FUNDAMENTALS OF VIBRATION ANALYSIS AND VIBROACOUSTICS

MODULE 1: FUNDAMENTALS OF VIBRATION ANALYSIS (PROF. STEFANO ALFI)

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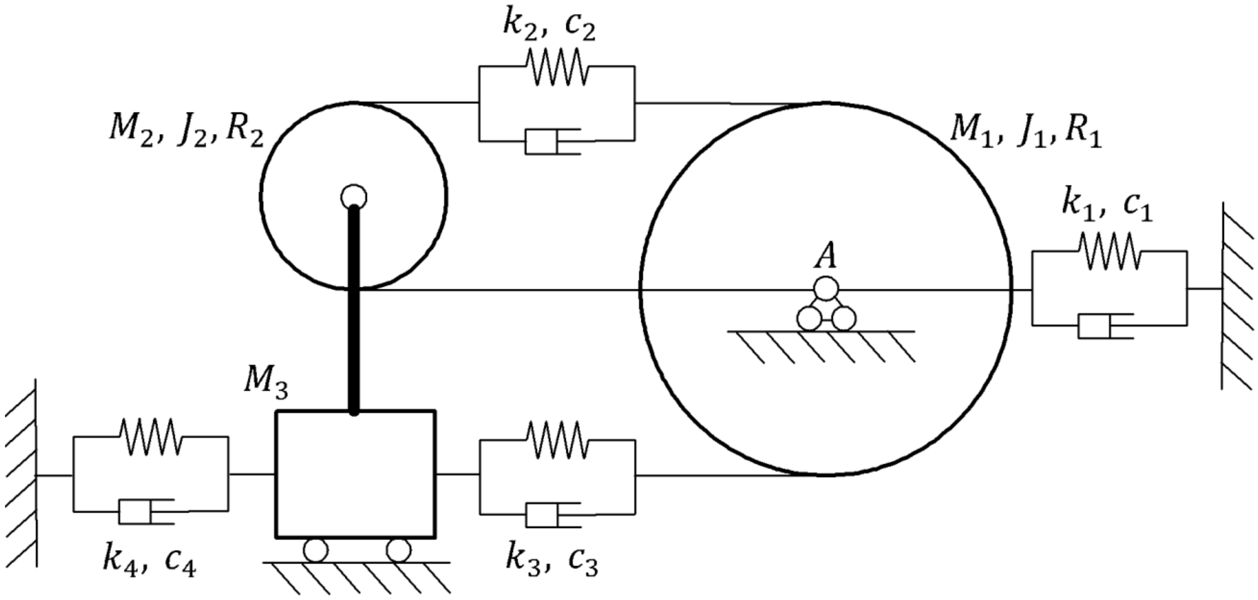
<u>Assignment 2</u>: Dynamics of n-dof systems

The system to be studied consists of 3 rigid bodies. A disk (M_2, J_2, R_2) is constrained through a hinge to the extremity of a mass (M_3) that slides in the horizontal plane. An inextensible rod rolls up on the disk and its extremity is rigidly connected to the centre of another disk (M_1, J_1, R_1) whose vertical motion is constrained. Springs (k_1, k_2, k_3, k_4) and dampers (c_1, c_2, c_3, c_4) realize together with the rigid bodies the mechanical system depicted below.



$M_1\ [kg]$	$J_1\ [kg\ m^2]$	$R_1\ [m]$	$M_2 [kg]$	$J_2\ [kg\ m^2]$	$R_2\ [m]$	$M_3 [kg]$	
5	2.5	1	1.25	0.16	0.5	10	
$k_1\ [N/m]$	$c_1\ [Ns/m]$	$k_2\ [N/m]$	$c_2\ [Ns/m]$	$k_3\ [N/m]$	$c_3\ [Ns/m]$	$k_4\ [N/m]$	$c_4\ [Ns/m]$
1000	0.5	100	0.5	560	1	800	4
$A_1\ [N]$	$A_2\ [N]$	$f_1\ [Hz]$	$f_2\ [Hz]$	$f_0\ [Hz]$			
15	7	1,5	3.5	0.75			
$x_{3,0}\ [m]$	$\theta_{1,0}\ [rad]$	$\theta_{2,0}\ [rad]$	$\dot{x}_{3,0}\ [m/s]$	$\dot{\theta}_{1,,0}\ [rad/s]$	$\dot{\theta}_{2,0}\ [rad/s]$		
0.1	$\pi/12$	$-\pi/12$	1	0.5	2		

According to the data defined in the above table, it is requested to:

1) Equations of motion and system matrices:
    a. Write the equations of motion for small vibrations around the represented configuration considering that the system is in its equilibrium position.
    b. Evaluate the eigenfrequencies and corresponding eigenvectors in case of **undamped** and **damped** system.
    c. Assuming Rayleigh damping, evaluate α and β to approximate the generalized damping matrix [C*] to be of the form α[M]+β[K].

2) Free motion of the system (considering the Rayleigh damping as in 1.c)
    a. Plot and comment the free motion of the system starting from the initial conditions reported in the table, being x_3 , θ_1 , θ_2 the displacement of mass M_3, and the rotation of disks 1 and 2 (the initial conditions for generic independent variables can be obtained through kinematic relations).
    b. Impose particular initial conditions so that only one mode contributes to the free motion of the system.

3) Forced motion of the system (considering the Rayleigh damping as in 1.c)
    a. Plot and comment the elements of the Frequency Response Matrix *H(Ω)*.
    b. Plot the co-located FRF of point A at the centre of the disk (co-located meaning the FRF between the displacement in A and a force applied in A).
    c. Plot the co-located FRF between the rotation of the disk of radius R_2 and the torque applied onto the disk itself.
    d. Starting from the initial condition defined in (2.a), evaluate the complete time response of the system for the three degrees of freedom to the horizontal force applied in A, considering that the force is harmonic of the form:

$$F(t) = A_1\, cos(2\pi f_1\, t) +\ A_2\, cos(2\pi f_2\, t)$$

    e. Evaluate the steady-state response of the system of horizontal displacement of point A to the horizontal force applied in A considering that the force is a periodic triangular wave, with fundamental frequency f_0, of the form:

$$F(t) = \frac{8}{\pi^2}\sum_{k=0}^{4}(-1)^k\,\frac{\sin\big((2k+1)\,2\pi f_0\, t\big)}{(2k+1)^2}$$

4) Modal approach (considering the Rayleigh damping as in 1.c)
    a. Derive the equations of motion in modal coordinates and plot the elements of the corresponding Frequency Response Matrix $H_q(\Omega)$.
    b. Reconstruct the co-located FRF of point A employing a modal approach and compare with the one obtained using physical coordinates in (3.b).
    c. Reconstruct the co-located FRF between the rotation of the disk or radius R_2 and the torque applied onto the disk employing a modal approach and compare with the one obtained using physical coordinates in (3.c).

OPTIONAL

i. Give a qualitative graphical representation of the mode shapes.