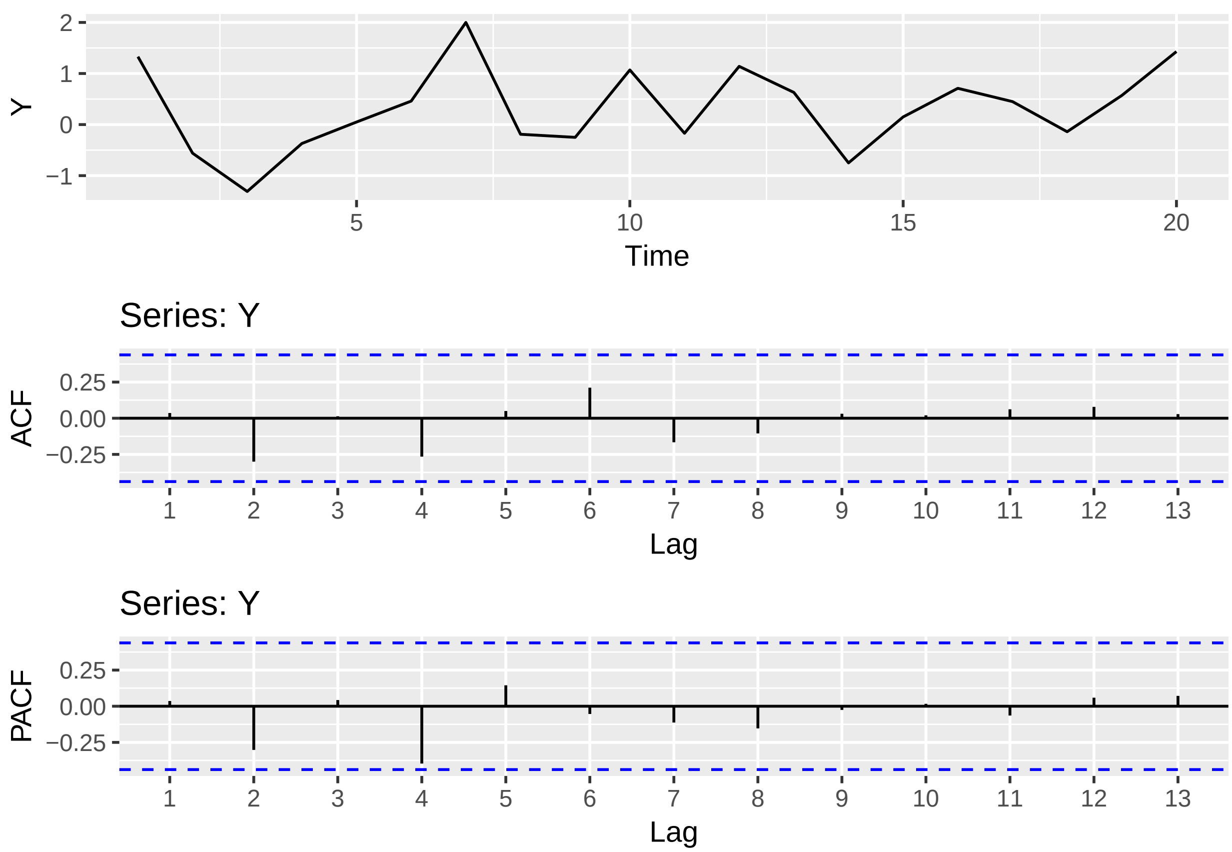
# 2018R2 Financial Time Series (STAT6104) Assignment 3

*Yiu Chung WONG 1155017920*

```r
Y <- ts(c(1.33,-0.56,-1.31,-0.37,0.05,0.46,2.00,-0.19,
         -0.25,1.07,-0.17,1.14,0.63,-0.75,0.15,0.71,
         0.45,-0.14,0.57,1.43));
```

