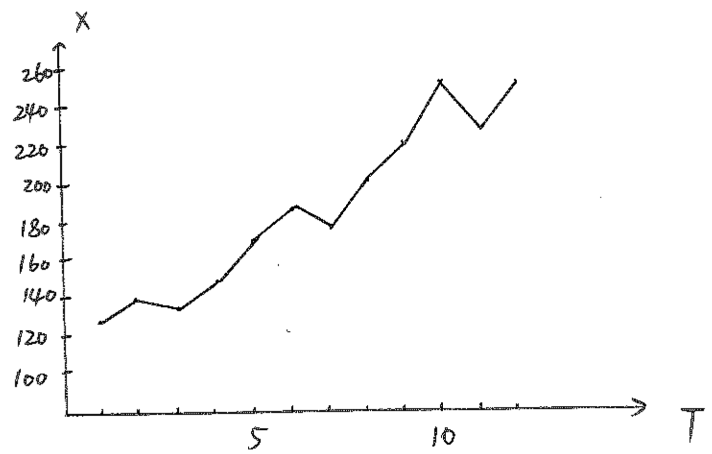
1a.



1b. period = 4

Filter : $(a_{-2}, a_{-1}, 0, a_1, a_2) = (\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8})$

$$\hat{T}_4 = \frac{138}{8} + \frac{132}{4} + \frac{144}{4} + \frac{169}{4} + \frac{188}{8} = 152$$

$$\hat{T}_9 = \frac{170}{8} + \frac{206}{4} + \frac{220}{4} + \frac{248}{4} + \frac{226}{8} = 218$$

1c. $D_t = X_t - \hat{T}_t$

D : 9 6 -12 -11 2 9 -21 3 5 22 -12 -2

$$\bar{D} = -\frac{1}{6}$$

$$\hat{S}_1 = \frac{D_1 - \bar{D} + D_5 - \bar{D} + D_9 - \bar{D}}{3} = \frac{11}{2}$$

$$\hat{S}_2 = \frac{D_2 - \bar{D} + D_6 - \bar{D} + D_{10} - \bar{D}}{3} = \frac{25}{2}$$

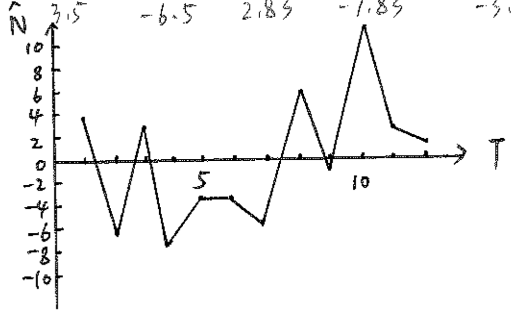
$$\hat{S}_3 = \frac{D_3 - \bar{D} + D_7 - \bar{D} + D_{11} - \bar{D}}{3} = -\frac{89}{6}$$ 14.8333

$$\hat{S}_4 = \frac{D_4 - \bar{D} + D_8 - \bar{D} + D_{12} - \bar{D}}{3} = -\frac{19}{6}$$ 3.16667

1d. $\hat{N} = X - \hat{T} - \hat{S}$

$\hat{N}$: $\frac{7}{2}$ $-\frac{13}{2}$ $\frac{17}{6}$ $-\frac{47}{6}$ $-\frac{7}{2}$ $-\frac{7}{2}$ $-\frac{37}{6}$ $\frac{37}{6}$ $-\frac{1}{2}$ $\frac{19}{2}$ $\frac{17}{6}$ $\frac{7}{6}$

3.5 -6.5 2.83 -7.83 -3.5 -3.5 -6.16 6.16 -0.5 9.5 2.83 1.16



1e. Sample $ACF_1 = \frac{c_1}{c_0}$

$$c_0 = \frac{1}{12}\sum_{t=1}^{12}(\hat{N}_t - \bar{N})^2 = 27$$

$$c_1 = \frac{1}{12}\sum_{t=1}^{11}(N_t - \bar{N})(N_{t+1} - \bar{N}) = -\frac{85}{54} \quad -1.574$$

$$r_1 = \frac{c_1}{c_0} = -\frac{85}{1458} < -\frac{2}{\sqrt{12}} \quad \therefore \text{ white noise.}$$

