

6 Structural Equation Modeling 3: Multisample Analysis

References:

- Beaujean (2014). Chapter 4.

6.1. Introduction

- When we have data from different groups (e.g., gender, countries, organizations), an interesting question to ask is whether a structural model would work the same for these groups.
- Universality vs. specificity

- Statistically, suppose we have G populations (groups) with covariance matrices $\Sigma_1, \Sigma_2, \dots, \Sigma_G$. Our task is to examine to what extent these covariance matrices share the same structure in terms of some latent variable models. We call this procedure multisample structural equation modeling (MSEM) analysis.
- For each individual group g , we have $\Sigma_g = \Sigma_g(\theta_g)$, $g = 1, 2, \dots, G$.
- If we treat these G groups separately, our task is
 - (1) to estimate $\theta_1, \theta_2, \dots, \theta_G$.
 - (2) to evaluate if the hypothesized model, $\Sigma_g = \Sigma_g(\theta_g)$, adequately fits the data in each group, $g = 1, 2, \dots, G$.
- In multisample SEM, however, our interest is to establish some kinds of universality (invariance) across the groups.

6.2. Types of Invariance in MSEM

Table 4.1 Types of Measurement Invariance.

| | Type of Invariance | Constraints | Between-Groups Comparisons Allowed |
|---|--------------------|---|--|
| 1 | Configural | Same model. No parameter constraints. | None |
| 2 | Weak | 1 + all loadings constrained to be equal between groups (but can vary within a group). Latent (co)variances allowed to vary between groups. | Latent (co)variances [weak evidence] |
| 3 | Strong | 2 + all intercepts are constrained to be equal between groups (but can vary within a group). Latent means allowed to vary between groups. | Latent means, latent (co)variances [strong evidence] |
| 4 | Strict | 3 + error variances are constrained to be the same between groups (but can vary within a group). | |

Models with larger numbers are nested within the models with smaller numbers.

(source: Beaujean, 2014)

6.3. Some Common Steps in MSEM

6.3.1. Step 1: Testing configural (structural form) invariance

- Want to know if the groups share a common structure
- The hypothesis of interest is

$$H_o: \Sigma_1 = \Sigma(\theta_1), \Sigma_2 = \Sigma(\theta_2), \dots, \Sigma_G = \Sigma(\theta_G)$$

- Once configural invariance is established, we can proceed to the next step.

6.3.2. Step 2: Testing weak factorial (measurement) invariance

- Want to know if the factor loadings are equal across the groups
- The hypothesis of interest is

$$H_o: \Lambda^{(1)} = \dots = \Lambda^{(G)}$$

- Once factorial invariance is established, we can proceed to the comparisons of structural coefficients.

6.3.3. Step 3: Comparing structural coefficients

- *Step 3a*: Testing equality of factors'/variables' variances and/or covariances
- The hypothesis of interest is

$$H_o: \Psi_f^{(1)} = \dots = \Psi_f^{(G)}$$

- *Step 3b*: Testing equality of structural/path coefficients
- The hypothesis of interest is

$$H_o: B^{(1)} = \dots = B^{(G)}$$

- Moderation effect exists if the path coefficients are different across the groups.

6.3.4. Step 4: Comparing error/disturbance variances/covariances

- In general, this is of less theoretical importance
- *Step 4a*: Testing equality of error variances and covariances
- The hypothesis of interest is

$$H_o: \Theta^{(1)} = \dots = \Theta^{(G)} \text{ (or } H_o: \Psi_e^{(1)} = \dots = \Psi_e^{(G)} \text{ in path analysis)}$$

- *Step 4b*: Testing equality of disturbance variances and covariances
- The hypothesis of interest is

$$H_o: \Psi_d^{(1)} = \dots = \Psi_d^{(G)}$$

- Acceptance of Steps 1 – 4 would imply that all of the parameters in the common model are equal across groups.
- This is a highly restrictive hypothesis (all covariance matrices as well as the structural models across groups are identical).
- We can also use MSEM to test the equality of (unstructured) population covariance matrices:

$$H_o: \Sigma_1 = \Sigma_2 = \dots = \Sigma_G$$

6.4 Goodness-of-fit Measures

- The chi-square goodness-of-fit statistic is

$$T = (N_1-1)F(\hat{\theta}_1) + (N_2-1)F(\hat{\theta}_2) + \dots + (N_G-1)F(\hat{\theta}_G)$$

- Under H_o , $T \overset{a}{\sim} \chi^2(p^*-q+c)$, where

p^* = no. of sample variances and covariances in all groups

q = no. of free parameters in all groups

c = no. of within- and between-group equality constraints

- Multiple group models are hard to fit because a single model misspecification in any one sample would result in a large overall test statistic

