STAT 6104 Financial Time Series
Midterm
7:00-9:15. Monday, 17 Oct 2016

Name: ______________________ **Major:** ______________________

1. (20 marks) Given nine observations $\mathbf{x} = (3.8, 3.1, 2.1, 1, 0, 0.9, 1.9, 3.2, 4)$ and a filter $\{a_{-2}, a_{-1}, a_0, a_1, a_2\} = \{0.1, 0.2, 0.4, 0.2, 0.1\}$
   a) (4 marks). Does the filter pass through a linear trend without distortion? Does the filter passes through a quadratic trend without distortion?
   b) (6 marks). Let T_t be the filtered value at time t. What are T_3, T_5 and T_9?
   c) (10 marks). Recall that in estimating the seasonal component, we have to estimate the trend first. If the period is 3, can the given filter be used for estimating the trend? Why? If no, suggest a filter. Find the trend estimated from the filter that you suggest, **and** estimate the 1st seasonal effect (the seasonal effect corresponds to time 1).

2. (20 marks) Suppose that $X_t = 2 + a_t + 0.3a_{t-1}$, $a_t \overset{i.i.d.}{\sim} N(0, 2)$.
   (a) (3 marks) Find $Var(X_t)$.
   (b) (8 marks) Find $Cov(X_t, X_{t+k})$ for $|k| \neq 0$.
   (c) (9 marks) Let $\bar{X} = \sum_{t=1}^{10} X_t/10$. Find $Var(\bar{X})$.

3. (30 marks) Consider the time series
$$Y_t = 0.5Y_{t-1} + a_t + 0.6a_{t-1}, \quad a_t \overset{i.i.d}{\sim} N(0, 1)$$
   (a) (10 marks) Find $Cov(Y_t, Y_{t+k})$ for all $|k|$.
   (b) (10 marks) Given nine observations $\{x_t\}_{t=1}^{9}$ with sample mean $\bar{x}$. Suppose the values of $\sum_{t=1}^{9-k}(x_t - \bar{x})(x_{t+k} - \bar{x})$ for $k = 0, 1, 2, 3, 4$ are $78.09, 58.26, 22.62, -7.03, -27.68$, respectively. Draw the sample ACF plot for the data for lags up to 4. Which lags are reliable?
   (c) (10 marks) Suppose that we have the representation $Y_t = \sum_{j=0}^{\infty} \psi_j a_{t-j}$. Find ψ_0, ψ_1, ψ_2, and ψ_3.

4. (25 marks) Given $a_t \overset{i.i.d.}{\sim} N(0, 1)$. Identify the following ARIMA models (state the order of the model):
   a) $Z_t = 1.4Z_{t-1} - 0.4Z_{t-2} + a_t - 2a_{t-2}$.
   b) $Z_t = -0.5Z_{t-1} + 0.14Z_{t-2} + a_t - 0.49a_{t-2}$.
   c) $Z_t = 1.6Z_{t-1} - 0.6Z_{t-2} + a_t - 3a_{t-1} + 3a_{t-2} - a_{t-3}$.
   d) $Z_t = -0.8Z_{t-1} + 0.2Z_{t-2} + 2a_t - 3.2a_{t-1} + 1.28a_{t-2}$.
   e) $Z_t = 6Z_{t-1} - 0.59Z_{t-2} + a_t - a_{t-1} - 0.25a_{t-2}$.

   State whether the models are causal and invertible.

5. (5 marks) Given $Z_t \overset{i.i.d.}{\sim} N(0, 1)$. Consider the model
$$Y_t = \begin{cases} Z_t & \text{with probability } 0.3 \\ 1 + 0.8Y_t + Z_t & \text{with probability } 0.7 \end{cases}.$$
   Find $E(Y_t)$ and $Cov(Y_t, Y_{t+1})$.

End of paper