-0.0583

2. a)

$$Z_t(1-0.6B)(1+0.4B) = a_t(1+\sqrt{3}B)(1-\sqrt{3}B)$$

$\therefore$ ARIMA(2, 0, 2) stationary, causal, not invertible

b)

$$Z_t(1-0.7B)(1+0.7B) = a_t(1-0.7B)(1+0.2B)$$

$$Z_t(1+0.7B) = a_t(1+0.2B)$$

$\therefore$ ARIMA(1, 0, 1) stationary, causal, invertible

c)

$$Z_t(1-B)(1-0.8B) = a_t(1-B)^4$$

$$Z_t(1-0.8B) = a_t(1-B)^3$$

$\therefore$ ARIMA(1, 0, 3) stationary, causal, not invertible

d)

$$Z_t(1-B)^3 = a_t(1-B)(1-0.2B)$$

$$Z_t(1-B)^2 = a_t(1-0.2B)$$

$\therefore$ ARIMA(0, 2, 1) not stationary, not causal, invertible

e)

$$Z_t(1-0.1B)(1+8.1B) = a_t(1-0.5B)(1+8.1B)$$

$$Z_t(1-0.1B) = a_t(1-0.5B)$$

$\therefore$ ARIMA(1, 0, 1) stationary, causal, invertible.

3a. $X_t - 0.5X_{t-1} = Z_t + 1.5Z_{t-1}$

$(1-0.5B)X_t = Z_t + 1.5Z_{t-1}$

$$X_t = \frac{1}{1-0.5B}(Z_t + 1.5Z_{t-1})$$

$$= [1 + 0.5B + (0.5B)^2 + \cdots](Z_t + 1.5Z_{t-1})$$

$\therefore \psi_1 = 1.5 + 0.5 = 2$

$\psi_2 = 1.5(0.5) + (0.5)^2 = 2(0.5)$

$\therefore \psi_k = 2(0.5)^{k-1} = 0.5^{k-2}$

$\psi_k = -0.5^{k-2}$ // (OR $-4(0.5)^k$ OR $-2(0.5)^{k-1}$)

3b. $Cov(X_t, Z_t) = 9$

$Cov(X_t, Z_{t-1}) - 0.5(9) = 1.5 \times 9$ $\therefore Cov(X_t, Z_{t-1}) = 18$

$\therefore Cov(X_t, X_t) - 0.5\,Cov(X_t, X_{t-1}) = 9 + 1.5(18) = 36$

$\gamma_0 = 0.5\gamma_1 + 36$

$\gamma_1 = 0.5\gamma_0 + 13.5$

$\therefore \gamma_0 = 57$, $\gamma_1 = 42$.

$\gamma_2 = 0.5\gamma_1$

$\vdots$

$$\therefore \gamma_k = \begin{cases} 57, & k=0 \\ 42(0.5)^{k-1}, & k=1,2,3,\ldots \end{cases}$$ //

3c. Roots in MA characteristic equation $= -\frac{2}{3}$,
so this model is not invertible.
$\therefore$ This model is not useful in practise as we cannot express it by past data.

4. $Y_t = Z_t + 0.5 Z_{t-1} Z_{t-2}$

$$E(Y_t) = E(Z_t) + 0.5 E(Z_{t-1}) E(Z_{t-2}) = 0$$

$$\therefore \gamma(0) = Cov(Y_t, Y_t) = E(Y_t^2) - [E(Y_t)]^2$$

$$= E(Z_t^2 + 0.25 Z_{t-1}^2 Z_{t-2}^2 + Z_t Z_{t-1} Z_{t-2}) - 0$$

$$= 1 + 0.25(1)(1) + 0 = 1.25_{\parallel}$$

$$\gamma(1) = Cov(Y_t, Y_{t-1}) = E[(Z_t + 0.5 Z_{t-1} Z_{t-2})(Z_{t-1} + 0.5 Z_{t-2} Z_{t-3})]$$

$$= 0.5 E(Z_{t-1}^2)(Z_{t-2}) + 0.5^2 E(Z_{t-1} Z_{t-2}^2 Z_{t-3})$$

$$= 0_{\parallel}$$

$$\gamma(2) = Cov(Y_t, Y_{t-2}) = E[(Z_t + 0.5 Z_{t-1} Z_{t-2})(Z_{t-2} + 0.5 Z_{t-3} Z_{t-4})]$$

$$= 0_{\parallel}$$

$$\therefore ACVF = \begin{cases} 1.25, & k=0 \\ 0, & \text{otherwise} \end{cases} \#.$$

5a. $Var(Y_t - 2Y_{t-1} + Y_{t-2})$

$= Cov(Y_t - 2Y_{t-1} + Y_{t-2}, Y_t - 2Y_{t-1} + Y_{t-2})$

$= 6\gamma(0) - 8\gamma(1) + 2\gamma(2)$

$= -0.4 < 0$

$\because$ Variance cannot be smaller then 0.

$\therefore$ It is not reasonable.

5b. i) $Var(Y_t - 2Y_{t-1} + Y_{t-2}) = 4.4 > 0$

ii) $Var(Y_t - 2Y_{t-1} + Y_{t-2}) = -1.6 < 0$

iii) $Var(Y_t - 2Y_{t-1} + Y_{t-2}) = -0.8 < 0$

$\therefore$ Set i is most reasonable. #.