6.5 An Example: Sales/Marketing and Administration (SMA) Ability

- Group 1: Males Officers ($N = 265$, filename = *sma_male.cov*)

| | | | | |
|--------|----|-----|----|----|
| sales1 | 63 | | | |
| sales2 | 70 | 110 | | |
| admin1 | 41 | 52 | 60 | |
| admin2 | 30 | 37 | 36 | 32 |

- Group 2: Females Officers ($N = 300$, filename = *sma_female.cov*)

| | | | | |
|--------|----|-----|----|----|
| sales1 | 67 | | | |
| sales2 | 72 | 107 | | |
| admin1 | 40 | 55 | 63 | |
| admin2 | 28 | 38 | 39 | 35 |

- Model 1: Testing if the two groups have equal factorial structure (configural invariance)
- Model 2: Model 1 + Equal factor loadings (weak measurement invariance)

- Model 3: Model 2 + Equal factor variances
- Model 4: Model 3 + Equal factor correlations

- Model 5: Model 4 + Equal error variances
- Model 6: Equality of unstructured covariance matrices ($\Sigma_M = \Sigma_F$)

| Model | χ^2_1 | χ^2_2 | χ^2 (df) | p | <i>NNFI</i> | <i>CFI</i> | <i>RMSEA</i> | <i>SRMR</i> | $\Delta\chi^2(\Delta df)$ | p |
|-------|------------|------------|---------------|------|-------------|------------|--------------|-------------|---------------------------|-------|
| 1 | 0.763 | 0.188 | 0.950 (2) | .622 | 1.004 | 1.000 | 0.000 | 0.002 | | |
| 2 | 1.806 | 1.114 | 2.921 (4) | .571 | 1.002 | 1.000 | 0.000 | 0.015 | 1.971 (2) | 0.373 |
| 3 | 2.035 | 1.310 | 3.346 (6) | .764 | 1.003 | 1.000 | 0.000 | 0.027 | 0.425 (2) | 0.808 |
| 4 | 2.849 | 2.084 | 4.933 (7) | .668 | 1.002 | 1.000 | 0.000 | 0.023 | 1.587 (1) | 0.208 |
| 5 | 7.342 | 5.326 | 12.668 (11) | .316 | 0.999 | 0.999 | 0.023 | 0.024 | 7.735 (4) | 0.102 |
| 6 | ? | ? | ? | | | | | | | |

• R script file: *sma.R*

```
# Example: Sales/Marketing and Administration Ability

# set work directory
setwd("c:/users/wchan/google drive/stat6108/data")

# load the lavaan package
library(lavaan)

# data preparation
varname <- c("sales1", "sales2", "admin1", "admin2")
cov1 <- scan("sma_male.cov")
male.cov <- getCov(cov1, names=varname)
cov2 <- scan("sma_female.cov")
female.cov <- getCov(cov2, names=varname)

# specify Model 1 (configural invariance)
modell <- "
# measurement model
S_M =~ sales1 + sales2
ADM =~ admin1 + admin2
# factor Variances and covariance
S_M ~~ 1*S_M
ADM ~~ 1*ADM
S_M ~~ ADM
# error Variances
sales1 ~~ sales1
sales2 ~~ sales2
admin1 ~~ admin1
admin2 ~~ admin2
"

# Fit Model 1 to data
fit1 <- lavaan(modell, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))
```

```

# specify Model 2 (weak factorial invariance)
model2 <- "
# measurement model
S_M =~ c(eq1,eq1)*sales1 + c(eq2,eq2)*sales2
ADM =~ c(eq3,eq3)*admin1 + c(eq4,eq4)*admin2
# factor Variances and covariance
S_M ~~ c(1,NA)*S_M
ADM ~~ c(1,NA)*ADM
S_M ~~ ADM
# error Variances
sales1 ~~ sales1
sales2 ~~ sales2
admin1 ~~ admin1
admin2 ~~ admin2
"

# Fit Model 2 to data
fit2 <-lavaan(model2, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))

# specify Model 3 (Model 2 + equal factor variances)
model3 <- "
# measurement model
S_M =~ c(eq1,eq1)*sales1 + c(eq2,eq2)*sales2
ADM =~ c(eq3,eq3)*admin1 + c(eq4,eq4)*admin2
# factor Variances and covariance
S_M ~~ c(1,1)*S_M
ADM ~~ c(1,1)*ADM
S_M ~~ ADM
# error Variances
sales1 ~~ sales1
sales2 ~~ sales2
admin1 ~~ admin1
admin2 ~~ admin2
"

# Fit Model 3 to data
fit3 <-lavaan(model3, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))

