R-Square:
                   Estimate
    time2             0.646
    time3             0.774
    time4             0.795
```

```
Chi-Squared Difference Test

     Df    AIC    BIC   Chisq Chisq diff Df diff Pr(>Chisq)
fit2  1 4751.9 4781.6  0.6874
fit1  3 4816.0 4839.1 68.7706     68.083       2  1.644e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
 Output for Model 3

Note: model contains equality constraints:

  lhs op rhs
1   4 ==   5
2   4 ==   6
3   4 ==   7

$lambda
      Proft1 Proft2 Proft3 Proft4
time1      0      0      0      0
time2      0      0      0      0
time3      0      0      0      0
time4      0      0      0      0

$theta
      time1 time2 time3 time4
time1 4
time2 0     5
time3 0     0     6
time4 0     0     0     7

$psi
        Proft1 Proft2 Proft3 Proft4
Profit1  8
Profit2  0      9
Profit3  0      0     10
Profit4  0      0      0     11

$beta
        Proft1 Proft2 Proft3 Proft4
Profit1      0      0      0      0
Profit2      1      0      0      0
Profit3      0      2      0      0
```

```
Profit4      0      0      3      0

lavaan 0.6-5 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         11
  Number of equality constraints                     3
  Row rank of the constraints matrix                 3

  Number of observations                           200

Model Test User Model:

  Test statistic                                 4.397
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.111

Model Test Baseline Model:

  Test statistic                               822.319
  Degrees of freedom                                 6
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.997
  Tucker-Lewis Index (TLI)                       0.991

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2368.824
  Loglikelihood unrestricted model (H1)      -2366.626

  Akaike (AIC)                                4753.648
```

```
  Bayesian (BIC)                               4780.034
  Sample-size adjusted Bayesian (BIC)          4754.689

Root Mean Square Error of Approximation:

  RMSEA                                           0.077
  90 Percent confidence interval - lower          0.000
  90 Percent confidence interval - upper          0.178
  P-value RMSEA <= 0.05                           0.234

Standardized Root Mean Square Residual:

  SRMR                                            0.008

Parameter Estimates:

  Information                                  Expected
  Information saturated (h1) model           Structured
  Standard errors                              Standard

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Profit1 =~
    time1             1.000                               5.767    0.912
  Profit2 =~
    time2             1.000                               6.782    0.934
  Profit3 =~
    time3             1.000                               7.317    0.942
  Profit4 =~
    time4             1.000                              10.013    0.968

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Profit2 ~
    Profit1           1.120    0.065   17.247    0.000    0.952    0.952
```

```
  Profit3 ~
    Profit2           1.050    0.046   22.710    0.000    0.973    0.973
  Profit4 ~
    Profit3           1.303    0.054   24.205    0.000    0.952    0.952

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .time1   (vare)    6.737    0.881    7.644    0.000    6.737    0.168
   .time2   (vare)    6.737    0.881    7.644    0.000    6.737    0.128
   .time3   (vare)    6.737    0.881    7.644    0.000    6.737    0.112
   .time4   (vare)    6.737    0.881    7.644    0.000    6.737    0.063
    Profit1          33.263    4.096    8.121    0.000    1.000    1.000
   .Profit2           4.294    2.048    2.097    0.036    0.093    0.093
   .Profit3           2.863    1.433    1.998    0.046    0.053    0.053
   .Profit4           9.377    2.625    3.572    0.000    0.094    0.094

R-Square:
                   Estimate
    time1             0.832
    time2             0.872
    time3             0.888
    time4             0.937
    Profit2           0.907
    Profit3           0.947
    Profit4           0.906
```

Example 3. Profit Growth

Model	*T*	*df*	*p*	*NNFI*	*CFI*	*RMSEA*	*SRMR*	$\Delta\chi^2(\Delta df)$	*p*	*AIC*