Fundamentals of Vibration Analysis and Vibroacoustics
Module 1 - Fundamentals of Vibration Analysis
Assignment 1 - One-degree-of-freedom systems

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1 Equation of motion

1.a Equation derivation

Step 1: number of degrees of freedom identification

We can verify the system has one degree of freedom since

$$
\begin{aligned}
n_b \cdot 3 \text{ DOF} = 6 \text{ DOF} - & \\
2 \text{ DOF} - & \quad \text{(hinge)} \\
2 \text{ DOF} - & \quad \text{(2 rollers)} \\
1 \text{ DOF} = & \quad \text{(string)} \\
1 \text{ DOF} &
\end{aligned}
$$

We chose to solve the problem directly using Lagrange equation, so to have one equation only, as the system has one degree of freedom.

Step 2: energy terms definition

$$E_c = \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} M_2 v_2^2$$

$$V_e = \frac{1}{2}\,k_1\,\Delta l_1^2 + \frac{1}{2}\,k_2\,\Delta l_2^2\,; \quad V_g = M_2\,g\,h_2$$

$$\Rightarrow V = V_e + V_g = \frac{1}{2}\,k_1\,\Delta l_1^2 + \frac{1}{2}\,k_2\,\Delta l_2^2 + M_2\,g\,h_2$$

$$D = \frac{1}{2}\,c_1\,\dot{\Delta l_1}^2 + \frac{1}{2}\,c_2\,\dot{\Delta l_2}^2$$

Because the assignment's requests define external forces to compute the system's forced motion later on, we're assuming a vertical force $F(t)$, directed upward, applied on M_2, so that we'll find a positive Lagrangian component.

$$\delta W = F(t)\,\delta y_2$$

Step 3: physical variables as functions of independent ones

The independent variable θ is chosen to be the one variable we need to describe the motion.

$\omega_1 = \dot{\theta}$

$v_2 = \dot{y_2} = \omega_1\,R_2 = \dot{\theta}\,R_2$

$\dot{\Delta l_1} = \dot{\theta}\,R_2 \quad$ (Rivals theorem)
$\Rightarrow \Delta l_1 = \theta\,R_2$

$\dot{\Delta l_2} = -\dot{\theta}\,R_1 \quad$ (Rivals theorem)
$\Rightarrow \Delta l_2 = -\theta\,R_1$

$h_2 = y_2 = \theta\,R_2 \quad$ (gravitational potential level = 0 at equilibrium point)

$\delta y_2 = \delta\theta\,R_2$

Step 4: resulting equation

$$\delta_W = Q_\theta\,\delta\theta = F(t)\,\delta y_2 = F(t)\,\delta\theta\,R_2 \Rightarrow Q_\theta = F(t)\,R_2$$

$$\frac{\partial}{\partial t}\Big(\frac{\partial E_c}{\partial\dot\theta}\Big) - \frac{\partial E_c}{\partial\theta} + \frac{\partial V}{\theta} + \frac{\partial D}{\partial\dot\theta} = Q_\theta$$

$$\underbrace{(J_1 + M_2\,R_2^2)}_{\text{generalized mass } m_g}\;\ddot\theta + \underbrace{(c_1\,R_2^2 + c_2\,R_1^2)}_{\text{generalized damping } c_g}\;\dot\theta + \underbrace{(k_1\,R_2^2 + k_2\,R_1^2)}_{\text{generalized stiffness } k_g}\;\theta = \underbrace{F(t)\,R_2 - M_2\,g\,R_2}_{\substack{\text{active forces } C_g(t)\\ \text{(conservative/non conservative)}}}$$

1.b Adimensional damping ratio

$$\xi = \frac{c_g}{c_{cr}} = \frac{c_g}{2\,\sqrt{k_g\,m_g}} = \frac{c_1\,R_2^2 + c_2\,R_1^2}{2\,\sqrt{(k_1\,R_2^2 + k_2\,R_1^2)\,(J_1 + M_2\,R_2^2)}} \approx 0.134$$

1.c Natural and damped frequency

$$\omega_n = \sqrt{\frac{k_g}{m_g}} = \sqrt{\frac{k_1\,R_2^2 + k_2\,R_1^2}{J_1 + M_2\,R_2^2}} \approx 7.16 \text{ rad/s}$$

$$\alpha = \xi\,\omega_n \approx 0.959 \text{ rad/s}$$

$$\omega_d = \sqrt{\omega_n^2 - \alpha^2} \approx 7.09 \text{ rad/s}$$

2 Free motion of the system

2.a Generic initial conditions

2.b Halved adimensional damping ratio

2.c Raised adimensional damping ratio

3 Forced motion of the system

3.a Frequency Response Function

Case 1: generic initial conditions

Case 2: halved adimensional damping ratio

Case 3: raised adimensional damping ratio

3.b Temporal evolution of complete response of the system

Case 1: f = 0.15Hz

Case 2: f = 4.5Hz

3.c Forced response of the system (steady-state)