```



```

# specify Model 4 (Model 3 + equal factor correlation)
model4 <- "
# measurement model
S_M =~ c(eq1,eq1)*sales1 + c(eq2,eq2)*sales2
ADM =~ c(eq3,eq3)*admin1 + c(eq4,eq4)*admin2
# factor Variances and covariance
S_M ~~ c(1,1)*S_M
ADM ~~ c(1,1)*ADM
S_M ~~ c(eq5,eq5)*ADM
# error Variances
sales1 ~~ sales1
sales2 ~~ sales2
admin1 ~~ admin1
admin2 ~~ admin2
"

# Fit Model 4 to data
fit4 <-lavaan(model4, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))

# specify Model 5 (Model 4 + equal error variances)
model5 <- "
# measurement model
S_M =~ c(eq1,eq1)*sales1 + c(eq2,eq2)*sales2
ADM =~ c(eq3,eq3)*admin1 + c(eq4,eq4)*admin2
# factor Variances and covariance
S_M ~~ c(1,1)*S_M
ADM ~~ c(1,1)*ADM
S_M ~~ c(eq5,eq5)*ADM
# error Variances
sales1 ~~ c(eq6,eq6)*sales1
sales2 ~~ c(eq7,eq7)*sales2
admin1 ~~ c(eq8,eq8)*admin1
admin2 ~~ c(eq9,eq9)*admin2
"

# Fit Model 5 to data
fit5 <-lavaan(model5, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))

```

```

# specify Model 6 (equality of unstructured covariance matrices)
model6 <- "
# measurement model
F1 =~ 1*sales1
F2 =~ 1*sales2
F3 =~ 1*admin1
F4 =~ 1*admin2
# factor Variance
F1 ~~ c(eq1,eq1)*F1
F2 ~~ c(eq2,eq2)*F2
F3 ~~ c(eq3,eq3)*F3
F4 ~~ c(eq4,eq4)*F4
# factor covariance
F2 ~~ c(eq5,eq5)*F1
F3 ~~ c(eq6,eq6)*F1
F3 ~~ c(eq7,eq7)*F2
F4 ~~ c(eq8,eq8)*F1
F4 ~~ c(eq9,eq9)*F2
F4 ~~ c(eq10,eq10)*F3
"
# Fit Model 6 to data
fit6 <-lavaan(model6, sample.cov=list(Group1=male.cov, Group2=female.cov), sample.nobs=c(265, 300))

# save the output
sink("sma.out", split=TRUE)
writeLines("\n Example: Sales/Marketing and Administration Ability\n")
writeLines("\n Output for Model 1 (Configural Invariance)\n")
summary(fit1, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 2 (Weak Factorial Invariance)\n")
summary(fit2, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 3 (Model 2 + equal factor variances)\n")
summary(fit3, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 4 (Model 3 + equal factor correlation)\n")
summary(fit4, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Output for Model 5 (Model 4 + equal error variances)\n")

```

```
summary(fit5, fit.measures=TRUE, standardized=TRUE)
writeLines("\n Model Comparisons\n")
lavTestLRT(fit1, fit2, fit3, fit4, fit5)
writeLines("\n Output for Model 6 (equality of unstructured covariance matrices)\n")
summary(fit6, fit.measures=TRUE, standardized=TRUE)
sink()
```

• R output file: *sma.out*

Example: Sales/Marketing and Administration Ability

Output for Model 1 (Configural Invariance)

lavaan 0.6-5 ended normally after 30 iterations

| | |
|---|----------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 18 |
| Number of observations per group: | |
| Group1 | 265 |
| Group2 | 300 |
| Model Test User Model: | |
| Test statistic | 0.950 |
| Degrees of freedom | 2 |
| P-value (Chi-square) | 0.622 |
| Test statistic for each group: | |
| Group1 | 0.763 |
| Group2 | 0.188 |
| Model Test Baseline Model: | |
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |
| User Model versus Baseline Model: | |
| Comparative Fit Index (CFI) | 1.000 |
| Tucker-Lewis Index (TLI) | 1.004 |
| Loglikelihood and Information Criteria: | |

| | |
|---------------------------------------|-----------|
| Loglikelihood user model (H0) | -6993.431 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14022.862 |
| Bayesian (BIC) | 14100.925 |
| Sample-size adjusted Bayesian (BIC) | 14043.783 |

Root Mean Square Error of Approximation:

| | |
|--|-------|
| RMSEA | 0.000 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.095 |
| P-value RMSEA <= 0.05 | 0.783 |

Standardized Root Mean Square Residual:

| | |
|------|-------|
| SRMR | 0.002 |
|------|-------|

Parameter Estimates:

| | |
|----------------------------------|------------|
| Information | Expected |
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

Latent Variables:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | |
| sales1 | 7.465 | 0.387 | 19.289 | 0.000 | 7.465 | 0.942 |
| sales2 | 9.342 | 0.526 | 17.745 | 0.000 | 9.342 | 0.892 |
| ADM =~ | | | | | | |
| admin1 | 7.033 | 0.387 | 18.151 | 0.000 | 7.033 | 0.910 |
| admin2 | 5.099 | 0.284 | 17.953 | 0.000 | 5.099 | 0.903 |

