95

2.1 a)
b)

	son's occupation					
father's occupation	farm	operatives	craftsmen	sales	professional	$\mathbb{P}(\text{Father}_i \mid \text{Son})$
farm	0.018	0.035	0.031	0.008	0.018	0.11
operatives	0.002	0.112	0.064	0.032	0.069	0.279
craftsmen	0.001	0.066	0.094	0.032	0.084	0.277
sales	0.001	0.018	0.019	0.010	0.051	0.099
professional	0.001	0.029	0.032	0.043	0.130	0.235
$\mathbb{P}(\text{Son} \mid \text{Father})$	0.023	0.26	0.24	0.125	0.352	1

2.1 c)

	son's occupation				
father's occupation	farm	operatives	craftsmen	sales	professional
farm	0.018	0.035	0.031	0.008	0.018

$\underbrace{0.018 \quad 0.035 \quad 0.031 \quad 0.008 \quad 0.018}_{0.11}$

= 0.1636 , 0.3181 , 0.2818 , 0.0727 , 0.1636

2.1d)

father's occupation	farm
farm	0.018
operatives	0.002
craftsmen	0.001
sales	0.001
professional	0.001

0.023

= 0.78260
0.08696
0.04348
0.04348
0.04348

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2.2a) $E[a_1 \cdot Y_1 + a_2 \cdot Y_2]$

$= a_1 \cdot \mu_1 + a_2 \cdot \mu_2$

$Var[a_1 \cdot Y_1 + a_2 \cdot Y_2]$

$= a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + 2 a_1 \cdot a_2 \cdot \underbrace{Cov(Y_1, Y_2)}_{= 0}$

b) $E[a_1 \cdot Y_1 - a_2 \cdot Y_2] = a_1 \cdot \mu_1 - a_2 \cdot \mu_2$

$Var[a_1 \cdot Y_1 - a_2 \cdot Y_2]$

$= a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 - 2 a_1 \cdot a_2 \cdot \underbrace{Cov(Y_1, Y_2)}_{= 0}$

use the definition

-5

2.3a) $$\mathbb{P}(x \mid y, z) = \frac{\mathbb{P}(x, y, z)}{\mathbb{P}(y, z)}$$

$$\propto \frac{\mathbb{P}(x, y, z)}{\int p(x, y, z)\, dx}$$

$$\propto \frac{f(x, z) \cdot g(y, z) \cdot h(z)}{\int f(x, z) \cdot g(y, z) \cdot h(z)\, dx}$$

$$\propto \frac{f(x, z) \cdot g(y, z) \cdot h(z)}{g(y, z) \cdot h(z) \int f(x, z)\, dx}$$

$$\propto \frac{f(x, z)}{\int f(x, z)\, dx}$$

b) $\mathbb{P}(y \mid x, z) = \mathbb{P}(y, x, z) / \mathbb{P}(x, z)$

$$\propto \frac{\mathbb{P}(y, x, z)}{\int \mathbb{P}(y, x, z)\, dy}$$

$$\propto \frac{f(x,z) \cdot g(y,z) \cdot h(z)}{\int f(x,z) \cdot g(y,z) \cdot h(z)\, dy}$$

$$\propto \frac{f(x,z) \cdot g(y,z) \cdot h(z)}{f(x,z) \cdot h(z) \int g(y,z)\, dy}$$

$$\propto \frac{g(y,z)}{\int g(y,z)\, dy}$$

c) Show $p(y|z) = p(y|z,x)$

$$p(y|z) = p(y,z)/p(z)$$

$$\propto \frac{\int p(x,y,z)\,dx}{\int\int p(x,y,z)\,dx\,dy}$$

$$\propto \frac{\int f(x,z)\cdot g(y,z)\cdot h(z)\,dx}{\int\int f(x,z)\cdot g(y,z)\cdot h(z)\,dx\,dy}$$

$$\propto \frac{g(y,z)\cdot h(z)\int f(x,z)\cdot dx}{h(z)\int\int f(x,z)\cdot g(y,z)\,dx\,dy}$$

$$\propto \frac{g(y,z)\int f(x,z)\,dx}{\left[\int f(x,z)\,dx\right]\cdot\left[\int g(y,z)\,dy\right]}$$

$$\propto \frac{g(y,z)}{\int g(y,z)\,dy} = p(y|z,x)$$

2.5 a)

Y \ X	H	T
Green	0.2	0.3
Red	0.3	0.2

b) $\frac{1}{2}$

c) $Var[Y \mid X=0]$

$$= E\left[(Y \mid X=0)^2\right] - E[Y \mid X=0] \cdot E[Y \mid X=0]$$

$$= \left[\begin{matrix} 1^2 \cdot P(Y \mid X=0) \\ + \\ 0^2 \cdot P(Y \mid X=0) \end{matrix}\right] - \left[\begin{matrix} 1 \cdot P(Y=1 \mid X=0) \\ + \\ 0 \cdot P(Y=0 \mid X=0) \end{matrix}\right]^2$$

$$= 0.6 - 0.6^2 = 0.24$$

$$Var[Y \mid X=1] = E\left[(Y \mid X=1)^2\right] - E[Y \mid X=1]^2$$

$$= \left[\begin{matrix} 1^2 \cdot P(Y \mid X=1) \\ + \\ 0^2 \cdot P(Y \mid X=1) \end{matrix}\right] - \left[\begin{matrix} 1 \cdot P(Y=1 \mid X=1) \\ + \\ 0 \cdot P(Y=0 \mid X=1) \end{matrix}\right]^2$$

$$= 0.2 - 0.2^2 = 0.16$$

$$Var[Y] = E[Y^2] - E[Y] \cdot E[Y]$$

$$= 0.5 - 0.25 = 0.25$$

$Var[Y]$ is the largest because we do not have information from the coin toss, which tells us which urn the balls will be drawn from.
Knowing which urn the balls are being drawn from narrows down the conditional prob.

