Poisson MLE

$$y_i \sim \text{Pois}(\mu)$$

$$f(y_i|\mu) = \frac{e^{-\mu} \cdot \mu^{y_i}}{y_i!}$$

$$L(\mu|y_1, y_2, \dots y_n) = \prod_{i=1}^{n} f(y_i|\mu)$$

$$= \prod_{i=1}^{n} \frac{e^{-\mu} \cdot \mu^{y_i}}{y_i!}$$

$$l(\mu|y_1, y_2, \dots y_n) = \sum_{i=1}^{n} \log\left[\frac{e^{-\mu} \cdot \mu^{y_i}}{y_i!}\right]$$

$$= \sum_{i=1}^{n} -\mu + y_i \cdot \log(\mu) - \log(y_i!)$$

$$l(\mu|y_1, y_2, \dots y_n) \propto \sum_{i=1}^{n} -\mu + y_i \cdot \log(\mu)$$

$$= -n\mu + \sum_{i=1}^{n} y_i \cdot \log(\mu)$$

$$l'(\mu \mid y_1, y_2, \ldots y_n) = -n + \frac{\sum_{i=1}^{n} y_i}{\mu} \stackrel{\text{set}}{=} 0$$

$$n = \frac{\sum^{n} y_i}{\mu}$$

$$\mu = \frac{\sum_{i=1}^{n} y_i}{n} = \bar{y}$$

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \bar{Y}_i}{n}$$

$$E(\hat{\mu}) = E\left(\frac{Y_1 + Y_2 + \ldots Y_n}{n}\right)$$

$$= \frac{E(Y_1) + E(Y_2) + \ldots E(Y_n)}{n}$$

$$= \frac{\mu_1 + \mu_2 + \mu_3 + \ldots \mu_n}{n}$$

$$= \frac{n\mu}{n} \qquad = \mu$$

$$Var(\hat{\mu}) = Var\left(\frac{Y_1 + Y_2 + \ldots Y_n}{n}\right)$$

$$= \frac{Var(Y_1) + Var(Y_2) + \ldots Var(Y_n)}{n^2}$$

$$= \frac{n\,Var(Y_1)}{n^2} = \frac{Var(Y_1)}{n} = \frac{\mu}{n}$$