Covariances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | |
| ADM | 0.781 | 0.031 | 25.260 | 0.000 | 0.781 | 0.781 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 7.038 | 2.098 | 3.355 | 0.001 | 7.038 | 0.112 |
| .sales2 | 22.314 | 3.693 | 6.043 | 0.000 | 22.314 | 0.204 |
| .admin1 | 10.304 | 2.086 | 4.941 | 0.000 | 10.304 | 0.172 |
| .admin2 | 5.878 | 1.114 | 5.277 | 0.000 | 5.878 | 0.184 |

Group 2 [Group2]:

Latent Variables:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | |
| sales1 | 7.238 | 0.390 | 18.571 | 0.000 | 7.238 | 0.886 |
| sales2 | 9.915 | 0.472 | 21.002 | 0.000 | 9.915 | 0.960 |
| ADM =~ | | | | | | |
| admin1 | 7.489 | 0.369 | 20.306 | 0.000 | 7.489 | 0.945 |
| admin2 | 5.190 | 0.285 | 18.228 | 0.000 | 5.190 | 0.879 |

Covariances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | |
| ADM | 0.738 | 0.032 | 22.839 | 0.000 | 0.738 | 0.738 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 14.390 | 2.206 | 6.524 | 0.000 | 14.390 | 0.215 |
| .sales2 | 8.346 | 3.568 | 2.339 | 0.019 | 8.346 | 0.078 |
| .admin1 | 6.705 | 2.197 | 3.051 | 0.002 | 6.705 | 0.107 |
| .admin2 | 7.945 | 1.211 | 6.562 | 0.000 | 7.945 | 0.228 |

Output for Model 2 (Weak Factorial Invariance)

lavaan 0.6-5 ended normally after 38 iterations

| | |
|---|-----------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 20 |
| Number of equality constraints | 4 |
| Row rank of the constraints matrix | 4 |
| Number of observations per group: | |
| Group1 | 265 |
| Group2 | 300 |
| Model Test User Model: | |
| Test statistic | 2.921 |
| Degrees of freedom | 4 |
| P-value (Chi-square) | 0.571 |
| Test statistic for each group: | |
| Group1 | 1.806 |
| Group2 | 1.114 |
| Model Test Baseline Model: | |
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |
| User Model versus Baseline Model: | |
| Comparative Fit Index (CFI) | 1.000 |
| Tucker-Lewis Index (TLI) | 1.002 |
| Loglikelihood and Information Criteria: | |
| Loglikelihood user model (H0) | -6994.416 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14020.832 |

| | |
|-------------------------------------|-----------|
| Bayesian (BIC) | 14090.221 |
| Sample-size adjusted Bayesian (BIC) | 14039.429 |

| | |
|--|-------|
| Root Mean Square Error of Approximation: | |
| RMSEA | 0.000 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.078 |
| P-value RMSEA \leq 0.05 | 0.816 |

| | |
|---|-------|
| Standardized Root Mean Square Residual: | |
| SRMR | 0.015 |

Parameter Estimates:

| | |
|----------------------------------|------------|
| Information | Expected |
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.306 | 0.366 | 19.938 | 0.000 | 7.306 | 0.931 |
| sales2 | (eq2) | 9.589 | 0.492 | 19.477 | 0.000 | 9.589 | 0.905 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.113 | 0.366 | 19.449 | 0.000 | 7.113 | 0.916 |
| admin2 | (eq4) | 5.035 | 0.263 | 19.117 | 0.000 | 5.035 | 0.897 |

Covariances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | |
| ADM | 0.783 | 0.031 | 25.436 | 0.000 | 0.783 | 0.783 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 8.258 | 1.840 | 4.488 | 0.000 | 8.258 | 0.134 |
| .sales2 | 20.420 | 3.416 | 5.977 | 0.000 | 20.420 | 0.182 |
| .admin1 | 9.731 | 1.924 | 5.057 | 0.000 | 9.731 | 0.161 |
| .admin2 | 6.180 | 1.019 | 6.065 | 0.000 | 6.180 | 0.196 |

Group 2 [Group2]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.306 | 0.366 | 19.938 | 0.000 | 7.423 | 0.898 |
| sales2 | (eq2) | 9.589 | 0.492 | 19.477 | 0.000 | 9.744 | 0.949 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.113 | 0.366 | 19.449 | 0.000 | 7.424 | 0.940 |
| admin2 | (eq4) | 5.035 | 0.263 | 19.117 | 0.000 | 5.255 | 0.885 |

Covariances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | |
| ADM | 0.787 | 0.104 | 7.572 | 0.000 | 0.742 | 0.742 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.032 | 0.135 | 7.666 | 0.000 | 1.000 | 1.000 |
| ADM | 1.089 | 0.144 | 7.572 | 0.000 | 1.000 | 1.000 |
| .sales1 | 13.274 | 2.026 | 6.553 | 0.000 | 13.274 | 0.194 |
| .sales2 | 10.439 | 3.069 | 3.401 | 0.001 | 10.439 | 0.099 |
| .admin1 | 7.306 | 1.940 | 3.767 | 0.000 | 7.306 | 0.117 |
| .admin2 | 7.655 | 1.116 | 6.859 | 0.000 | 7.655 | 0.217 |