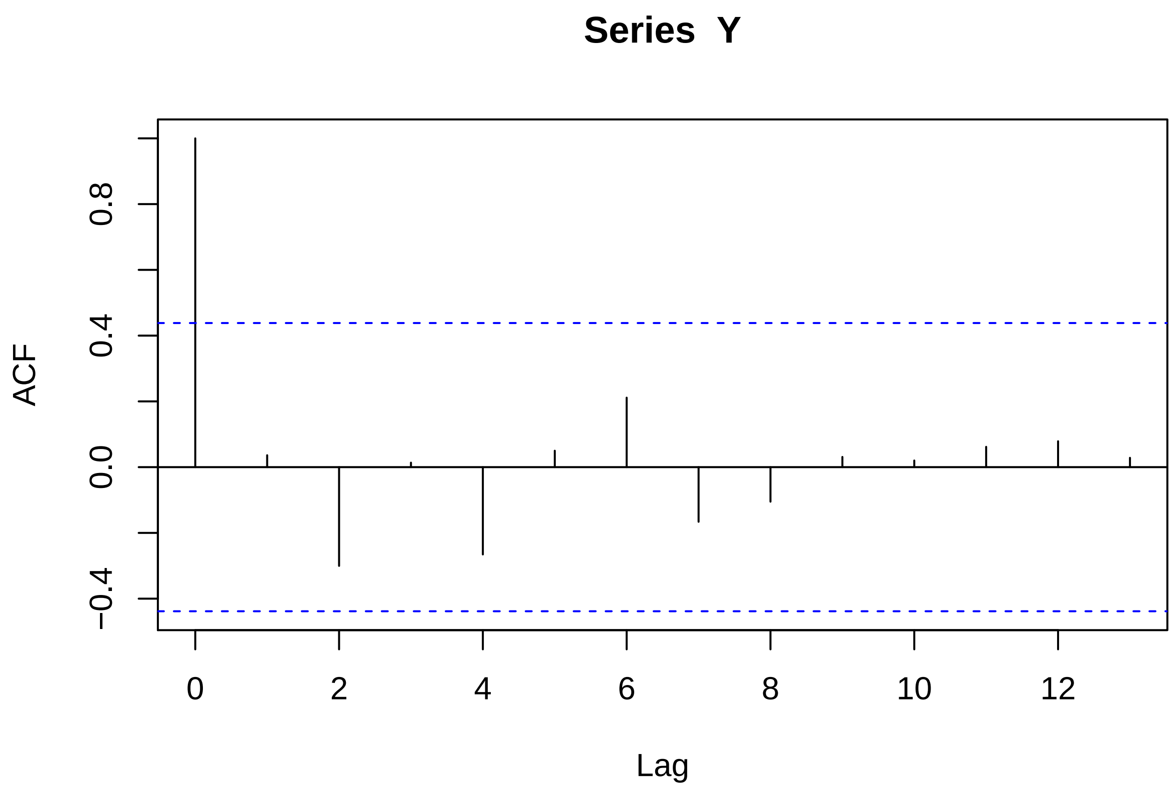
### 1

```r
plot1 <- forecast::autoplot(Y)
plot2 <- forecast::ggAcf(Y)
plot3 <- forecast::ggPacf(Y)
gridExtra::grid.arrange(plot1, plot2, plot3, nrow=3)
```




### 2

```r
c2 <- acf(Y)$acf[3];
```




```
theta <- uniroot(f = function(x){ (x / (x^2 + 1) ) - c2}, interval = c(-1, 0))$root;
```

$$
\begin{aligned}
\theta &= -0.333744 \\
\sigma^2 * \hat{\theta} &= C_2 \\
\sigma^2 &= C_2/\hat{\theta} \\
&= -0.3002958 \;/ -0.333744 \\
&= 0.8997788
\end{aligned}
$$

### 3

```
p = 2;
X = matrix(nrow = length(Y) - p, ncol = p);
for (i in 1:(length(Y)-p))
{
  X[i,] <- Y[(i):(i+1)]
}
X[,1:2] = X[,2:1];
model <- lm(Y[-seq_len(p)]~X-1)
phi_1 <- coef(model)[1];
phi_2 <- coef(model)[2];

z <- rep(0, length(Y));
for (i in 3:length(Y))
```

```
{
  z[i] <- Y[i] - phi_1 * Y[i - 1] - phi_2 * Y[i - 2];
}
z <- z[-seq_len(p)];
```

$$\phi_1 = 0.2353662$$
$$\phi_2 = -0.2230385$$

95% CI for $\phi_1$ = (-0.2689644, 0.7396968)

95% CI for $\phi_2$ = (-0.69717, 0.251093)

$$\sigma^2 = 0.6432305$$

## 4

```
sampleACF <- acf(x = Y, lag.max = 2, plot = FALSE)$acf
YWalker <- matrix(c(1, sampleACF[2], sampleACF[2], 1), 2)
r <- c(sampleACF[2], sampleACF[3])
phi <- solve(YWalker, r)
```

$$\phi_1 = 0.0471812$$
$$\phi_2 = -0.3020055$$

## 5

```
CLS <- function(parameters, findSigma = FALSE)
{
  z <- rep(0, length(Y));
  z[1] <- Y[1];
  for (i in 2:length(Y))
  {
    z[i] <- Y[i] - parameters[1] * Y[i - 1] - parameters[2] * z[i - 1];
  }
  if (findSigma)
  {
    return (z)
  }

  return (sum(z ** 2));
}

phi <- optim(par = c(.1,.1), fn = CLS)$par[1]
theta <- optim(par = c(.1,.1), fn = CLS)$par[2]
z <- CLS(c(phi, theta), findSigma = TRUE)
```

$$\phi = -0.5108904$$

$$\theta = 0.8439972$$
$$\sigma^2 = 0.6469631$$

## 6

```
MLE <- arima(Y, order = c(1, 0, 1));
phi <- MLE$coef[1];
theta <- MLE$coef[2];
z <- rep(0, length(Y));
for (i in 2:length(Y))
{
  z[i] <- Y[i] - phi * Y[i - 1] - theta * z[i - 1];
}
```