d) $P(X=0 \mid Y=1)$

$$= \frac{P(X=0 \text{ and } Y=1)}{P(Y=1)}$$

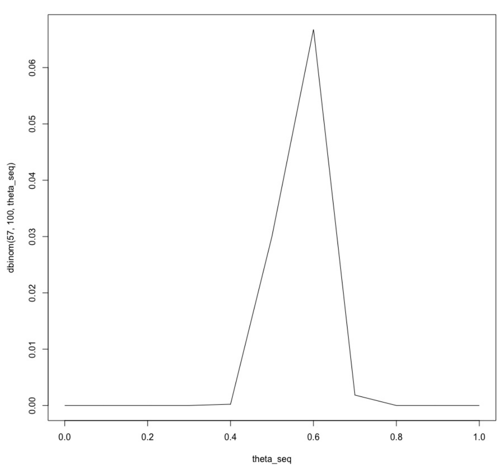
$$= 0.3/0.5 = \frac{3}{5} = 0.6$$

3.1a) $\theta^{\sum_i y_i} \cdot (1-\theta)^{100-\sum_i y_i}$

$$P\left(\sum_i Y_i = y \mid \theta\right) = \binom{N}{y} \theta^{\sum_i Y_i} (1-\theta)^{100-\sum_i Y_i}$$

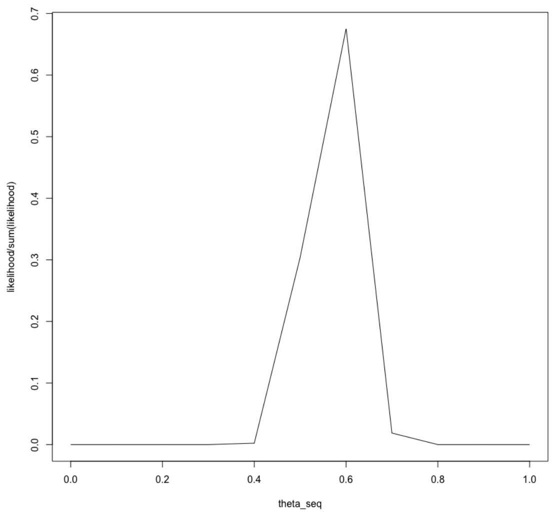
b)

```
> theta_seq <- seq(0, 1, by = .1)
> likelihood <- dbinom(57, 100, theta_seq)
> plot(theta_seq, dbinom(57, 100, theta_seq), type = 'l')
> likelihood
 [1] 0.000000e+00 4.107157e-31 3.738459e-16 1.306895e-08 2.285792e-04 3.006864e-02 6.672895e-02 1.853172e-03 1.003535e-07 9.395858e-18
[11] 0.000000e+00
```



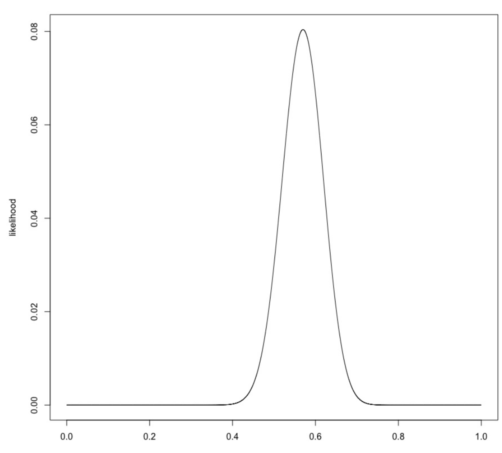
c)

```
> likelihood / sum(likelihood)
 [1] 0.000000e+00 4.153701e-30 3.780824e-15 1.321705e-07 2.311695e-03 3.040939e-01 6.748515e-01 1.874172e-02 1.014907e-06 9.502335e-17
[11] 0.000000e+00
```



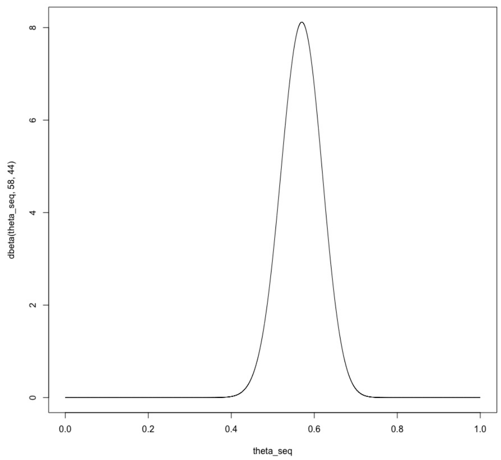
d)

```
> theta_seq <- seq(0, 1, length.out = 10000)
> likelihood <- dbinom(57, 100, theta_seq)
> plot(theta_seq, likelihood, type = 'l')
```



e)

```
> theta_seq <- seq(0, 1, length.out = 10000)
> plot(theta_seq, dbeta(theta_seq, 58, 44), type = 'l')
```



e is normalized,
d is not