Output for Model 3 (Model 2 + equal factor variances)

lavaan 0.6-5 ended normally after 27 iterations

| | |
|---|-----------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 18 |
| Number of equality constraints | 4 |
| Row rank of the constraints matrix | 4 |
| Number of observations per group: | |
| Group1 | 265 |
| Group2 | 300 |
| Model Test User Model: | |
| Test statistic | 3.346 |
| Degrees of freedom | 6 |
| P-value (Chi-square) | 0.764 |
| Test statistic for each group: | |
| Group1 | 2.035 |
| Group2 | 1.310 |
| Model Test Baseline Model: | |
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |
| User Model versus Baseline Model: | |
| Comparative Fit Index (CFI) | 1.000 |
| Tucker-Lewis Index (TLI) | 1.003 |
| Loglikelihood and Information Criteria: | |
| Loglikelihood user model (H0) | -6994.629 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14017.257 |

| | |
|-------------------------------------|-----------|
| Bayesian (BIC) | 14077.973 |
| Sample-size adjusted Bayesian (BIC) | 14033.530 |

Root Mean Square Error of Approximation:

| | |
|--|-------|
| RMSEA | 0.000 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.053 |
| P-value RMSEA <= 0.05 | 0.940 |

Standardized Root Mean Square Residual:

| | |
|------|-------|
| SRMR | 0.027 |
|------|-------|

Parameter Estimates:

| | |
|----------------------------------|------------|
| Information | Expected |
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.374 | 0.274 | 26.890 | 0.000 | 7.374 | 0.932 |
| sales2 | (eq2) | 9.678 | 0.352 | 27.501 | 0.000 | 9.678 | 0.906 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.280 | 0.267 | 27.231 | 0.000 | 7.280 | 0.920 |
| admin2 | (eq4) | 5.151 | 0.201 | 25.641 | 0.000 | 5.151 | 0.901 |

Covariances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | |
| ADM | 0.788 | 0.029 | 27.610 | 0.000 | 0.788 | 0.788 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--|----------|---------|---------|---------|--------|---------|
|--|----------|---------|---------|---------|--------|---------|

| | | | | | | |
|---------|--------|-------|-------|-------|--------|-------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 8.269 | 1.837 | 4.500 | 0.000 | 8.269 | 0.132 |
| .sales2 | 20.417 | 3.409 | 5.990 | 0.000 | 20.417 | 0.179 |
| .admin1 | 9.673 | 1.938 | 4.992 | 0.000 | 9.673 | 0.154 |
| .admin2 | 6.165 | 1.028 | 5.999 | 0.000 | 6.165 | 0.189 |

Group 2 [Group2]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.374 | 0.274 | 26.890 | 0.000 | 7.374 | 0.897 |
| sales2 | (eq2) | 9.678 | 0.352 | 27.501 | 0.000 | 9.678 | 0.949 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.280 | 0.267 | 27.231 | 0.000 | 7.280 | 0.937 |
| admin2 | (eq4) | 5.151 | 0.201 | 25.641 | 0.000 | 5.151 | 0.881 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|--|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | | |
| ADM | | 0.738 | 0.031 | 23.899 | 0.000 | 0.738 | 0.738 |

Variances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|--|----------|---------|---------|---------|--------|---------|
| S_M | | 1.000 | | | | 1.000 | 1.000 |
| ADM | | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | | 13.278 | 2.028 | 6.547 | 0.000 | 13.278 | 0.196 |
| .sales2 | | 10.422 | 3.071 | 3.394 | 0.001 | 10.422 | 0.100 |
| .admin1 | | 7.415 | 1.914 | 3.874 | 0.000 | 7.415 | 0.123 |
| .admin2 | | 7.638 | 1.105 | 6.915 | 0.000 | 7.638 | 0.223 |

Output for Model 4 (Model 3 + equal factor correlation)

lavaan 0.6-5 ended normally after 28 iterations

| | |
|---|-----------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 18 |
| Number of equality constraints | 5 |
| Row rank of the constraints matrix | 5 |
| Number of observations per group: | |
| Group1 | 265 |
| Group2 | 300 |
| Model Test User Model: | |
| Test statistic | 4.933 |
| Degrees of freedom | 7 |
| P-value (Chi-square) | 0.668 |
| Test statistic for each group: | |
| Group1 | 2.849 |
| Group2 | 2.084 |
| Model Test Baseline Model: | |
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |
| User Model versus Baseline Model: | |
| Comparative Fit Index (CFI) | 1.000 |
| Tucker-Lewis Index (TLI) | 1.002 |
| Loglikelihood and Information Criteria: | |
| Loglikelihood user model (H0) | -6995.422 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14016.844 |