$$\phi = -0.4424074$$
$$\theta = 0.9999995$$
$$\sigma^2 = 0.7733874$$

Maximized log-likelihood = -21.8708459

## 7

```
IC <- function(x, AR = 1)
{
  fit <- arima(x, c(AR, 0, 0));
  n <- length(x);
  p <- AR;
  q <- 0
  sig <- fit$sigma2
  FPE <- sig*(n+p)/(n-p);
  FPE
}

FPEs <- rep(0, 5);
for (i in 1:6)
{
  FPEs[i] <- IC(Y, i)
}
```

AR(4) is the best in terms of FPE.

## 8

```
IC <- function(x, MA = 1)
{
  fit <- arima(x, c(0, 0, MA));
  n <- length(x);
  p <- 0;
  q <- MA;
```

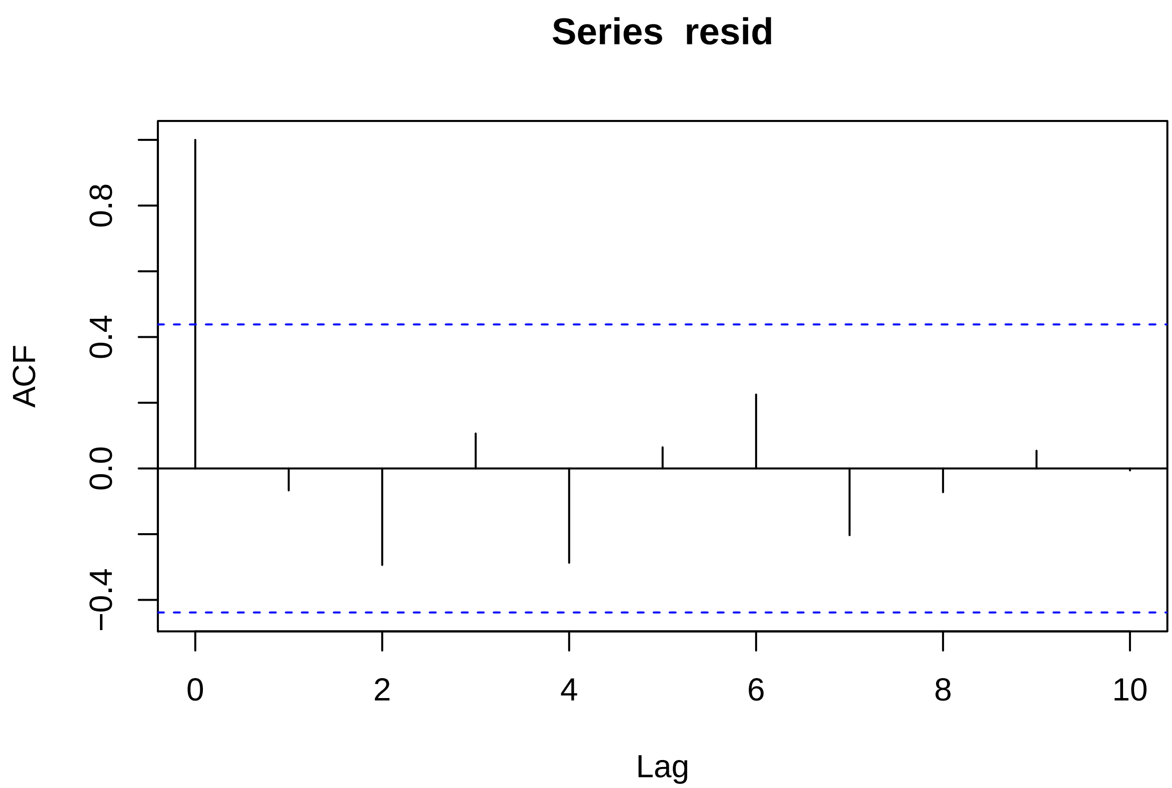
```
  sig <- fit$sigma2
  fit$aic
}

AICC <- rep(0, 5);
for (i in 1:5)
{
  AICC[i] <- IC(Y, i)
}
```

MA(2) is the best in terms of AICC.

## 9

```
p <- 0;
q <- 1;
h <- 10;
n <- length(Y)
MA <- arima(Y, order = c(0, 0, q));
resid <- MA$residuals;
r.z=as.numeric(acf(resid, h)$acf);
```




```
portmanteau.stat <- n*(n+2)*sum((r.z[-1]^2)/(n-(1:h)));
portmanteau.stat > qchisq(0.95,h - p - q)
```

```
## [1] FALSE
```

$$H_0 : Z_t \sim WN$$

$$H_1 : Z_t \ not \sim WN$$

Portmanteau statistic is 8.1936863; we fail to reject $H_0$.

## 10

White Noise