| | |
|-------------------------------------|-----------|
| Bayesian (BIC) | 14073.223 |
| Sample-size adjusted Bayesian (BIC) | 14031.954 |

| | |
|--|-------|
| Root Mean Square Error of Approximation: | |
| RMSEA | 0.000 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.058 |
| P-value RMSEA <= 0.05 | 0.916 |

| | |
|---|-------|
| Standardized Root Mean Square Residual: | |
| SRMR | 0.023 |

| | |
|----------------------------------|------------|
| Parameter Estimates: | |
| Information | Expected |
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

| Latent Variables: | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|-------------------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.375 | 0.275 | 26.866 | 0.000 | 7.375 | 0.933 |
| sales2 | (eq2) | 9.662 | 0.351 | 27.502 | 0.000 | 9.662 | 0.906 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.276 | 0.267 | 27.212 | 0.000 | 7.276 | 0.921 |
| admin2 | (eq4) | 5.153 | 0.201 | 25.609 | 0.000 | 5.153 | 0.901 |

| Covariances: | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------------|-------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | | |
| ADM | (eq5) | 0.760 | 0.022 | 34.245 | 0.000 | 0.760 | 0.760 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 8.074 | 1.888 | 4.276 | 0.000 | 8.074 | 0.129 |
| .sales2 | 20.407 | 3.508 | 5.817 | 0.000 | 20.407 | 0.179 |
| .admin1 | 9.483 | 1.994 | 4.757 | 0.000 | 9.483 | 0.152 |
| .admin2 | 6.141 | 1.059 | 5.797 | 0.000 | 6.141 | 0.188 |

Group 2 [Group2]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.375 | 0.275 | 26.866 | 0.000 | 7.375 | 0.897 |
| sales2 | (eq2) | 9.662 | 0.351 | 27.502 | 0.000 | 9.662 | 0.946 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.276 | 0.267 | 27.212 | 0.000 | 7.276 | 0.934 |
| admin2 | (eq4) | 5.153 | 0.201 | 25.609 | 0.000 | 5.153 | 0.882 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | | |
| ADM | (eq5) | 0.760 | 0.022 | 34.245 | 0.000 | 0.760 | 0.760 |

Variances:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|----------|---------|---------|---------|--------|---------|
| S_M | 1.000 | | | | 1.000 | 1.000 |
| ADM | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | 13.179 | 1.974 | 6.676 | 0.000 | 13.179 | 0.195 |
| .sales2 | 10.858 | 2.953 | 3.677 | 0.000 | 10.858 | 0.104 |
| .admin1 | 7.692 | 1.850 | 4.158 | 0.000 | 7.692 | 0.127 |
| .admin2 | 7.607 | 1.079 | 7.047 | 0.000 | 7.607 | 0.223 |

Output for Model 5 (Model 4 + equal error variances)

lavaan 0.6-5 ended normally after 21 iterations

| | |
|------------------------------------|--------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 18 |
| Number of equality constraints | 9 |
| Row rank of the constraints matrix | 9 |

Number of observations per group:

| | |
|--------|-----|
| Group1 | 265 |
| Group2 | 300 |

Model Test User Model:

| | |
|--------------------------------|--------|
| Test statistic | 12.668 |
| Degrees of freedom | 11 |
| P-value (Chi-square) | 0.316 |
| Test statistic for each group: | |
| Group1 | 7.342 |
| Group2 | 5.326 |

Model Test Baseline Model:

| | |
|--------------------|----------|
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |

User Model versus Baseline Model:

| | |
|-----------------------------|-------|
| Comparative Fit Index (CFI) | 0.999 |
| Tucker-Lewis Index (TLI) | 0.999 |

Loglikelihood and Information Criteria:

| | |
|---------------------------------------|-----------|
| Loglikelihood user model (H0) | -6999.290 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14016.579 |

| | |
|-------------------------------------|-----------|
| Bayesian (BIC) | 14055.611 |
| Sample-size adjusted Bayesian (BIC) | 14027.040 |

Root Mean Square Error of Approximation:

| | |
|--|-------|
| RMSEA | 0.023 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.069 |
| P-value RMSEA <= 0.05 | 0.792 |

Standardized Root Mean Square Residual:

| | |
|------|-------|
| SRMR | 0.024 |
|------|-------|

Parameter Estimates:

| | |
|----------------------------------|------------|
| Information | Expected |
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.341 | 0.276 | 26.645 | 0.000 | 7.341 | 0.911 |
| sales2 | (eq2) | 9.646 | 0.352 | 27.400 | 0.000 | 9.646 | 0.928 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.280 | 0.268 | 27.208 | 0.000 | 7.280 | 0.929 |
| admin2 | (eq4) | 5.145 | 0.202 | 25.490 | 0.000 | 5.145 | 0.889 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | | |
| ADM | (eq5) | 0.758 | 0.022 | 33.724 | 0.000 | 0.758 | 0.758 |

Variances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|-------|----------|---------|---------|---------|--------|---------|
| S_M | | 1.000 | | | | 1.000 | 1.000 |
| ADM | | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | (eq6) | 11.004 | 1.520 | 7.237 | 0.000 | 11.004 | 0.170 |
| .sales2 | (eq7) | 14.978 | 2.531 | 5.917 | 0.000 | 14.978 | 0.139 |
| .admin1 | (eq8) | 8.374 | 1.530 | 5.473 | 0.000 | 8.374 | 0.136 |
| .admin2 | (eq9) | 6.998 | 0.834 | 8.391 | 0.000 | 6.998 | 0.209 |

Group 2 [Group2]:

Latent Variables:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M =~ | | | | | | | |
| sales1 | (eq1) | 7.341 | 0.276 | 26.645 | 0.000 | 7.341 | 0.911 |
| sales2 | (eq2) | 9.646 | 0.352 | 27.400 | 0.000 | 9.646 | 0.928 |
| ADM =~ | | | | | | | |
| admin1 | (eq3) | 7.280 | 0.268 | 27.208 | 0.000 | 7.280 | 0.929 |
| admin2 | (eq4) | 5.145 | 0.202 | 25.490 | 0.000 | 5.145 | 0.889 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|-------|----------|---------|---------|---------|--------|---------|
| S_M ~~ | | | | | | | |
| ADM | (eq5) | 0.758 | 0.022 | 33.724 | 0.000 | 0.758 | 0.758 |

Variances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|-------|----------|---------|---------|---------|--------|---------|
| S_M | | 1.000 | | | | 1.000 | 1.000 |
| ADM | | 1.000 | | | | 1.000 | 1.000 |
| .sales1 | (eq6) | 11.004 | 1.520 | 7.237 | 0.000 | 11.004 | 0.170 |
| .sales2 | (eq7) | 14.978 | 2.531 | 5.917 | 0.000 | 14.978 | 0.139 |
| .admin1 | (eq8) | 8.374 | 1.530 | 5.473 | 0.000 | 8.374 | 0.136 |
| .admin2 | (eq9) | 6.998 | 0.834 | 8.391 | 0.000 | 6.998 | 0.209 |

Model Comparisons

Chi-Squared Difference Test

| | Df | AIC | BIC | Chisq | Chisq diff | Df diff | Pr(>Chisq) |
|------|----|-------|-------|---------|------------|---------|------------|
| fit1 | 2 | 14023 | 14101 | 0.9504 | | | |
| fit2 | 4 | 14021 | 14090 | 2.9205 | 1.9701 | 2 | 0.3734 |
| fit3 | 6 | 14017 | 14078 | 3.3459 | 0.4253 | 2 | 0.8084 |
| fit4 | 7 | 14017 | 14073 | 4.9327 | 1.5869 | 1 | 0.2078 |
| fit5 | 11 | 14017 | 14056 | 12.6678 | 7.7351 | 4 | 0.1018 |

Output for Model 6 (equality of unstructured covariance matrices)

lavaan 0.6-5 ended normally after 100 iterations

| | |
|------------------------------------|--------|
| Estimator | ML |
| Optimization method | NLMINB |
| Number of free parameters | 20 |
| Number of equality constraints | 10 |
| Row rank of the constraints matrix | 10 |
| Number of observations per group: | |
| Group1 | 265 |
| Group2 | 300 |

Model Test User Model:

| | |
|--------------------------------|--------|
| Test statistic | 11.750 |
| Degrees of freedom | 10 |
| P-value (Chi-square) | 0.302 |
| Test statistic for each group: | |
| Group1 | 6.579 |
| Group2 | 5.171 |

Model Test Baseline Model:

| | |
|--------------------|----------|
| Test statistic | 1739.457 |
| Degrees of freedom | 12 |
| P-value | 0.000 |

User Model versus Baseline Model:

| | |
|-----------------------------|-------|
| Comparative Fit Index (CFI) | 0.999 |
| Tucker-Lewis Index (TLI) | 0.999 |

Loglikelihood and Information Criteria:

| | |
|---------------------------------------|-----------|
| Loglikelihood user model (H0) | -6998.831 |
| Loglikelihood unrestricted model (H1) | -6992.956 |
| Akaike (AIC) | 14017.662 |
| Bayesian (BIC) | 14061.030 |
| Sample-size adjusted Bayesian (BIC) | 14029.285 |

Root Mean Square Error of Approximation:

| | |
|--|-------|
| RMSEA | 0.025 |
| 90 Percent confidence interval - lower | 0.000 |
| 90 Percent confidence interval - upper | 0.072 |
| P-value RMSEA \leq 0.05 | 0.764 |

Standardized Root Mean Square Residual:

| | |
|------|-------|
| SRMR | 0.024 |
|------|-------|

Parameter Estimates:

| Information | Expected |
|----------------------------------|------------|
| Information saturated (h1) model | Structured |
| Standard errors | Standard |

Group 1 [Group1]:

Latent Variables:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|--------|----------|---------|---------|---------|--------|---------|
| F1 =~ | | | | | | |
| sales1 | 1.000 | | | | 8.056 | 1.000 |
| F2 =~ | | | | | | |
| sales2 | 1.000 | | | | 10.393 | 1.000 |
| F3 =~ | | | | | | |

| | | | |
|--------|-------|-------|-------|
| admin1 | 1.000 | 7.834 | 1.000 |
| F4 =~ | | | |
| admin2 | 1.000 | 5.786 | 1.000 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|-------|--------|----------|---------|---------|---------|--------|---------|
| F1 ~~ | | | | | | | |
| F2 | (eq5) | 70.811 | 4.613 | 15.350 | 0.000 | 0.846 | 0.846 |
| F3 | (eq6) | 40.326 | 3.151 | 12.799 | 0.000 | 0.639 | 0.639 |
| F2 ~~ | | | | | | | |
| F3 | (eq7) | 53.404 | 4.097 | 13.036 | 0.000 | 0.656 | 0.656 |
| F1 ~~ | | | | | | | |
| F4 | (eq8) | 28.835 | 2.306 | 12.506 | 0.000 | 0.619 | 0.619 |
| F2 ~~ | | | | | | | |
| F4 | (eq9) | 37.398 | 2.979 | 12.553 | 0.000 | 0.622 | 0.622 |
| F3 ~~ | | | | | | | |
| F4 | (eq10) | 37.460 | 2.474 | 15.142 | 0.000 | 0.826 | 0.826 |

Variances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|-------|----------|---------|---------|---------|--------|---------|
| F1 | (eq1) | 64.894 | 3.861 | 16.808 | 0.000 | 1.000 | 1.000 |
| F2 | (eq2) | 108.023 | 6.427 | 16.808 | 0.000 | 1.000 | 1.000 |
| F3 | (eq3) | 61.375 | 3.652 | 16.808 | 0.000 | 1.000 | 1.000 |
| F4 | (eq4) | 33.474 | 1.992 | 16.808 | 0.000 | 1.000 | 1.000 |
| .sales1 | | 0.000 | | | | 0.000 | 0.000 |
| .sales2 | | 0.000 | | | | 0.000 | 0.000 |
| .admin1 | | 0.000 | | | | 0.000 | 0.000 |
| .admin2 | | 0.000 | | | | 0.000 | 0.000 |

Group 2 [Group2]:

Latent Variables:

| | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|-------|----------|---------|---------|---------|--------|---------|
| F1 =~ | | | | | | |

| | | | |
|--------|-------|--------|-------|
| sales1 | 1.000 | 8.056 | 1.000 |
| F2 =~ | | | |
| sales2 | 1.000 | 10.393 | 1.000 |
| F3 =~ | | | |
| admin1 | 1.000 | 7.834 | 1.000 |
| F4 =~ | | | |
| admin2 | 1.000 | 5.786 | 1.000 |

Covariances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|-------|--------|----------|---------|---------|---------|--------|---------|
| F1 ~~ | | | | | | | |
| F2 | (eq5) | 70.811 | 4.613 | 15.350 | 0.000 | 0.846 | 0.846 |
| F3 | (eq6) | 40.326 | 3.151 | 12.799 | 0.000 | 0.639 | 0.639 |
| F2 ~~ | | | | | | | |
| F3 | (eq7) | 53.404 | 4.097 | 13.036 | 0.000 | 0.656 | 0.656 |
| F1 ~~ | | | | | | | |
| F4 | (eq8) | 28.835 | 2.306 | 12.506 | 0.000 | 0.619 | 0.619 |
| F2 ~~ | | | | | | | |
| F4 | (eq9) | 37.398 | 2.979 | 12.553 | 0.000 | 0.622 | 0.622 |
| F3 ~~ | | | | | | | |
| F4 | (eq10) | 37.460 | 2.474 | 15.142 | 0.000 | 0.826 | 0.826 |

Variances:

| | | Estimate | Std.Err | z-value | P(> z) | Std.lv | Std.all |
|---------|-------|----------|---------|---------|---------|--------|---------|
| F1 | (eq1) | 64.894 | 3.861 | 16.808 | 0.000 | 1.000 | 1.000 |
| F2 | (eq2) | 108.023 | 6.427 | 16.808 | 0.000 | 1.000 | 1.000 |
| F3 | (eq3) | 61.375 | 3.652 | 16.808 | 0.000 | 1.000 | 1.000 |
| F4 | (eq4) | 33.474 | 1.992 | 16.808 | 0.000 | 1.000 | 1.000 |
| .sales1 | | 0.000 | | | | 0.000 | 0.000 |
| .sales2 | | 0.000 | | | | 0.000 | 0.000 |
| .admin1 | | 0.000 | | | | 0.000 | 0.000 |
| .admin2 | | 0.000 | | | | 0.000 | 0.000 |